Correlations and coherence in quantum systems
Évora, Portugal, October 11 2012

Electrical Control of the Kondo Effect at the Edge of a Quantum Spin Hall System

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supported by the
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Outline

Quantum spin Hall effect... some basics

At the edge: A new kind of electron liquid

Adding a magnetic impurity...

... and a Rashba spin-orbit interaction

Electrical control of the Kondo effect!

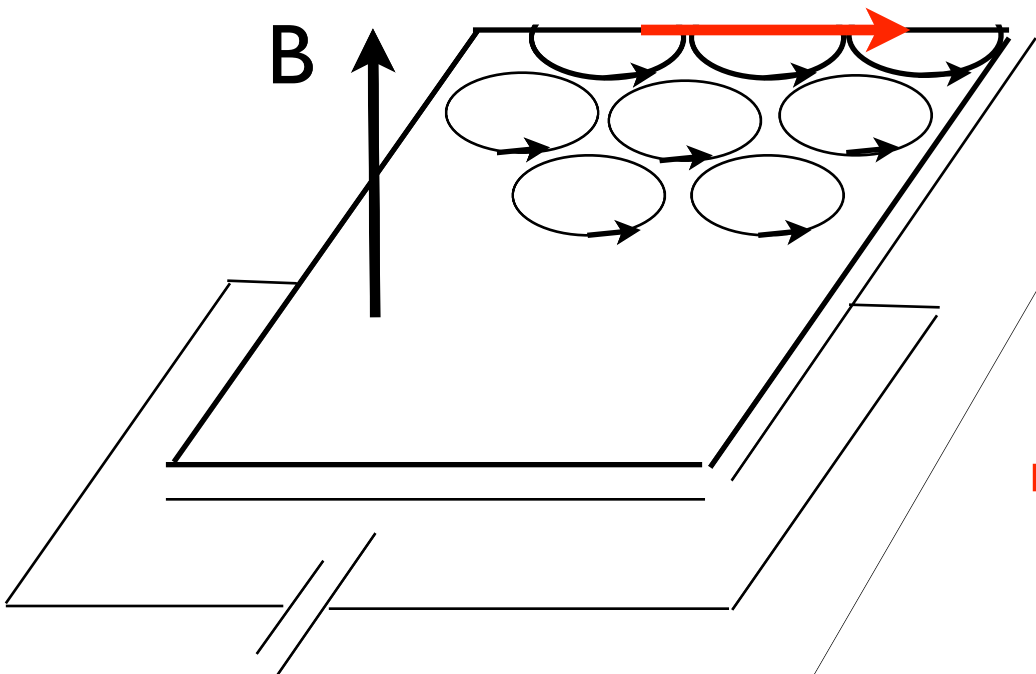
Summary and outlook

Quantum spin Hall effect... some basics

Quantum ~~spin~~ Hall effect

Integer

Quantum Hall effect



"skipping currents"

quantization

chiral edge states

B. I. Halperin, PRB **25**, 2185 (1982)

no channel for backscattering

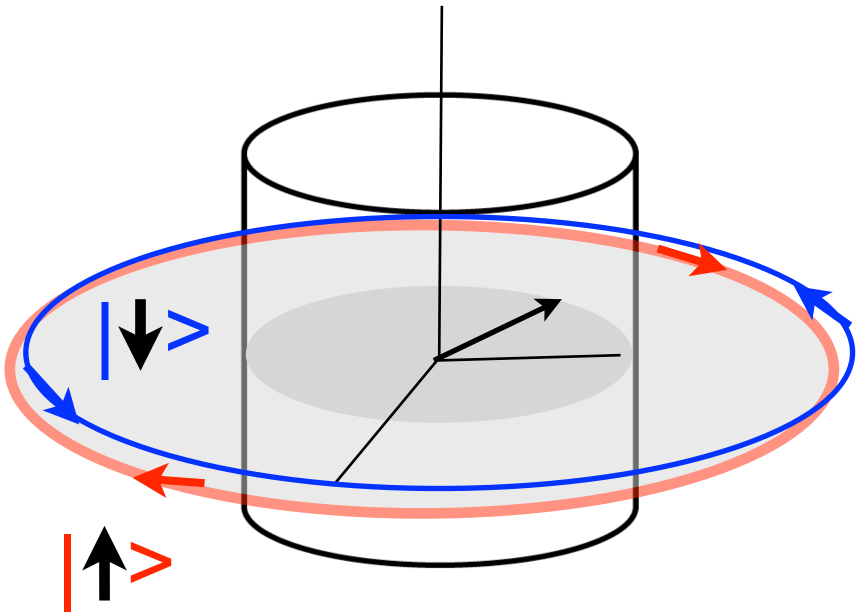
ballistic transport along the edge, accounts for the quantization of the Hall conductance

M. Büttiker, PRB **38**, 9375 (1988)

Can a system be stable against local perturbations without breaking time-reversal invariance?

Consider a Gedanken experiment...

B. A. Bernevig and S.-C. Zhang, PRL **96**, 106802 (2006)

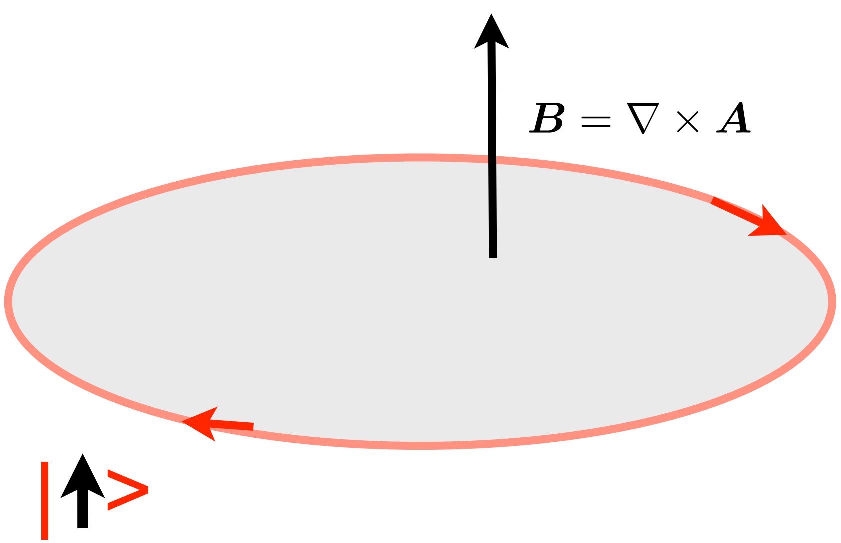


uniformly charged cylinder with electric field

$$\boldsymbol{E} = E(x, y, 0)$$

spin-orbit interaction

$$(\boldsymbol{E} \times \boldsymbol{k}) \cdot \boldsymbol{\sigma} = E\sigma^z (k_y x - k_x y)$$



cf. with the IQHE in a symmetric gauge

$$\boldsymbol{A} = \frac{B}{2}(y, -x, 0)$$

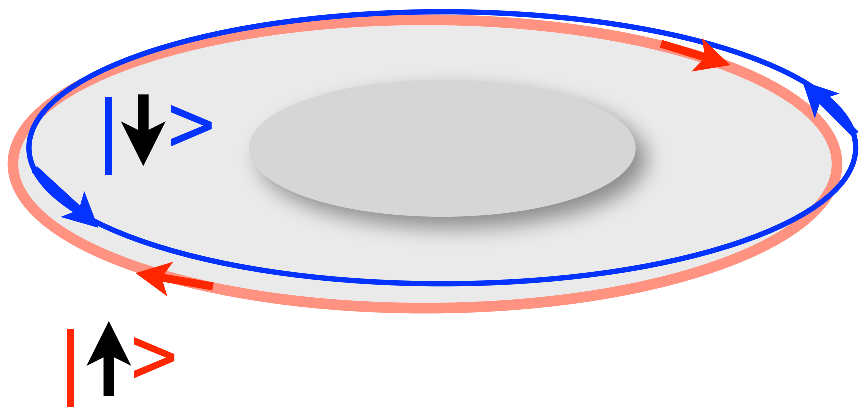
Lorentz force

$$\boldsymbol{A} \cdot \boldsymbol{k} \sim eB(k_y x - k_x y)$$

Quantum spin Hall (QSH) system

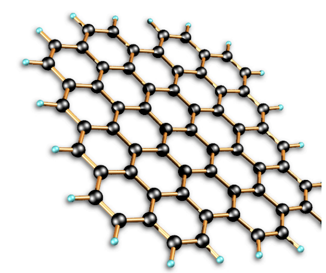
single Kramers pair

Two copies of a IQH system, bulk insulator with **helical edge states**

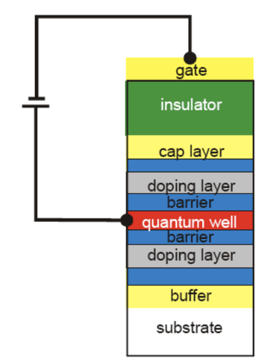


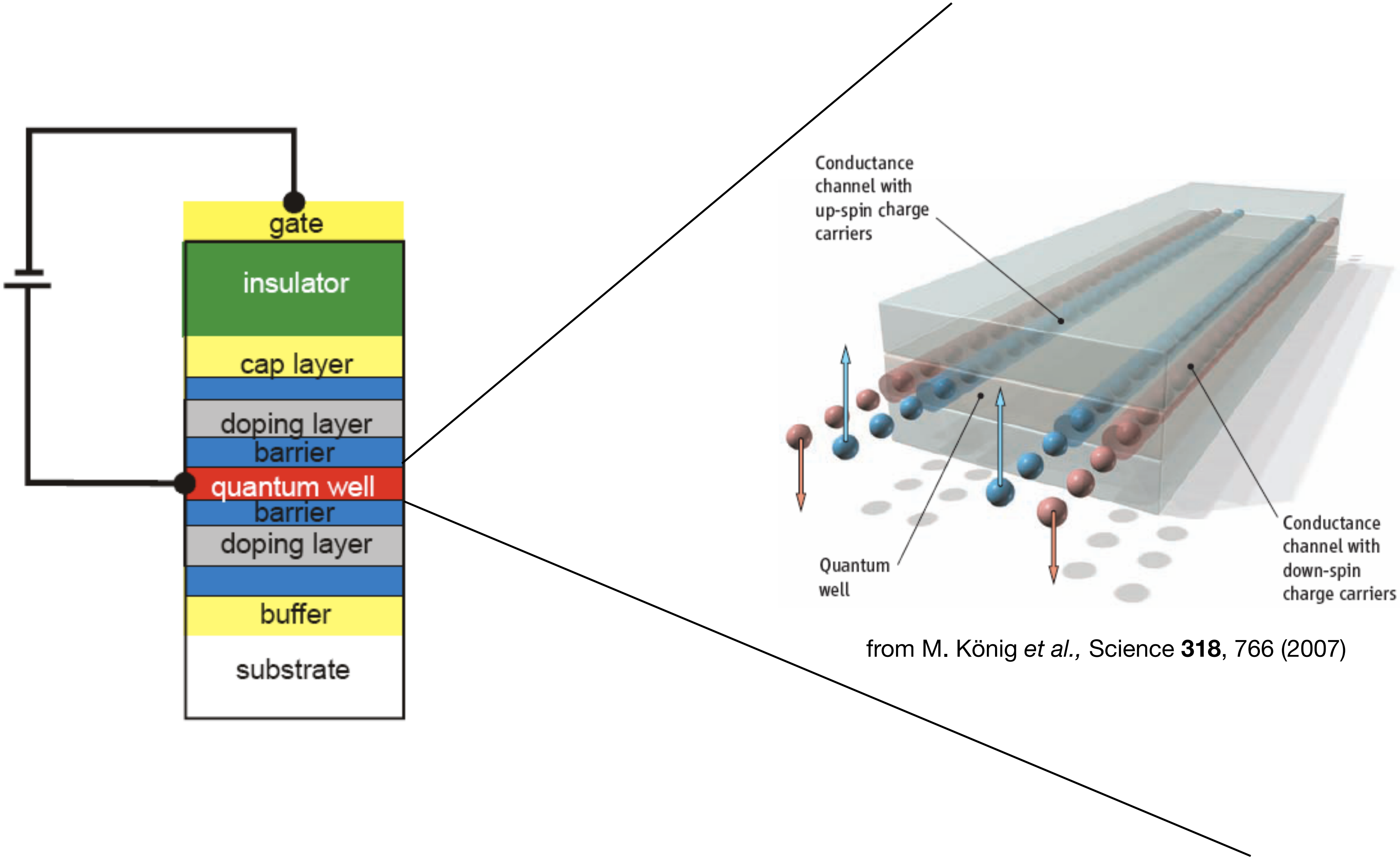
First proposed by Kane and Mele for graphene (2005)

too weak spin-orbit interaction, doesn't quite work...



Bernevig *et al.* proposal for HgTe quantum wells (2006)
Experimental observation by König *et al.* (2007)





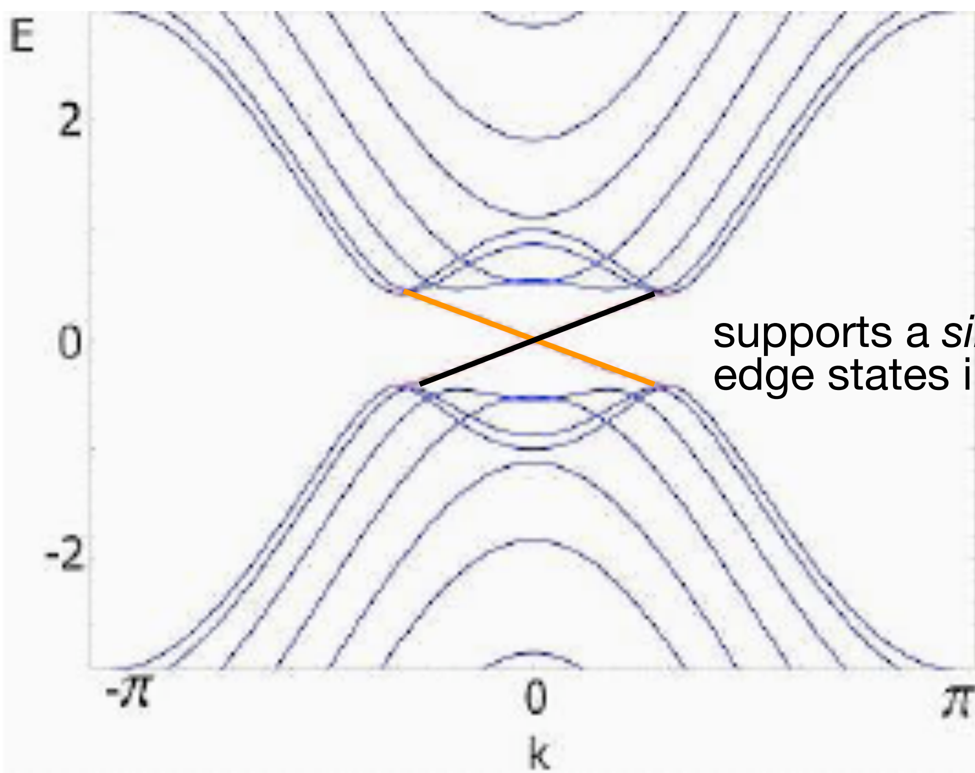
from M. König *et al.*, Science **318**, 766 (2007)

Bernevig *et al.* proposal for HgTe quantum wells (2006)
Experimental observation by König *et al.* (2007)

Are the helical edge states stable against local perturbations?

Yes! As long as the perturbations are time-reversal invariant! Look at the band structure of an HgTe quantum well…

strong spin-orbit interactions in atomic p-orbitals create an *inverted* band gap (p-band on top of s-band)



B. A. Bernevig *et al.,* PRL **95**, 066601 (2005)

supports a *single* Kramers pair of helical edge states inside the inverted gap

Kramers degeneracy at k=0 protects the stability of the edge states

ballistic transport

$$G = \frac{2e^2}{h}$$

4-terminal measurement, equilibration in contacts

At the edge: A new kind of electron liquid

of Kramers pair in a **nontrivial** (trivial) helical liquid

$N_K = 1 \mod 2$ $\quad = 0 \mod 2$

$\nu = 0, 1$ is a "Z_2" **topological invariant** and can be calculated from the band structure of the bulk ("bulk-edge correspondence")

L. Fu and C. L. Kane, PRB **76**, 045302 (2007)

2D topological insulator

(a.k.a. quantum spin Hall system)

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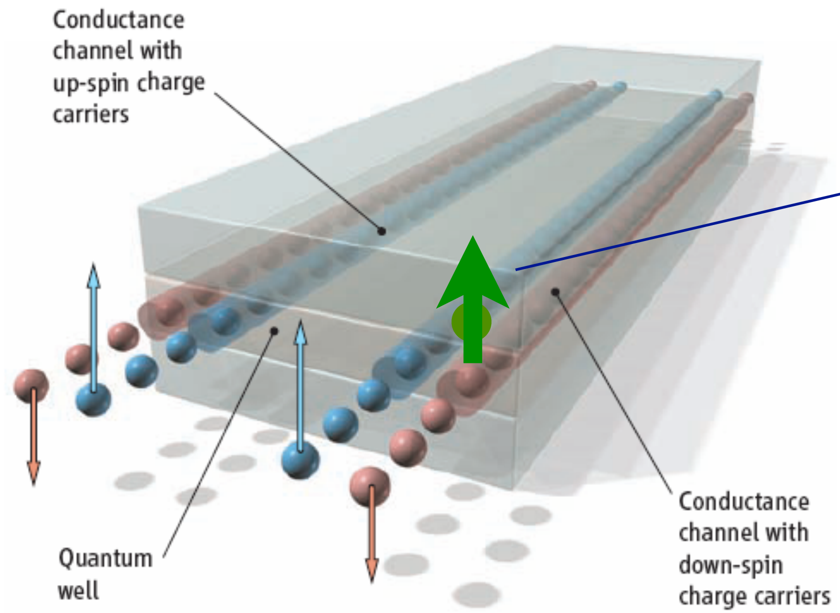
What if time-reversal symmetry is broken...?

... for example, by the presence of a magnetic impurity?



from M. König *et al.*, Science **318**, 766 (2007)

case study:

$$\mathrm{Mn}^{2+}$$

large and positive single-ion anisotropy $(S^z)^2$

$$S = 5/2 \xrightarrow[\text{low } T]{} S_{\text{eff}} = 1/2$$

anisotropic spin exchange with the edge electrons

R. Zitko *et al.*, PRB **78**, 224404 (2008)

$$H_{\mathrm{K}} = \Psi^\dagger(0)\left[J_\perp(\sigma^+ S^-_{\text{eff}} + \sigma^- S^+_{\text{eff}}) + J_z \sigma^z S^z_{\text{eff}}\right]\Psi(0)$$

$$\Psi^T = (\psi_\uparrow, \psi_\downarrow)$$

Adding a magnetic impurity...

The Kondo interaction is time-reversal invariant!
Could it still cause a *spontaneous* breaking of time reversal invariance and collapse the QSH state?

Adding a magnetic impurity...

Recall the Kondo effect

One-loop RG equations:

P. W. Anderson, J. Phys. C **3**, 2436 (1970)

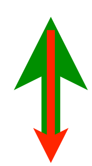
$$\begin{aligned}\frac{\partial J_\perp}{\partial D} &= -\nu J_\perp J_z + \ldots \\ \frac{\partial J_z}{\partial D} &= -\nu J_\perp^2 + \ldots\end{aligned}$$

strong-coupling physics for $T << T_K$

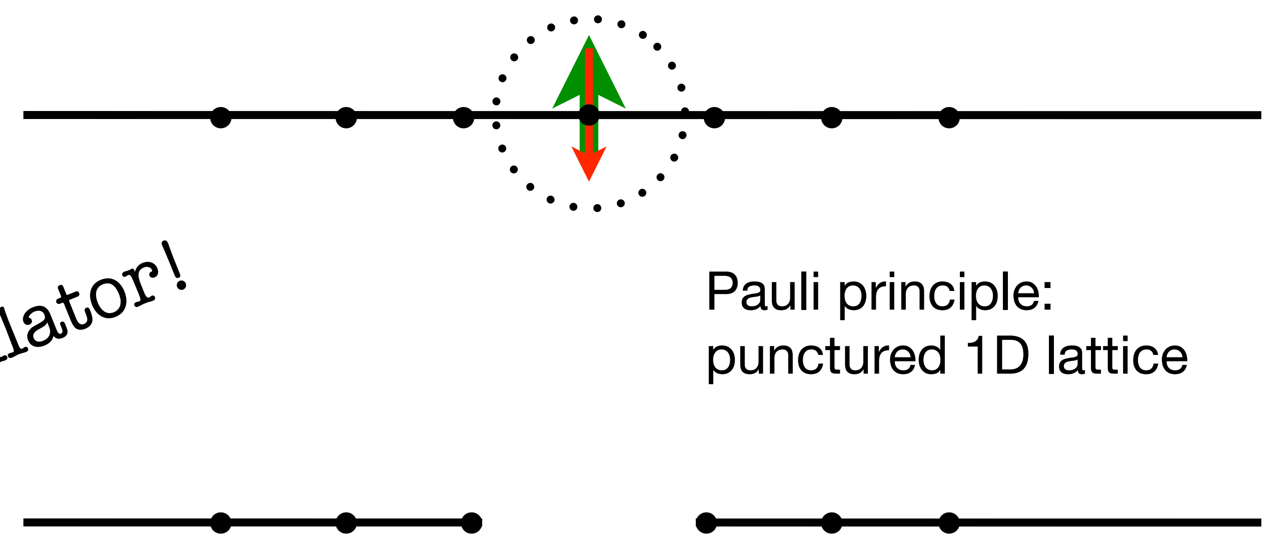
$$T_K = D_0 \exp(-\mathrm{const.}/J_0)$$

$$J_0 \equiv \max(J_\perp, J_z)_{D=D_0}$$

formation of impurity-electron singlet **("Kondo screening")**



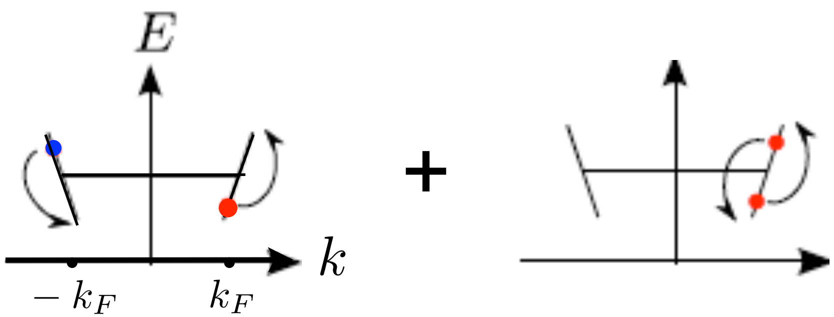
Adding a magnetic impurity...



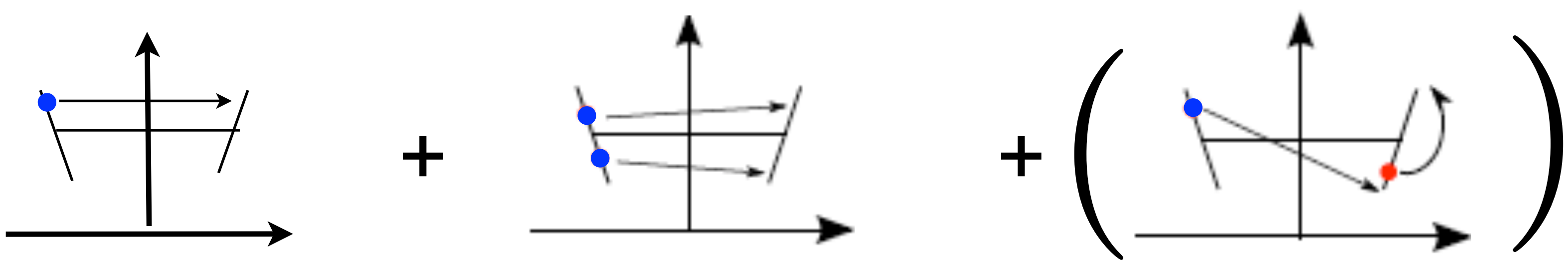
Adding a magnetic impurity...

Does this really happen for the helical liquid?
To find out, first add e-e interactions.... important in 1D!

bulk



local (at impurity site)



from Kondo

≠ 0 if U(1) spin symmetry is broken

T. L. Schmidt *et al*., PRL **108**, 156402 (2012)

Adding a magnetic impurity...

Adding the kinetic energy and bosonizing...

"Luttinger liquid parameter"

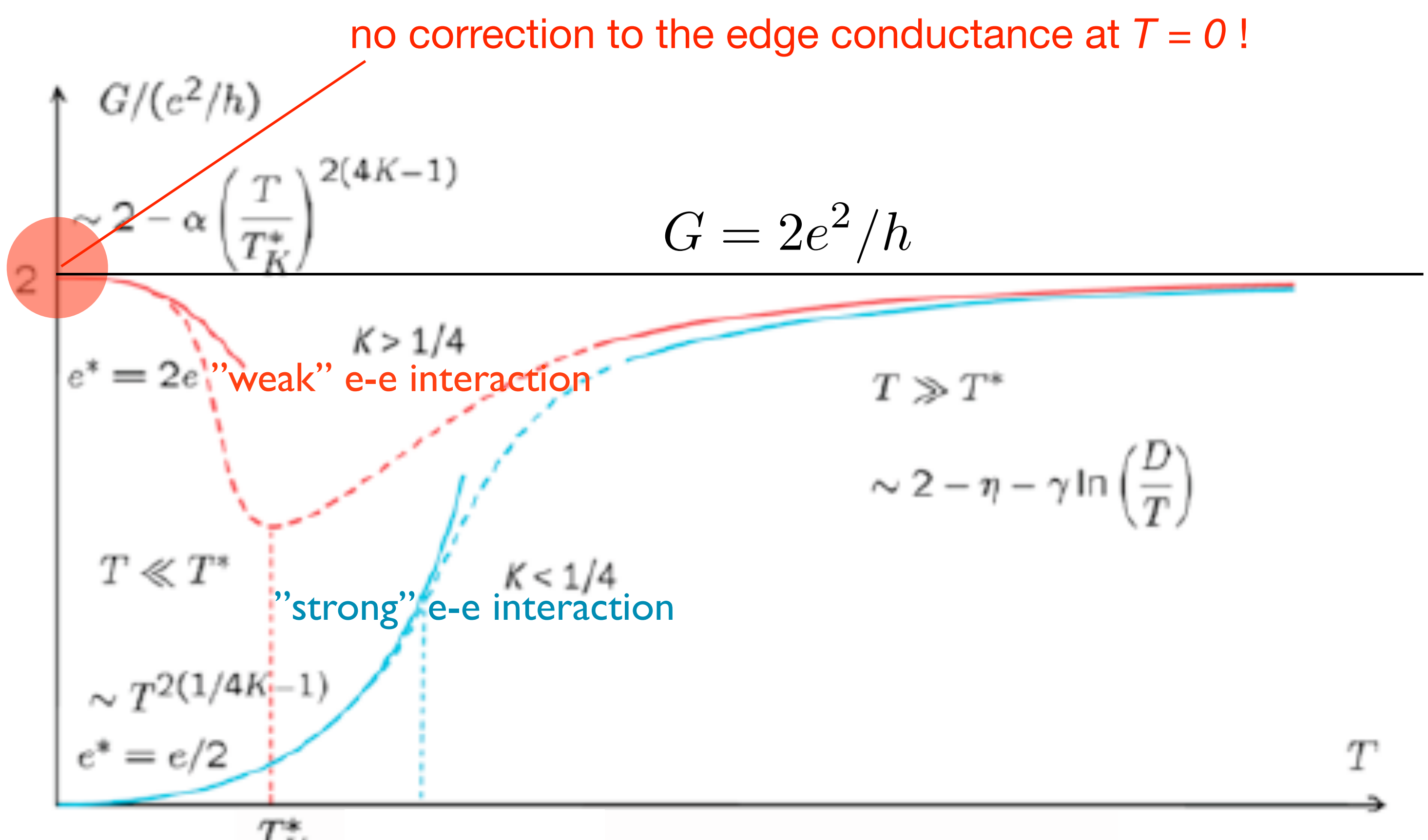
functions of Kondo couplings

$$
\begin{aligned}
H &= (v/2)\int \mathrm{d}x\big((\partial_x\varphi)^2 + (\partial_x\vartheta)^2\big) \\
&+ \frac{A}{\kappa}\cos(\sqrt{4\pi K}\varphi) + \frac{B}{\kappa}\sin(\sqrt{4\pi K}\varphi) + \frac{C}{\sqrt{K}}\partial_x\vartheta \\
&+ \frac{g_U}{2(\pi\kappa)^2}\cos(\sqrt{16\pi K}\varphi)
\end{aligned}
$$

...perturbative RG and linear response

J. Maciejko *et al.*, PRL **102**, 256803 (2009)

Adding a magnetic impurity...



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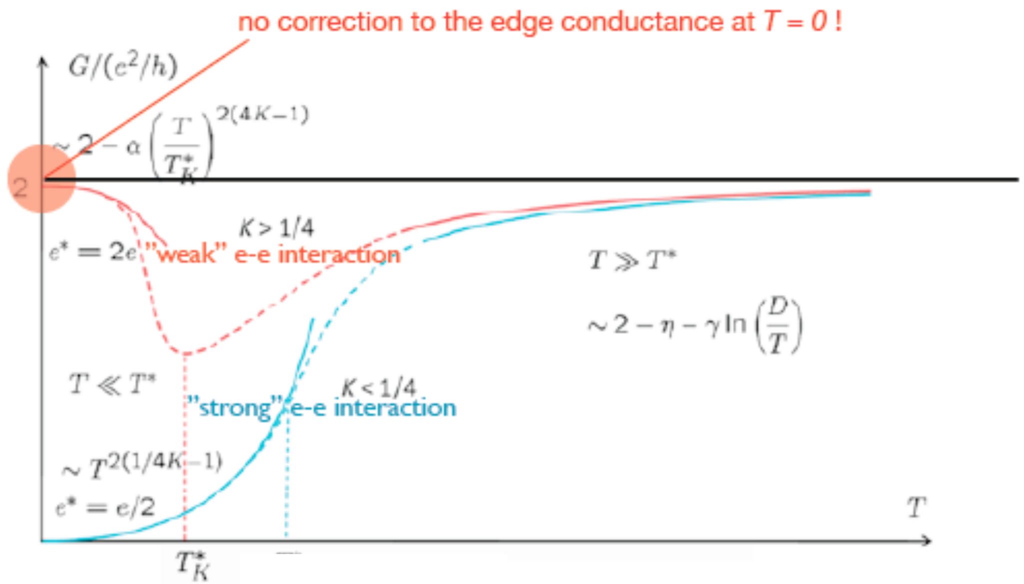
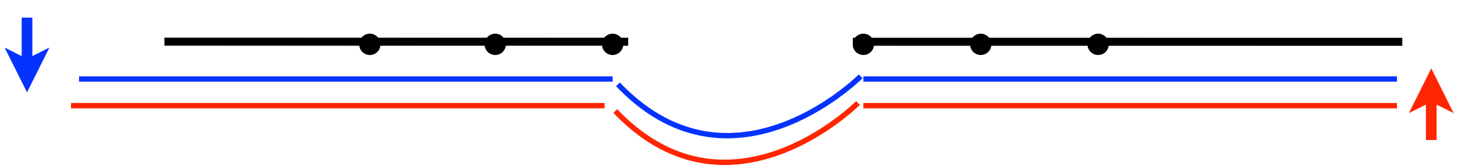
Adding a magnetic impurity...



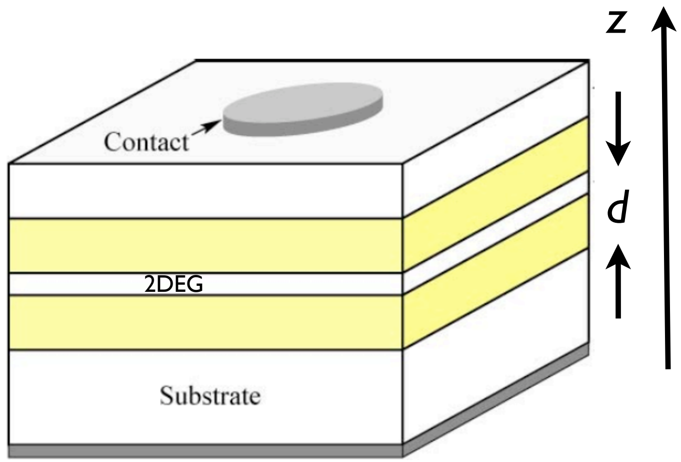


Due to its topological nature, the QSH state follows the new shape of the edge.
Weak coupling QSH states are robust against local breaking of time-reversal symmetry!

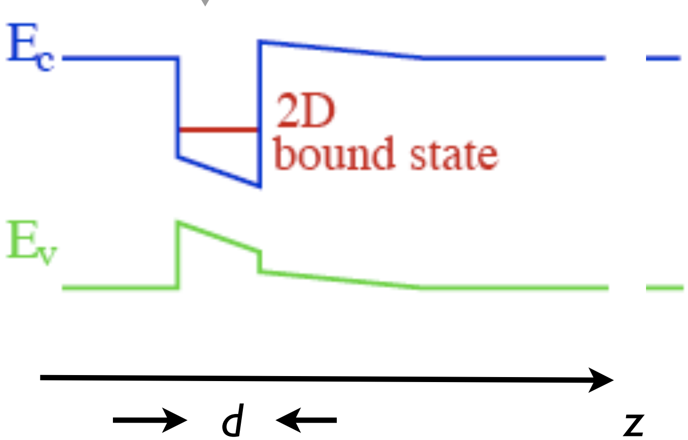
J. Maciejko *et al.*, PRL **102**, 256803 (2009)

But… one important thing is missing from the analysis!

Rashba spin-orbit interaction!



semiconductor heterostructure

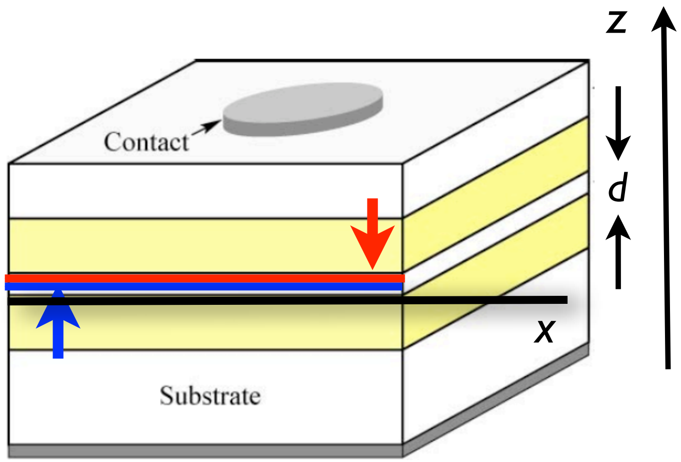


Spatial asymmetry of band edges mimics an ***E***-field in the *z*-direction

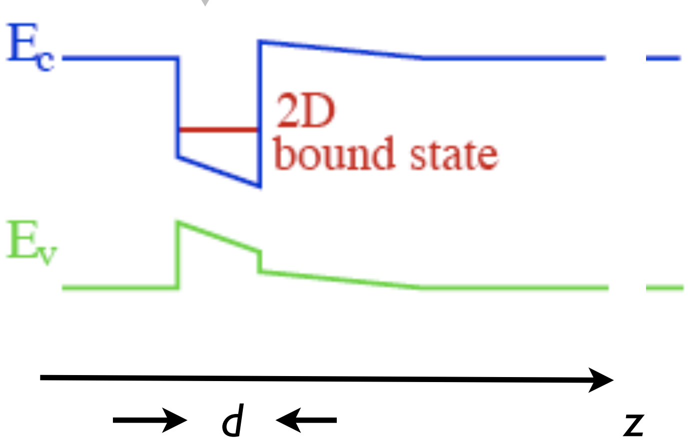
$$H_R = \alpha(k_x\sigma^y - k_y\sigma^x)$$

Yu. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984)

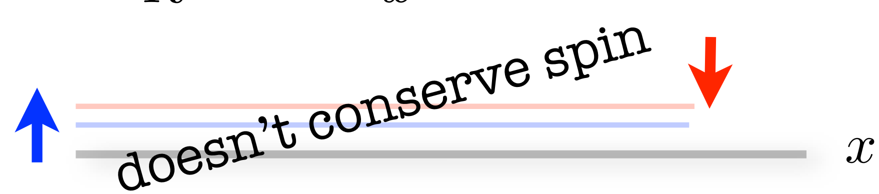
Rashba spin-orbit interaction!



semiconductor heterostructure



$$H_R = \alpha\, k_x \sigma^y$$



Adding the Rashba interaction...

... breaks the locking of spin to momentum. However, there is still a single Kramers pair on the QSH edge, and this is all that matters!

C. L. Kane and E. J. Mele, PRL **95**, 226801 (2005)

kinetic term — Rashba

$$H = v_F \int \mathrm{d}x \; \Psi^\dagger(x) \left[-\mathrm{i}\sigma^z \partial_x\right] \Psi(x) + \alpha \int \mathrm{d}x \; \Psi^\dagger(x) \left[-\mathrm{i}\sigma^y \partial_x\right] \Psi(x)$$

$$\Psi^T = (\psi_\uparrow, \psi_\downarrow)$$

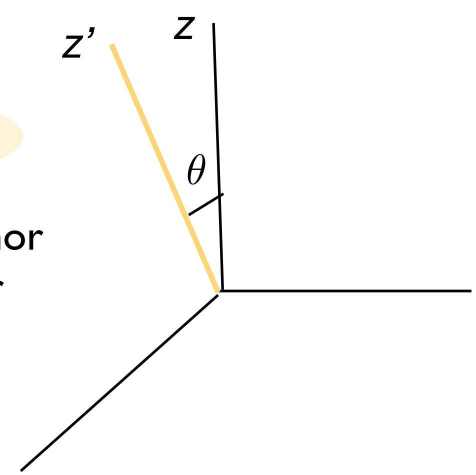
$$\Psi' = \mathrm{e}^{-\mathrm{i}\sigma^x \theta/2} \Psi$$

$$\cos\theta = v_F / v_\alpha \qquad v_\alpha = \sqrt{v_F^2 + \alpha^2}$$

$$H' = v_\alpha \int \mathrm{d}x \; \Psi'^\dagger(x) \left[-\mathrm{i}\sigma^{z'} \partial_x\right] \Psi'(x)$$

$$\Psi'^T = (\psi_{\uparrow'}, \psi_{\downarrow'})$$

The "Rashba-rotated" spinor still defines a Kramers pair



Adding the Rashba interaction...

e-e interaction is invariant under $\Psi \to \Psi'$

Kondo interaction $H_{\mathrm{K}} = \Psi^{\dagger}(0)\left[J_{\perp}(\sigma^{+}S^{-}_{\mathrm{eff}} + \sigma^{-}S^{+}_{\mathrm{eff}}) + J_{z}\sigma^{z}S^{z}_{\mathrm{eff}}\right]\Psi(0)$

$$\Psi' = \mathrm{e}^{-\mathrm{i}\sigma^{x}\theta/2}\Psi$$
$$\boldsymbol{S}' = \mathrm{e}^{-\mathrm{i}S^{x}\theta/2}\boldsymbol{S}\mathrm{e}^{\mathrm{i}S^{x}\theta/2}$$

$$H'_{K} = \Psi'^{\dagger}(0)[J_{x}\sigma^{x}S^{x} + J'_{y}\sigma^{y'}S^{y'} + J'_{z}\sigma^{z'}S^{z'} + J_{\mathrm{NC}}(\sigma^{y'}S^{z'} + \sigma^{z'}S^{y'})]\Psi'(0)$$

XYZ Kondo

Non-Collinear term

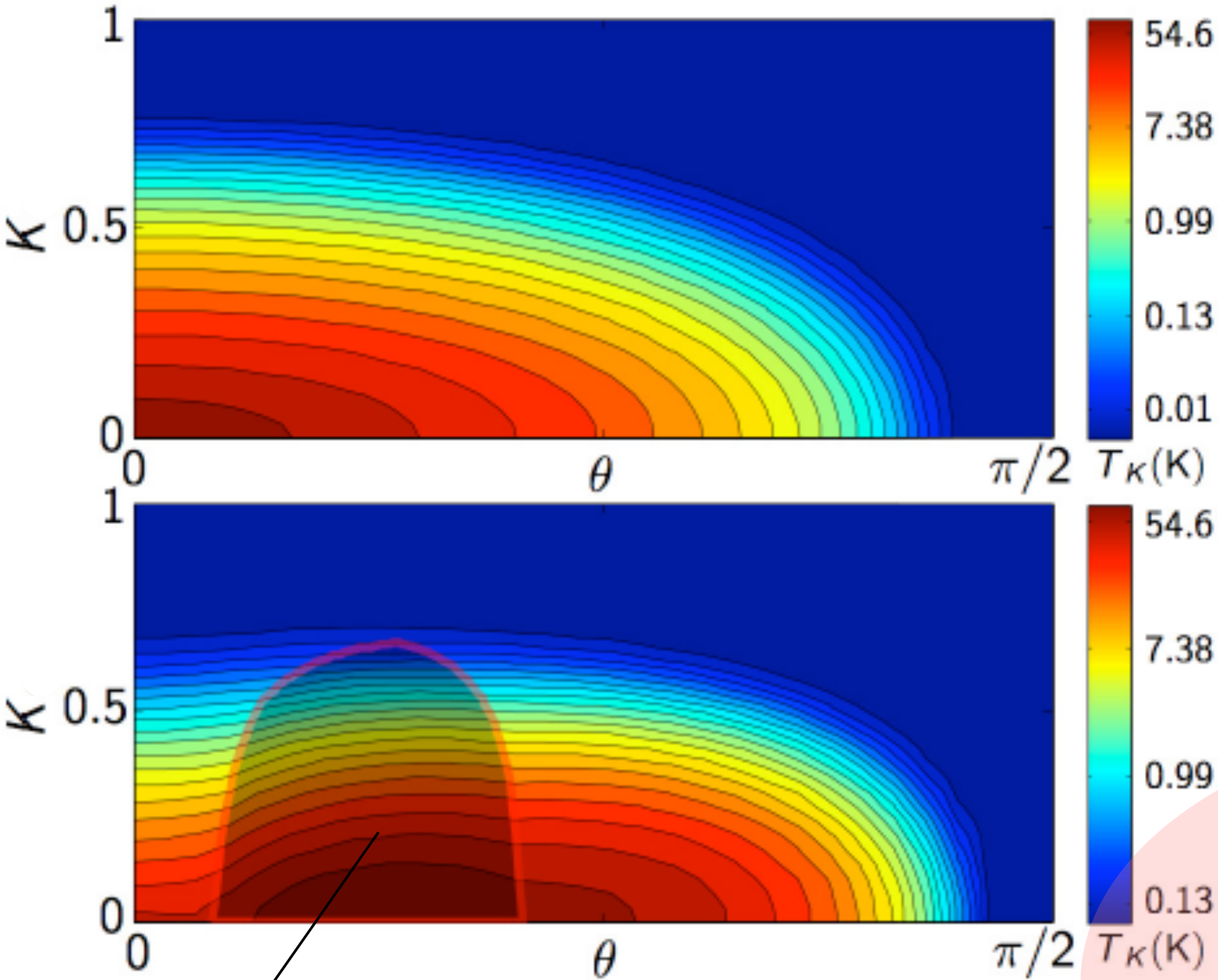
depend on the Rashba coupling α
controllable by a gate voltage

...bosonization and perturbative RG

E. Eriksson *et al.*, PRB **86**, 161103(R) (2012)

Electrical control of the Kondo temperature

via the "Rashba angle" θ ~ gate voltage



easy-plane Kondo

$J_x = J_y = 20\ \text{meV},\ J_z = 10\ \text{meV}$

easy-axis Kondo

$J_x = J_y = 5\ \text{meV},\ J_z = 50\ \text{meV}$

Region where J_{NC} dominates the RG flow

→ obstruction of Kondo screening!

challenges the Meir-Wingreen conjecture that the Kondo effect is blind to time-reversal invariant perturbations

Y. Meir and N.S. Wingreen, PRB 50, 4947 (1994)

Low-temperature transport, $T \ll T_K$ (away from the "dome")

"weak" e-e interaction $K > 1/4$

$T = 0$

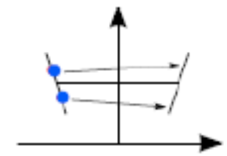
$$G = \frac{2e^2}{h}$$

Low-temperature transport, $T \ll T_K$ (away from the "dome")

"weak" e-e interaction

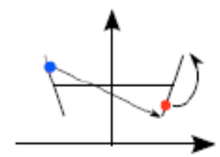
$$G = \frac{2e^2}{h} - \delta G$$

$1/4 < K < 2/3$: $\sim (T/T_K)^{8K-2}$



$K > 2/3$: $\sim (T/T_K)^{2K+2}$

signature of Rashba!



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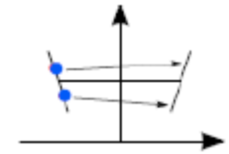

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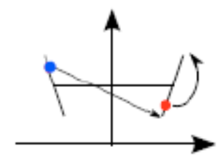
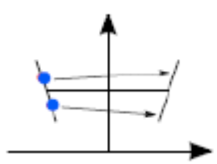


"strong" e-e interaction $K < 1/4$

$T = 0$

$G = 0$

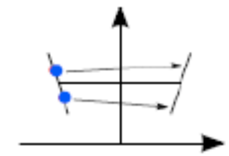


Low-temperature transport, $T \ll T_K$ (away from the "dome")

"weak" e-e interaction

$$G = \frac{2e^2}{h} - \delta G$$

$1/4 < K < 2/3$: $\sim (T/T_K)^{8K-2}$



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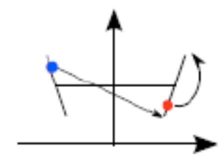


"strong" e-e interaction $K < 1/4$

$$\mathrm{G} \sim (T/T_K)^{2(1/4K-1)}$$ from instanton processes

blind to Rashba

J. Maciejko *et al.*, PRL **102**, 256803 (2009)

"High-temperature" transport, $T \gg T_K$

$$I = G_0 V - \delta I$$

$$J_{\perp,z} \ll \omega \ll T$$

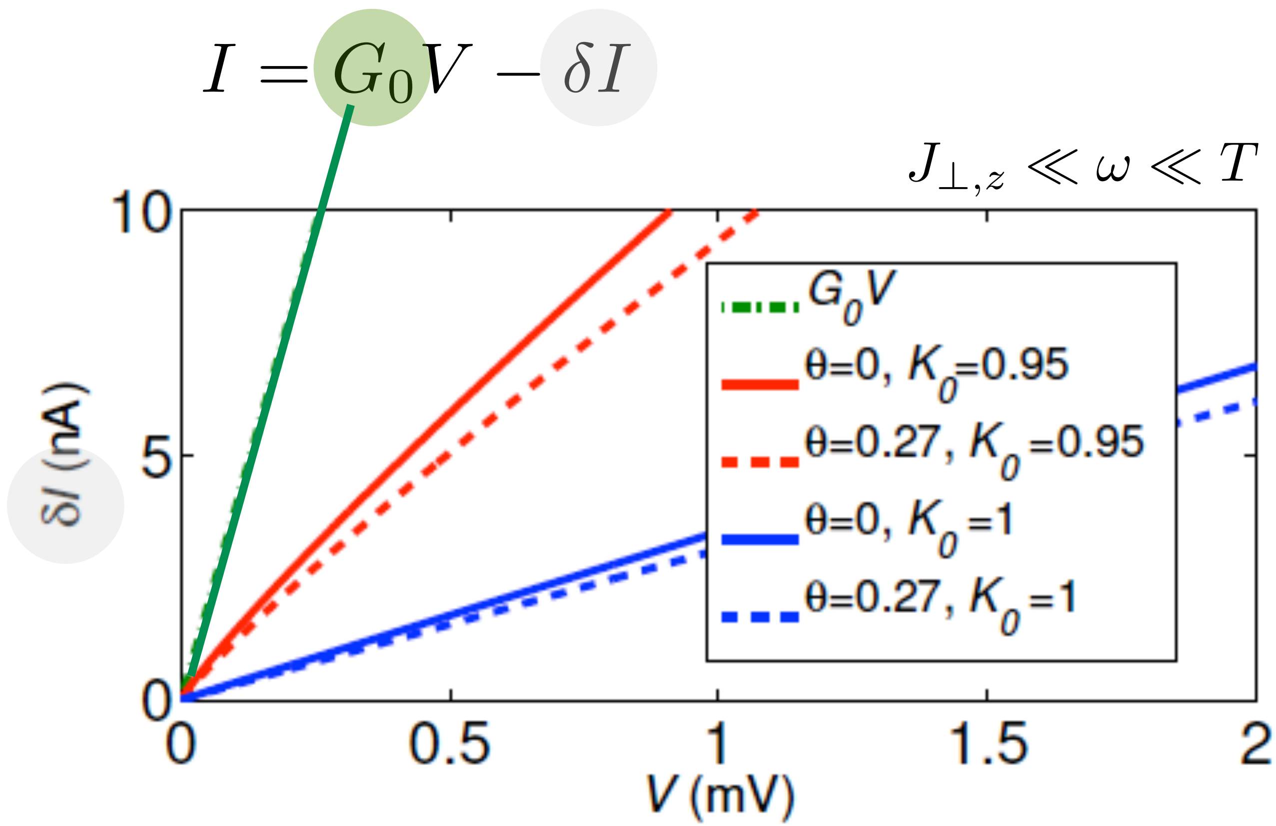
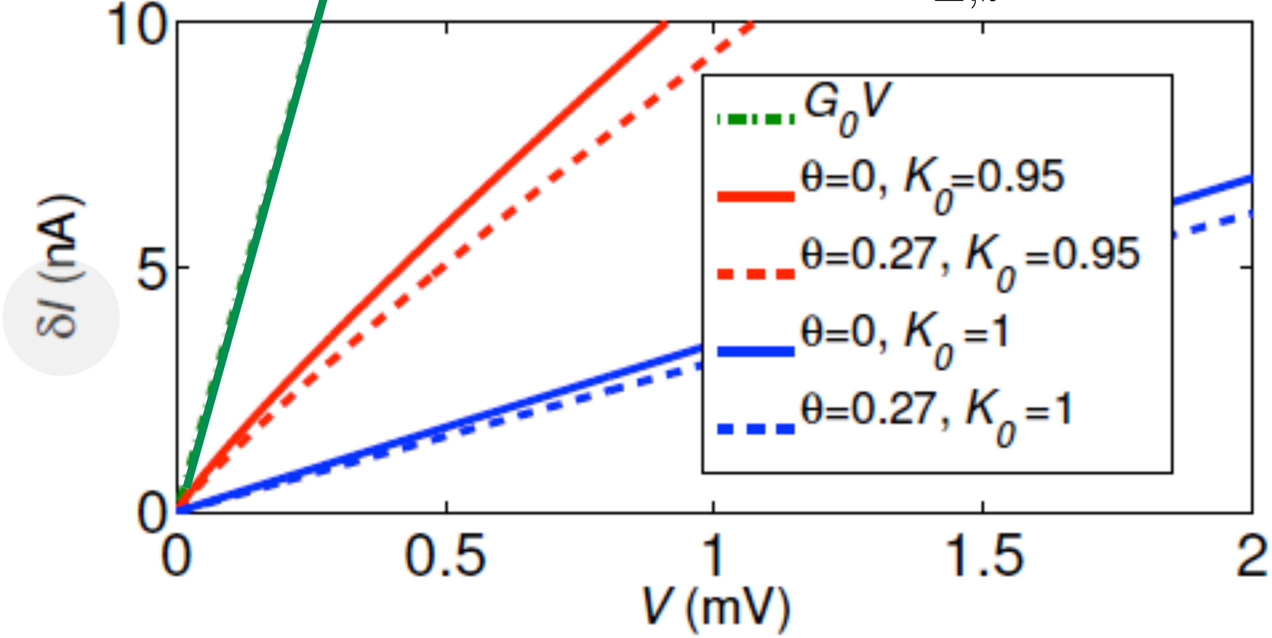


FIG. 2: The RG-improved current correction (12) at $T = 30$ mK as a function of applied voltage, for different values of K_0 and θ. The dashed lines represent $\theta \approx 0.27$, corresponding to $\hbar\alpha = 10^{-10}$ eVm. Other parameters are defined in the text. The QSH edge current $G_0 V$ is plotted as a reference.

"High-temperature" transport, $T \gg T_K$

$$I = G_0 V - \delta I$$

$$J_{\perp,z} \ll \omega \ll T$$



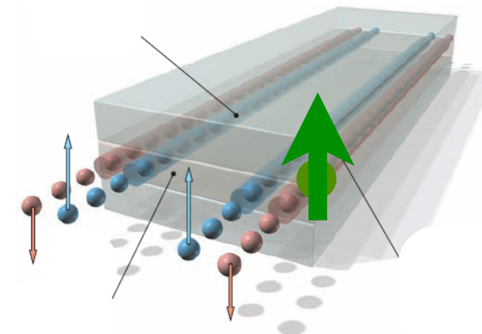
conductance correction in dc limit:

$$\delta G = -\frac{e^2 \cos^2\theta}{2T}\Big[\frac{4\gamma_0\gamma_0' + (\gamma_0+\gamma_0')(\gamma_0^E+\tilde{\gamma}_0^E)+\tilde{\gamma}_0^E\gamma_0^E}{\gamma_0+\gamma_0'+\tilde{\gamma}_0^E}\Big]$$

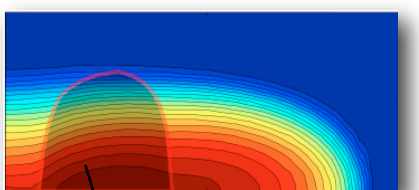
$$\gamma_0 \sim (J_x + J_y')^2 T^{2(\sqrt{K}-\lambda/2)^2-1}, \gamma_0' \sim (J_x - J_y')^2 T^{2(\sqrt{K}+\lambda/2)^2-1}, \gamma_0^E \sim J_{NC}^2 T^{2K-1}, \tilde{\gamma}_0^E \sim J_{NC}^2 T$$

Summary and outlook

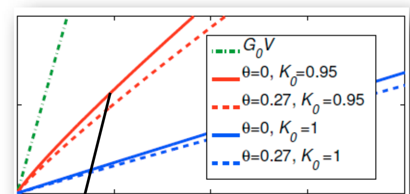
E. Eriksson *et al.*, PRB **86**, 161103(R) (2012)



Magnetic impurity in a helical edge liquid:
Rashba coupling allows electrical control of
Kondo temperature and **IV-characteristics**



blocking of Kondo screening!?



Rashba-induced
impurity correction
accessible in experiment?

Interesting open problems:
Two-loop RG, thermal transport, effects from Dresselhaus spin-orbit interactions, Kondo lattice in a helical liquid, higher-spin impurities... and more!