# Phase Separation and FFLO phases in ultra-cold gas of fermionic atoms with attractive potential in a one-dimensional trap

P. Schlottmann

Florida State University
Dept. of Physics and NHMFL
Tallahassee, Florida

In collaboration with Andrei Zvyagin

Supported by the US Department of Energy
Grant No. DE-FG02-98ER45707

# I. Motivation

(Liao et al, Nature 2010)

- Array of 1D tubes within a 2D optical lattice
  Lattice potential : $V = V_0 [\cos^2(kx) + \cos^2(ky)]$
  $V_0$ is potential depth and $k=2\pi/\lambda$ with $\lambda$ being the laser wavelength

- Evaporative cooling
  Confinement-induced (Feshbach-type) resonance [Bergman et al, PRL 2003]
  Interaction between fermions can be fine-tuned, $\delta(z)$-potential with interaction strength

$$g_{1D} = \frac{2\hbar^2 a}{\mu a_\perp^2} \frac{1}{1 - Ca/a_\perp} \qquad C=1.4603$$

  $a$ is 3D scattering strength , $a_\perp$ is the confinement length scale

- If $Ca/a_\perp > 1$ the potential is attractive

- Fermionic atoms (half-integer total angular momentum, nuclear+ electronic)

S=1/2 $^{6}$Li
S=3/2 $^{9}$Be , $^{135}$Ba , $^{137}$Ba , $^{53}$Cr
S=5/2 $^{173}$Yb
S=7/2 $^{43}$Ca
S=9/2 $^{40}$K , $^{87}$Sr

# II. Statement of problem

Hamiltonian:

$$\mathcal{H} = -\sum_{i=1}^{N_p} \frac{\partial^2}{\partial x_i^2} - 2|c| \sum_{i<j} \delta(x_i - x_j)$$

- Gaudin, C.N. Yang (1967) : S=1/2
- Sutherland (1968) : arbitrary spin S
- Takahashi (1970) : ground state for attractive potential, arbitrary S
- Schlottmann (1993, 1994) : classification of all states and thermodynamics
- Orso (2007) : numerical solution S=1/2
- Guan et al (2009) : numerical solution S=3/2 (fixed density)
- Schlottmann and Zvyagin (2012) : numerical solution for S=3/2,…,9/2

Classification of states for attractive potential at T=0

- S=1/2: three states: (i) Cooper pairs, (ii) spin-polarized atoms, (iii) mixed phase
- arbitrary S: (i) single (pure) phases consisting of bound states of up to N=2S+1 states (generalized Cooper pairs), (ii) mixed phases of up to N phases

Note: the model we consider is SU(N) symmetric

This assumes that the 3D scattering length is the same for all pairs of atoms, independently of their spin projections. This is the simplest model for interacting fermions in 1D.

# III. Bethe ansatz equations

- Integrable model: many-particle scattering process can be factorized into two-particle processes; the order of the individual scattering processes is arbitrary (Yang-Baxter relation for scattering matrix)
- N=2S+1 successive Bethe ansätze, each step eliminates one color.
- Each Bethe ansatz yields one set of generalized momenta (rapidities), ξ, for the bound states.
- 2S+1 energy bands, one for each class of bound states, $\varepsilon^{(l)}(\xi)$, $l$=1,…,2S
  $l$ is the "string" length, $l$+1 particles in bound state.

Integral equations for energy potentials:

$$\varepsilon^{(l)}(\xi) = D_l(\xi) - \sum_{q=0}^{2S} \int_{-B_q}^{B_q} d\xi' K_{lq}(\xi - \xi')\varepsilon^{(q)}(\xi')$$

$$K_{lq}(\xi) = \int \frac{d\omega}{2\pi} \exp[i\xi\omega - (l + q - p_{l,q})|\omega c|/2] \frac{\sinh[(p_{l,q}+1)\omega c/2]}{\sinh(\omega c/2)}$$

$$D_l(\xi) = (l+1)\left[\xi^2 - \frac{l(l+2)}{12}c^2 - \mu + \frac{2S-l}{2}H\right]$$

$$p_{l,q} = \min(l,q) - \delta_{l,q} \quad , \quad K_{lq}(\xi) = K_{ql}(\xi)$$

$\mu$ is chemical potential and $H$ Zeeman splitting; Fermi points $\pm B_l$ such that $\varepsilon^{(l)}(\pm B_l) = 0$.

- Scaling with c: $\varepsilon^{(l)} \to \varepsilon^{(l)}/c^2$ , $\mu \to \mu/c^2$ , $H \to H/c^2$ , $B_l \to B_l/|c|$ , $\xi \to \xi/|c|$ universality
- Kernel of integral equations specific to SU(N) symmetry and attractive interactions
  Same for Anderson impurity for U → ∞ and supersymmetric t-J model

# IV. Phase diagrams

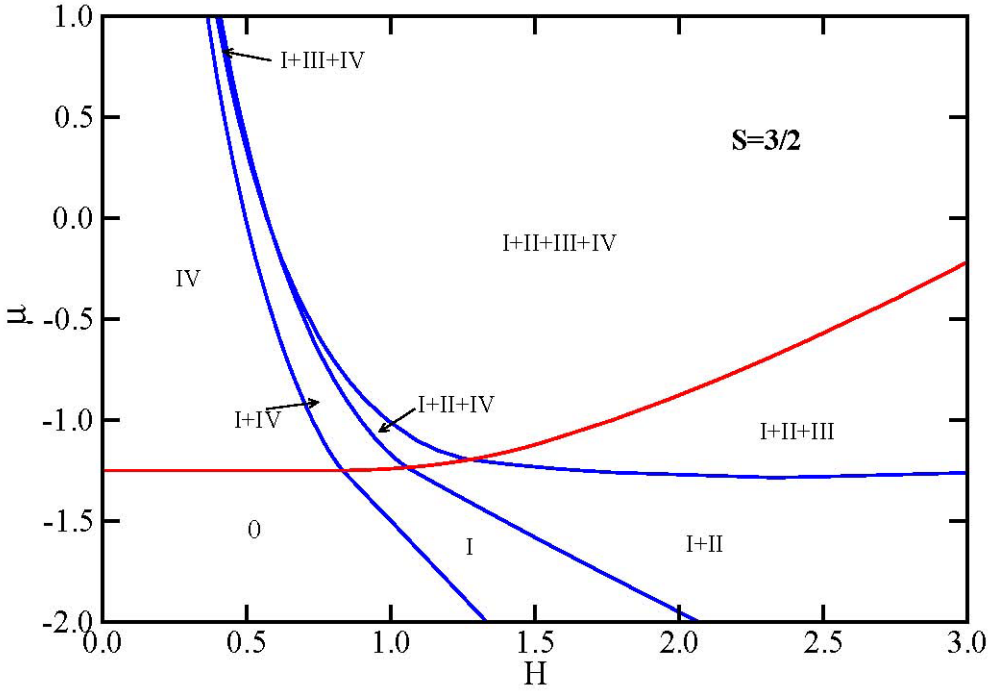
Phase boundaries: $\varepsilon^{(l)}(B_l=0) = 0$

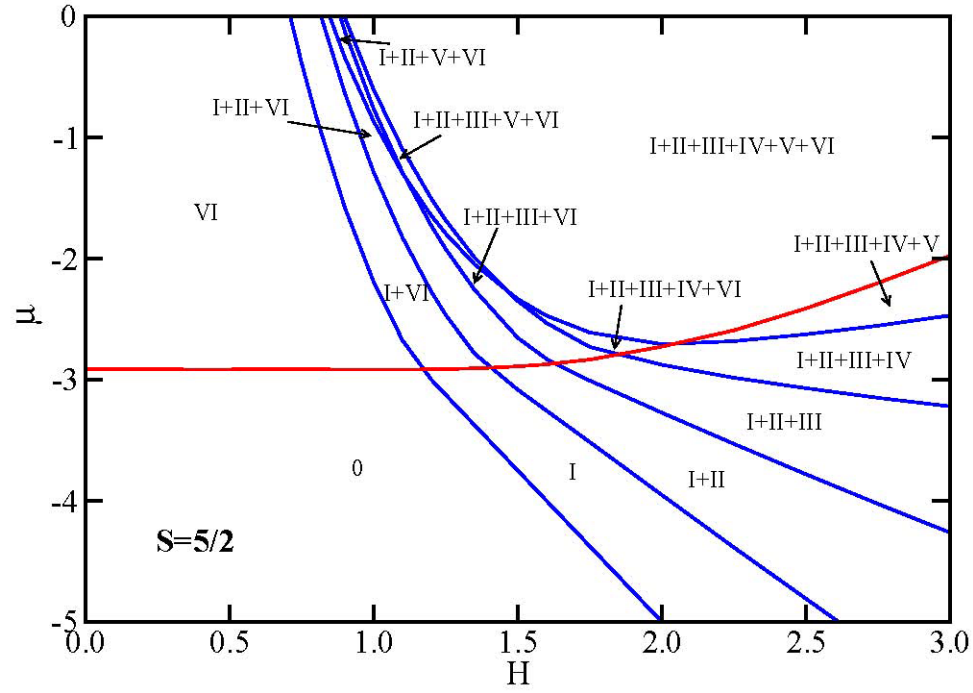
2S+1 phase boundaries

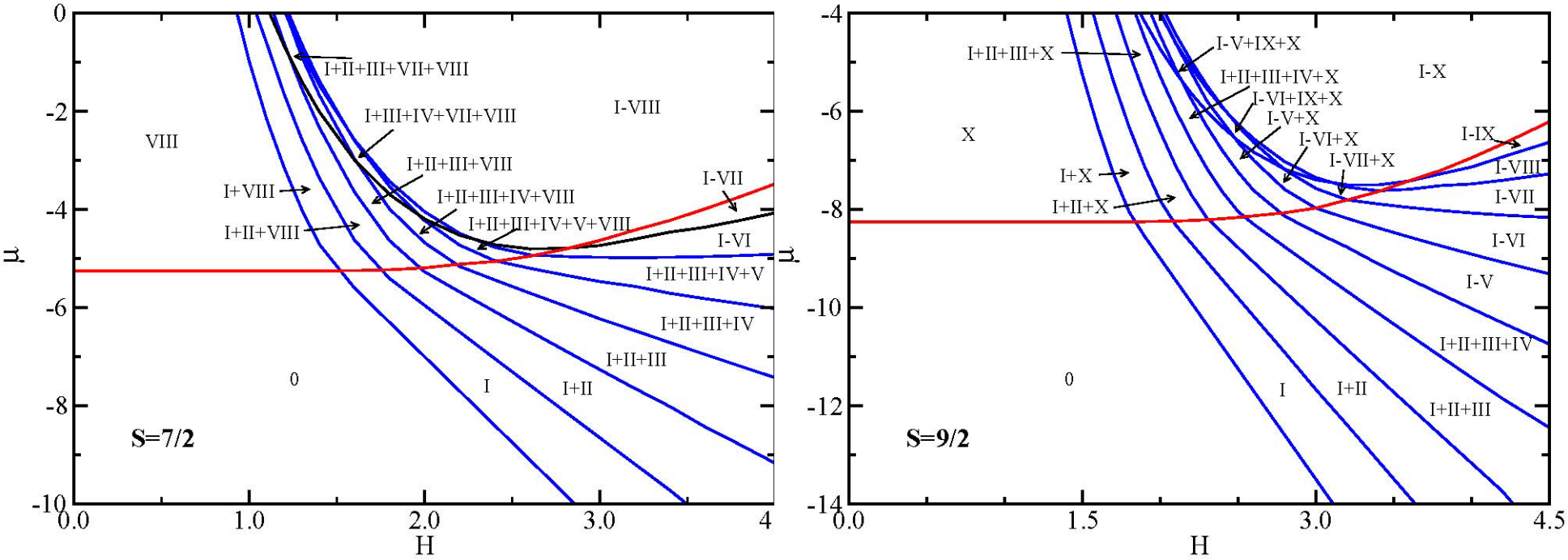
Level crossings rather than phase transitions

No long range order

Roman numbers denote the number of particles of bound state






Red curves: boundary for bound states of (2S+1) atoms; driving term has no magnetic field dependence; separates regions with and without (2S+1) atom clusters; horizontal curve (μ ~ const)

Blue curves: boundaries for bound states of less than (2S+1) atoms; driving terms depend on magnetic field; almost vertical curves (H ~ const)

Same pattern expected for larger spins

# V. Density profiles and phase separation

- Densities of rapidities $\rho^{(l)}(\xi)$, $l = 0, \cdots, 2S$, satisfy integral equations with the same kernel as the energy potentials $\varepsilon^{(l)}(\xi)$ but with a different driving term

$$D_l = (l+1)/(2\pi)$$

- Number of bound states of $(l+1)$ particles per unit length

$$n_l = \int_{-B_l}^{B_l} d\xi \; \rho^{(l)}(\xi)$$

- density of total number of particles $\quad N_t = \sum_{l=0}^{2S}(l+1)n_l$

magnetization density $\quad M/L = (1/2)\sum_{l=0}^{2S}(l+1)(2S-l)n_l$

- Local density approximation: diameter of tube gradually changes with position from center of trap to its boundaries. Harmonic confinement potential of frequency $\omega_{ho}$

$$\mu(x) + \tfrac{1}{2}m\omega_{ho}^2 x^2 = \mu(0) \quad , \quad \frac{x}{L/2} = \sqrt{\frac{\mu(x)-\mu(0)}{\mu(L/2)-\mu(0)}} \quad , \quad L = \text{length of trap}$$

Phase separation along the tube

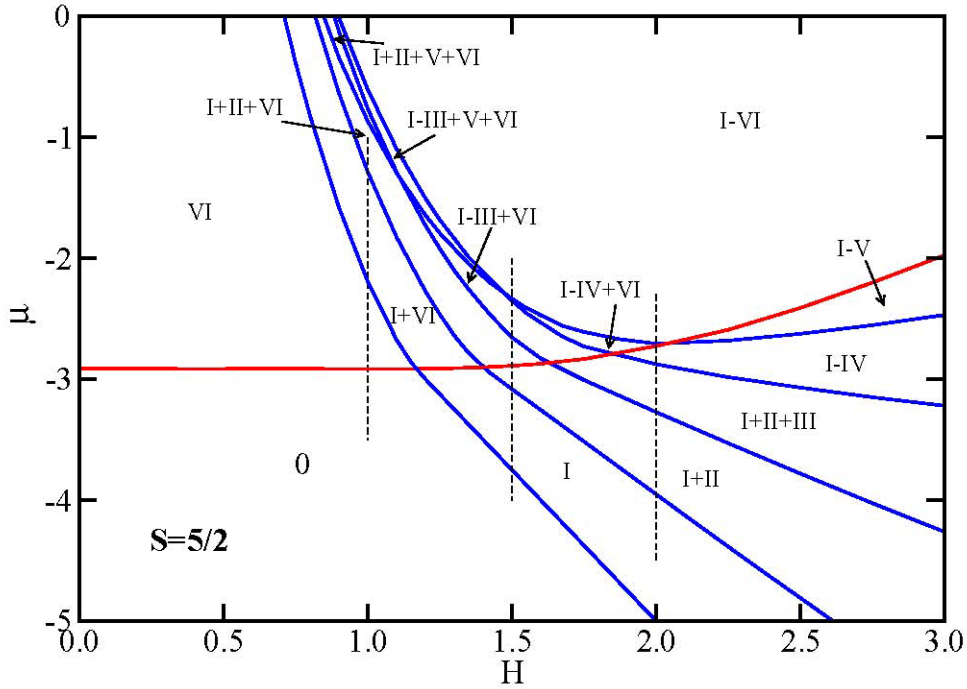
Phase separation along the tube

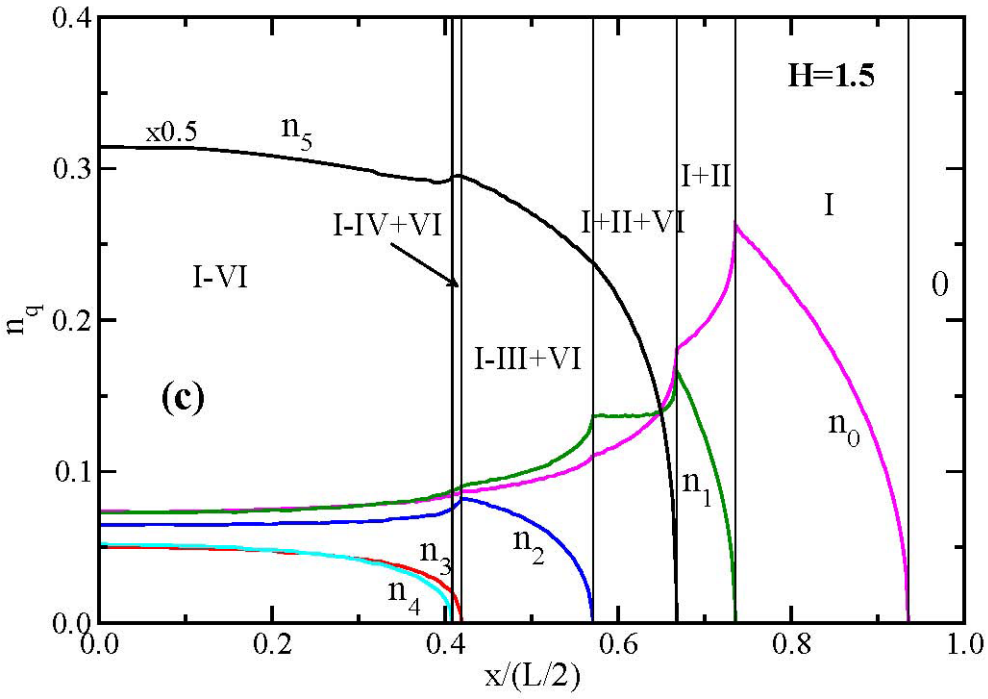
S=5/2  H=1.5  constant

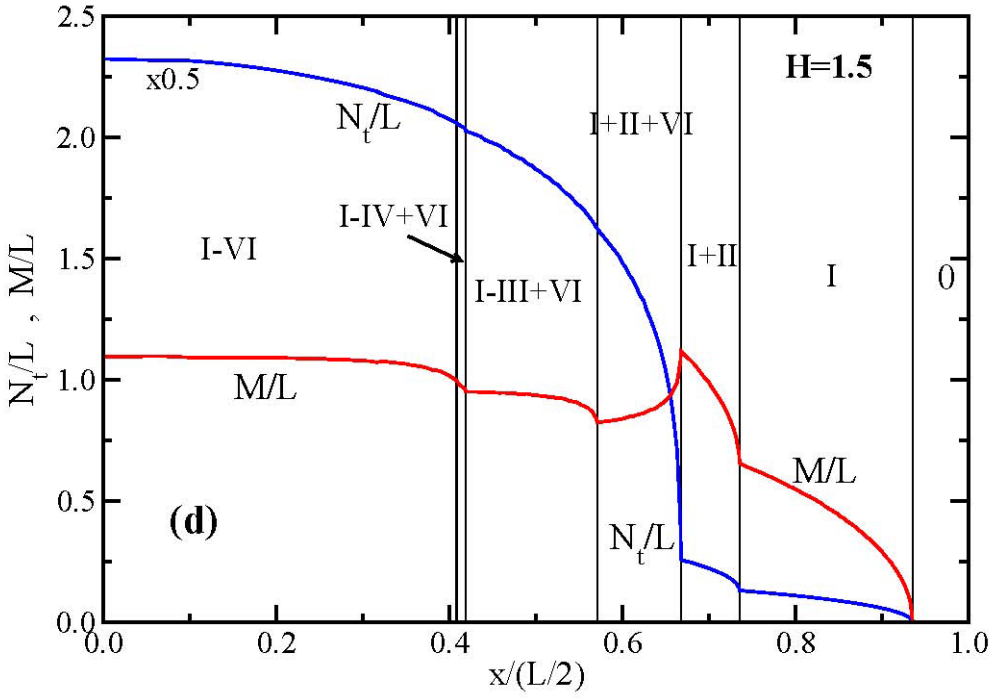
μ(x)  varies along the tube

experimentally accessible:

$N_t/L$ and M/L



Onset of magnetization

$M(H) \sim [H - H_C(\mu)]^{1/2}$

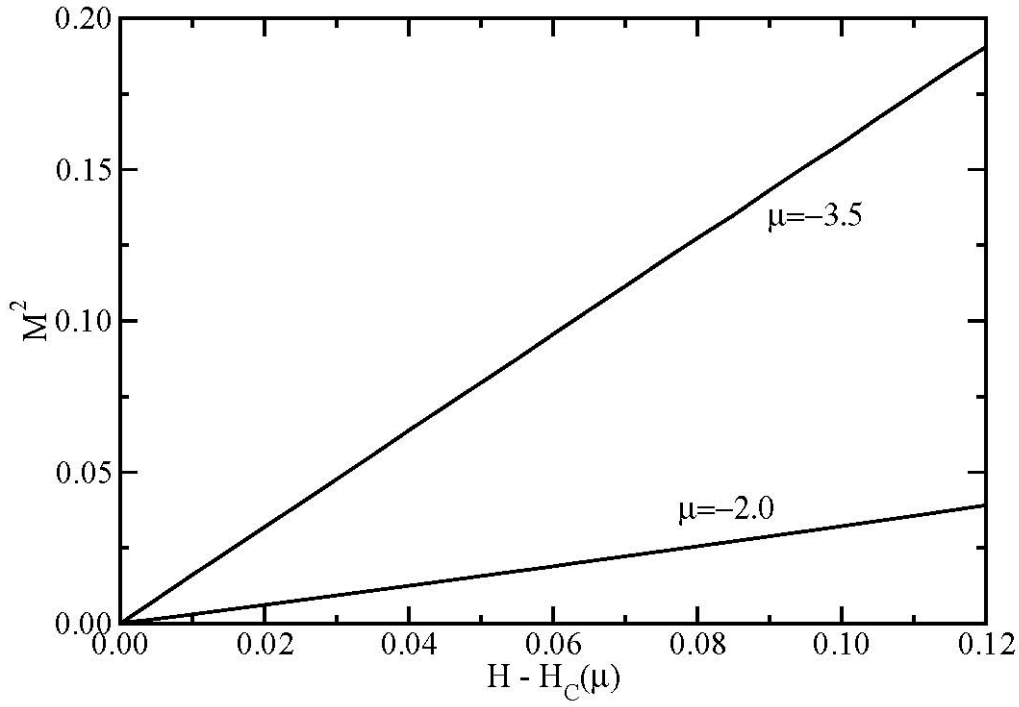
S=5/2

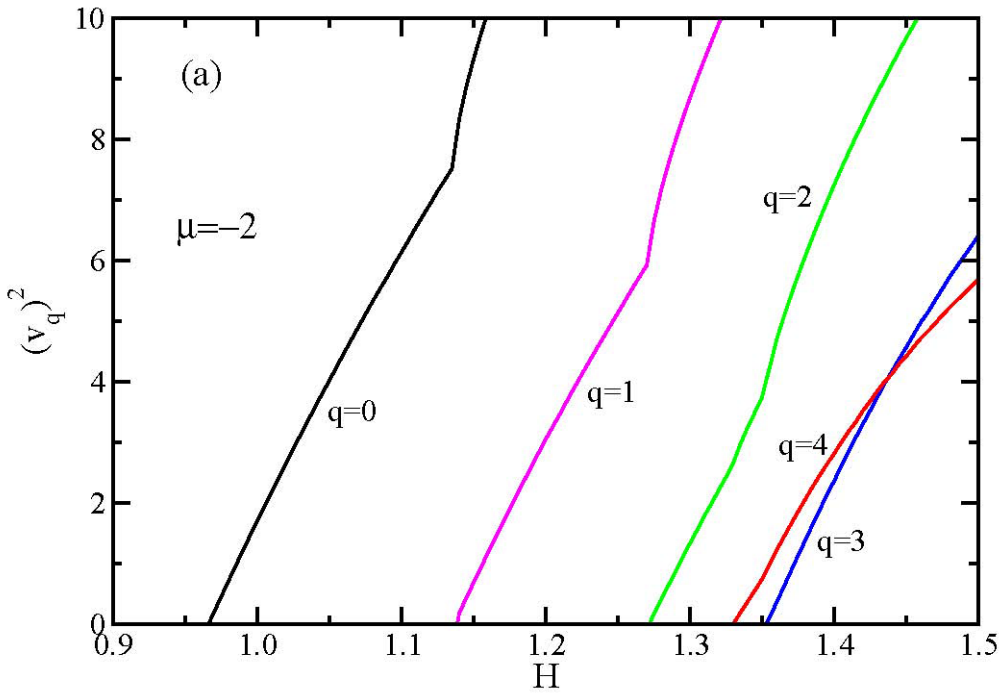
Group velocities $v_q$

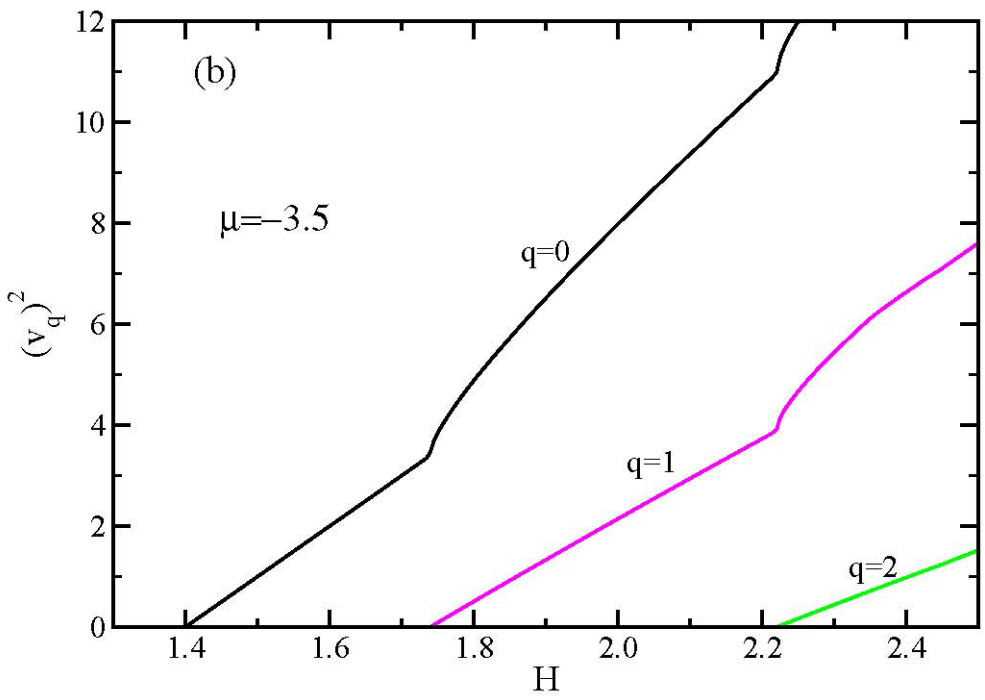
$v_q \sim [H - H_C(\mu)]^{1/2}$

1D van Hove singularities

Prokovsky-Talapov transition



# VI. Conformal Towers

Critical behavior determined by low-energy excitations

Finite size corrections to ground state energy (conformal towers)

$$E = E_{GS} + \sum_{l=0}^{2S} \frac{\pi v_l}{2L} \left[ \sum_{q=0}^{2S} (\hat{z}^{-1})_{lq} \Delta N_q \right]^2 + \sum_{l=0}^{2S} \frac{2\pi v_l}{L} \left\{ \left[ \sum_{q=0}^{2S} z_{ql} D_q \right]^2 + n_l^+ + n_l^- - \frac{1}{12} \right\}$$

and the corresponding change in momentum for the excitations is

$$\Delta P = \frac{2\pi}{L} \sum_{l=0}^{2S} \left[ D_l \Delta N_l + n_l^+ - n_l^- \right]$$

Four quantum numbers for each band: $\Delta N_l$, $D_l$, $n_l^+$, $n_l^-$, $l = 0, \cdots, 2S$

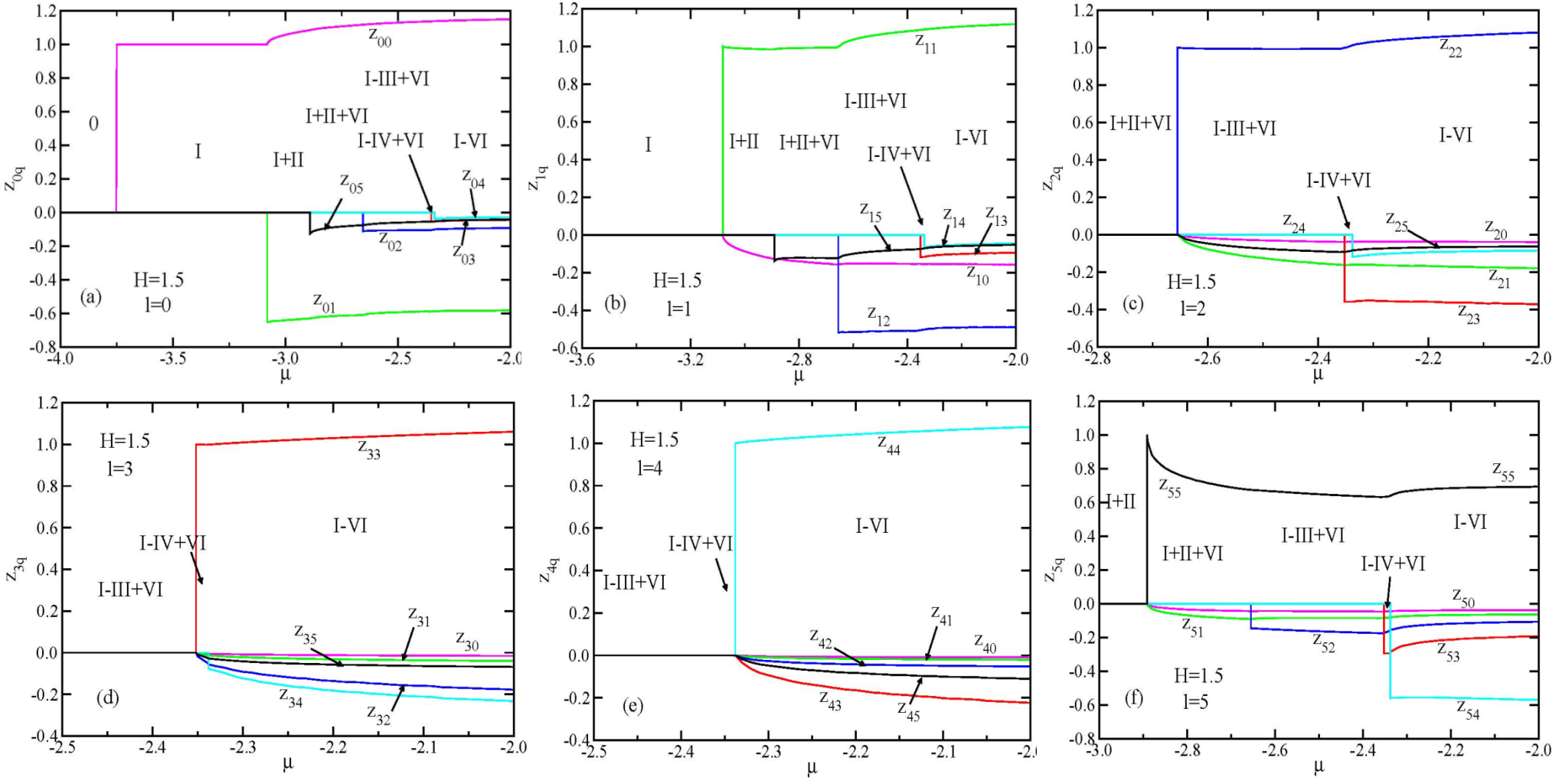
$$D_l = \tfrac{1}{2}[\min(l,q) + 1]\Delta N_q \pmod 1$$

$v_l$ is group velocity, $z_{lq} = \xi_{lq}(B_q)$ is $N \times N$ matrix of generalized dressed charges

The $X^{(q)}(\xi) = \xi_{lq}(B_q)$ satisfy similar integral equations as $\varepsilon^{(l)}(\xi)$ but with driving terms $D_q(\xi) = \delta_{l,q}$

The dressed generalized charges determine the interrelation between the different Fermi points when an excitation is introduced. The $z_{lq}$ can vary discontinuously at the transitions between phases.

## Dressed generalized charges for $S = 5/2$ and $H = 1.5$: $z_{ll} > 0$ , $z_{lq} < 0$ if $l \neq q$




# Superfluidity

Weak Josephson tunneling between tubes

interactions between particles in different tubes

dimensional crossover from 1D to very anisotropic 3D

possible superfluid long-range order

# Correlation functions

**Long-time large-distance asymptote at $T = 0$**

$$\langle \mathcal{O}^\dagger(x,t)\mathcal{O}(0,0)\rangle = \frac{\exp[-2i(\sum_{q=0}^{2S} D_q p_q)x]}{\prod_{q=0}^{2S}(x - iv_q t)^{2\Delta_q^+}(x + iv_q t)^{2\Delta_q^-}}$$

$p_q = \pi n_q$ **Fermi momentum,** $\Delta_q^\pm$ **conformal dimensions for the band** $q$

$$2\Delta_q^\pm = 2n_q^\pm + \left[\sum_{l=0}^{2S} z_{lq} D_l \pm \frac{1}{2}\sum_{l=0}^{2S} (\hat{z}^{-1})_{ql} \Delta N_l\right]^2$$

$2N$ **factors, two for each band (forward and backward movers)**

**Equal time correlation function for operator** $\mathcal{O}$ **is**

$$\langle \mathcal{O}^\dagger(x,0)\mathcal{O}(0,0)\rangle = Ax^{-\theta}\cos[\pi x/\lambda + \phi]\ ,\ \theta = 2\sum_{q=0}^{2S}(\Delta_q^+ + \Delta_q^-)\ ,\ n_q^\pm = 0$$

**Amplitude** $A$ **cannot be determined from conformal field theory,** $\phi$ **is an arbitrary phase (to be neglected), distance between nodes** $\lambda = 1/|2\sum_{q=0}^{2S} D_q n_q|$.

## 28 operators for <u>superfluidity</u> for $S = 5/2$

TABLE I: Most relevant superfluidity operators

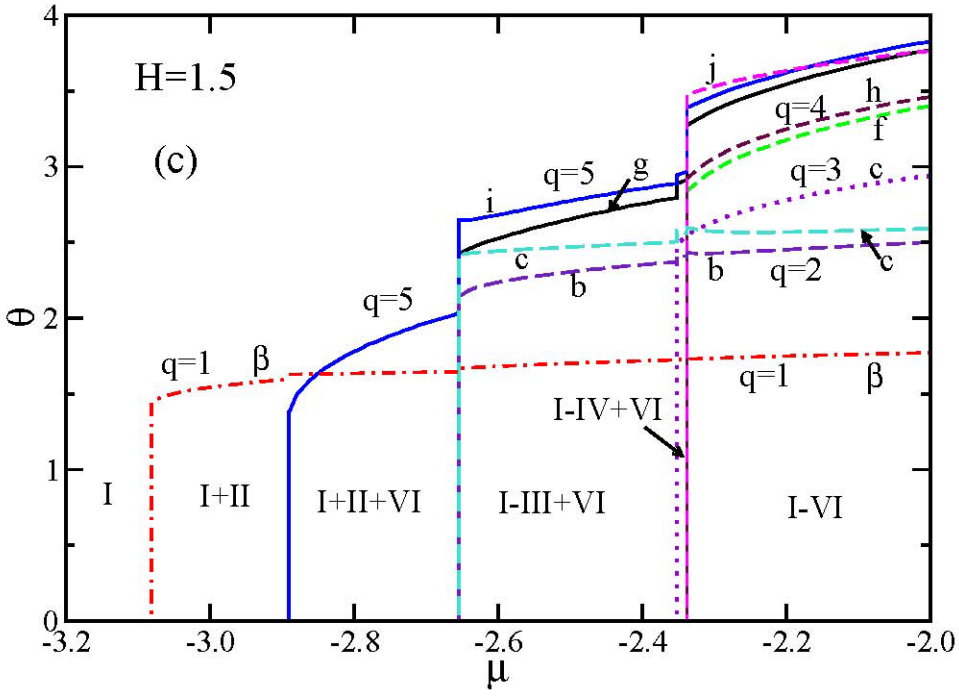
| phase | operator | $\Delta N_q = -1$ | $D_0$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|---|---|---|---|---|---|---|---|---|
| I+II | $\mathcal{O}_\beta^{(1)} = c_{5/2,+}c_{3/2,-}$ | q=1 | $\pm\frac{1}{2}$ | 0 | - | - | - | - |
| I+II+VI | $\mathcal{O}_g^{(5)} = c_{5/2,+}c_{3/2,-}c_{1/2,+}c_{-1/2,-}c_{-3/2,+}c_{-5/2,-}$ | q=5 | $\pm\frac{1}{2}$ | 0 | - | - | - | 0 |
| I+II+VI | $\mathcal{O}_\beta^{(1)} = c_{5/2,+}c_{3/2,-}$ | q=1 | $\pm\frac{1}{2}$ | 0 | - | - | - | 0 |
| I-III+VI | $\mathcal{O}_g^{(5)} = c_{5/2,+}c_{3/2,-}c_{1/2,+}c_{-1/2,-}c_{-3/2,+}c_{-5/2,-}$ | q=5 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | - | - | 0 |
| I-III+VI | $\mathcal{O}_i^{(5)} = c_{5/2,+}c_{3/2,-}c_{1/2,-}c_{-1/2,+}c_{-3/2,-}c_{-5/2,+}$ | q=5 | $\pm\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | - | - | 0 |
| I-III+VI | $\mathcal{O}_b^{(2)} = c_{5/2,+}c_{3/2,-}c_{1/2,+}$ | q=2 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | - | - | $\pm\frac{1}{2}$ |
| I-III+VI | $\mathcal{O}_c^{(2)} = c_{5/2,+}c_{3/2,-}c_{1/2,-}$ | q=2 | $\pm\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | - | - | $\mp\frac{1}{2}$ |
| I-III+VI | $\mathcal{O}_\beta^{(1)} = c_{5/2,+}c_{3/2,-}$ | q=1 | $\pm\frac{1}{2}$ | 0 | 0 | - | - | 0 |
| I-VI | $\mathcal{O}_g^{(5)} = c_{5/2,+}c_{3/2,-}c_{1/2,+}c_{-1/2,-}c_{-3/2,+}c_{-5/2,-}$ | q=5 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | 0 |
| I-VI | $\mathcal{O}_i^{(5)} = c_{5/2,+}c_{3/2,-}c_{1/2,-}c_{-1/2,+}c_{-3/2,-}c_{-5/2,+}$ | q=5 | $\pm\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | 0 |
| I-VI | $\mathcal{O}_f^{(4)} = c_{5/2,+}c_{3/2,-}c_{1/2,+}c_{-1/2,-}c_{-3/2,+}$ | q=4 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | $\pm\frac{1}{2}$ |
| I-VI | $\mathcal{O}_h^{(4)} = c_{5/2,+}c_{3/2,-}c_{1/2,-}c_{-1/2,+}c_{-3/2,-}$ | q=4 | $\pm\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | $\mp\frac{1}{2}$ |
| I-VI | $\mathcal{O}_j^{(4)} = c_{5/2,+}c_{3/2,+}c_{1/2,-}c_{-1/2,-}c_{-3/2,-}$ | q=4 | $\pm\frac{1}{2}$ | $\pm1$ | $\pm\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | $\mp\frac{1}{2}$ |
| I-VI | $\mathcal{O}_c^{(3)} = c_{5/2,+}c_{3/2,-}c_{1/2,-}c_{-1/2,+}$ | q=3 | $\pm\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | 0 | 0 | 0 |
| I-VI | $\mathcal{O}_b^{(2)} = c_{5/2,+}c_{3/2,-}c_{1/2,+}$ | q=2 | $\pm\frac{1}{2}$ | 0 | $\pm\frac{1}{2}$ | $\pm\frac{1}{2}$ | $\pm\frac{1}{2}$ | $\pm\frac{1}{2}$ |
| I-VI | $\mathcal{O}_c^{(2)} = c_{5/2,+}c_{3/2,-}c_{1/2,-}$ | q=2 | $\pm\frac{1}{2}$ | 0 | $\mp\frac{1}{2}$ | $\mp\frac{1}{2}$ | $\mp\frac{1}{2}$ | $\mp\frac{1}{2}$ |
| I-VI | $\mathcal{O}_\beta^{(1)} = c_{5/2,+}c_{3/2,-}$ | q=1 | $\pm\frac{1}{2}$ | 0 | 0 | 0 | 0 | 0 |

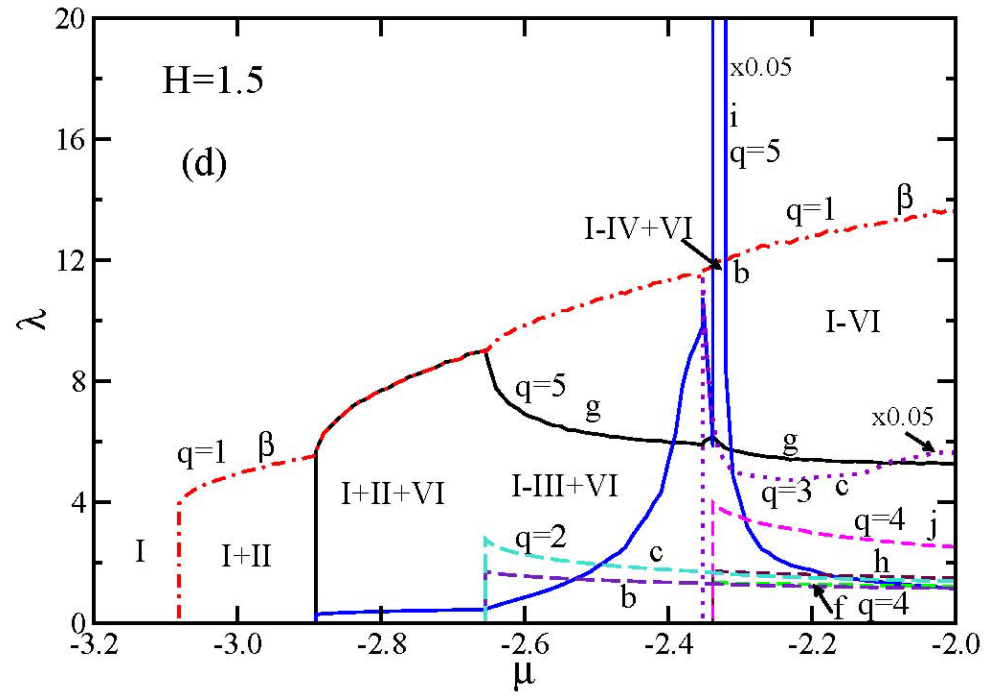
## criteria for superfluidity

- smallest exponent $\theta$ ($< 2$ = critical dimension in 1D)
- large $\lambda$ (distance between FFLO nodes)
- small momentum of bound states

$\beta$-phase (pairing) most favorable

Gradient term of the Ginzburg-Landau expansion



# VII. Conclusions

Inhomogeneous phases due to:

- phase separation along the tube
- first superfluid phase accessed from normal (Luttinger) phase is pairing of atoms with spin components S and S-1
- FFLO phase - modulation $\lambda$ (spin imbalance) - excellent conditions : system is very pure (no impurities) and has low effective dimension (extreme anisotropy)