Dynamics and description after relaxation of disordered quantum systems after a sudden quench

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in transition to

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Correlations and coherence in quantum systems
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Collaborators

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Supported by:

Foundations of quantum statistical mechanics

Quantum ergodicity: John von Neumann ‘29
(Proof of the ergodic theorem and the H-theorem in quantum mechanics)

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Quantum ergodicity: John von Neumann '29
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Related to ideas on typicality and eigenstate thermalization:

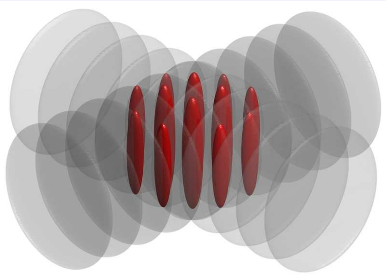
Goldstein, Lebowitz, Tumulka, and Zanghi '06
(Canonical Typicality)

Popescu, Short, and A. Winter '06
(Entanglement and the foundation of statistical mechanics)

Goldstein, Lebowitz, Mastrodonato, Tumulka, and Zanghi '10
(Normal typicality and von Neumann's quantum ergodic theorem)

MR and Srednicki '12
(Alternatives to Eigenstate Thermalization)

Experiments with ultracold gases in 1D



Effective one-dimensional δ potential

M. Olshanii, PRL **81**, 938 (1998).

$$U_{1D}(x) = g_{1D}\delta(x)$$

where

$$g_{1D} = \frac{2\hbar a_s \omega_\perp}{1 - C a_s \sqrt{\frac{m\omega_\perp}{2\hbar}}}$$

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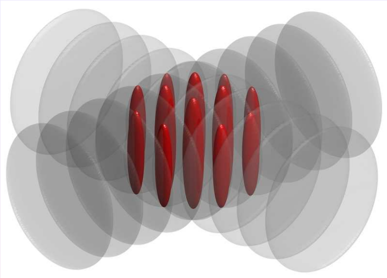

Girardeau '60

T. Kinoshita, T. Wenger, and D. S. Weiss, Science **305**, 1125 (2004).

T. Kinoshita, T. Wenger, and D. S. Weiss, Phys. Rev. Lett. **95**, 190406 (2005).

$$2 \gtrsim \gamma_{\text{eff}} = \frac{m g_{1D}}{\hbar^2 \rho} \gtrsim 20$$

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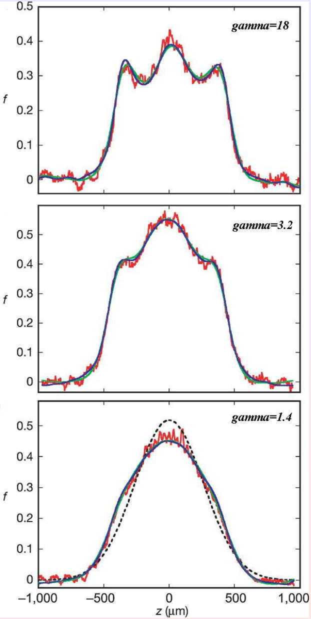
Lieb, Schulz, and Mattis '61

B. Paredes *et al.*,

Nature **429**, 277 (2004).

$$\gamma_{\text{eff}} = \frac{U}{J} \approx 5\text{–}200$$

Absence of thermalization in 1D?



T. Kinoshita, T. Wenger, and D. S. Weiss, Nature **440**, 900 (2006).

$$\gamma = \frac{m g_{1D}}{\hbar^2 \rho}$$

g_{1D}: Interaction strength
ρ: One-dimensional density

If $\gamma \gg 1$ the system is in the strongly correlated Tonks-Girardeau regime

If $\gamma \ll 1$ the system is in the weakly interacting regime

Gring *et al.*, Science **337**, 1318 (2012).

Quenches in disordered quantum systems

Exact results from quantum mechanics

If the initial state is not an eigenstate of $\widehat{H}$

$$|\psi_0\rangle \neq |\alpha\rangle \quad \text{where} \quad \widehat{H}|\alpha\rangle = E_\alpha|\alpha\rangle \quad \text{and} \quad E_0 = \langle\psi_0|\widehat{H}|\psi_0\rangle,$$

then a generic observable O will evolve in time following

$$O(\tau) \equiv \langle\psi(\tau)|\widehat{O}|\psi(\tau)\rangle \quad \text{where} \quad |\psi(\tau)\rangle = e^{-i\widehat{H}\tau}|\psi_0\rangle.$$

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What is it that we call thermalization?

$$\overline{O(\tau)} = O(E_0) = O(T) = O(T, \mu).$$

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One can rewrite

$$O(\tau) = \sum_{\alpha',\alpha} C^\star_{\alpha'} C_\alpha e^{i(E_{\alpha'}-E_\alpha)\tau} O_{\alpha'\alpha} \quad \text{where} \quad |\psi_0\rangle = \sum_\alpha C_\alpha|\alpha\rangle,$$

and taking the infinite time average (diagonal ensemble)

$$\overline{O(\tau)} = \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau d\tau' \langle\Psi(\tau')|\hat{O}|\Psi(\tau')\rangle = \sum_\alpha |C_\alpha|^2 O_{\alpha\alpha} \equiv \langle\hat{O}\rangle_{\text{diag}},$$

which depends on the initial conditions through $C_\alpha = \langle\alpha|\psi_0\rangle$.

Width of the energy density, sudden quench

Initial state $|\psi_0\rangle = \sum_\alpha C_\alpha |\alpha\rangle$ is an eigenstate of $\widehat{H}_0$. At $\tau = 0$

$$\widehat{H}_0 \to \widehat{H} = \widehat{H}_0 + \widehat{W} \qquad \text{with} \quad \widehat{W} = \sum_{j \in \sigma} \hat{w}(j) \quad \text{and} \quad \widehat{H}|\alpha\rangle = E_\alpha |\alpha\rangle.$$

MR, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008).

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The width of the energy density ΔE is then

$$\Delta E = \sqrt{\sum_\alpha E_\alpha^2 |C_\alpha|^2 - (\sum_\alpha E_\alpha |C_\alpha|^2)^2} = \sqrt{\langle\psi_0|\widehat{W}^2|\psi_0\rangle - \langle\psi_0|\widehat{W}|\psi_0\rangle^2},$$

or

$$\Delta E = \sqrt{\sum_{j_1,j_2\in\sigma} \left[\langle\psi_0|\hat{w}(j_1)\hat{w}(j_2)|\psi_0\rangle \; - \; \langle\psi_0|\hat{w}(j_1)|\psi_0\rangle\langle\psi_0|\hat{w}(j_2)|\psi_0\rangle\right]} \overset{L\rightarrow\infty}{\propto} L^{d_\sigma/2}$$

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or

$$\Delta E = \sqrt{\sum_{j_1, j_2 \in \sigma} [\langle\psi_0|\hat{w}(j_1)\hat{w}(j_2)|\psi_0\rangle - \langle\psi_0|\hat{w}(j_1)|\psi_0\rangle\langle\psi_0|\hat{w}(j_2)|\psi_0\rangle]} \overset{L\to\infty}{\propto} L^{d_\sigma/2}$$

Since the width of the full spectrum diverges as L^{d_L}

$$\Delta\epsilon = \frac{\Delta E}{L^{d_L}} \overset{L\to\infty}{\propto} \frac{1}{L^{d_L - d_\sigma/2}},$$

$d_L(d_\sigma)$ is the dimensionality of the lattice (of the region affected by the quench).
since $d_L \geq d_\sigma$ then $\Delta\epsilon$ vanishes in the thermodynamic limit.

MR, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008).

Quenches in disordered quantum systems

Description after relaxation

Hard-core boson Hamiltonian

$$\hat{H} = \sum_{i=1}^{L} -t\left(\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.}\right) + V\hat{n}_i\hat{n}_{i+1} - t'\left(\hat{b}_i^\dagger \hat{b}_{i+2} + \text{H.c.}\right) + V'\hat{n}_i\hat{n}_{i+2} + \mu_i\hat{n}_i$$

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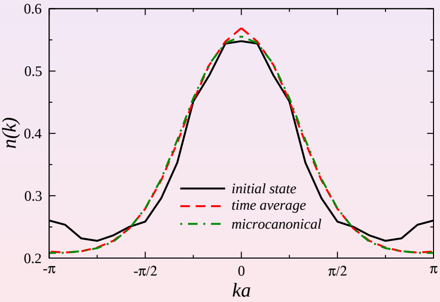
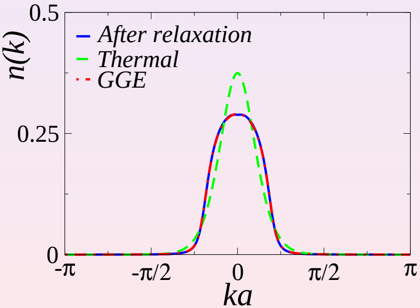
Dynamics vs statistical ensembles

Nonintegrable: $t' = V' \neq 0$, $\mu_i = 0$



MR, PRL **103**, 100403 (2009);
PRA **80**, 053607 (2009), . . .

Integrable: $V = t' = V' = 0$, $\mu_i \neq 0$



MR, V. Dunjko, V. Yurovsky, and
M. Olshanii, PRL **98**, 050405 (2007), . . .

Eigenstate thermalization

Eigenstate thermalization hypothesis

[Deutsch, PRA **43** 2046 (1991); Srednicki, PRE **50**, 888 (1994).]

- The expectation value $\langle\alpha|\widehat{O}|\alpha\rangle$ of a few-body observable $\widehat{O}$ in an eigenstate of the Hamiltonian $|\alpha\rangle$, with energy E_α, of a many-body system equals the thermal average of $\widehat{O}$ at the mean energy E_α:

$$\langle\alpha|\widehat{O}|\alpha\rangle = \langle\widehat{O}\rangle_{\text{microcan.}}(E_\alpha).$$

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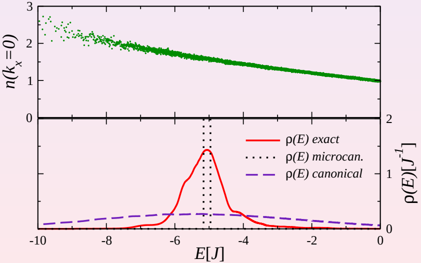
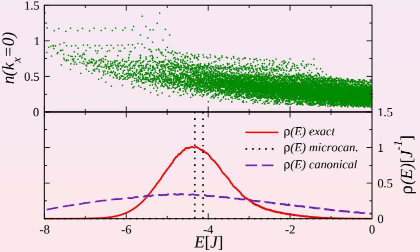
Nonintegrable



Integrable $(\hat{\rho}_{\text{GGE}} = \frac{1}{Z_{\text{GGE}}} e^{-\sum_m \lambda_m \hat{I}_m})$



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What changes in the presence of disorder?

Many-body localization

- D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Ann. Phys. **321**, 1126 (2006).
- V. Oganesyan and D. A. Huse, Phys. Rev. B **75**, 155111 (2007).
- A. Pal and D. A. Huse, Phys. Rev. B **82**, 174411 (2010).
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Some questions we would like to address

- How is the relaxation dynamics?
- Will observables fail to equilibrate?

$$O(\tau) \neq \overline{O(\tau)}$$

- If an observable equilibrates, will it fail to thermalize?

$$\overline{O(\tau)} \neq O(E_0) = O(T) = O(T, \mu)$$

Model Hamiltonian and the localization transition

Spinless fermion Hamiltonian in 1D

$$\hat{H} = \sum_{ij} J_{ij} \left(\hat{f}_i^\dagger \hat{f}_j + \text{H.c.} \right) + V \sum_i \left(\hat{n}_i - \frac{1}{2} \right) \left(\hat{n}_{i+1} - \frac{1}{2} \right)$$

E. Khatami, MR, A. Relaño, and A. M. García-García, PRE **85**, 050102(R) (2012).

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Hopping amplitudes

Gaussian random distribution $\langle J_{ij} \rangle = 0$

$$\langle (J_{ij})^2 \rangle = \left[1 + \left(\frac{|i-j|}{\beta} \right)^{2\alpha} \right]^{-1}$$

Limit $V = 0$:

- Properties depend on α but not on $\beta > 0$
- $\alpha < 1$, eigenstates are delocalized
- $\alpha > 1$, eigenstates are localized
- $\alpha = 1$, eigenstates are multifractal

Mirlin *et al.*, PRE **54**, 3221 (1996).

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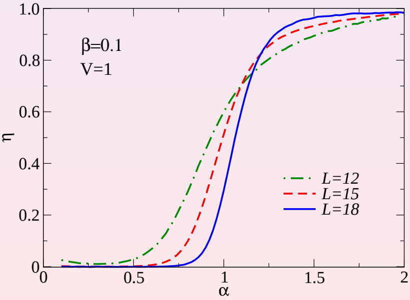
Metal-insulator transition

$$\eta = [\text{var} - \text{var}_{\text{WD}}]/[\text{var}_{\text{P}} - \text{var}_{\text{WD}}]$$

var: variance of level spacing distribution



Dynamics after a quench

Quench protocol

- Start from an eigenstate of $\hat{H}$ ($|\psi_0\rangle$) in a certain disorder realization.
- Evolve under another disorder realization with the same α.
- $E = \langle\psi_0|\hat{H}_{\text{fin}}|\psi_0\rangle$ is the energy of a thermal state with temperature $T = 10$.

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Microcanonical vs diagonal

Observables:

$$\hat{n}(k) = \frac{1}{L}\sum_{l,m} e^{ik(l-m)}\hat{f}_l^\dagger \hat{f}_m$$

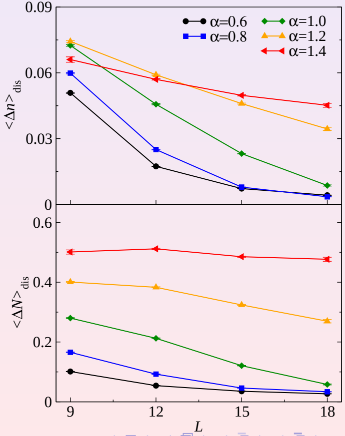
$$\hat{N}(k) = \frac{1}{L}\sum_{l,m} e^{ik(l-m)}\hat{n}_l\hat{n}_m$$

Normalized difference:

$$\Delta O = \frac{\sum_k |O_{\text{mic}}(k) - O_{\text{diag}}(k)|}{\sum_k O_{\text{diag}}(k)}$$

Disorder average:

$$\langle\Delta O\rangle_{\text{dis}}$$



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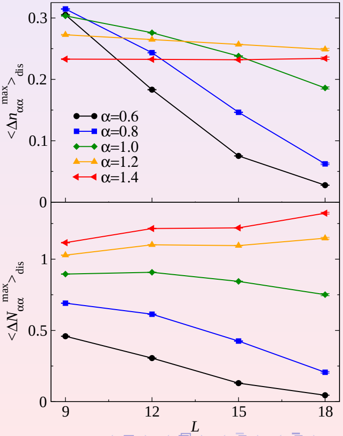
$$\hat{N}(k) = \frac{1}{L}\sum_{l,m} e^{ik(l-m)}\hat{n}_l\hat{n}_m$$

Maximal normalized difference:

$$\Delta O_{\alpha\alpha}^{\text{max}} = \frac{\sum_k |O_{\alpha\alpha}^{\text{max}}(k) - O_{\text{mic}}(k)|}{\sum_k O_{\text{mic}}(k)}$$

Disorder average:

$$\langle \Delta O_{\alpha\alpha}^{\text{max}}\rangle_{\text{dis}}$$



Quenches in disordered quantum systems

Bose-Fermi mapping

Hard-core boson Hamiltonian in an external potential

$$\hat{H} = -J \sum_i \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.} \right) + \sum_i v_i \ \hat{n}_i$$

Constraints on the bosonic operators

$$\hat{b}_i^{\dagger 2} = \hat{b}_i^2 = 0$$

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$$\Downarrow$$

Map to spins and then to fermions (Jordan-Wigner transformation)

$$\sigma_i^+ = \hat{f}_i^\dagger \prod_{\beta=1}^{i-1} e^{-i\pi \hat{f}_\beta^\dagger \hat{f}_\beta}, \quad \sigma_i^- = \prod_{\beta=1}^{i-1} e^{i\pi \hat{f}_\beta^\dagger \hat{f}_\beta} \hat{f}_i$$

$$\Downarrow$$

Non-interacting fermion Hamiltonian

$$\hat{H}_F = -J \sum_i \left(\hat{f}_i^\dagger \hat{f}_{i+1} + \text{H.c.} \right) + \sum_i v_i \; \hat{n}_i^f$$

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$$\Downarrow$$

Integrals of motion

(underlying noninteracting fermions)

$$\hat{H}_F \hat{\gamma}_m^{f\dagger} |0\rangle = E_m \hat{\gamma}_m^{f\dagger} |0\rangle$$

$$\left\{ \hat{I}_m^f \right\} = \left\{ \hat{\gamma}_m^{f\dagger} \hat{\gamma}_m^f \right\}$$

Lagrange multipliers

(can be calculated analytically)

$$\lambda_m = \ln \left[\frac{1 - \langle \hat{I}_m \rangle_{\tau=0}}{\langle \hat{I}_m \rangle_{\tau=0}} \right]$$

One-particle density matrix

One-particle Green's function

$$G_{ij} = \langle \Psi_{HCB} | \sigma_i^- \sigma_j^+ | \Psi_{HCB} \rangle = \langle \Psi_F | \prod_{\beta=1}^{i-1} e^{i\pi \hat{f}_\beta^\dagger \hat{f}_\beta} \hat{f}_i \hat{f}_j^\dagger \prod_{\gamma=1}^{j-1} e^{-i\pi \hat{f}_\gamma^\dagger \hat{f}_\gamma} | \Psi_F \rangle$$

$$\Downarrow$$

Time evolution

$$|\Psi_F(\tau)\rangle = e^{-i\hat{H}_F \tau/\hbar} |\Psi_F^I\rangle = \prod_{\delta=1}^{N} \sum_{\sigma=1}^{L} P_{\sigma\delta}(\tau) \hat{f}_\sigma^\dagger \, |0\rangle$$

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$$\Downarrow$$

Exact Green's function

$$G_{ij}(\tau) = \det \left[\left(\mathbf{P}^l(\tau) \right)^\dagger \mathbf{P}^r(\tau) \right]$$

Computation time $\sim L^2 N^3$

3000 lattice sites, 300 particles

MR and A. Muramatsu, PRL **93**, 230404 (2004); PRL **94**, 240403 (2005).

Finite temperature

One-particle density matrix (grand-canonical ensemble)

$$\rho_{ij} \equiv \frac{1}{Z}\mathrm{Tr}\left\{\hat{b}_i^\dagger \hat{b}_j e^{-\frac{\hat{H}_{HCB}-\mu\sum_m \hat{b}_m^\dagger \hat{b}_m}{k_B T}}\right\}, \quad Z = \mathrm{Tr}\left\{e^{-\frac{\hat{H}_{HCB}-\mu\sum_m \hat{b}_m^\dagger \hat{b}_m}{k_B T}}\right\}$$

$$\Downarrow$$

Mapping to noninteracting fermions

$$\rho_{ij} = \frac{1}{Z}\mathrm{Tr}\left\{\hat{f}_i^\dagger \hat{f}_j \prod_{k=1}^{j-1} e^{i\pi \hat{f}_k^\dagger \hat{f}_k} e^{-\frac{\hat{H}_F-\mu\sum_m \hat{f}_m^\dagger \hat{f}_m}{k_B T}} \prod_{l=1}^{i-1} e^{-i\pi \hat{f}_l^\dagger \hat{f}_l}\right\}$$

$$\Downarrow$$

Exact one-particle density matrix

$$\rho_{ij} = \frac{1}{Z}\left\{\det\left[\mathbf{I} + (\mathbf{I}+\mathbf{A})\mathbf{O}_1\mathbf{U}e^{-(\mathbf{E}-\mu\mathbf{I})/k_BT}\mathbf{U}^\dagger\mathbf{O}_2\right] - \det\left[\mathbf{I} + \mathbf{O}_1\mathbf{U}e^{-(\mathbf{E}-\mu\mathbf{I})/k_BT}\mathbf{U}^\dagger\mathbf{O}_2\right]\right\}$$

Computation time $\sim L^5$: 1000 sites

MR, PRA **72**, 063607 (2005).

Model Hamiltonian and the localization transition

Hard-core boson Hamiltonian in 1D ($\lambda_c = 2J$)

$$\hat{H} = -J \sum_{i=1}^{L-1} (\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.}) + \lambda \sum_i \cos(2\pi\sigma i + \delta)\, \hat{n}_i^b \quad \text{where} \quad \sigma = (\sqrt{5} - 1)/2$$

C. Gramsch and MR, arXiv:1206.3570.

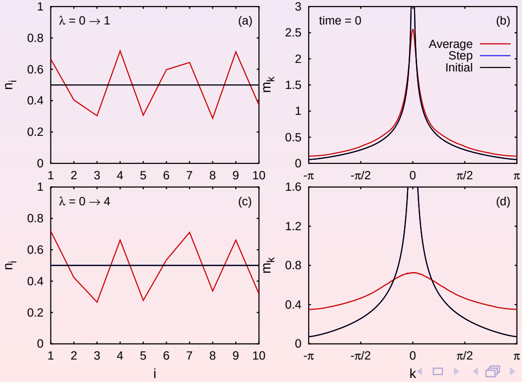
Model Hamiltonian and the localization transition

Hard-core boson Hamiltonian in 1D ($\lambda_c = 2J$)

$$\hat{H} = -J \sum_{i=1}^{L-1} (\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.}) + \lambda \sum_i \cos(2\pi\sigma i + \delta)\, \hat{n}_i^b \quad \text{where} \quad \sigma = (\sqrt{5}-1)/2$$

C. Gramsch and MR, arXiv:1206.3570.

Dynamics after a quench from the ground state ($\lambda_I = 0 \rightarrow \lambda_F \neq 0$)



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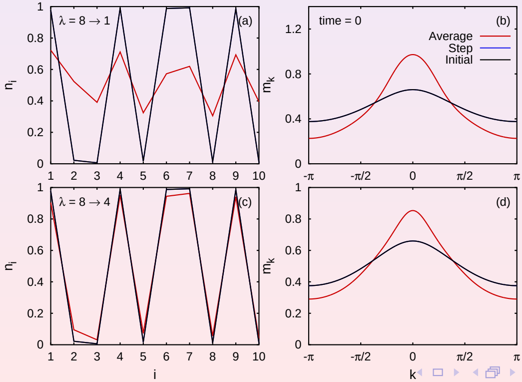
C. Gramsch and MR, arXiv:1206.3570.

Dynamics after a quench from the ground state ($\lambda_I \neq 0 \rightarrow \lambda_F < \lambda_I$)



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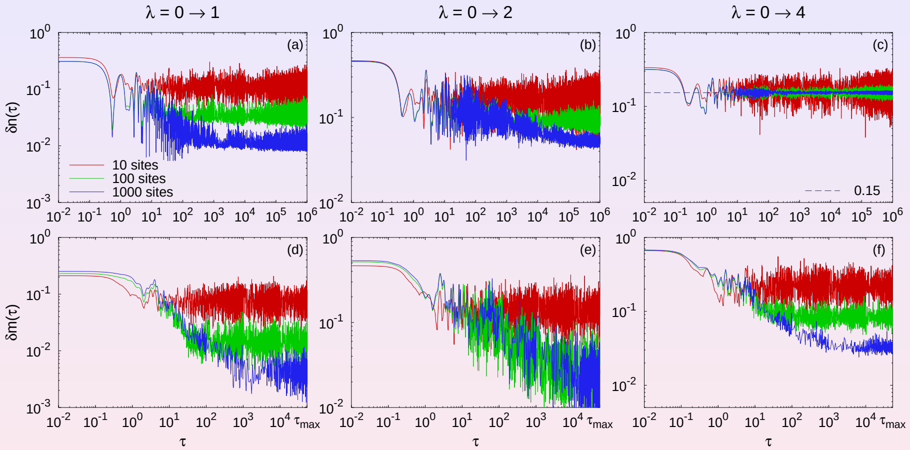
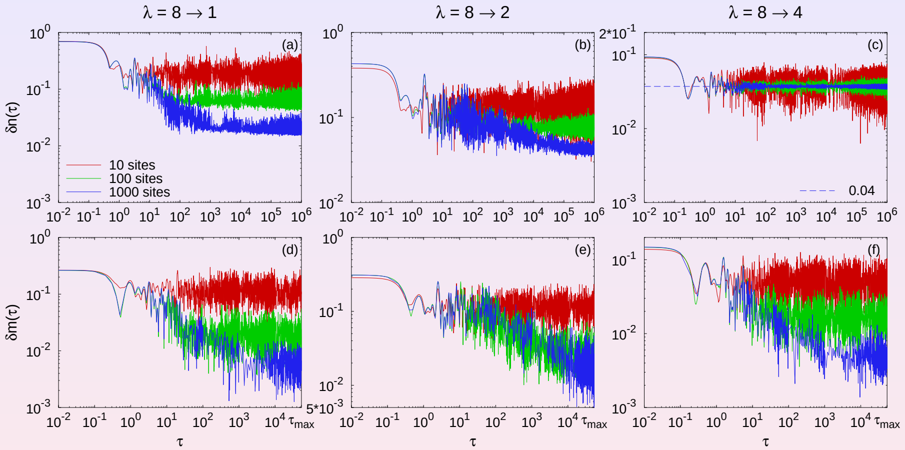


$$\delta n(\tau) = \frac{\sum_i |n_i(\tau) - \overline{n_i(\tau)}|}{\sum_i \overline{n_i(\tau)}} \qquad \delta m(\tau) = \frac{\sum_k |m_k(\tau) - \overline{m_k(\tau)}|}{\sum_k \overline{m_k(\tau)}}$$

Dynamics after a quench from the ground state



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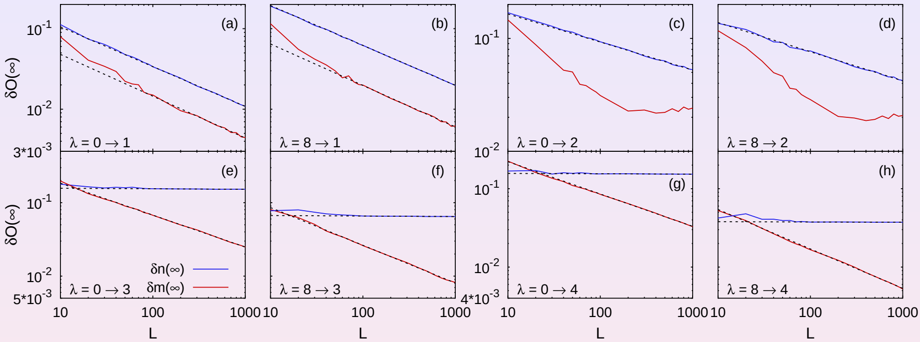
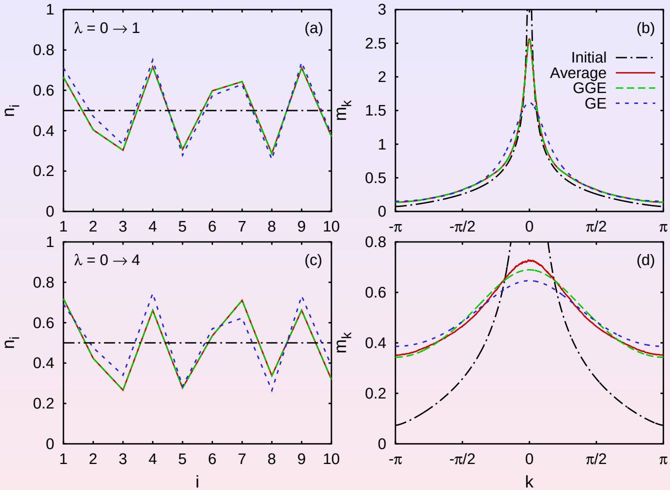


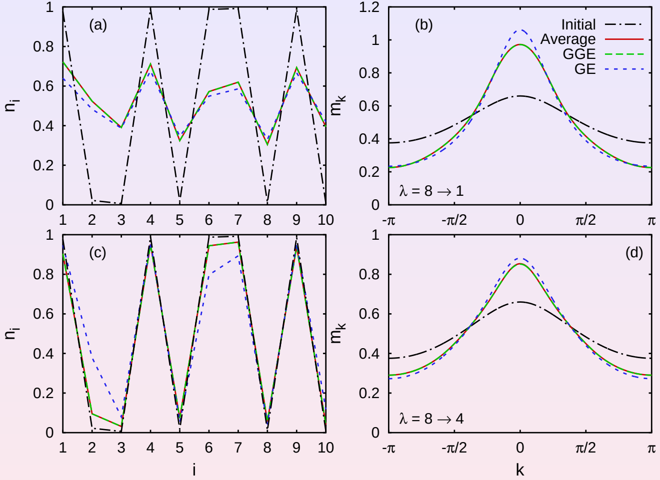
- Delocalized phase ($\lambda_F < 2$): $\delta n(\infty) \sim \delta m(\infty) \propto 1/\sqrt{L}$
- Critical point ($\lambda_F = 2$): $\delta n(\infty) \propto 1/L^{1/4}$
- Localized phase ($\lambda_F > 2$): $\delta n(\infty) =$ const, $\delta m(\infty) \propto 1/\sqrt{L}$.
- Recent analytic results (proof for a specific class of observables):
  L. Campos Venuti and P. Zanardi, arXiv:1208.1121.

Results after relaxation vs statistical mechanics



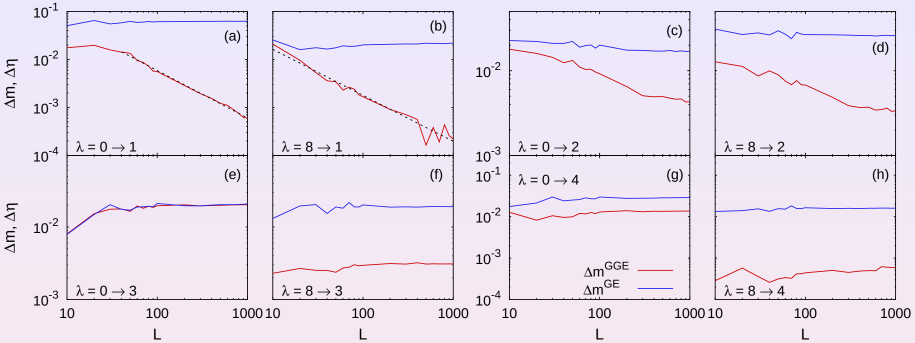
- Delocalized phase ($\lambda_F < 2$): GGE describes one-body observables, GE fails.
- Localized phase ($\lambda_F > 2$): GGE describes n_i but fails for m_k, GE fails.

Results after relaxation vs statistical mechanics



- Delocalized phase ($\lambda_F < 2$): GGE describes one-body observables, GE fails.
- Localized phase ($\lambda_F > 2$): GGE describes n_i and m_k (?), GE fails.

Scaling of Δm with L



- Delocalized phase ($\lambda_F < 2$): GGE describes one-body observables ($\Delta m^{\text{GGE}} \propto 1/L$), GE fails.
- Critical point ($\lambda_F = 2$): GGE describes one-body observables, GE fails.
- Localized phase ($\lambda_F > 2$): GGE describes n_i but fails for m_k, GE fails.

Summary

Nonintegrable case

- Delocalized regime: Eigenstate thermalization holds and the system thermalizes. Power law relaxation?
- Localized regime: Eigenstate thermalization fails and the system does not thermalize

Integrable case

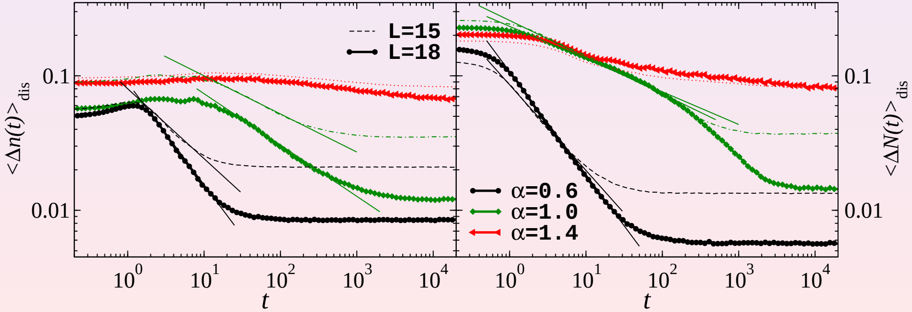
- Delocalized regime: n_i and m_k equilibrate and they are described by GGE, despite the lack of translational invariance!
  Power law relaxation?
- Critical point: Slower relaxation dynamics. GGE describes observables after relaxation
- Localized regime: m_k equilibrates but GGE fails to describe it after relaxation

Dynamics after a quench

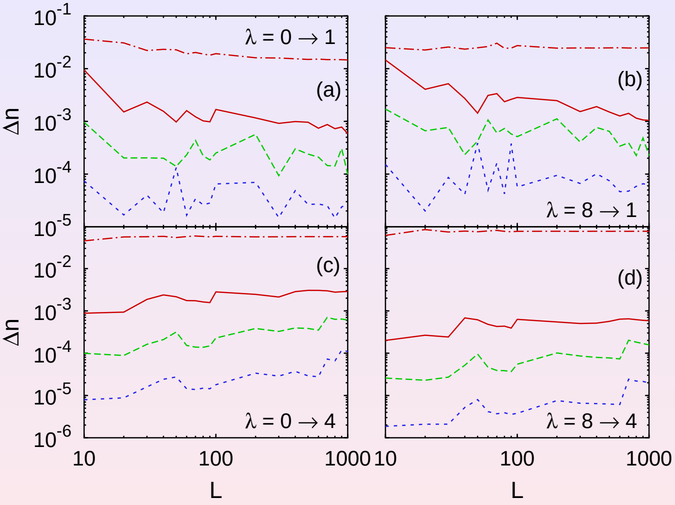
Quench protocol

- Start from an eigenstate of $\hat{H}$ ($|\psi_0\rangle$) in a certain disorder realization.
- Evolve under another disorder realization with the same α.
- $E = \langle\psi_0|\hat{H}_{\text{fin}}|\psi_0\rangle$ is the energy of a thermal state with temperature $T = 10$.
- Everything is computed by means of full exact diagonalization.

Time evolution $[\Delta O(t) = \sum_k |O(k,t) - O_{\text{diag}}(k)| / \sum_k O_{\text{diag}}(k)]$



Scaling of Δn with L



- In all regimes: the differences go to zero as the accuracy in the calculation of the time average is increased.