Anomalous Hall effect in Superconductors with spin-orbit interaction

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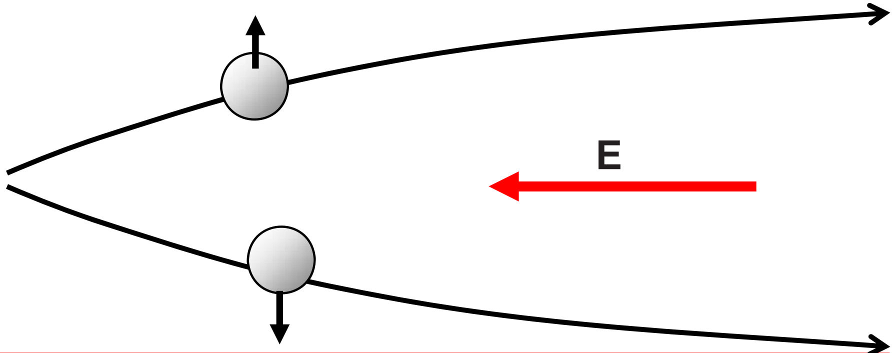
Anomalous Hall effect

$$\rho_{xy} = R_0 H_z + R_s M_z$$

(Karplus, Luttinger, 1954)

a) Intrinsic deflection

Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.

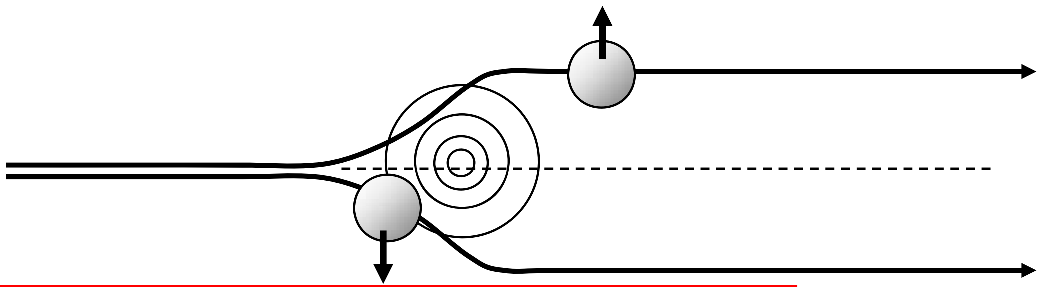


$$\frac{d\langle\vec{r}\rangle}{dt} = \frac{\partial E}{\hbar\partial\vec{k}} + \frac{e}{\hbar}E\times b_n$$

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry's phase curvature

b) Side jump

(Berger, 1970)

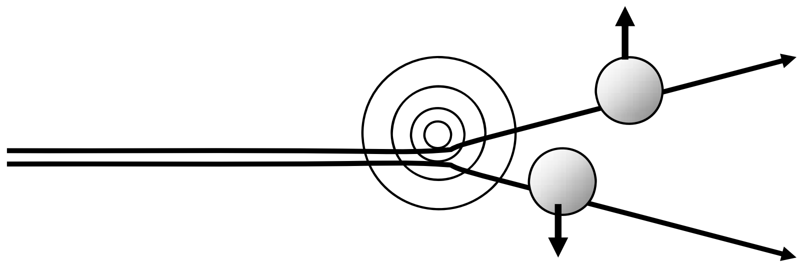


(Smit, 1955,58)

The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity. The time-integrated velocity deflection is the side jump.

c) Skew scattering

Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.



$$H(\boldsymbol{R})|n(\boldsymbol{R})\rangle = \varepsilon_n(\boldsymbol{R})|n(\boldsymbol{R})\rangle$$

$$|\psi_n(t)\rangle = e^{i\gamma_n(t)}\exp\left[-\frac{i}{\hbar}\int_0^t dt'\,\varepsilon_n(\boldsymbol{R}(t'))\right]|n(\boldsymbol{R}(t))\rangle$$

Berry connection

$$\mathcal{A}_n(\boldsymbol{R}) = i\langle n(\boldsymbol{R})|\frac{\partial}{\partial \boldsymbol{R}}|n(\boldsymbol{R})\rangle$$

Berry phase

$$\gamma_n = \oint_{\mathcal{C}} d\boldsymbol{R}\cdot\mathcal{A}_n(\boldsymbol{R})$$

Berry curvature

$$\boldsymbol{\Omega}_n(\boldsymbol{R}) = \boldsymbol{\nabla}_{\boldsymbol{R}} \times \mathcal{A}_n(\boldsymbol{R})$$

$$\gamma_n = \int_{\mathcal{S}} d\boldsymbol{S} \cdot \boldsymbol{\Omega}_n(\boldsymbol{R}) .$$

$$\begin{aligned}\Omega^n_{\mu\nu}(\boldsymbol{R}) &= \frac{\partial}{\partial R^\mu}\mathcal{A}^n_\nu(\boldsymbol{R}) - \frac{\partial}{\partial R_\nu}\mathcal{A}^n_\mu(\boldsymbol{R}) \\ &= i\left[\left\langle \frac{\partial n(\boldsymbol{R})}{\partial R^\mu} \middle| \frac{\partial n(\boldsymbol{R})}{\partial R^\nu} \right\rangle - (\nu \leftrightarrow \mu)\right]\end{aligned}$$

$$\Omega^n_{\mu\nu}(\boldsymbol{R}) = i \sum_{n' \neq n} \frac{\langle n|\partial H/\partial R^\mu|n'\rangle\langle n'|\partial H/\partial R^\nu|n\rangle - (\nu \leftrightarrow \mu)}{(\varepsilon_n - \varepsilon_{n'})^2}$$

$$\Omega^n_{\mu\nu} = \epsilon_{\mu\nu\xi}(\boldsymbol{\Omega}_n)_\xi$$

$$\boldsymbol{\Omega}_n(\boldsymbol{k}) = i\langle \boldsymbol{\nabla}_{\boldsymbol{k}} u_n(\boldsymbol{k})| \times |\boldsymbol{\nabla}_{\boldsymbol{k}} u_n(\boldsymbol{k})\rangle$$

$$\gamma_n = \int_{\mathrm{BZ}} d\boldsymbol{q} \cdot \langle u_n(\boldsymbol{q})|i\boldsymbol{\nabla}_{\boldsymbol{q}}|u_n(\boldsymbol{q})\rangle$$

$$\boldsymbol{v}_n(\boldsymbol{k}) = \frac{\partial \varepsilon_n(\boldsymbol{k})}{\hbar \partial \boldsymbol{k}} - \frac{e}{\hbar}\boldsymbol{E} \times \boldsymbol{\Omega}_n(\boldsymbol{k})$$

(Thouless et al., 1982)

n=Chern number

$$\sigma_{xy} = \frac{e^2}{\hbar}\int_{\mathrm{BZ}} \frac{d^2k}{(2\pi)^2}\Omega_{k_x k_y}$$

$$\sigma_{xy} = n\frac{e^2}{h}$$

$$\sigma_H = ie^2\hbar \sum_{n>0} \frac{\langle \Phi_0|v_1|\Phi_n\rangle\langle \Phi_n|v_2|\Phi_0\rangle - (1 \leftrightarrow 2)}{(\varepsilon_0 - \varepsilon_n)^2}$$

Kubo formula

Anomalous velocity

The general form of the Berry curvature $\boldsymbol{\Omega}_n(\boldsymbol{k})$ can be obtained via symmetry analysis. The velocity formula (3.6) should be invariant under time-reversal and spatial inversion operations if the unperturbed system has these symmetries. Under time reversal, $\boldsymbol{v}_n$ and $\boldsymbol{k}$ change sign while $\boldsymbol{E}$ is fixed. Under spatial inversion, $\boldsymbol{v}_n$, $\boldsymbol{k}$, and $\boldsymbol{E}$ change sign. If the system has time-reversal symmetry, the symmetry condition on Eq. (3.6) requires that

$$\boldsymbol{\Omega}_n(-\boldsymbol{k}) = -\boldsymbol{\Omega}_n(\boldsymbol{k}). \tag{3.8}$$

If the system has spatial inversion symmetry, then

$$\boldsymbol{\Omega}_n(-\boldsymbol{k}) = \boldsymbol{\Omega}_n(\boldsymbol{k}). \tag{3.9}$$

Therefore, for crystals with simultaneous time-reversal and spatial inversion symmetry the Berry curvature vanishes identically throughout the Brillouin zone. In this

Symmetry	Berry curvature	σ_{xy}
$\mathcal{T}$	$\boldsymbol{\Omega}_n(\mathbf{k}) = -\boldsymbol{\Omega}_n(-\mathbf{k})$	0
$\mathcal{I}$	$\boldsymbol{\Omega}_n(\mathbf{k}) = \boldsymbol{\Omega}_n(-\mathbf{k})$	non-zero
$\mathcal{T}\mathcal{I}$	$\boldsymbol{\Omega}_n(\mathbf{k}) = -\boldsymbol{\Omega}_n(\mathbf{k}) = 0$	0

Superconductors + Magnetism

V. M. Edelstein, Phys. Rev. Lett. **75**, 2004 (1995).

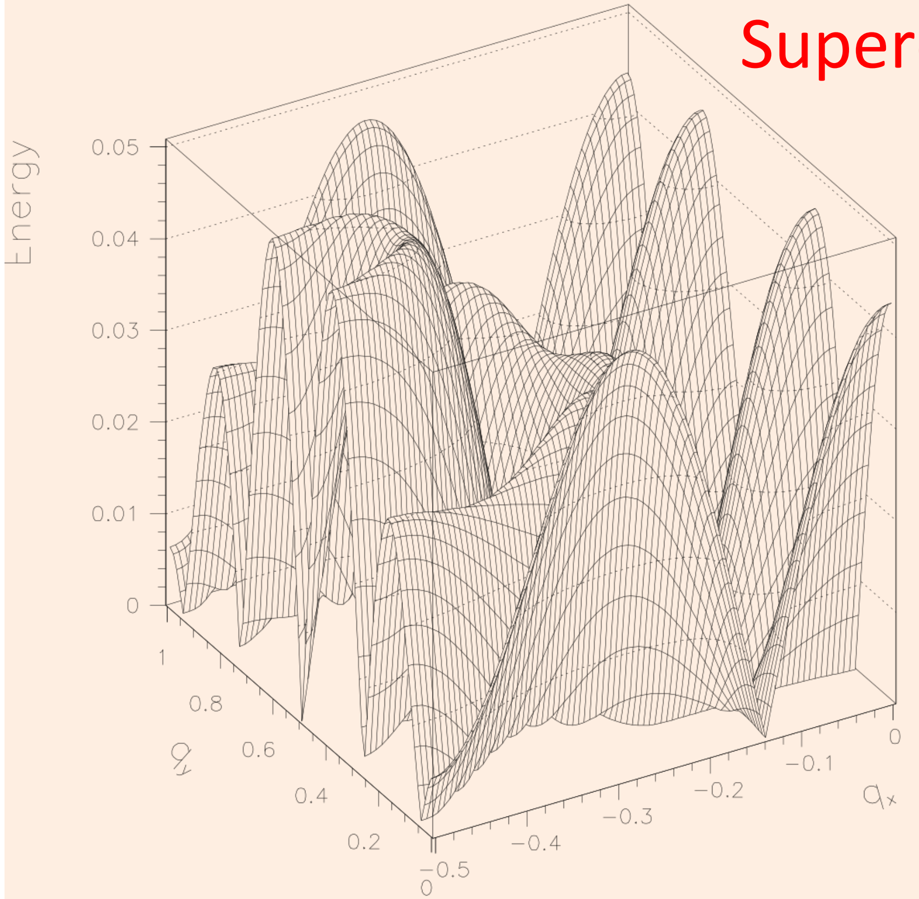
S. K. Yip, Phys. Rev. B **65**, 144508 (2002).

V. M. Edelstein, Phys. Rev. B **67**, 020505(R) (2003).

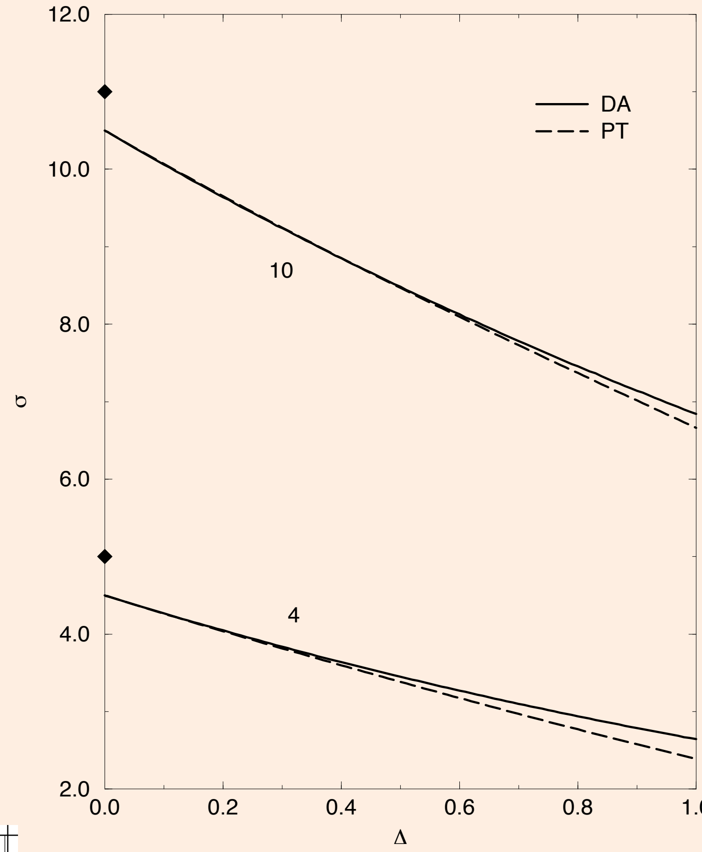
K. V. Samokhin, Phys. Rev. B **70**, 104521 (2004).

S. Fujimoto, Phys. Rev. B **72**, 024515 (2005).

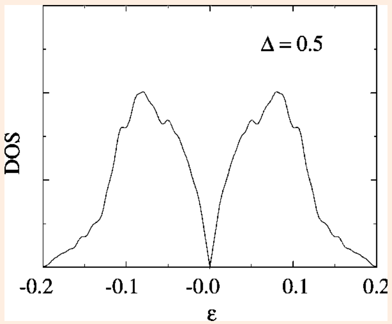
Superconductor in high magnetic field

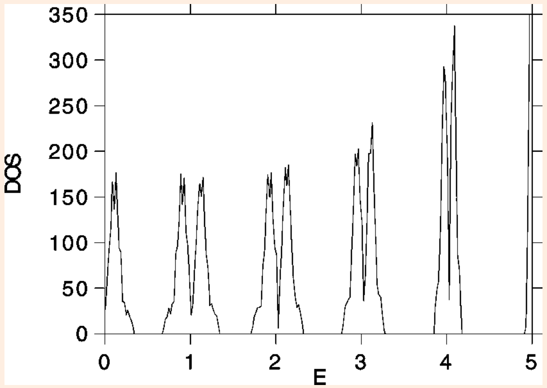


(PDS, 1999)



(Tesanovic, PDS, 1998)





s-wave in field

Orbital effect

Landau levels

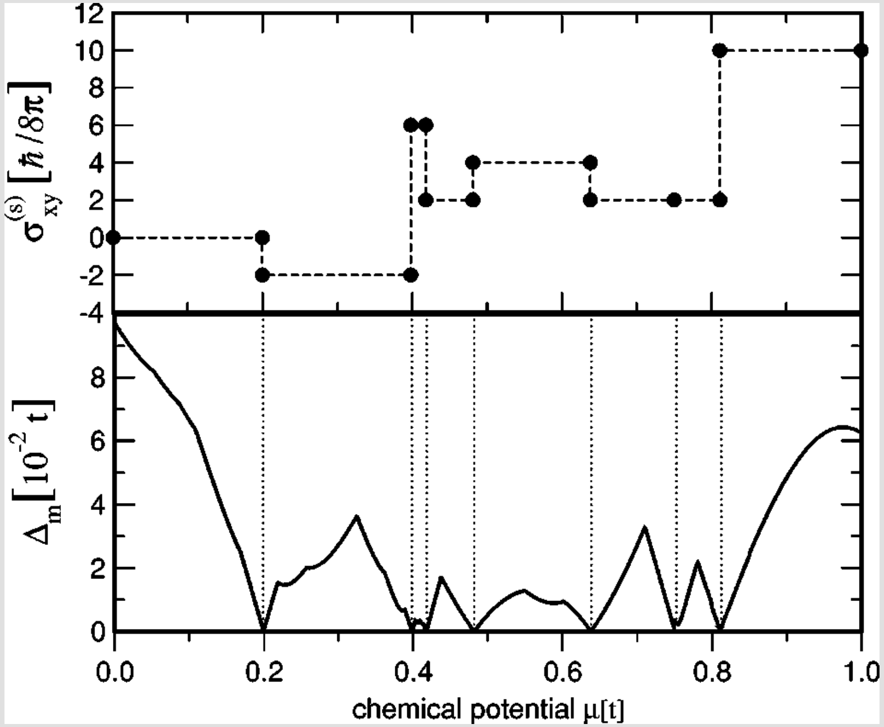
Not quantized

d-wave

(Vafek, Melikyan, Tesanovic, 2001)

$$\sigma_{xy}^{s,m} = \frac{\hbar}{8\pi} N$$

Spin Hall conductance quantized



$$\sigma_0 = \frac{e^2}{\hbar \pi^2} \frac{v_f}{v_2} \beta_{VC} {\alpha_{FL}^{s}}^2,$$

$$\frac{\kappa_0}{T} = \frac{[(\pi^2/3)k_B^2]}{\hbar \pi^2} \left(\frac{v_f}{v_2} + \frac{v_2}{v_f} \right),$$

$$\kappa_{xy} = (4\pi^2/3)(k_B/\hbar)^2 T \sigma_{xy}^s$$

$$\sigma_0^s = \frac{s^2}{\hbar \pi^2} \left(\frac{v_f}{v_2} + \frac{v_2}{v_f} \right) {\alpha_{FL}^{a}}^2,$$

Thermal Hall conductance quantized
Wiedemann-Franz law

Zero field + impurity scattering

(Durst, Lee, 2000)

Magnetic Impurity in s-wave superconductor

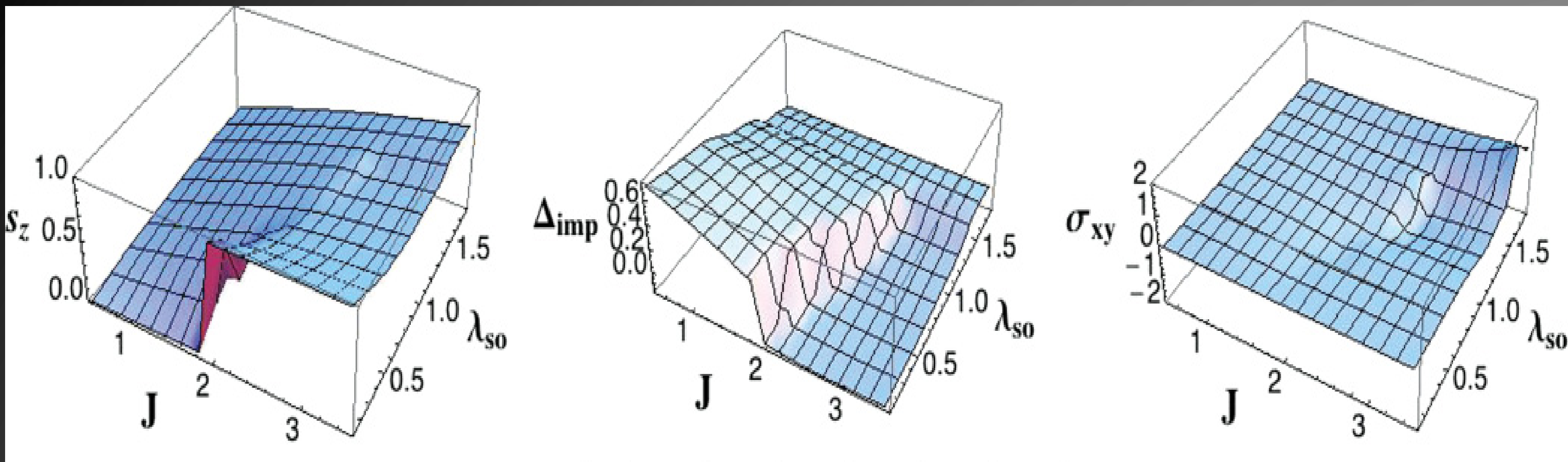
$$H = -\sum_{\langle i,j\rangle,\sigma} t_{i,j} c^{\dagger}_{i\sigma} c_{j\sigma} - \mu \sum_{i\sigma} c^{\dagger}_{i\sigma} c_{i\sigma} + \sum_{i} (\Delta_i c^{\dagger}_{i\uparrow} c^{\dagger}_{i\downarrow} + \Delta^{*}_{i} c_{i\downarrow} c_{i\uparrow})$$

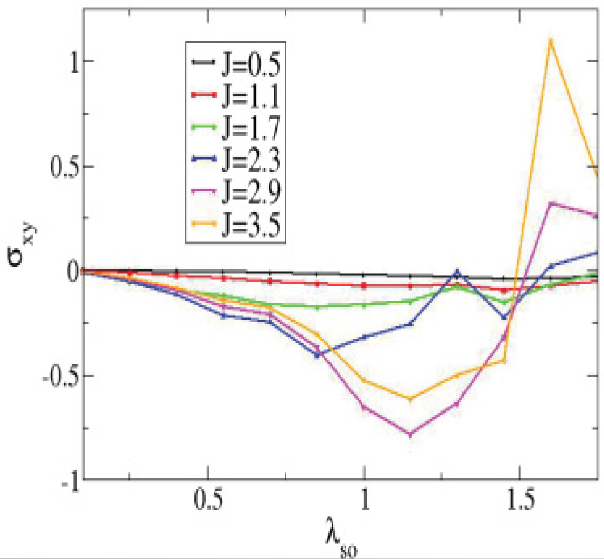
$$- \sum_{i,,l,\sigma,\sigma'} J_{i,l} (\cos\varphi_l c^{\dagger}_{i\sigma} \sigma^{x}_{\sigma,\sigma'} c_{i\sigma'} + \sin\varphi_l c^{\dagger}_{i\sigma} \sigma^{z}_{\sigma,\sigma'} c_{i\sigma'}), \quad (2)$$

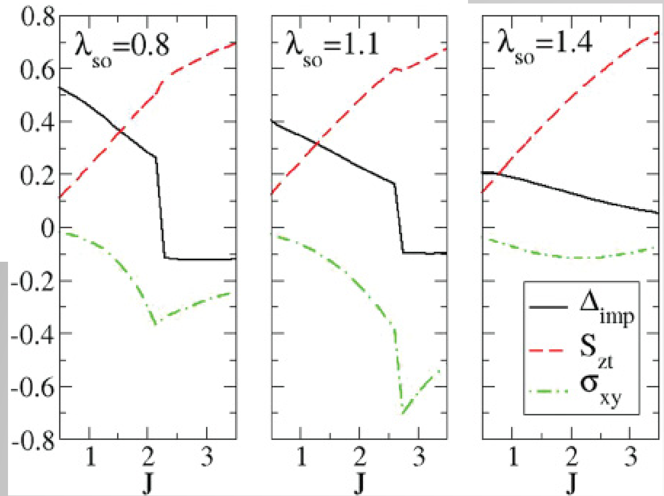
J: coupling of impurity spin to spin density

(Yu, 1965; Russinov, 1968; Shiba, 1969; Sakurai, 1970; Schlottmann, 1976)

$$\begin{pmatrix} -h-\epsilon_F - J\delta_{\vec r,\vec l_c} & \Delta_{\vec r} & \lambda_{so}(-\eta_x + i\eta_y) & 0 \\ \Delta^{*}_{\vec r} & h+\epsilon_F - J\delta_{\vec r,\vec l_c} & 0 & \lambda_{so}(\eta_x - i\eta_y) \\ \lambda_{so}(\eta_x + i\eta_y) & 0 & -h-\epsilon_F + J\delta_{\vec r,\vec l_c} & \Delta_{\vec r} \\ 0 & \lambda_{so}(-\eta_x - i\eta_y) & \Delta^{*}_{\vec r} & h+\epsilon_F + J\delta_{\vec r,\vec l_c} \end{pmatrix} \begin{pmatrix} u_n(\vec r,\uparrow) \\ v_n(\vec r,\downarrow) \\ u_n(\vec r,\downarrow) \\ v_n(\vec r,\uparrow) \end{pmatrix} = \epsilon_n \begin{pmatrix} u_n(\vec r,\uparrow) \\ v_n(\vec r,\downarrow) \\ u_n(\vec r,\downarrow) \\ v_n(\vec r,\uparrow) \end{pmatrix}$$







(PDS, Araújo, Vieira, Dugaev, Barnas, 2012)

Quantum phase transition

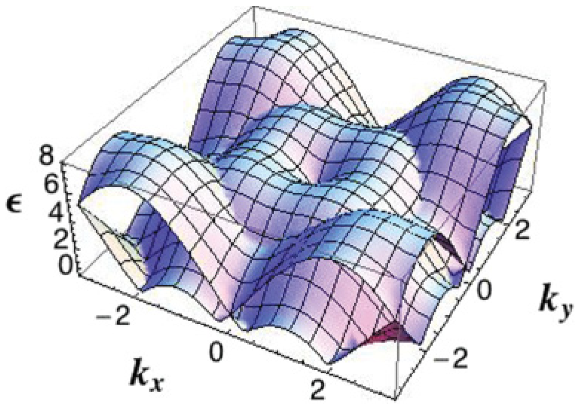
p-wave triplet superconductor + Rashba spin-orbit + Zeeman non-centrosymmetric

$$\begin{pmatrix} \epsilon_{\vec{k}} - h_z & \alpha(\sin k_y + i \sin k_x) & -d_x + i d_y & d_z + \Delta_s \\ \alpha(\sin k_y - i \sin k_x) & \epsilon_{\vec{k}} + h_z & d_z - \Delta_s & d_x + i d_y \\ -d_x - i d_y & d_z - \Delta_s & -\epsilon_{\vec{k}} + h_z & \alpha(\sin k_y - i \sin k_x) \\ d_z + \Delta_s & d_x - i d_y & \alpha(\sin k_y + i \sin k_x) & -\epsilon_{\vec{k}} - h_z \end{pmatrix}$$

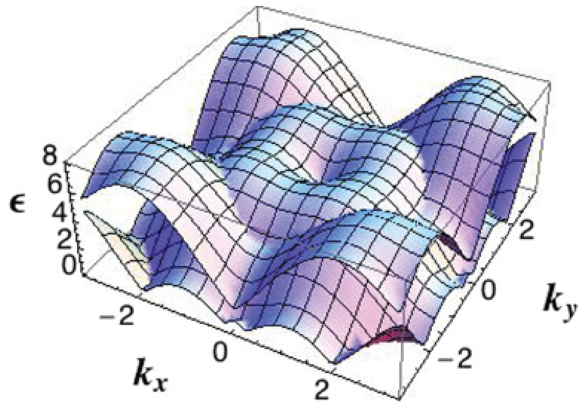
Spin-orbit

$$\vec{s} \cdot \vec{\sigma} = \alpha(\sin k_y \sigma_x - \sin k_x \sigma_y) \qquad \Delta = \begin{pmatrix} \Delta_{\uparrow,\uparrow} & \Delta_{\uparrow,\downarrow} \\ \Delta_{\downarrow,\uparrow} & \Delta_{\downarrow,\downarrow} \end{pmatrix} = \begin{pmatrix} -d_x + i d_y & d_z \\ d_z & d_x + i d_y \end{pmatrix}$$

(Sato, Fujimoto, 2009)



Spin-orbit

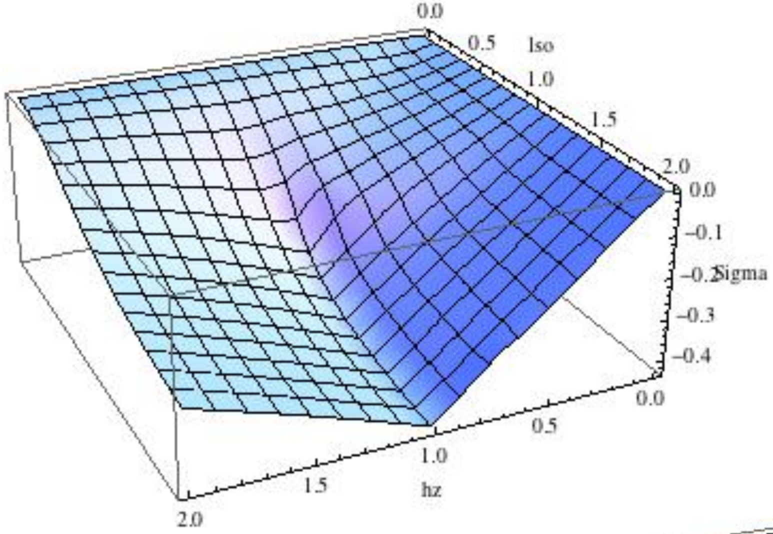


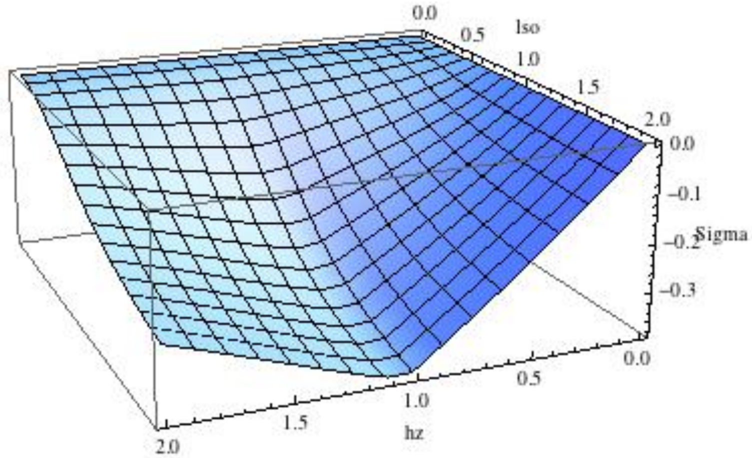
Zeeman

$$\Delta = \begin{pmatrix} \Delta_{\uparrow,\uparrow} & \Delta_{\uparrow,\downarrow} \\ \Delta_{\downarrow,\uparrow} & \Delta_{\downarrow,\downarrow} \end{pmatrix} = \begin{pmatrix} -d_x + i d_y & d_z \\ d_z & d_x + i d_y \end{pmatrix}$$

Unitary pairing

Normal phase



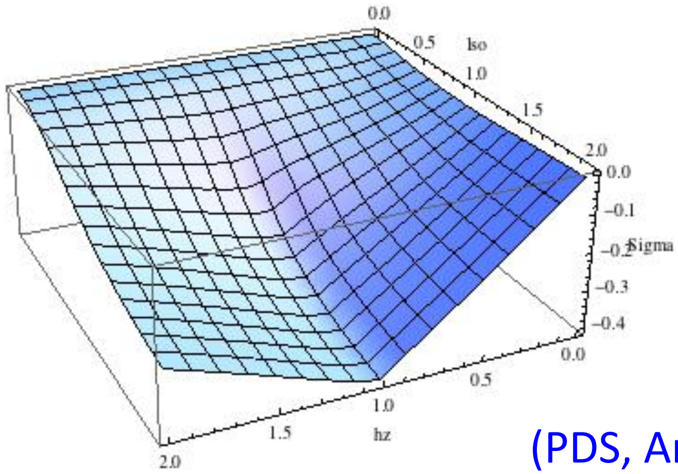


$$\Delta_{\uparrow,\uparrow} = d(-\sin k_y + i \sin k_x), \Delta_{\uparrow,\downarrow} = \Delta_{\downarrow,\uparrow} = 0$$

$$\Delta_{\downarrow,\downarrow} = d(\sin k_y + i \sin k_x)$$

$$\hat{\Delta}\hat{\Delta}^{\dagger} = |\mathbf{d}|^2 \hat{\sigma}_0 + \mathbf{q} \cdot \hat{\boldsymbol{\sigma}}$$

$$\mathbf{q} = i(\mathbf{d} \times \mathbf{d}^*)$$

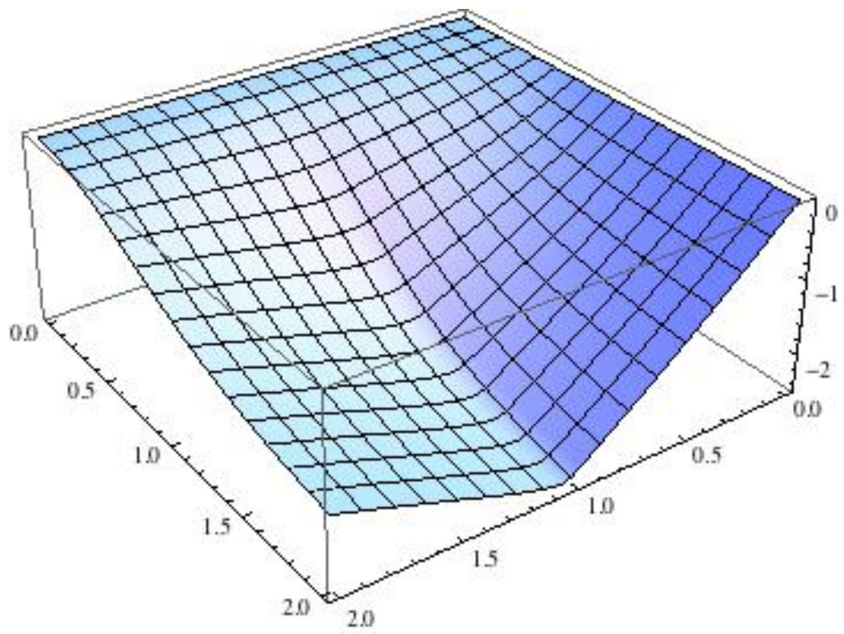


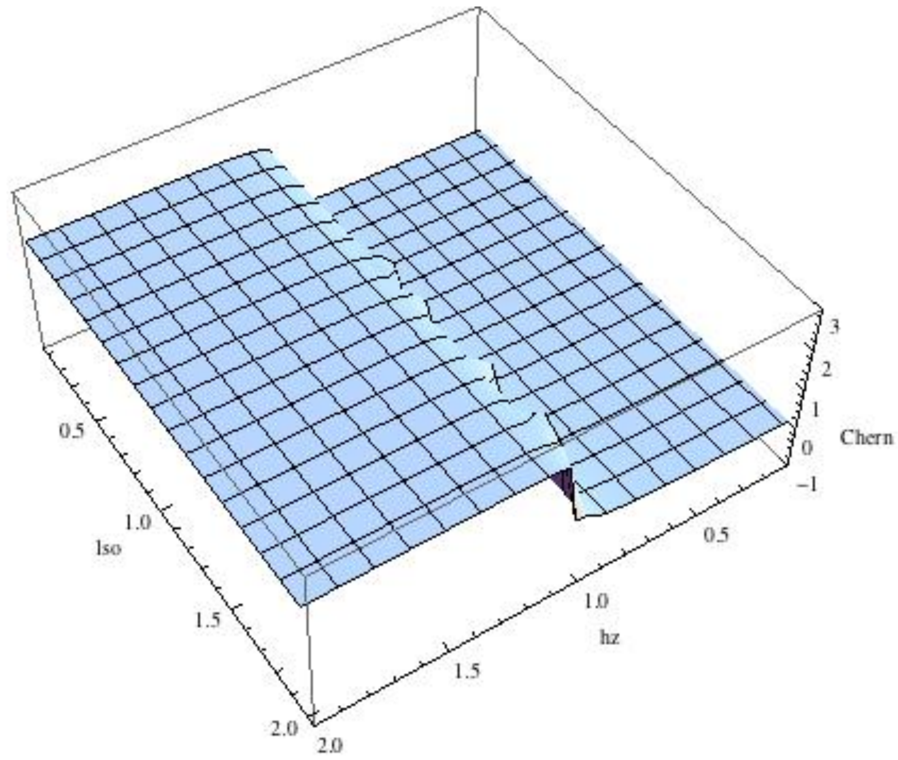
Non-unitary pairing

(PDS, Araújo, Vieira, Dugaev, Barnas, 2012)

$$\Delta_{\uparrow,\uparrow} = d \ \sin k_x, \quad \Delta_{\uparrow,\downarrow} = \Delta_{\downarrow,\uparrow} = 0, \quad \Delta_{\downarrow,\downarrow} = 0$$

Unitary pairing



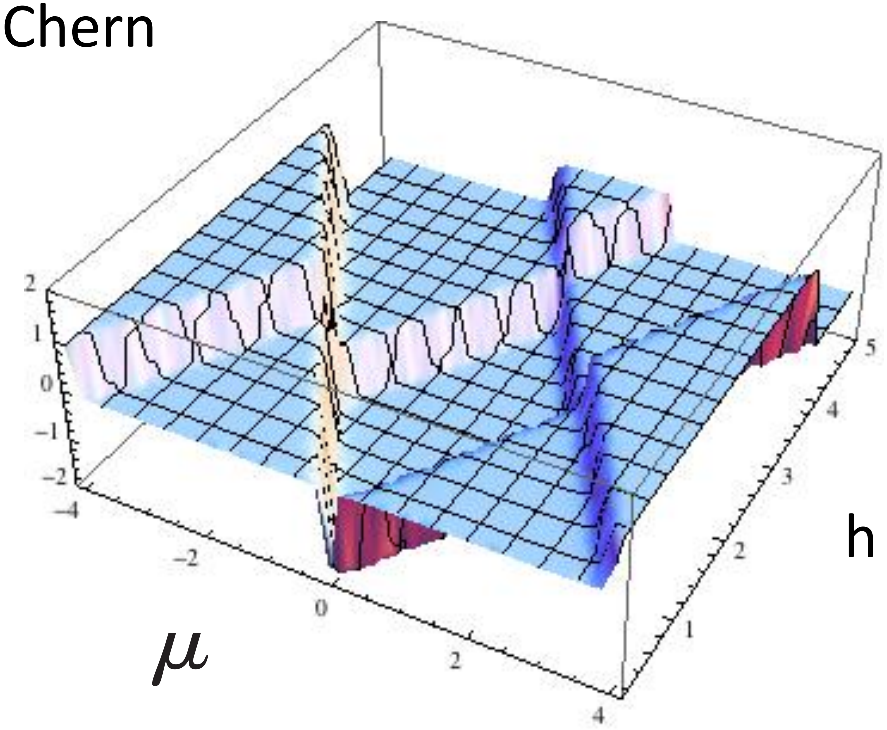


Hall conductance

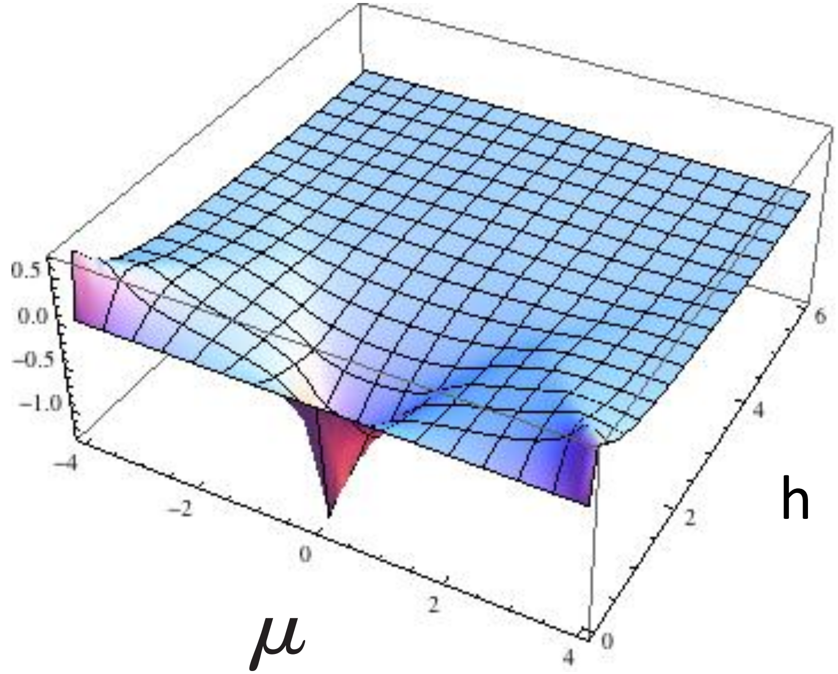
Chern number

Strong spin-orbit coupling

Chern



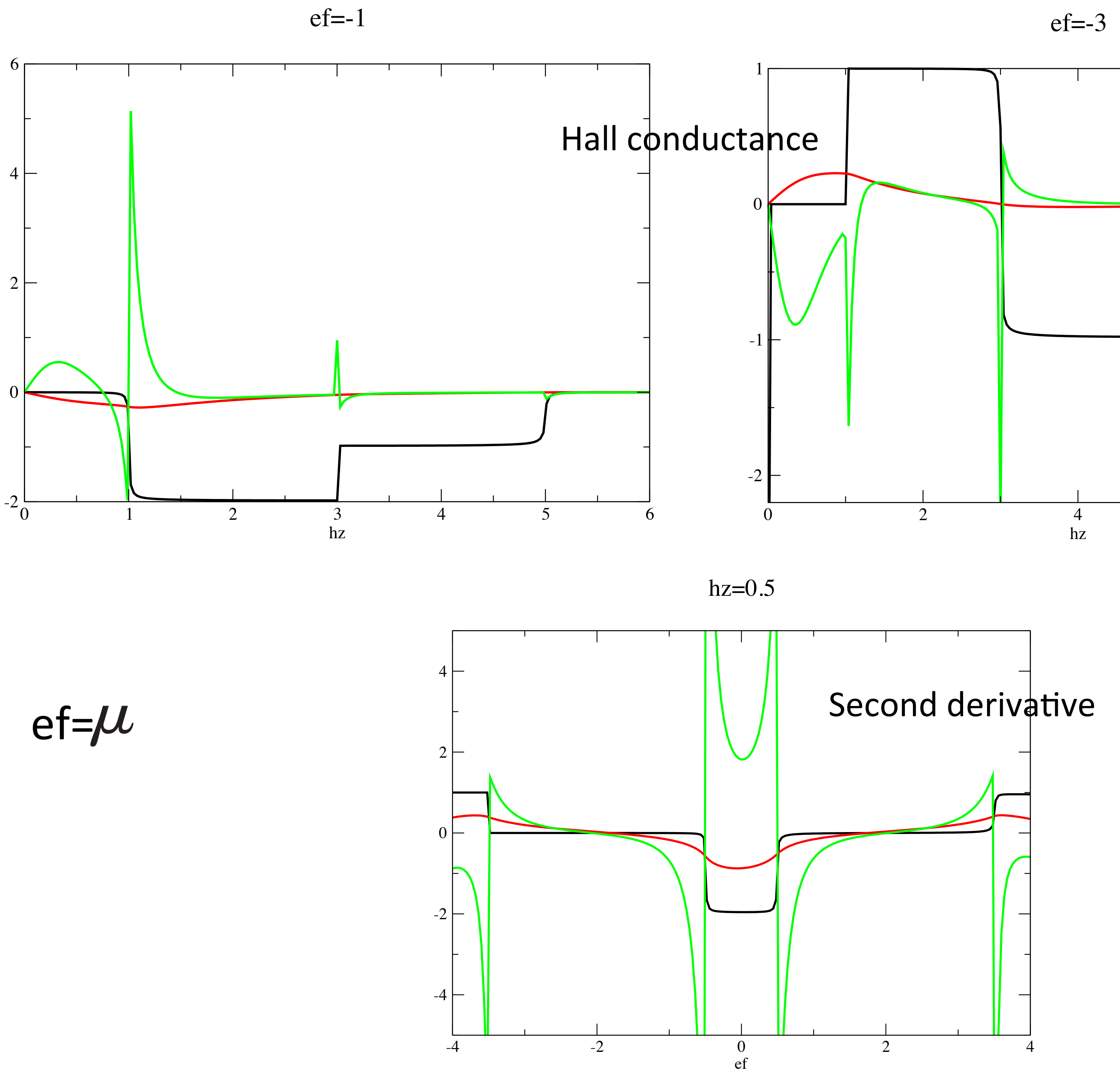
Hall conductance



$$\Delta(\boldsymbol{k}) = i\psi(\boldsymbol{k})\sigma_y + i\boldsymbol{d}(\boldsymbol{k})\boldsymbol{\sigma}\sigma_y$$

$$\mathcal{L}_0(\boldsymbol{k}) = (\sin k_y, -\sin k_x)$$

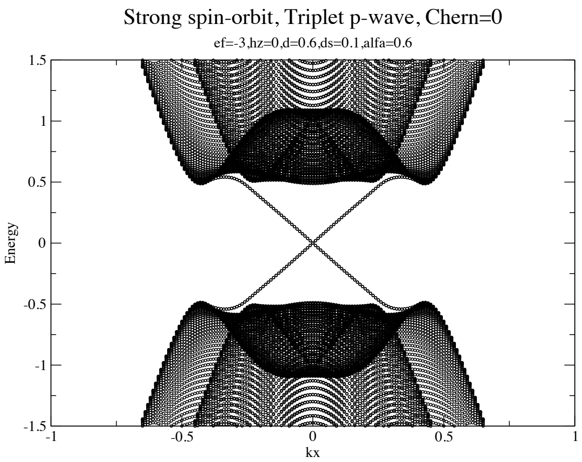
$$\boldsymbol{d}(\boldsymbol{k}) = \Delta_t \mathcal{L}_0(\boldsymbol{k})$$

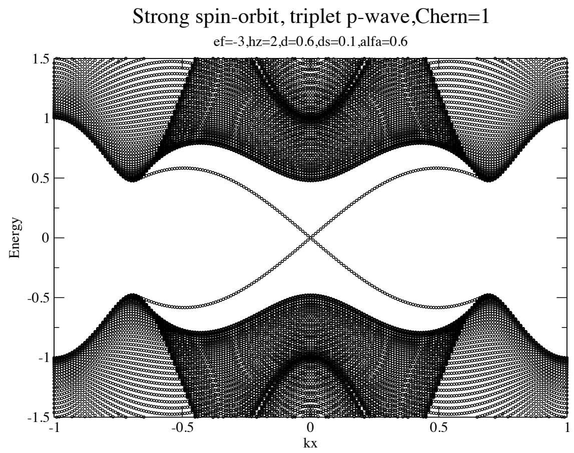


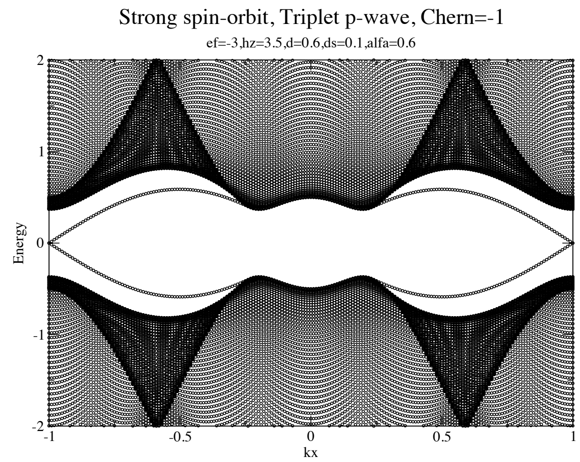
ef=-1
ef=-3
Hall conductance
Chern
hz
hz
hz=0.5
Second derivative
ef=μ
ef

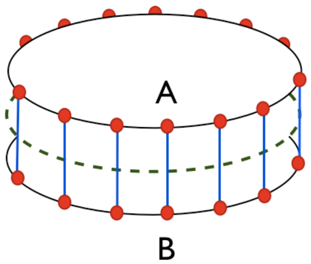
Gapless edge states (p-wave triplet pairing)

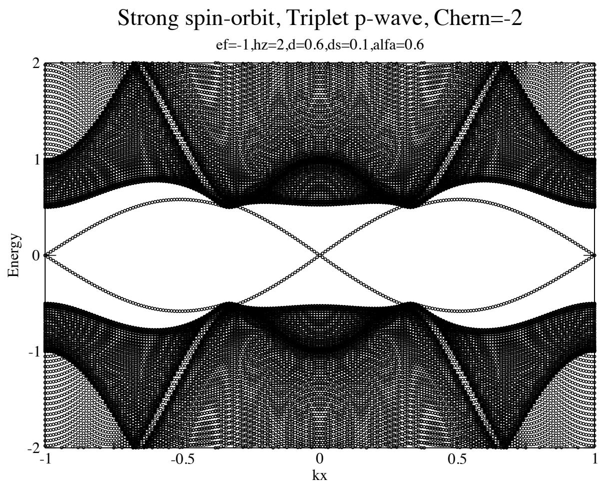
(Sato, Fujimoto, 2009)

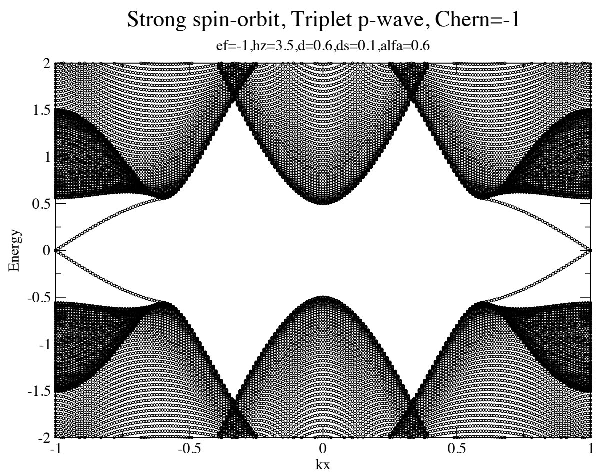






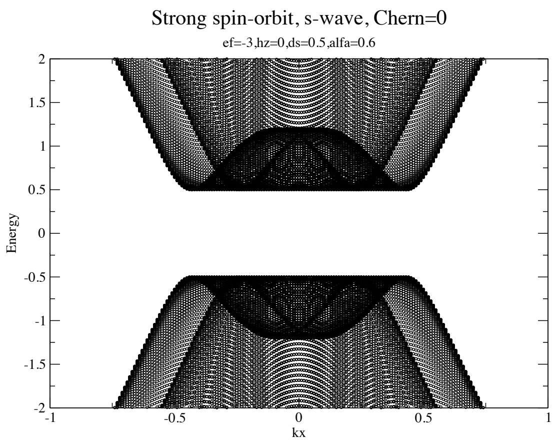


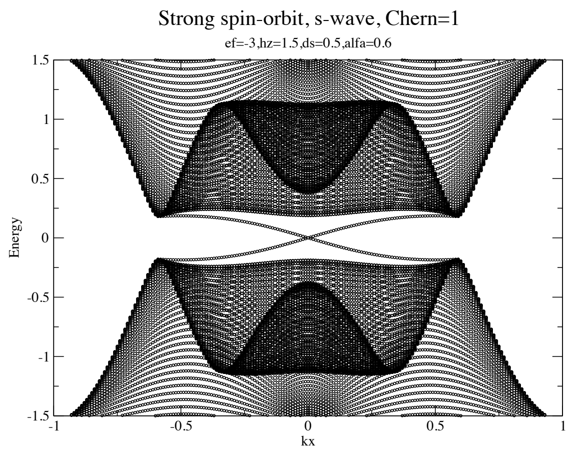


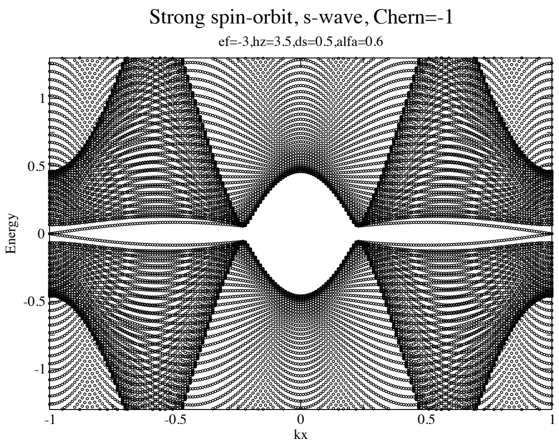


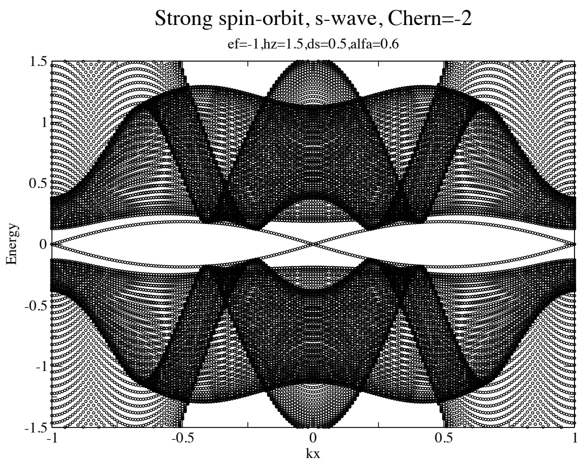
Detection through conductance or tunneling: subtract supercurrents

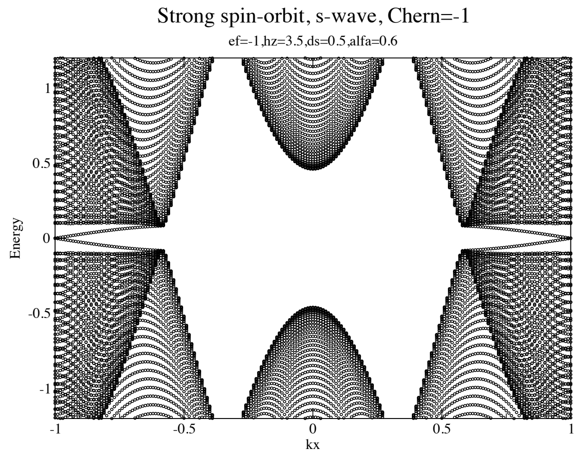
s-wave pairing





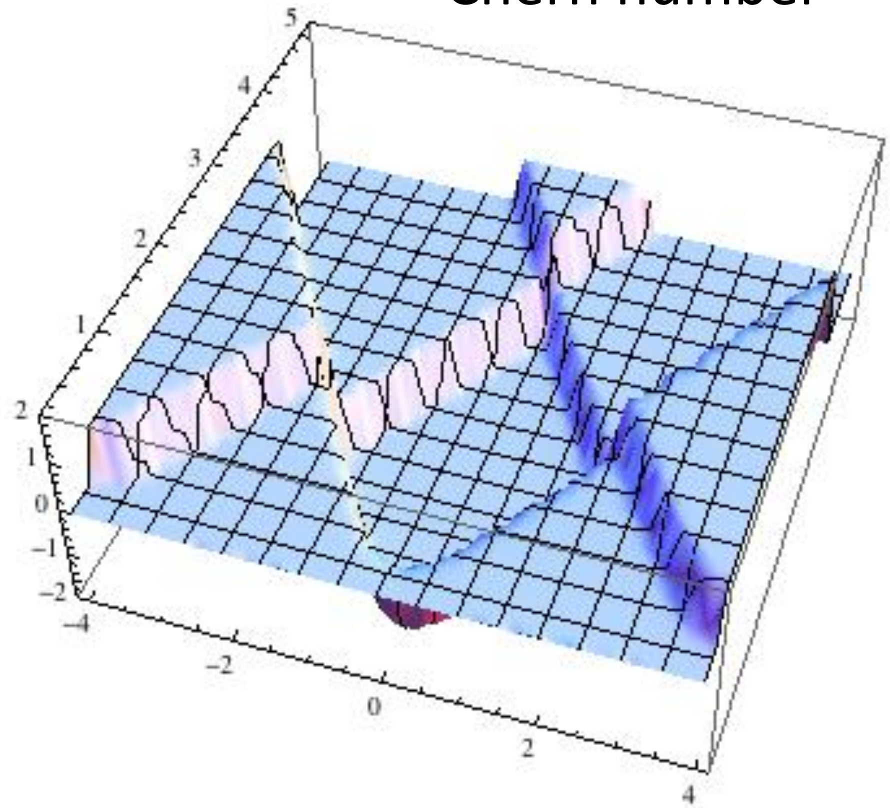




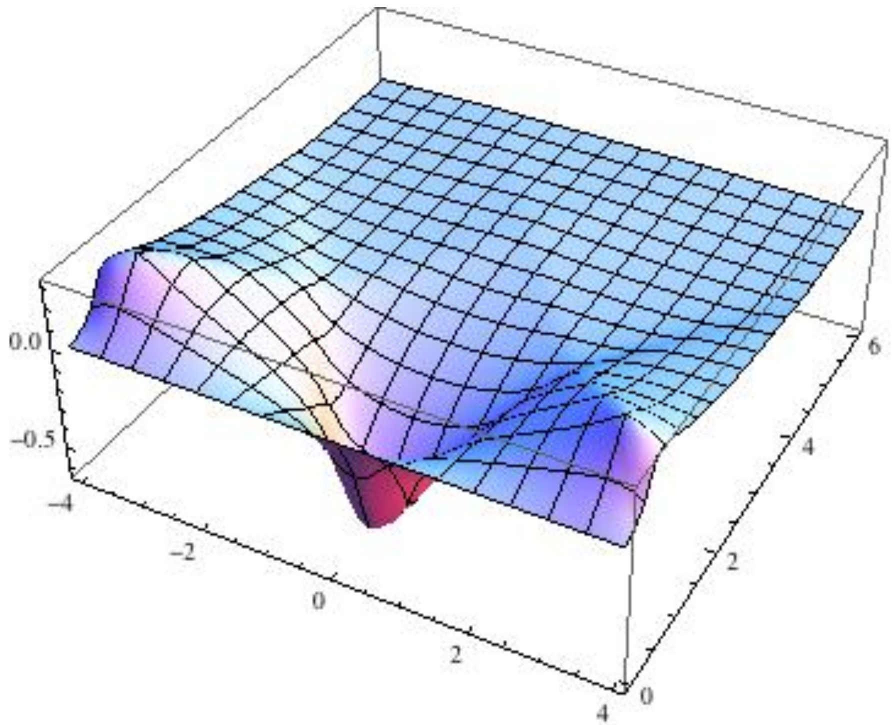


s-wave pairing

Chern number



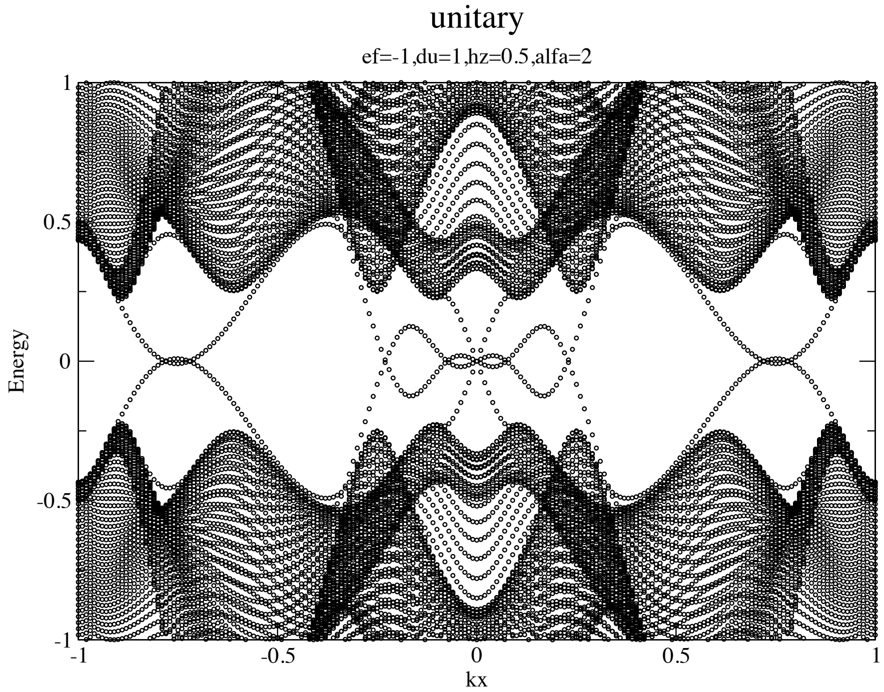
Hall conductance

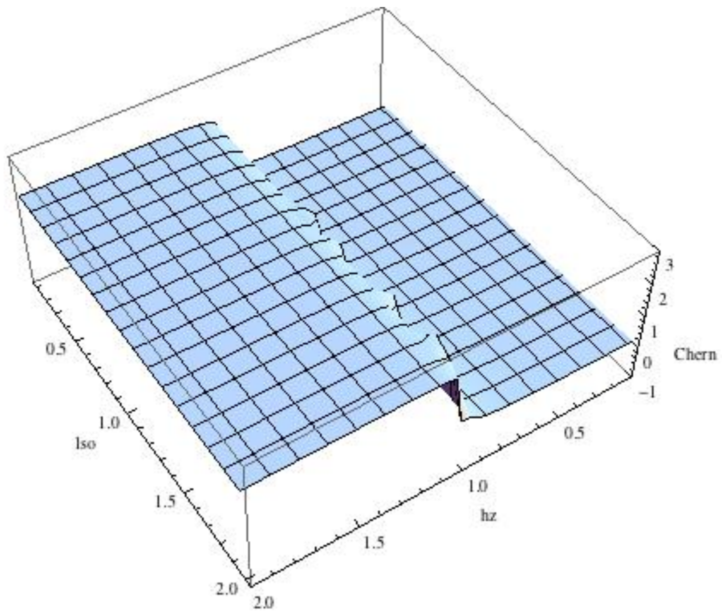


p-wave triplet pairing

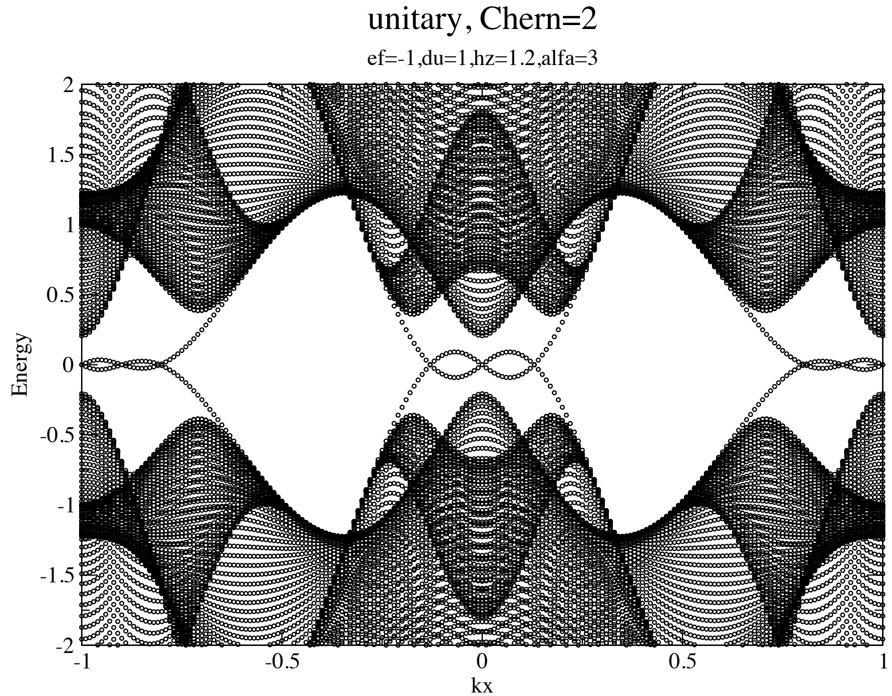
Unitary pairing

Chern=0





Chern=2



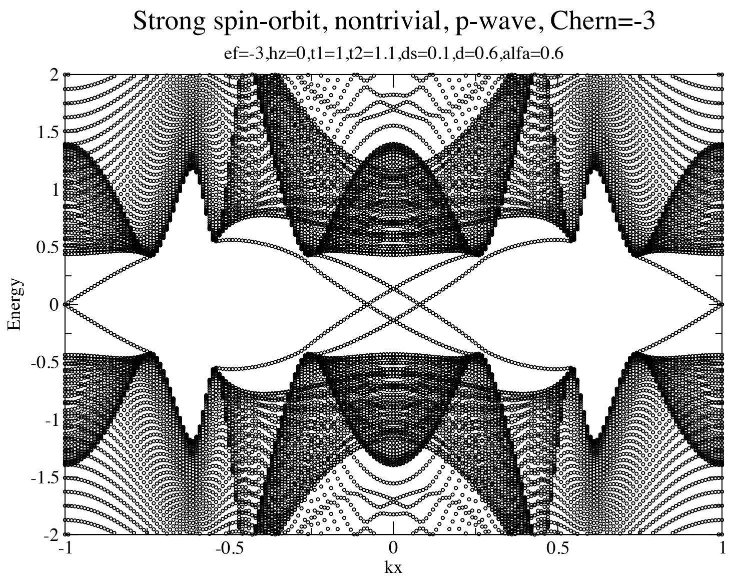
Nontrivial topology in normal phase

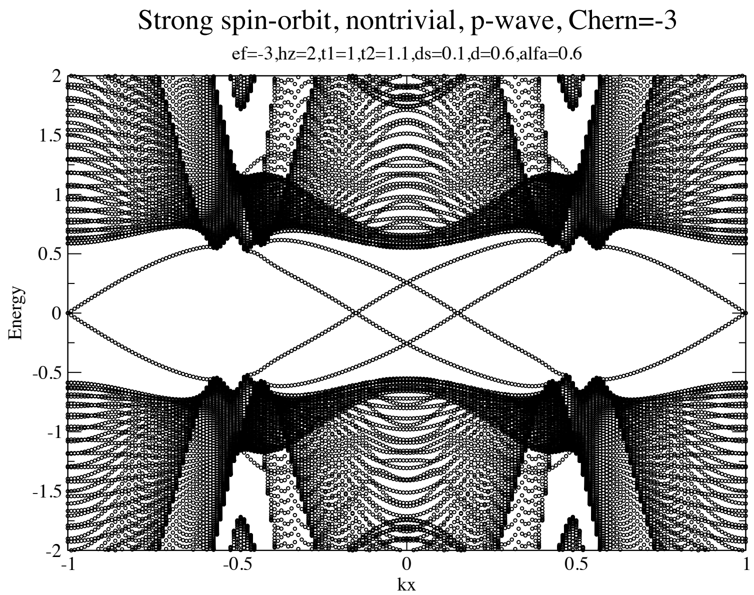
Role of interactions

(Araújo, Castro, PDS, 2012)

$$\hat{H}(\boldsymbol{h}) = \boldsymbol{h}(\boldsymbol{k}) \cdot \boldsymbol{\tau} + h_0(\boldsymbol{k})\tau_0$$

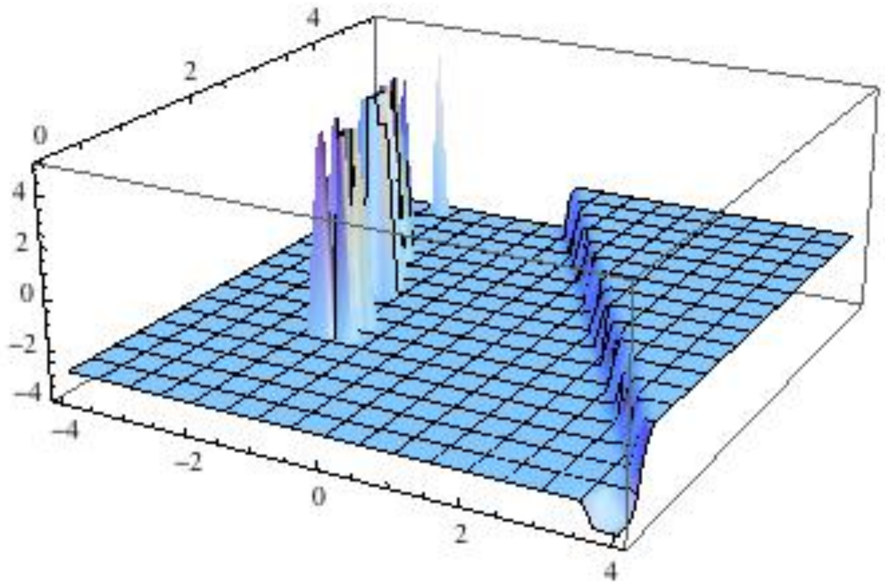
$$h_z = 4t_2 \cos k_x \cos k_y + 2t_1 \left(\cos k_x + \cos k_y\right)$$



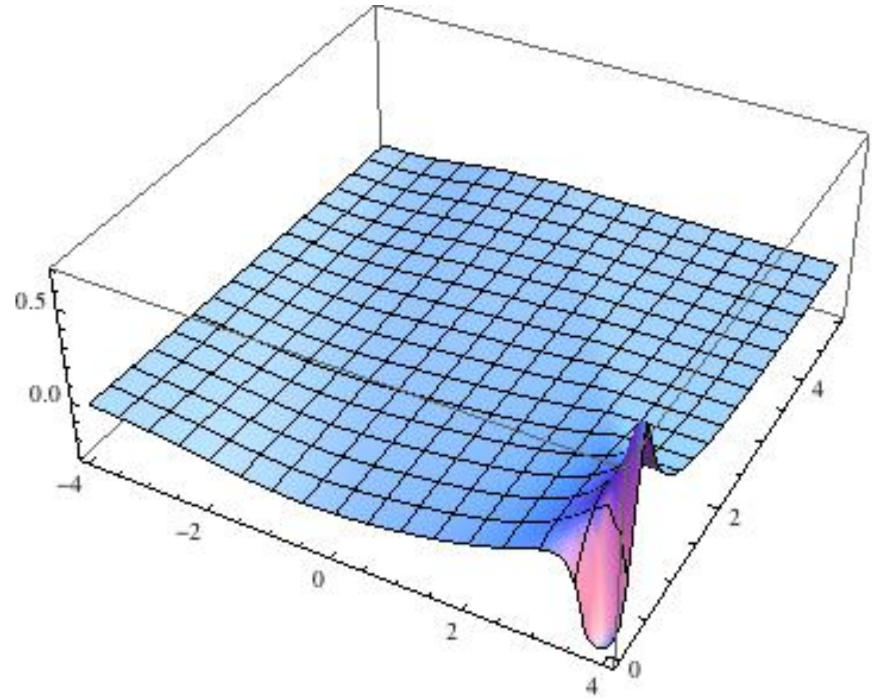


Robust in superconducting phase

Chern number



Hall conductance



Detection of gapless states

. Transport measurements with Hall geometry
  To subtract supercurrent contribution thermal transport

. Tunneling: zero bias peak

McEuen et al. (1990)
Wang, Goldman (1991)

Iniotakis et al (2007)

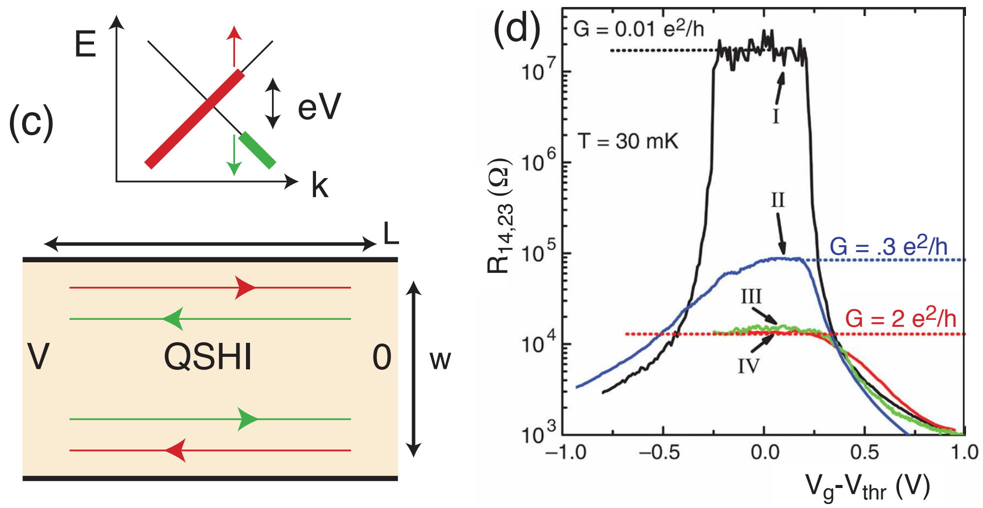
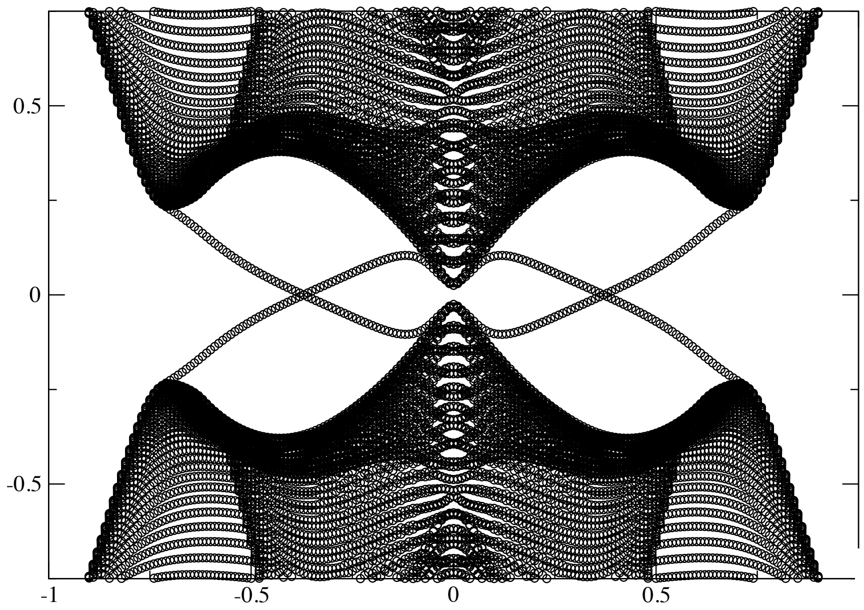


FIG. 6 (a) A HgCdTe quantum well structure. (b) As a

Conclusions

. Hall conductance not quantized. Sensitive to quantum phase transition?
. Magnetic impurity induces anomalous Hall effect
. Hall conductance signals quantum phase transition
. Hall conductance signals topological phase transitions
. First and second derivatives of Hall conductance signal
change of Chern number
. Normal phase nontrivial topology robust in superconducting phase

Nodal noncentrosymmetric superconductors



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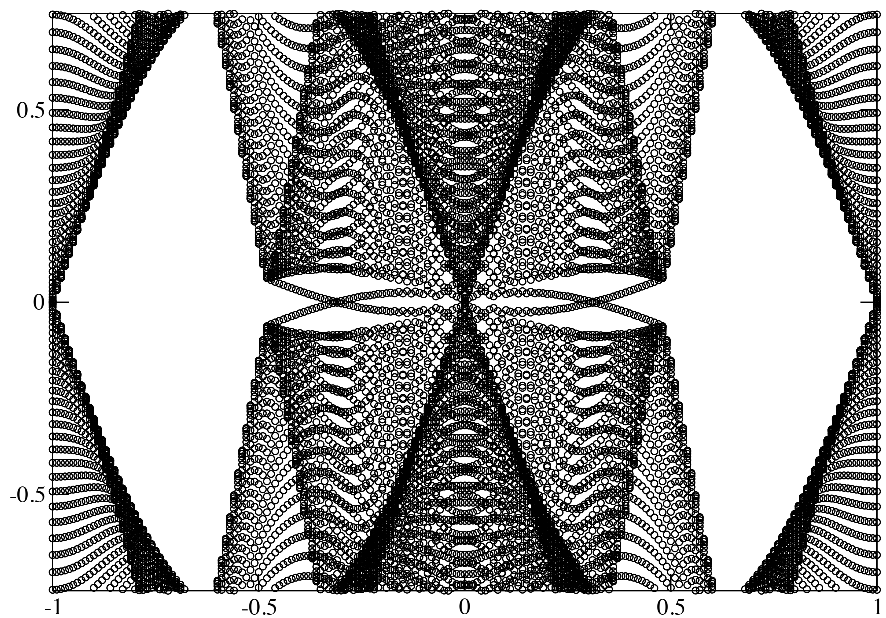

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