DECOHERENCE IN THE CENTRAL SPIN MODEL

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Outline

- Problem
- Methods
  - Algebraic Bethe Ansatz
  - Numerical Methods
- Results
- Conclusions

Hamiltonian:

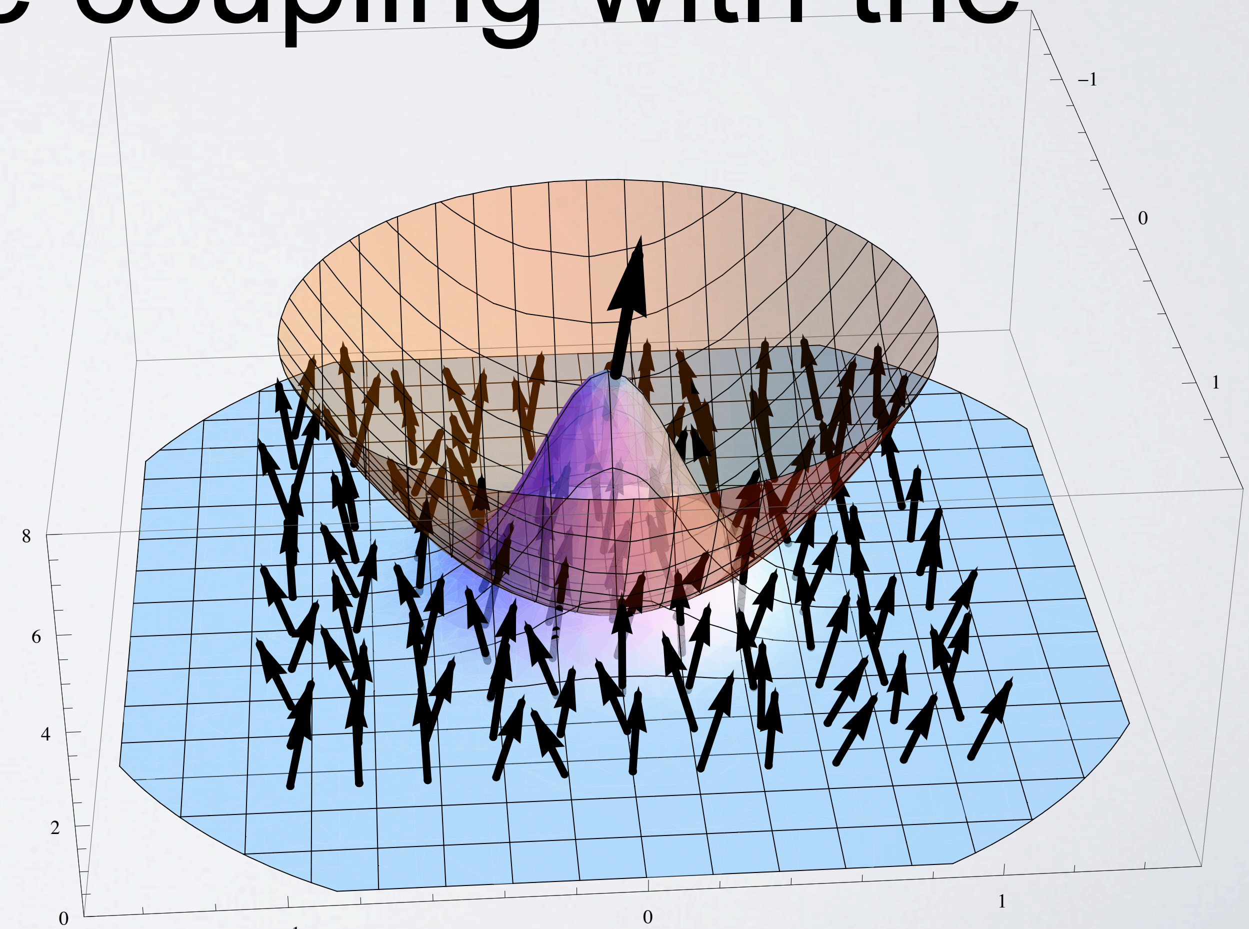
$$H = BS_0^z + \sum_{j=1}^{N} A_j \vec{S}_0 \cdot \vec{S}_j$$

Single electron trapped in a quantum dot.

- Relaxation and decoherence are dominated by hyperfine coupling with the nuclear spins.

Experiments:

- H. Bluhm - RWTH Aachen
- M. Bayer -TU Dortmund
- L. Vandersypen - TU Delft
- D. Steel - Michigan
- S. Tarucha - Tokyo
- A.C. Gossard - UCSB
- M. Pioro-Ladrière - Sherbrooke ...



Non-equilibrium dynamics:

Initial condition: $|\Psi_0\rangle = |\Downarrow; \downarrow, \uparrow, \downarrow, \uparrow, \downarrow, \uparrow ...\rangle + |\Uparrow; \downarrow, \uparrow, \downarrow, \uparrow, \downarrow, \uparrow ...\rangle$

Unitary time evolution (projected on the true eigenbasis):

$$|\Psi(t)\rangle = e^{-iHt} |\Psi_0\rangle = \sum_n \langle n | \Psi_0\rangle e^{-i\omega_n t} |n\rangle$$

Coherence factor:

Overlaps

$$\left\langle S_0^+(t) \right\rangle = \langle \Psi(t)| S_0^+ |\Psi(t)\rangle = \sum_{n,m} \langle \Psi_0 | m\rangle \langle m| S_0^+ |n\rangle \langle n | \Psi_0\rangle e^{-i(\omega_n - \omega_m)t}$$

Form factors

- Algebraic Bethe Ansatz gives us EXACT eigenstates at any B (non-perturbative)

$$|\{\lambda_1 ... \lambda_M\}\rangle \propto \prod_{i=1}^{M} \mathrm{S}^+(\lambda_i)\, |\Downarrow; \downarrow\downarrow ... \downarrow\rangle = \prod_{i=1}^{M} \left(\sum_{k=0}^{N} \frac{S_k^+}{\lambda_i - \epsilon_k} \right) |\Downarrow; \downarrow\downarrow ... \downarrow\rangle$$

$$\epsilon_k = -1/A_k \qquad \epsilon_0 = 0$$

Bethe Equations: $$-2B + \sum_{k=0}^{N} \frac{1}{\lambda_i - \epsilon_k} - \sum_{j=1(\neq i)}^{M} \frac{2}{\lambda_i - \lambda_j} = 0.$$

Eigenenergy: $$E(\{\lambda_1 ... \lambda_M\}) = \frac{1}{2} \sum_{i=1}^{M} \frac{1}{\lambda_i} - \frac{B}{2} - \sum_{j=1}^{N} \frac{1}{4\epsilon_j}$$

For unpolarized N=36 nuclear spins:

Dim H = 17 672 631 900 vs M =18

Find solutions to a small system
of M (< N) non-linear algebraic equations

- Non-linear system so we need an iterative procedure (Newton's method) and therefore a good guess.

$$-2B + \sum_{k=0}^{N} \frac{1}{\lambda_i - \epsilon_k} - \sum_{j=1(\neq i)}^{M} \frac{2}{\lambda_i - \lambda_j} = 0.$$

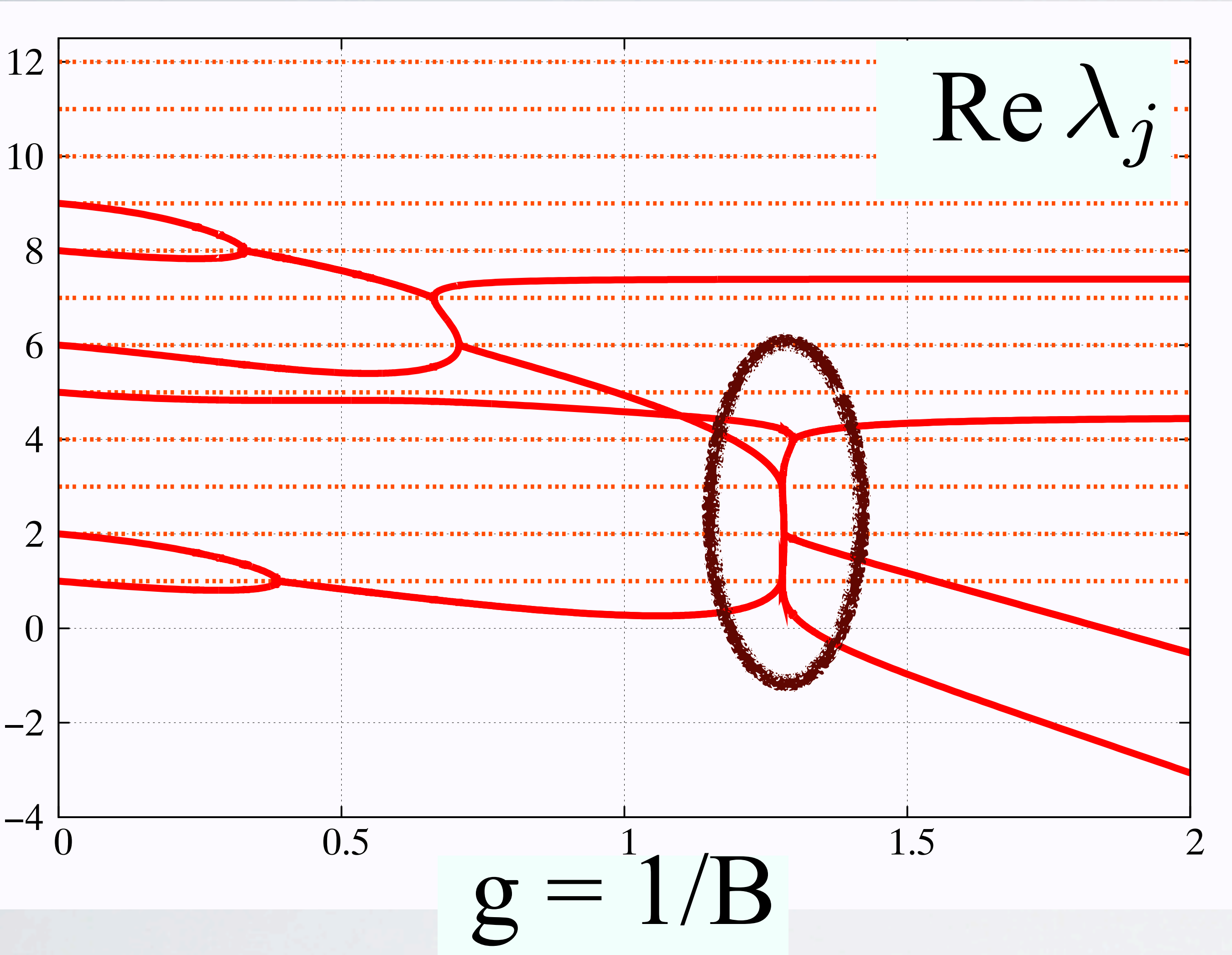
- Deforming the trivial large B solutions

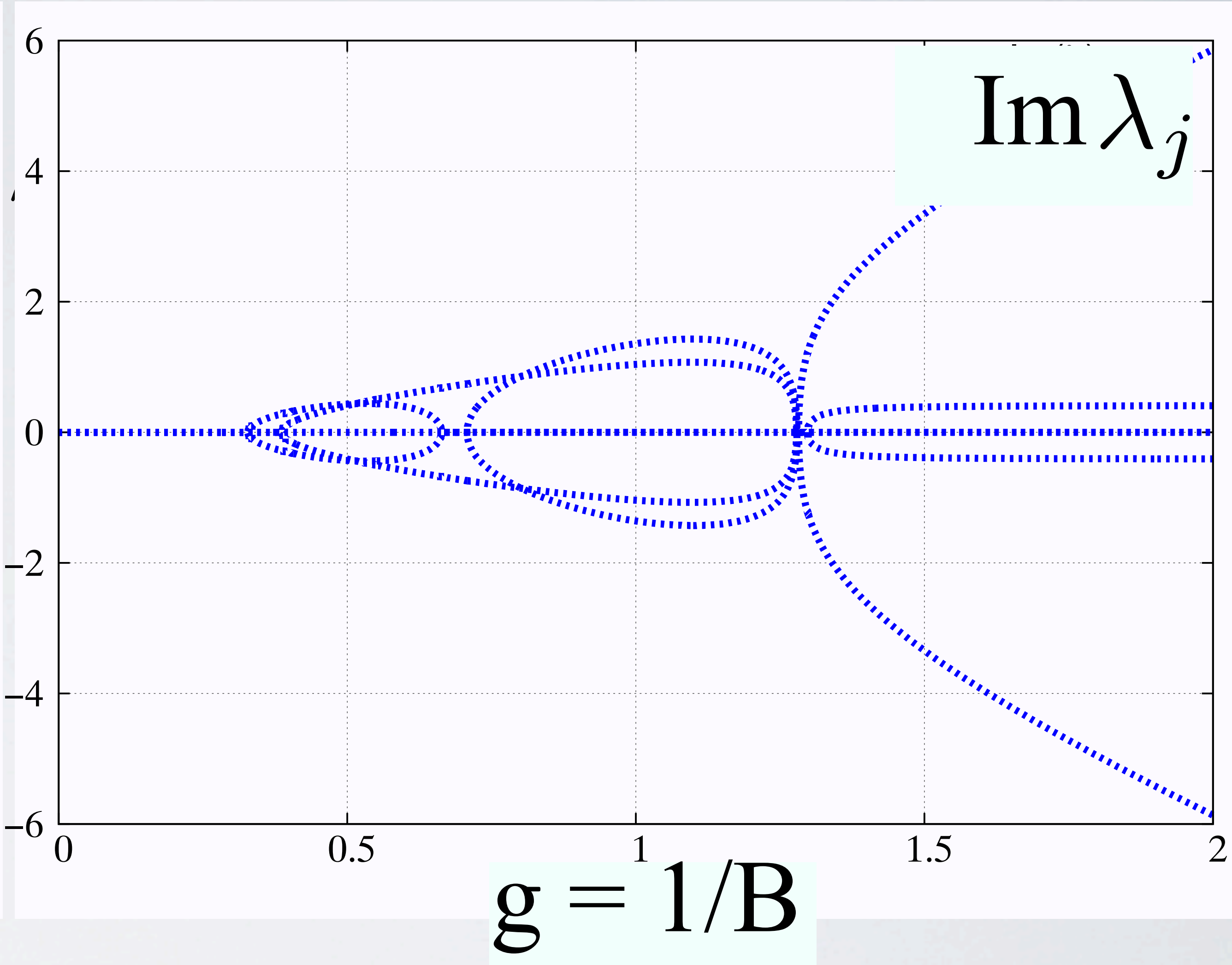
$$\{\lambda_1 ... \lambda_M\} \in \{\epsilon_0, \epsilon_1, \epsilon_2, ..., \epsilon_N\} \equiv \{0, 1/A_1, 1/A_2, ..., 1/A_N\}$$

to the desired magnetic field.

$$H = BS_0^z + \sum_{j=1}^{N} A_j \vec{S}_0 \cdot \vec{S}_j$$

Numerics





New variables: $\Lambda(\epsilon_i) = \sum_{j=1}^{M} \frac{1}{\epsilon_i - \lambda_j}$

N quadratic Bethe Eqs: $\Lambda^2(\epsilon_j) = \sum_{i \neq j} \frac{\Lambda(\epsilon_j) - \Lambda(\epsilon_i)}{\epsilon_j - \epsilon_i} + \frac{1}{g}\Lambda(\epsilon_j)$

- O Babelon and D Talalaev, *On the Bethe ansatz for the Jaynes–Cummings–Gaudin model*, J. Stat. Mech, P06013 (2007)

- A. Faribault, O. El Araby, C. Sträter, and V. Gritsev, *Gaudin models solver based on the correspondence between Bethe ansatz and ordinary differential equations*, Phys Rev. B **83**, 235124 (2011)

- O. El Araby, V. Gritsev and A. Faribault, *Bethe ansatz and ordinary differential equation correspondence for degenerate Gaudin models*, Phys. Rev. B **85**, 115130 (2012)

$$\left\langle S_0^+(t)\right\rangle = \sum_{n,m} \langle\Psi_0 | m\rangle \langle m| S_0^+ |n\rangle \langle n|\Psi_0\rangle e^{-i(\omega_n-\omega_m)t}$$

Overlaps: $\langle n|\Psi_0\rangle = \det_{M\times M} G(\{\lambda\})$

Form factors: $\langle m| S_0^+ |n\rangle = \det_{M\times M} J(\{\lambda\})$

N. A. Slavnov, *Calculation of scalar products of wave functions and form factors in the framework of the algebraic Bethe ansatz*, Teor. Mat. Fiz. **79**, 502 (1989)

New representation in terms of Λ_j

$$\langle n|\Psi_0\rangle = \det_{N\times N} G(\{\Lambda\})$$
$$\langle m| S_0^+ |n\rangle = \det_{N\times N} J(\{\Lambda\})$$

A. Faribault and D. Schuricht, On the determinant representations of Gaudin models' scalar products and form factors, arXiv: 1207.2352

- Sum over the contributions:

Monte Carlo sampling of pairs (m,n)

- Dynamics for N = 36 (Spin 1/2)

- Couplings (Gaussian wavefunction for 2D dot):

$$A_j = \frac{A}{N} e^{-\frac{j-1}{N}}$$

- Monte-Carlo sample 10^7 configurations

Results

- Initial state: $|\Psi_0\rangle = |\Downarrow; \downarrow, \uparrow, \downarrow, \uparrow, \downarrow, \uparrow ...\rangle + |\Uparrow; \downarrow, \uparrow, \downarrow, \uparrow, \downarrow, \uparrow ...\rangle$

"Maximal entropy state"

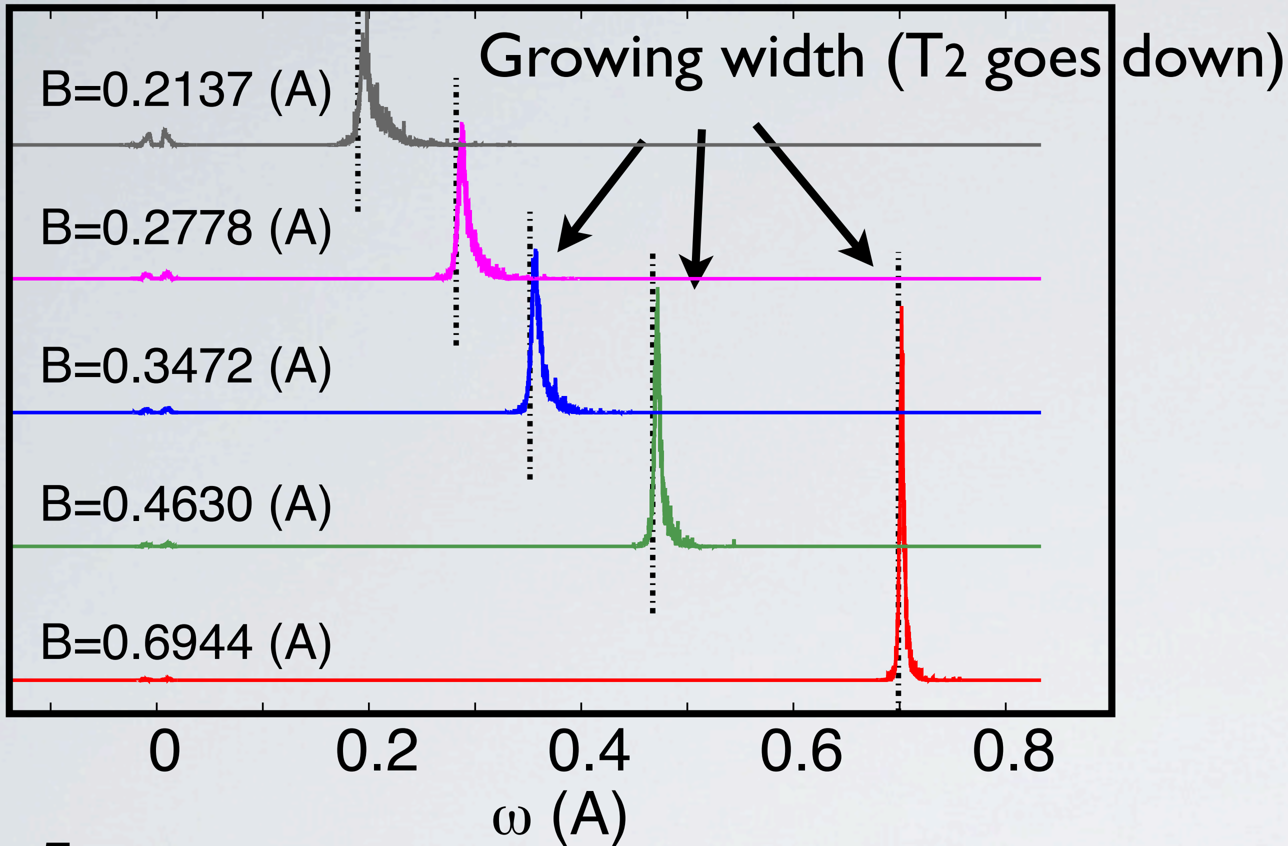
$$\langle S_0^+(t)\rangle = \sum_{n,m} \langle \Uparrow; \downarrow, \uparrow, \downarrow, \uparrow, \downarrow, \uparrow ... |m\rangle \langle m| S_0^+ |n\rangle \langle n |\Downarrow; \downarrow, \uparrow, \downarrow, \uparrow, \downarrow, \uparrow ...\rangle \, e^{-i(\omega_n - \omega_m)t}$$

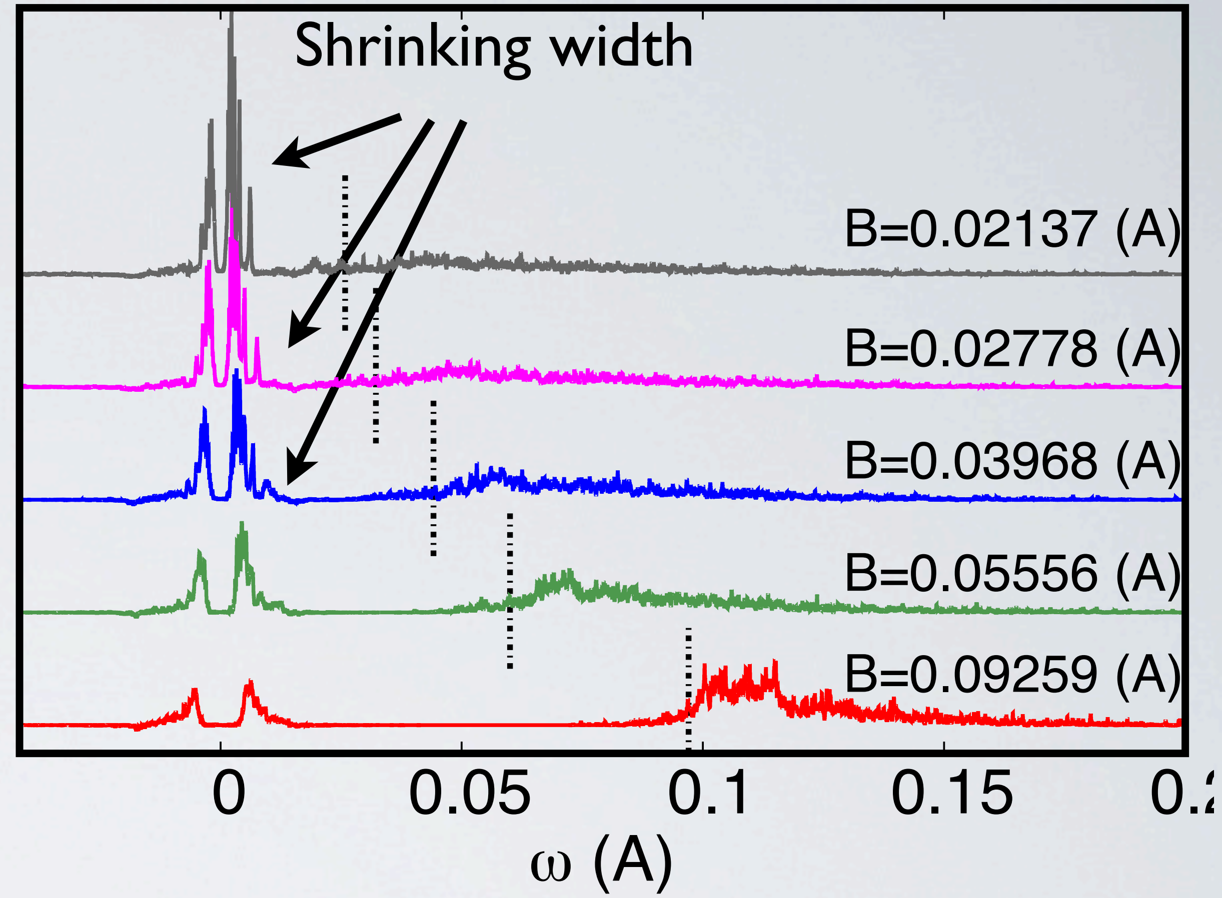
M   M-1

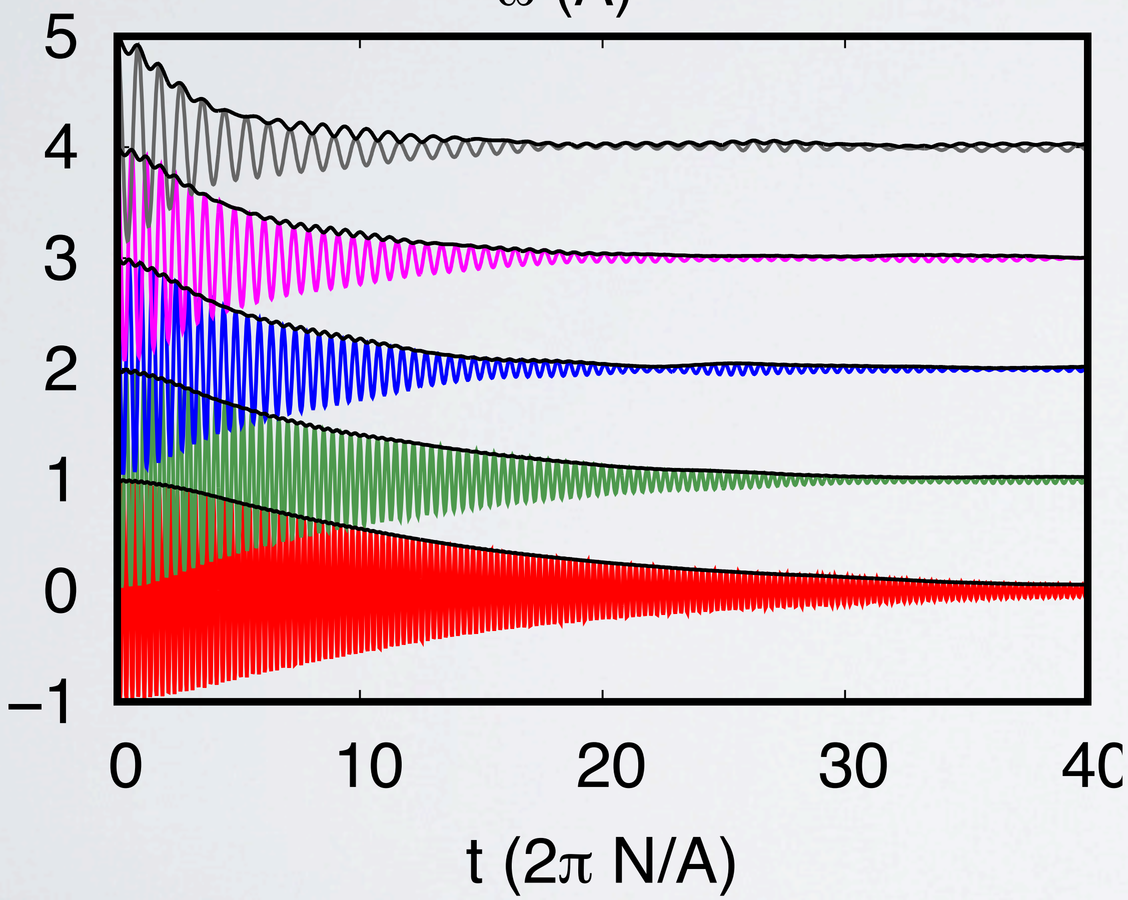
up spins

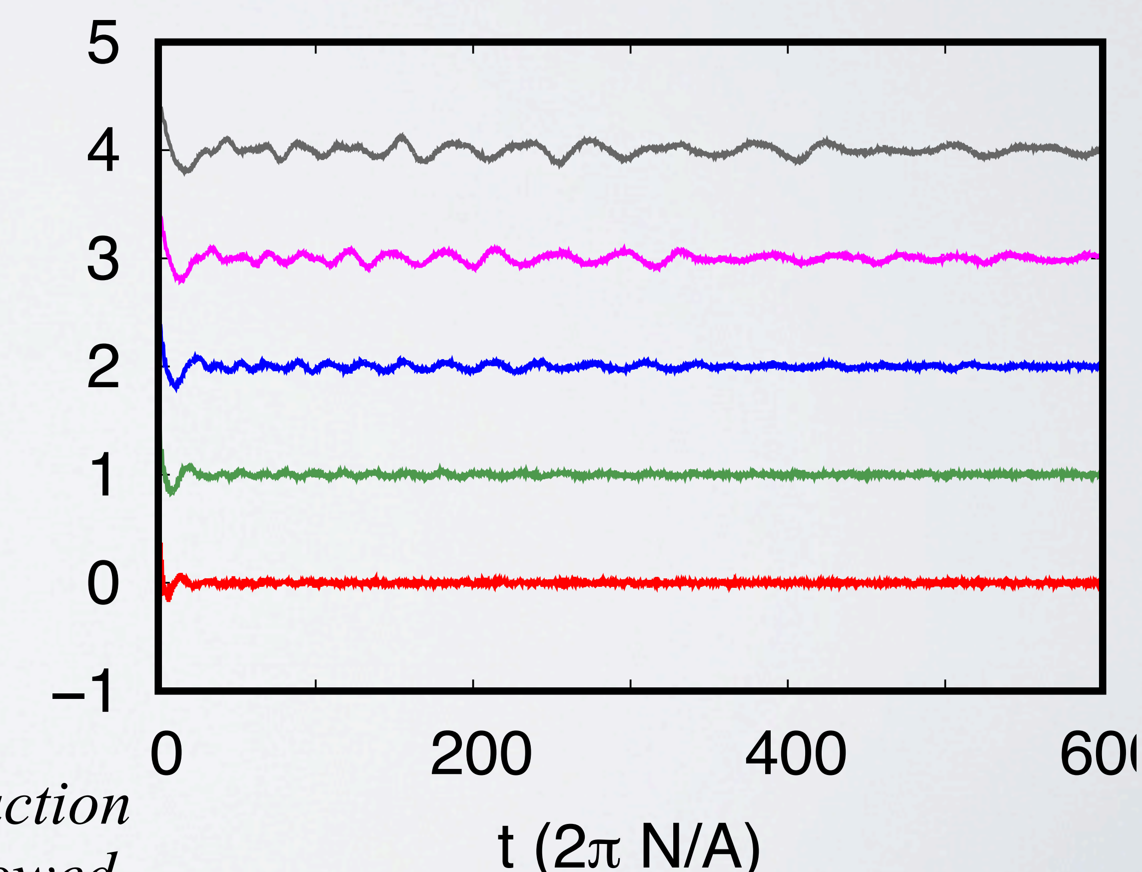
- Time evolution (and spectrum) of $\langle S_0^+(t)\rangle$ for a wide range of magnetic fields.

Results



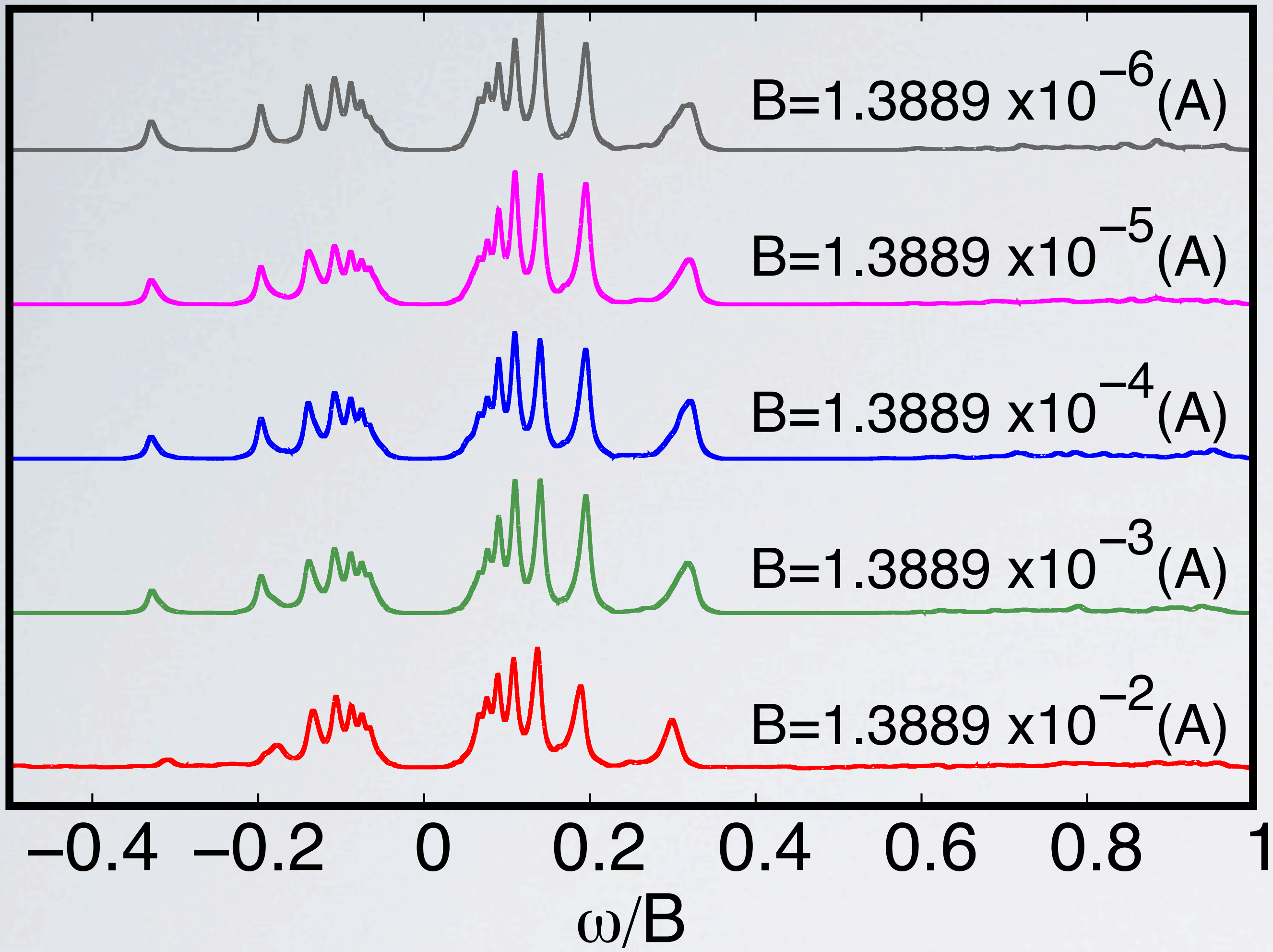


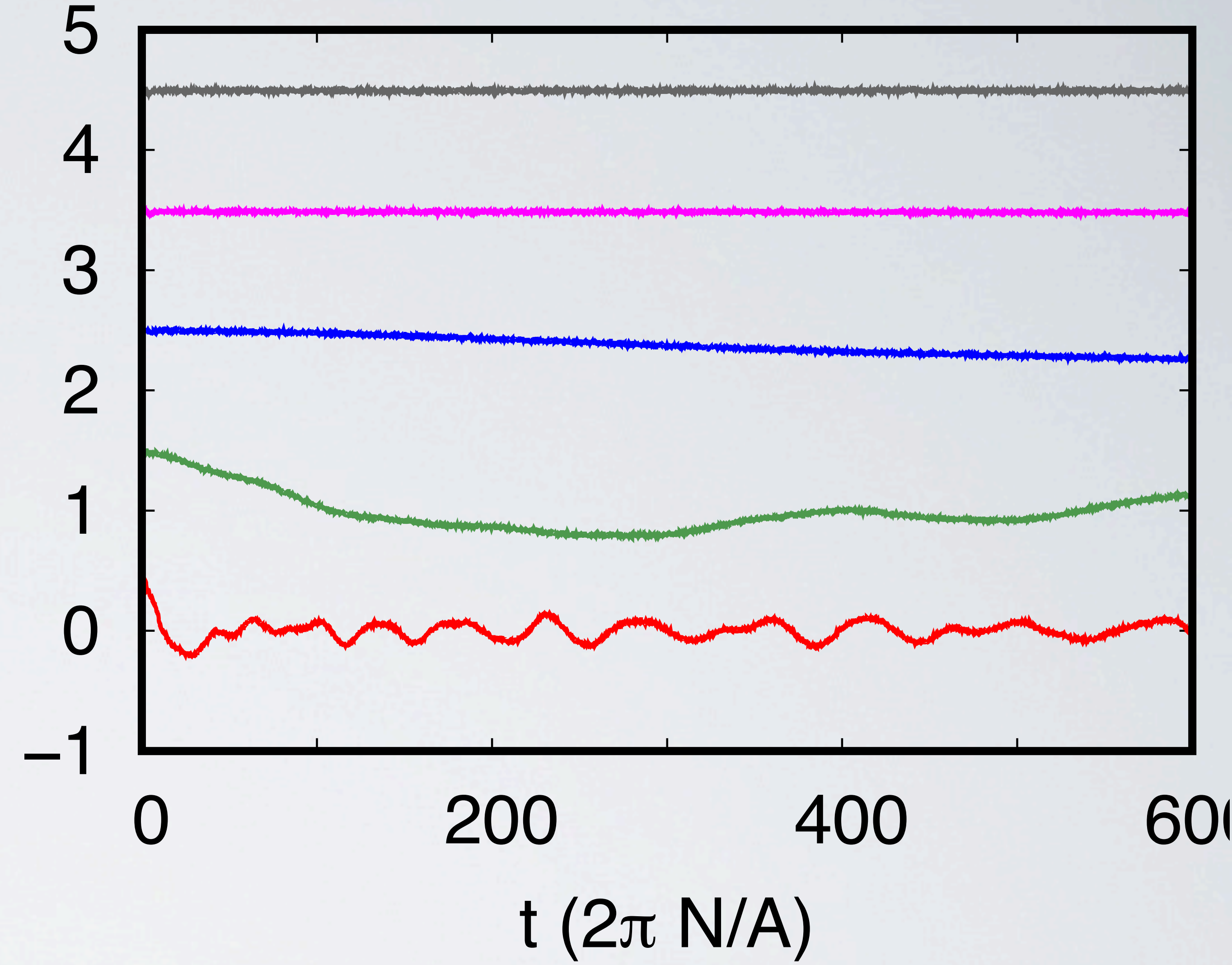




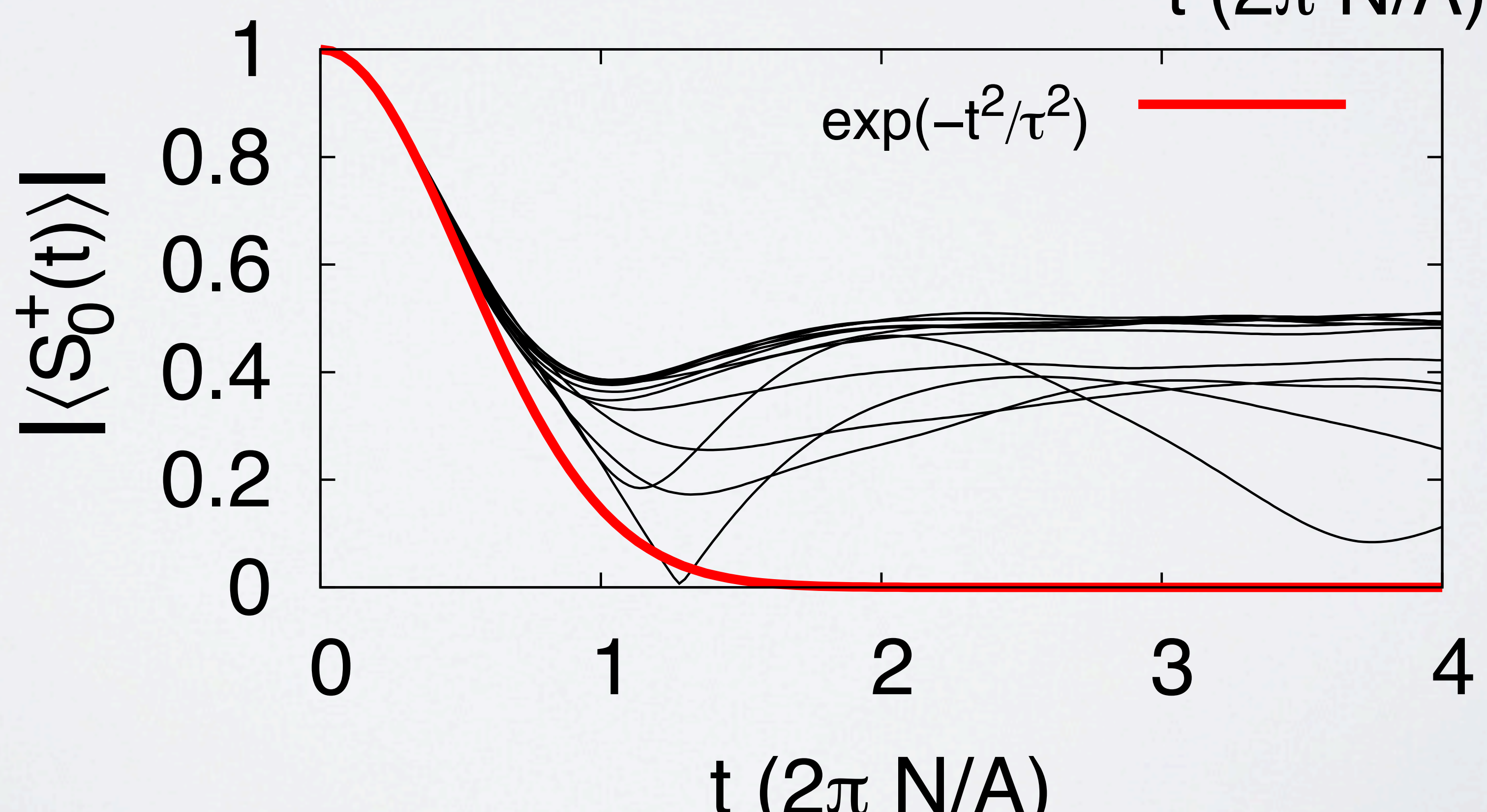
As in: W. A. Coish, J. Fischer, D. Loss, *Free-induction decay and envelope modulations in a narrowed nuclear spin bath*, Phys. Rev. B **81**, 165315 (2010)

- Weak field regime





Short time:



B=0 eigenstates are split into independent subsets

of localized: $\lambda_i \approx \lambda_i^0 + \mathcal{O}(B)$

and delocalized: $\lambda_i \approx \frac{L_i}{B} + \mathcal{O}(1)$

E. A. Yuzbashyan, A. A. Baytin, and B. L. Altshuler, *Strong-coupling expansion for the pairing Hamiltonian for small superconducting metallic grains*, Phys. Rev. B **68,** 214509 (2003)

Delocalized

$$\lambda_i = \infty$$

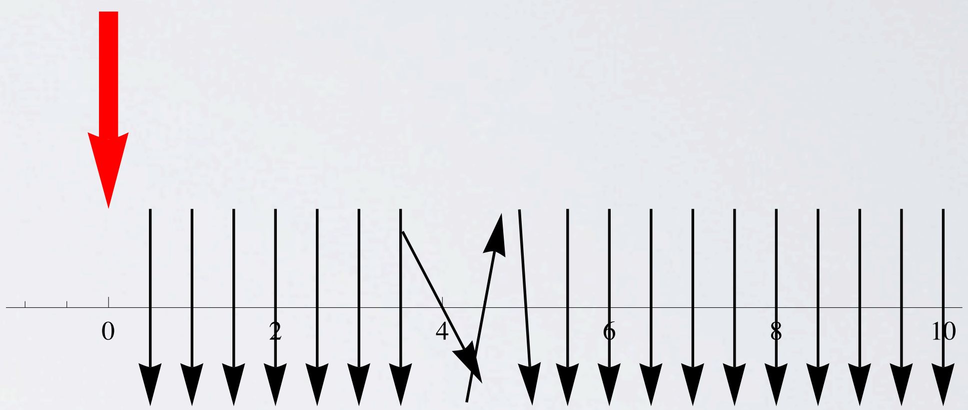
$$\left(\sum_{k=0}^{N} \frac{S_k^+}{\lambda_i - \epsilon_k}\right) |\Downarrow; \downarrow\downarrow \dots \downarrow\rangle \propto \left(\sum_{k=0}^{N} S_k^+\right) |\Downarrow; \downarrow\downarrow \dots \downarrow\rangle$$

Zero-energy: $\frac{1}{\lambda_i} = 0$

Localized

$$\lambda_i = \lambda_i^0 \approx \epsilon_j$$

$$\left(\sum_{k=0}^{N} \frac{S_k^+}{\lambda_i - \epsilon_k}\right) |\Downarrow; \downarrow\downarrow \dots \downarrow\rangle$$

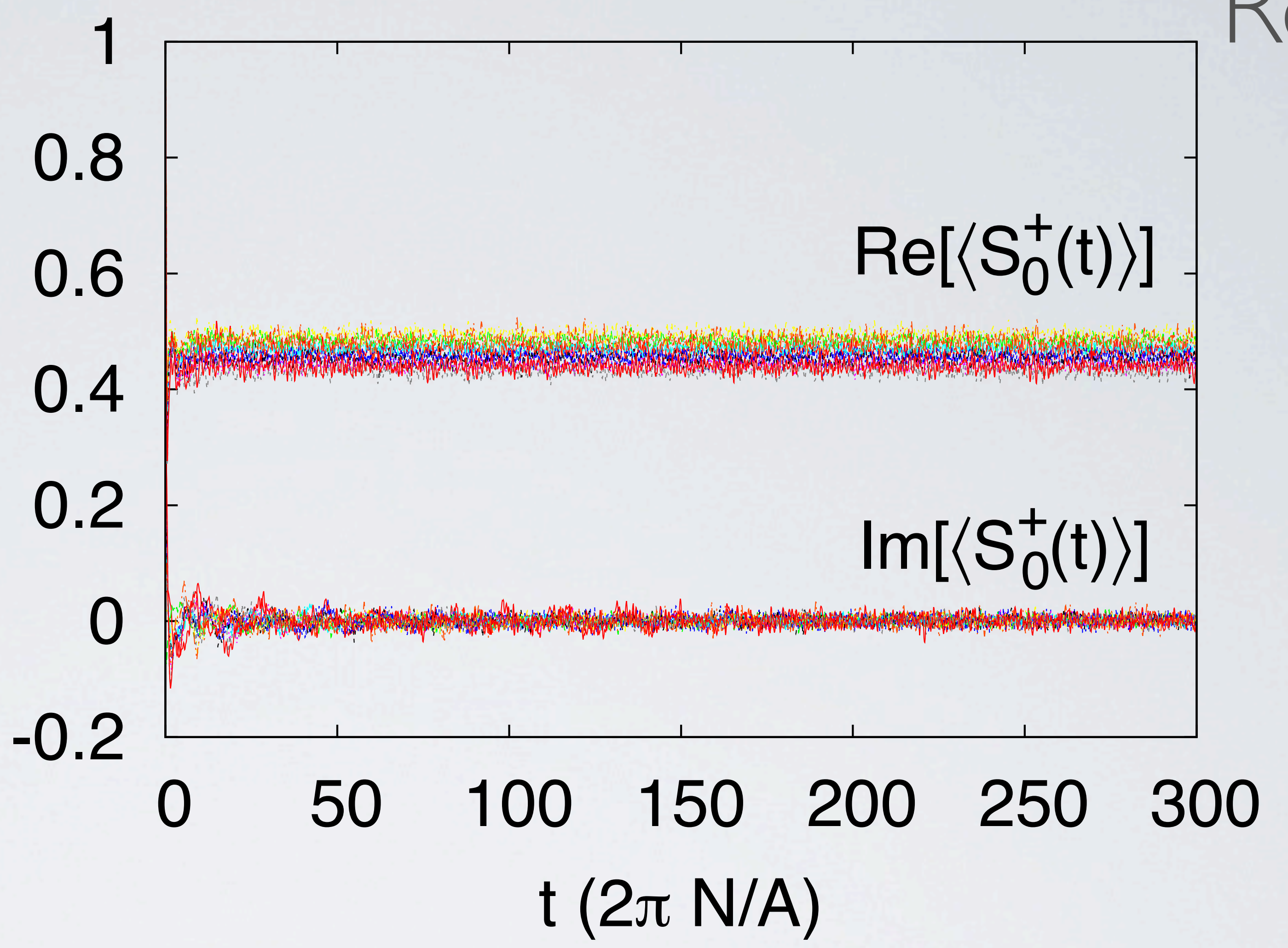


Finite energy

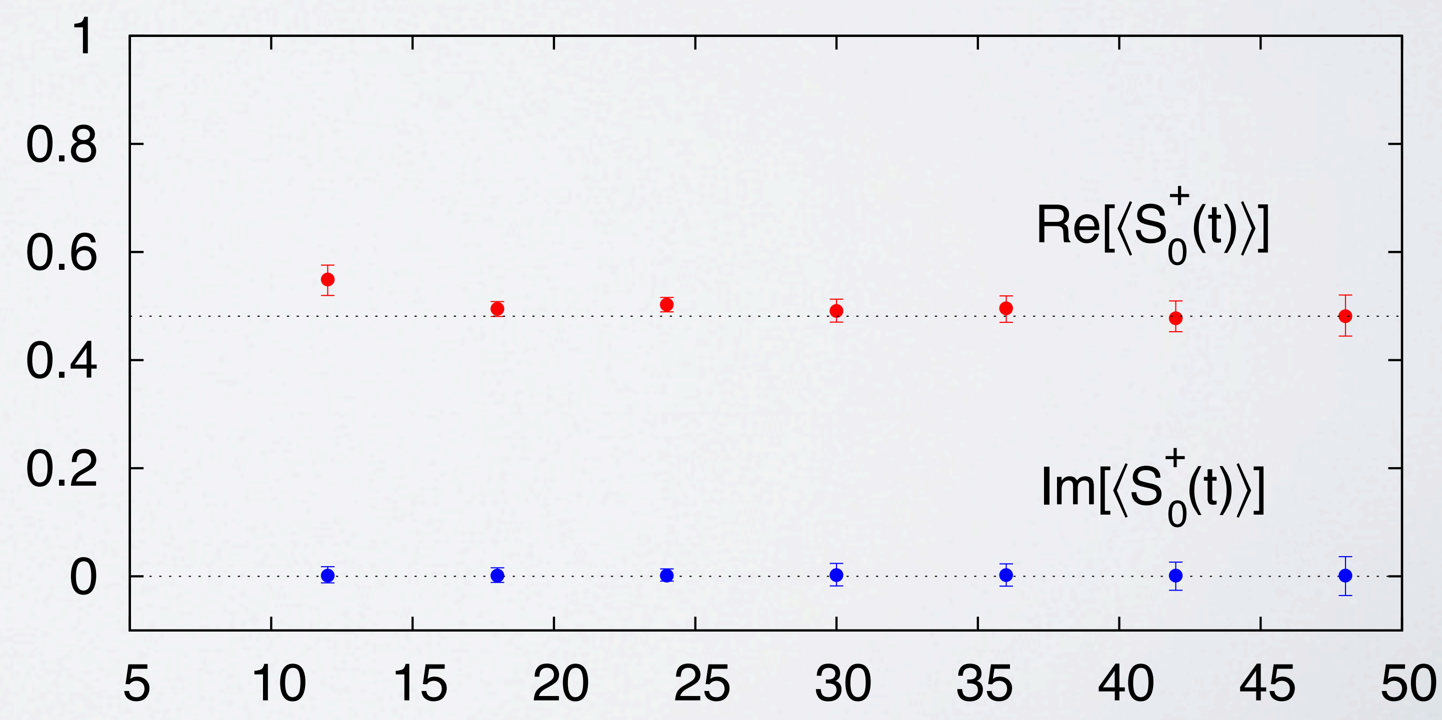
$$\langle S_0^+(t)\rangle = \sum_{n,m} \langle \Uparrow; \downarrow, \uparrow, \downarrow, \uparrow, \downarrow, \uparrow \ldots |m\rangle \langle m| S_0^+ |n\rangle \langle n | \Downarrow; \downarrow, \uparrow, \downarrow, \uparrow, \downarrow, \uparrow \ldots\rangle \, e^{-i(\omega_n - \omega_m)t}$$

EVERY M-1 eigenstate (n)
has a degenerate M partner (m)
(by adding one delocalized QP)

10 random nuclear states:



Finite size effects play no role:



Conclusions

- By exploiting the integrability of the central spin model, we were able to numerically explore the decoherence properties for any magnetic field.

- Large B: Larmor precession + weakly modulated exponential decay.

- Intermediate B: crossover

- Weak field: rapid Gaussian decay followed by non-decaying fraction.