Non-Markovian dynamics of an open quantum system in fermionic environments

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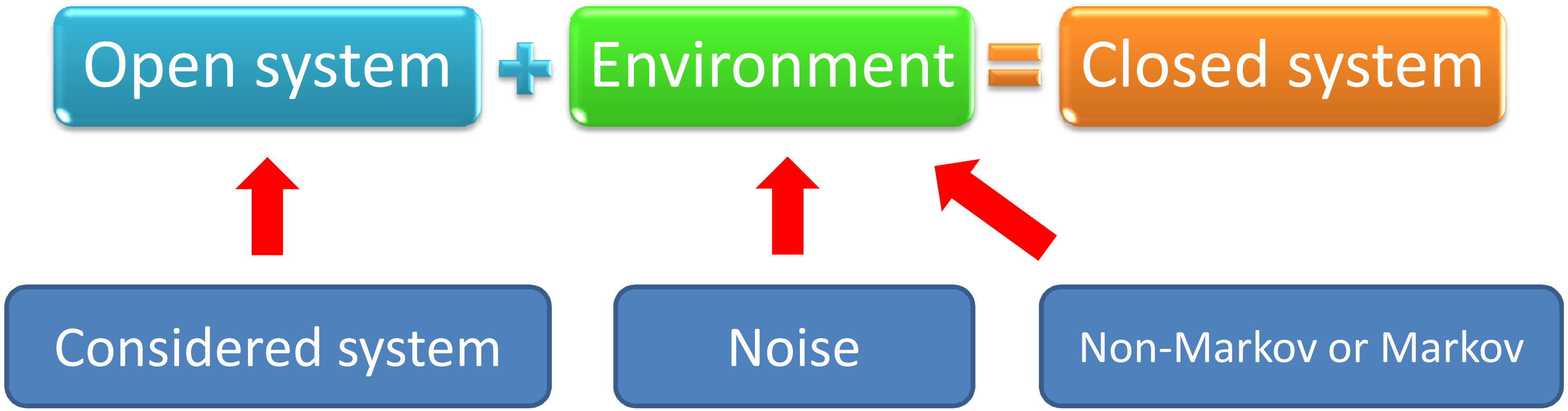
Outline

- Open quantum systems and the master equation formalism: Non-Markov vs. Markov
- Non-Markovian quantum state diffusion (QSD) in a bosonic bath: Some exactly solvable models
- Non-Markovian QSD in fermionic environments
- Summary and outlook

What is an open quantum system?
How to describe its dynamics?

System plus environment framework



Non-Markov is the rule, Markov is the exception (N. G. van Kampen)

Reduced density operator: $\rho_{\text{sys}}(t) = Tr_{\text{env}}\left\{\rho_{\text{tot}}(t)\right\}$

Non-Markov vs. Markov

- Non-Markovian dynamics (memory environment):

  The time evolution of the system's state depends on its history.

$$\frac{\partial \rho(t)}{\partial t} = -\frac{i}{\hbar}[H_{\mathrm{sys}}, \rho(t)] + \int_0^t \kappa(t,s)\rho(s)ds$$

- Markov approximation (memoryless environment):

  Replace $\rho(s)$ by $\rho(t)$ $\Rightarrow$ Lindblad form:

$$\frac{\partial \rho_t}{\partial t} = -\frac{i}{\hbar}[H_{\mathrm{sys}}, \rho_t] + \frac{\Gamma}{2}([L, \rho_t L^\dagger] + [L\rho_t, L^\dagger])$$

The reservoir is assumed to be of a broadband spectrum, so as to have a correlation time (memory time) much shorter than the dynamical (evolution) time of the considered system.

Master equation under Born-Markov approximation

Born approximation (2nd-order approximation): Weak interaction

Typical assumption: $\rho_{tot}(0) = \rho_{sys}(0) \otimes \rho_{env}(0)$

$$\frac{\partial \rho(t)}{\partial t} = -\frac{i}{\hbar}[H_{sys}, \rho(t)] - \frac{1}{\hbar^2}\int_0^t dt' \left\langle [H_{\text{int}}(t), [H_{\text{int}}(t'), \rho_{\text{int}}(t')]] \right\rangle_{env}$$

One can use it to derive a non-Markovian master equation, but this method is perturbative (i.e., it applies to a weak system-bath coupling).

Markov approximation: Replace $\rho_{\text{int}}(t')$ by $\rho_{\text{int}}(t)$

$$\frac{\partial \rho(t)}{\partial t} = -\frac{i}{\hbar}[H_{sys}, \rho(t)] - \frac{1}{\hbar^2}\int_0^t dt' \left\langle [H_{\text{int}}(t), [H_{\text{int}}(t'), \rho_{\text{int}}(t)]] \right\rangle_{env}$$

H. Carmichael, *An open systems approach to quantum optics* (Springer 1993)

Two non-pertutbative methods for non-Markovian dynamics

- Feynman-Vernon influence functional approach

R. P. Feynman and F. L. Vernon, *Ann. Phys. (N.Y.)* 24, 118 (1963)

Non-Markovian master equation for quantum Brown motion model :

B. L. Hu, J. P. Paz, and Y. Zhang, *Phys. Rev. D* 45, 2843 (1992)

Non-Markovian master equation of a double-quantum-dot system:

M. W. Y. Tu and W. M. Zhang, *Phys. Rev. B* 78, 235311 (2008).

- Non-markovian quantum trajectories

Non-Markovian quantum state diffusion (NMQSD)

L. Diosi, N. Gisin, and W. T. Strunz, *Phys. Rev. A* 58, 1699 (1998); W. T. Strunz, L. Diosi, and N. Gisin, *Phys. Rev. Lett.* 82, 1801 (1999); T. Yu, L. Diosi, N. Gisin, and W. T. Strunz, *Phys. Rev. A* 60, 91 (1999).

Quantum three-level system:

J. Jing and T. Yu, *Phys. Rev. Lett.* 105, 240403 (2010)

Extension to the fermionic-bath case:

Mi Chen and J.Q. You, arXiv:1203.2217; Wufu Shi, Xinyu Zhao and Ting Yu, arXiv:1203.2219.

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Some exactly solvable models

Total Hamiltonian: $H_{tot} = H_{sys} + \sum_k (g_k^* L b_k^\dagger + g_k L^\dagger b_k) + \sum_k \omega_k b_k^\dagger b_k$

Stochastic Schrödinger equation (Diosi-Gisin-Strunz equation)

$$\frac{\partial}{\partial t}|\psi_t(z^*)\rangle = [-iH_{sys} + Lz_t^* + L^+ \bar{O}(t,z^*)]|\psi_t(z^*)\rangle$$

$$\bar{O}(t,z^*) = \int_0^t \alpha(t-s) O(t,s,z^*) ds$$

$$\rho_t \equiv \mathrm{M}\{|\psi_t(z^*)\rangle\langle\psi_t(z)|\}$$

$$\alpha(t-s) = \int_0^\infty J(\omega) e^{-i\omega(t-s)} d\omega$$

- Spin-boson pure dephasing model

$$H_{sys} = \frac{\omega}{2}\sigma_z \quad L = \sigma_z \quad \Rightarrow \quad O(t,s,z^*) = L$$

- Spin-boson dissipative model

$$H_{sys} = \frac{\omega}{2}\sigma_z \quad L = \sigma_- \quad \Rightarrow \quad O(t,s,z^*) = f(t,s)\sigma_-$$

$$\frac{\partial}{\partial t} f(t,s) = [i\omega + \int_0^t \alpha(t-s') f(t,s') ds'] f(t,s) \qquad f(t,t) = 1$$

Some exactly solvable models

- Damped harmonic oscillator model

$$H_{sys} = \omega a^{\dagger} a \qquad L = a \quad \Rightarrow \quad O(t,s,z^*) = f(t,s)a$$

$$\frac{\partial}{\partial t} f(t,s) = [i\omega + \int_0^t \alpha(t-s')f(t,s')ds']f(t,s) \qquad f(t,t) = 1$$

- Quantum Brown motion model

$$H_{sys} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2 \qquad L = q$$

$$\Rightarrow \quad O(t,s,z^*) = f(t,s)q + g(t,s)p - i\int_0^t j(t,s,s')z_{s'}ds'$$

Exact non-Markovian master equation

$$\frac{\partial}{\partial t}\rho_t = -i[H_{sys},\rho_t] + \frac{a(t)}{2i}[q^2,\rho_t] + \frac{b(t)}{2i}[q,\{p,\rho_t\}] + c(t)[q,[p,\rho_t]] - d(t)[q,[q,\rho_t]]$$

B. L. Hu et. al., *Phys. Rev. D* 45, 2843 (1992);

W. T. Strunz and T. Yu, *Phys. Rev. A* 69, 052115 (2004).

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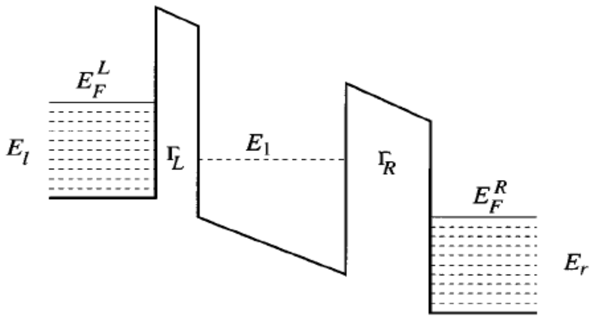
  Solid-state quantum circuits (see, e.g., JQY and Nori, *Nature* 474, 589-597 (2011) ; Xiang, Ashhab, JQY and Nori, arXiv: 1204.2137, to appear in *Rev. Mod. Phys*.)

  Fermionic baths: Electric leads, background charge fluctuations, ……

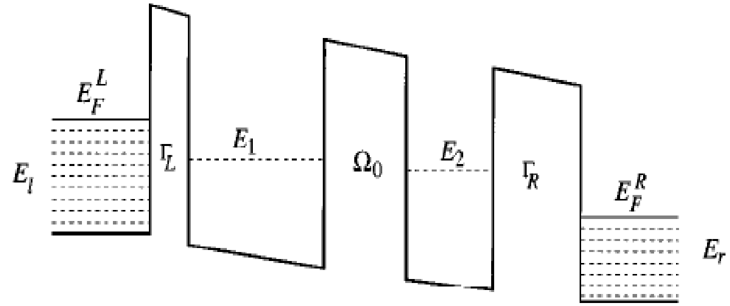
- Summary and outlook

Quantum confined system coupled to fermionic reservoirs

Quantum dot



Double quantum dot



- The total Hamiltonian of a quantum confined system coupled to two fermionic reservoirs

$$H_{tot} = H_{sys} + H_{\text{int}} + H_{env}$$

$$H_{\text{int}} = \sum_k (g_{Lk} c_L^\dagger a_{Lk} + g_{Rk} c_R^\dagger a_{Rk} + H.c.)$$

$$H_{env} = \sum_k (\omega_{Lk} a_{Lk}^\dagger a_{Lk} + \omega_{Rk} a_{Rk}^\dagger a_{Rk})$$

In a quantum state diffusion approach, environments are required to be initially at zero temperature, so as to conveniently represent the environmental degrees of freedom with the coherent state basis.

Bogoliubov transformation: Converting a nonzero- to a zero-temperature problem

As for environments initially with a nonzero temperature, one can map the nonzero-temperature density operator to a zero-temperature density operator using a Bogoliubov transformation [T. Yu, *Phys. Rev. A* 69, 062107 (2004)].

By including the part involving holes in the electric leads, the total Hamiltonian can be written as

$$H_{tot} = H_{sys} + \sum_{\lambda k} (g_{\lambda k} c_\lambda^\dagger a_{\lambda k} + H.c.) + \sum_{\lambda k} \omega_{\lambda k} a_{\lambda k}^\dagger a_{\lambda k} + \sum_{\lambda k} \omega_{\lambda k} b_{\lambda k} b_{\lambda k}^\dagger$$

Performing Bogoliubov transformation for fermionic operators:

$$a_{\lambda k} = \sqrt{1-\bar{n}_{\lambda k}}\, d_{\lambda k} - \sqrt{\bar{n}_{\lambda k}}\, e_{\lambda k}^\dagger \qquad b_{\lambda k} = \sqrt{1-\bar{n}_{\lambda k}}\, e_{\lambda k} + \sqrt{\bar{n}_{\lambda k}}\, d_{\lambda k}^\dagger$$

$$H'_{tot} = H_{sys} + \sum_{\lambda k} \sqrt{1-\bar{n}_{\lambda k}} (g_{\lambda k} c_\lambda^\dagger d_{\lambda k} + H.c.) + \sum_{\lambda k} \sqrt{\bar{n}_{\lambda k}} (g_{\lambda k}^* c_\lambda e_{\lambda k} + H.c.) + \sum_{\lambda k} \omega_{\lambda k} d_{\lambda k}^\dagger d_{\lambda k} + \sum_{\lambda k} \omega_{\lambda k} e_{\lambda k} e_{\lambda k}^\dagger$$

$$H'_{tot}(t) = H_{sys} + \sum_{\lambda k} (\sqrt{1-\bar{n}_{\lambda k}}\, g_{\lambda k} c_\lambda^\dagger d_{\lambda k} e^{-i\omega_{\lambda k} t} + \sqrt{\bar{n}_{\lambda k}}\, g_{\lambda k}^* c_\lambda e_{\lambda k} e^{i\omega_{\lambda k} t} + H.c.)$$

The fermionic baths with nonzero initial temperatures are mapped to virtual fermionic baths with zero initial temperature.

Fermionic coherent states representation

- Initial condition $\left|\Psi_0\right\rangle = \left|\psi_0\right\rangle \otimes \left|0\right\rangle \quad \left|0\right\rangle = \otimes_\lambda \left|0\right\rangle_{\lambda d} \otimes \left|0\right\rangle_{\lambda e}$

$$d_{\lambda k}\left|0\right\rangle = 0 \qquad e_{\lambda k}\left|0\right\rangle = 0$$

- Fermionic coherent state basis for the environments

$$\left|z\right\rangle_\lambda = \otimes_k \left|z_k\right\rangle_\lambda = e^{-\sum_k z_{\lambda k} d_{\lambda k}^\dagger}\left|0\right\rangle \quad \left|w\right\rangle_\lambda = \otimes_k \left|w_k\right\rangle_\lambda = e^{-\sum_k w_{\lambda k} e_{\lambda k}^\dagger}\left|0\right\rangle \quad \left|zw\right\rangle = \otimes_\lambda \left|z\right\rangle_\lambda \otimes \left|w\right\rangle_\lambda$$

Grassmann variables $z_{\lambda k}, w_{\lambda k}$

- In the coherent state representation $\left|\psi_t(z^*, w)\right\rangle \equiv \left\langle zw \middle| \Psi_t \right\rangle$

$$\frac{\partial}{\partial t}\left|\Psi_t\right\rangle = -iH_{tot}(t)\left|\Psi_t\right\rangle \quad \Longrightarrow \quad \frac{\partial}{\partial t}\left|\psi_t(z^*, w)\right\rangle = -i\left\langle zw\right|H_{tot}(t)\left|\Psi_t\right\rangle$$

Use the following equations:

$$d_{\lambda k}\left\langle zw\right| = z_{\lambda k}^*\left\langle zw\right| \qquad d_{\lambda k}^\dagger\left\langle zw\right| = \frac{\partial}{\partial z_{\lambda k}^*}\left\langle zw\right| \qquad e_{\lambda k}\left\langle zw\right| = w_{\lambda k}^*\left\langle zw\right| \qquad e_{\lambda k}^\dagger\left\langle zw\right| = \frac{\partial}{\partial w_{\lambda k}^*}\left\langle zw\right|$$

$$\frac{\partial}{\partial z_{\lambda k}^*} = \int \frac{\partial z_\lambda^*(s)}{\partial z_{\lambda k}^*}\frac{\delta}{\delta z_\lambda^*(s)}ds \qquad \frac{\partial}{\partial w_{\lambda k}^*} = \int \frac{\partial w_\lambda^*(s)}{\partial w_{\lambda k}^*}\frac{\delta}{\delta w_\lambda^*(s)}ds \qquad \text{(chain rules)}$$

Non-Markovian quantum state diffusion (QSD) equation

Non-Markovian QSD equation (stochastic Schrödinger equation)

$$\frac{\partial}{\partial t}|\psi_t\rangle = -iH_{sys}|\psi_t\rangle - \sum_\lambda [c_\lambda z_\lambda^*(t) + c_\lambda^\dagger w_\lambda^*(t)]|\psi_t\rangle$$
$$-\sum_\lambda c_\lambda^\dagger \int_0^t \alpha_{\lambda 1}(t-s)\frac{\delta}{\delta z_\lambda^*(s)}|\psi_t\rangle ds - \sum_\lambda c_\lambda \int_0^t \alpha_{\lambda 2}(t-s)\frac{\delta}{\delta w_\lambda^*(s)}|\psi_t\rangle ds$$

Noise function

$$z_\lambda^*(t) = -i\sum_k \sqrt{1-\bar{n}_{\lambda k}}\, g_{\lambda k}^* z_{\lambda k}^* e^{i\omega_{\lambda k} t} \qquad w_\lambda^*(t) = -i\sum_k \sqrt{\bar{n}_{\lambda k}}\, g_{\lambda k} w_{\lambda k}^* e^{-i\omega_{\lambda k} t}$$

Statistical properties of noise function

$$M\{z_\lambda(t)\} = M\{z_\lambda(t) z_\lambda(s)\} = 0 \qquad M\{z_\lambda(t) z_\lambda^*(s)\} = \alpha_{\lambda 1}(t-s) = \int d\omega [1-\bar{n}_\lambda(\omega)] J_\lambda(\omega) e^{-i\omega(t-s)}$$

$$M\{w_\lambda(t)\} = M\{w_\lambda(t) w_\lambda(s)\} = 0 \qquad M\{w_\lambda(t) w_\lambda^*(s)\} = \alpha_{\lambda 2}(t-s) = \int d\omega \bar{n}_\lambda(\omega) J_\lambda(\omega) e^{i\omega(t-s)}$$

$$M\{\cdots\} \equiv \int e^{-z^* z - w^* w}\{\cdots\} d^2 z d^2 w$$

$$z^* z = \sum_{\lambda k} z_{\lambda k}^* z_{\lambda k} \qquad w^* w = \sum_{\lambda k} w_{\lambda k}^* w_{\lambda k} \qquad d^2 z = \prod_{\lambda k} dz_{\lambda k}^* dz_{\lambda k} \qquad d^2 w = \prod_{\lambda k} dw_{\lambda k}^* dw_{\lambda k}$$

The completeness relation for coherent states: $\int e^{-z^* z - w^* w} |zw\rangle\langle zw| d^2 z d^2 w = 1$

The O-operator and its equation of motion

Non-Markovian QSD equation (stochastic Schrödinger equation)

$$\frac{\partial}{\partial t}|\psi_t\rangle = -iH_{sys}|\psi_t\rangle - \sum_{\lambda}[c_\lambda z_\lambda^*(t) + c_\lambda^\dagger w_\lambda^*(t)]|\psi_t\rangle$$
$$-\sum_{\lambda} c_\lambda^\dagger \int_0^t \alpha_{\lambda 1}(t-s)\frac{\delta}{\delta z_\lambda^*(s)}|\psi_t\rangle ds - \sum_{\lambda} c_\lambda \int_0^t \alpha_{\lambda 2}(t-s)\frac{\delta}{\delta w_\lambda^*(s)}|\psi_t\rangle ds$$

- Like the Diosi-Gisin-Strunz equation in the bosonic-bath case, we introduce the O-operators:

$$\frac{\delta}{\delta z_\lambda^*(s)}\left|\psi_t(z^*,w^*)\right\rangle = O_{\lambda 1}(t,s,z^*,w^*)\left|\psi_t(z^*,w^*)\right\rangle \qquad O_{\lambda 1}(t,t,z^*,w^*) = c_\lambda$$
$$\frac{\delta}{\delta w_\lambda^*(s)}\left|\psi_t(z^*,w^*)\right\rangle = O_{\lambda 2}(t,s,z^*,w^*)\left|\psi_t(z^*,w^*)\right\rangle \qquad O_{\lambda 2}(t,t,z^*,w^*) = c_\lambda^\dagger$$

- Equation of motion for the O-operators:

$$\frac{\partial O_{\lambda n}}{\partial t} = [-iH_{sys} - \sum_{\lambda'}(c_{\lambda'}^\dagger \bar{O}_{\lambda' 1} + c_{\lambda'}\bar{O}_{\lambda' 2}), O_{\lambda n}] + Q_n + \sum_{\lambda'}(\{c_{\lambda'}, O_{\lambda n}\} z_{\lambda'}^*(t) + \{c_{\lambda'}^{+}, O_{\lambda n}\} w_{\lambda'}^*(t))$$

$$Q_n = c_L^\dagger \frac{\delta \bar{O}_{L1}}{\delta \Lambda_n} + c_R^\dagger \frac{\delta \bar{O}_{R1}}{\delta \Lambda_n} + c_L \frac{\delta \bar{O}_{L2}}{\delta \Lambda_n} + c_R \frac{\delta \bar{O}_{R2}}{\delta \Lambda_n} \qquad \Lambda_1 = z_\lambda^*(s) \qquad \Lambda_2 = w_\lambda^*(s)$$

Non-Markovian master equation

Stochastic reconstruction of the reduced density operator:

$$\rho_t = Tr_{env}\langle \Psi_t | \Psi_t \rangle = \int e^{-z^*z-w^*w} |\psi_t(z^*,w^*)\rangle\langle\psi_t(-z,-w)| d^2z d^2w \equiv M\{|\psi_t(z^*,w^*)\rangle\langle\psi_t(-z,-w)|\}$$

- Stochastic projection operator

$$P_t \equiv |\psi_t(z^*,w^*)\rangle\langle\psi_t(-z,-w)| \quad \Rightarrow \quad \rho_t = M\{P_t\}$$

- Non-Markovian master equation

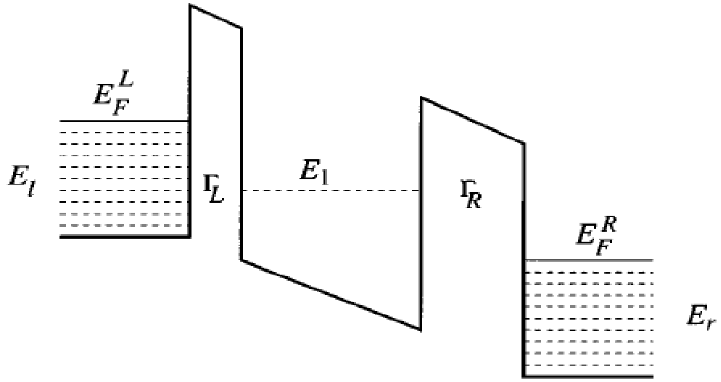
$$\frac{\partial}{\partial t}\rho_t = -i[H_{sys},\rho_t] + \sum_\lambda ([c_\lambda, \mathrm{M}\{P_t\overline{O}^\dagger_{\lambda 1}(t,-z,-w)\}] - [c^\dagger_\lambda, \mathrm{M}\{\overline{O}_{\lambda 1}(t,z^*,w^*)P_t\}]$$
$$-[c_\lambda, \mathrm{M}\{\overline{O}_{\lambda 2}(t,z^*,w^*)P_t\}] + [c^\dagger_\lambda, \mathrm{M}\{P_t\overline{O}^\dagger_{\lambda 2}(t,-z,-w)\}]$$

Generally, it is complex, but can have a simple form for a given system.

- Markov limit $\quad \alpha_{\lambda 1}(t-s) \to [1-\bar{n}_\lambda]\Gamma_\lambda \delta(t-s) \qquad \alpha_{\lambda 2}(t-s) \to \bar{n}_\lambda \Gamma_\lambda \delta(t-s)$

$$\frac{\partial}{\partial t}\rho_t = -i[H_{sys},\rho_t] + \sum_\lambda \frac{\Gamma_\lambda}{2}[\bar{n}_\lambda(2c^\dagger_\lambda\rho_t c_\lambda - c_\lambda c^\dagger_\lambda \rho_t - \rho_t c_\lambda c^\dagger_\lambda) + (1-\bar{n}_\lambda)(2c_\lambda\rho_t c^\dagger_\lambda - c^\dagger_\lambda c_\lambda \rho_t - \rho_t c^\dagger_\lambda c_\lambda)]$$

Applications: (1) Single quantum dot



- Single quantum dot

$$H_{tot} = H_{sys} + H_{\text{int}} + H_{env}$$

$$H_{env} = \sum_k (\omega_{Lk} a_{Lk}^\dagger a_{Lk} + \omega_{Rk} a_{Rk}^\dagger a_{Rk})$$

$$H_{\text{int}} = \sum_k (g_{Lk} c_L^\dagger a_{Lk} + g_{Rk} c_R^\dagger a_{Rk} + H.c.)$$

$$H_{sys} = \omega_0 c^\dagger c \qquad c_L = c_R = c$$

Strong Coulomb blockade:

$$U \to \infty$$

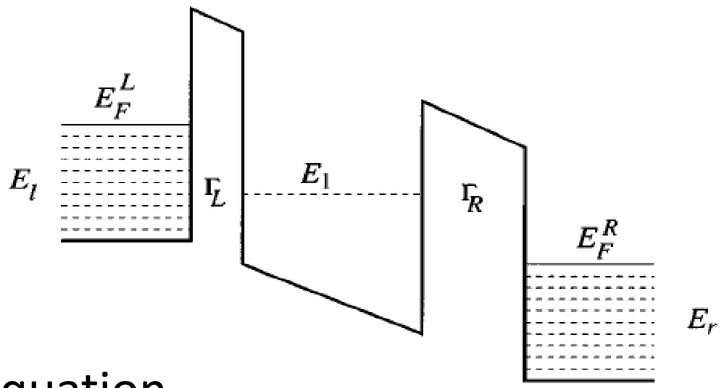
Constraint: Only one electron is allowed in the single quantum dot

- O-operators

$$O_{\lambda 1}(t,s,z^*,w^*) = f_1(t,s)c + \int_0^t q_1(t,s,s')[w_L^*(s') + w_R^*(s')]ds'$$

$$O_{\lambda 2}(t,s,z^*,w^*) = f_2(t,s)\ c^\dagger + \int_0^t q_2(t,s,s')[z_L^*(s') + z_R^*(s')]ds'$$

Applications: (1) Single quantum dot



- Exact non-Markovian master equation

$$\frac{\partial}{\partial t}\rho_t = -i[H_{sys},\rho_t] + \Gamma_1(t)[c,\rho_t c^\dagger] + \Gamma_2(t)[c,c^\dagger\rho_t] - \Gamma_1^*(t)[c^\dagger, c\rho_t] - \Gamma_2^*(t)[c^\dagger,\rho_t c]$$

with time-dependent rates:

$$\Gamma_j(t) = \int_0^t [\alpha_1(t-s)A_j(t,s) - \alpha_2(t-s)B_j(t,s)]ds \qquad \alpha_j(t-s) = \alpha_{Lj}(t-s) + \alpha_{Rj}(t-s)$$

$$(\frac{\partial}{\partial s} - i\omega_0)A_j(t,s) + \int_0^s \beta(s-s')A_j(t,s')ds' = U(t,s) \qquad U(t,s) = \int_0^t \alpha_2(t-s)h(t,s')ds'$$

$$(\frac{\partial}{\partial s} - i\omega_0)B_j(t,s) + \int_0^s \beta(s-s')B_j(t,s')ds' = V(t,s) \qquad V(t,s) = \int_0^t \alpha_1(t-s)h(t,s')ds'$$

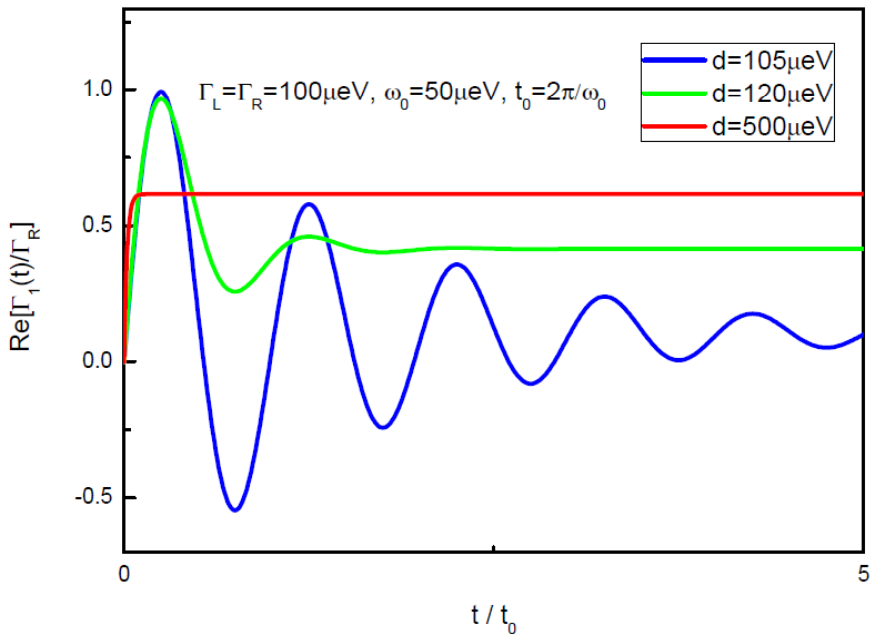
$$(\frac{\partial}{\partial s} - i\omega_0)h(t,s) - \int_s^t \beta(t-s')h(t,s')ds' = 0 \qquad \beta(s-s') = \alpha_1(s'-s) + \alpha_2(s-s')$$

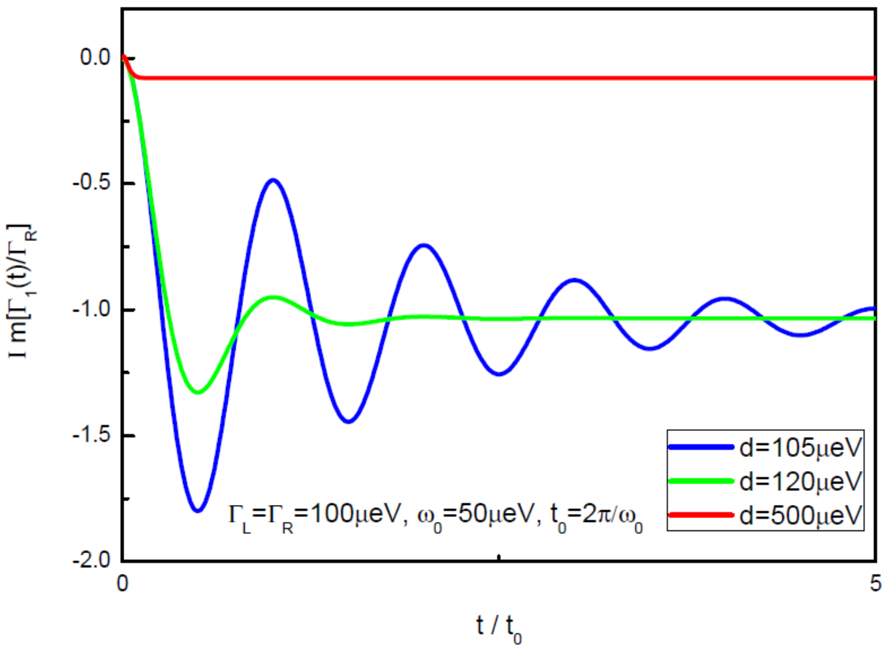
Final condition at $s = t$: $A_1(t,t) = B_1(t,t) = h(t,t) = 1 \qquad A_2(t,t) = B_2(t,t) = 0$

Applications: (1) Single quantum dot

- Exact non-Markovian master equation

$$\frac{\partial}{\partial t}\rho_t = -i[H_{sys},\rho_t] + \Gamma_1(t)[c,\rho_t c^\dagger] + \Gamma_2(t)[c,c^\dagger\rho_t] - \Gamma_1^*(t)[c^\dagger, c\rho_t] - \Gamma_2^*(t)[c^\dagger,\rho_t c]$$



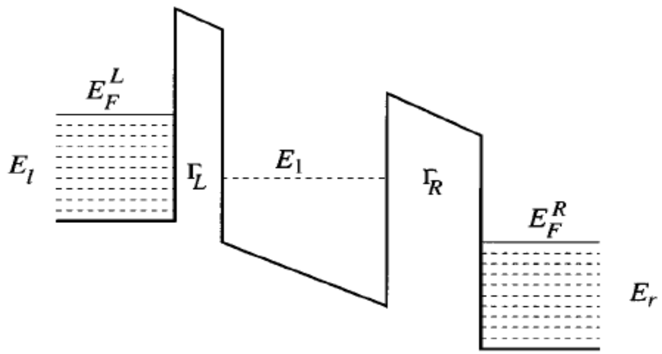


Applications: (1) Single quantum dot

Markov limit:

$$\Gamma_1(t) \to \frac{1}{2}[1-\bar{n}_L(\omega_0)]\Gamma_L + \frac{1}{2}[1-\bar{n}_R(\omega_0)]\Gamma_R$$

$$\Gamma_2(t) \to -\frac{1}{2}\bar{n}_L(\omega_0)\Gamma_L - \frac{1}{2}\bar{n}_R(\omega_0)\Gamma_R$$



Master equation in the Lindblad form:

$$\frac{\partial}{\partial t}\rho_t = -i[H_{sys},\rho_t] + \sum_\lambda \frac{\Gamma_\lambda}{2}[\bar{n}_\lambda(2c_\lambda^\dagger\rho_t c_\lambda - c_\lambda c_\lambda^\dagger\rho_t - \rho_t c_\lambda c_\lambda^\dagger) + (1-\bar{n}_\lambda)(2c_\lambda\rho_t c_\lambda^\dagger - c_\lambda^\dagger c_\lambda\rho_t - \rho_t c_\lambda^\dagger c_\lambda)]$$

Large bias limit: $\mu_L > \omega_0 > \mu_R$ Zero temperature: $\bar{n}_L(\omega_0) = 1$ $\bar{n}_R(\omega_0) = 0$

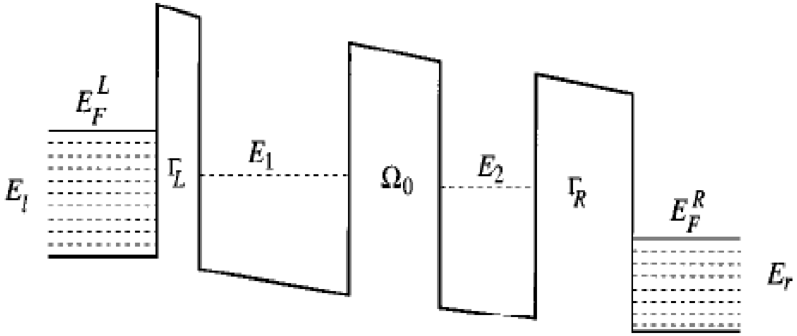
$$\dot{\rho}_{00} = -\Gamma_L\rho_{00} + \Gamma_R\rho_{11}$$
$$\dot{\rho}_{11} = \Gamma_L\rho_{00} - \Gamma_R\rho_{11}$$
$$\dot{\rho}_{10} = -[i\omega_0 + \Gamma_L + \Gamma_R]\rho_{10}$$

$|0\rangle$ empty dot state

$|1\rangle$ occupied dot state

S. A. Gurvitz and Ya. S. Prager, *Phys. Rev. B* 53, 15932 (1996)

Applications: (2) Double quantum dot



(a) $U \to \infty,\ V = 0$

Constraint: At most two electrons are allowed in the DQD.

(b) $U \to \infty,\ V \to \infty$

Constraint: Only one electron is allowed in the DQD.

- Double quantum dot $H_{tot} = H_{sys} + H_{\text{int}} + H_{env}$

$$H_{env} = \sum_k (\omega_{Lk} a_{Lk}^\dagger a_{Lk} + \omega_{Rk} a_{Rk}^\dagger a_{Rk}) \qquad H_{\text{int}} = \sum_k (g_{Lk} c_L^\dagger a_{Lk} + g_{Rk} c_R^\dagger a_{Rk} + H.c.)$$

$$H_{sys} = \omega_1 c_1^\dagger c_1 + \omega_2 c_2^\dagger c_2 + \Omega_0 (c_2^\dagger c_1 + c_1^\dagger c_2) \qquad c_L = c_1 \qquad c_R = c_2$$

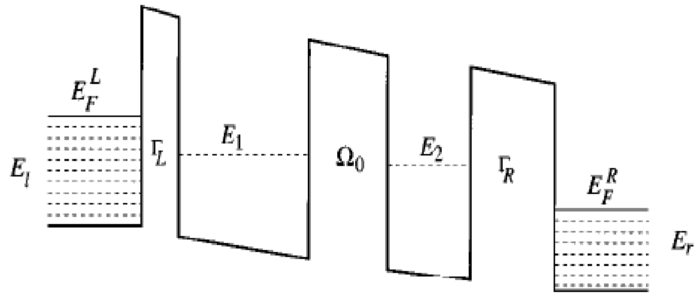
- O-operators

$$O_{\lambda 1}(t,s,z^*,w^*) = f_{\lambda 1}(t,s) c_1 + f_{\lambda 2}(t,s) c_2 + \int_0^t [f_{\lambda 3}(t,s,s') w_L^*(s') + f_{\lambda 4}(t,s,s') w_R^*(s')] ds'$$

$$O_{\lambda 2}(t,s,z^*,w^*) = m_{\lambda 1}(t,s) c_1 + m_{\lambda 2}(t,s) c_2 + \int_0^t [m_{\lambda 3}(t,s,s') w_L^*(s') + m_{\lambda 4}(t,s,s') w_R^*(s')] ds'$$

Applications: (2) Double quantum dot

Exact non-Markovian master equation



$$
\begin{aligned}
\frac{\partial}{\partial t}\rho_t = -i[H_{sys},\rho_t] &+ \Gamma_{L1}(t)[c_1,\rho_t c_1^\dagger] + \Gamma_{L2}(t)[c_1,c_1^\dagger\rho_t] + \Gamma_{L3}(t)[c_1,\rho_t c_2^\dagger] + \Gamma_{L4}(t)[c_1,c_2^\dagger\rho_t] \\
&+ \Gamma_{R1}(t)[c_2,\rho_t c_1^\dagger] + \Gamma_{R2}(t)[c_2,c_1^\dagger\rho_t] + \Gamma_{R3}(t)[c_2,\rho_t c_2^\dagger] + \Gamma_{R4}(t)[c_2,c_2^\dagger\rho_t] \\
&- \Gamma_{L1}^*(t)\,[c_1^\dagger,\rho_t c_1] - \Gamma_{L2}^*(t)\,[c_1^\dagger,c_1\rho_t] - \Gamma_{L3}^*(t)\,[c_1^\dagger,c_2\rho_t] - \Gamma_{L4}^*(t)\,[c_1^\dagger,\rho_t c_2] \\
&- \Gamma_{R1}^*(t)\,[c_2^\dagger,\rho_t c_1] - \Gamma_{R2}^*(t)\,[c_2^\dagger,c_1\rho_t] - \Gamma_{R3}^*(t)[c_2^\dagger,c_2\rho_t] - \Gamma_{R4}^*(t)\,[c_2^\dagger,\rho_t c_2]
\end{aligned}
$$

with time-dependent rates

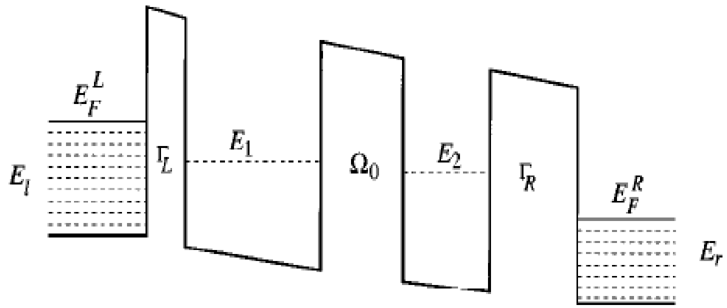
$$\Gamma_{\lambda j}(t) = \int_0^t [\alpha_{\lambda 1}(t-s)A_{\lambda j}(t,s) - \alpha_{\lambda 2}(t-s)B_{\lambda j}(t,s)]ds$$

$A_{\lambda j}(t,s)$ $B_{\lambda j}(t,s)$ satisfy a set of integro-differential equations, with final conditions

$$A_{L1}(t,t) = A_{R3}(t,t) = B_{L2}(t,t) = B_{R4}(t,t) = 1$$

$A_{\lambda j}(t,t) = B_{\lambda j}(t,t) = 0$ for other λ, j

Applications: (2) Double quantum dot



Markov limit:

$$\Gamma_{L1}(t) = \frac{1}{2}[1 - \bar{n}_L(\omega_1)]\Gamma_L \qquad \Gamma_{L2}(t) = -\frac{1}{2}\bar{n}_L(\omega_1)\Gamma_L$$

$$\Gamma_{R3}(t) = \frac{1}{2}[1 - \bar{n}_R(\omega_2)]\Gamma_R \qquad \Gamma_{R4}(t) = -\frac{1}{2}\bar{n}_R(\omega_2)\Gamma_R$$

$$\Gamma_{R1}(t) = \Gamma_{R2}(t) = \Gamma_{L3}(t) = \Gamma_{L4}(t) = 0$$

Master equation in the Lindblad form:

$$\frac{\partial}{\partial t}\rho_t = -i[H_{sys}, \rho_t] + \sum_{\lambda} \frac{\Gamma_\lambda}{2}[\bar{n}_\lambda(2c_\lambda^\dagger \rho_t c_\lambda - c_\lambda c_\lambda^\dagger \rho_t - \rho_t c_\lambda c_\lambda^\dagger) + (1 - \bar{n}_\lambda)(2c_\lambda \rho_t c_\lambda^\dagger - c_\lambda^\dagger c_\lambda \rho_t - \rho_t c_\lambda^\dagger c_\lambda)]$$

Applications: (2) Double quantum dot

Master equation in the Lindblad form:

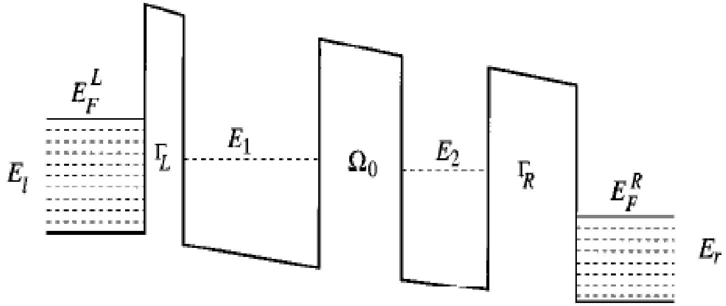
$$\frac{\partial}{\partial t}\rho_t = -i[H_{sys},\rho_t] + \sum_\lambda \frac{\Gamma_\lambda}{2}[\bar{n}_\lambda(2c_\lambda^\dagger\rho_t c_\lambda - c_\lambda c_\lambda^\dagger\rho_t - \rho_t c_\lambda c_\lambda^\dagger) + (1-\bar{n}_\lambda)(2c_\lambda\rho_t c_\lambda^\dagger - c_\lambda^\dagger c_\lambda\rho_t - \rho_t c_\lambda^\dagger c_\lambda)]$$

- Large bias limit and zero temperature condition

$$\mu_L > \omega_1,\ \omega_2 > \mu_R \qquad \bar{n}_L(\omega_1) = 1 \qquad \bar{n}_R(\omega_2) = 0$$

$|0\rangle$ empty dot $\qquad |1\rangle, |2\rangle$ left (right)dot occupied

$|3\rangle$ both dots occupied



- Strong Coulomb-blockade regime

(a) Strong interdot Coulomb repulsion

$$\dot\rho_{00} = -\Gamma_L\rho_{00} + \Gamma_R\rho_{22}$$
$$\dot\rho_{11} = \Gamma_L\rho_{00} + \Gamma_R\rho_{33} + i\Omega_0(\rho_{12} - \rho_{21})$$
$$\dot\rho_{22} = -(\Gamma_L + \Gamma_R)\rho_{22} - i\Omega_0(\rho_{12} - \rho_{21})$$
$$\dot\rho_{33} = \Gamma_L\rho_{22} - \Gamma_R\rho_{33}$$
$$\dot\rho_{12} = -i(\omega_1 - \omega_2)\rho_{12} + i\Omega_0(\rho_{11} - \rho_{22}) - \frac{\Gamma_L + \Gamma_R}{2}\rho_{12}$$

S. A. Gurvitz and Y. S. Prager, *PRB* 53,15932 (1996);
M. W. Y. Tu and W. M. Zhang, *PRB* 78, 235311 (2008).

(b) Both strong intra- and interdot Coulomb repulsions

$$\dot\rho_{00} = -\Gamma_L\rho_{00} + \Gamma_R\rho_{22}$$
$$\dot\rho_{11} = \Gamma_L\rho_{00} + i\Omega_0(\rho_{12} - \rho_{21})$$
$$\dot\rho_{22} = -\Gamma_R\rho_{22} - i\Omega_0(\rho_{12} - \rho_{21})$$
$$\dot\rho_{12} = -i(\omega_1 - \omega_2)\rho_{12} + i\Omega_0(\rho_{11} - \rho_{22}) - \frac{\Gamma_R}{2}\rho_{12}$$

T. H. Stoof and Y. V. Nazarov, *PRB* 53,1050 (1996)

Summary and outlook

Summary

- Non-Markovian quantum state diffusion (QSD) approach is extended to deal with the fermionic environments.
- Non-Markovian master equation is derived.
- The approach is applied to single quantum dot and double quantum dot, each coupled to two electric leads.

Outlook

- Connection to Feynman-Vernon influence functional approach?
- Non-Markovian QSD for a spin environment?
- Non-Markovian QSD for the system nonlinearly coupled to either a bosonic or fermionic bath ?