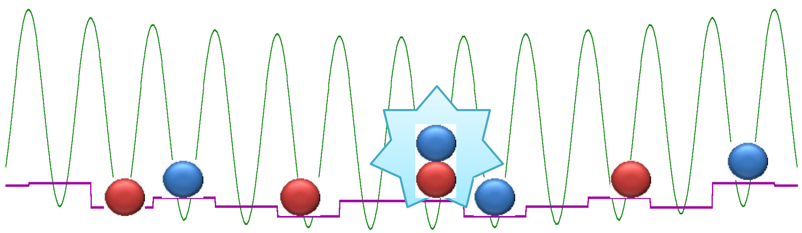
PRA **85**, 031602(R) (2012); PRA **82**, 043613 (2010)

Dynamics of interacting fermions on a bichromatic optical lattice



"Correlations and coherence in quantum systems"

Évora, 8 October 2012

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Table of contents

- Introduction:
  - Optical lattice with a quasiperiodic potential
  - Experiment with bosons
- Phase diagram for 1D two-component Fermi system
- Dynamics after release from a box trap
- Conclusion

Introduction: Fermi condensates

Inhomogeneity:
enhance?
suppress?

Optical trap shape
Speckle pattern; holography;
bichromatic potential

Superconductivity

Superfluidity

Interaction:
pairing force?
other phase?

Feshbach resonance

Population imbalance
= "magnetic field" : FFLO

Now widely controllable in cold atomic gases

n: site index of the main optical lattice

Bichromatic potential

$$V(n) \equiv \lambda \cos(2\pi\omega n + \theta)$$

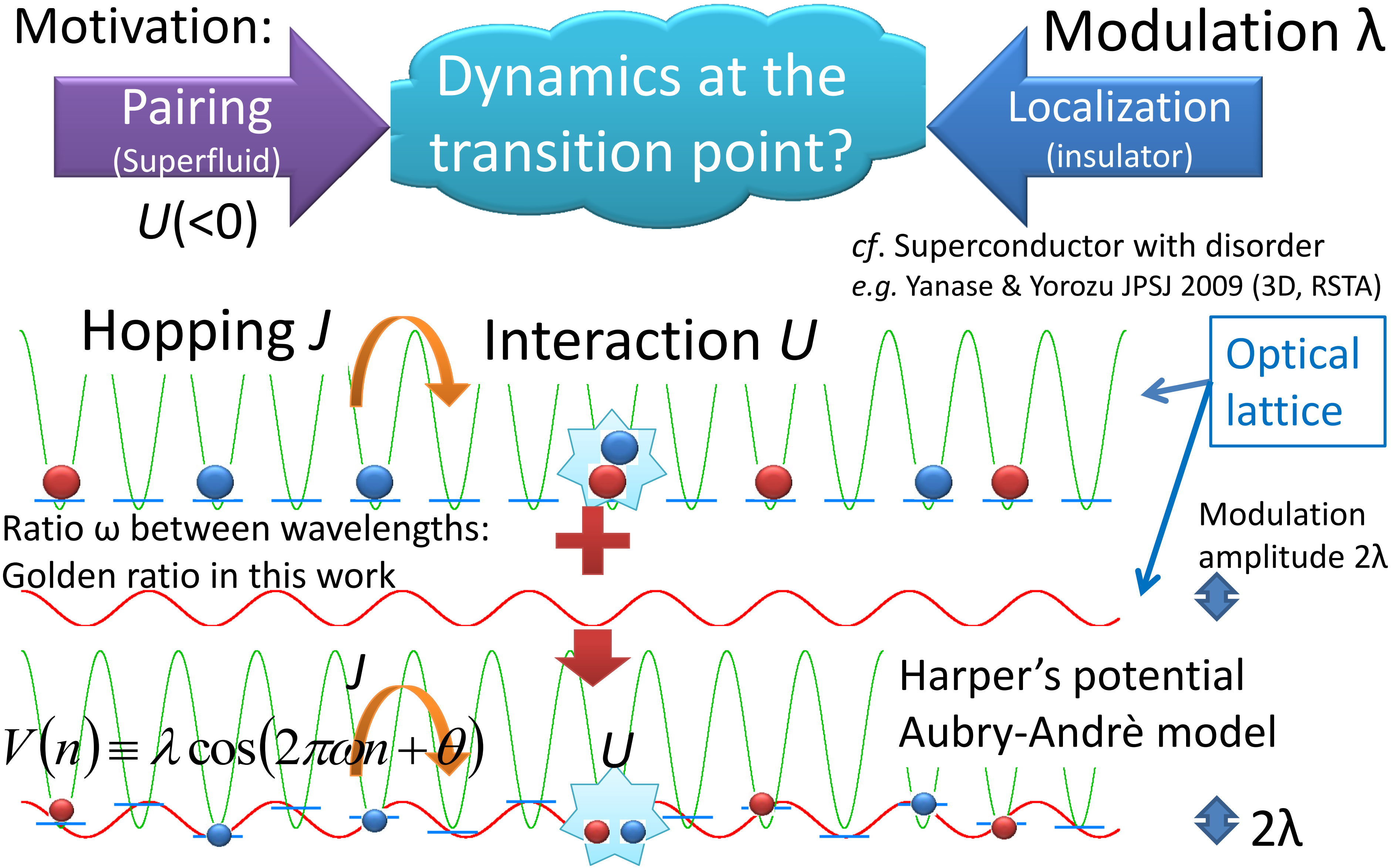
"Aubry-Andre model"

(*cf*. talk by Marcos Rigol this morning)

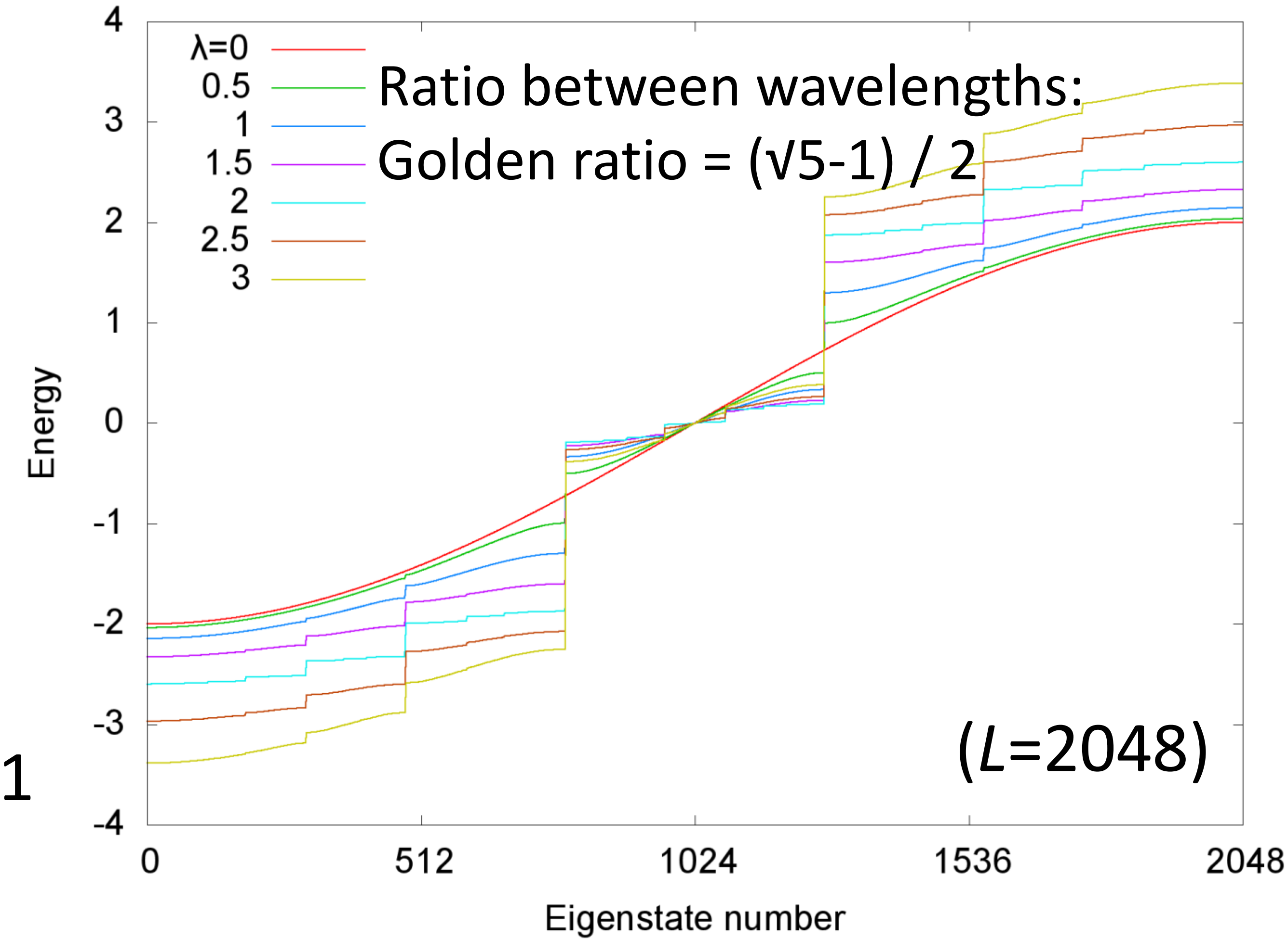
non-interacting : "metal"-insulator transition at λ=2*J* (*J*: hopping)

- Numerical studies of interacting systems
  - Bose-Hubbard (DMRG): Deng *et al.*: PRA **78**, 013625 (2008); Roux *et al.*: PRA **78**, 023628 (2008); *cf*. studies of BEC with non-linear Schrödinger eq.
  - Spinless Fermions : Chaves and Satija (ED, PRB **55**, 14076 (1997)); Schuster *et al.* (PBC DMRG, PRB **65**, 115114 (2002))

S=1/2 Fermions in bichromatic potential

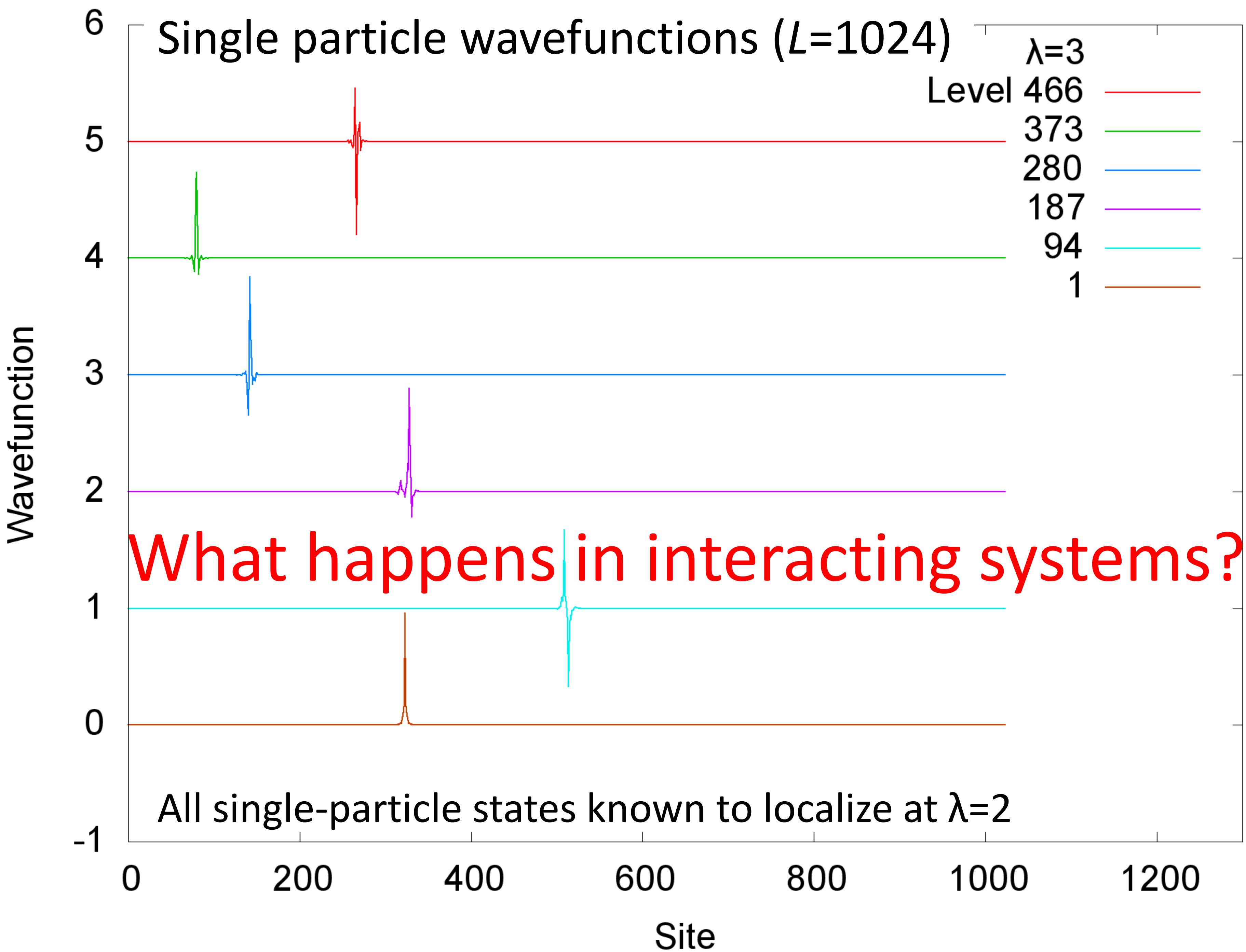


Single-particle level scheme



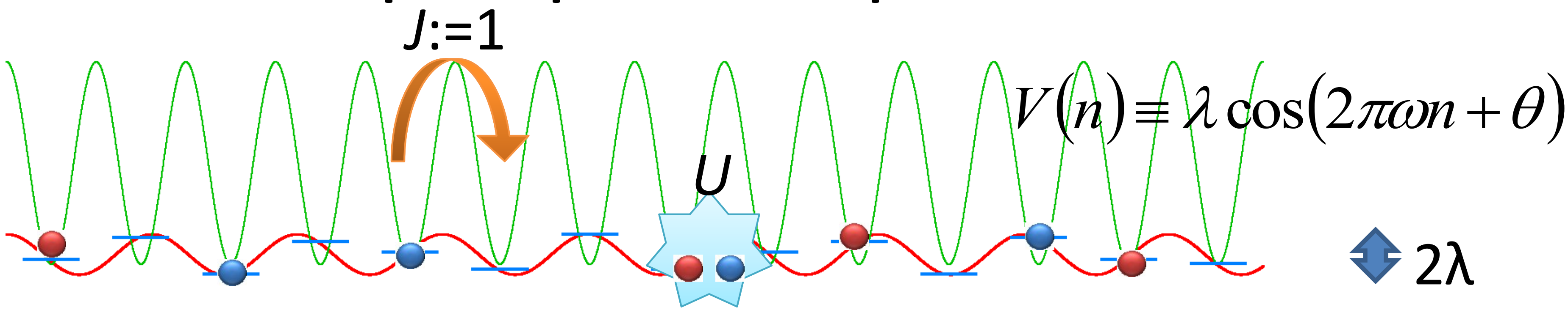
J=1

λ < 2: bands; λ > 2: separate, localized energy levels (as $L \to \infty$)



Single particle wavefunctions (L=1024)
λ=3
Level 466
373
280
187
94
1
What happens in interacting systems?
All single-particle states known to localize at λ=2
Wavefunction
Site
6
5
4
3
2
1
0
-1
0
200
400
600
800
1000
1200

Interacting fermions in quasiperiodic potential



- U=0 : All eigenstates localized for $\lambda > \lambda_c = 2$
- U>0 : Repulsive interaction
  - $\lambda_c > 2$, but not much increase [MT and García-García, in prep.]
- U<0 : Superfluidity; quasi-condensate (in 1D)
  - $\lambda_c < 2$ (for $|U| >> 1$, $\lambda_c \sim 2/|U| << 1$): Dynamics?
  - Does the potential inhomogeneity enhance pairing?

➔ Study the system numerically with DMRG

(Density-matrix renormalization group)

How to detect pairing and delocalization?

Pairing

On-site pair correlation function:
easy to calculate with DMRG
But depends on the site potentials of the site pair

$$\Gamma(x,r) \equiv \left\langle \hat{c}^{\dagger}_{x+r,\downarrow} \hat{c}^{\dagger}_{x+r,\uparrow} \hat{c}_{x,\uparrow} \hat{c}_{x,\downarrow} \right\rangle$$

Averaged equal-time pair structure factor

Sum of pair correlation for all lengths

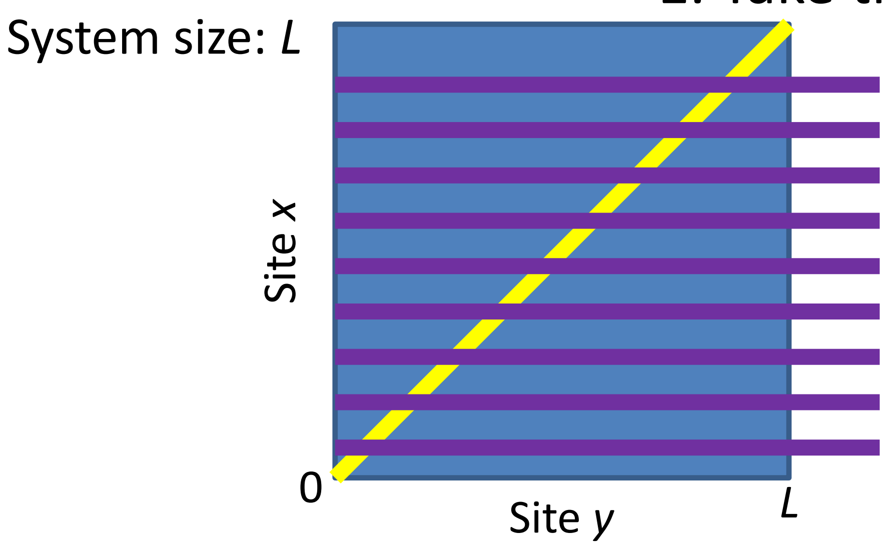
→ average over sites

cf. Hurt *et al.*: PRB **72**, 144513 (2005);
Mondaini *et al.*: PRB **78**, 174519 (2008)

$$P_s \equiv \left\langle \sum_{r>0} \Gamma(x,r) \right\rangle_x$$

Increasing function of *L*
if decay of correlation is slow

1. Take the sum over distance $r=|x-y| > 0$



2. Average over sites *x*

Compare between different system sizes using DMRG

How to detect pairing and delocalization?

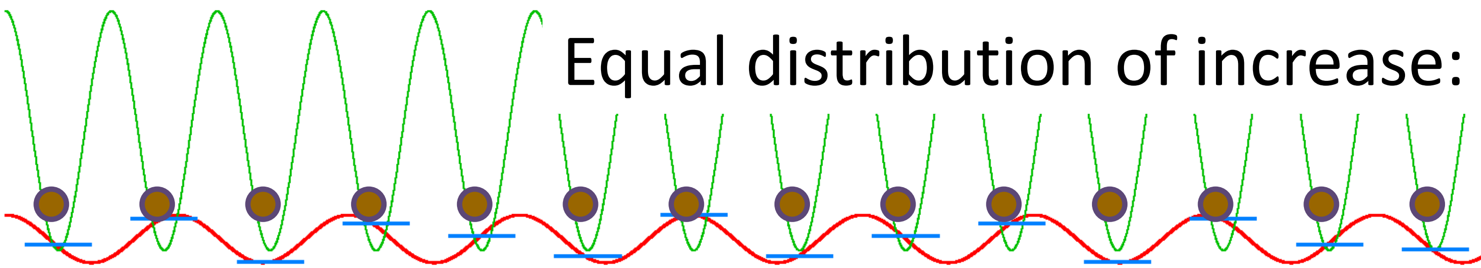
(De)localization

Inverse participation ratio (IPR)

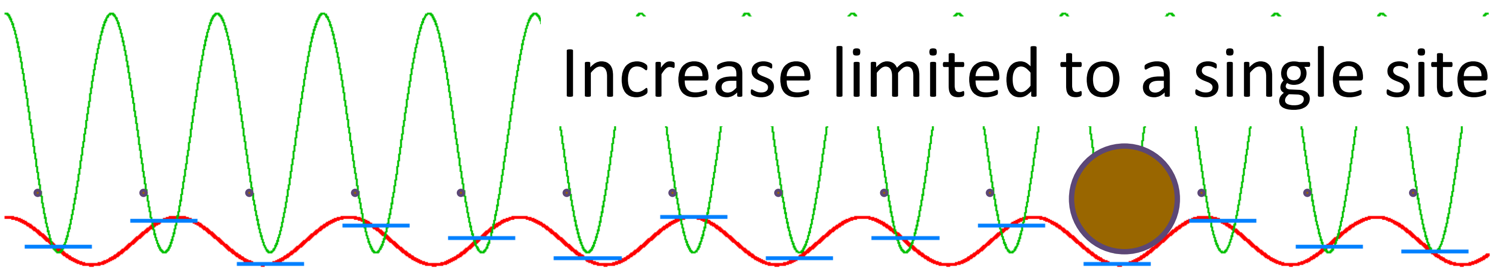
$$I_E \equiv \left(\sum_i \left(\langle \hat{n}_i \rangle_{N+1,N+1} - \langle \hat{n}_i \rangle_{N,N} \right)^2 \right)^{-1}$$

Add 2 atoms ➔ How uniformly is the population increase distributed?

$(n_\uparrow, n_\downarrow) = (N, N) \rightarrow (N+1, N+1)$: obtain $\Delta(\langle n_{i,\uparrow} + n_{i,\downarrow} \rangle)$ at each lattice site i



Equal distribution of increase: $I_E = L/4 \propto L$



Increase limited to a single site: $I_E \rightarrow 1/4$

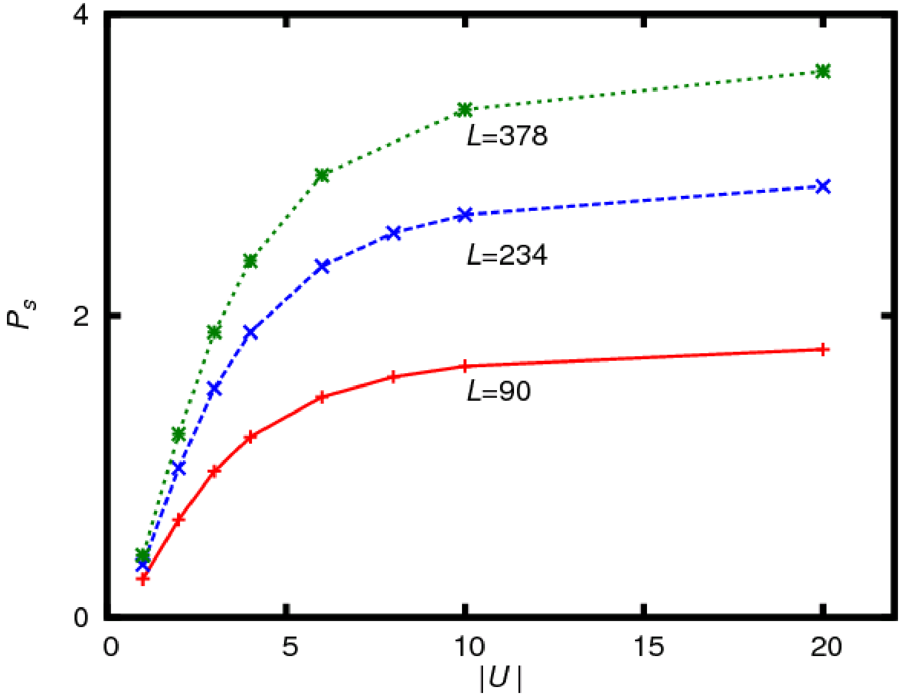
Compare between different system sizes using DMRG

cf. Phase sensitivity: requires (anti-)periodic condition [see *e.g.* Schuster *et al.*: PRB **65**, 115114 (2002)]
Hard to calculate within DMRG (not open BC) in large systems (OK for small systems)

The case without lattice modulation (λ=0)

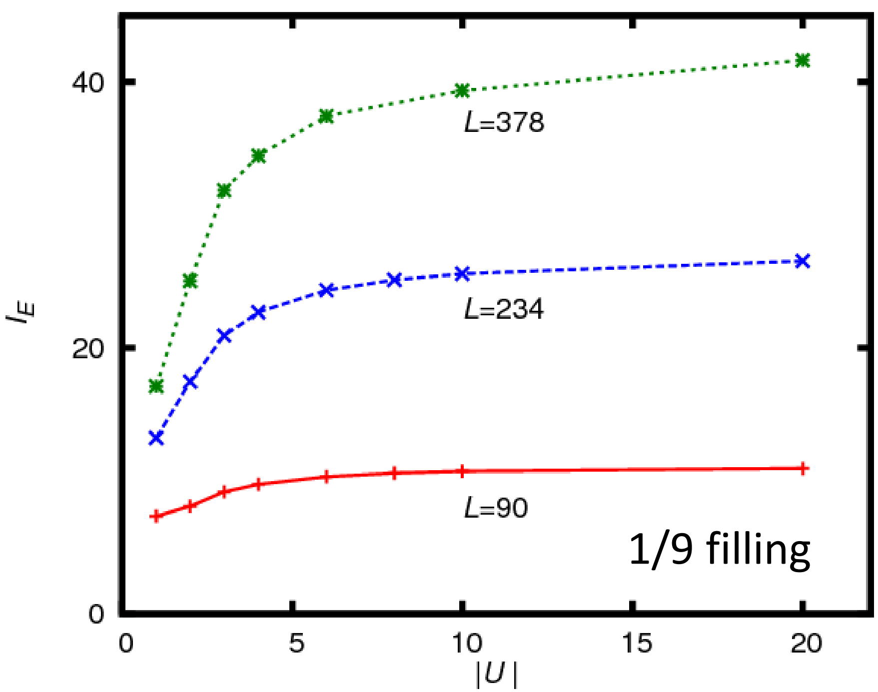
Pair structure factor

indicator of global (quasi long-range) superfluidity

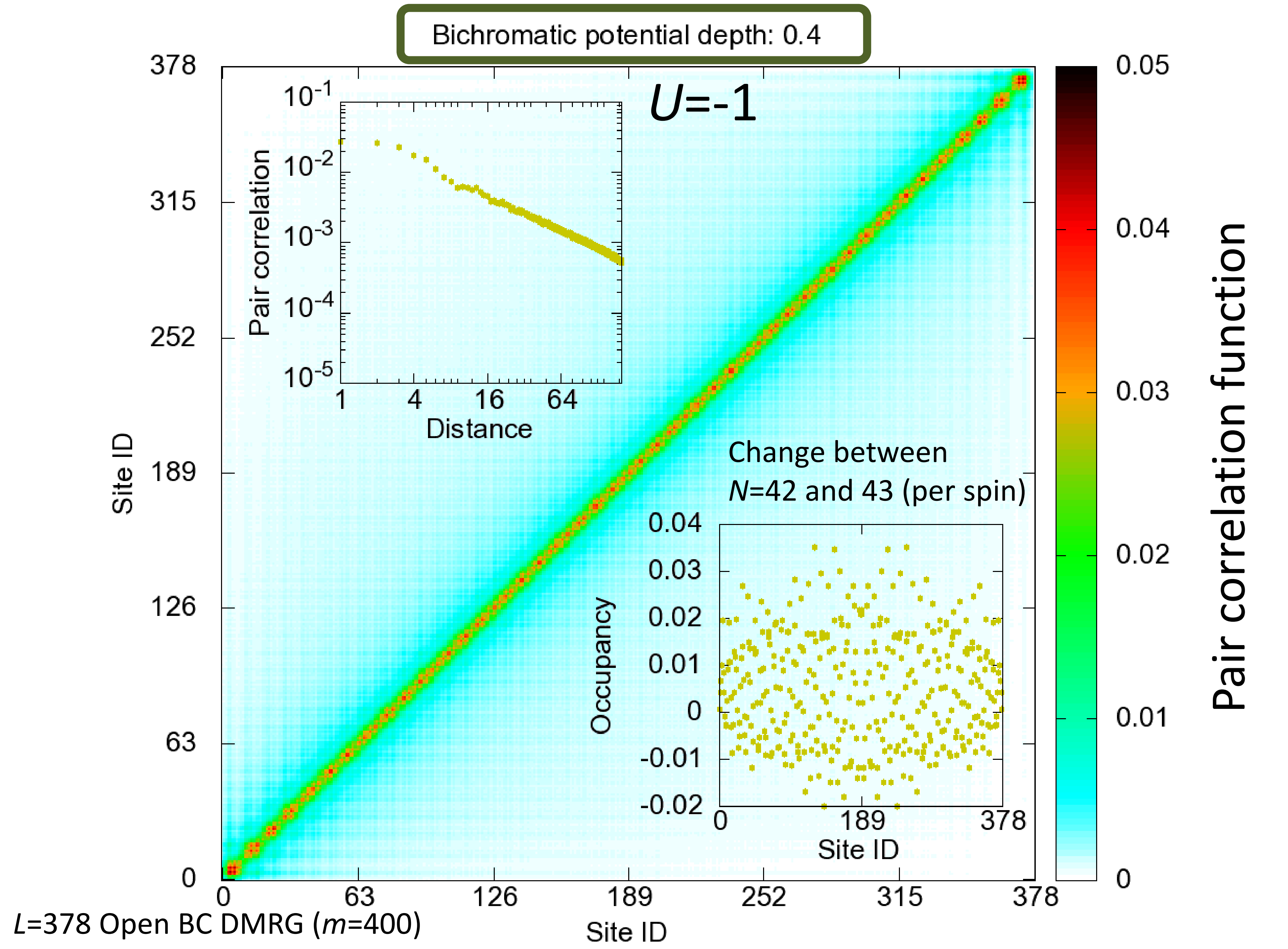


Inverse participation ratio

indicator of atom delocalization



Both increase with $|U|$, and system size L



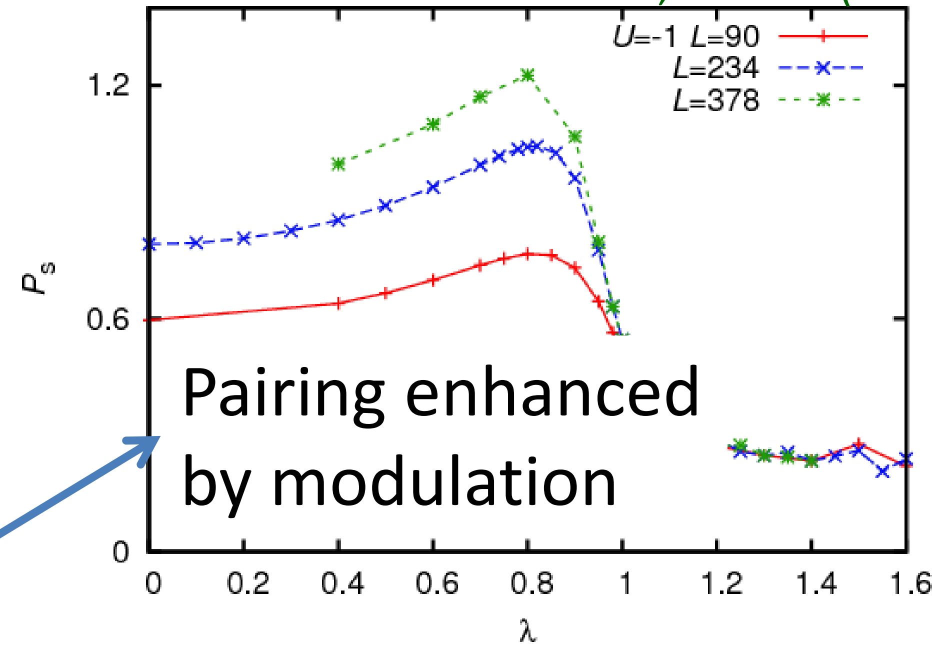
Bichromatic potential depth: 0.4
U=-1
Pair correlation
Distance
Change between
N=42 and 43 (per spin)
Occupancy
Site ID
Pair correlation function
Site ID
Site ID
L=378 Open BC DMRG (m=400)

Small |U|: weak attractive int.

Tezuka and García-García: PRA **82**, 043613 (2010)

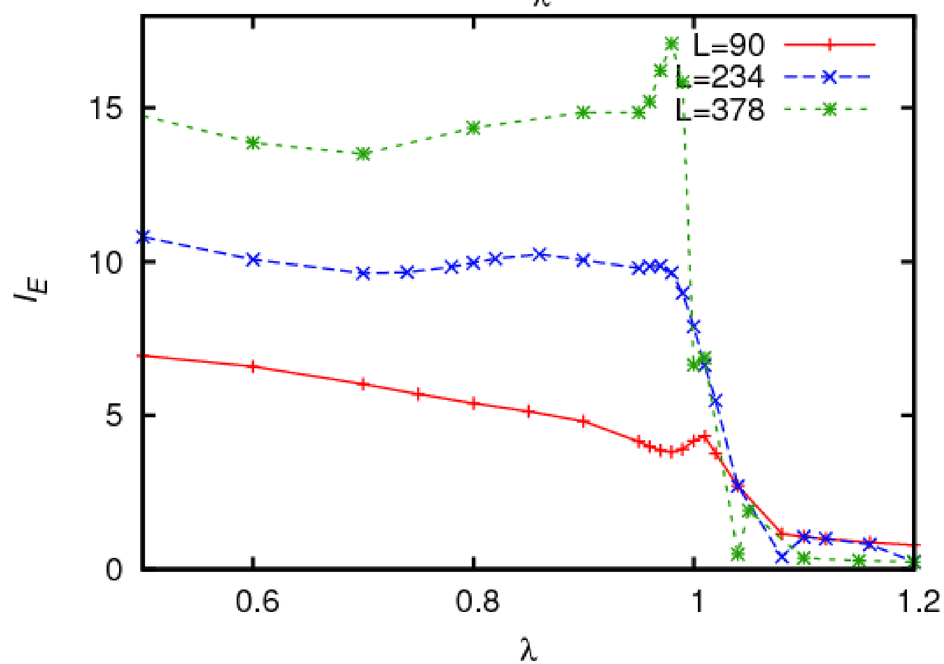
Pair structure factor

U=-1 :
Quasi long-range pairing disappears (λ_p~0.95) before localization (λ_c~1.00)



cf. Pair correlation enhanced in 1D by random disorder
E. Gambetti: PRB **72**, 165338 (2005);
T. Shirakawa *et al.*: J. Phys. Conf. Ser. **150**, 052238 (2009)

Inverse participation ratio

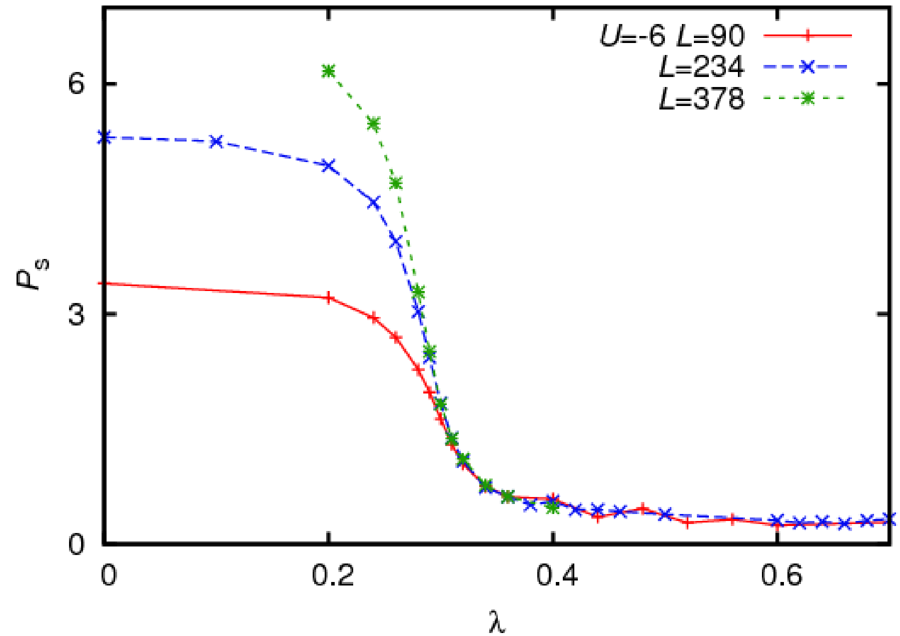


1/9 filling

Stronger attractive interaction

Tezuka and García-García: PRA **82**, 043613 (2010)

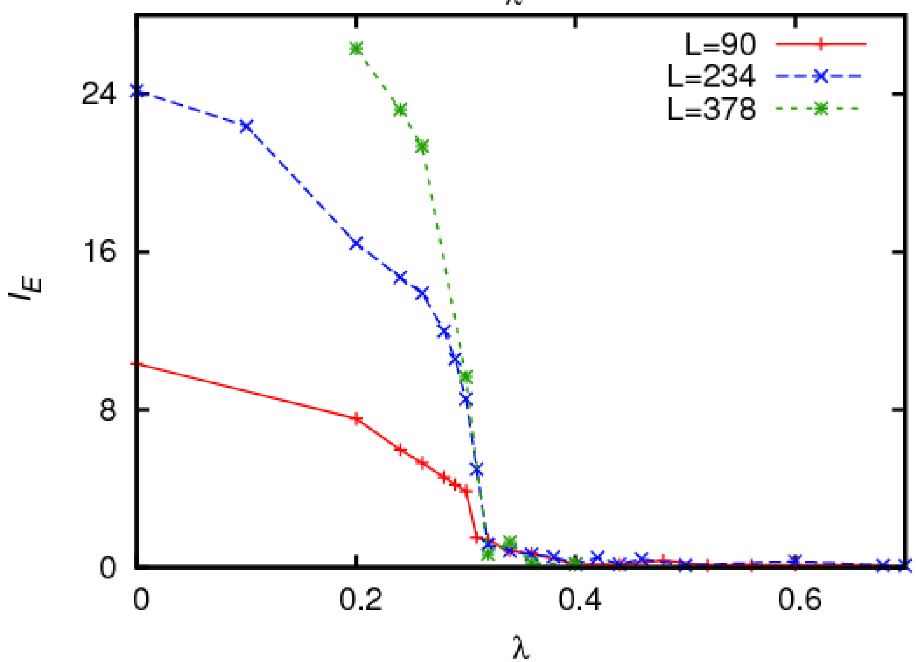
Pair structure factor



U=-6 :

Quasi long-range pairing disappears at localization (λ_c~0.30)

Inverse participation ratio



1/9 filling

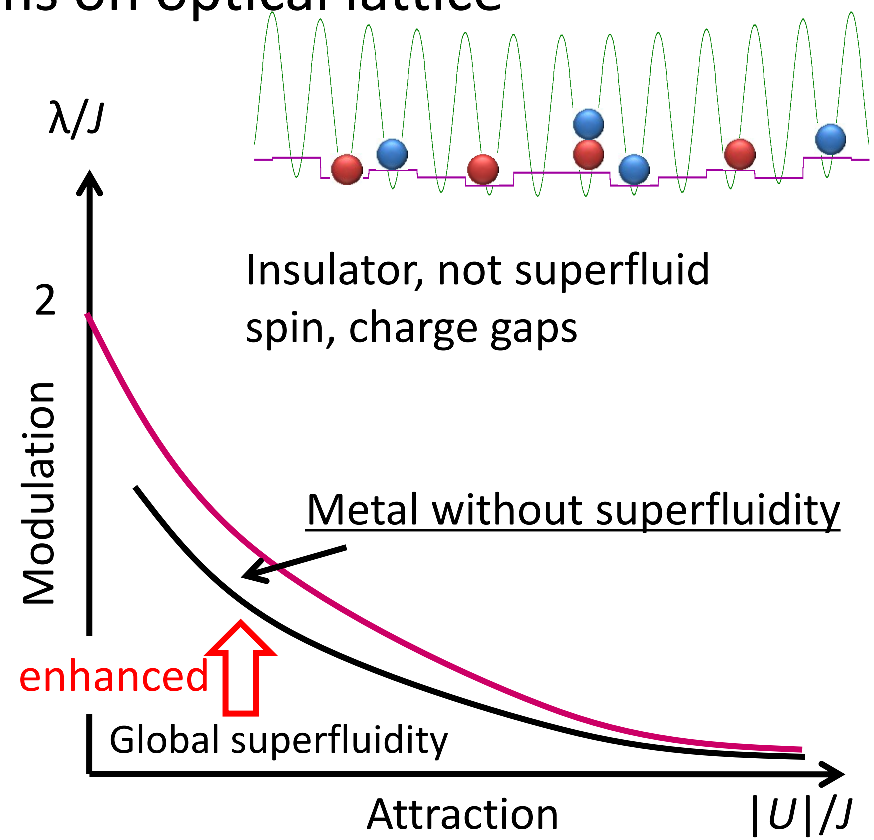
Tezuka and Garcia-Garcia: PRA **82**, 043613 (2010)

Summary (1): Schematic phase diagram

- Effect of coexisting modulation (bichromatic potential) and short-range attractive interaction
  - Studied for 1D fermionic atoms on optical lattice

- For strong interaction ($|U| \gg J$), pairing decreases as modulation amplitude λ is increased, and localizes at ~ insulating transition λ_c

- For weaker interaction ($|U| \sim J$), pairing has a peak as a function of λ, but localizes before λ_c



Dynamics

- Simulate trap-release experiments
- At localization transition point, does the dynamics after release depend on the value of U?

Disorder + interaction: subdiffusion?

For a potential which localizes single-particle states in 1D…

- 1D random potential, discrete version of Gross-Pitaevskii equation (BEC, mean-field approximation):
  - "Long-time Anderson localization": concluded that "displacement of the wave front slower than t^{α} for any $\alpha>0$"

    [W.-M. Wang and Z. Zhang: J. Stat. Phys. **134**, 953 (2009)]

Debated

  - "Anderson localization destroyed by nonlinearlity; subdiffusion continues"

    [Pikovsky and Shepelyansky: PRL **100**, 094101 (2008)]

Dynamics: experiments with bosons

Trap-release experiments: dynamics of the atomic clouds observed

Bosons: E. Lucioni *et al.* (LENS, Florence): PRL **106**, 230403 (2011)

Subdiffusion (slower than random walk) observed in bichromatic lattice (3D)

$V(x) = V_1\cos^2(k_1 x) + V_2\cos^2(k_2 x)$, $k_1=2\pi/(1064.4\text{nm})$, $k_2=2\pi/(859.6\text{nm})$

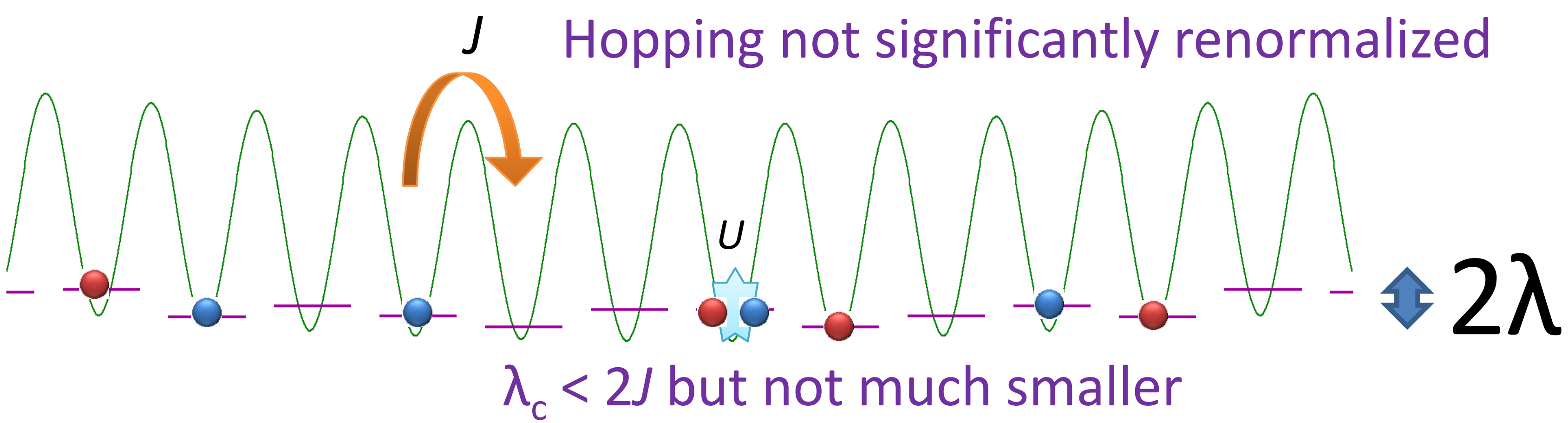
50 thousand ^{39}K atoms, almost spherical trap switched off at $t=0$

➔ What happens for interacting fermions in a bichromatic potential?

Very weakly attractive interaction

Modulation governs the conductance

Effect of modulation: relatively strong ($|U| \ll \lambda$)

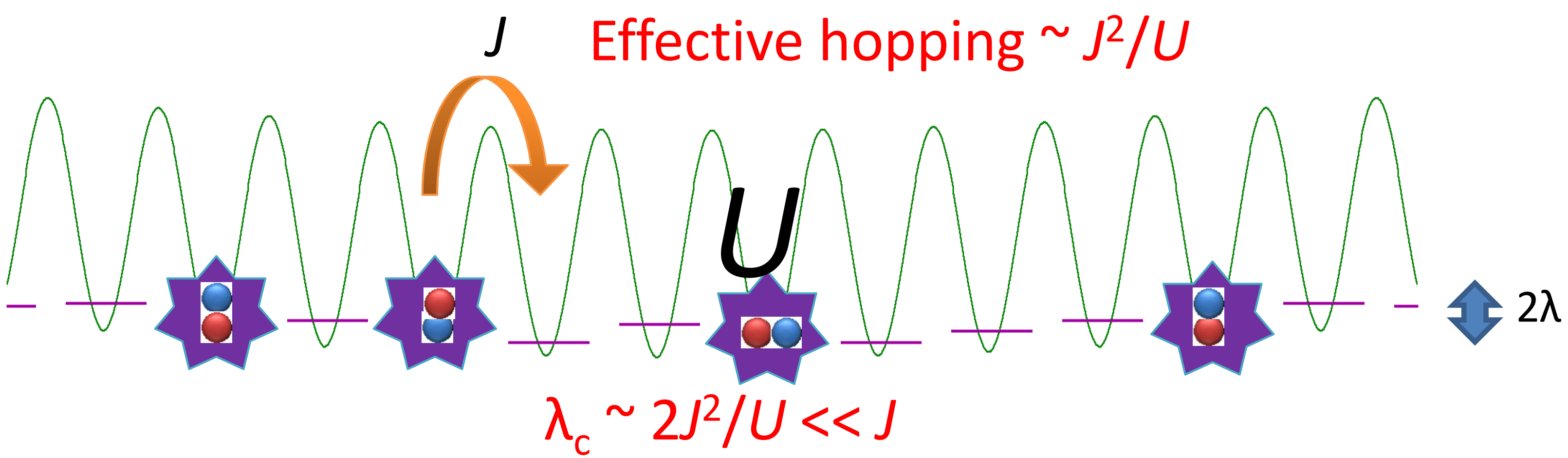


At transition point: spectrum still fractal; random walk-like motion ($\langle x^2 \rangle \sim t$) expected

Strongly attractive interaction

Two fermions ~ tightly bound hard-core bosons

Effect of modulation: relatively weak ($\lambda \ll |U|$)



At transition point: spectrum should be almost normal

Is particle motion almost ballistic? ($\langle x^2 \rangle \sim t^2$?)

One parameter scaling theory

Abrahams *et al.*: PRL **42**, 673 (1979)

see also Garcia-Garcia and Wang: PRL **100**, 070603 (2008)

L : system size

Dimensionless conductance $g(L) = E_T / \delta$: behavior as $L \to \infty$?

E_T : Thouless energy

~ 1 / (typical time for particle to travel L)

δ : (1 particle) mean level spacing

(d>2)D Normal metal:

$g(L) \propto L^{d-2} \to \infty$

$\because E_T \propto L^{-2}$, $\delta \propto L^{-d}$

Insulator:

$g(L) \propto \exp(-L/\xi) \to 0$

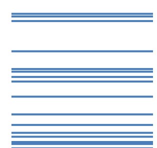
ξ: localization length

Metal-insulator transition

$g(L) = g_c$ (constant)

Disordered system

$\langle x^2(t) \rangle \propto t^{\alpha}$ $(0<\alpha<2)$



Motion slowed down for $\alpha<1$,

~ $L^{2/\alpha}$ time to propagate L, $E_T \propto L^{-2/\alpha}$

Multifractal spectrum with

Hausdorff dimension d_H

➔ $\delta \propto L^{-d/d_H}$; $g(L) \propto L^{d/d_H - 2/\alpha}$

Insulating transition should occur at $\alpha/2 = d_H/d = d_H$ (d=1)

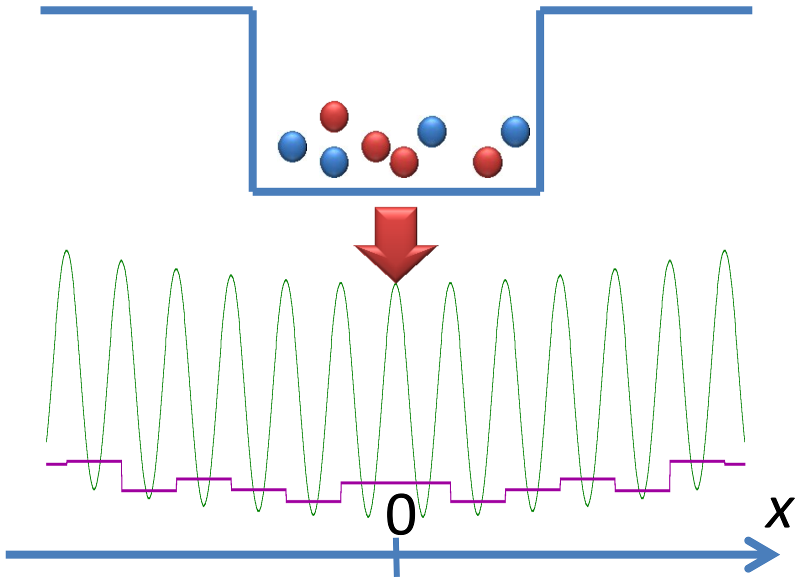
(has been checked for the non-interacting case:

Artuso *et al.*: PRL **68**, 3826 (1992); Piechon *et al.*: PRL **76**, 4372 (1996))

(d_H ~ 1/2 at U=0; d_H = 1 if not fractal)

Setup

- Optical lattice + incommensurate potential (Aubry-André model)
- On-site attractive interaction
- Initially trapped in a box potential <u>without</u> q.p. potential (initial condition does not depend on λ)



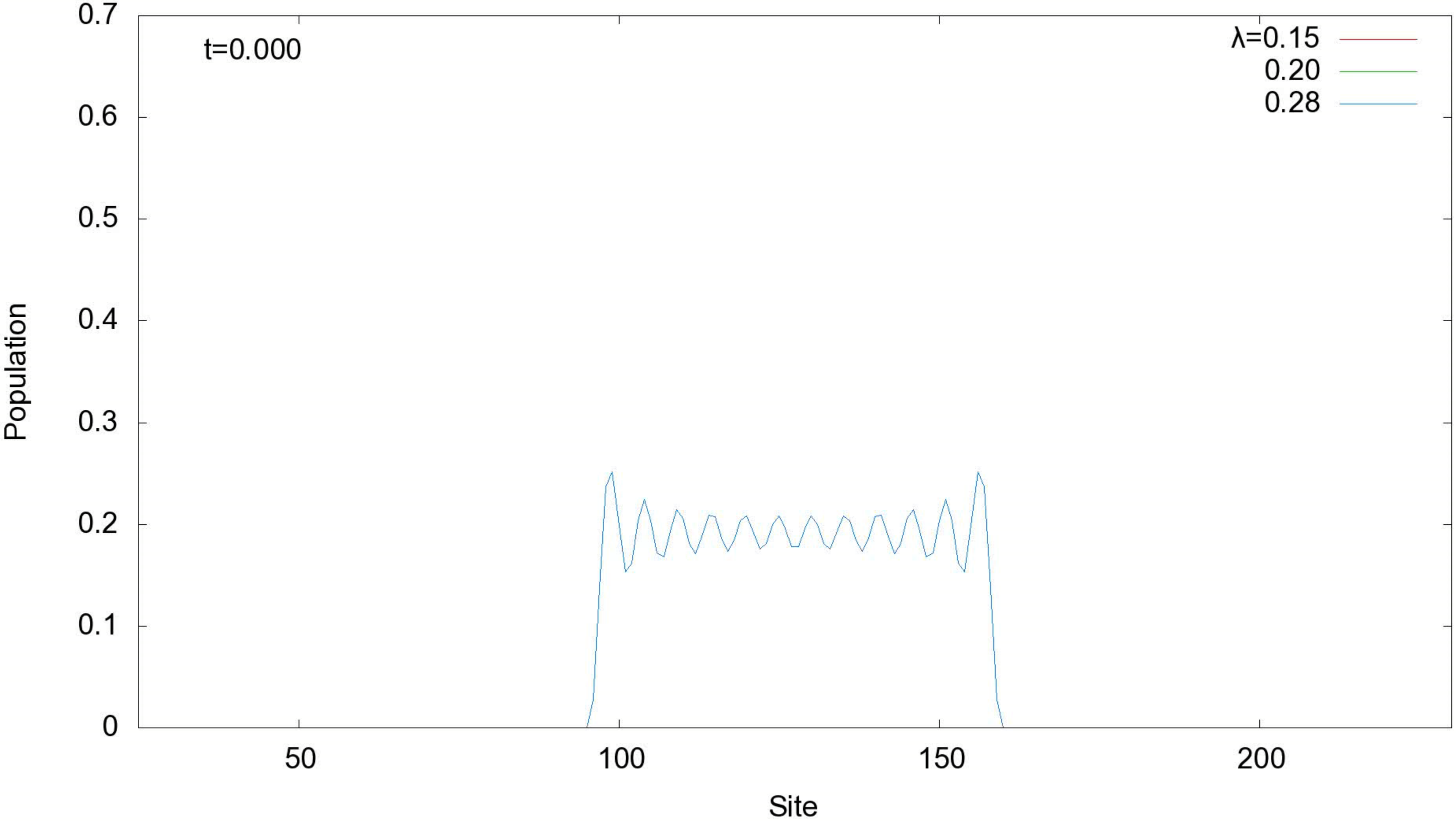
- Remove the box potential and switch the incommensurate potential on: what happens?

➔ Study by time-dependent DMRG for Hubbard model

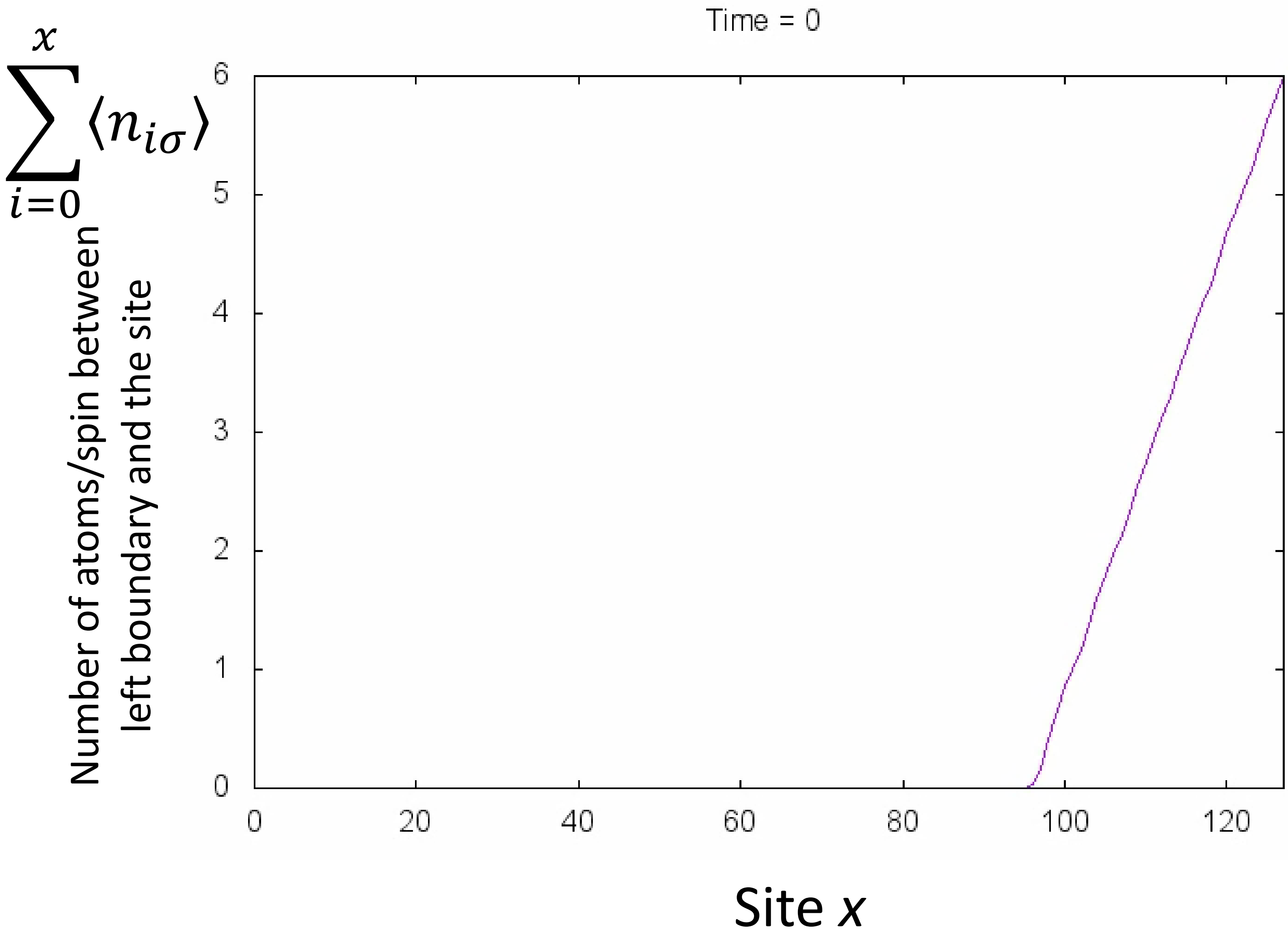
Example: U=-10 (λ_c~0.17)

$\langle n_{x\sigma} \rangle$

12 particles per spin



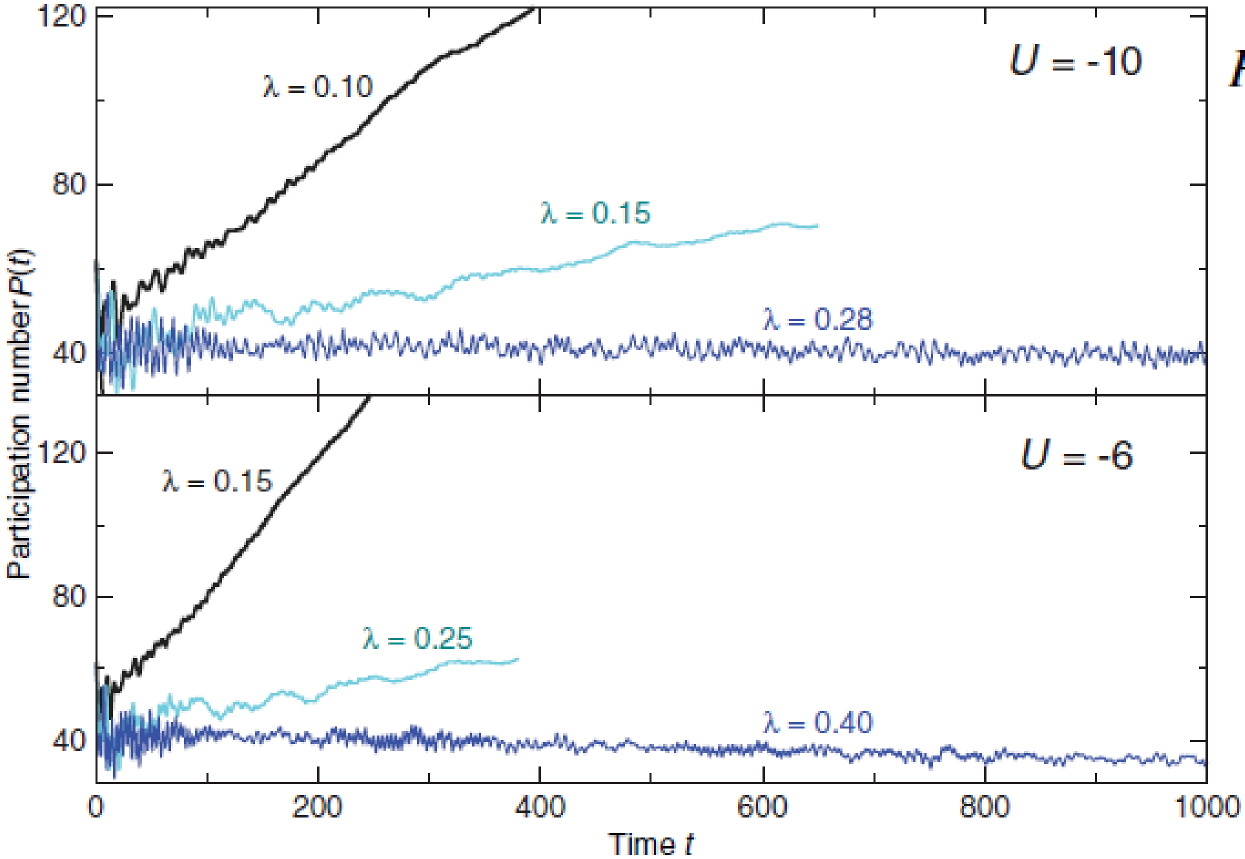
Example: U=-10 (λ_c~0.17)



U=-10
λ=0.15,
0.20,
0.28
(λ_c~0.17)

Tezuka and García-García: PRA **85**, 031602 (R) (2012)

Participation number



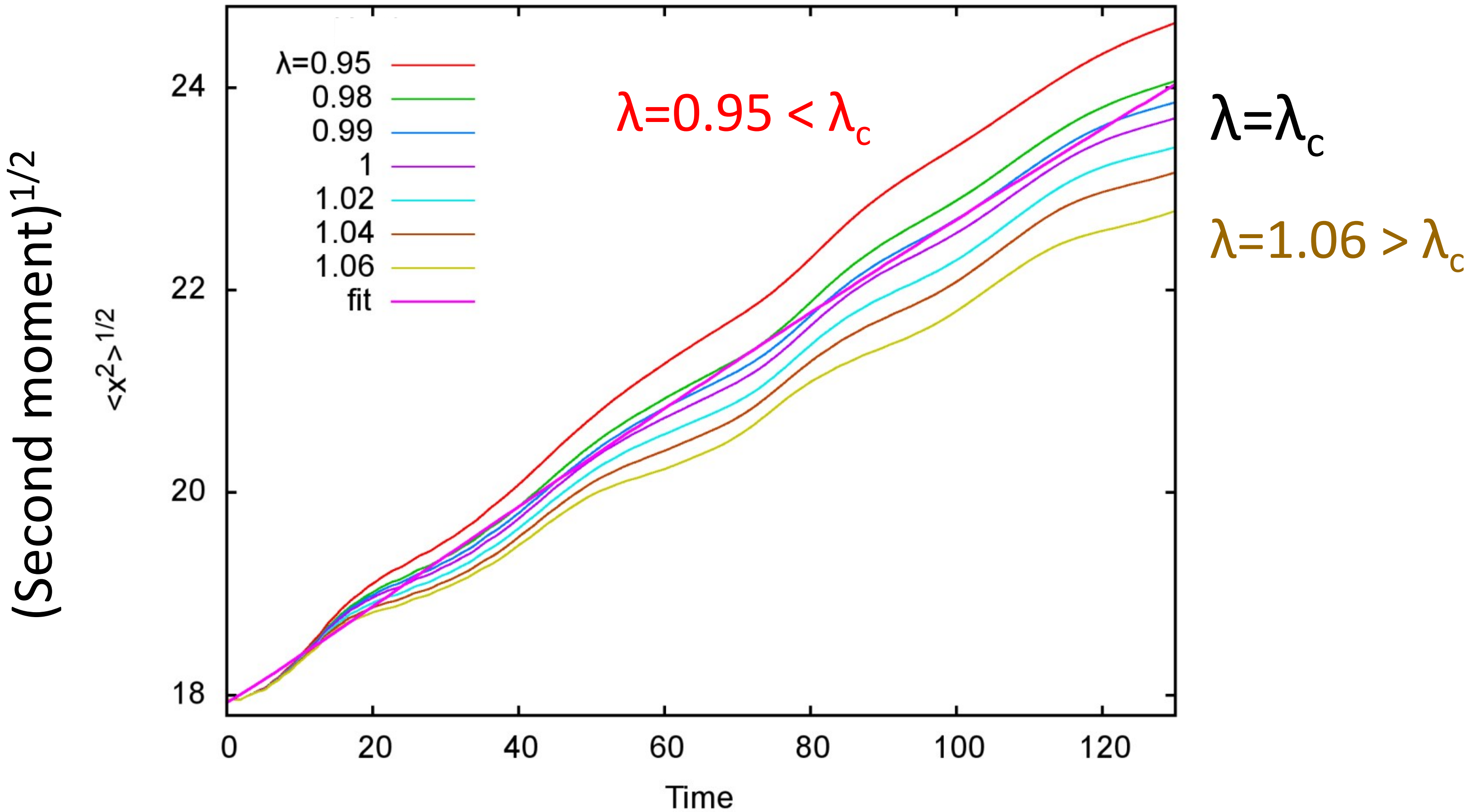
$$P(t) \equiv \frac{\left(\sum_i \langle \Psi(t)|\hat{n}_i|\Psi(t)\rangle\right)^2}{\sum_i \langle \Psi(t)|\hat{n}_i|\Psi(t)\rangle^2}$$

Estimation of the number of sites occupied

➔ Shows complete localization for $\lambda >> \lambda_c$

Tezuka and García-García: PRA **85**, 031602 (R) (2012)

Second moment: U=-1



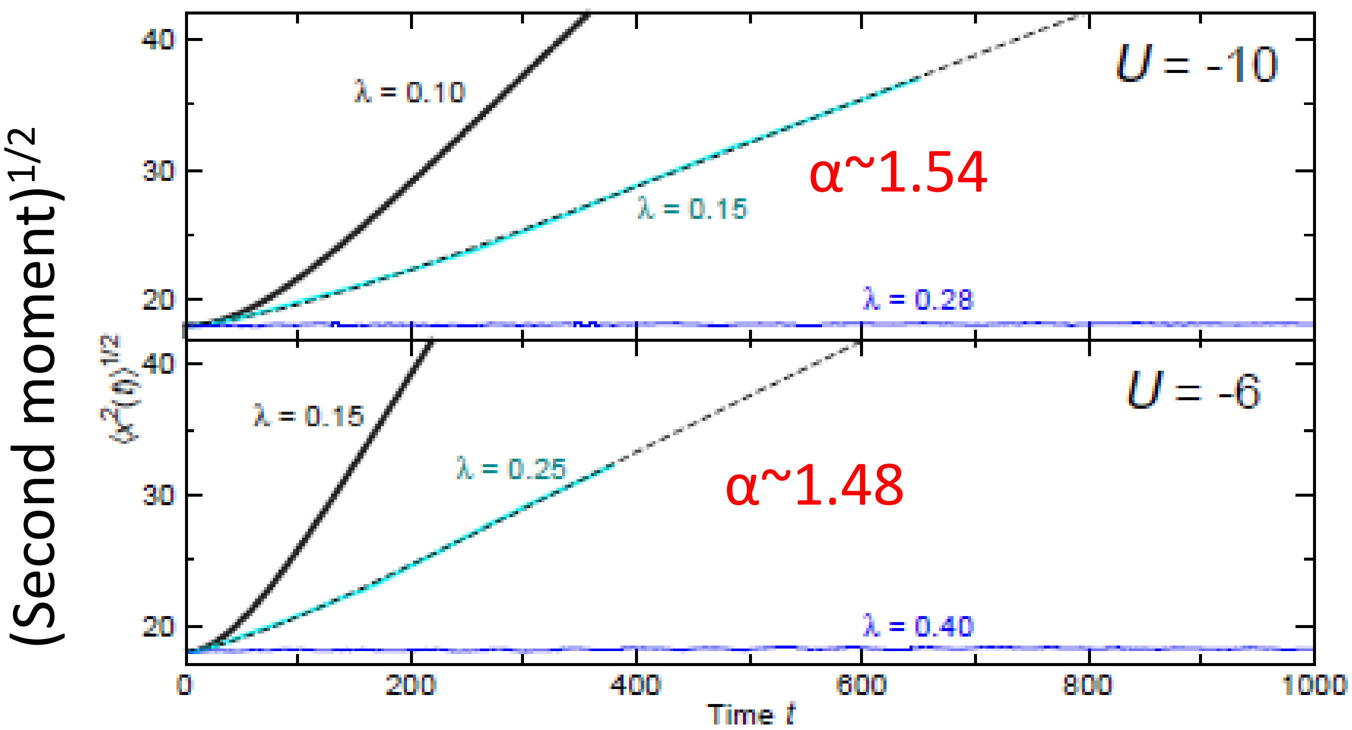
$<x^2(t)>$ fit by $x_0\sqrt{1+(t/t_0)^{\alpha}}$

α ~ 1.06 (larger than α=1 for U=0) at transition point

Tezuka and García-García: PRA **85**, 031602 (R) (2012)

Larger *U*: longer timescale needed

Fit by $x_0\sqrt{1+\left(t/t_0\right)^{\alpha}}$



Value of α at transition increasing as |*U*| increases:
anomalous exponent! (between random walk and ballistic)

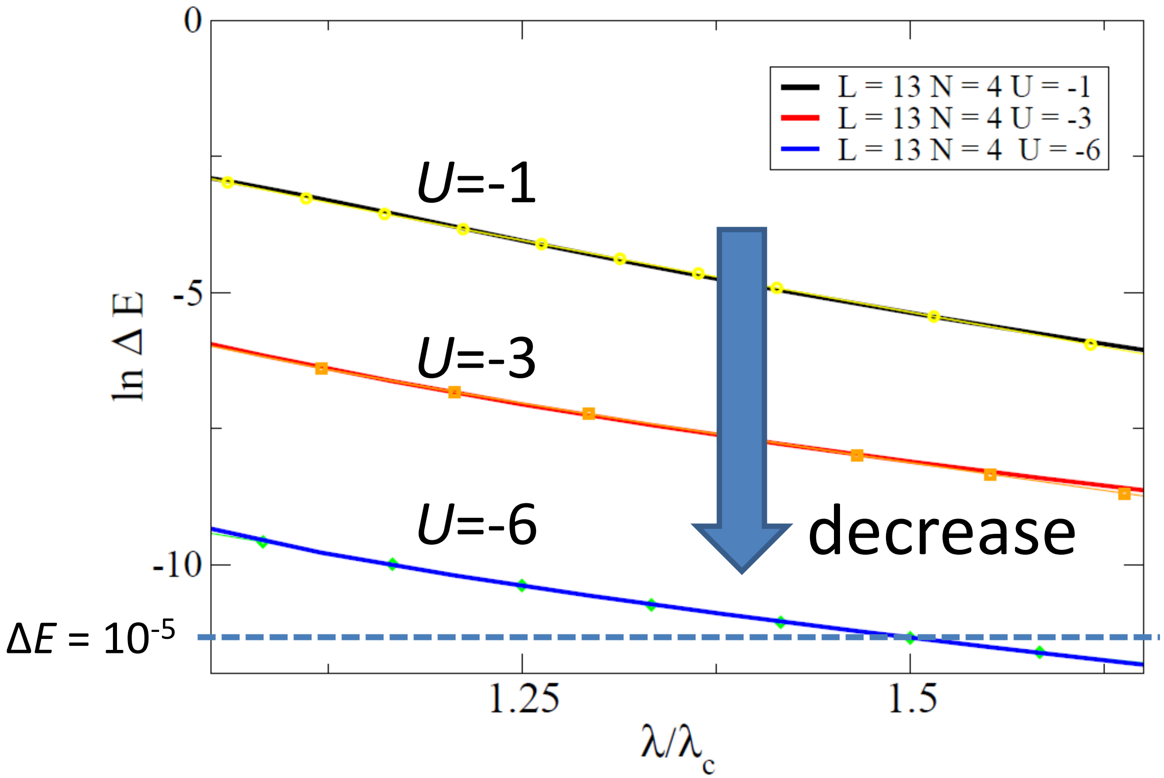
Scaling of localization length

Localization length ξ should diverge as $|\lambda-\lambda_c|^{-\nu}$ as MIT is approached from insulator side (ν=1 at U=0; ν=1/2 in mean field limit)

Sensitivity of the ground state energy to a change of boundary conditions (b.c.)

$\Delta E = |E_P - E_A| \propto e^{-L/\xi}$ in the insulator side ($\lambda > \lambda_c$) of the transition

$E_{P(A)}$: ground state energy for periodic (antiperiodic) b.c.



fit with

$\xi \propto |\lambda - \lambda_c|^{-\nu}$

$\ln \Delta E \sim -L\,|\lambda - \lambda_c|^{\nu}$

1D system: $\alpha/2 = d_H$

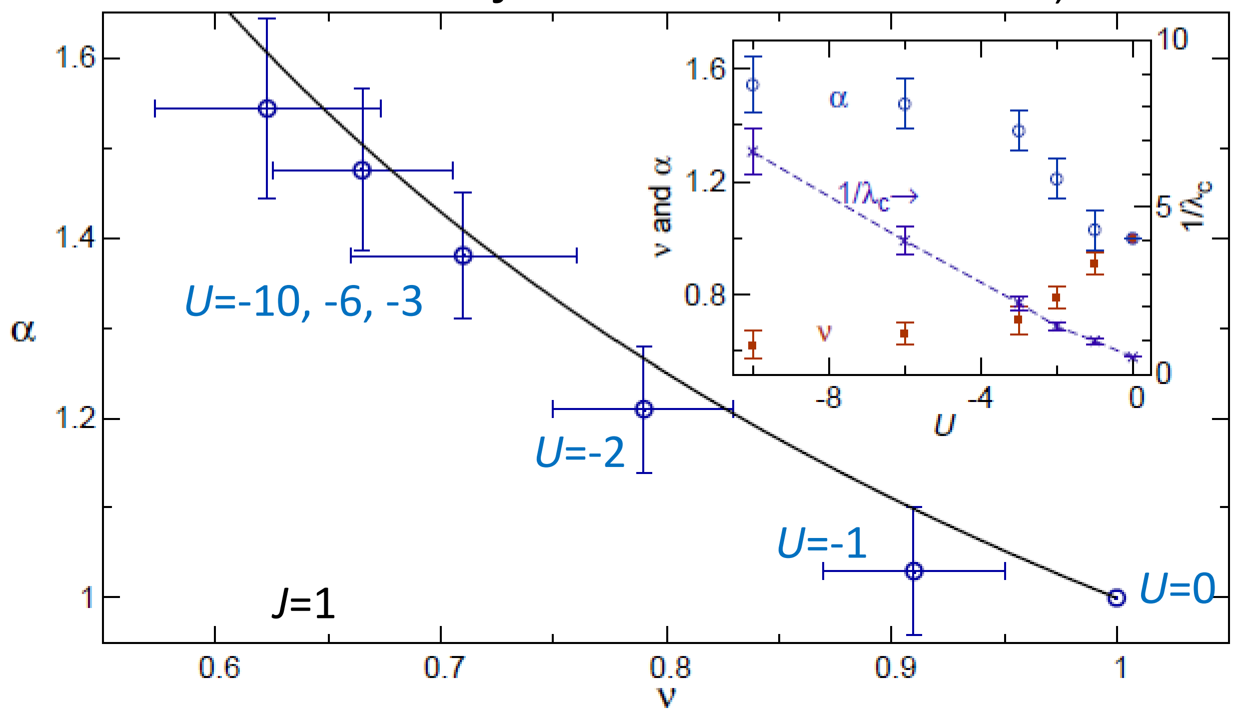
Our conjecture from phenomenological arguments: $\nu\alpha$=1

Tezuka and García-García: PRA **85**, 031602 (R) (2012)

Exponents

$\nu\alpha = 1$: our conjecture

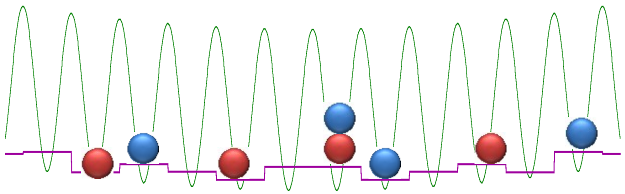
12+12 fermions, 64→160 or 256 sites



Exponent of localization length (static property)

➔ α indeed increases at least up to $|U|$ ~ bandwidth (=4J) while ν decreases; $\nu\alpha = 1$?

Tezuka and García-García: PRA **85**, 031602 (R) (2012)



Summary (2)

(Bichromatic lattice)

Modulated 1D system, $U < 0$, at metal-insulator transition

	$\|U\| \to 0$	intermediate $\|U\|$	$\|U\| \to \infty$
Diffusion $\langle x^2(t)\rangle \propto t^\alpha$	$\alpha=1$ brownian motion	**increases** as $\|U\|$ increases	α~2? ballistic motion?
Hausdorff dimension of the spectrum d_H	$d_H=0.5$ see *e.g.* Artuso *et al.*: PRL **68**, 3826 (1992)	One parameter scaling $\alpha = 2d_H$ at MIT	d_H~1? Not fractal?
Localization length close to transition $\xi \propto \|\lambda - \lambda_c\|^{-\nu}$	$\nu=1$	**decreases** as $\|U\|$ increases	**(1) λ_c strongly suppressed! (~1/|*U*|)** ν~1/2?

Our conjecture: $\nu\alpha=1$

→**Anomalous diffusion** in modulated, interacting 1D Fermi gas observed; interesting relation between the **dynamic** and **static** behavior

Conclusion

- Static and dynamic behavior of Fermi cold atom gases in 1D inhomogeneous potential – DMRG study
  - Quasiperiodic modulation

    MT and A. M. García-García: PRA **82**, 043613 (2010)
    - Can enhance condensation for weak attraction
    - Trap-release dynamics close to metal-insulator transition: anomalous diffusion observed

    MT and AMG: PRA **85**, 031602 (R) (2012)

Other recent works

MT and N. Kawakami: PRB **85**, 140508 (R) (2012)

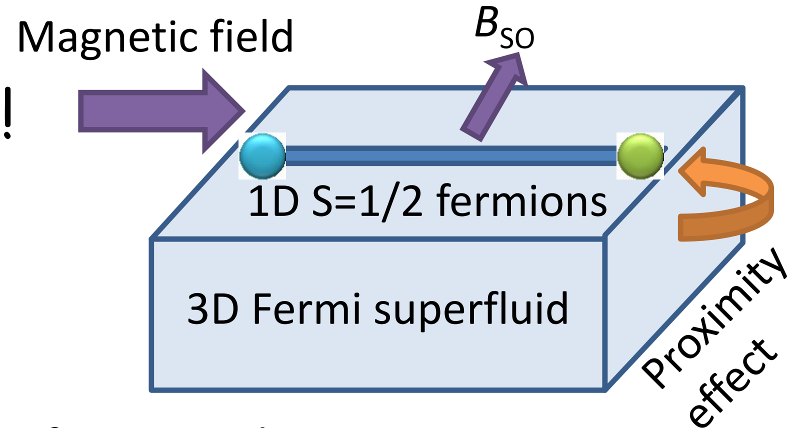
- 1D topological superconductor in quasiperiodically modulated systems

J. Ozaki, MT, and N. Kawakami: PRA **86**, 033621 (2012)

- Collision of spin clusters: more atoms pass through than quasi-classically expected = emergent many-body behavior

MT and Norio Kawakami: Phys. Rev. B **85**, 140508(R) (2012)

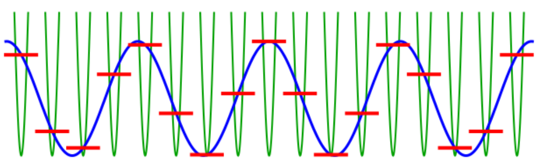
See http://ngf.jp/MF12/ for details!

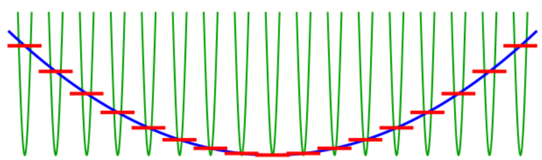


Majorana fermions (MF) at the ends of 1D topological superconductor (TS)

Q. Effect of lattice modulation?

1. Quasiperiodic lattice 2. Harmonic trap

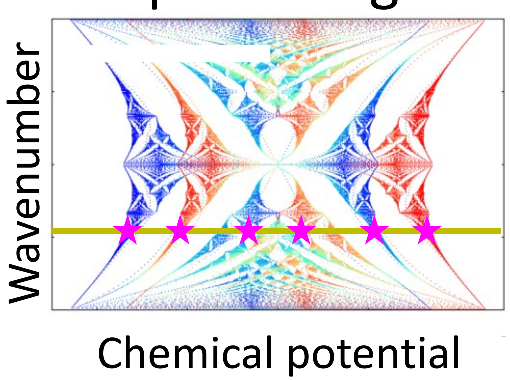




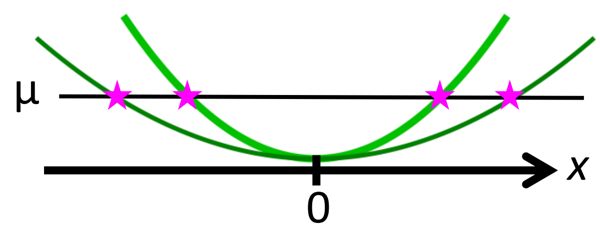
cf. Semiconductor experiment
Mourik *et al.* : Science **336**, 1003 (2012)

BdG and DMRG analysis

Multiple TS regions



MF at effective boundaries



☑ Pair of E=0 BdG eigenstates; DMRG ground state degeneracy

☑ Reduced density matrix: eigenvalue degeneracy

☑ Localized Majorana modes