Correlations and Coherence in Quantum Systems, Évora, October 8-12, 2012

# Exploring Quantum Fluctuations and Quantum Phase transitions in Spin Systems

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- Arnab Sen (BU → Max Planck, Dresden)

**Related review article:**

**- R. K. Kaul, R. G. Melko, A. W. Sandvik, arXiv:1204.5405 (to appear in Annual Review of Condensed Matter Physics)**

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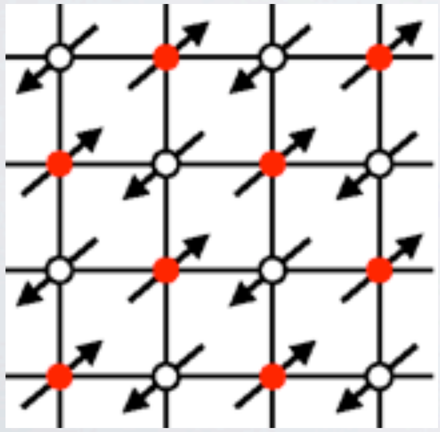
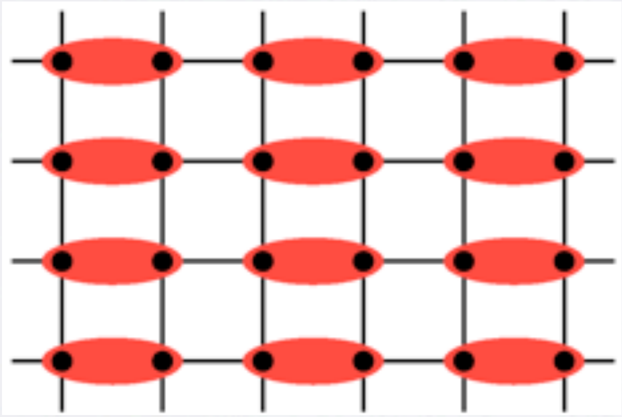
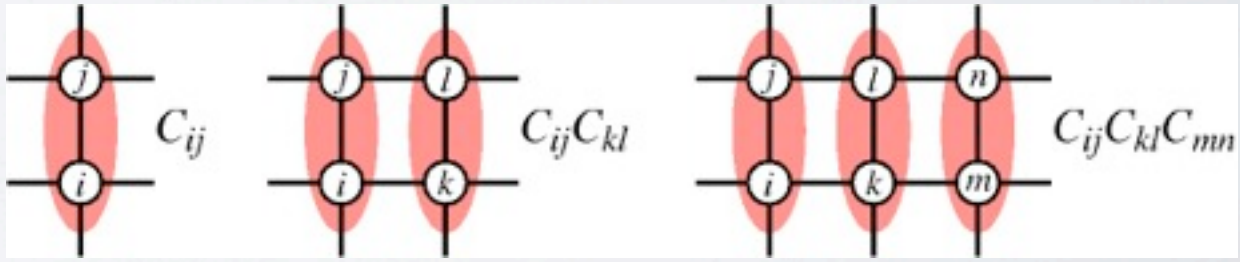
# Outline

- Antiferromagnet-paramagnet quantum phase transition
- Valence-bonds-solid (VBS) order and “deconfined” criticality
- Microscopic realizations; J-Q model
- Insights from QMC simulations; SU(2) and SU(N) models
- Time permitting: Emergent U(1) symmetry of the near-critical VBS



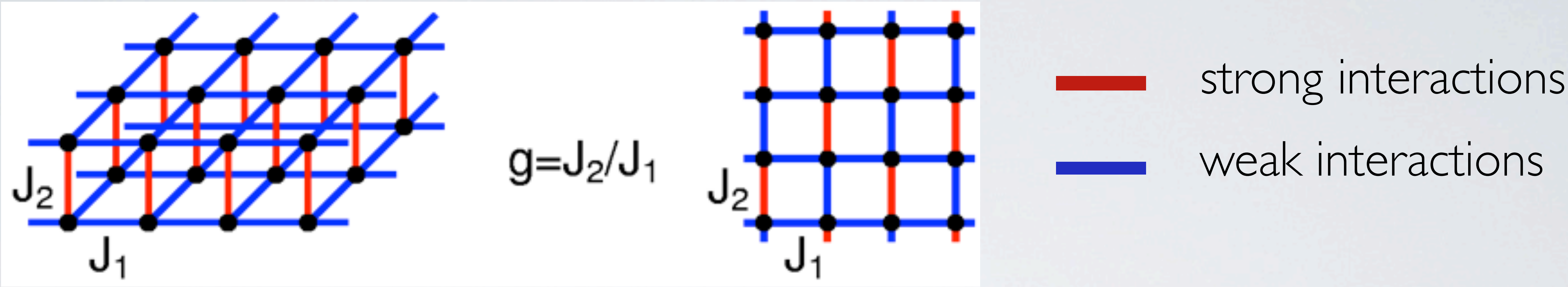




# Conventional Neel-paramagnet quantum phase transition

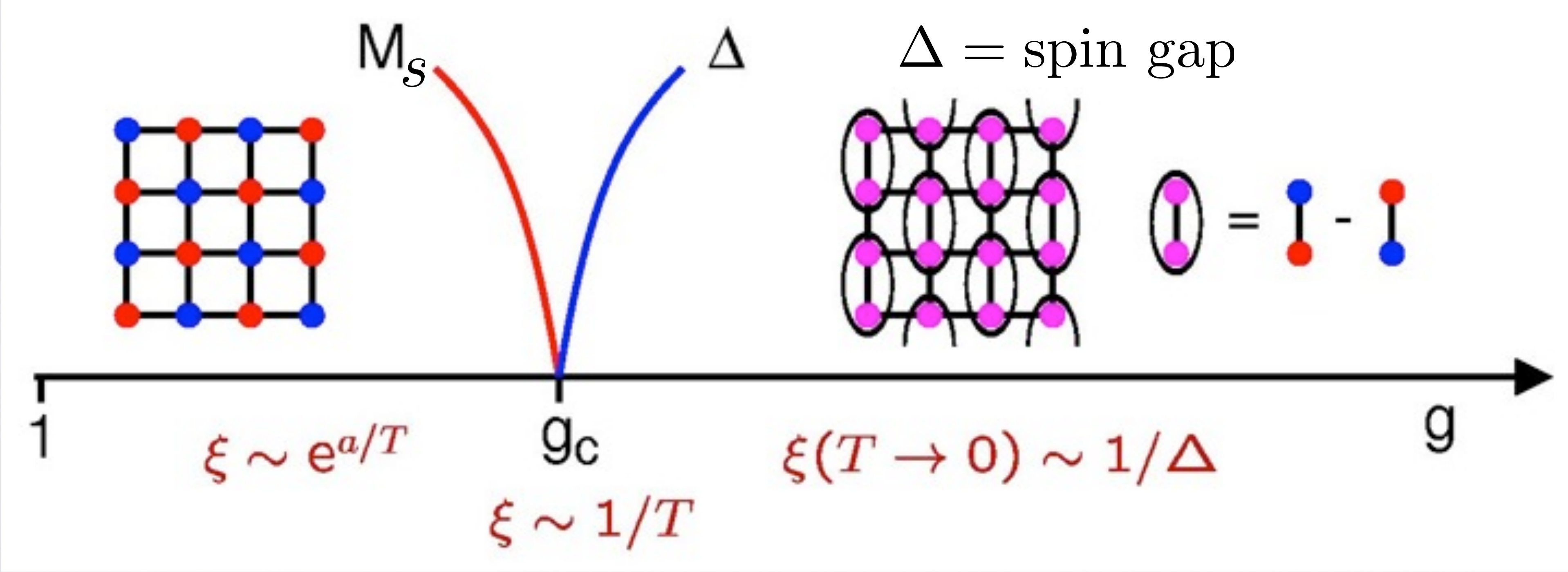
**Example: Dimerized S=1/2 Heisenberg models**

- every spin belongs to a dimer (strongly-coupled pair)
- many possibilities, e.g., bilayer, dimerized single layer



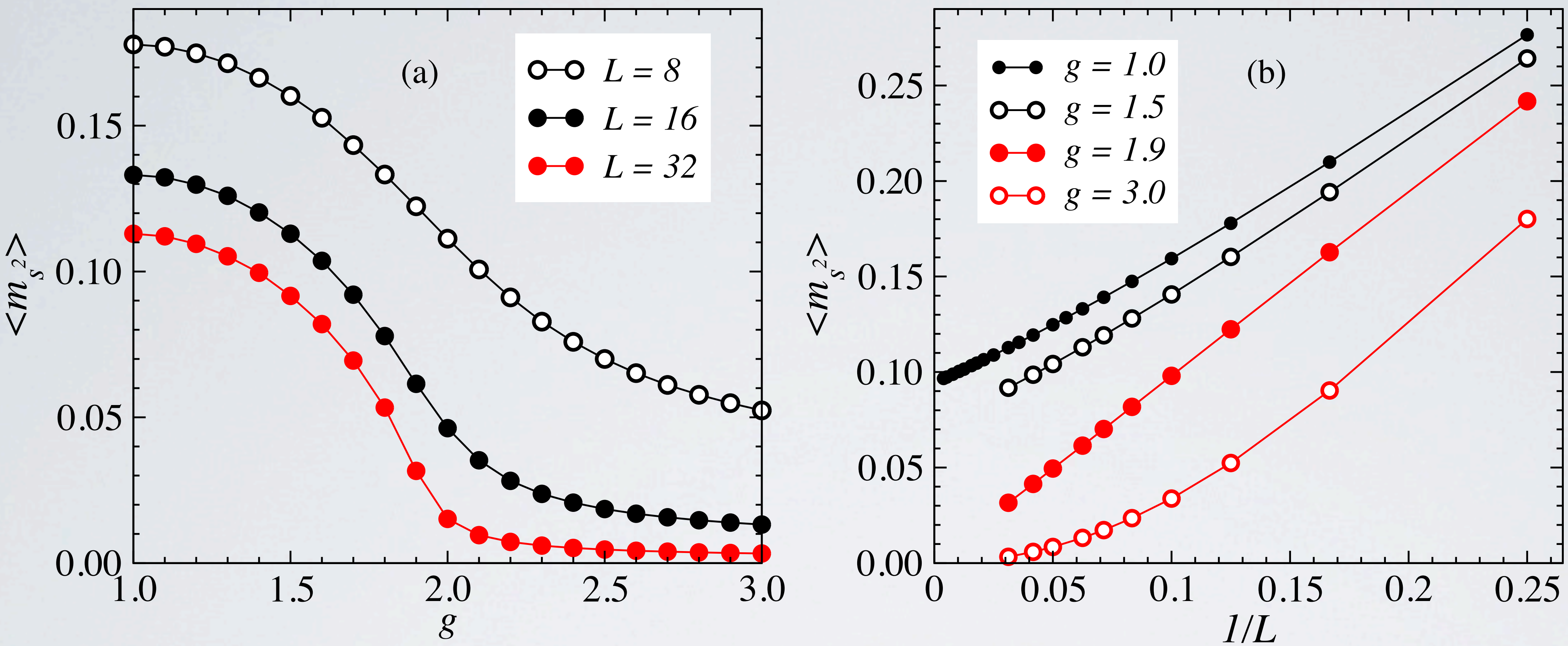

Singlet formation on strong bonds ➙ Neel - disordered transition

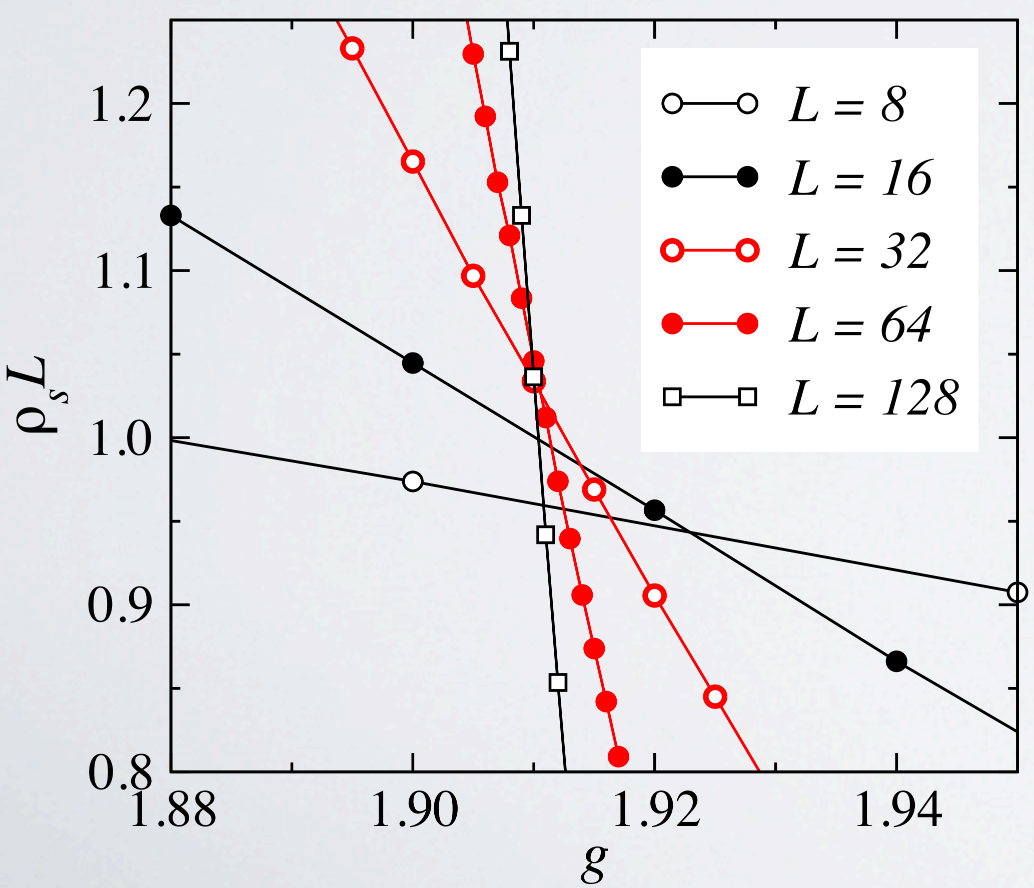
**Ground state (T=0) phases**




⇒ 3D classical Heisenberg (O3) universality class; QMC confirmed

# Example of QMC finite-size scaling scaling with QMC data dimerized single-layer Heisenberg model




According to theory, spin stiffness at the critical point should scale according to (T=0)

$$\rho_s \sim \frac{1}{L} \rightarrow L\rho_s \text{ constant}$$

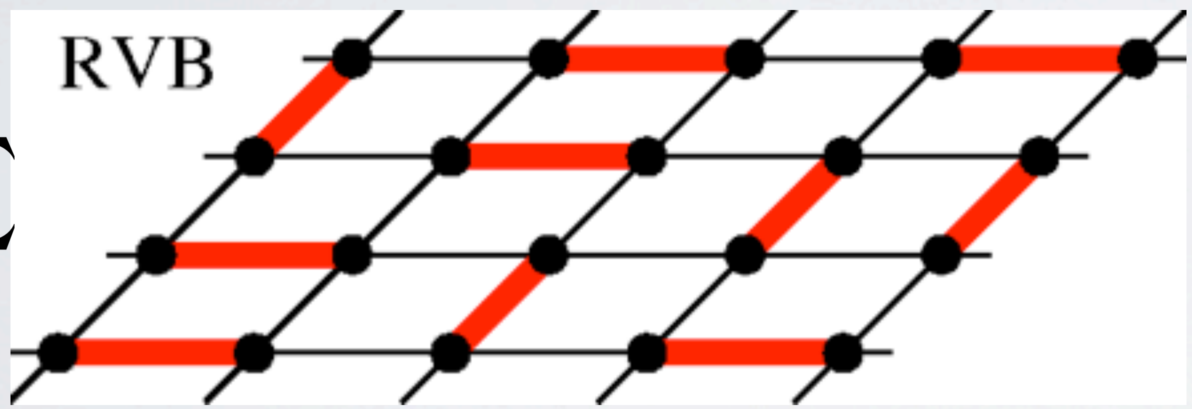
Allows accurate determination of the critical point (curve crossings)

## More complex non-magnetic states; systems with 1 spin per unit cell

$$\mathbf{H} = \mathbf{J}\sum_{\langle \mathbf{i},\mathbf{j}\rangle} \mathbf{S_i}\cdot\mathbf{S_j} + \mathbf{g}\times\cdots$$

- **non-trivial non-magnetic ground states are possible, e.g.,**
  - ➡ resonating valence-bond (RVB) spin liquid
  - ➡ valence-bond solid (VBS)

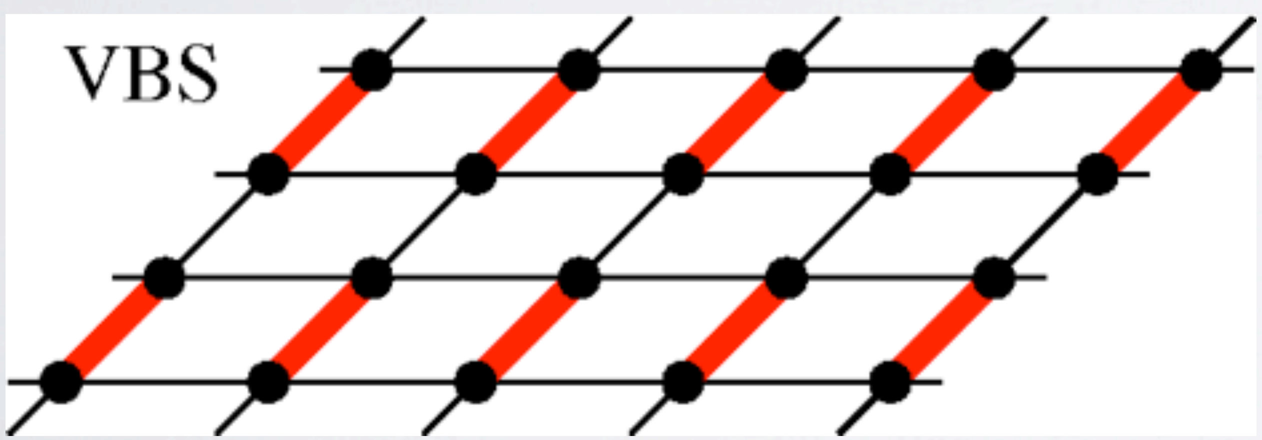
Non-magnetic states often have natural descriptions with **valence bonds**

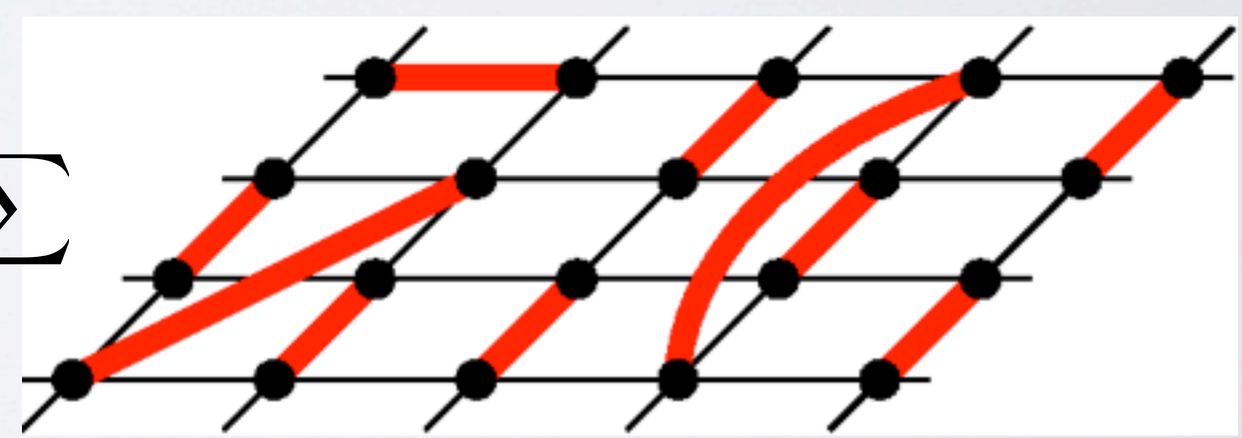
$\sum$ 



$$= (\uparrow_i\downarrow_j - \downarrow_i\uparrow_j)/\sqrt{2}$$

The basis including bonds of all lengths is **overcomplete** in the singlet sector



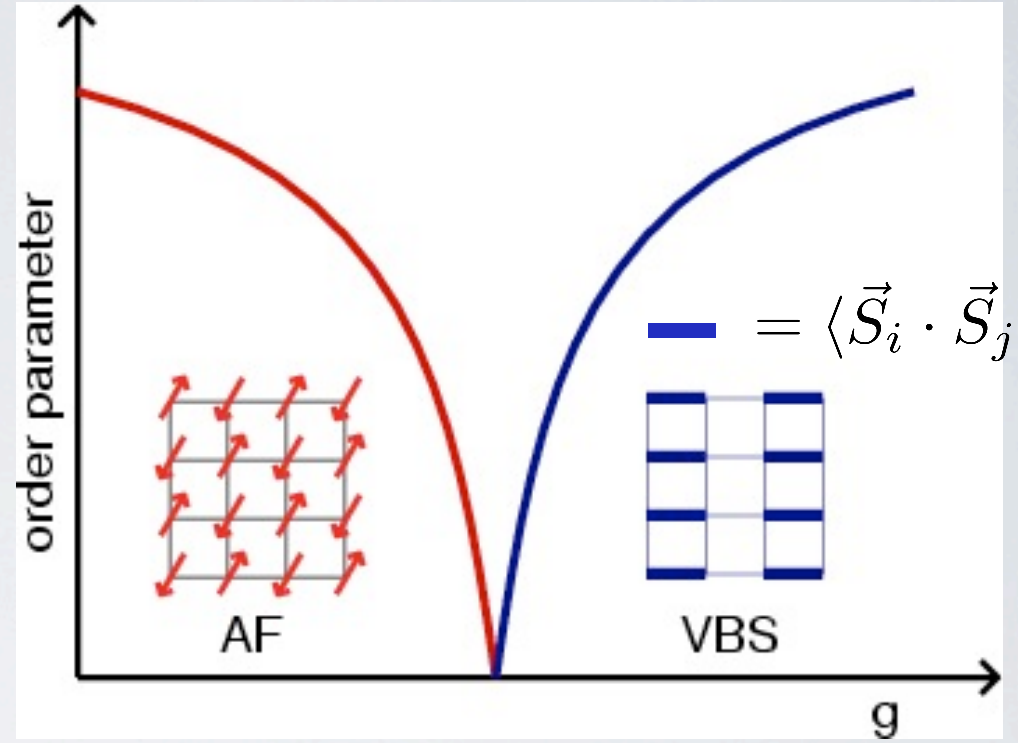

$\sum$ 

- non-magnetic states dominated by short bonds

# VBS states and "deconfined" quantum criticality

Read, Sachdev (1989),....,Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)

$$\mathbf{H} = \mathbf{J} \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \mathbf{S_i} \cdot \mathbf{S_j} + \mathbf{g} \times \cdots$$




Neel-VBS transition in 2D
- generically continuous
- violating the "Landau rule" stating 1st-order transition

Description with spinor field (2-component complex vector)

$$\mathbf{\Phi} = z^*_\alpha \boldsymbol{\sigma}_{\alpha\beta} z_\beta \qquad \text{gauge redundancy: } z \longrightarrow e^{i\gamma(r,\tau)} z$$

$$\mathcal{S}_z = \int \mathrm{d}^2 r \mathrm{d}\tau \left[ |(\partial_\mu - iA_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

A is a U(1) symmetric gauge field

- $CP^1$ action (non-compact)

- large-N calculations for $CP^{N-1}$ theory

- proposed as critical theory separating Neel and VBS states
- describes VBS state when additional terms are added

**Competing scenario: first-order transition** (Kuklov et al., 2008)

## In what systems can Neel-VBS transition be studied with QMC?

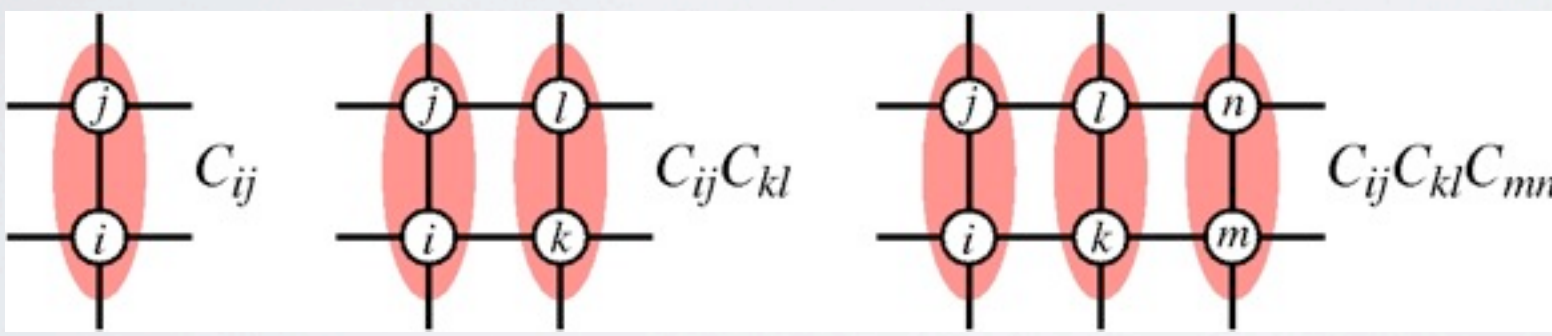
**VBS states from multi-spin interactions** (Sandvik, 2007)

**The Heisenberg interaction is equivalent to a singlet-projector**

$$C_{ij} = \frac{1}{4} - \vec{S}_i \cdot \vec{S}_j$$

$$C_{ij}|\phi^s_{ij}\rangle = |\phi^s_{ij}\rangle, \qquad C_{ij}|\phi^{tm}_{ij}\rangle = 0 \quad (m = -1, 0, 1)$$

- we can construct models with products of singlet projectors
- no frustration in the conventional sense (QMC can be used)
- correlated singlet projection reduces the antiferromagnetic order




+ all translations and rotations

**The "J-Q₂" model with two projectors is**

$$H = -J \sum_{\langle ij \rangle} C_{ij} - Q \sum_{\langle ijkl \rangle} C_{ij} C_{kl}$$

- Has Néel-VBS transition, appears to be continuous
- Not a realistic microscopic model for materials
- Intended to study Néel-VBS transition (universal physics)

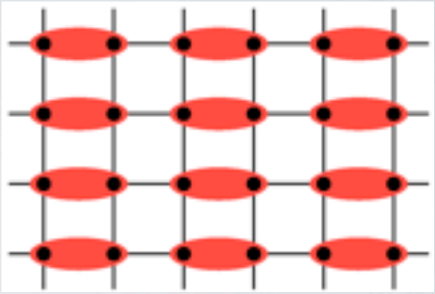
# Néel-VBS transition in the J-Q model

**T=0 projector QMC results** (no approximations; finite size)

(Sandvik, 2007; Lou, Sandvik, Kawashima, 2009)

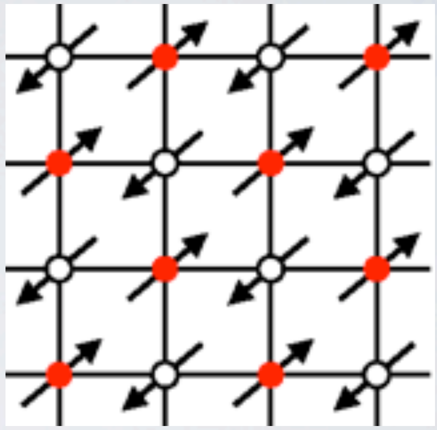
VBS vector order parameter ($D_x$,$D_y$) (x and y lattice orientations)

$$D_x = \frac{1}{N}\sum_{i=1}^{N}(-1)^{x_i}\mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}}, \quad D_y = \frac{1}{N}\sum_{i=1}^{N}(-1)^{y_i}\mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}}$$



Néel order parameter (staggered magnetization)

$$\vec{M} = \frac{1}{N}\sum_{i}(-1)^{x_i+y_i}\vec{S}_i$$



No symmetry-breaking in simulations; study the squares

$$M^2 = \langle \vec{M} \cdot \vec{M} \rangle, \qquad D^2 = \langle D_x^2 + D_y^2 \rangle$$

Finite-size scaling: a critical squared order parameter (A) scales as

$$A(L,q) = L^{-(1+\eta)} f[(q-q_c)L^{1/\nu}]$$

coupling ratio

$$q = \frac{Q}{J+Q}$$

Data “collapse” for different system sizes L of **$AL^{1+\eta}$** graphed vs **$(q-q_c)L^{1/\nu}$**

**J-$Q_2$ model; $q_c$=0.961(1)**

$$\eta_s = 0.35(2)$$

$$\eta_d = 0.20(2)$$

$$\nu = 0.67(1)$$

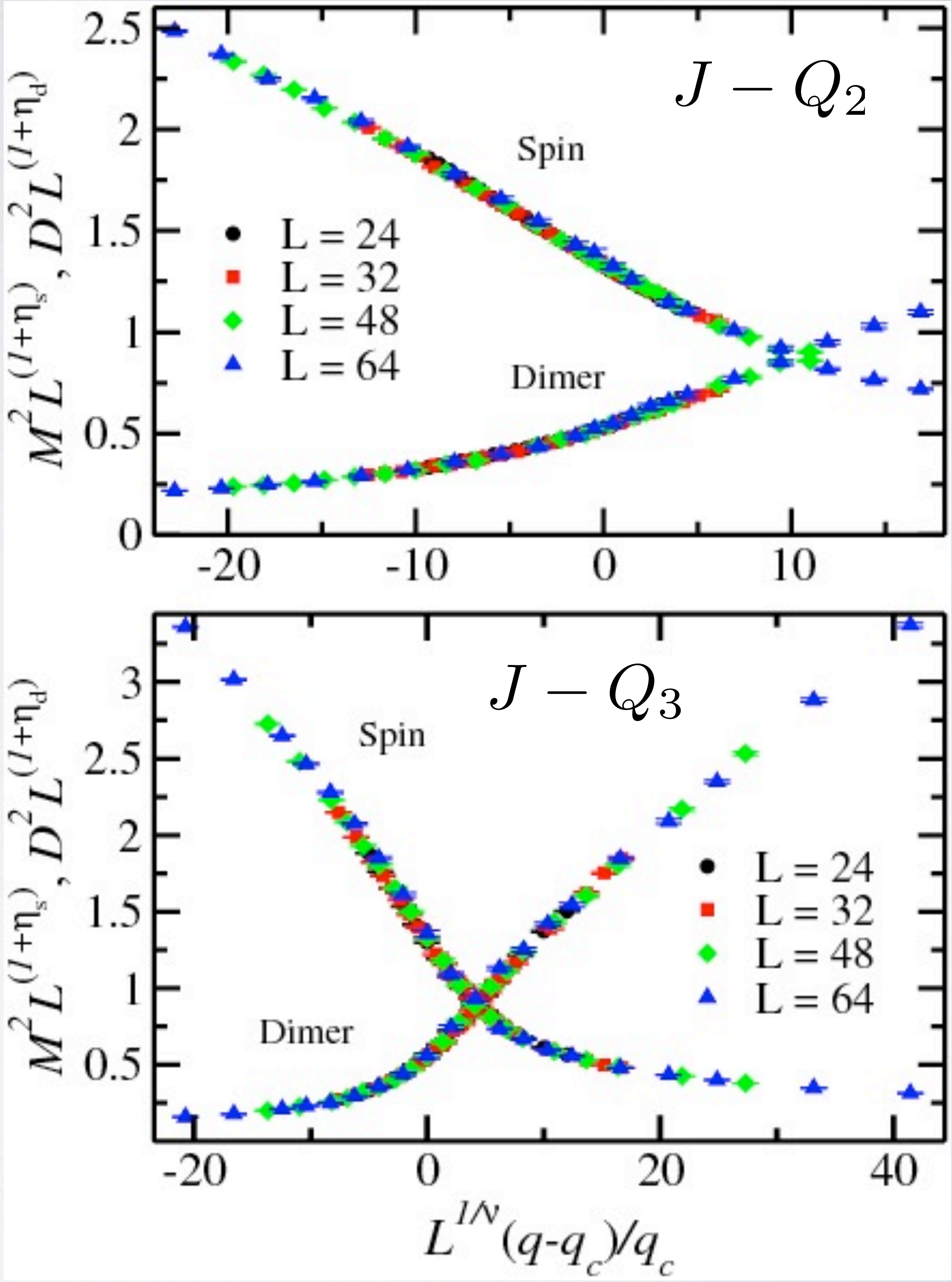
**J-$Q_3$ model; $q_c$=0.600(3)**

$$\eta_s = 0.33(2)$$

$$\eta_d = 0.20(2)$$

$$\nu = 0.69(2)$$

**Exponents universal**
**(same within error bars)**

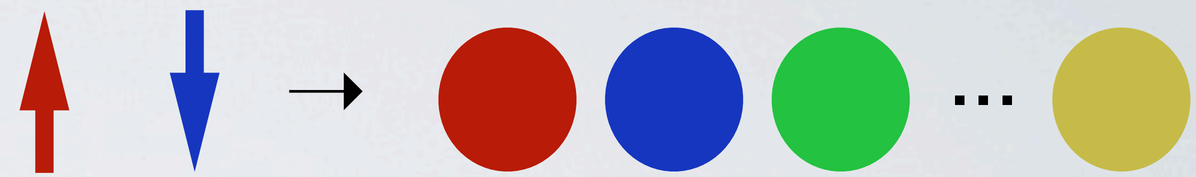


# Making connections with field theory

The non-compact CP$^{N-1}$ model has been studied for large N

- large-N expansion, SU(N) symmetry

Senthil et al. (2004), Kaul & Sachdev (2009)



SU(2) → SU(N)

$$\eta_s = 1 - \frac{32}{\pi^2 N} + \ldots$$

- older results, using relationship between monopoles in the field theory and the VBS order parameter Read & Sachdev (1989)

$$\eta_d = 0.2492 \times N - 1 + \ldots$$

How can we test these results?

## QMC studies of spin hamiltonians with SU(N) spins

2D SU(N) Heisenberg model [Harada et al. (2003), Beach et al. (2010)]

- Fundamental and conjugate repr. of SU(N) on A,B sublattices
- No sign problem in QMC
- Same repr. used in analytical large-N calculations
- Neel ground state for N<5, VBS for N=5,6,...

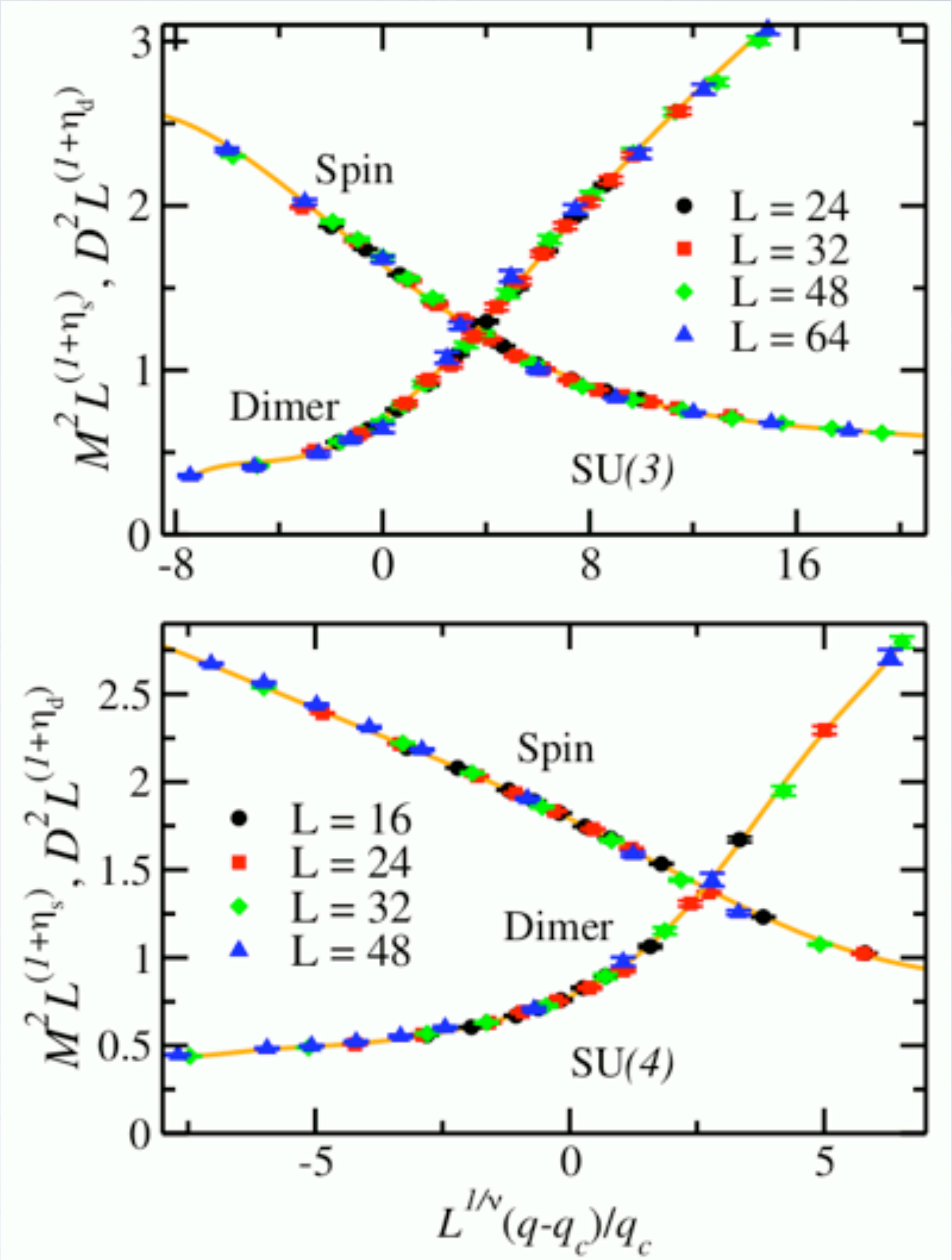
# J-Q models with SU(N) spins

Lou, Sandvik, Kawashima, PRB (2009)

Heisenberg model (Q=0) has Neel ground state for N=2,3,4 ⇒

Neel - VBS transition vs Q/J

| Model, symmetry | $\eta_s$ | $\eta_d$ | $\nu$ |
|---|---|---|---|
| $J$-$Q_2$, SU(2) | 0.35(2) | 0.20(2) | 0.67(1) |
| $J$-$Q_3$, SU(2) | 0.33(2) | 0.20(2) | 0.69(2) |
| $J$-$Q_2$, SU(3) | 0.38(3) | 0.42(3) | 0.65(3) |
| $J$-$Q_2$, SU(4) | 0.42(5) | 0.64(5) | 0.70(2) |




How can we reach larger N to really study the large-N limit?

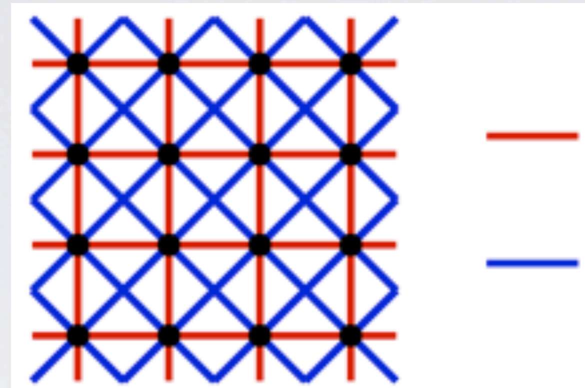
$$\eta_s = 1 - \frac{32}{\pi^2 N} + \ldots$$
$$\eta_d = 0.2492 \times N - 1 + \ldots$$

## $J_1$-$J_2$ Heisenberg model with SU(N) spins

(Kaul, Sandvik, PRL 2012)

Ferromagnetic 2nd-neighbor couplings enhance Neel order

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



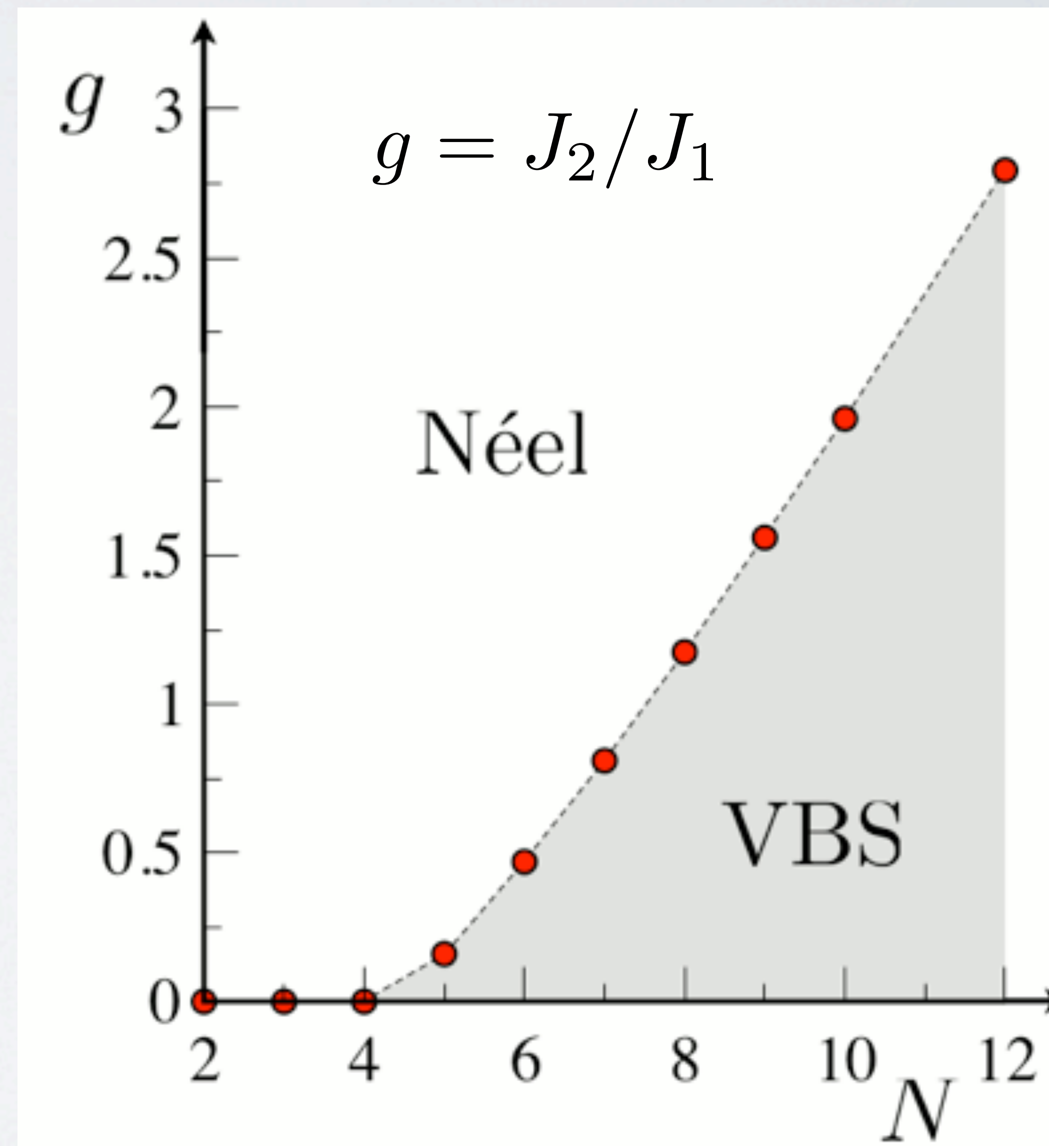

SU(N) generalization:

$$H = -\frac{J_1}{N} \sum_{\langle ij \rangle} P_{ij} - \frac{J_2}{N} \sum_{\langle\langle ij \rangle\rangle} \Pi_{ij}$$

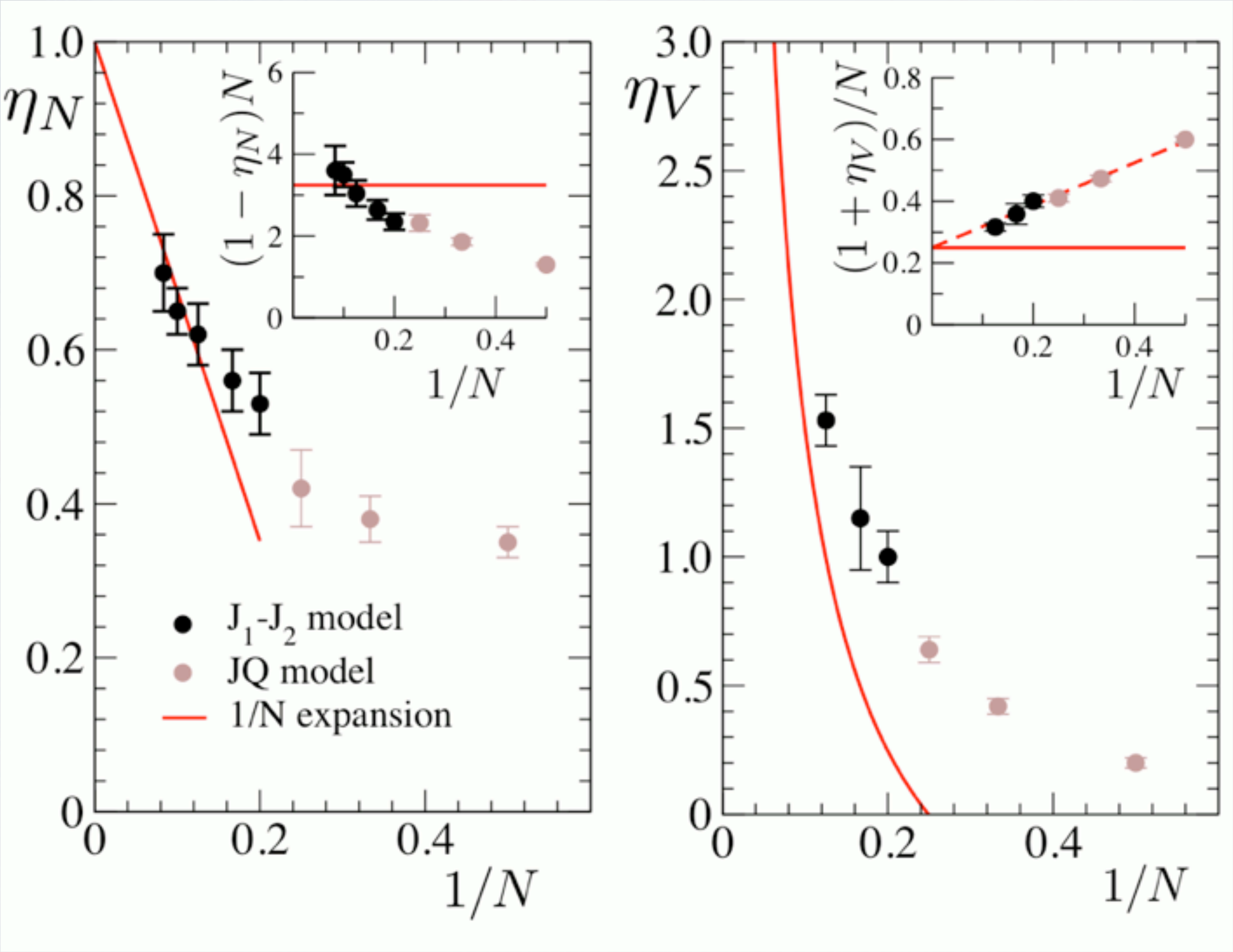
$P_{ij}$ = SU(N) singlet projector
$\Pi_{ij}$ = permutation operator

There is Neel order for all N>4

- Neel - VBS transition accessible with QMC for large N



# Comparing results: $J_1$-$J_2$, J-Q, $NCCP^{N-1}$




Conclusion: Trends for large N show excellent agreement

- QMC results predict size of the next 1/N corrections
- Field-theory challenge: Compute the next correction

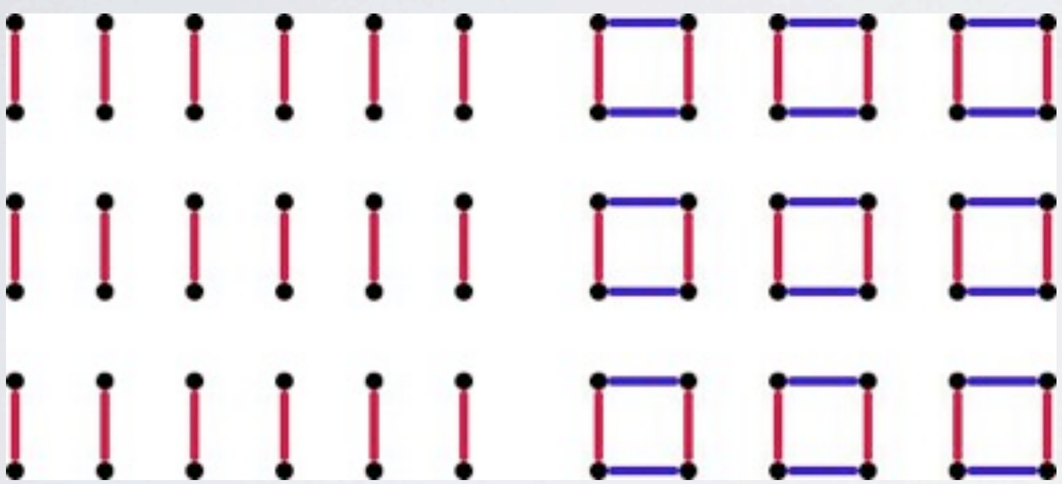
# Nature of the VBS fluctuations in the J-Q model - SU(2)

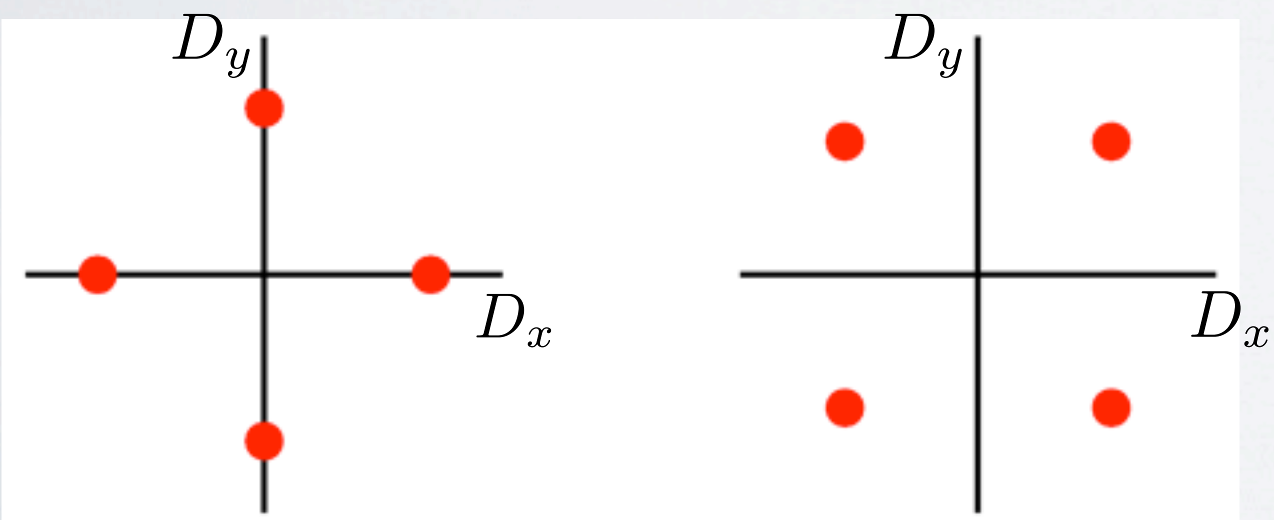
Joint probability distribution **$P(D_x,D_y)$** of x and y VBS order

$$D^2 = \langle D_x^2 + D_y^2 \rangle, \quad D_x = \frac{1}{N}\sum_{i=1}^{N}(-1)^{x_i}\mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}}, \quad D_y = \frac{1}{N}\sum_{i=1}^{N}(-1)^{y_i}\mathbf{S}_i \cdot \mathbf{S}_{i+\hat{y}}$$

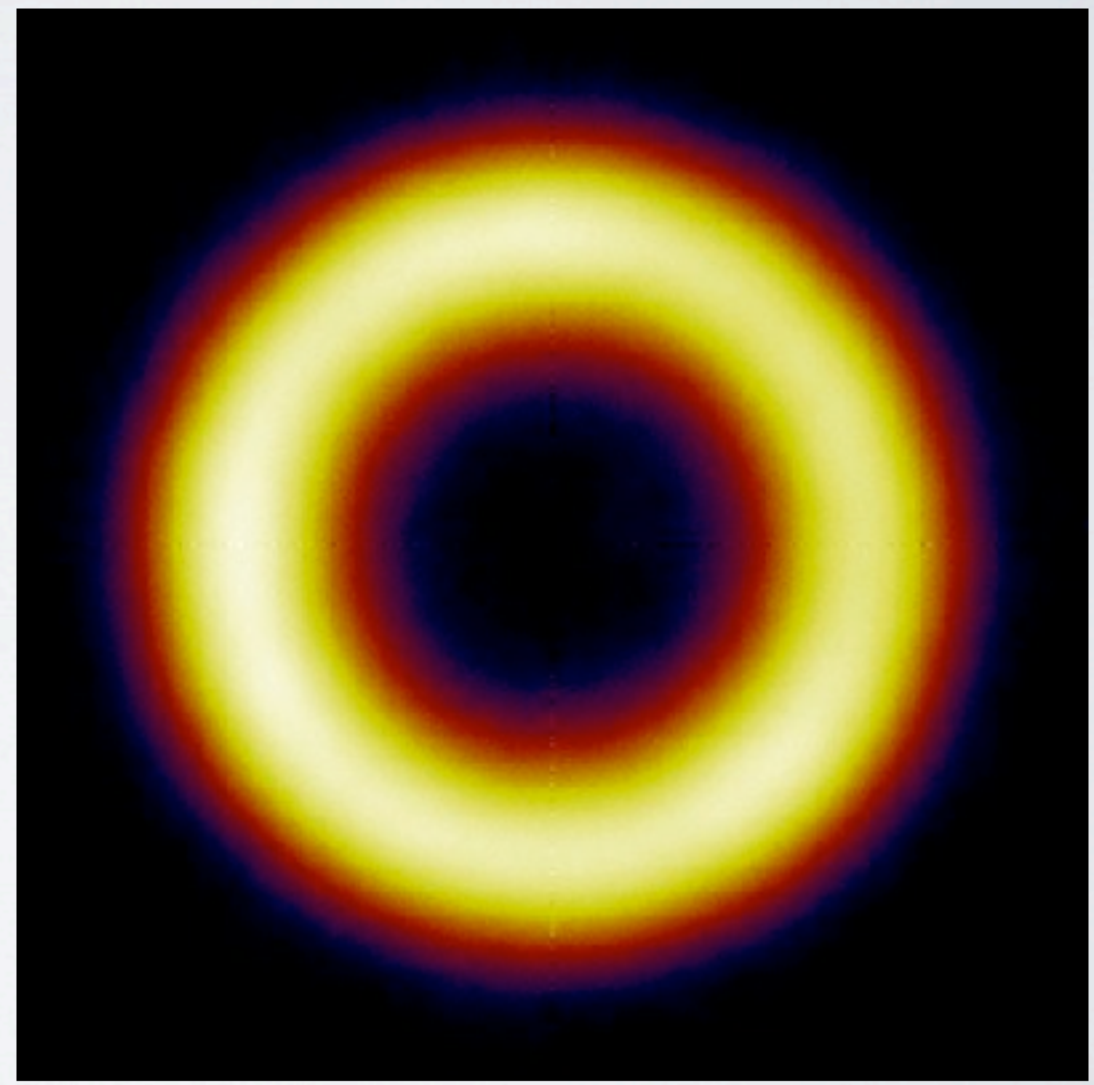
The squared order parameter cannot distinguish between:

columnar plaquette






J-$Q_2$ model, J=0, L=128



Magnitude of D has formed but the VBS “angle” is fluctuating

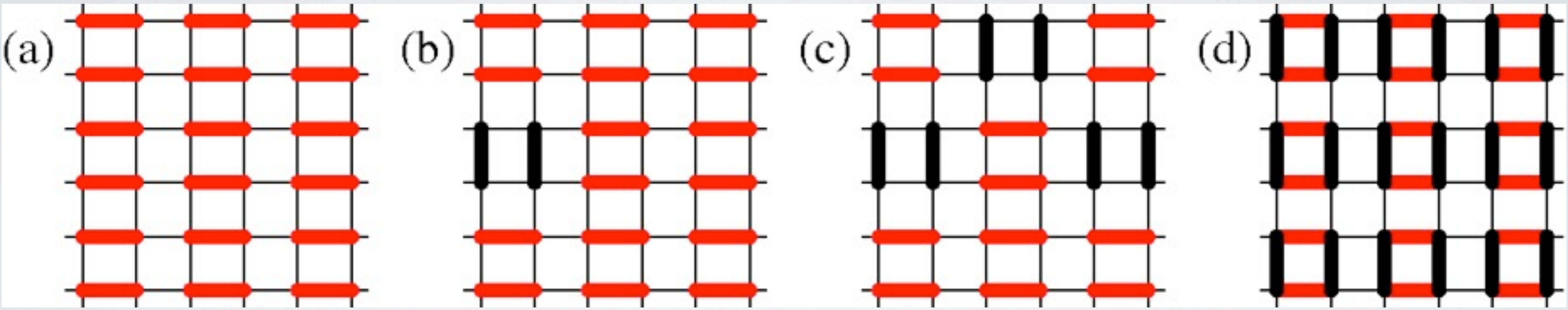
# VBS fluctuations in the theory of deconfined quantum-critical points

[Senthil et al., 2004]

➢ plaquette and columnar VBS are almost degenerate

➢ tunneling barrier seperating the two

- barrier increases with increasing system size L
- barrier decreases as the critical point is approached



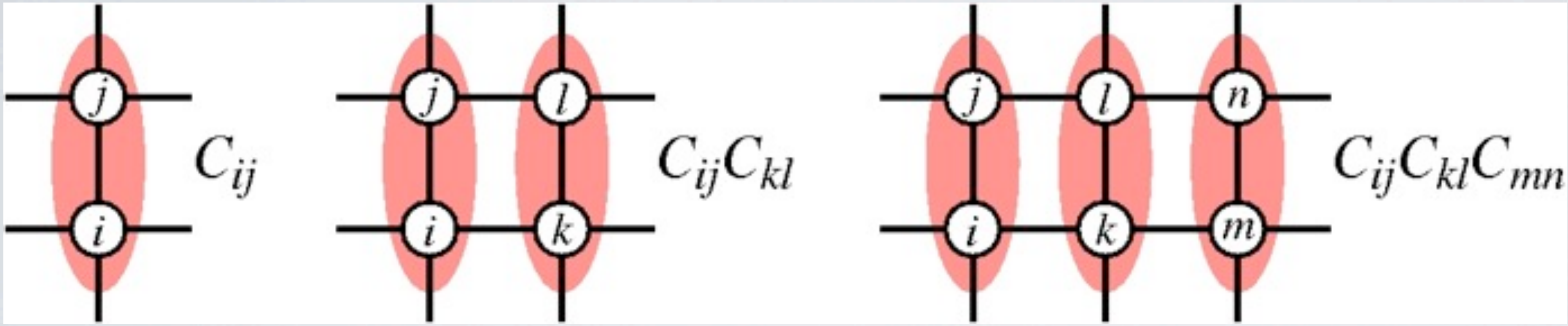

➢ **emergent U(1) symmetry**

➢ **ring-shaped distribution expected in the VBS phase for small systems**

$L < \Lambda \sim \xi^a$, $a>1$ (related to spinon confinement length)

# Creating a more rubust VBS order - the J-$Q_3$ model

J. Lou, A.W. Sandvik, N. Kawashima, PRB (2009)




$$H = -J \sum_{\langle ij \rangle} C_{ij} - Q_3 \sum_{\langle ijklmn \rangle} C_{ij} C_{kl} C_{mn} \qquad C_{ij} = \frac{1}{4} - \vec{S}_i \cdot \vec{S}_j$$

$$q = \frac{Q_3}{J + Q_3}$$

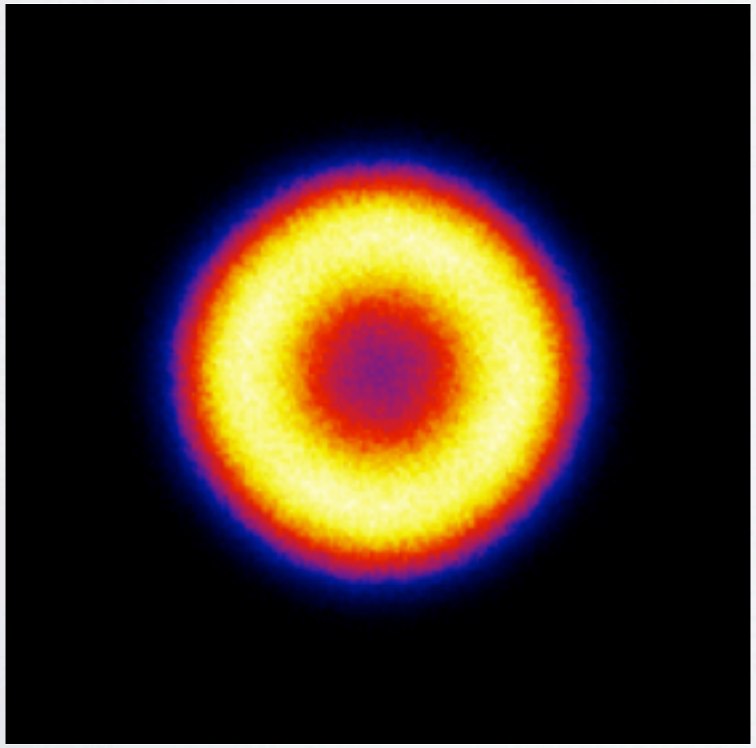
This model has a more robust VBS phase

- can the symmetry cross-over be detected?

$q = 0.635$

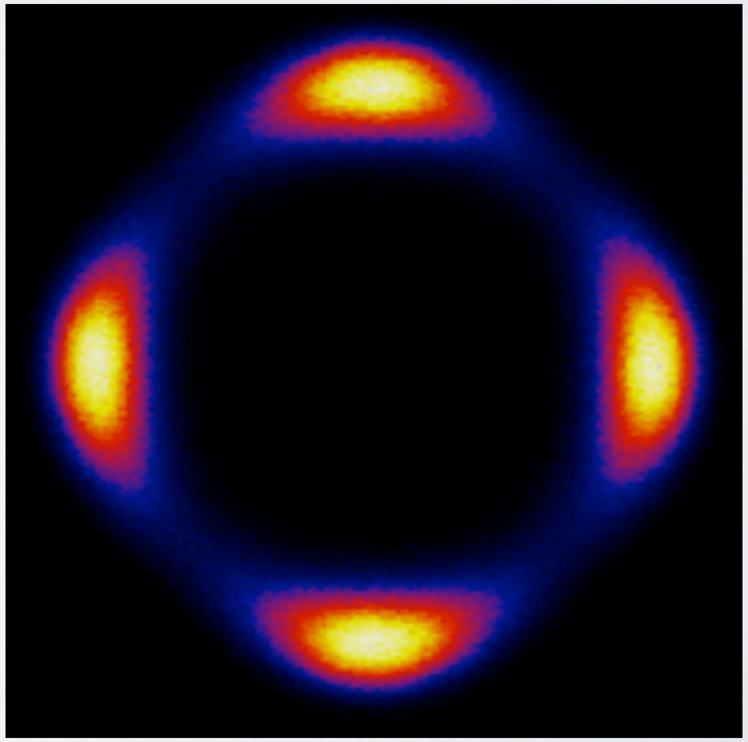
$(q_c \approx 0.60)$

$L = 32$



$q = 0.85$

$L = 32$

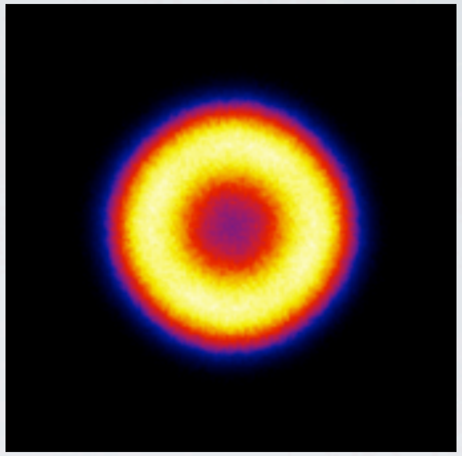


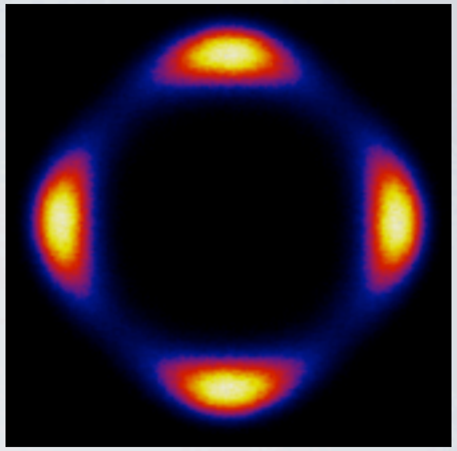
# Analysis of the VBS symmetry cross-over (J-$Q_3$ model)

J. Lou, A.W. Sandvik, N. Kawashima, PRB (2009)

$Z_4$-sensitive VBS order parameter

$$D_4 = \int r dr \int d\phi P(r,\phi)\cos(4\phi)$$





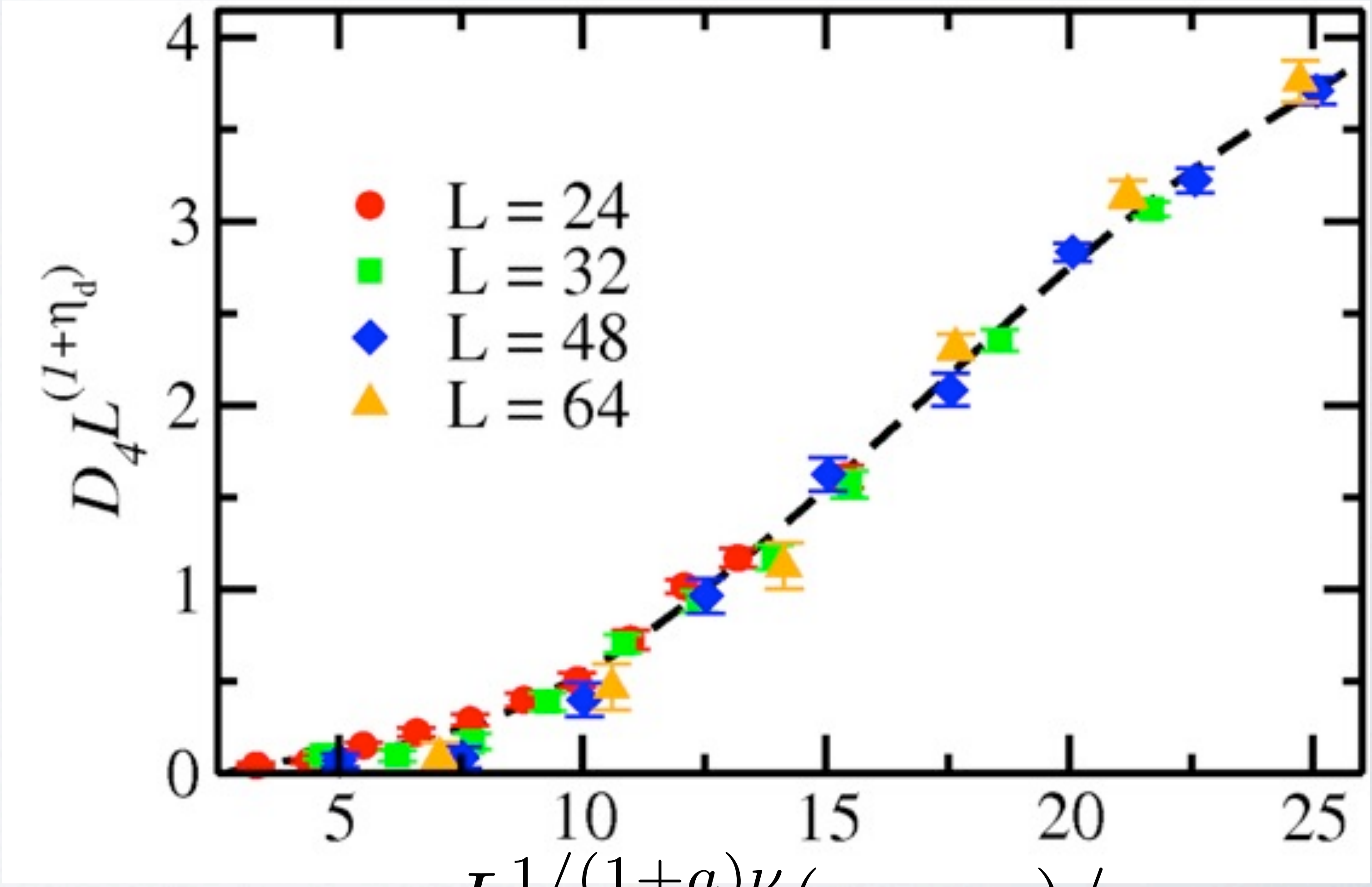
Finite-size scaling gives U(1) length-scale

$$\Lambda \sim \xi^{1+a}$$
$$\sim (q-q_c)^{-(1+a)\nu}$$

$$a = 0.20 \pm 0.05$$

$$SU(3) : a = 0.6 \pm 0.2$$

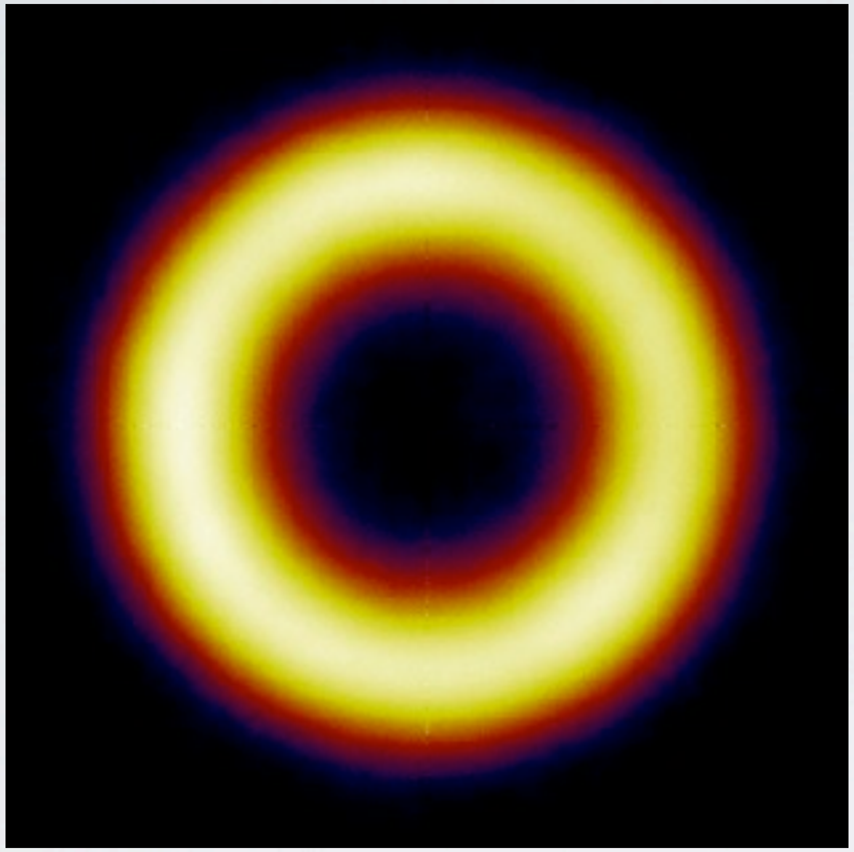
$$SU(4) : a = 0.5 \pm 0.2$$

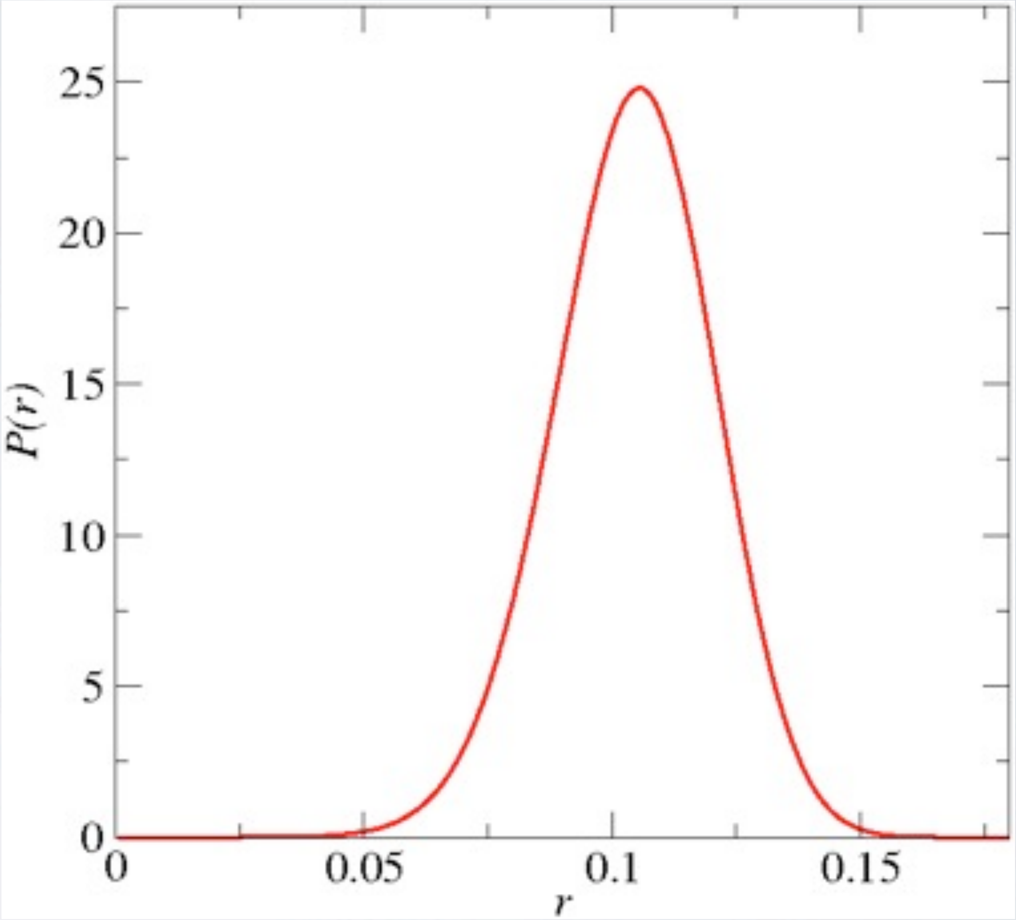



$L^{1/(1+a)\nu}(q-q_c)/q_c$

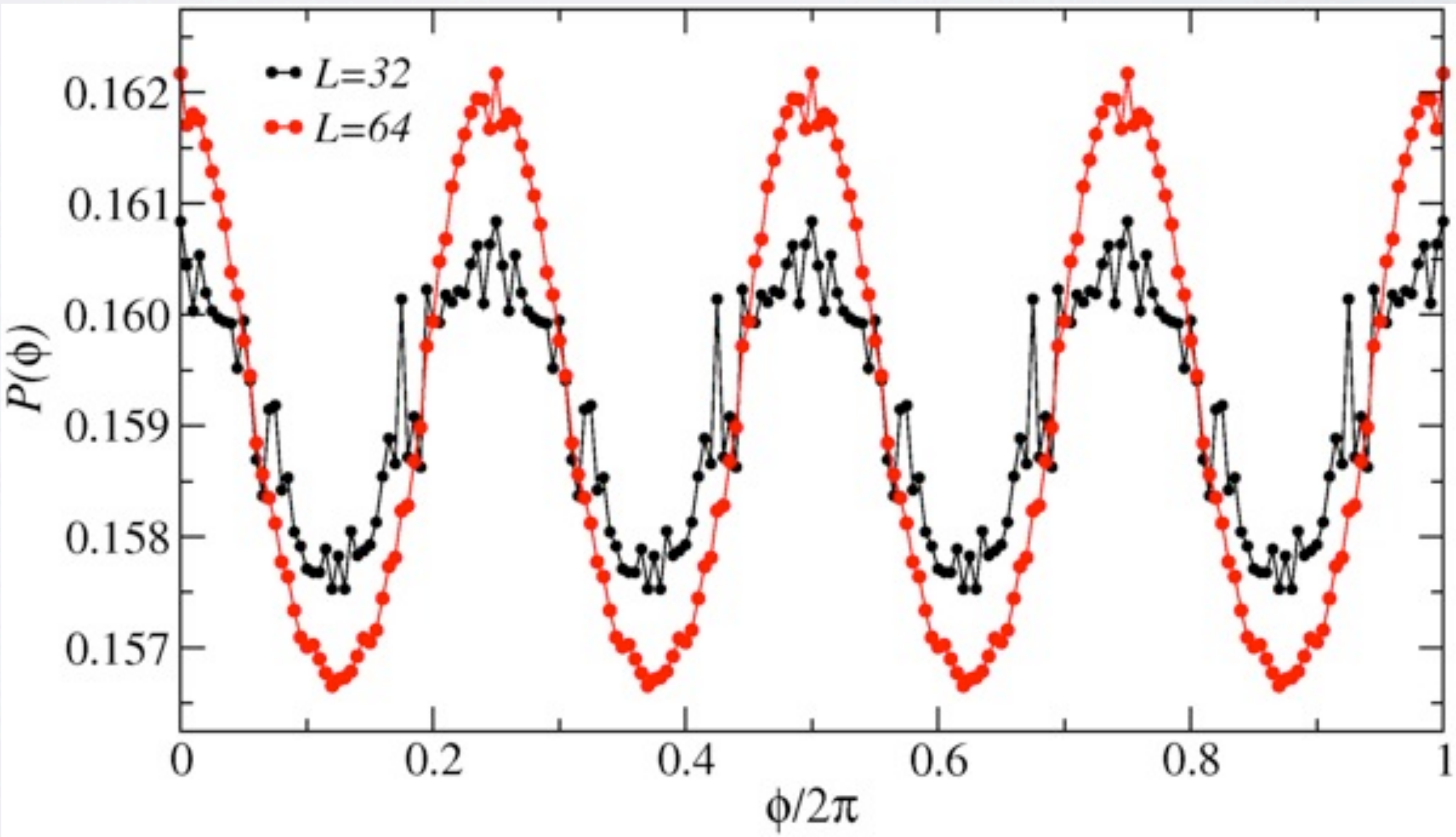
# Signs of $Z_4$ symmetry in the original J-Q model?

**L=128, J=0**
**$P(D_x,D_y)$**



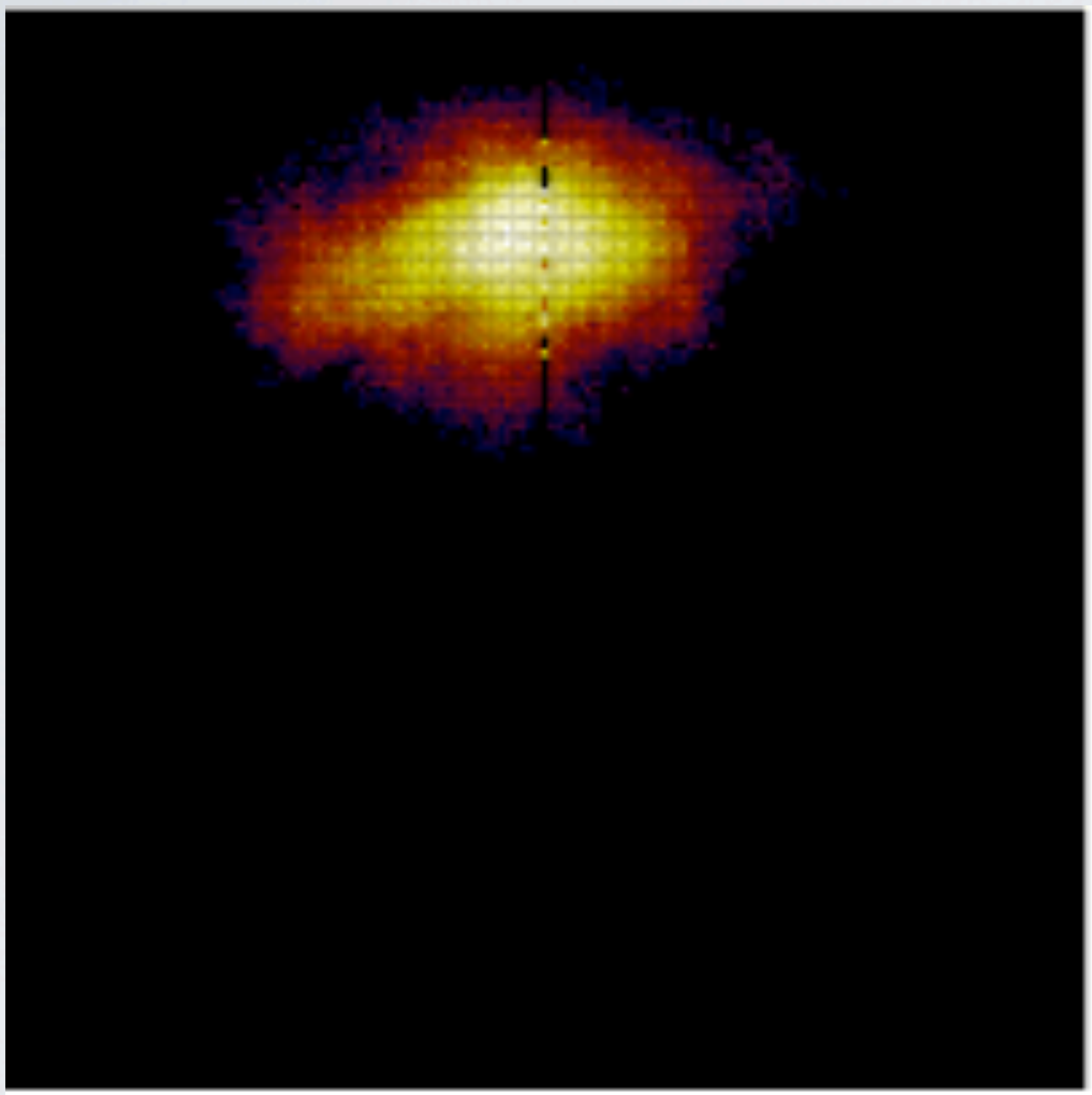



**L=32, L=64; J=0**
Weak but statistically significant angular dependence consistent with **columnar VBS**
(L=128 still too noisy)

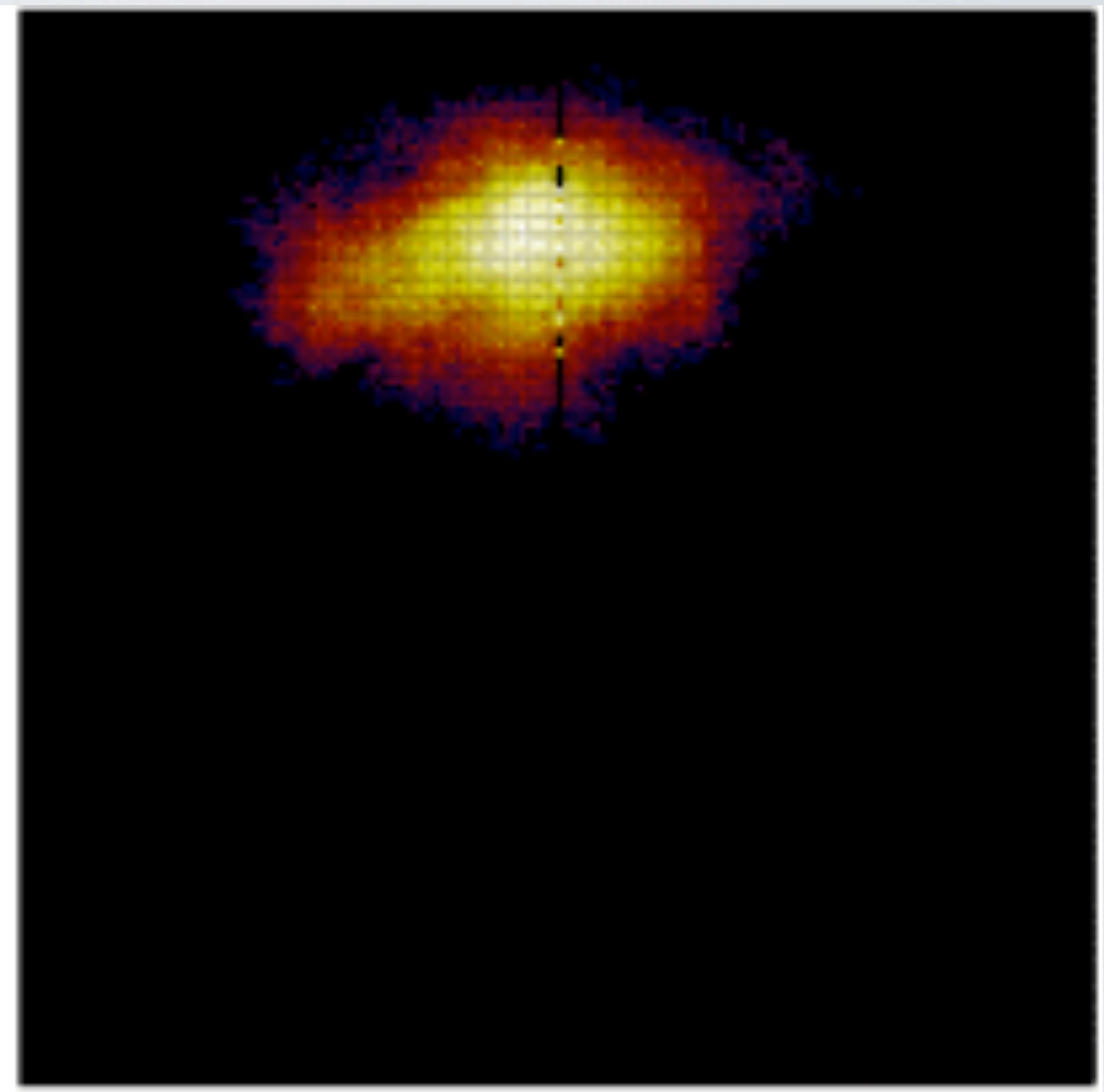


The simulations take a long time to rotate the VBS angle

L=128: $10^5$ measurements require > 1 day of computation



building $100\times10^5$ measurements



$10^5$ measurements

# Conclusions

## Large-scale QMC calculations of the J-Q model

- **scaling behavior consistent with a continuous Neel-VBS transition**
  - with weak scaling corrections; maybe logarithmic
- **no signatures of first-order behavior**
  - cannot be ruled out as a matter of principle, but seems unlikely
- **emergent U(1) symmetric VBS order parameter**

## SU(N) J-Q model and $J_1$-$J_2$ Heisenberg model

- **critical correlation exponents approach large-N results**

## Relation to deconfined quantum-criticality of Senthil et al.

- **Main features in good agreement**
  - z=1 scaling
  - "large" anomalous dimension $\eta_{spin}$
  - emergent U(1) symmetry
- **$NCCP^{N-1}$ field theory for large N**
  [Senthil et al. (PRB 2004), Kaul & Sachdev (PRB 2008)]
  - no log-corrections found analytically
  - difficult to extend to N=2 (3,4) in analytical work
  - could there be log-corrections for N=2 (or general "small" N)?
    - claimed recently by Nogueira & Sudbo (arXiv 2011)

# Could the transition be first-order?

Jiang, Nyfeler, Chandrasekharan, Wiese, JSTAT, P02009 (2008)

**From an antiferromagnet to a valence bond solid: evidence for a first order phase transition**

Kuklov, Matsumoto, Prokof'ev, Svistunov, Troyer, PRL 101, 050405 (2008)

**Deconfined Criticality: Generic First-Order Transition in the SU(2) Symmetry Case**

One can never, strictly speaking, rule out a very weak first-order transition

- **but are there any real signs of this in the J-Q model?**

The above studies were based on scaling of winding numbers

- claimed signs of phase coexistence (finite spin stiffness and susceptibility)

$$\langle W^2 \rangle = \langle W_x^2 \rangle + \langle W_y^2 \rangle + \langle W_\tau^2 \rangle$$
$$= 2\beta\rho_s + \frac{4N}{\beta}\chi$$

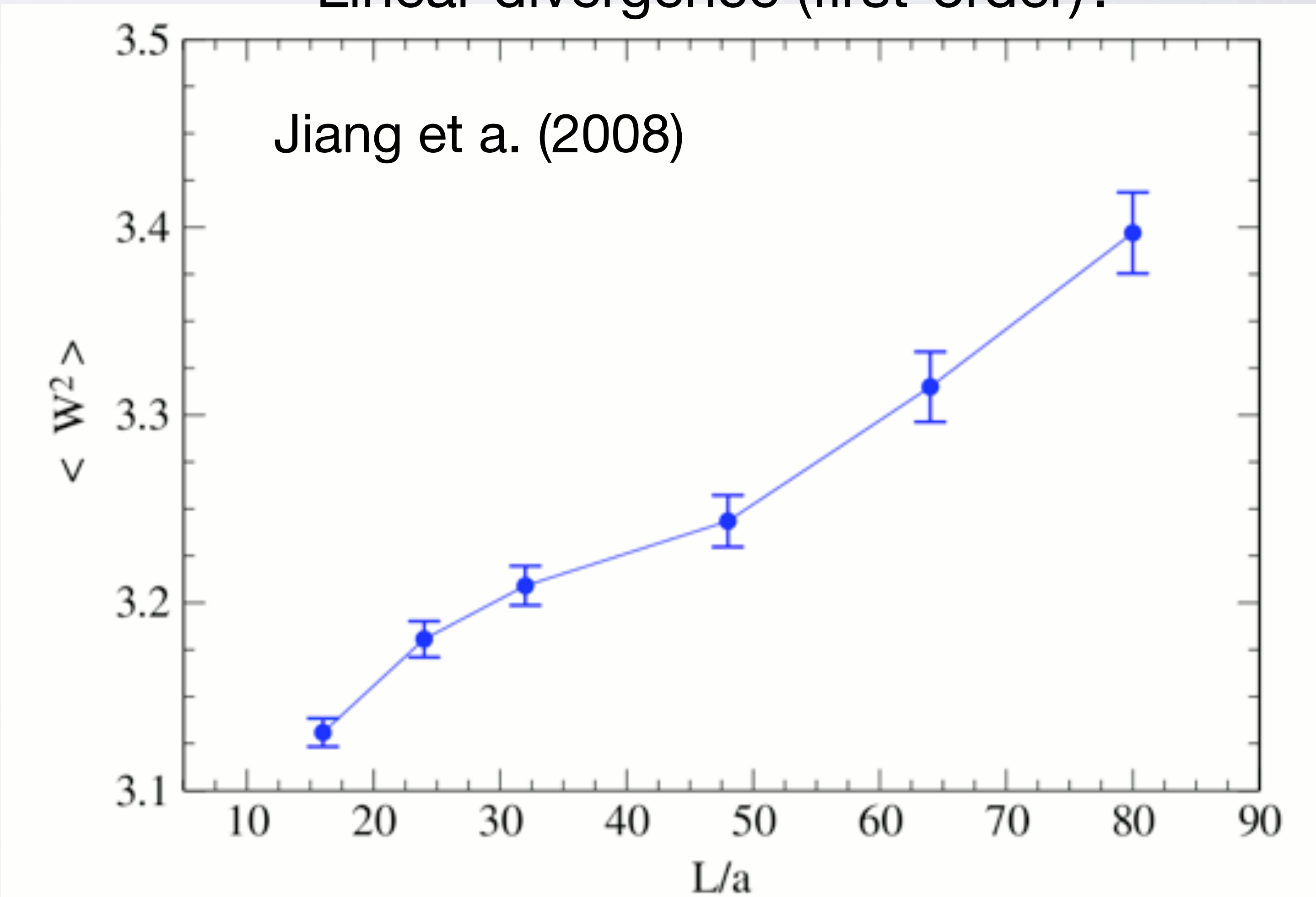
At at a critical point

$$z = 1, \beta \propto L \rightarrow$$

$$\rho_s \propto L^{-1}, \quad \chi \propto L^{-1}$$

$$\rightarrow \langle W^2 \rangle = \text{constant}$$

Linear divergence (first-order)?



# Recent large-scale QMC results

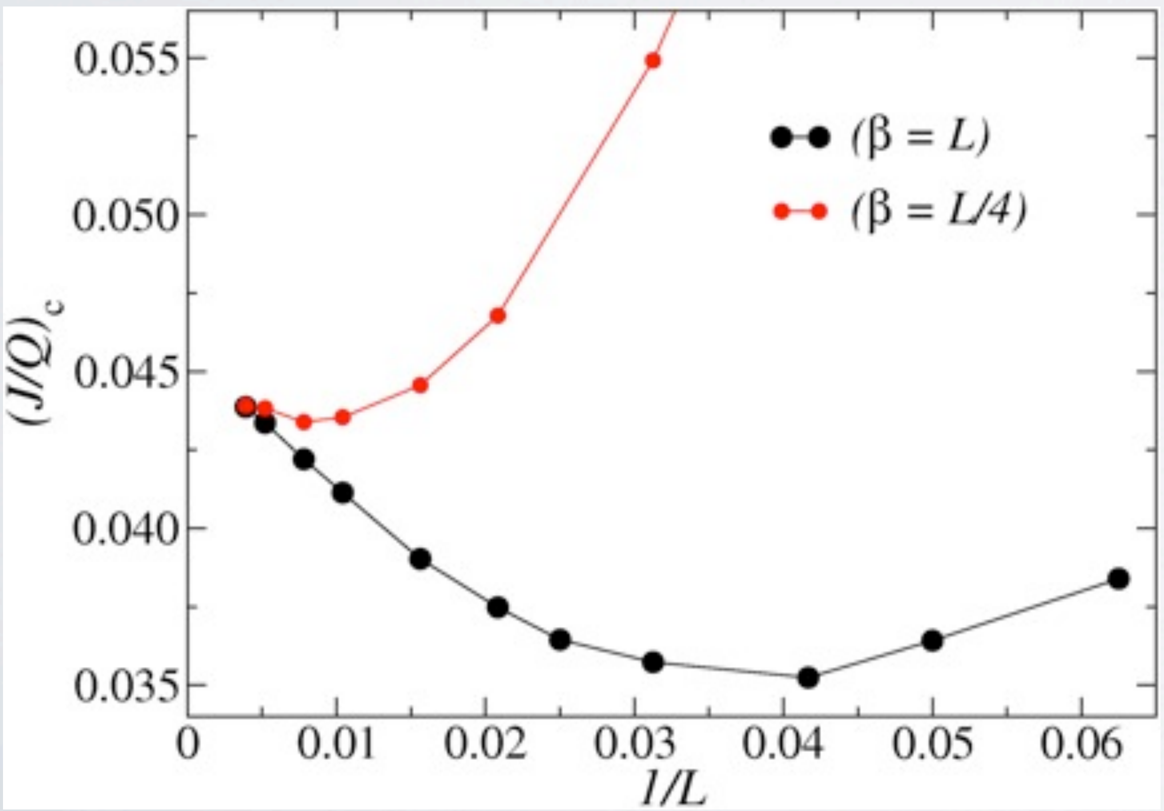
Sandvik, PRL 104, 177201 (2010)

- Stochastic series expansion
- up to 256×256 lattices

$$\beta \propto L \ (\beta = L, \ \beta = L/4)$$

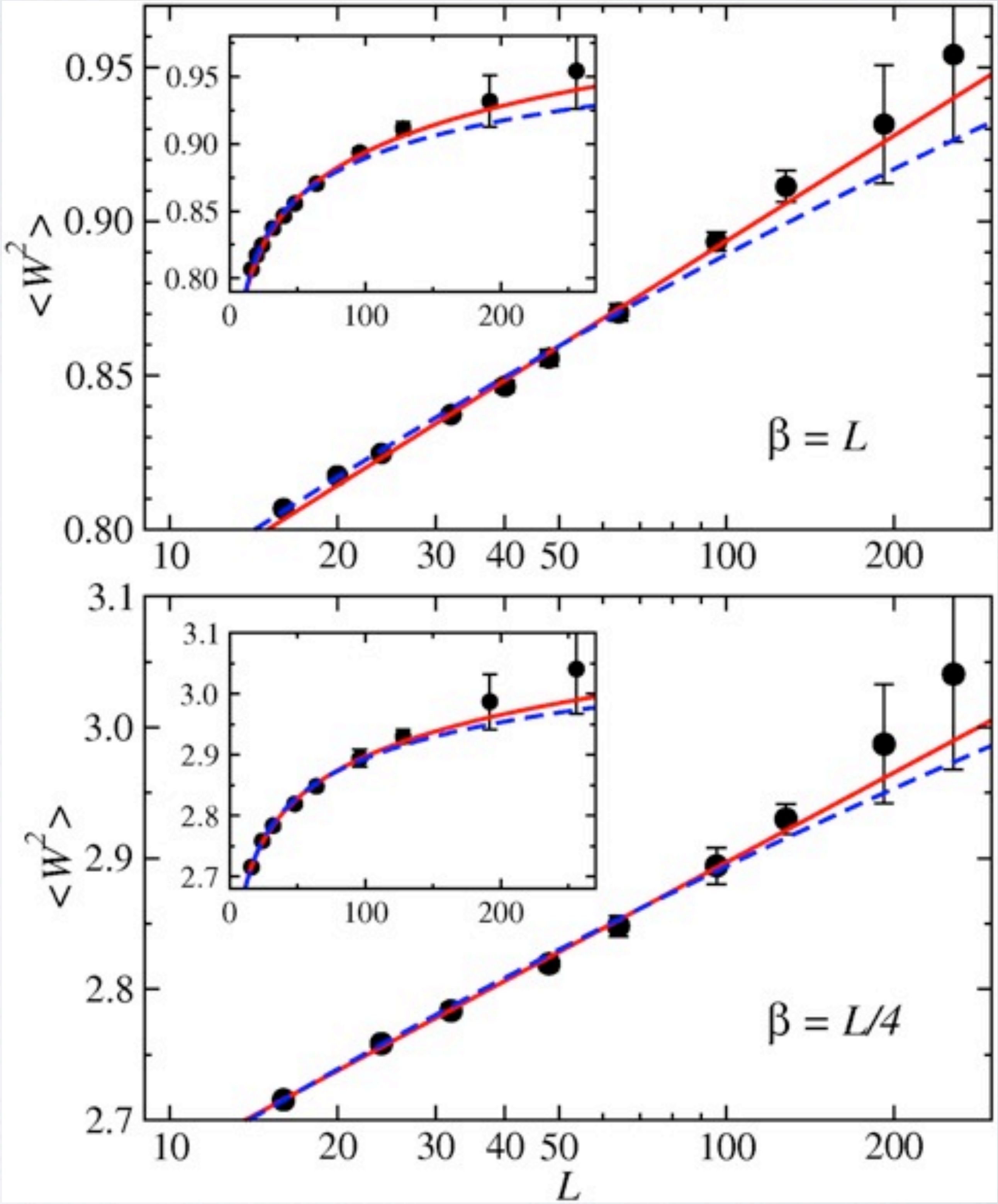
Same finite-size definition of critical point as used by Kuklov et al. and Jiang et al.

- fixed probability of the generated configurations having $W_x = W_y = W_\tau = 0$




## Logarithmic divergence of <W²>

- scaling correction (not 1st-order)



Let's look at a well known signal of a first-order transition:

**Binder ratio**

$$Q_2 = \frac{\langle m^4 \rangle}{\langle m^2 \rangle^2}$$

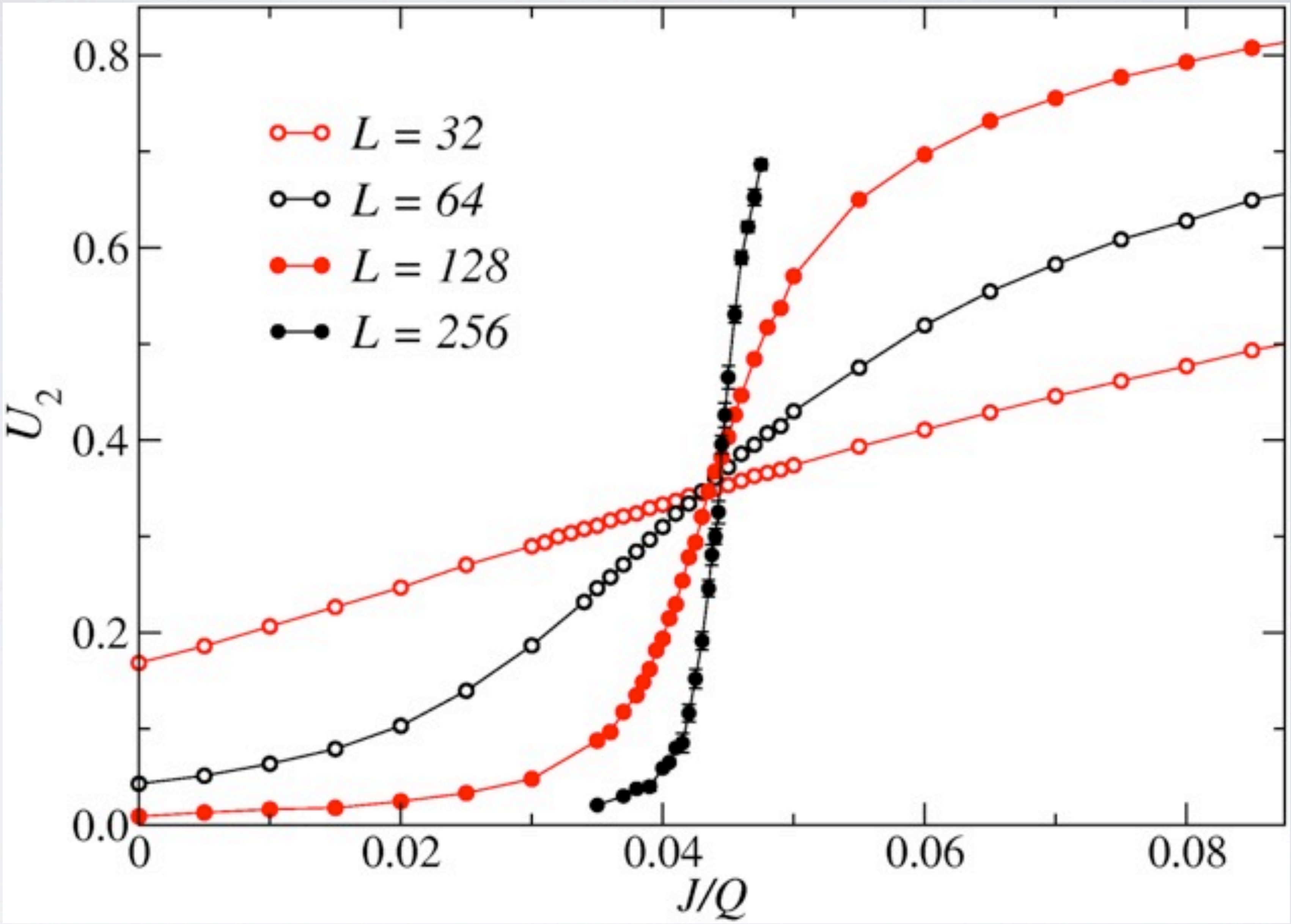
**Binder cumulant**

$$U_2 = (5 - 3Q_2)/2$$

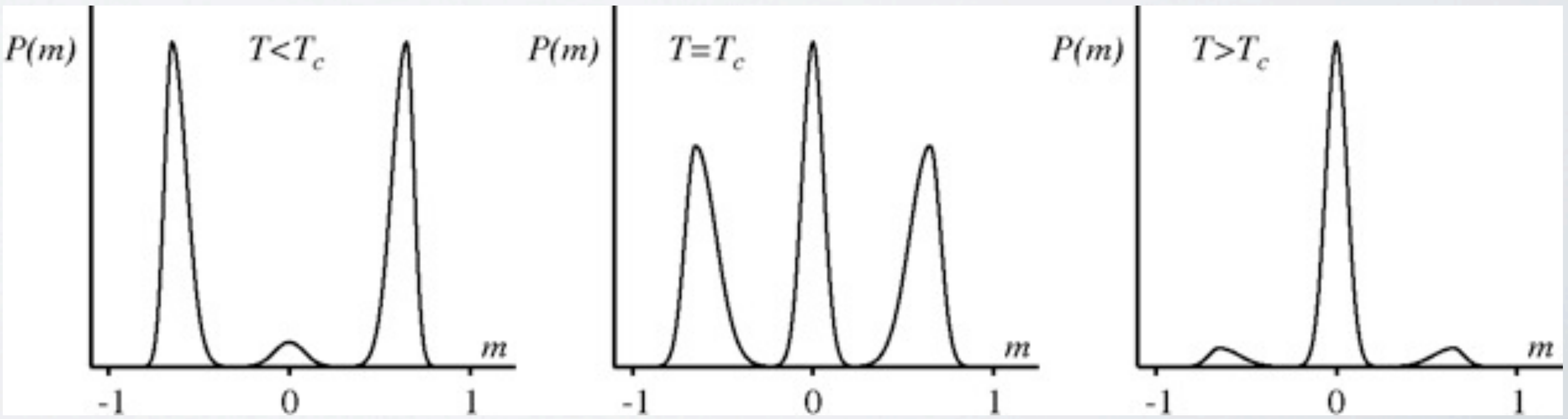
Size independent (curve crossings) at criticality

$U_2 < 0$ at a first-order transition

- no signs of $U_2<0$ in SSE QMC results for L up to 256




Example: Scalar order parameter at classical transition



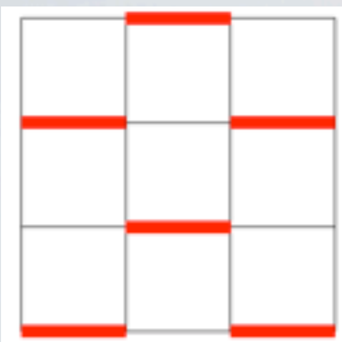

**Phase coexistence** leads to $U_2 \to -\infty$ at 1st-order trans

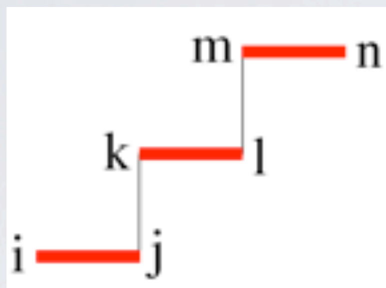
# Example of a first-order Néel - VBS transition

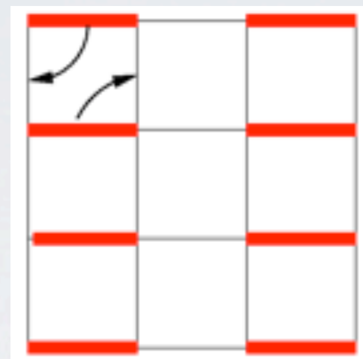
J-Q model with staggered VBS phase [A. Sen, A. Sandvik, PRB (2010)]

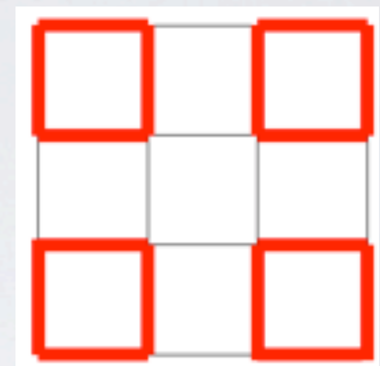
- no local VBS fluctuations favoring emergent U(1) symmetry

$$H = -J \sum_{\langle ij \rangle} C_{ij} - Q_3 \sum_{\langle ijklmn \rangle} C_{ij} C_{kl} C_{mn} \qquad C_{ij} = \frac{1}{4} - \vec{S}_i \cdot \vec{S}_j$$

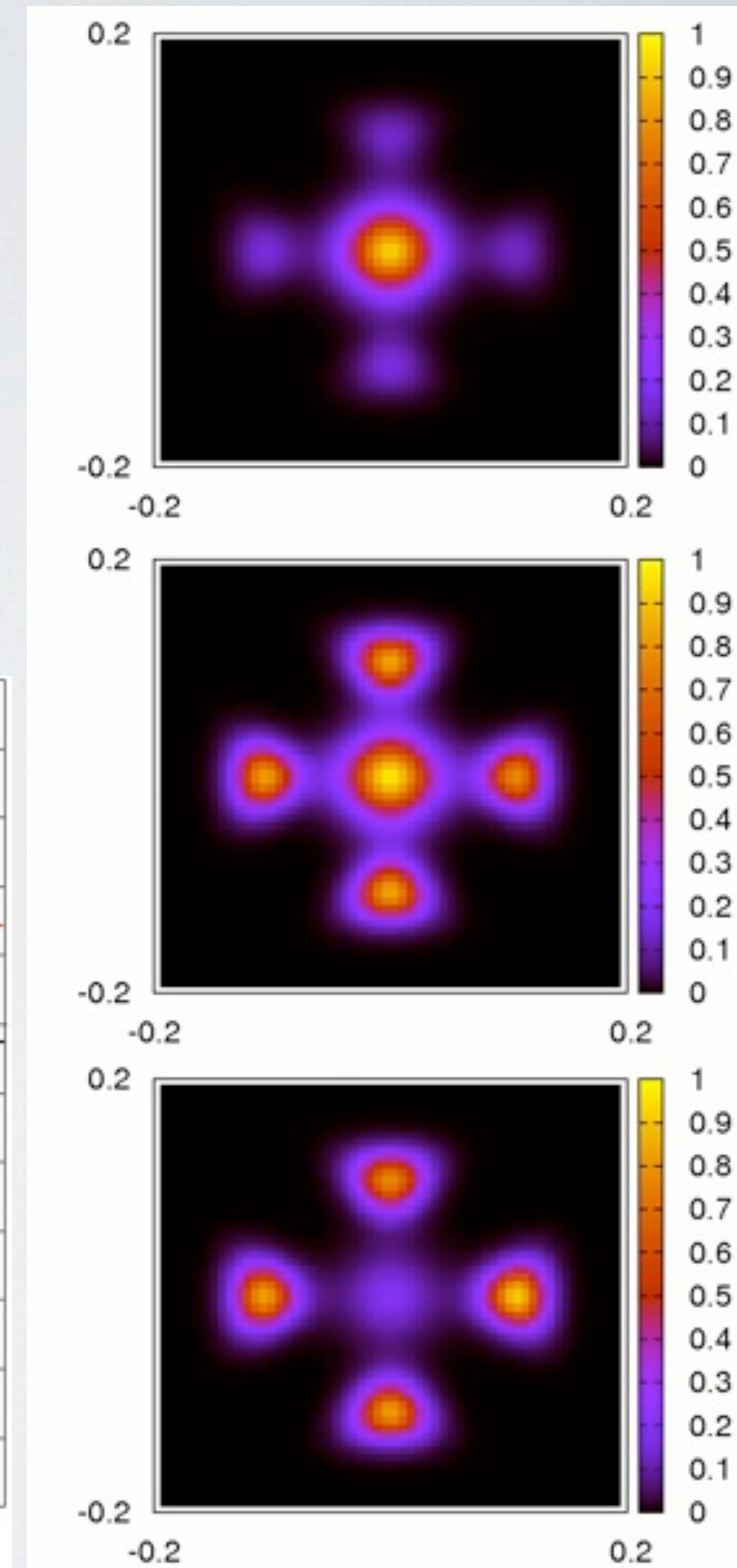




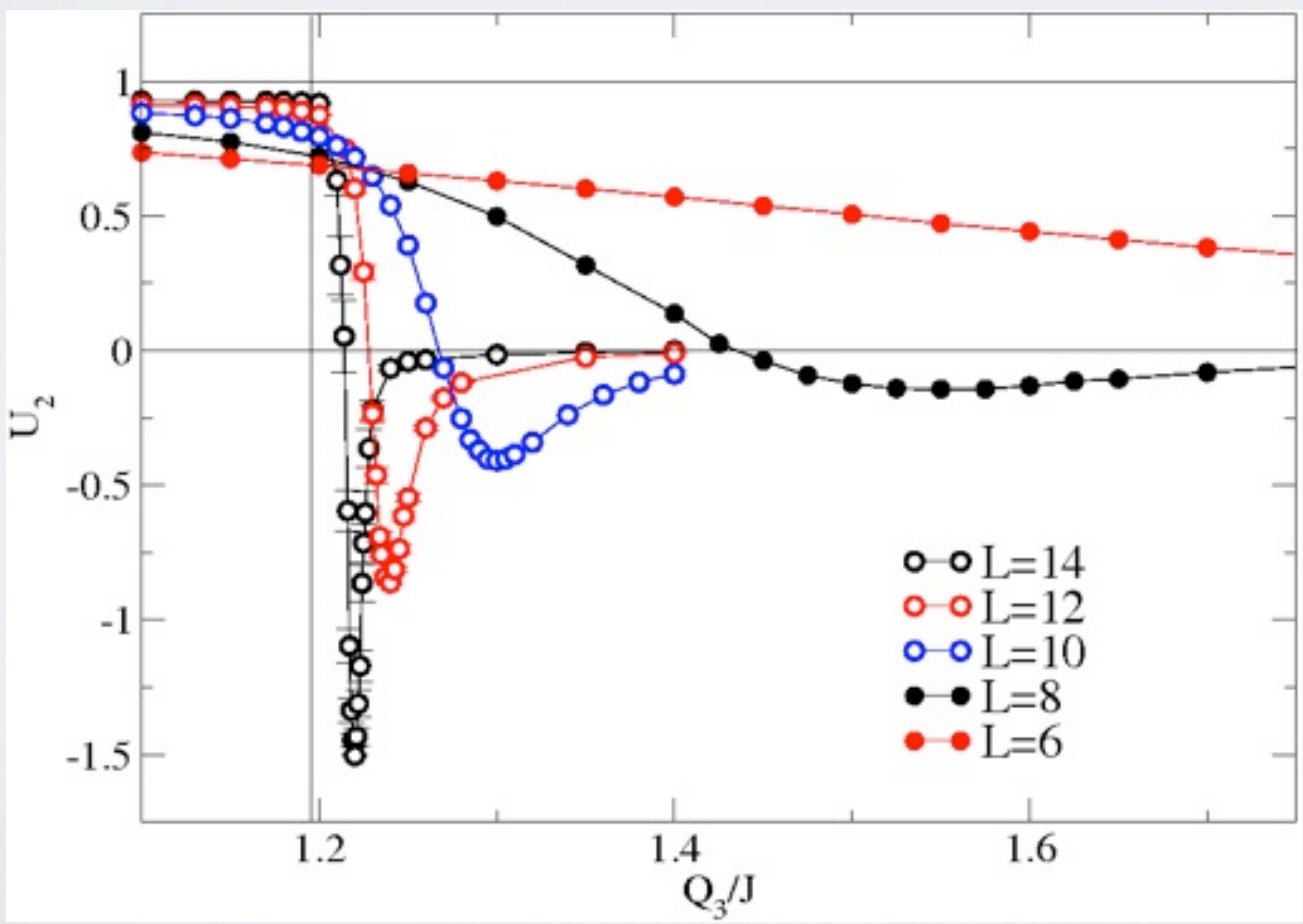


VBS




- clear signs of phase coexistence

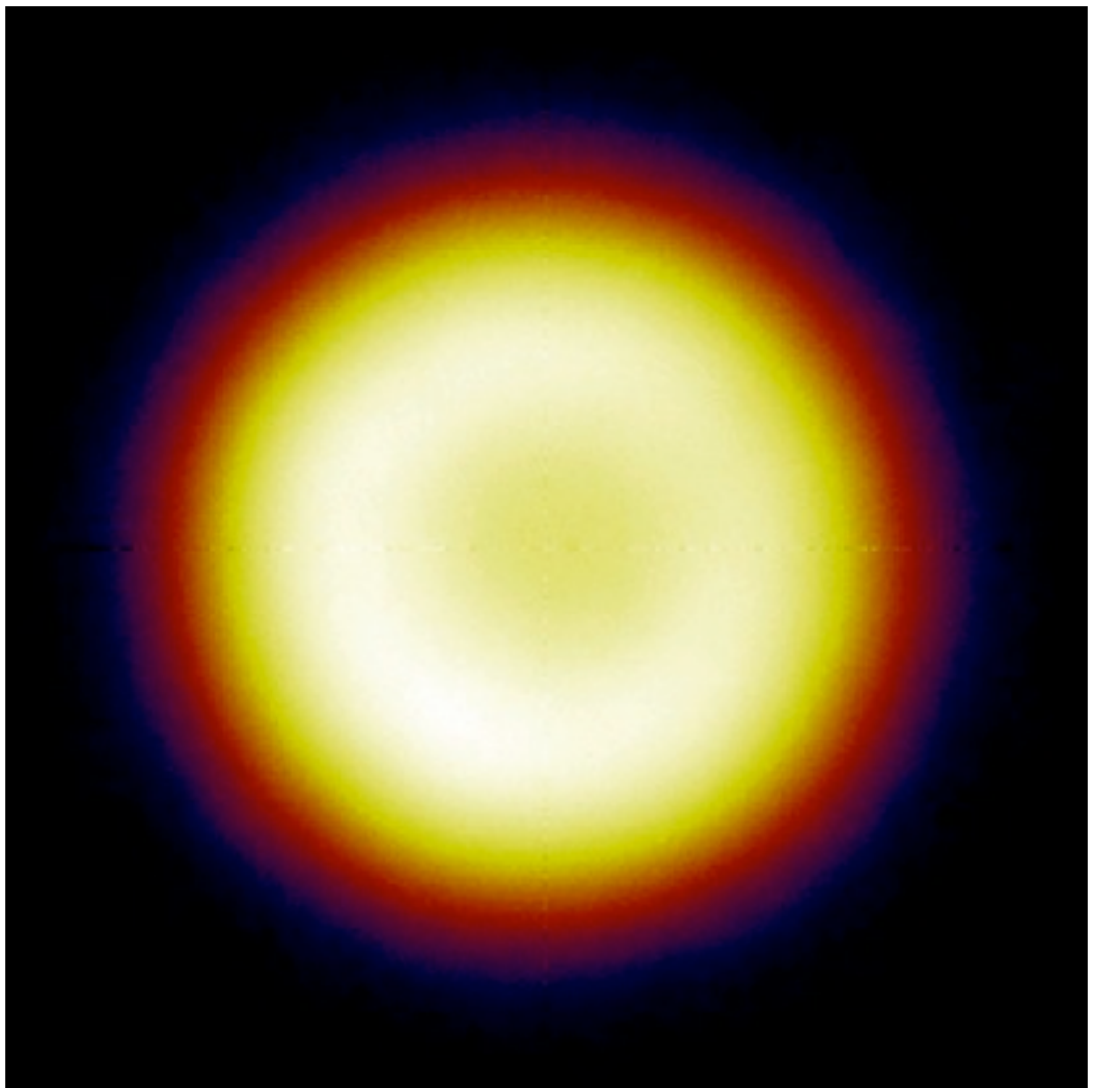
Binder cumulant of the Neel order

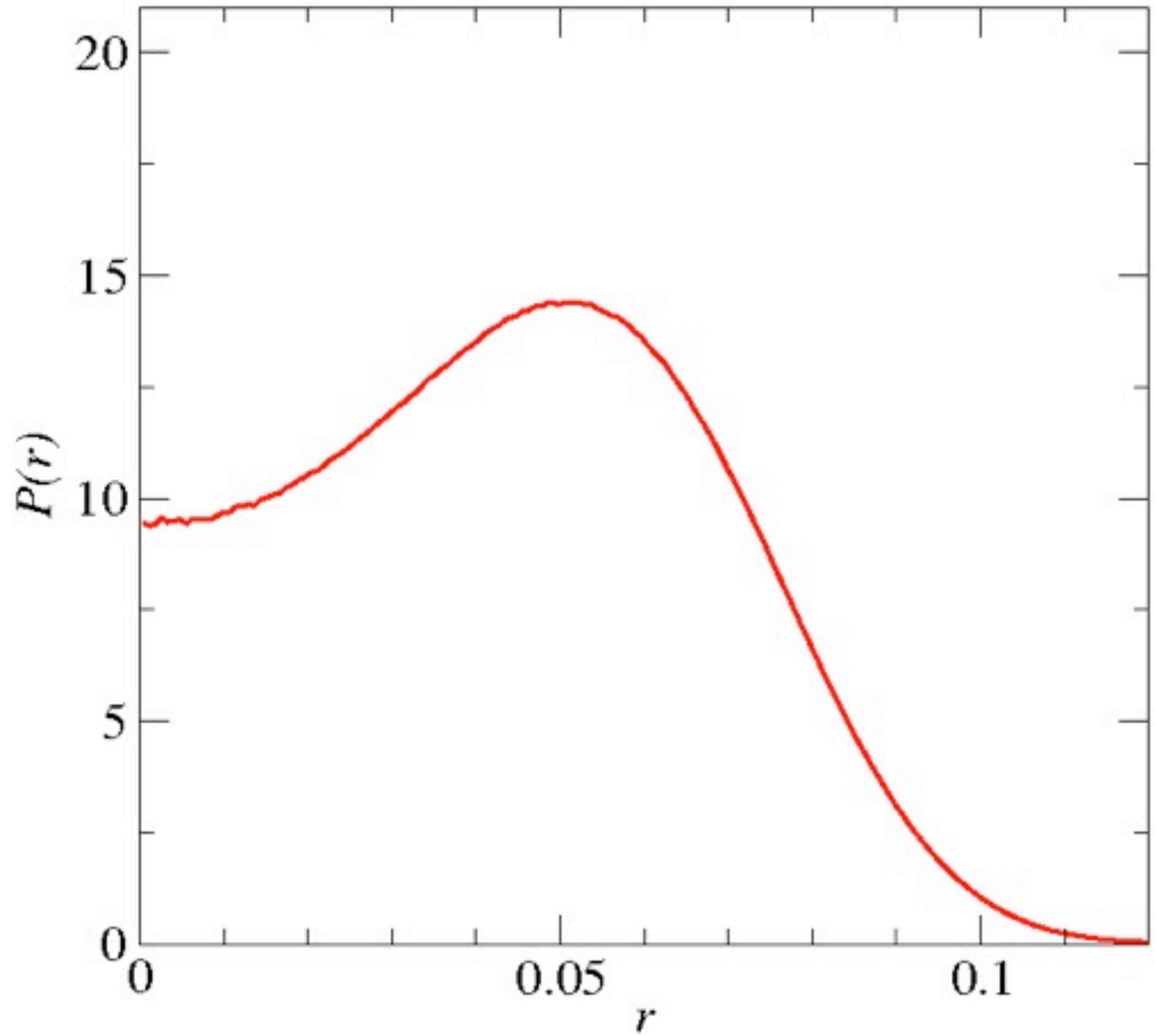


Any signs of coexistence in the standard J-Q VBS distributions?

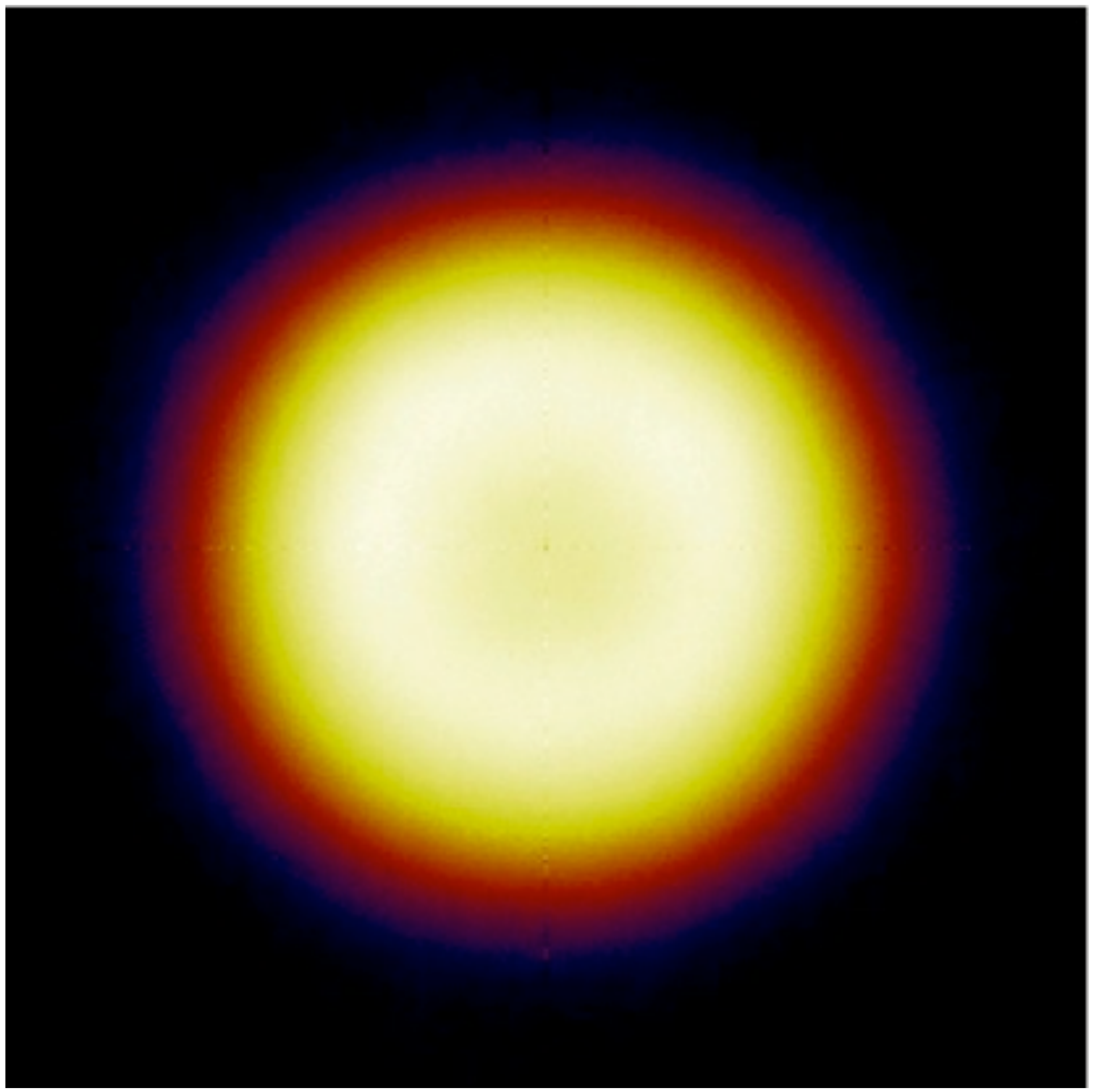
- L=128 data close to the transition

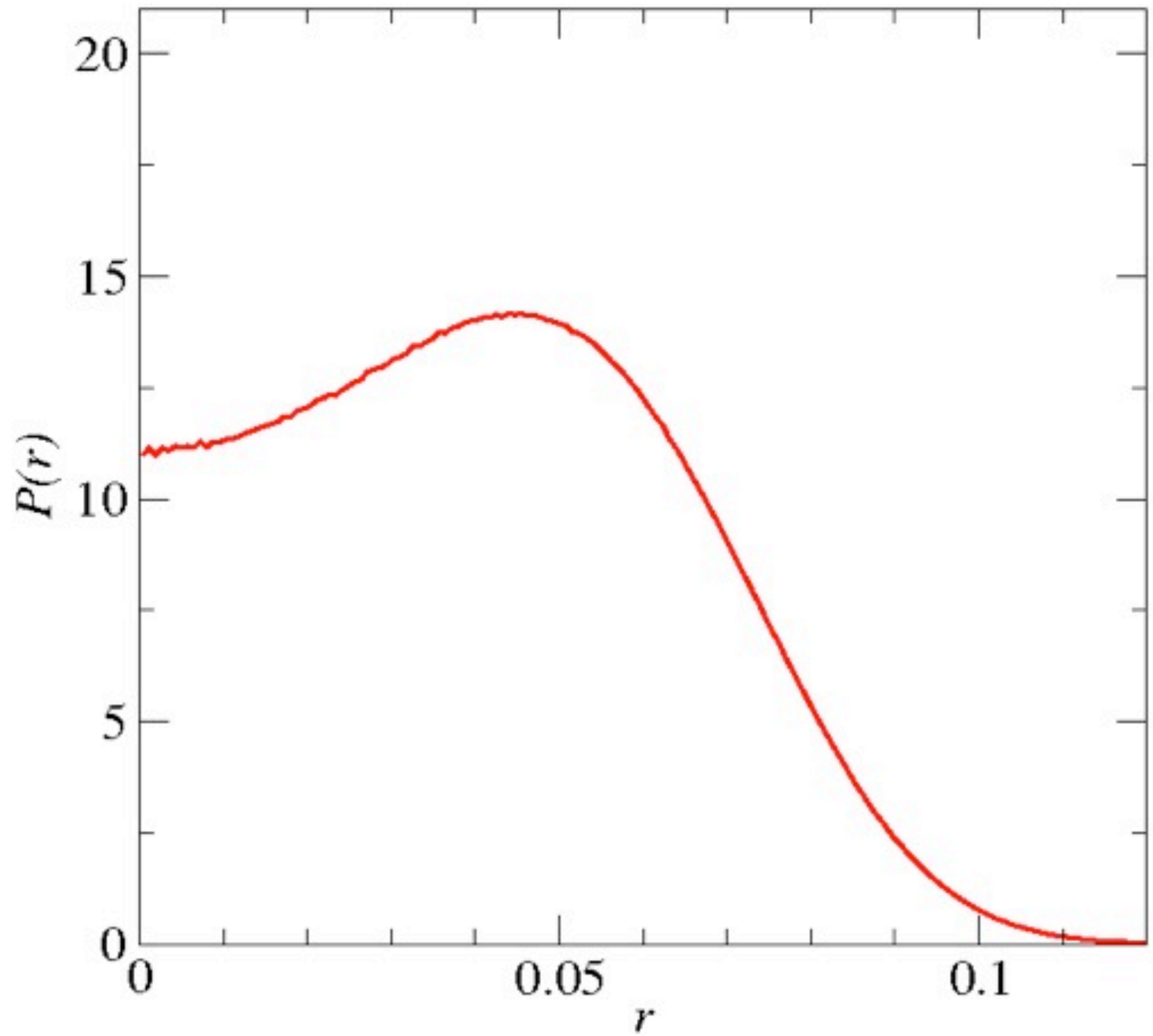
**J/Q=0.040**



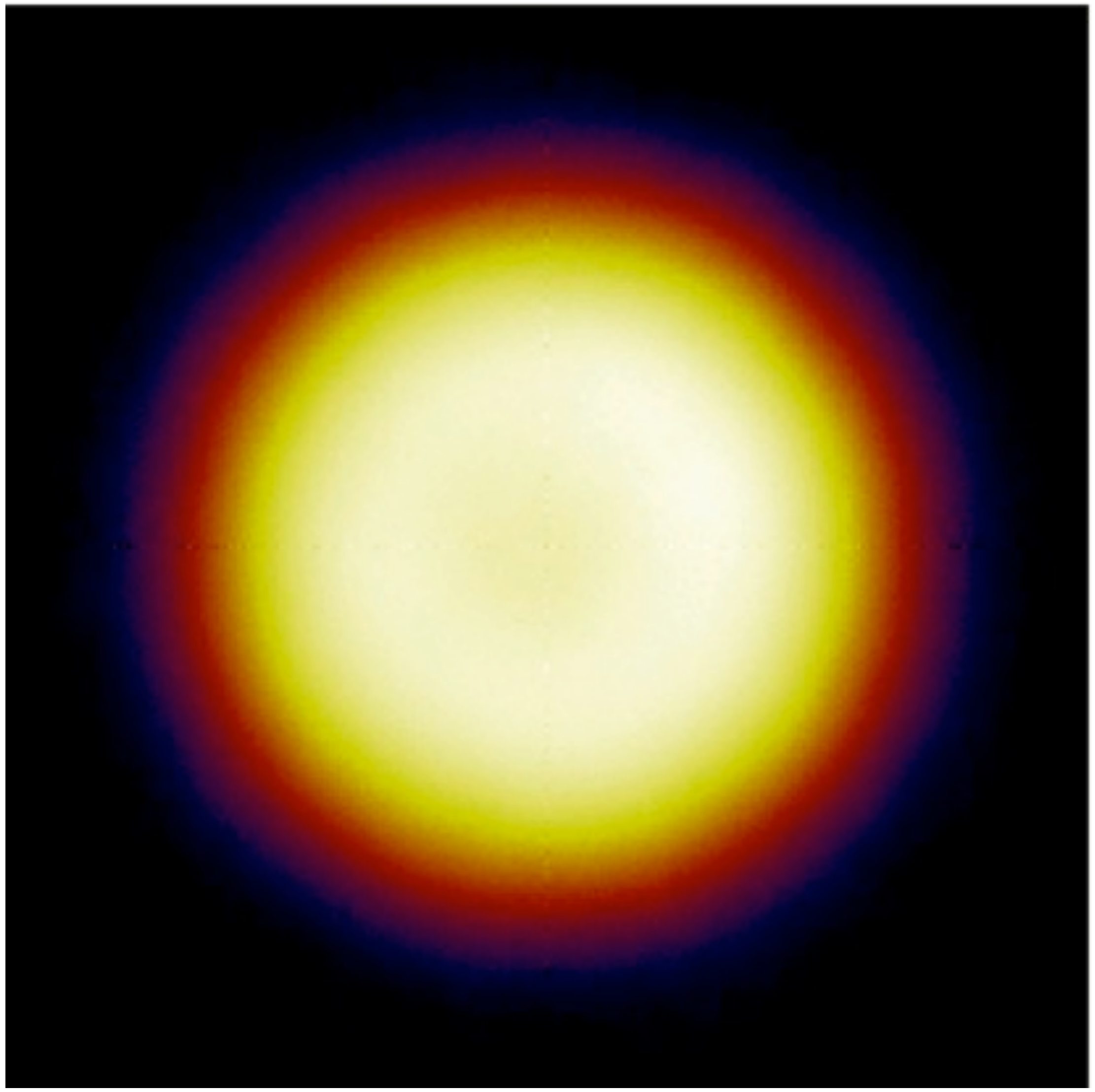


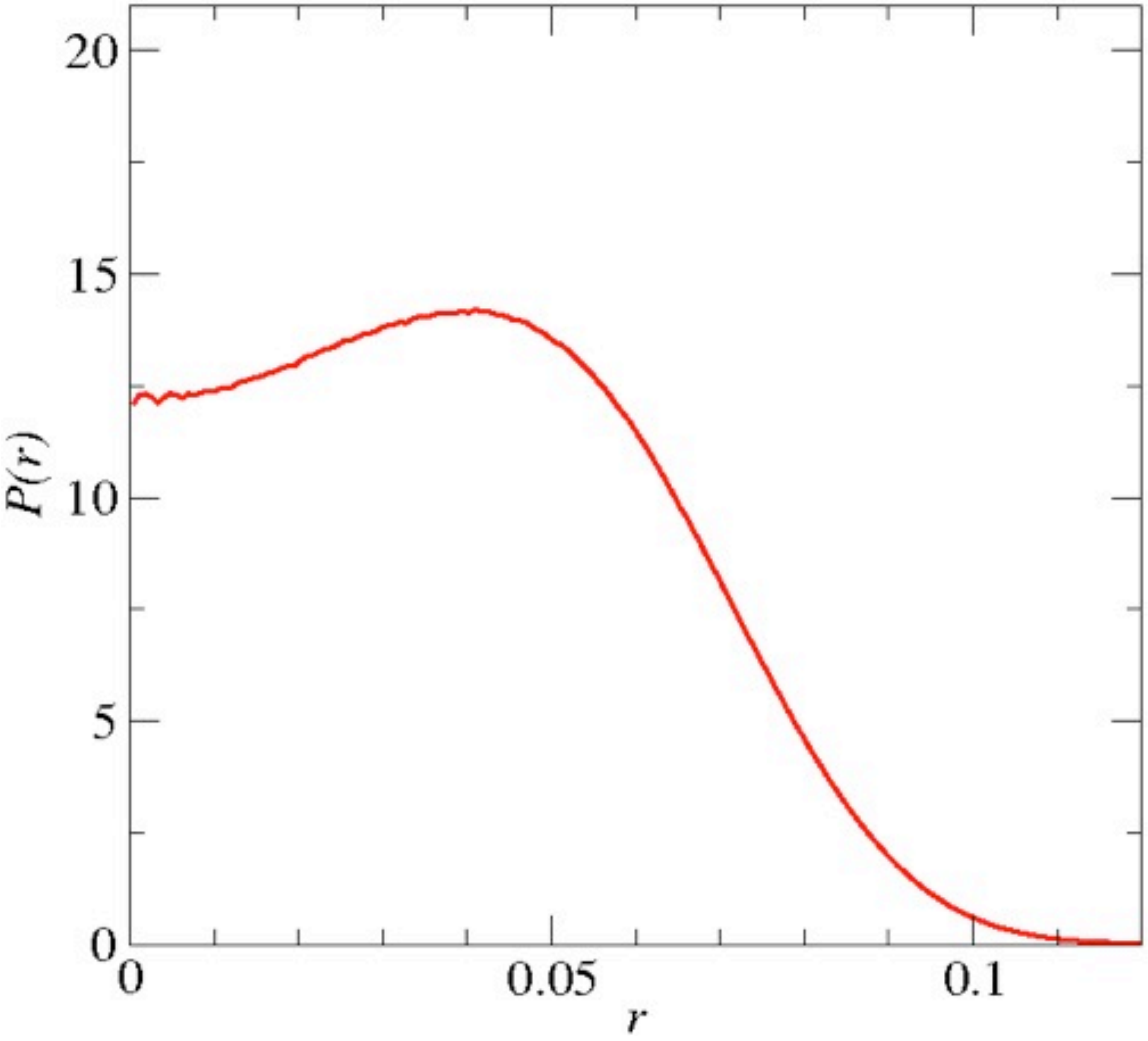
**J/Q=0.041**



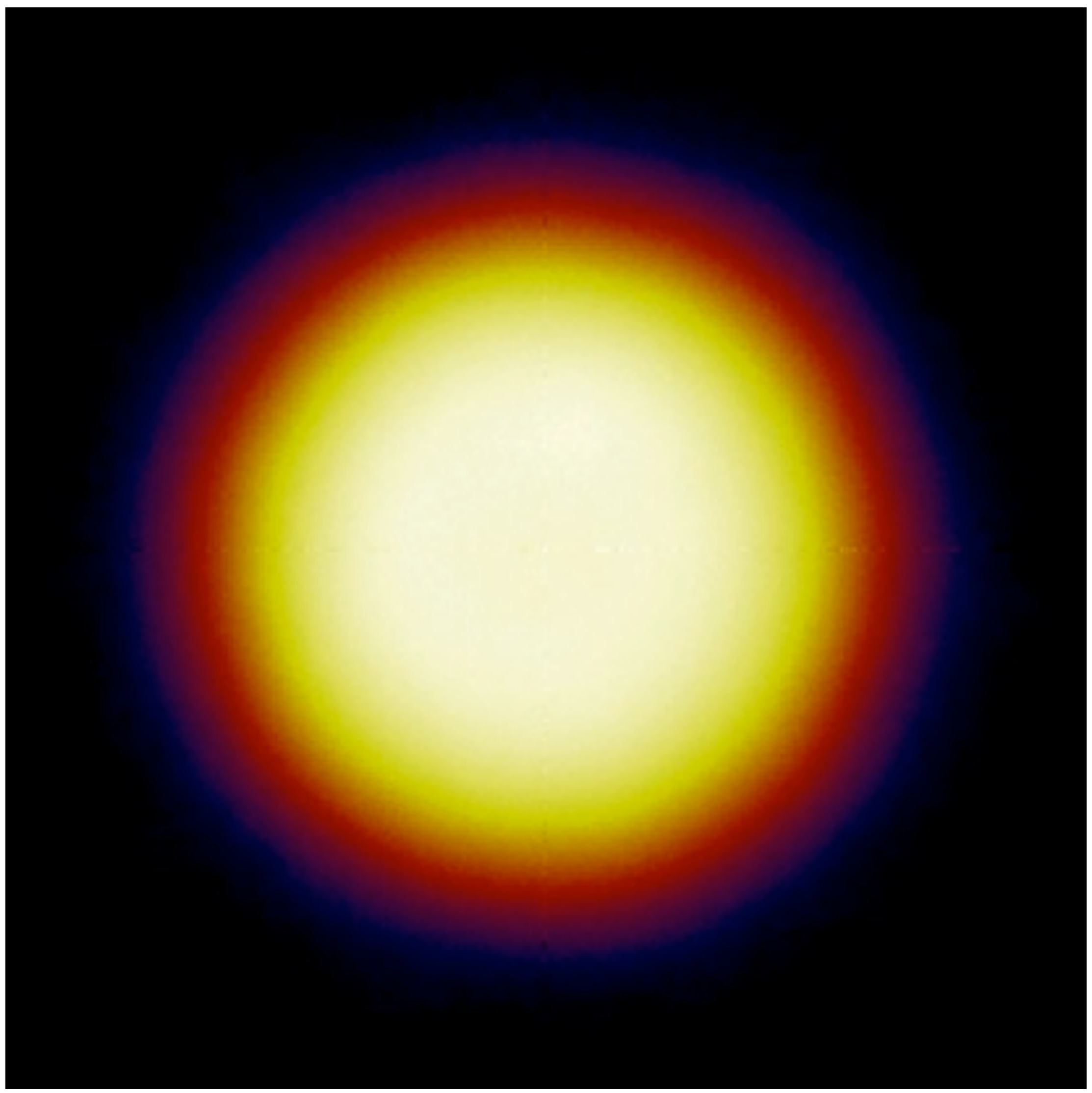


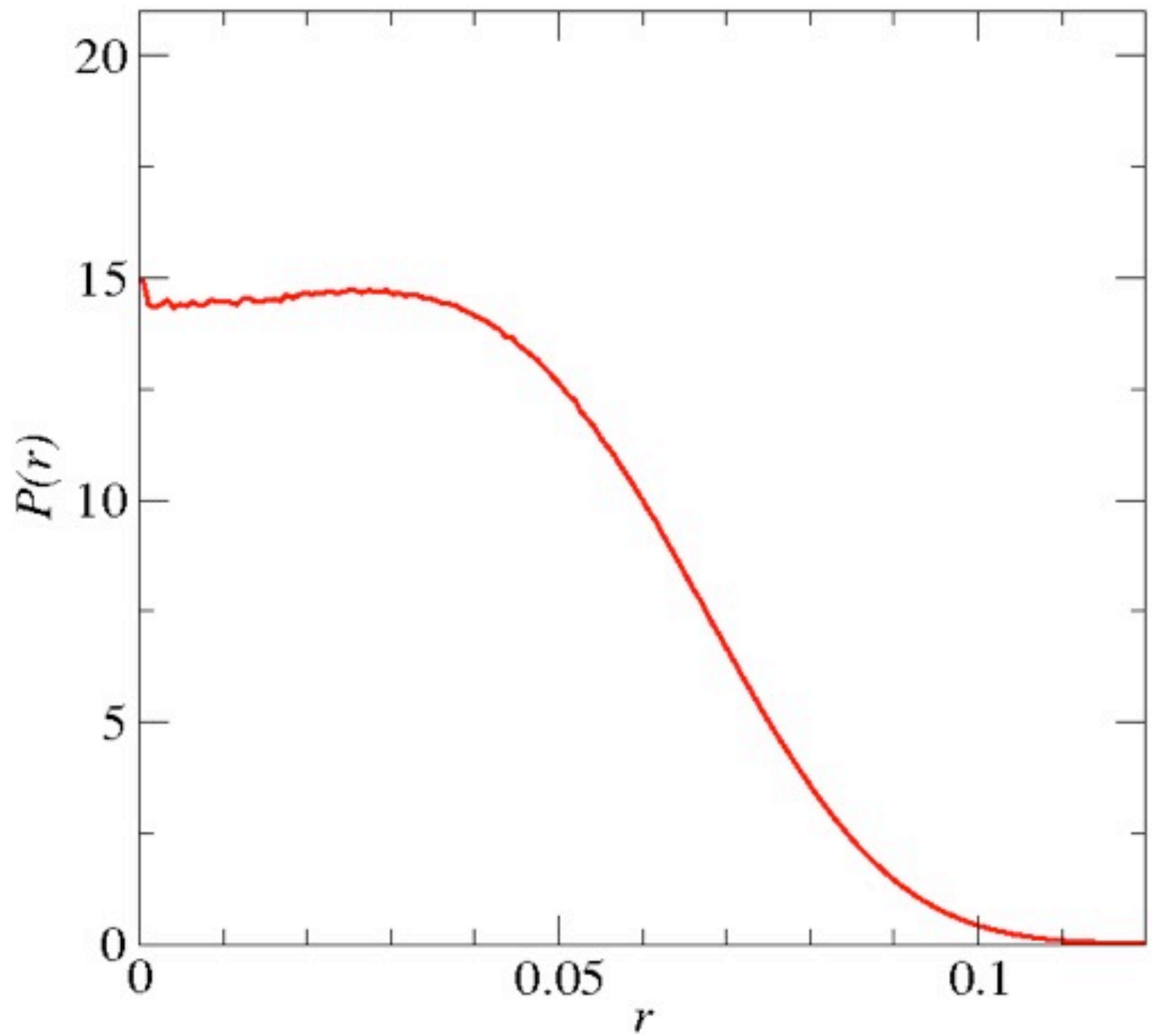
**J/Q=0.042**



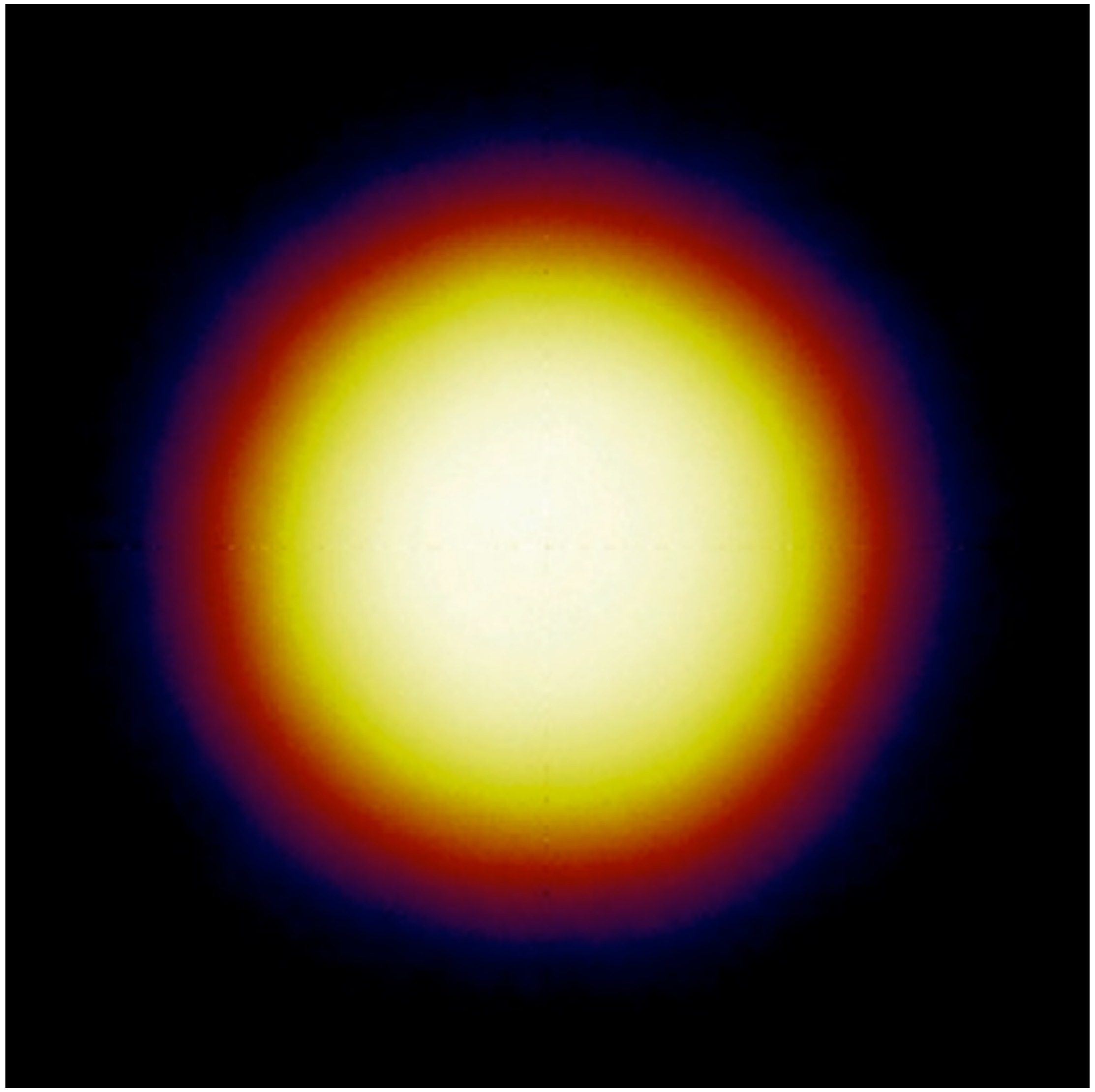


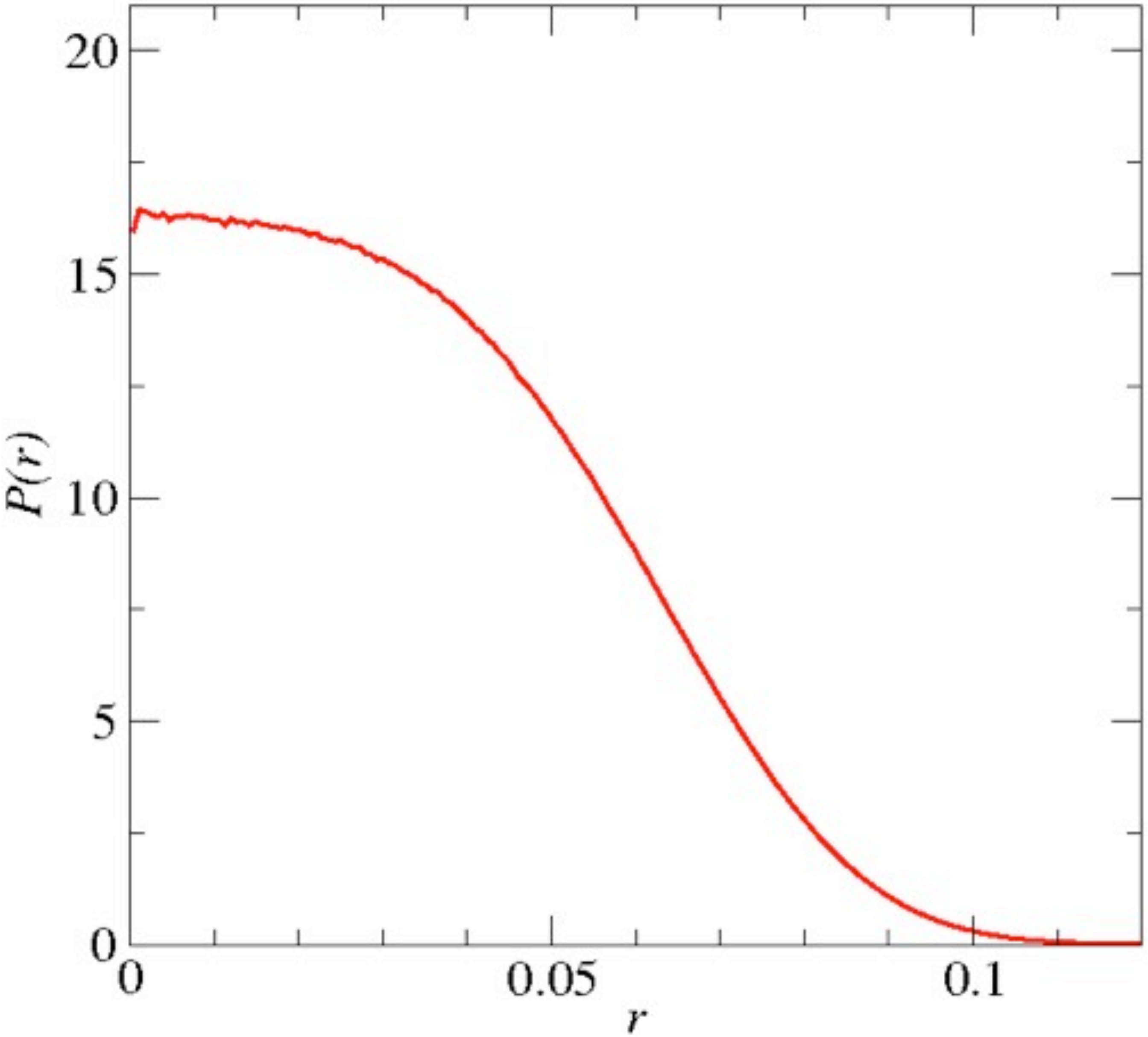
**J/Q=0.043**



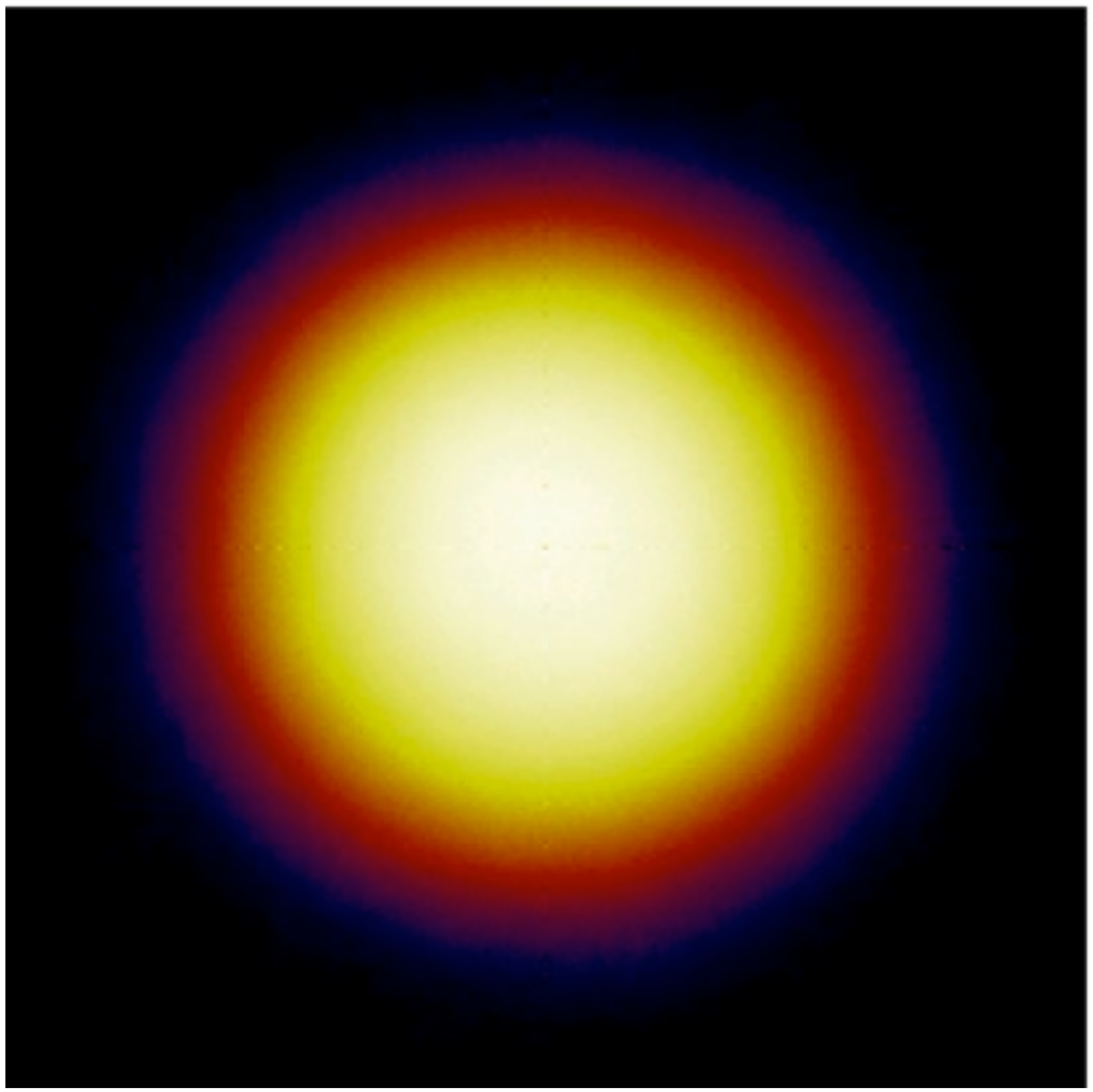


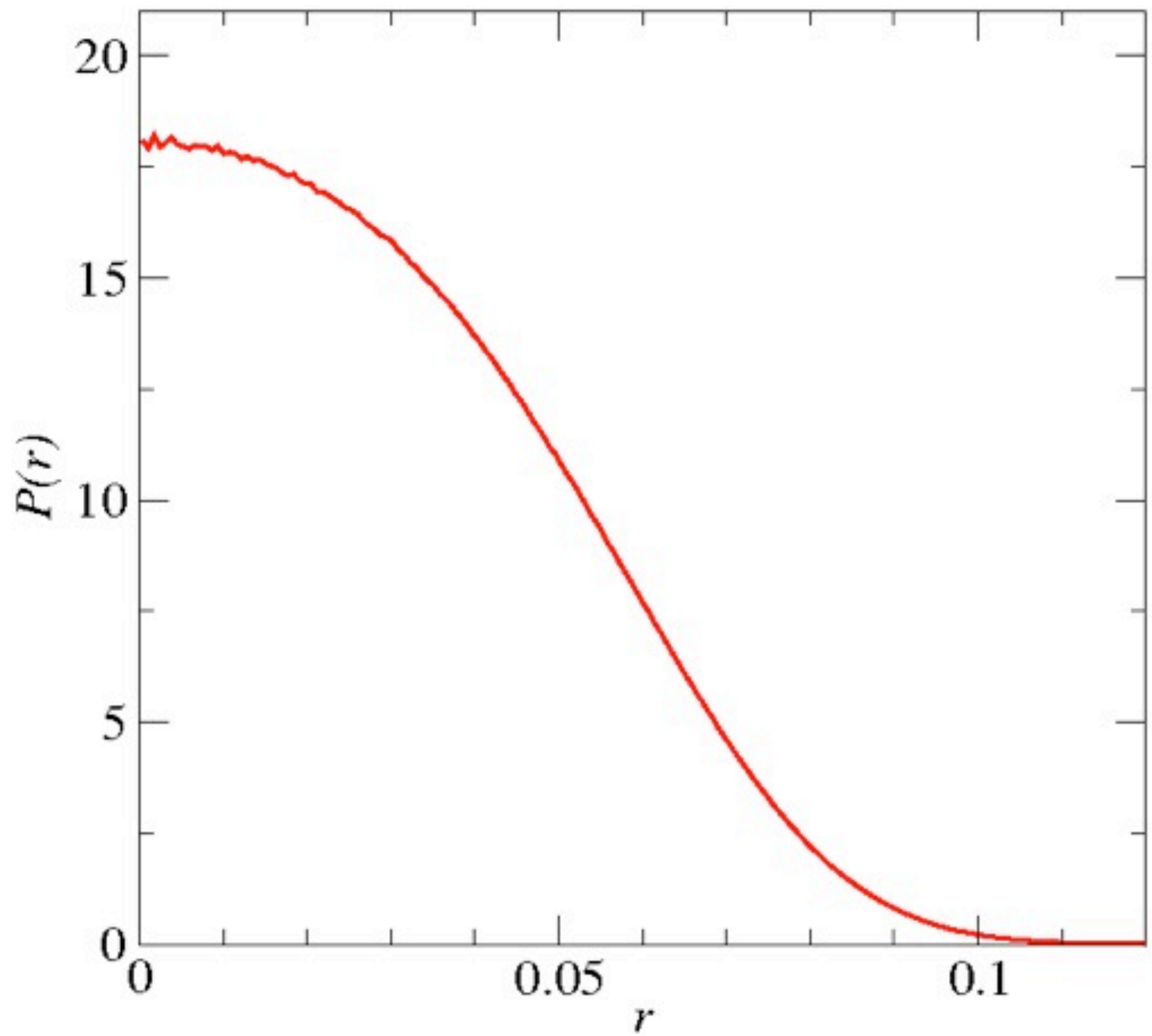
## J/Q=0.044



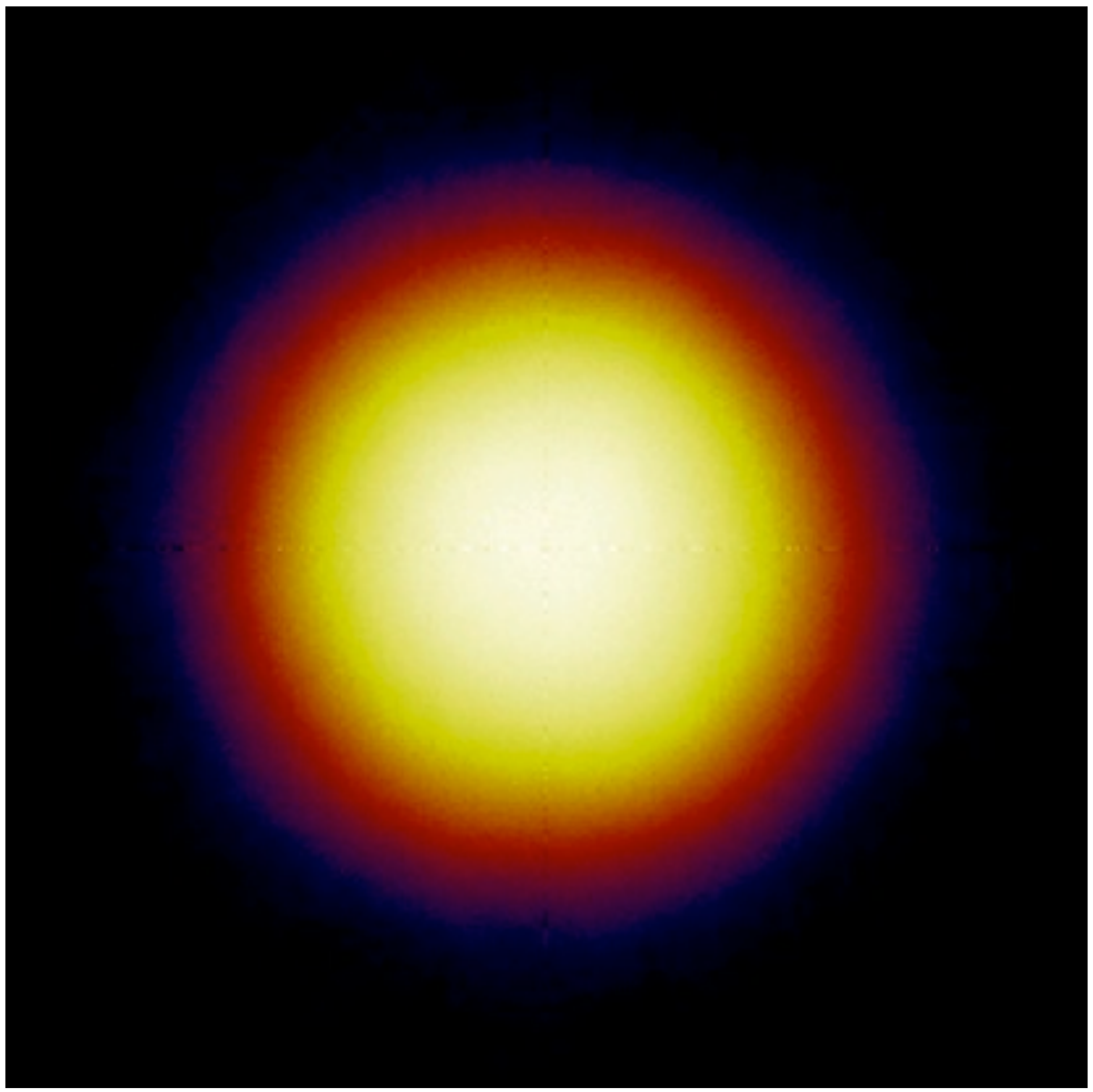


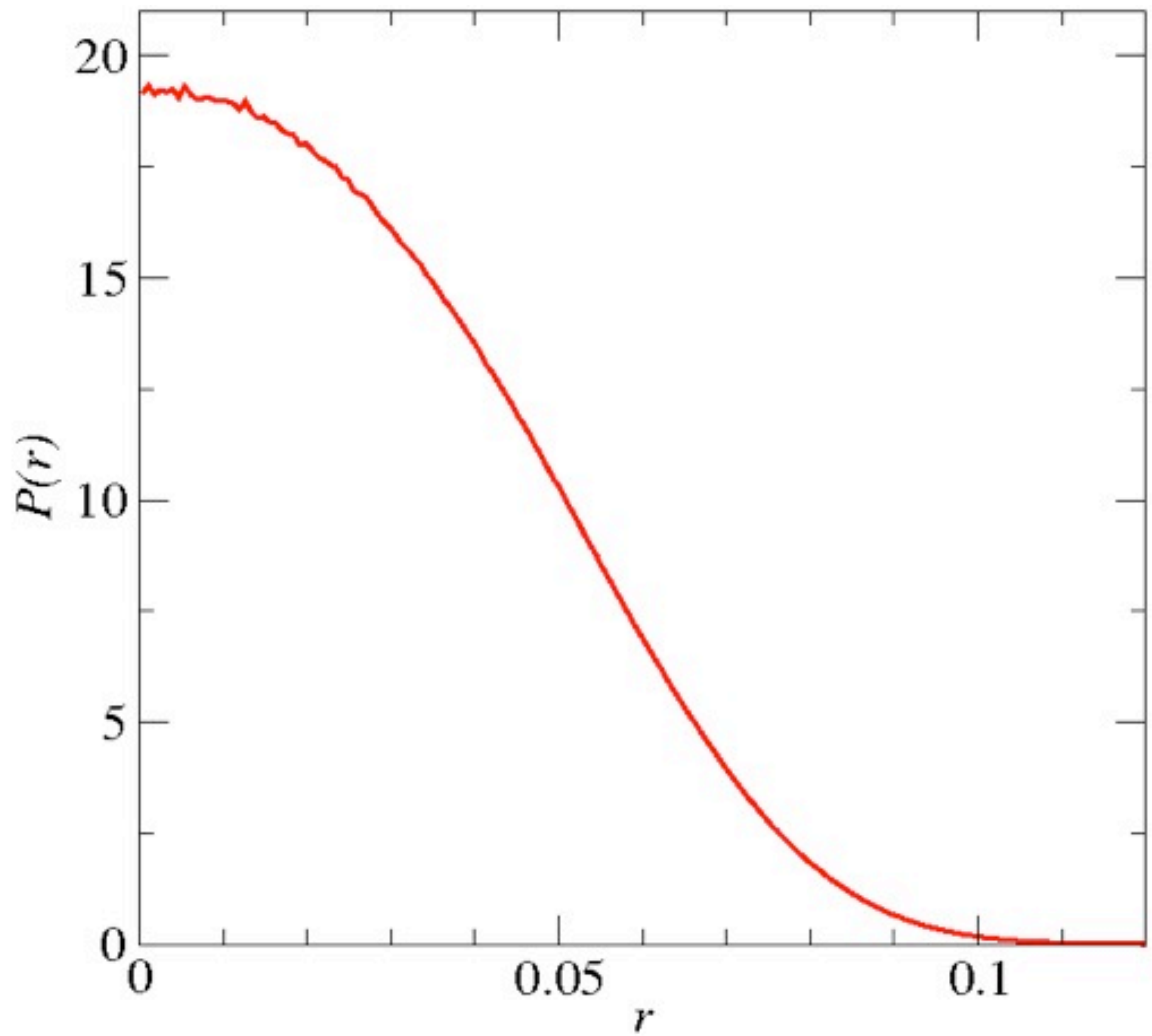
J/Q=0.045





**J/Q=0.046**





# logarithmic corrections

AWS, PRL 104, 177201 (2010)

$$\rho_s \sim \frac{\ln(L/L_0)}{L} \quad (T \to 0)$$

$$\chi \sim T[1 + a\ln(1/T)] \quad (L \to \infty)$$

**Governed by the dynamic exponent z (=1 in the theory)**

**Could the behavior indicate z≠1?**

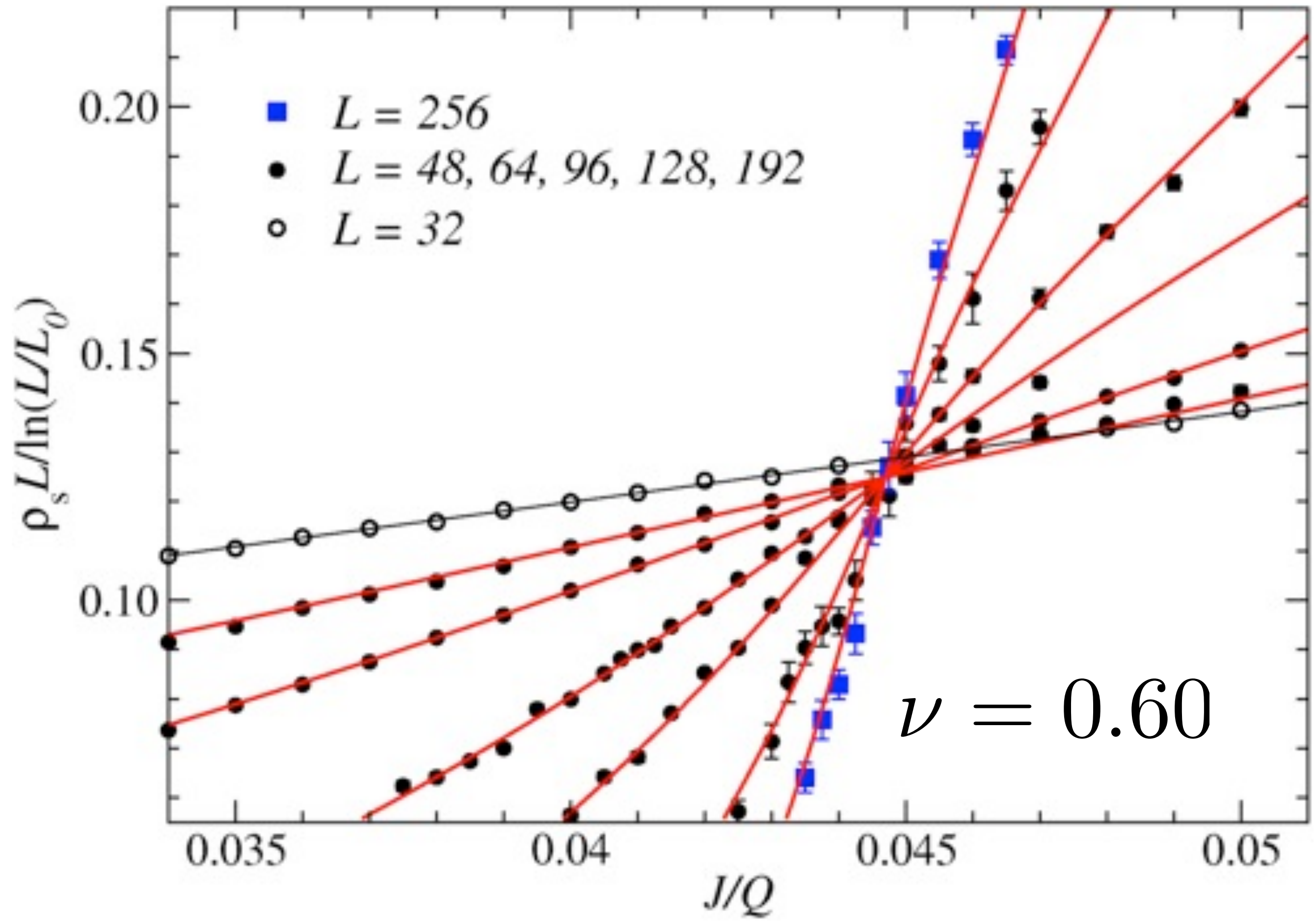
$$\xi \sim T^{-(1/z)}$$
$$\chi \sim T^{2/z-1}$$
$$\rho_s \sim L^{-z}$$

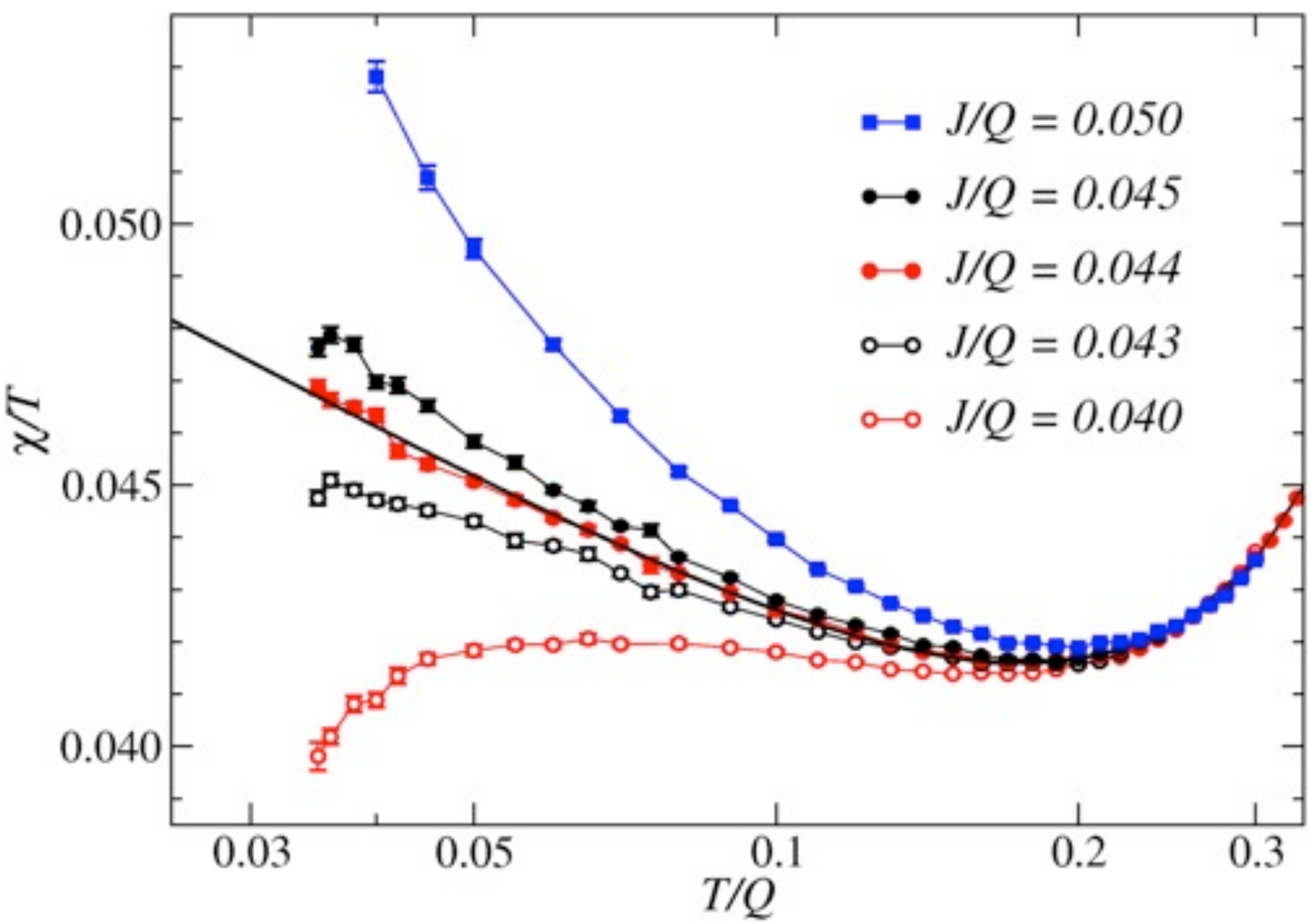
**ξ gives z≈0.82**

- consistent with $\rho_s(L)$
- inconsistent with χ(T)
  - demands χ/T→0 for T→0

**Most likely z=1**

- logs also in impurity response
  Banerjee, Damle, Alet, PRB 2010
- marginal operator causing logs?

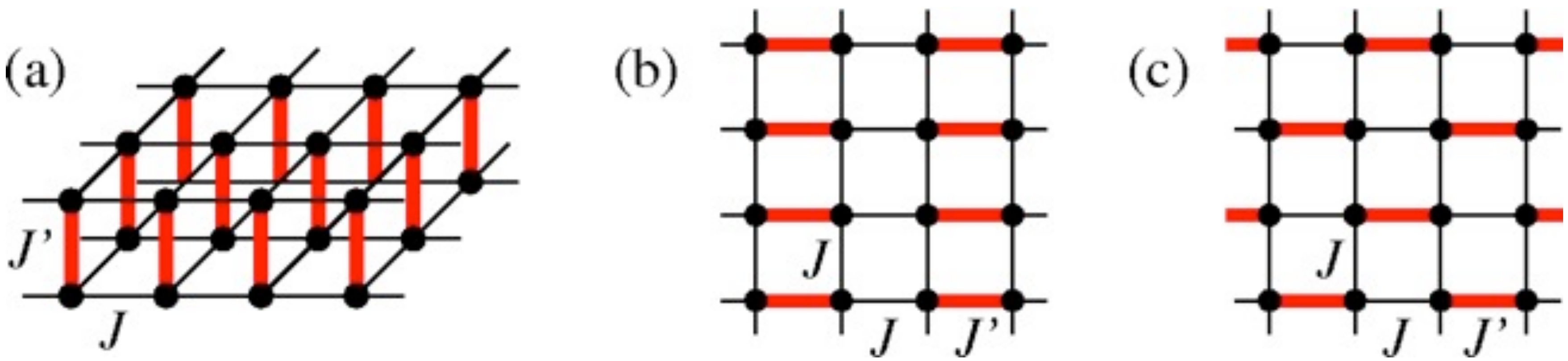


# T>0 quantum-criticality - conventional O(3) case

Theory: Chakravarty, Halperin, Nelson (1989), Chubukov, Sachdev, Ye (1994)

Realized in various dimerized S=1/2 Heisenberg models

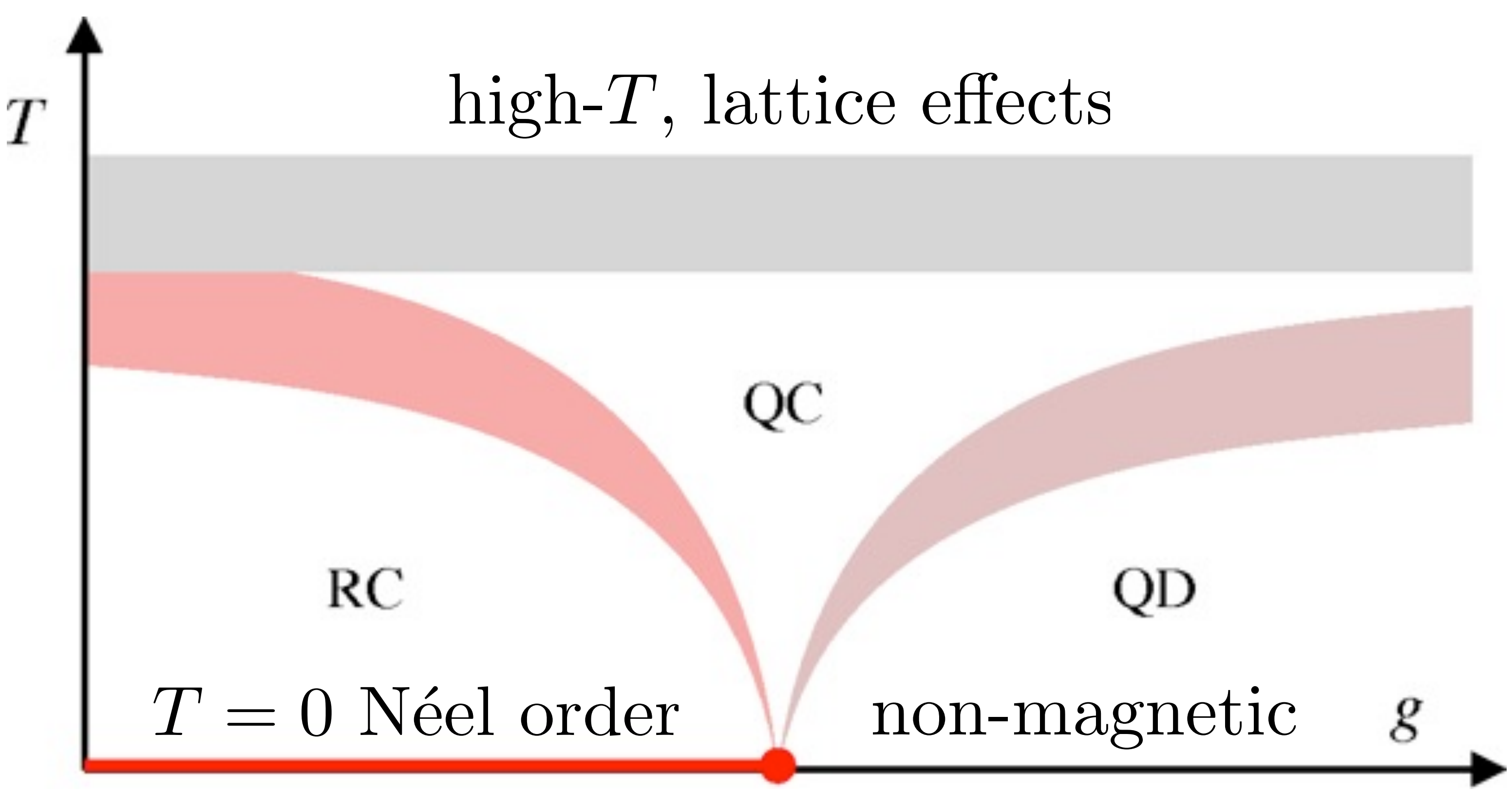



$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{\langle ij \rangle'} \mathbf{S}_i \cdot \mathbf{S}_j$$

Neel - non-magnetic T=0 transition vs **g=J'/J**

- plain singlet-product (+ fluct) state for $g>g_c$

**cross-over "phase diagram"**




3D O(3) (Heisenberg) universality

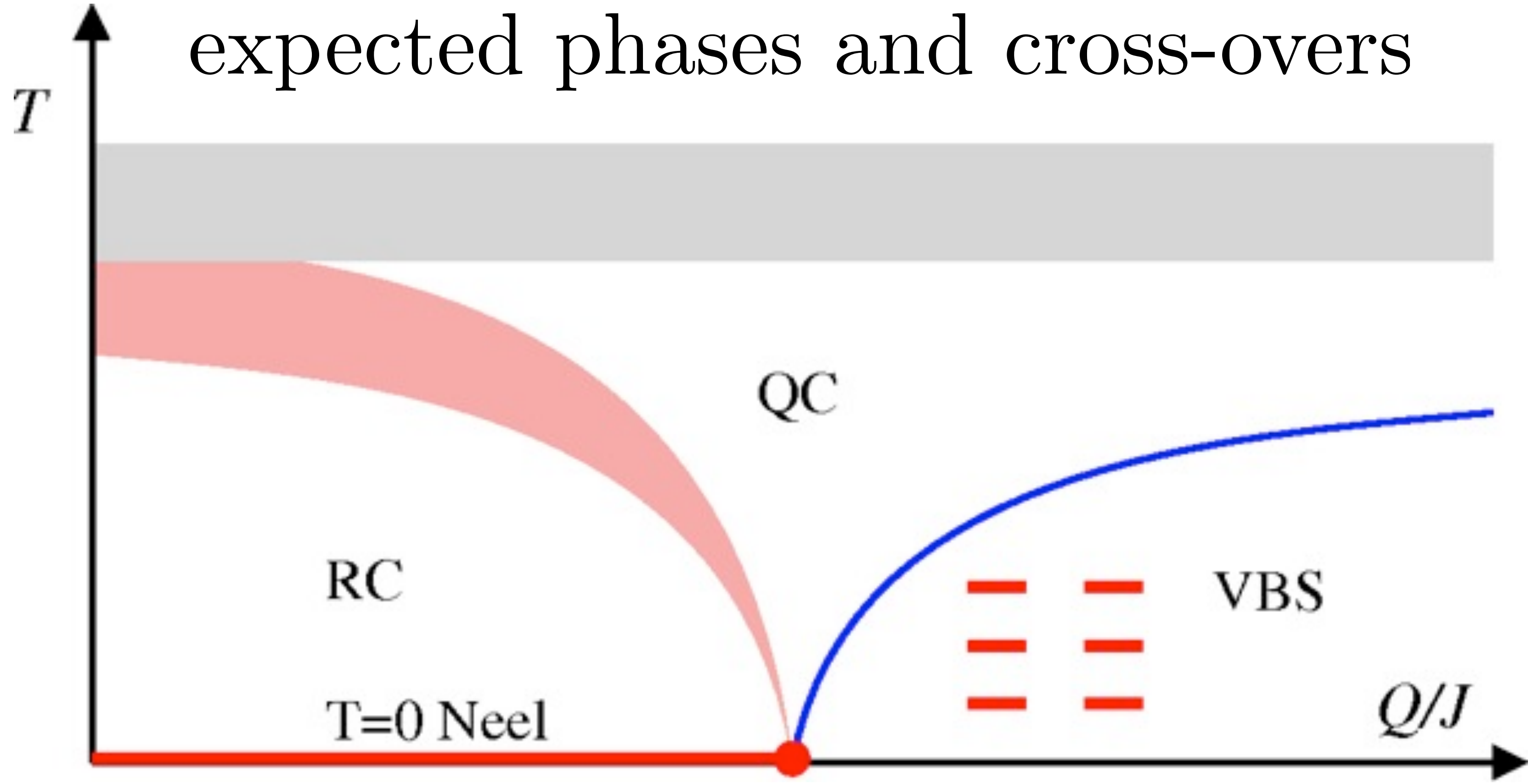
T>0 **quantum-critical regime**

- magnons (S=1) remain as the elementary excitations at the critical point
- dynamic exponent z=1
- scaling behavior:

$$\xi \propto T^{-1}$$
$$\chi \propto T$$
$$C \propto T^2$$

- confirmed by QMC
- some issues remain in (c)

# Consequences of spinons at T>0 in the J-Q model?




**J-Q QMC results:**

Standard QC forms

$$\xi \propto T^{-1}$$

$$\chi \propto T$$

are **weakly violated.** Specific heat obeys the standard form

$$C \propto T^2$$

**Phenomenological model of a spinon gas at T>0**

- bosonic spinons, linearly dispersing at T=0; **ε(k)=ck**
- thermal length ξ(T); assuming free spinons for momenta q>1/ξ
  - contributions to thermodynamics from these spinons

Infrared momentum cut-off 1/ξ equivalent to thermal "gap" Δ=1/ξ

$$\epsilon(\mathbf{k}) = \sqrt{\mathbf{c^2 k^2 + \Delta^2}}$$

**J-Q model:** critical **ξ diverges faster than 1/T** as T→0 (Δ/T→0) ➡

- infrared divergent integral leads to weak T→0 divergence (log) of χ/T
- weaker correction to $T^2$ form of C

# Spin correlation lengths; J-J' (columnar) and J-Q models

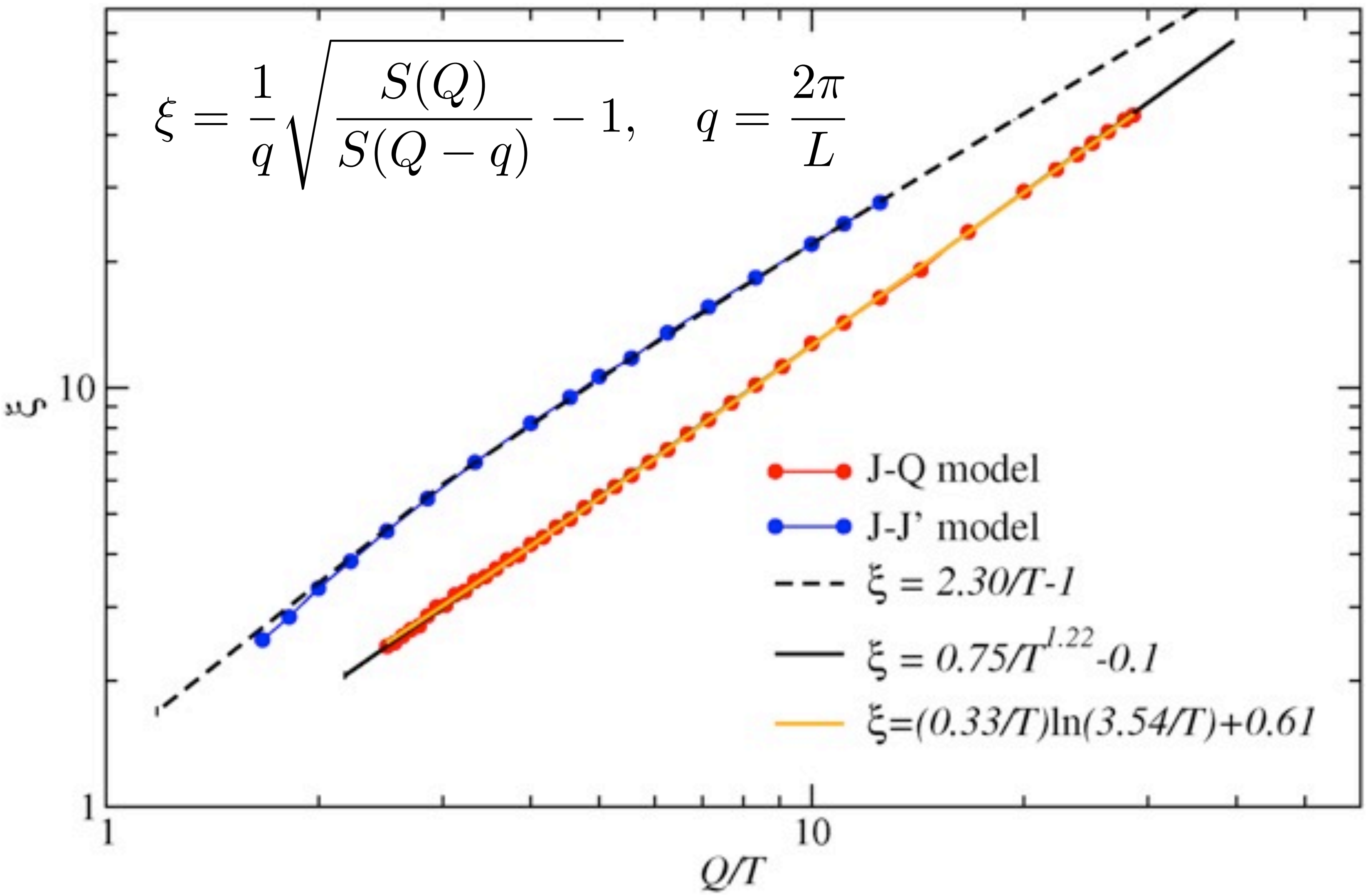
## Critical-point estimates

J-J' model: $(J'/J)_c$=1.90948(4), (using J'/J=1.9095)

J-Q model: $(J/Q)_c$=0.04498(3), (using J/Q=0.0450)

## T>0 critical spin correlation length

- L up to 512; converged to thermodynamic limit for T considered




**J-J' model:** expected 1/T divergence

**J-Q model:** faster than 1/T divergence

- logarithmic or power correction (data consistent with either form)

## Can we find relationships between the different anomalies?

- can this provide a fingerprint for spinons?

## Gas of non-interacting spinons (S=1/2) or magnons (S=1) at T>0

$$\epsilon_\pm(k) = \sqrt{c^2k^2+\Delta^2} \pm \mu B \equiv \epsilon(k) \pm \mu B$$ (B = magnetic field)

$$\mu = 1/2 \text{ (spinons)}, \quad \mu = 1 \text{ (magnons)}$$

Magnetization to linear order (bosonic excitations)

$$\begin{aligned} M &= \mu F \int \left( \frac{1}{e^{\epsilon_-/T}-1} - \frac{1}{e^{\epsilon_+/T}-1} \right) \frac{d^2k}{(2\pi)^2} \\ &= -2\mu^2 FB \int \frac{\partial n}{\partial \epsilon} \frac{d^2k}{(2\pi)^2} \\ &= \mu^2 F \frac{TB}{4\pi c^2} \int_0^\infty \frac{x\,dx}{\sinh^2[\frac{1}{2}\sqrt{x^2+(\Delta/T)^2}]} \end{aligned}$$

**F is a degeneracy factor**; F=2 (spinons/anti-spinons), F=1 (magnons)

Conventional quantum-criticality: **$\Delta/T \to m \approx 0.96$** (Chubukov & Sachdev 1994)

- computed using large-N calculations (nonlinear σ-model)

In the J-Q model (deconfined criticality?): **$\Delta/T \to 0$** ($\log^{-1}(1/T)$ or $T^a$)

- infrared divergent integral; significant consequences

$$\int_0^\infty \frac{x\,dx}{\sinh^2(\frac{1}{2}\sqrt{x^2+p^2})} = \frac{4p}{1-\mathrm{e}^{-p}} - 4\ln(\mathrm{e}^p - 1) \qquad p = \Delta/T$$

Using these gaps for spinon (S=1/2) and magnon (S=1) calculations:

$$\Delta_{1/2}/T \;=\; 1/(T\xi) \;=\; (T/mc)^a$$ (***mc*** and ***a*** from J-Q QMC data)

$$\Delta_1/T \;=\; m \;=\; 0.96$$ (Chubukov & Sachdev)

Gives the low-T **magnetic susceptibility**

$$\chi_1 \;=\; (1.0760/\pi c^2)T$$

$$\chi_{1/2} \;=\; \frac{T}{2\pi c^2}\left[1 + a\ln\left(\frac{mc}{T}\right) + \frac{1}{24}\left(\frac{T}{mc}\right)^{2a}\right]$$

Specific heat

$$C_S = (2S+1)F\int \epsilon(k)\frac{\partial n(\epsilon)}{\partial T}\frac{d^2k}{(2\pi)^2}$$

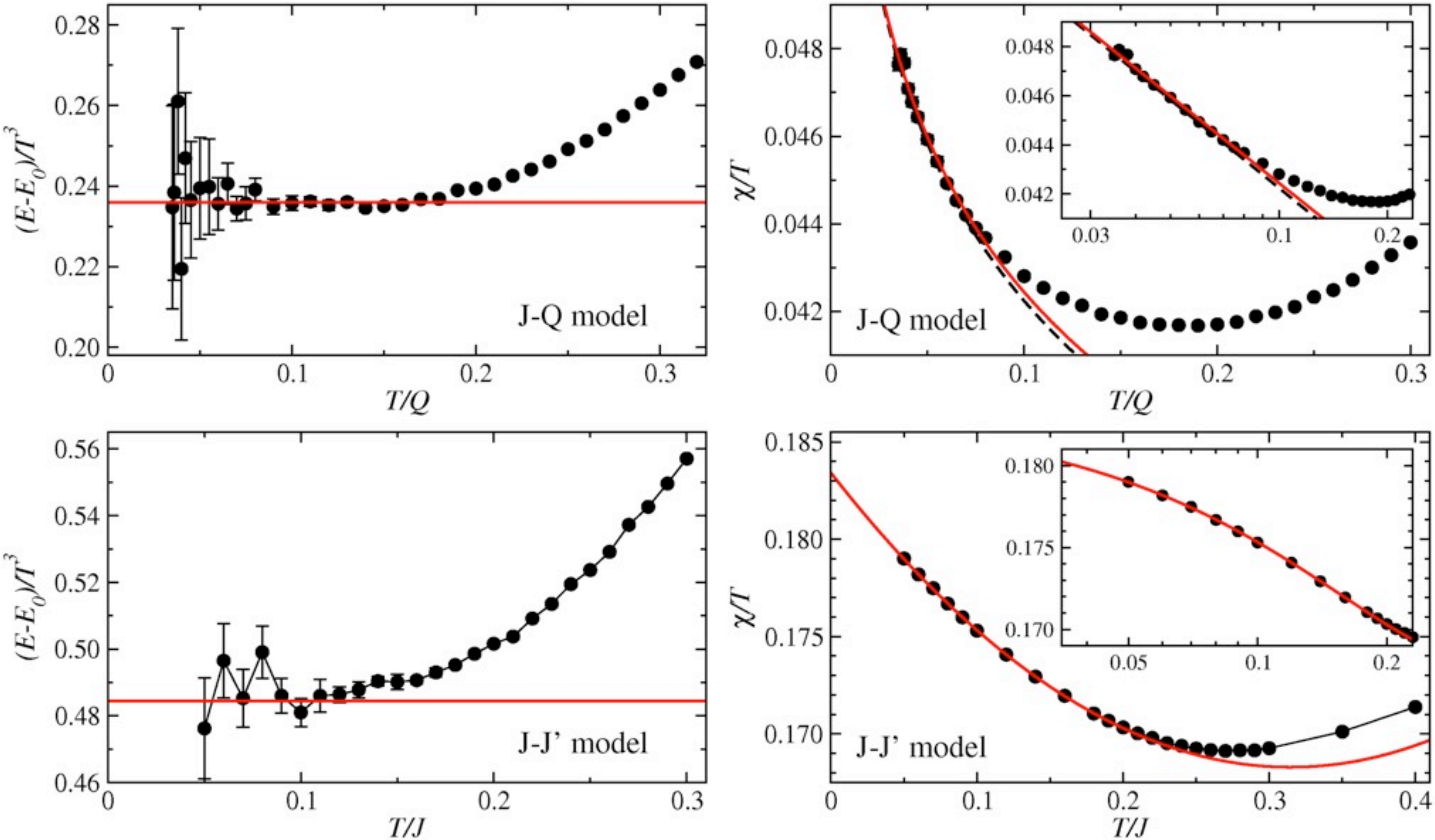
$$C_1 = [36\zeta(3)/5\pi c^2]T^2$$ (Chubukov & Sachdev)

$$C_{1/2} = \frac{2T^2}{\pi c^2}\left[6\zeta(3) - \left(\frac{T}{c}\right)^{2a}\left[\frac{3}{2} + a + a(1+a)\ln\left(\frac{c}{T}\right)\right]\right]$$

## QMC data fits: J-J' (magnon forms) and J-Q models (spinon forms)

- **J-J':** velocity fitted in $E/T^3$, polynomial fit for X/T (velocities agree to 2%)
- **J-Q:** only velocity is fitted; values from X/T and C agree within 2%

$(\mathbf{E} - \mathbf{E_0})/\mathbf{T^3}$ $\chi/\mathbf{T}$



# J-Q model: effective spin of the excitations

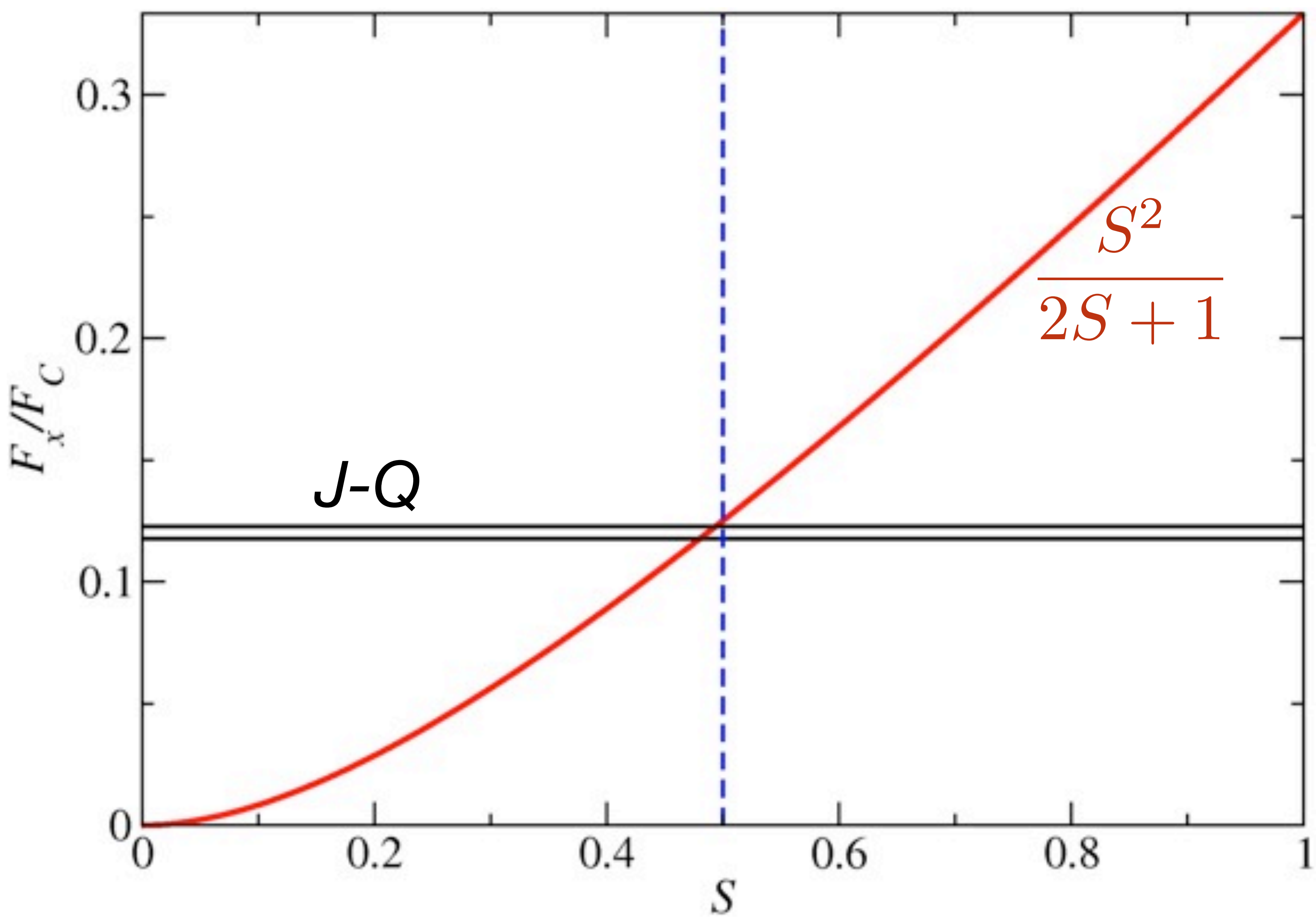
Under the assumption of spinons, S=1/2, μ=1/2, F=2 (spinon/anti-spinon):

$$F_\chi = \frac{\mu^2 F}{c_\chi^2} \approx 0.074, \qquad F_C = \frac{(2S+1)F}{c_C^2} \approx 0.615 \qquad \begin{aligned} c_\chi &= 2.60 \\ c_C &= 2.55 \end{aligned}$$

Should have **$c_\chi = c_C$.** S≠1/2? For both spinons (S=1/2) and magnons (S=1)

$$\mu = S, \quad F = 1/S \quad \rightarrow \quad \frac{F_\chi}{F_C} = \frac{S^2}{2S+1}$$

Treat S as continuous variable and find effective S given the J-Q data:




The J-Q results are consistent with S=1/2 (spinons) but not consistent with S=1 (magnons)

**Could this be a coincidence?**

- assumed Δ=1/ξ
- may be Δ=*d*/ξ, *d*≈1
- results depend weakly on *d*

Independent estimate of the velocity would be good

- can be done
  - imaginary time correlations