

Novel macroscopic quantum states in dipolar systems

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Outline

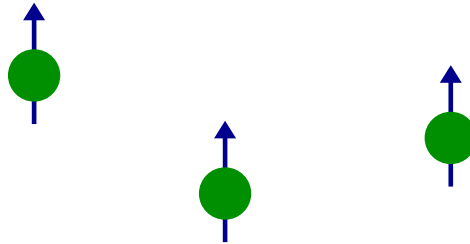
- Introduction
- RF-dressed fermionic polar molecules in 2D
- Topological $p_x + ip_y$ phase
- p -wave and d -wave pairing in bilayered systems
- Conclusions and outlook

Collaborations: N.R. Cooper and J. Levinsen (Cambridge), M. Efremov (Orsay)

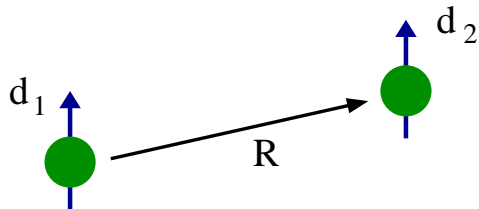
Evora, October 12, 2012

Dipolar gas

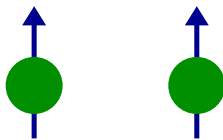
Polar molecules or atoms with a large magnetic moment



Dipole-dipole interaction $V_d = \frac{\vec{d}_1 \vec{d}_2 R^2 - 3(\vec{d}_1 \vec{R})(\vec{d}_2 \vec{R})}{R^5} \sim \frac{1}{R^3}$



long-range, anisotropic



repulsion



attraction

Different physics compared to ordinary atomic ultracold gases

Alkali-atom molecules d from $0.6 D$ for KRb to $5.5 D$ for LiCs

Atoms with large μ

Remarkable experiments with Cr atoms ($\mu = 6\mu_B \Rightarrow d \approx 0.05 \text{ D}$)

T. Pfau group (Stuttgart)

Effects of the dipole-dipole interaction in the dynamics
Stability diagram of trapped dipolar BEC

Spinor physics in chromium experiments at Villetaneuse, B. Laburthe-Tolra

Dysprosium BEC ($\mu = 10\mu_B$, (B. Lev, Stanford))

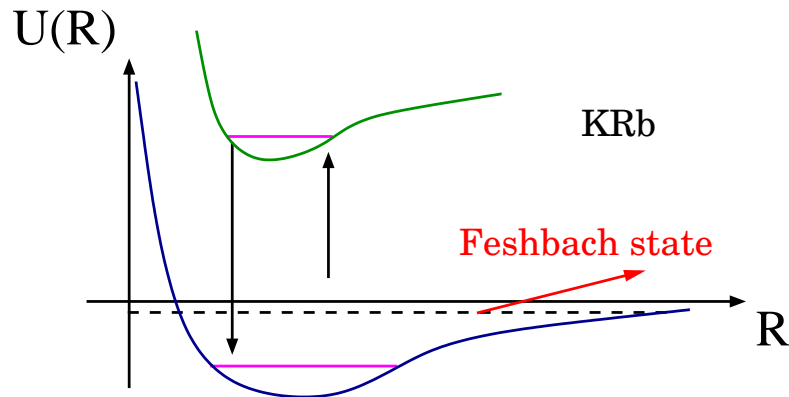
Erbium BEC ($\mu = 7\mu_B$, (F. Ferlaino, Inscbruck))

Polar molecules. Creation of ultracold clouds

Photoassociation

Transfer of weakly bound KRb fermionic molecules to the ground rovibrational state

JILA, D. Jin, J. Ye groups



$$n \sim 10^{12} - 10^{13} \text{ cm}^{-3}$$

$$T \approx 200 \text{ nK} \sim E_F$$

Ground-state LiCs molecules at Heidelberg

Ground-state RbCs molecules in Innsbruck

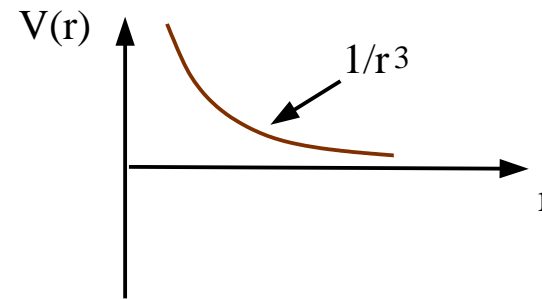
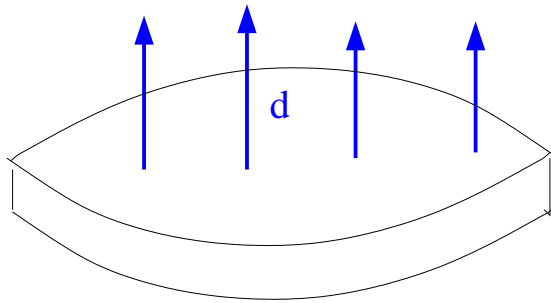
Ground-state KRb bosonic molecules in Tokyo

Experiments with NaK (MIT, Munich, Trento, Hannover) and KCs (Innsbruck) molecules

Ultracold chemistry

Ultracold chemical reactions $\text{KRb} + \text{KRb} \Rightarrow \text{K}_2 + \text{Rb}_2$

Suppress instability \rightarrow induce intermolecular repulsion
For example, 2D geometry with dipoles perpendicular to the plane



Reduction of the decay rate by 2 orders of magnitude at JILA

Select non-reactive molecules, like NaK, KCs, RbCs

What are prospects for novel physics ?

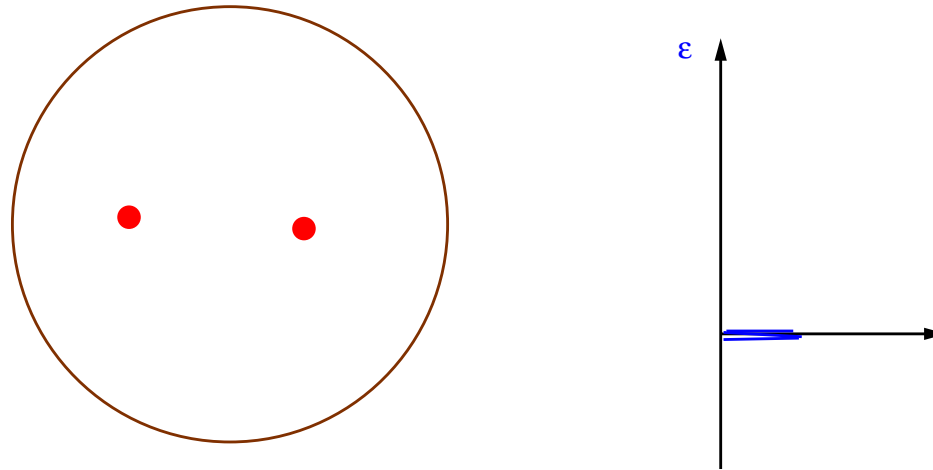
Theoretical studies

- Innsbruck group (P. Zoller, G. Pupillo, M.A. Baranov et al). Large variety of proposals including bilayer systems, Rydberg atoms etc.
- Trento group (S. Stringari et al). Excitation modes etc
- Harvard group (E. Demler, M. Lukin et al). Multilayer systems etc
- Hannover group (L. Santos et al). Spinor and dipolar systems
- Tokyo group (M. Ueda et al) Spinor and dipolar systems
- Cambridge group (N.R. Cooper, Jesper Levinsen). Novel states
- Rice group (H. Pu et al). Excitations and stability etc
- Maryland group (S. Das Sarma et al) Fermi liquid behavior etc
- Taipei group (D.-W. Wang et al)
- Barcelona group (M. Lewenstein et al)

Why single-component fermions are interesting?

Topological aspects of $p_x + ip_y$ state in 2D

Vortices. Zero-energy mode related to two vortices. (Read/Green, 2000)



The number of zero-energy states exponentially grows with the number of vortices $2^{(N_v/2-1)}$

Non-abelian statistics \Rightarrow Exchanging vortices creates a different state!

Non-local character of the state. Local perturbation does not cause decoherence

Topologically protected state for quantum information processing

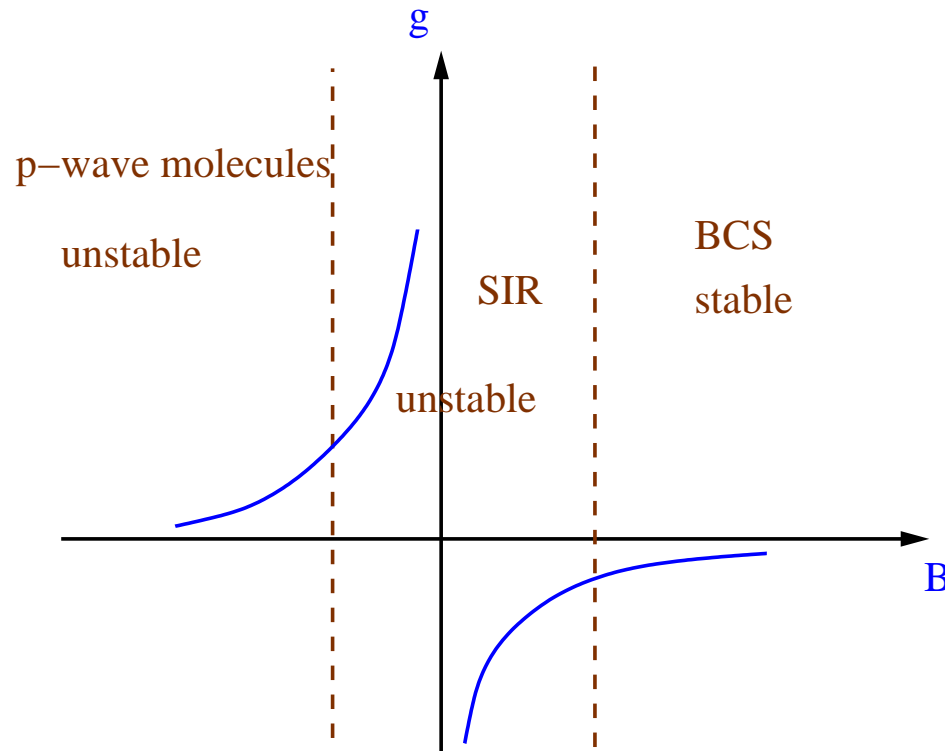
p-wave resonance for fermionic atoms

p-wave resonance Experiments at JILA, ENS, Melbourne, Tokyo, elsewhere

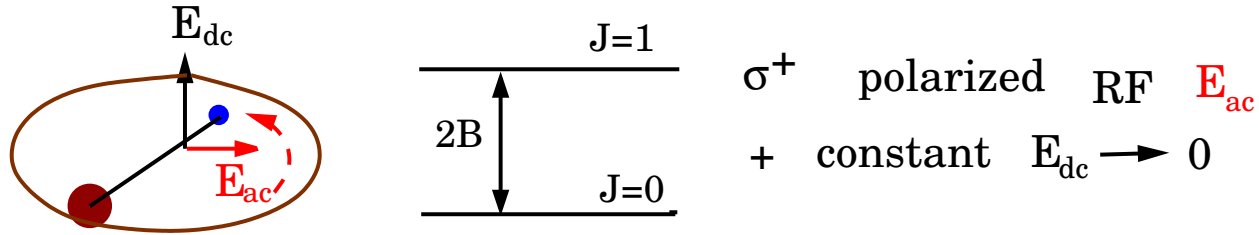
$$\text{BCS} \Rightarrow T_c \sim \exp\left(-\frac{1}{(k_F b)^2}\right) \text{ practically zero}$$

Molecular and strongly interacting regimes \Rightarrow rather high T_c , but collisional instability

Gurarie/Radzihovsky; Gurarie/Cooper; Castin/Jona-Lazinio



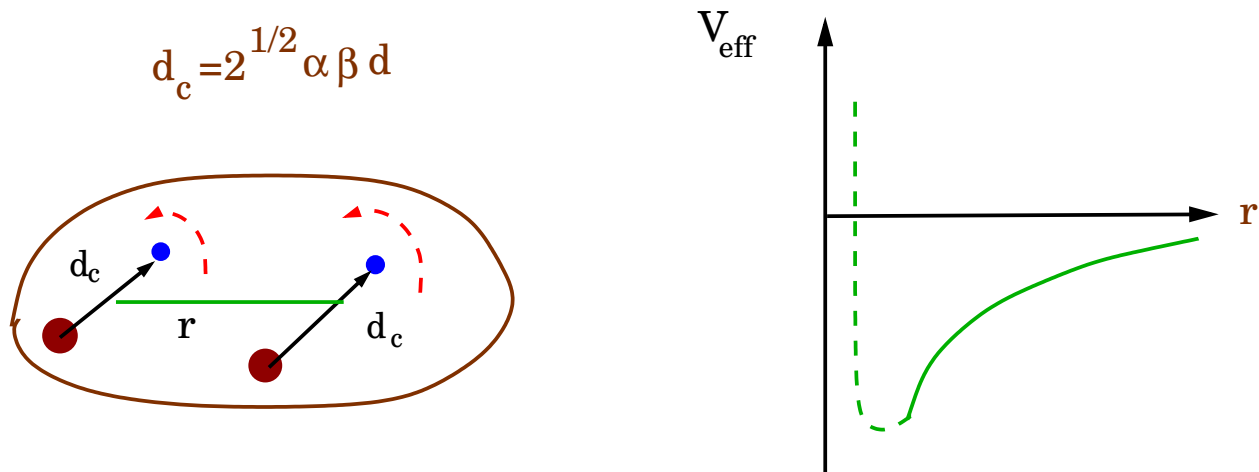
RF-dressed polar molecules in 2D; Gorshkov et al (2008)



Dressed states $|+\rangle = \alpha|0,0\rangle + \beta|1,1\rangle$; $|-\rangle = \beta|0,0\rangle - \alpha|1,1\rangle$

$$\alpha = -\frac{A}{\sqrt{A^2 + \Omega^2}}; \quad \beta = \frac{\Omega}{\sqrt{A^2 + \Omega^2}}; \quad A = \frac{1}{2}(\delta + \sqrt{\delta^2 + 4\Omega^2})$$

Two RFD molecules in 2D. The dipole moment is rotating with RF frequency



Large $r \rightarrow V_{eff} = \langle (1 - 3 \cos^2 \phi) \rangle \frac{d_c^2}{r^3} = -\frac{d_c^2}{2r^3}; \quad r_* = md_c^2/2\hbar^2$

Fermionic RFD molecules. Superfluid transition; Cooper/G.S. (2009)

Fermionic RFD molecules in a single quantum state in 2D

Attractive interaction for the p -wave scattering ($l = \pm 1$)

$$\hat{H} = \int d^2r \hat{\Psi}^\dagger(\mathbf{r}) \left\{ -(\hbar^2/2m)\Delta + \int d^2r' \hat{\Psi}^\dagger(\mathbf{r}') V_{eff}(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}') - \mu \right\} \hat{\Psi}(\mathbf{r})$$

$$\Delta(\mathbf{r} - \mathbf{r}') = \langle V_{eff}(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}) \hat{\Psi}(\mathbf{r}') \rangle$$

Gap equation
$$\Delta(\mathbf{k}) = - \int \frac{d^2k'}{(2\pi)^2} V_{eff}(\mathbf{k} - \mathbf{k}') \Delta(\mathbf{k}') \frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')}$$

$$\epsilon(k) = \sqrt{(\hbar^2 k^2 / 2m - \mu)^2 + |\Delta(k)|^2}; \quad \mu \approx E_F$$

$$T_c \approx E_F \exp(-3\pi/4k_F r_*)$$

$$\Delta(\mathbf{k}) = \Delta \exp(i\phi_k) \quad p_x + ip_y \text{ state } (l = \pm 1)$$

Superfluid transition. Role of anomalous scattering

For short-range potentials should be $V_{eff} \propto k^2$ and $T_c \propto \exp(-1/(k_F b)^2)$

This is the case for the atoms

Anomalous scattering in $1/r^3$ potential \rightarrow Contribution from $r \sim 1/k$

$$V_{eff}(k) = -\frac{8\hbar^2}{3m}(kr_*); \quad |k| = |k'|$$

$$T_c \propto \exp\left(-\frac{1}{\nu(k_F)|V_{eff}(k_F)|}\right); \quad \nu = \frac{m}{2\pi\hbar^2}$$

$$T_C \propto E_F \exp\left(-\frac{3\pi}{4k_F r_*}\right)$$

Second order diagrams \Rightarrow Get a correct preexponential factor?

Transition temperature

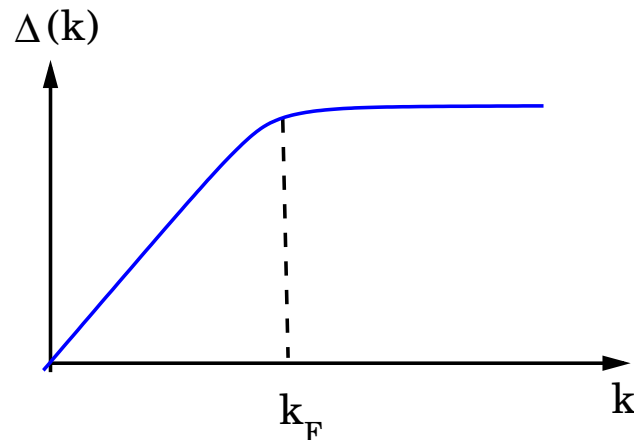
Do better than simple BCS. Reveal the role of short-range physics

Renormalized gap equation

$$\Delta(\mathbf{k}') = - \int f(\mathbf{k}', \mathbf{k}) \Delta(\mathbf{k}) \left\{ \frac{\tanh[\epsilon(k)/2T]}{2\epsilon(k)} - \frac{1}{(E_k - E_{k'} - i0)} \right\} \frac{d^2k}{(2\pi)^2}$$

$\Delta(\mathbf{k}) = \Delta(k) \exp(i\phi_k)$; $f(\mathbf{k}', \mathbf{k}) = f(k', k) \exp[i(\phi_k - \phi_{k'})]$ scattering amplitude

$$\Delta(k) = \Delta(k_F) f(k, k_F) / f(k_F, k_F)$$



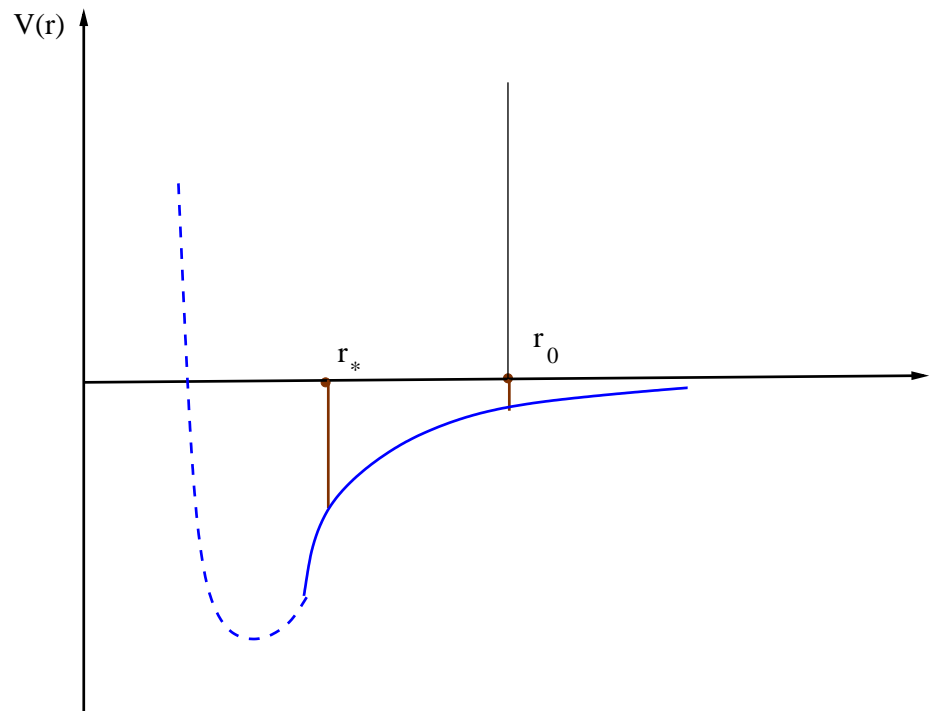
2D scattering in the potential with a $1/r^3$ tail

Scattering amplitude. No transparent exact solution for a finite k

Asymptotic method for slow scattering ($kr_* \ll 1$)

Divide the range of distances into two parts, $r < r_0$ and $r > r_0$

The distance r_0 is such that $r_0 \gg r_*$, but $kr_0 \ll 1$



$r < r_0$ Match exact zero-energy with free finite- k solution at $r = r_0$: $f \Rightarrow (\pi/2)d^2 r_* k^2 \ln k$

$r > r_0$ interaction as perturbation: $f = -(8\pi/3)d^2 k + (\pi/2)d^2 r_* k^2 \ln k$

Related results for the off-shell scattering amplitude

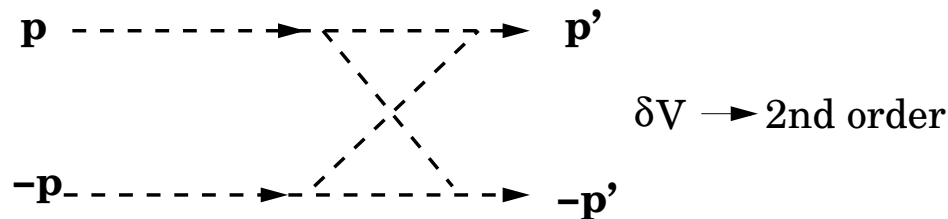
Manipulate T_c ?

$$f(k', k) = -\pi d^2 k_F \left(\frac{1}{2}, -\frac{1}{2}, 2, \frac{k^2}{k'^2} \right); \quad k \leq k'; \quad kr_* \ll 1$$

Include k^2 -term $f = \frac{1}{2} \pi d^2 r_* k^2 \ln[kr_* u]$

$$T_c = \frac{2e^C}{\pi} E_F \exp \left\{ -\frac{3\pi}{4k_F r_*} - \frac{9\pi^2}{64} \ln[k_F r_* u] \right\}$$

Take into account second-order Gor'kov-Melik-Barkhudarov processes



$$\Delta(\mathbf{k}) = - \int \frac{d^2 k'}{(2\pi)^2} f(\mathbf{k}, \mathbf{k}') \left\{ \frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')} - \frac{1}{2(E_{k'} - E_k)} \right\} \Delta(\mathbf{k}') \\ - \int \frac{d^2 k'}{(2\pi)^2} \delta V(\mathbf{k}, \mathbf{k}') \frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')} \Delta(\mathbf{k}')$$

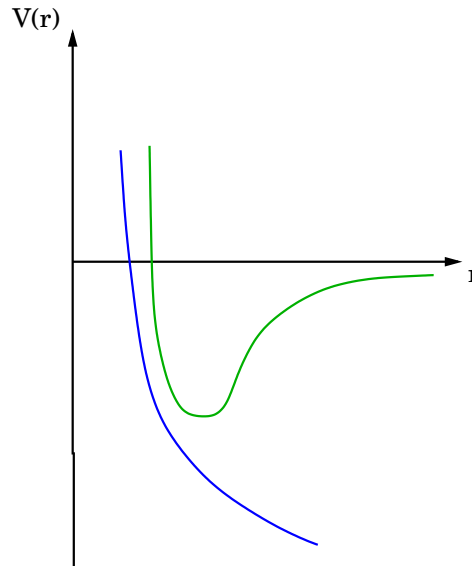
$$T_c = \kappa E_F^{0.3} E_*^{0.7} \exp \left\{ -\frac{3\pi}{4k_F r_*} \right\}; \quad E_* = \frac{\hbar^2}{2mr_*^2} \gg E_F$$

κ depends on short-range physics and can be varied within 2 orders of magnitude

Collisional stability and T_c

p -wave atomic superfluids: BCS $\Rightarrow T_c \rightarrow 0$ Resonance \Rightarrow collisional instability

Polar molecules \Rightarrow sufficiently large T_c and collisional stability



$$\alpha_{in} = A \frac{\hbar}{m} (kr_*)^2; \quad A \Rightarrow 10^{-3} - 10^{-4} \quad \alpha_{in} \rightarrow (10^{-8} - 10^{-9}) \text{ cm}^2/\text{s}$$

$$\text{LiK molecules} \rightarrow d \simeq 3.5 \text{ D} \quad r_* \approx 4000a_0$$

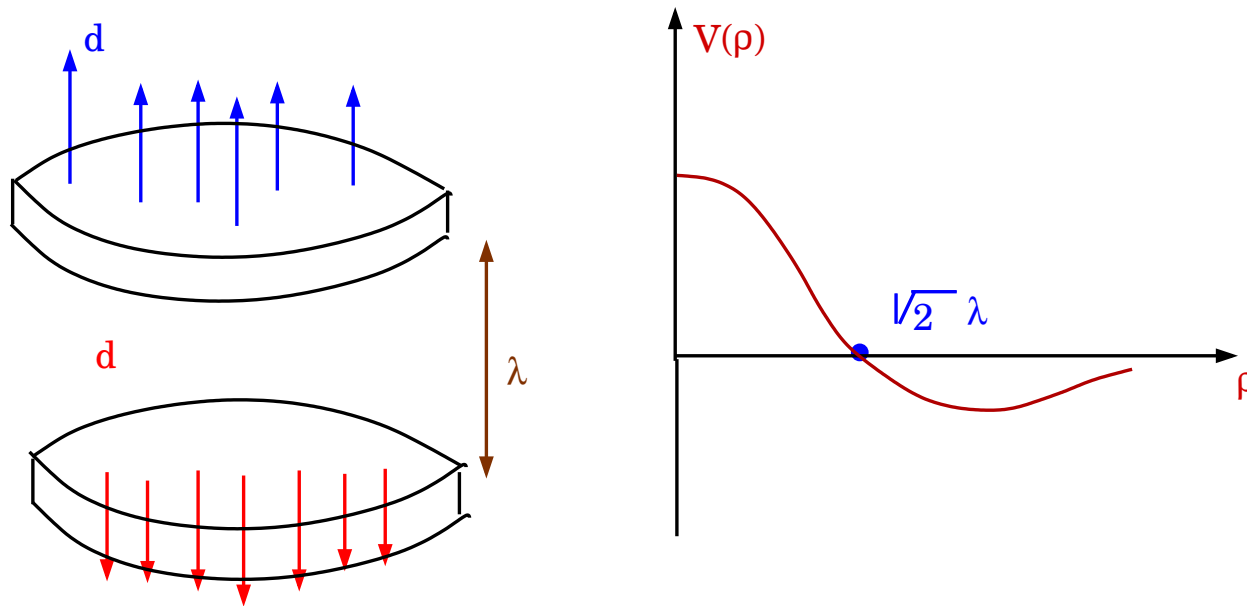
$$n = 2 \times 10^8 \text{ cm}^{-2} \Rightarrow E_F = 2\pi\hbar^2 n/m = 120 \text{ nK} \quad T_c \approx 10 \text{ nK}; \quad \tau \sim 2\text{s}$$

Bilayer system of \uparrow and \downarrow dipoles

Put $J = 0$ molecules in one layer and $J = 1$ in the other

Apply an electric field perpendicularly to the layers

Slightly non-uniform to prevent resonant dipolar flips leading to a rapid decay



Always a bound state of \uparrow and \downarrow dipoles

$$\beta \lesssim 1 \Rightarrow \epsilon_b \simeq \frac{\hbar^2}{m\lambda^2} \exp[-8/\beta^2 - 8/\beta - (5 + 2C - 2 \ln 2)]$$

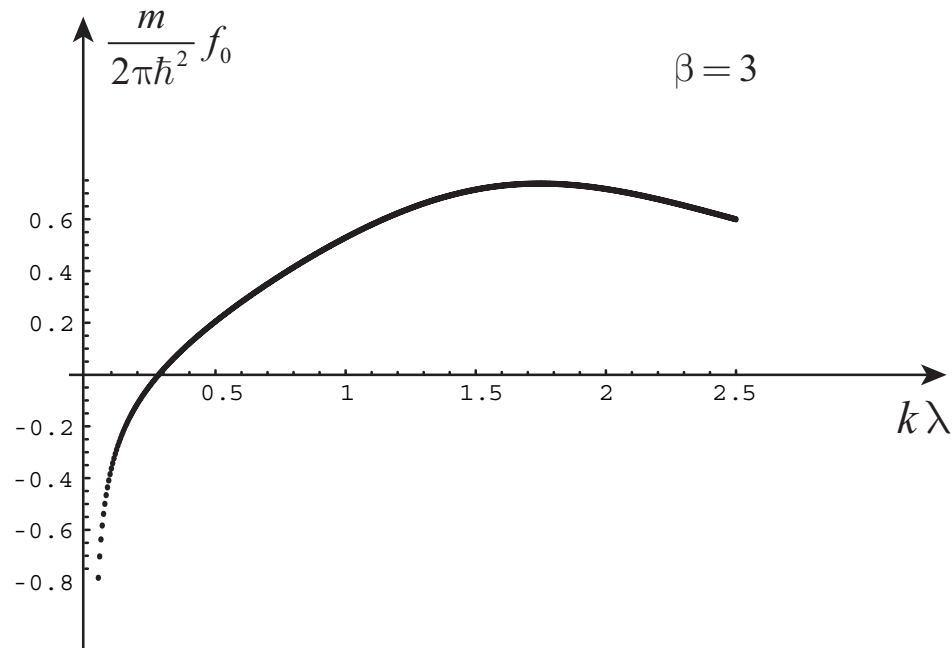
$$\beta = r_*/\lambda$$

Interlayer interaction. Scattering amplitudes

$$s\text{-wave amplitude } k \rightarrow 0 \quad f_0(k) = \frac{4\pi\hbar^2}{m \ln(\epsilon_b/\epsilon)} + \frac{8\hbar^2}{m} kr_*$$

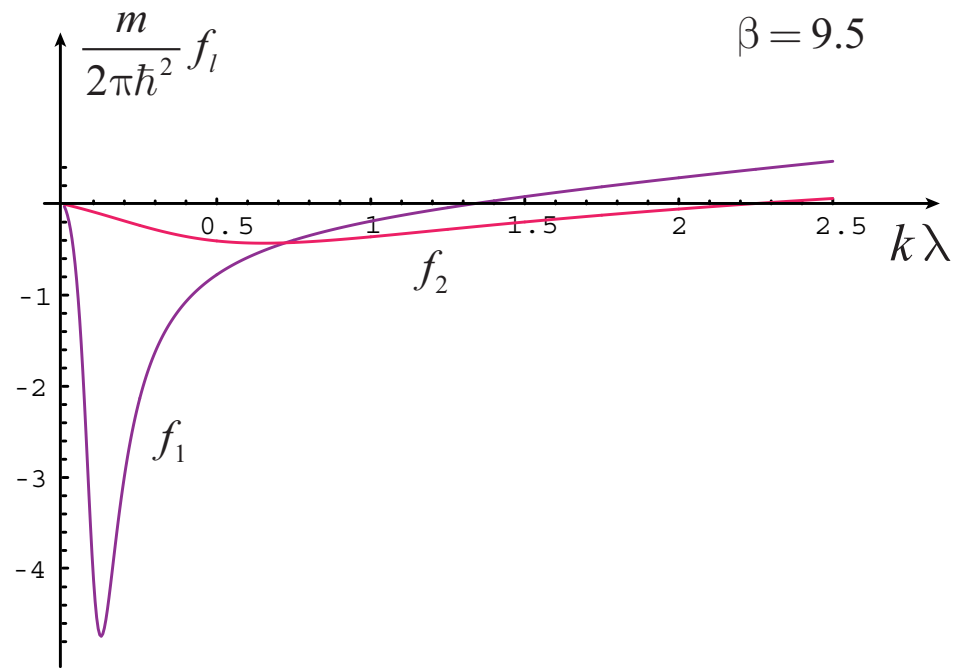
$$\epsilon = \hbar^2 k^2 / m \quad r_* = md^2 / \hbar^2$$

$f_0 > 0$ for reasonable k . No interlayer superfluid pairing



Interlayer interaction. Scattering amplitudes

p -wave and d -wave amplitudes are < 0



Interlayer *p*-wave and *d*-wave pairing

For $k_F r_* \gtrsim 1$ the effective mass significantly decreases

$$\text{Transition temperature } T_c \sim E_F^* \exp\left(\frac{2\pi\hbar^2}{m_* |f(k_F)|}\right)$$

The quasiparticle Fermi energy increases

Compensate the decrease of m_* in the exponent by increasing d^2 and, hence, f

p-wave interlayer superfluid with $T_c \sim$ tens of nK

d-wave superfluids with $T_c \sim$ nK. Analogy with high-temperature superconductors

LiCs with $n > 10^9 \text{ cm}^{-2}$

Conclusions

Creation of ultracold polar molecules opens wide avenues to make new quantum states

- $p_x + ip_y$ topological state for identical fermions
- p -wave and d -wave interlayer superfluids in bilayered fermionic dipolar systems