Efimov effect in quantum magnets

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Workshop on “correlations and coherence in quantum systems”

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LA-UR-12-25153

Plan of this talk

1. Universality in physics

2. What is the Efimov effect?

   Keywords: universality, discrete scale invariance, RG limit cycle

3. Efimov effect in solid state systems*

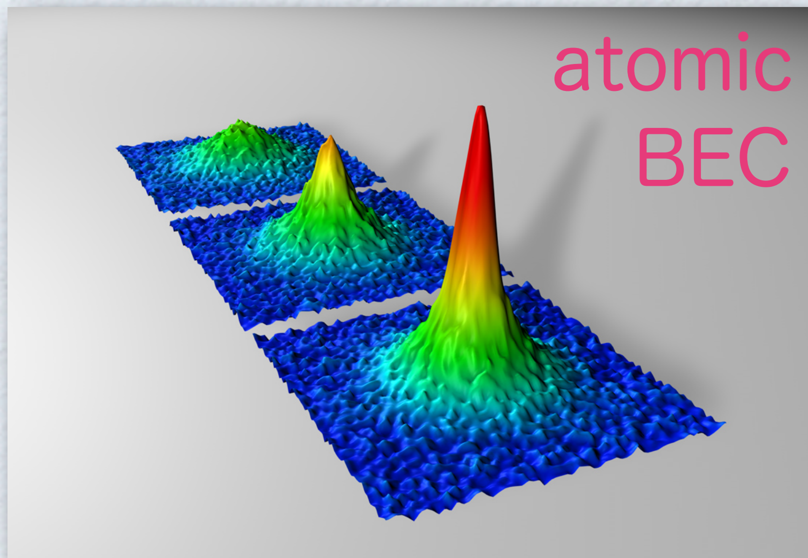
* based on collaboration with
Y. Kato and C. D. Batista, arXiv:1208.6214
"Efimov effect in quantum magnets"

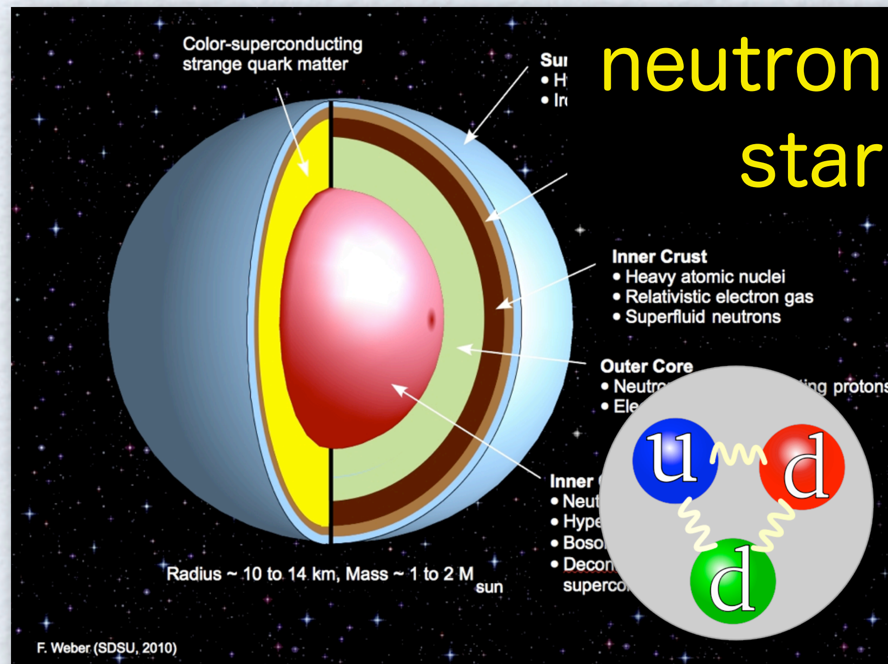
Introduction

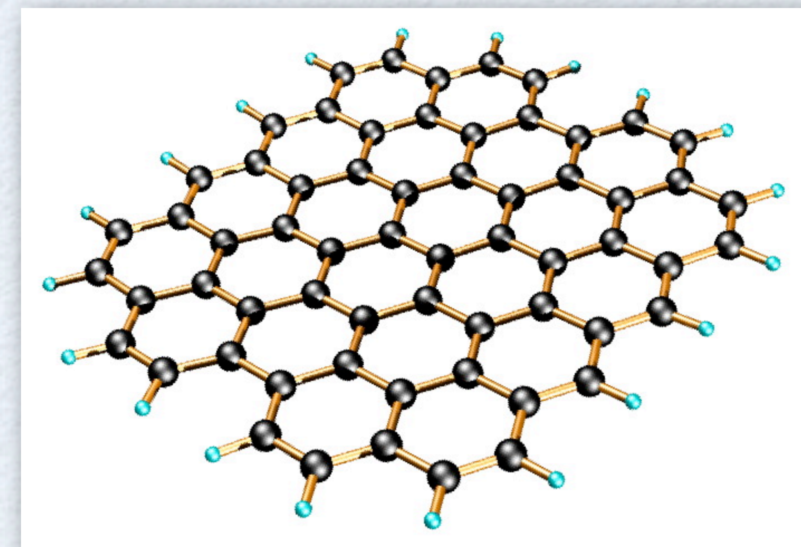
1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect in solid state systems

(ultimate) Goal of research

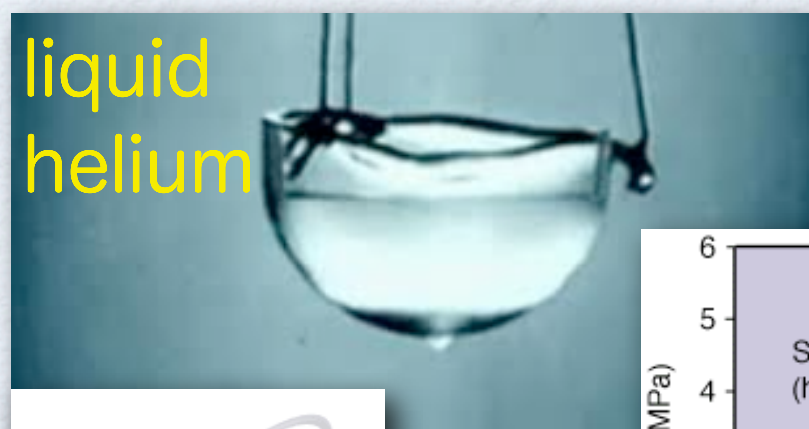
Understand physics of few and many particles governed by quantum mechanics

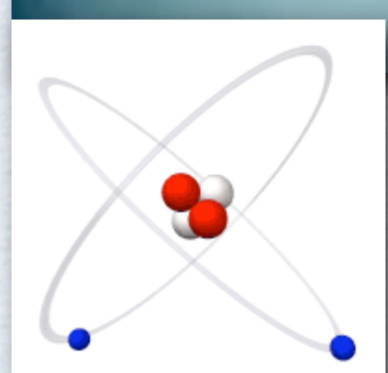


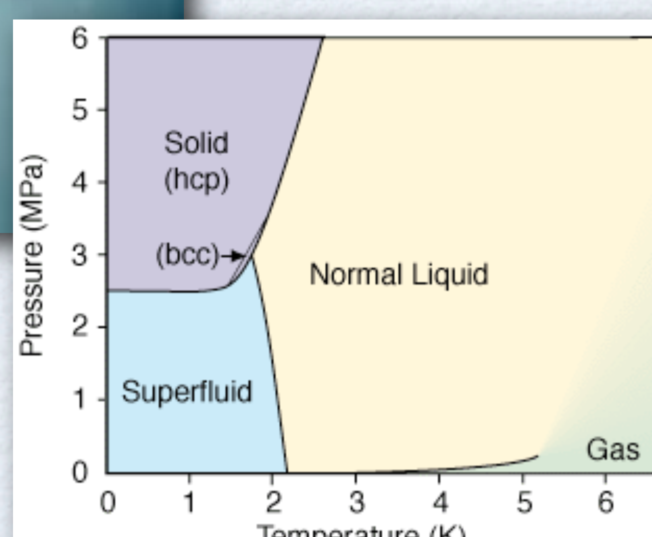




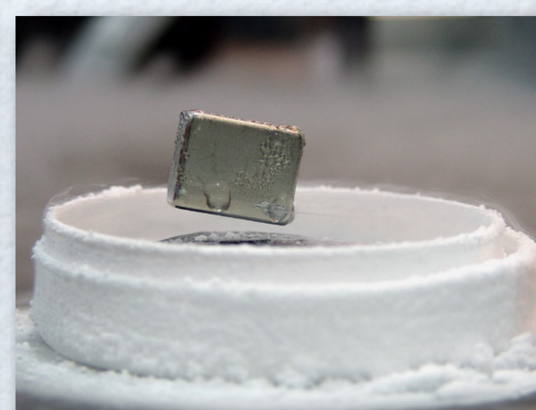
graphene

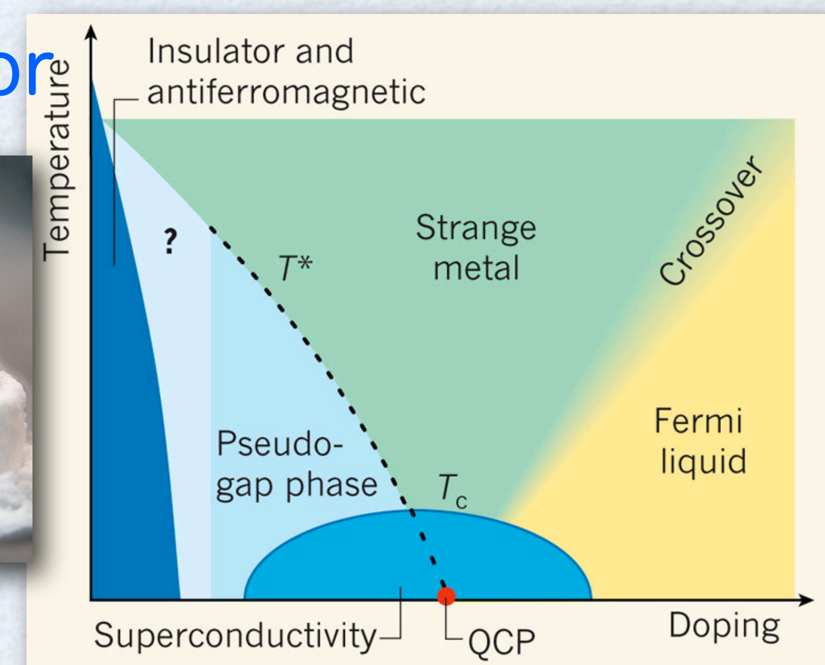






superconductor

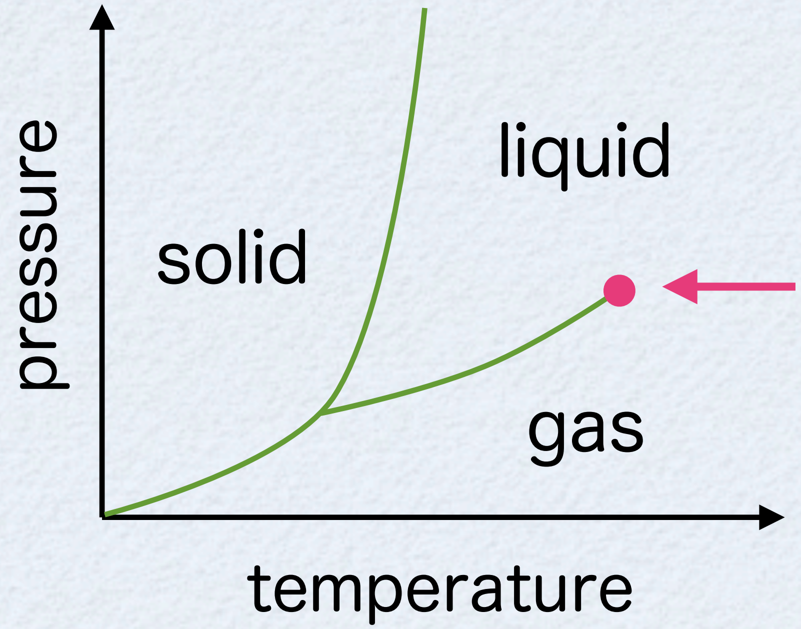


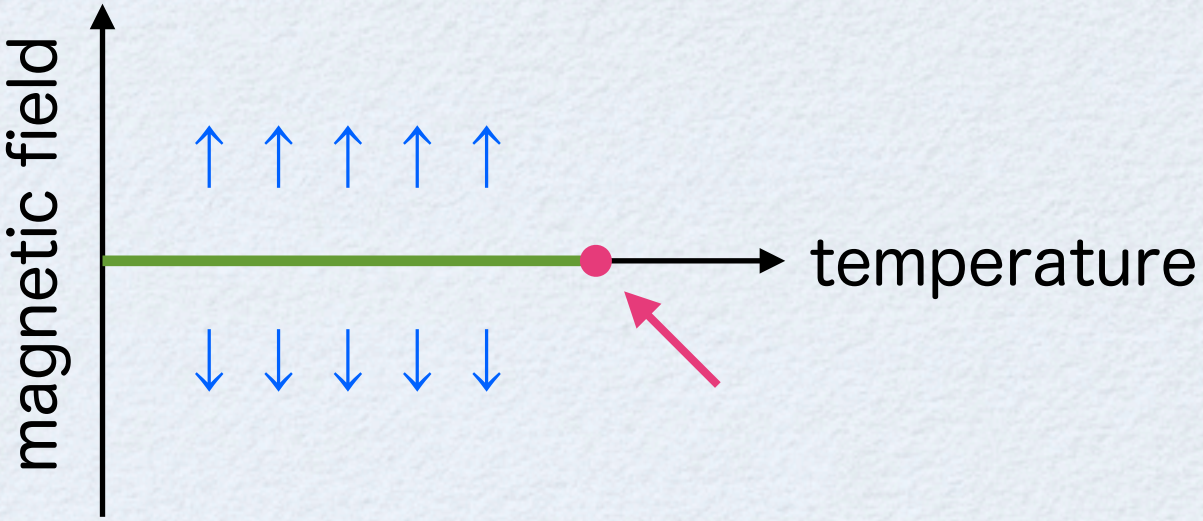


When physics is universal ?

A1. Continuous phase transitions $\Leftrightarrow$ $\xi / r_0 \to \infty$

E.g. Water  vs. Magnet 





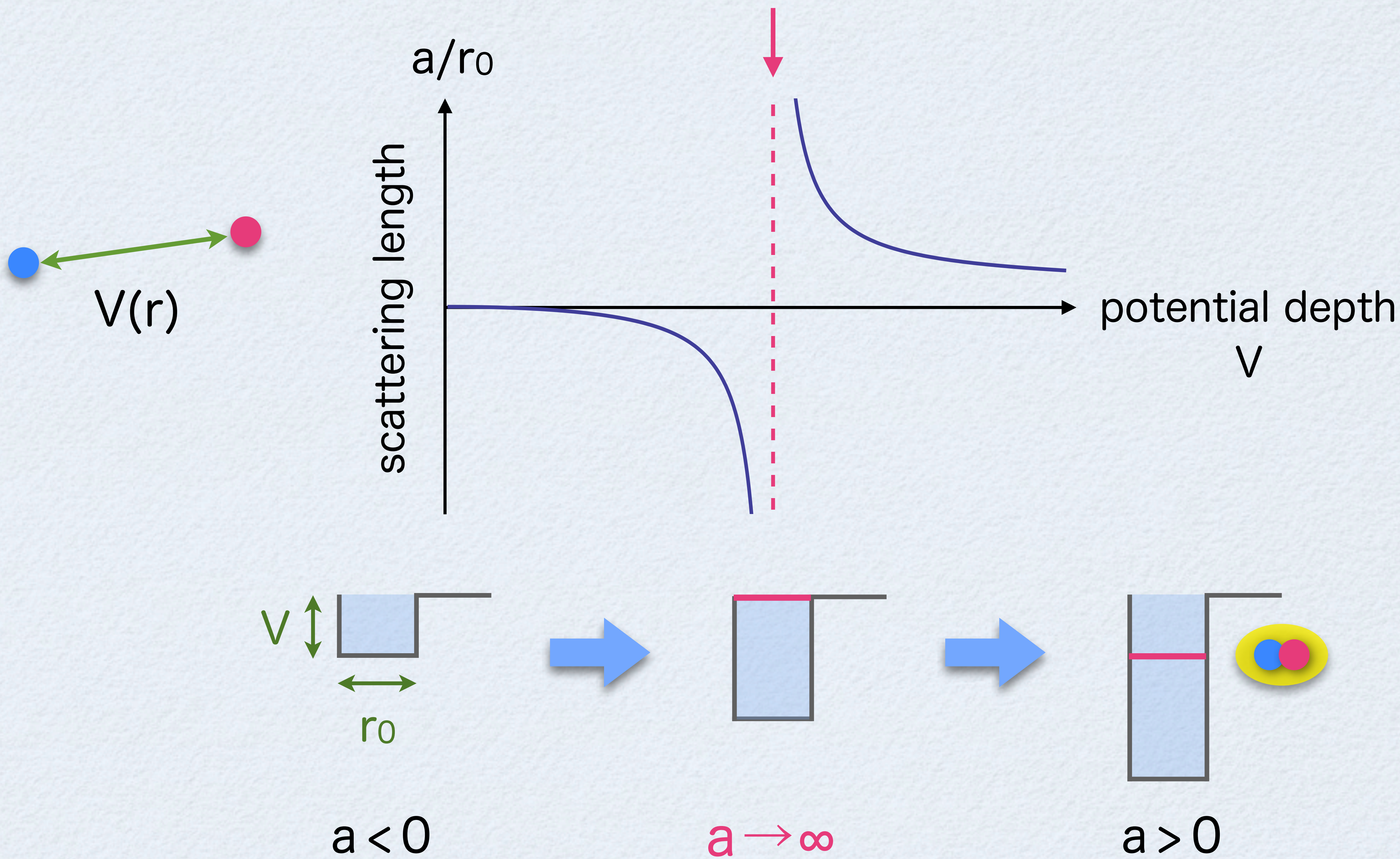
Water and magnet have the same exponent $\beta \approx 0.325$

$$\rho_{\rm liq} - \rho_{\rm gas} \sim (T_{\rm c} - T)^{\beta}$$

$$M_{\uparrow} - M_{\downarrow} \sim (T_{\rm c} - T)^{\beta}$$

When physics is universal ?

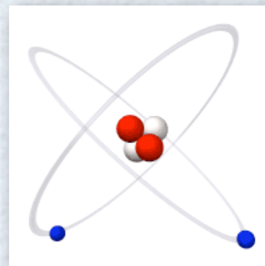
A2. Scattering resonances $\Leftrightarrow$ $a/r_0 \to \infty$



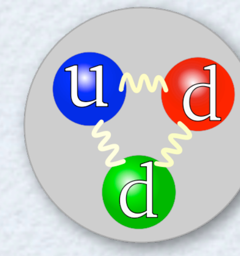
When physics is universal ?

A2. Scattering resonances ⇔ $a/r_0 \to \infty$

E.g. ^{4}He atoms



vs. proton/neutron



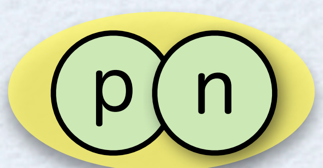
van der Waals force:

$a \approx 1\times10^{-8}$ m $\approx 20\ r_0$

$E_{\mathrm{binding}} \approx 1.3\times10^{-3}$ K

nuclear force:

$a \approx 5\times10^{-15}$ m $\approx 4\ r_0$



$E_{\mathrm{binding}} \approx 2.6\times10^{10}$ K

Atoms and nucleons have the same form of binding energy

$$E_{\mathrm{binding}} \to -\frac{\hbar^2}{m\,a^2} \qquad (a/r_0 \to \infty)$$

Physics only depends on the scattering length “a”

Efimov effect

1. Universality in physics
2. **What is the Efimov effect?**
3. Efimov effect in solid state systems

Efimov effect

Efimov (1970)

Volume 33B, number 8 PHYSICS LETTERS 21 Decembe

ENERGY LEVELS ARISING FROM RESONANT TWO-BODY FORCES IN A THREE-BODY SYSTEM

V. EFIMOV
A.F.Ioffe Physico-Technical Institute, Leningrad, USSR

Received 20 October 1970

Resonant two-body forces are shown to give rise to a series of levels in three-particle systems. The number of such levels may be very large. Possibility of the existence of such levels in systems of three α-particles (^{12}C nucleus) and three nucleons (^{3}H) is discussed.

The range of nucleon-nucleon forces r_0 is known to be considerably smaller than the scattering lengts a. This fact is a consequence of the resonant character of nucleon-nucleon forces. Apart from this, many other forces in nuclear physics are resonant. The aim of this letter is to expose an interesting effect of resonant forces in a three-body system. Namely, for $a \gg r_0$ a series of bound levels appears. In a certain case, the number of levels may become infinite.

Let us explicitly formulate this result in the simplest case. Consider three spinless neutral

ticle bound states emerge one after the other. At $g = g_0$ (infinite scattering length) their number is infinite. As g grows on beyond g_0, levels leave into continuum one after the other (see fig. 1).

The number of levels is given by the equation

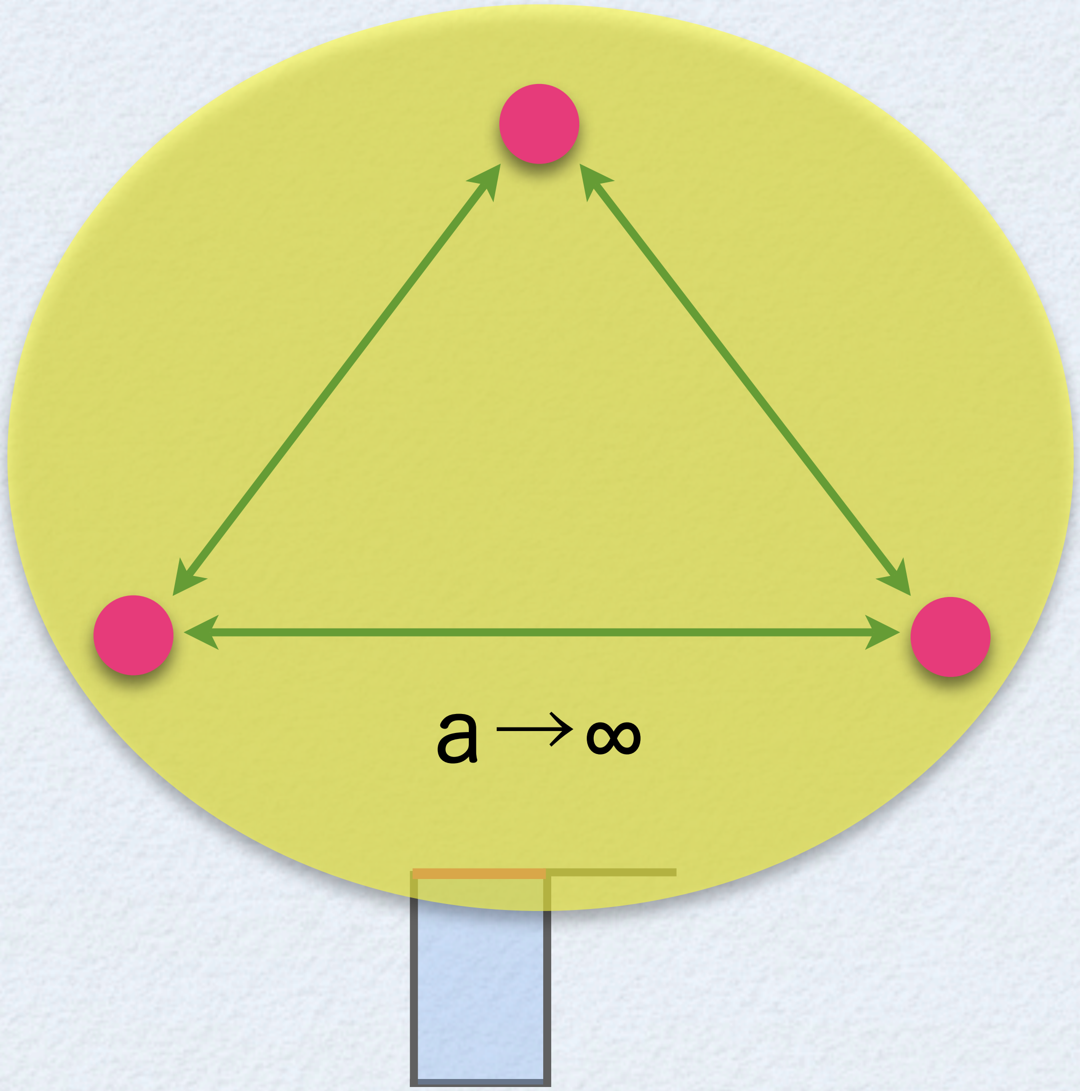
$$N \approx \frac{1}{\pi} \ln(|a|/r_0) \qquad (1)$$

All the levels are of the 0^+ kind; corresponding wave functions are symmetric; the energies $E_N \ll 1/r_0^2$ (we use $\hbar = m = 1$); the range of these bound states is much larger than r_0.

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)



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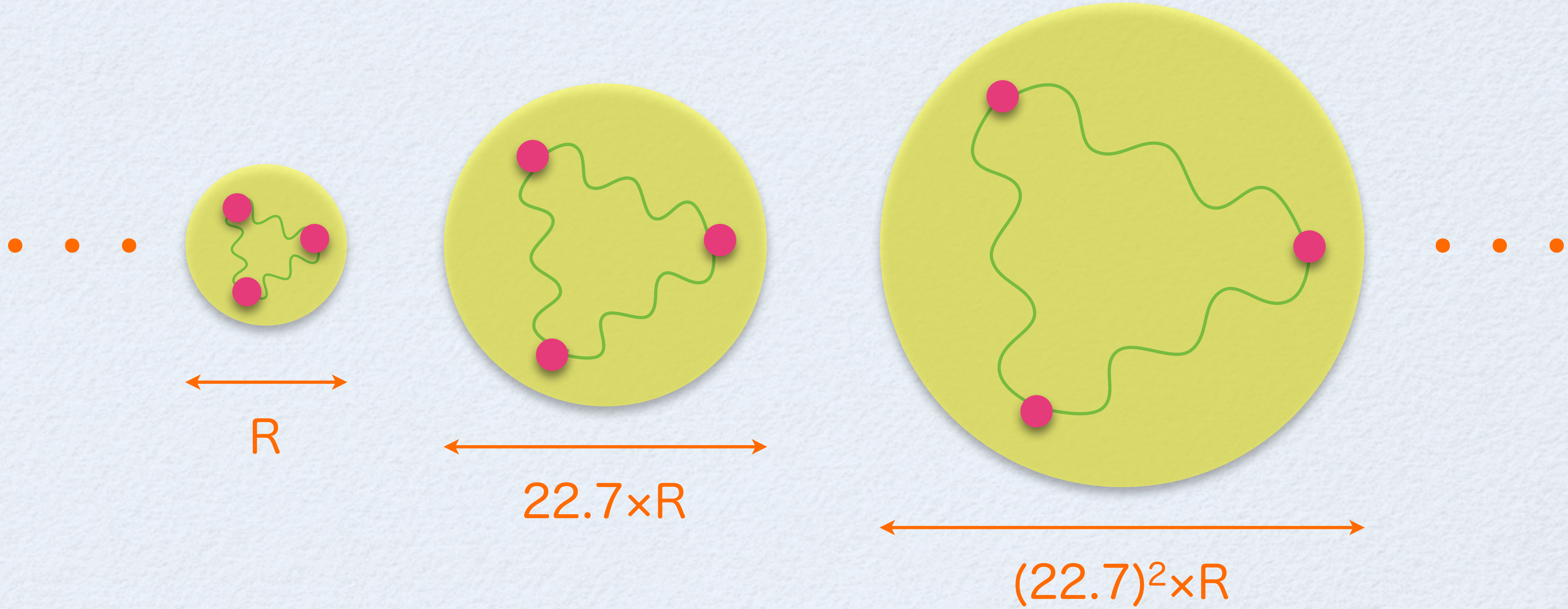
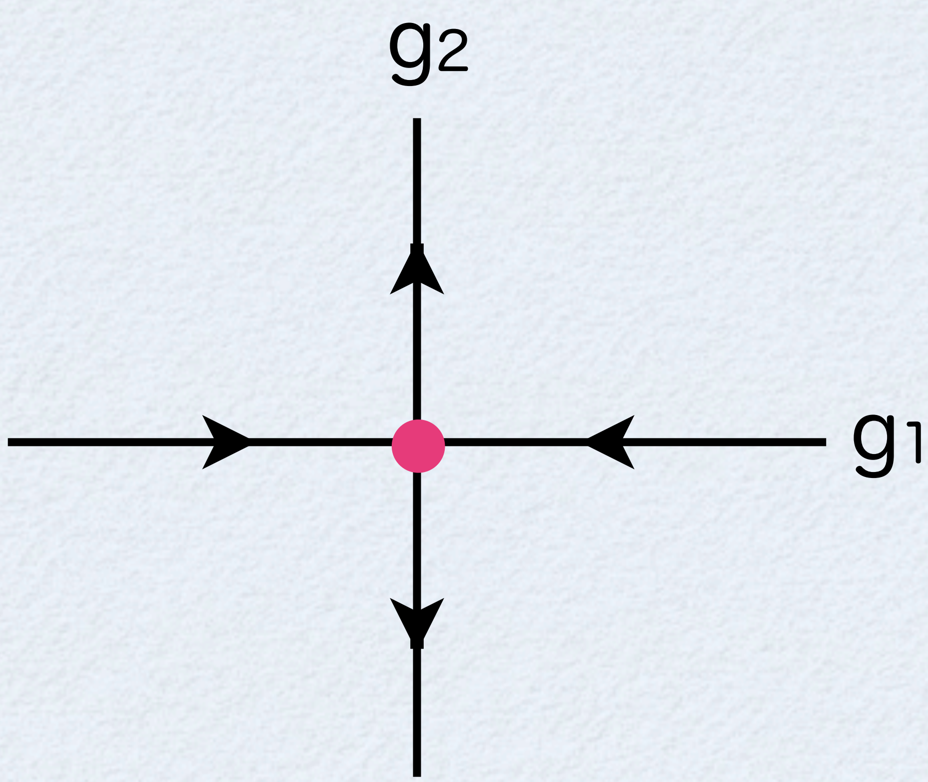


Discrete scaling symmetry

Renormalization group limit cycle

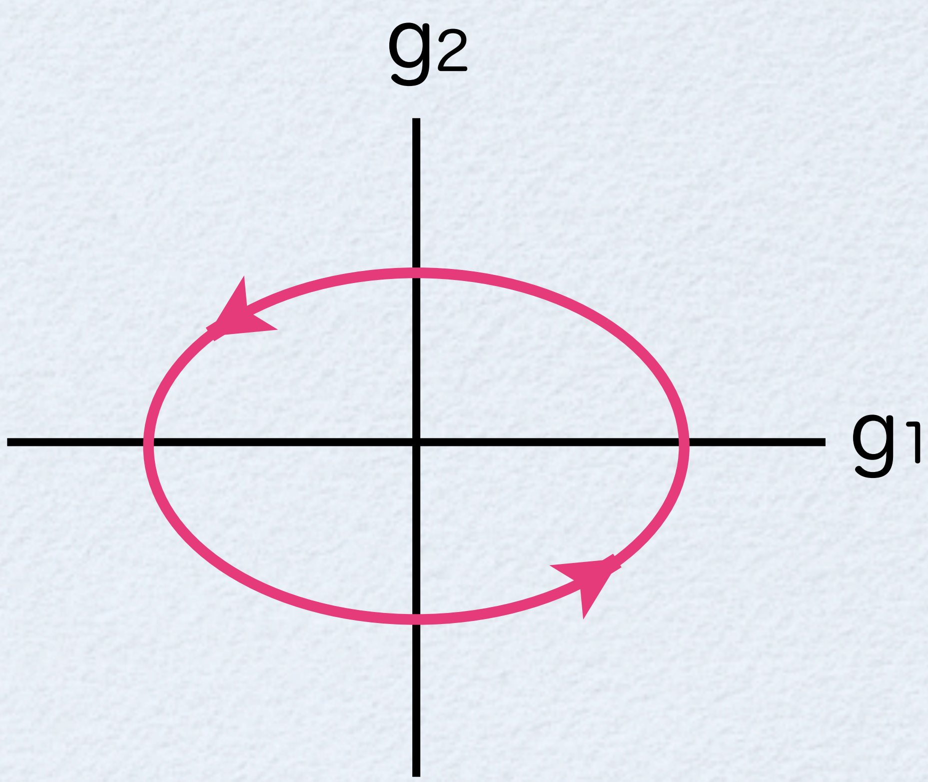
Renormalization group flow diagram in coupling space



RG fixed point

⇒ Scale invariance

E.g. critical phenomena



RG limit cycle

⇒ Discrete scale invariance

E.g. Efimov effect

Rare manifestation in physics!

Where Efimov effect appears ?

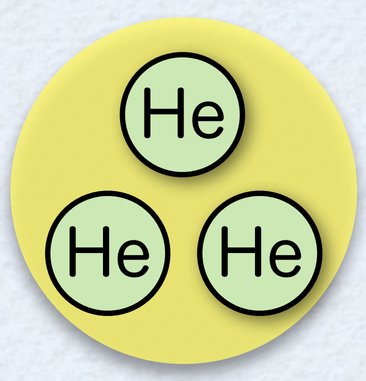
× Originally, Efimov considered

^{3}H nucleus (≈3n) and ^{12}C nucleus (≈3α)

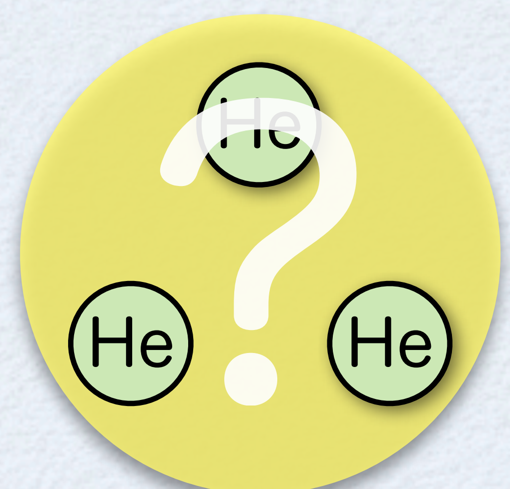
△ ^{4}He atoms ($a \approx 1\times10^{-8}$ m $\approx 20r_0$) ?

2 trimer states were predicted

1 was observed (1994)



$E_b = 125.8$ mK

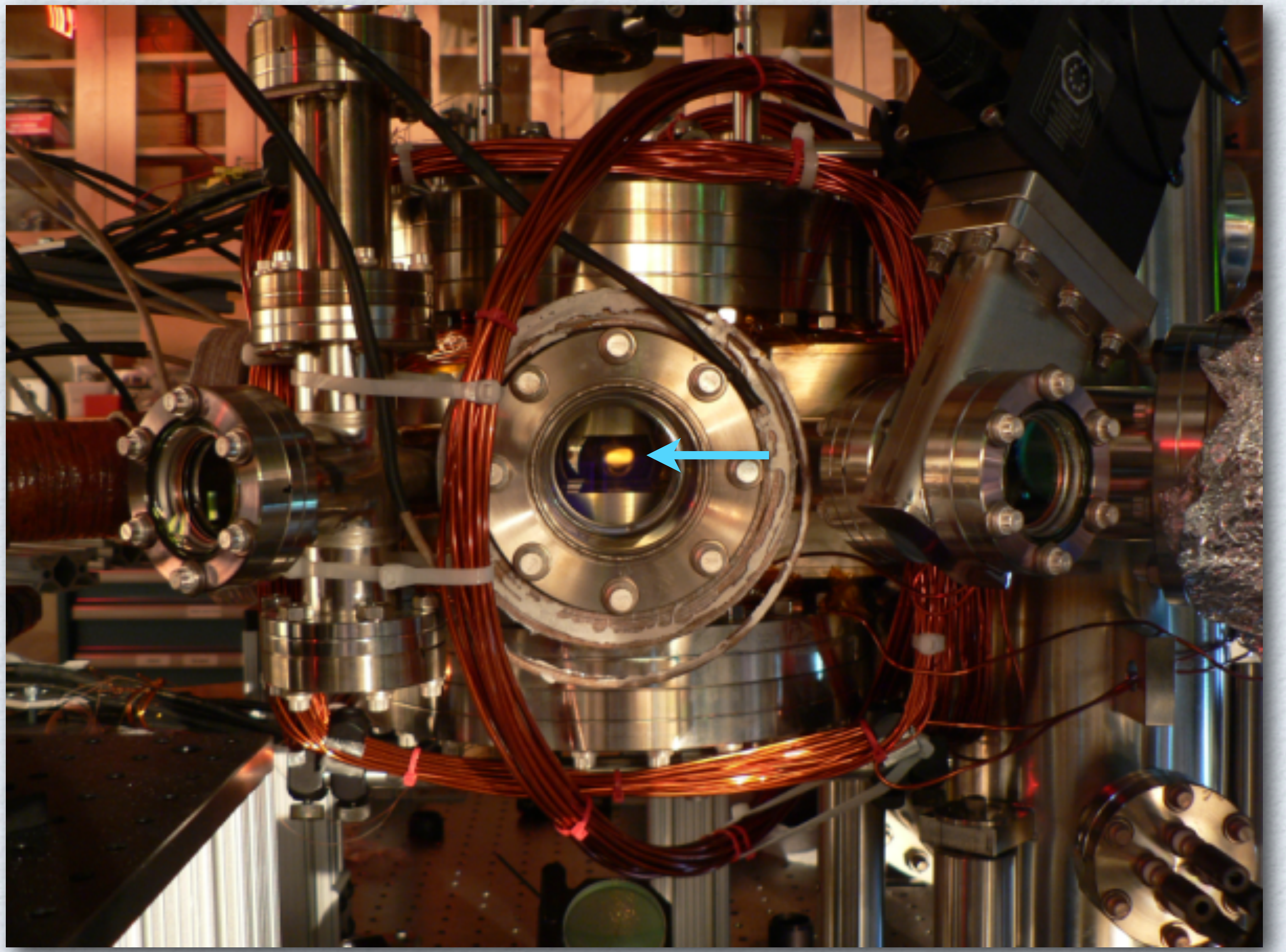


($E_b = 2.28$ mK)

→ Ultracold atoms !

Ultracold atom experiments

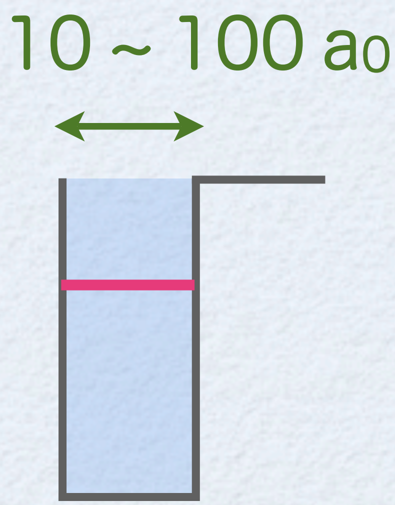
Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

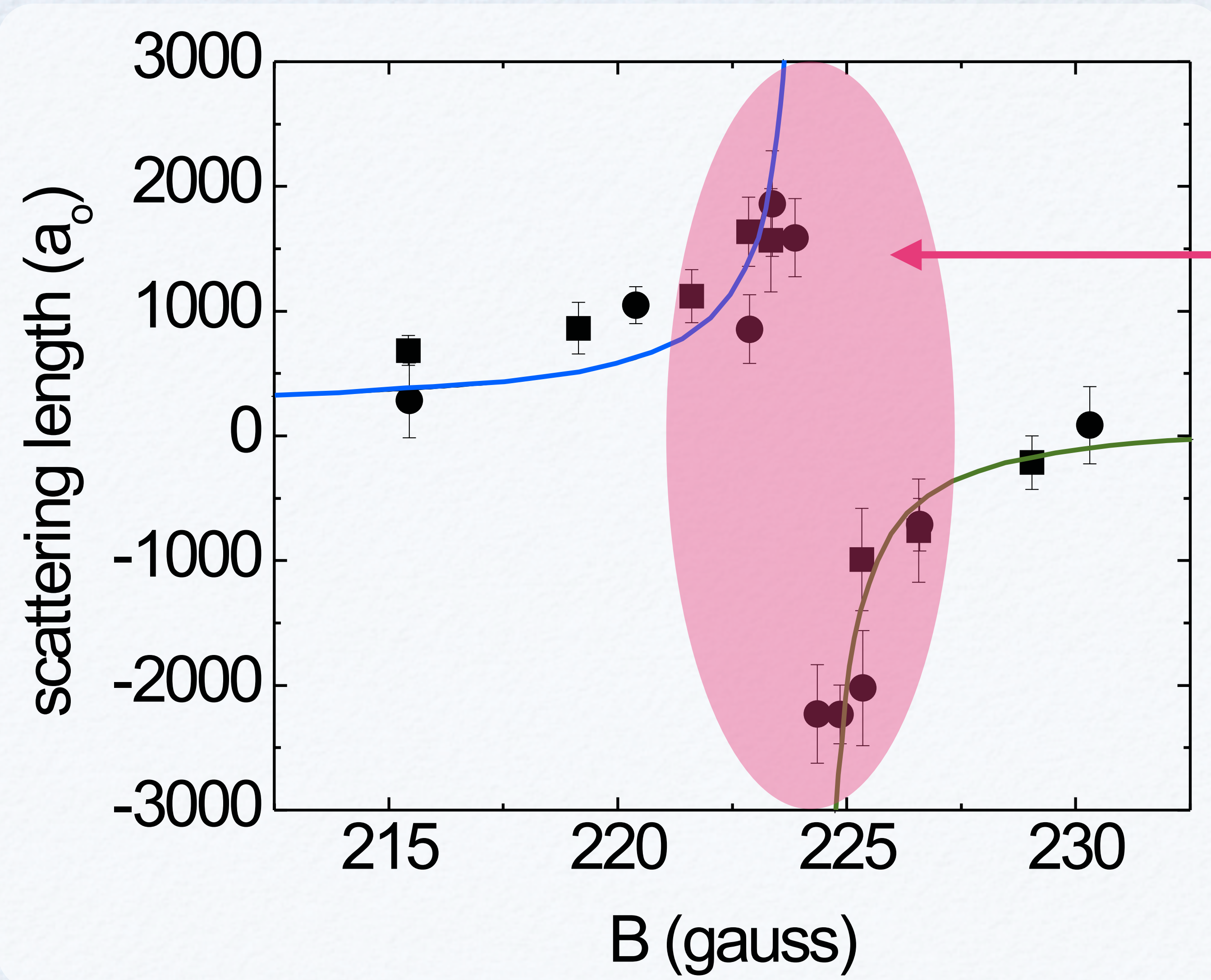


Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

✓ Interaction strength by Feshbach resonances

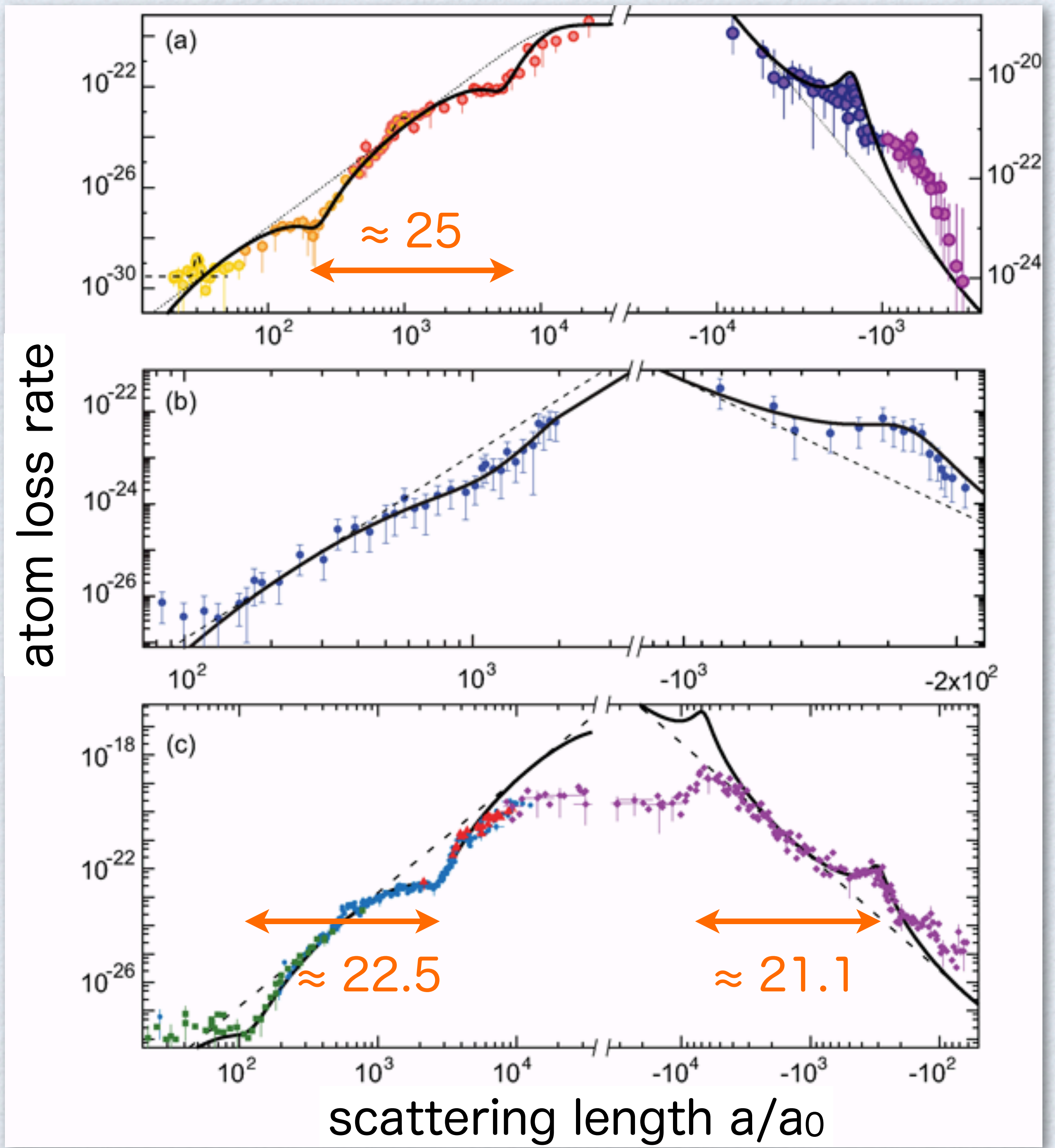




Universal regime

C.A. Regal & D.S. Jin
PRL90 (2003)

Ultracold atom experiments



Florence group for ^{39}K (2009)

Bar-Ilan University for ^{7}Li (2009)

Rice University for ^{7}Li (2009)

Discrete scaling & Universality !

Efimov effect is “universal” ?

- Efimov effect is “universal”

  = appears regardless of microscopic details

  (physics technical term)

- Efimov effect is not “universal”

  universal = present or occurring everywhere

  (Merriam-Webster Online)

→ Can we find the Efimov effect in other physical systems ?

Efimov effect in quantum magnets

1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect in solid state systems

Quantum magnet

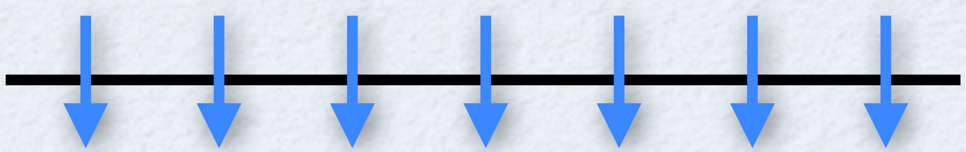
Anisotropic Heisenberg spins on a 3D lattice

$$H = -\sum_{r}\Big[\sum_{\hat{e}}(J S_r^{+} S_{r+\hat{e}}^{-} + J_z S_r^{z} S_{r+\hat{e}}^{z}) + D(S_r^{z})^{2} - B S_r^{z}\Big]$$

exchange anisotropy

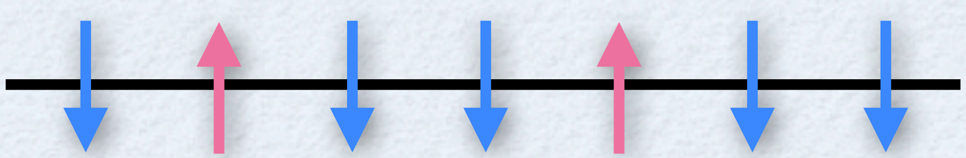
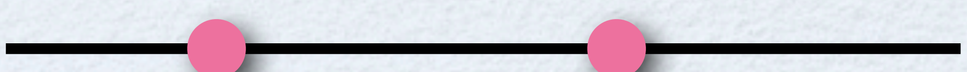
single-ion anisotropy

Spin-boson correspondence

 ⇔

fully polarized state (B→∞)

No boson = vacuum

 ⇔ 

N spin-flips

N bosons = magnons

Quantum magnet

Anisotropic Heisenberg spins on a 3D lattice

$$H = -\sum_{r}\Big[\sum_{\hat{e}}(J S^+_r S^-_{r+\hat{e}} + J_z S^z_r S^z_{r+\hat{e}}) + D(S^z_r)^2 - B S^z_r\Big]$$

xy-exchange coupling

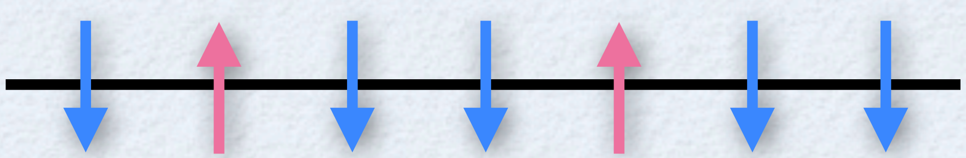
⇔ hopping

single-ion anisotropy

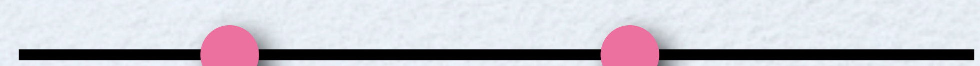
⇔ on-site attraction

z-exchange coupling

⇔ neighbor attraction



⇔



N spin-flips

N bosons = magnons

Quantum magnet

Anisotropic Heisenberg spins on a 3D lattice

$$H = -\sum_{r}\left[\sum_{\hat{e}}(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z) + D(S_r^z)^2 - B S_r^z\right]$$

xy-exchange coupling
⇔ hopping

single-ion anisotropy
⇔ on-site attraction

z-exchange coupling
⇔ neighbor attraction

By tuning the attraction
to induce a resonance between two magnons,
three magnons show the Efimov effect

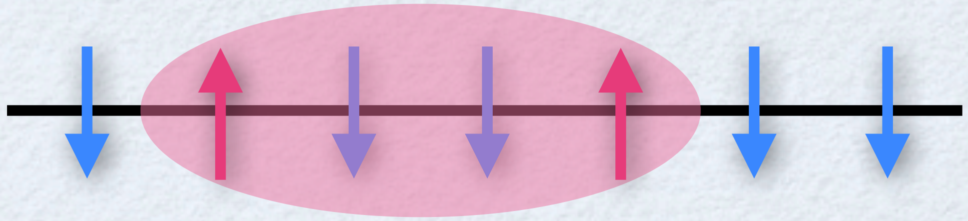
Two-magnon resonance

Scattering length between two magnons

$$\frac{a_s}{a} = \frac{\frac{3}{2\pi}\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}{2S - 1 + \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right) + 1.52\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}$$

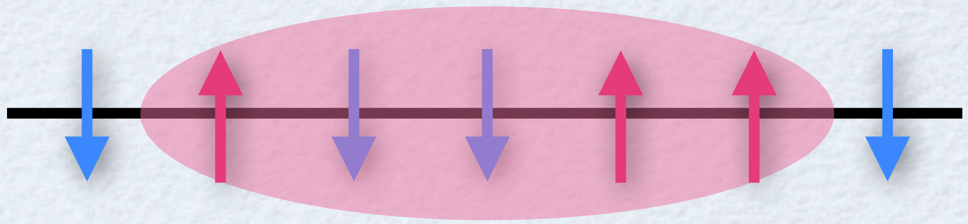
→ Two-magnon resonance ($a_s \to \infty$)

- $J_z/J = 2.94$ (spin-1/2)
- $J_z/J = 4.87$ (spin-1, D=0)
- $D/J = 4.77$ (spin-1, ferro $J_z=J>0$)
- $D/J = 5.13$ (spin-1, antiferro $J_z=J<0$)
- …



Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- **Spin-1/2**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- **Spin-1, D=0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

- **Spin-1, J_z=J>0**

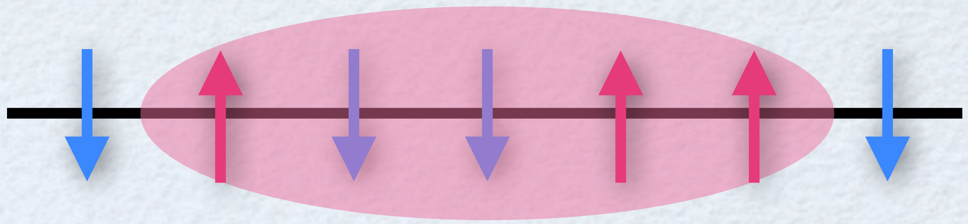
n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.50×10^{-2}	—
1	-1.16×10^{-4}	21.8

- **Spin-1, J_z=J<0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-4.36×10^{-3}	—
1	-8.88×10^{-6}	22.2

Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



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Universal scaling law by ~ 22.7 confirms they are Efimov states!

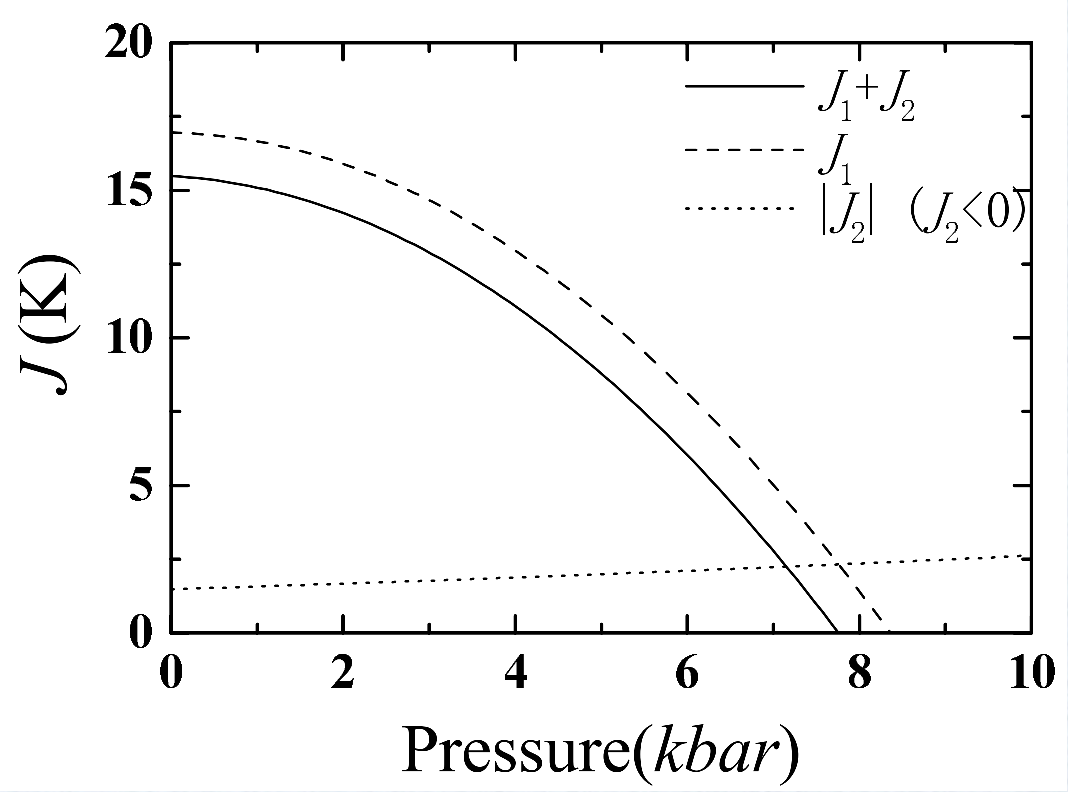
Toward experimental realization

1. Find a good compound
   whose anisotropy is close to the critical value

   E.g. Ni-based organic ferromagnet with D/J~3 (critical 4.8)

   R. Koch et al., Phys. Rev. B 67, 094407 (2003)

C.f. TDAE-C_{60}



T. Kawamoto et al, JPSJ (2001)

2. Tune the exchange coupling
   with pressure to induce
   the two-magnon resonance

3. Observe the Efimov states
   of three magnons with

- far-infrared absorption
- inelastic neutron scattering
- electron spin resonance

(many references since 1966)

Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic physics

nuclear physics

condensed matter

Efimov effect in quantum magnets induced by

- exchange anisotropy
- single-ion anisotropy
- spatial anisotropy
- fructration

(arXiv:1208.6214)

Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic physics

nuclear physics

condensed matter

Atomic BEC (1995)

↓

Efimov effect (2006)

Magnon BEC (2000)

↓

Efimov effect (201?)

Summary

Efimov effect: universality, discrete scale invariance, RG limit cycle

atomic physics

nuclear physics

condensed matter

few-body physics

many-body physics

- superfluidity
- superconductivity
- magnetism
- ...

How interplay ?