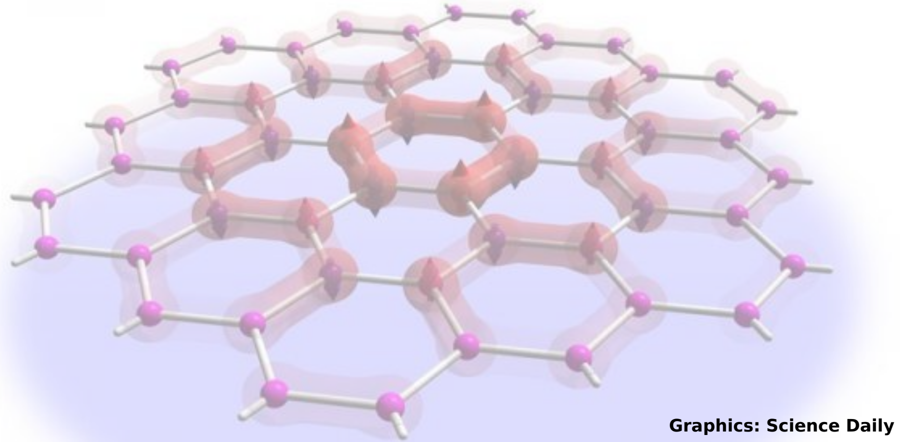
Spin liquid phases of alkaline earth atoms at finite temperatures



Graphics: Science Daily

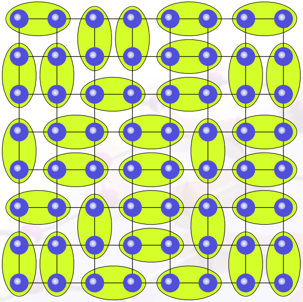
P. Sinkovicz, A. Zamora, E. Szirmai, G. Szirmai, M. Lewenstein

Wigner Research Centre of the Hungarian Academy of Sciences

ICFO - The Institute of Photonic Sciences, Barcelona

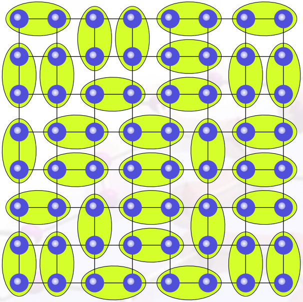
The Institute of Photonic Sciences

Motivation: magnetic systems without long range order



$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

$J > 0$	antiferromagnetic coupling



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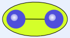
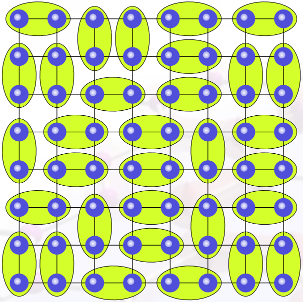
Singlet pairs

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

rotational symmetry preserved



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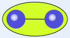 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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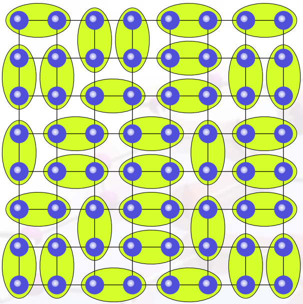
Schwinger fermions

$$\vec{S}_i = \sum_{\alpha,\beta} c^\dagger_{i,\alpha} \, \vec{\sigma}_{\alpha,\beta} \, c_{i,\beta} \qquad |\uparrow\rangle , |\downarrow\rangle$$

$$\{ c_{i,\alpha}, c^\dagger_{j,\beta} \} = \delta_{i,j} \delta_{\alpha,\beta} \qquad \begin{cases} |\uparrow\rangle , |\downarrow\rangle \\ |\varnothing\rangle , |\uparrow\downarrow\rangle \end{cases}$$

1 particle / site



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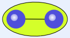 $= \frac{1}{\sqrt{2}} \left(\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle \right)$

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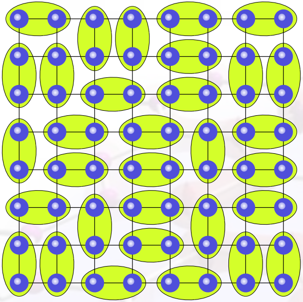

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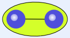 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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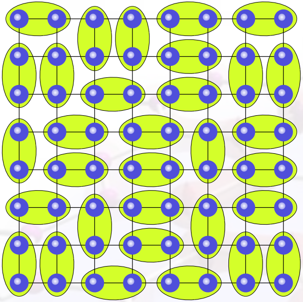
$$H = -J \sum_{\langle i,j \rangle} c^\dagger_{i,\alpha} c_{j,\alpha} c^\dagger_{j,\beta} c_{i,\beta} + \sum_i \varphi_i \left(c^\dagger_{i,\alpha} c_{i,\alpha} - 1 \right)$$



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Local gauge invariance

$$c_{j,\alpha} \to e^{i\theta_j}\, c_{j,\alpha}$$

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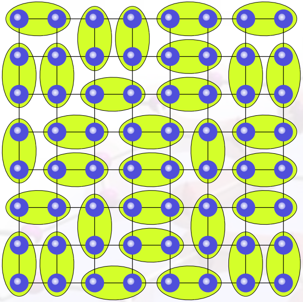

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Mean field theory

$$\chi_{i,j} = \left\langle c^\dagger_{i,\alpha} c_{j,\alpha} \right\rangle = \chi^*_{j,i}$$

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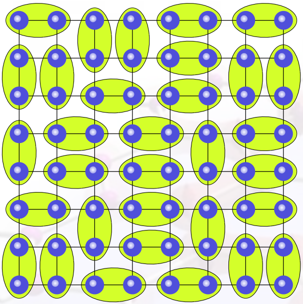

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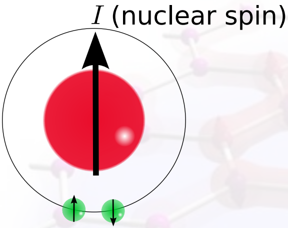
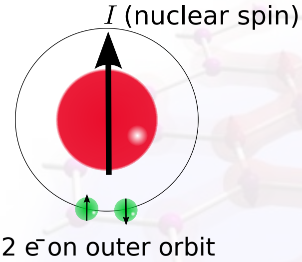
$$H_{\mathrm{mf}} = -J \sum_{\langle i,j \rangle} \left(\chi_{i,j} c^\dagger_{j,\alpha} c_{i,\alpha} + \chi_{j,i} c^\dagger_{i,\alpha} c_{j,\alpha} - |\chi_{i,j}|^2 \right) + \sum_i \varphi_i \left(c^\dagger_{i,\alpha} c_{i,\alpha} - 1 \right)$$

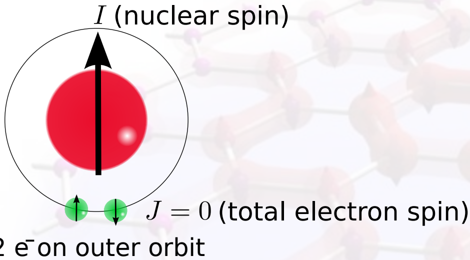
Works well for SU(N) spins when $\mathrm{N} \to \infty$

J. B. Marston, I. Affleck, Phys. Rev. B **39**, 11538 (1989)


I (nuclear spin)


I (nuclear spin)
2 e⁻ on outer orbit

SU(N) systems are realized with alkaline earth atoms



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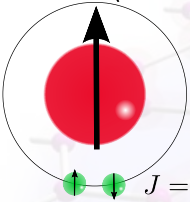


$F = I$ (total hyperfine spin)

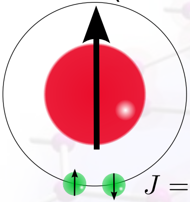
$J = 0$ (total electron spin)

2 e⁻ on outer orbit

Collisions are spin independent

SU(N) systems are realized with alkaline earth atoms

I (nuclear spin)



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$\mathrm{SU}(N = 2I + 1)$ symmetric models

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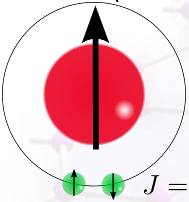

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Example $^{173}\mathrm{Yb}$: $I = \frac{5}{2} \Rightarrow 2I + 1 = 6$ spin components

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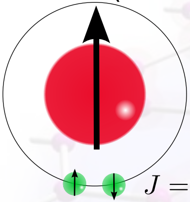

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2 e^- on outer orbit

Collisions are spin independent

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Example $^{173}\mathrm{Yb}$: $I = \frac{5}{2} \Rightarrow 2I + 1 = 6$ spin components

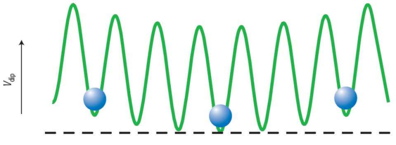
for square lattice: M. Hermele et al, Phys. Rev. Lett. 103, 135301 (2009)

[See also the talks of Karlo Penc and Edina Szirmai!]

Optical lattice

periodic potential created by standing wave laser light

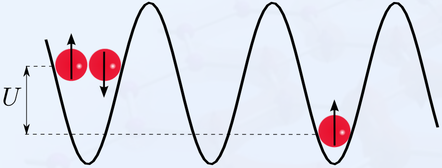
$$V_{1d}(r,z) = V_0 e^{-2r^2/w^2(z)} \sin^2(k_L z)$$



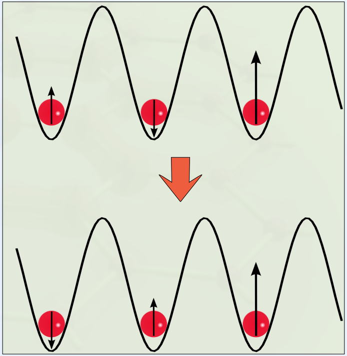
Tight binding Hamiltonian / Hubbard model

$$H = -t \sum_{\langle i,j \rangle} \left(c^\dagger_{i\alpha} c_{j\alpha} + \text{H.c.} \right) + \frac{U}{2} \sum_i c^\dagger_{i\alpha} c^\dagger_{i\beta} c_{i\beta} c_{i\alpha},$$

1 particle per site



$U \gg t$



particle transport is forbidden, **but**

spins can exchange without current

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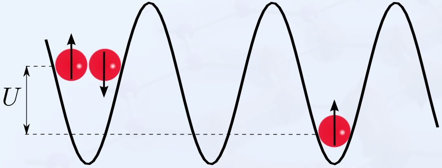

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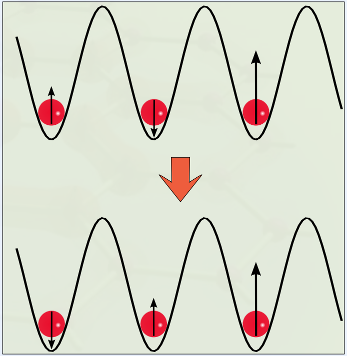

Low energy effective Hamiltonian (2nd order)

$$H = -J \sum_{\langle i,j \rangle} c^{\dagger}_{i,\alpha} c_{j,\alpha} c^{\dagger}_{j,\beta} c_{i,\beta} + \sum_i \varphi_i \left(c^{\dagger}_{i,\alpha} c_{i,\alpha} - 1 \right), \qquad J = \frac{4t^2}{U}$$

Partition function after a Hubbard-Stratonovich transformation

$$Z = \int D[c, \bar{c}, \chi, \chi^*] e^{-S_{\mathrm{HS}}[c,\bar{c},\chi,\chi^*]}$$

$$S_{\mathrm{HS}}[c, \bar{c}] = \int_0^\beta \mathrm{d}\tau \Big[\sum_i \bar{c}_{i,\alpha}(\partial_\tau + \varphi_i) c_{i,\alpha} - \sum_{\langle i,j \rangle} \Big(\chi_{i,j} \bar{c}_{j,\alpha} c_{i,\alpha} + \chi^*_{i,j} \bar{c}_{i,\alpha} c_{j,\alpha} - \frac{1}{J} |\chi_{i,j}|^2 \Big) \Big]$$

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Integrating out the fermions

$$Z = \int D[\chi, \chi^*] e^{-\int_0^\beta \sum_{\langle i,j \rangle} [\frac{1}{J} |\chi_{i,j}|^2 + \ln \det \mathcal{G}_{i,j}(i\omega_n)]}$$

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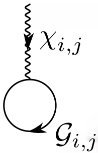
finite temperature Green's function

The saddle point provides the mean field equations

$$\chi_{i,j} = J \cdot \mathrm{tr}\left(\mathcal{G}\frac{\delta\mathcal{G}^{-1}}{\delta\chi^*_{i,j}}\right)$$



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For evaluation we need to choose a periodically repeating unit cell.

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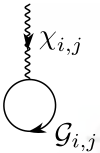

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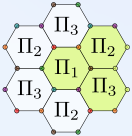
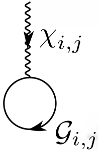
Solutions on a honeycomb lattice @ T=0

E	Π_1	Π_2	Π_3





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-6.148	$-0.159-0.276i$	$-0.159-0.276i$	$-0.159-0.276i$
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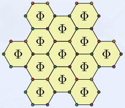

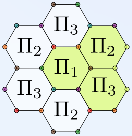

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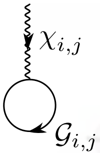

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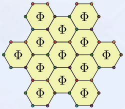
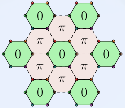
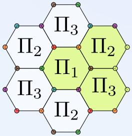
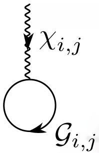
-6.062	0.460	-0.223	-0.223
-6.062	-0.223	0.460	-0.223
-6.062	-0.223	-0.223	0.460









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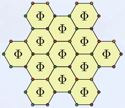
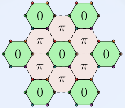
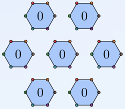
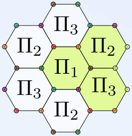
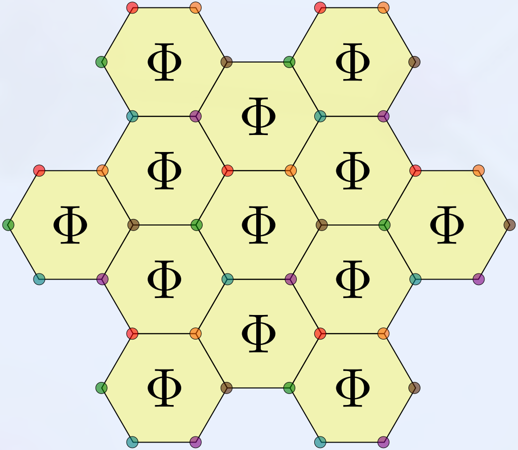
-6.062	-0.223	-0.223	0.460
-6	1	0	0
-6	0	1	0
-6	0	0	1











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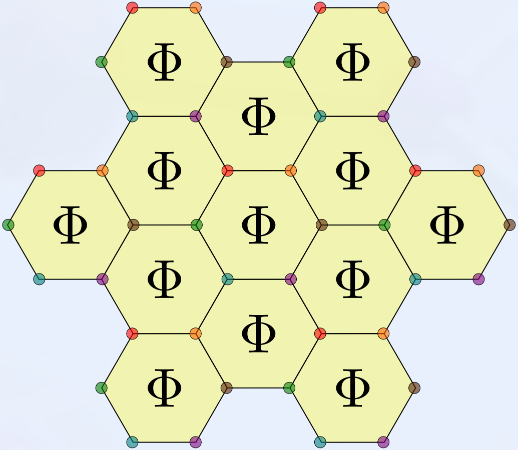

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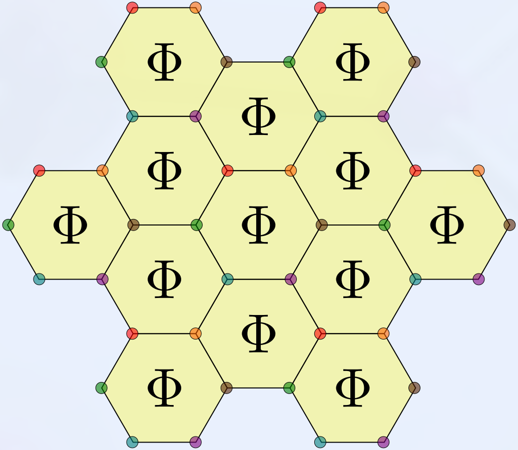

- uniform, lattice and SU(6) rotational symmetric,
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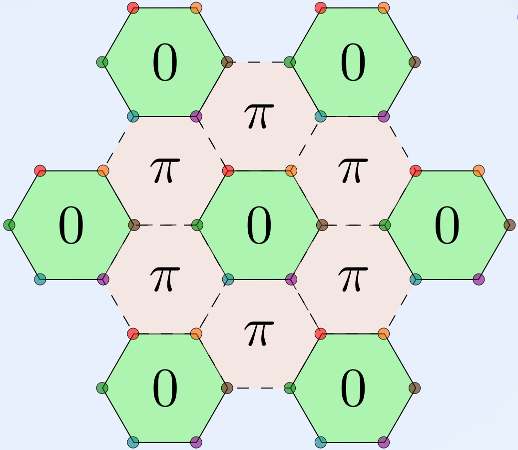
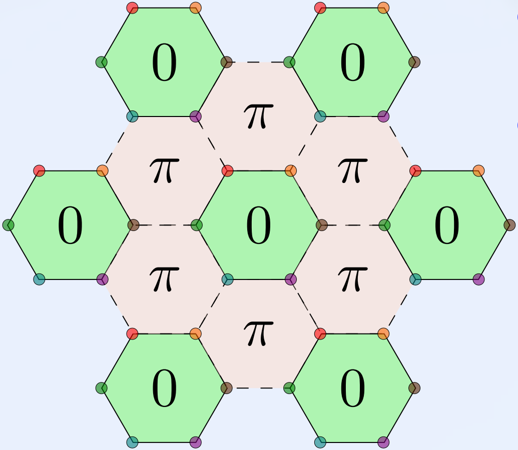
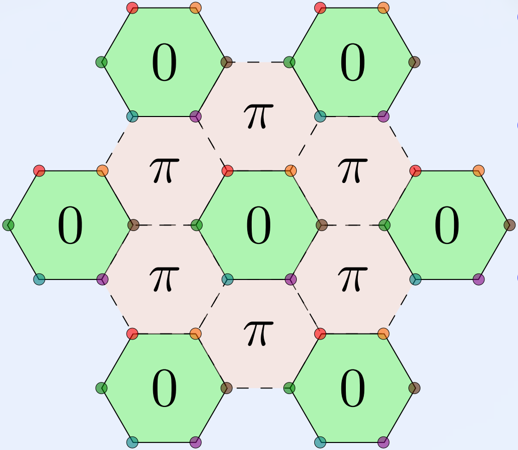


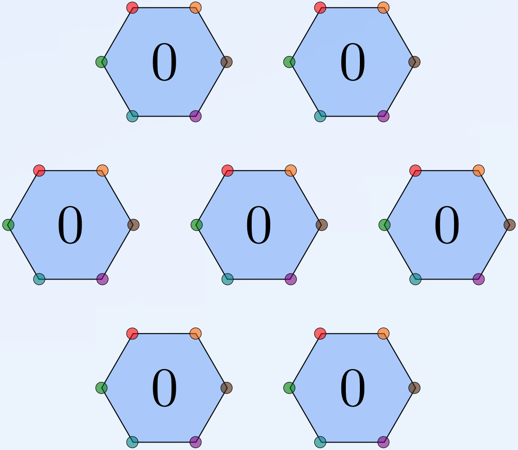
- has a triple degeneracy,



- has a triple degeneracy,
- is the honeycomb analog of the pi-flux phase



- has a triple degeneracy,
- is the honeycomb analog of the pi-flux phase
- due to the frustrated nature of the dual lattice alternating fluxes are unfavorable here.



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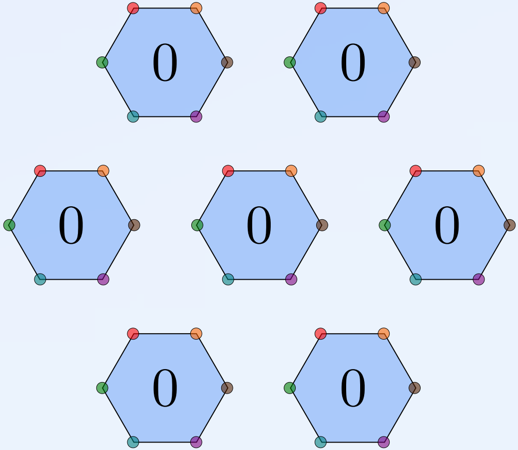

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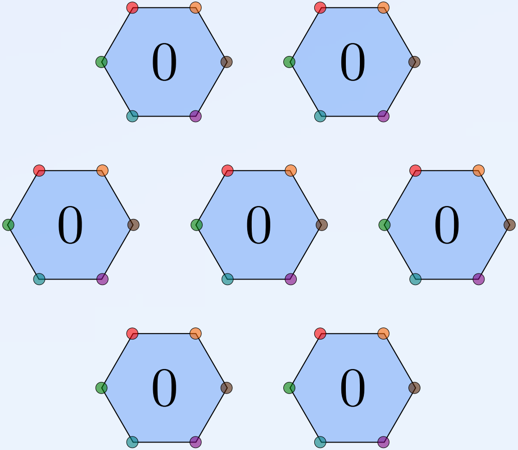

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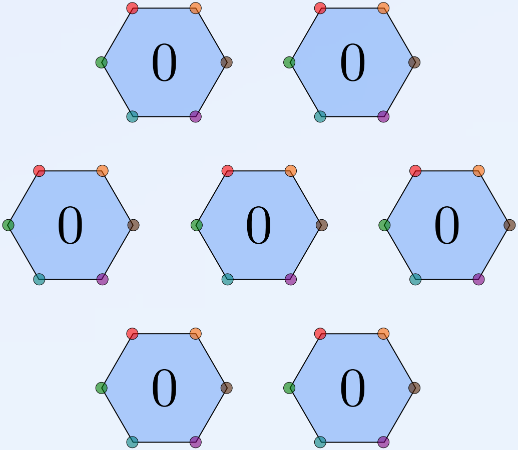

- also has a triple degeneracy,
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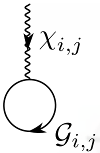
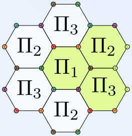


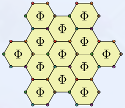
$$\chi_{i,j} = J \cdot \mathrm{tr}\left(\mathcal{G}\frac{\delta \mathcal{G}^{-1}}{\delta \chi^*_{i,j}}\right)$$

For evaluation we need to choose a periodically repeating unit cell.

Solutions on a honeycomb lattice @ T=0

E	Π_1	Π_2	Π_3
-6.148	$-0.159-0.276i$	$-0.159-0.276i$	$-0.159-0.276i$
-6.148	$-0.159+0.276i$	$-0.159+0.276i$	$-0.159+0.276i$
-6.062	0.460	-0.223	-0.223
-6.062	-0.223	0.460	-0.223
-6.062	-0.223	-0.223	0.460
-6	1	0	0
-6	0	1	0
-6	0	0	1





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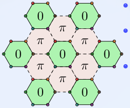

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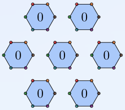

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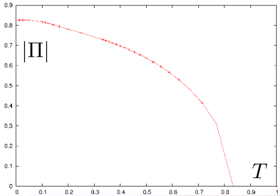
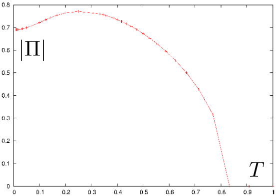
The spin liquid phases become unstable at finite temperatures

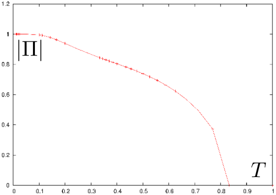
chiral phase



π-flux phase



plaquette phase



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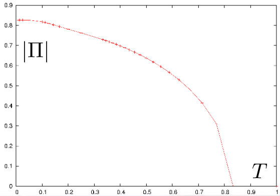

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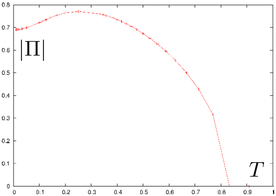

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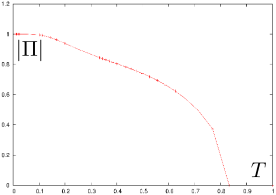

- all the spin liquid phases "melt" around the same temperature

The spin liquid phases become unstable at finite temperatures

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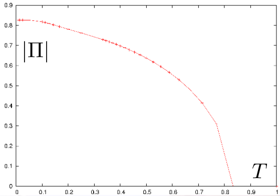

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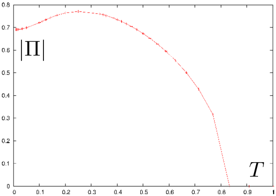

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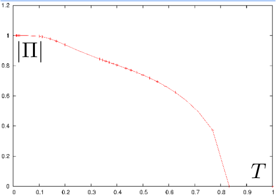
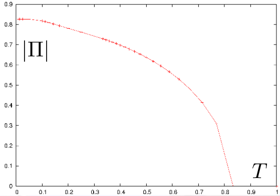


- all the spin liquid phases "melt" around the same temperature
- the stability of the phase can be decided by the second order term of the effective action

$$D_{i,j;k,l} = \mathrm{tr}\left(\mathcal{G}\frac{\delta\mathcal{G}^{-1}}{\delta\chi^*_{i,j}}\mathcal{G}\frac{\delta\mathcal{G}^{-1}}{\delta\chi^*_{k,l}}\right)$$

chiral phase



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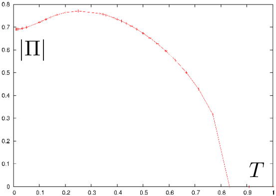
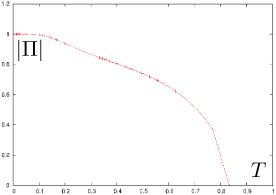


plaquette phase



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- the excitations are given by zeroes of the dielectric function:

$$0 = \det\left(\delta_{i,j;k,l} + J D_{i,j;k,l}\right)$$

Summary

- We considered thermodynamic properties of spin-5/2 alkaline-earth-metal fermions in a honeycomb lattice.
- At low temperatures the charge degrees of freedom are frozen, and the spin dynamics realizes a chiral spin liquid state with a dynamically generated flux that violates time reversal invariance.
- The low energy excitations in an infinite system are gauge bosons of U(1) Chern-Simons field theory.
- The higher energy spin liquid states are also interesting generalizations of their square lattice counterparts.

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