BEC-BCS cross-over in the exciton gas

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1)Dilute limit:
BEC condensate of
linearly polarized dark excitons

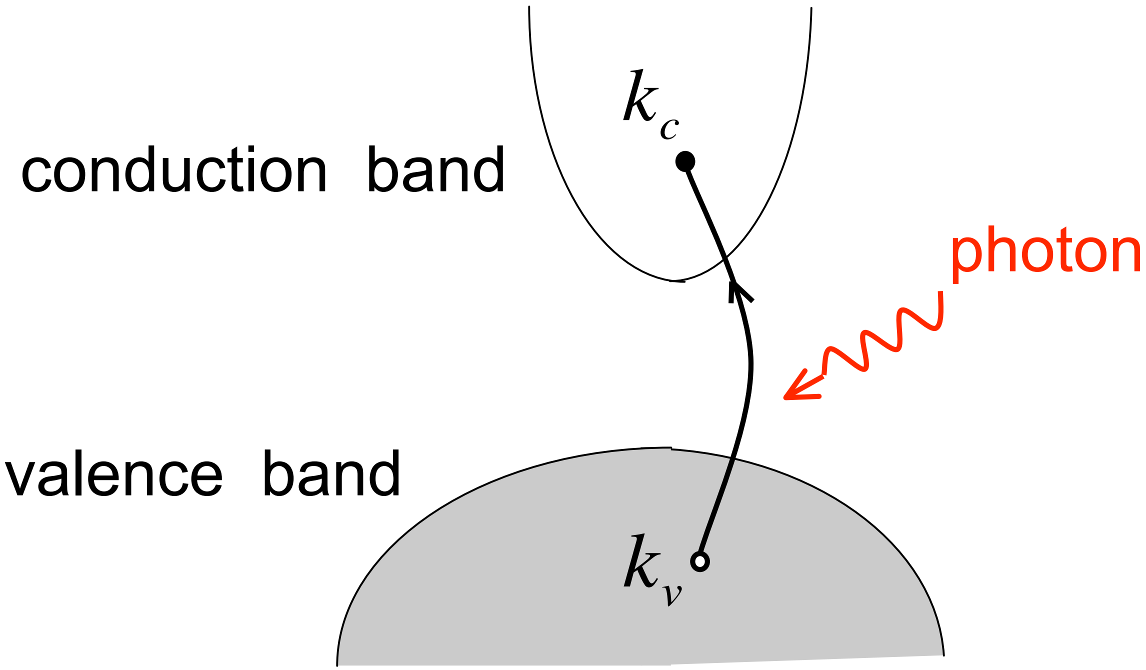
2)Under a density increase:
- mixture of dark and bright condensates
- phase separation between BEC and BCS condensates

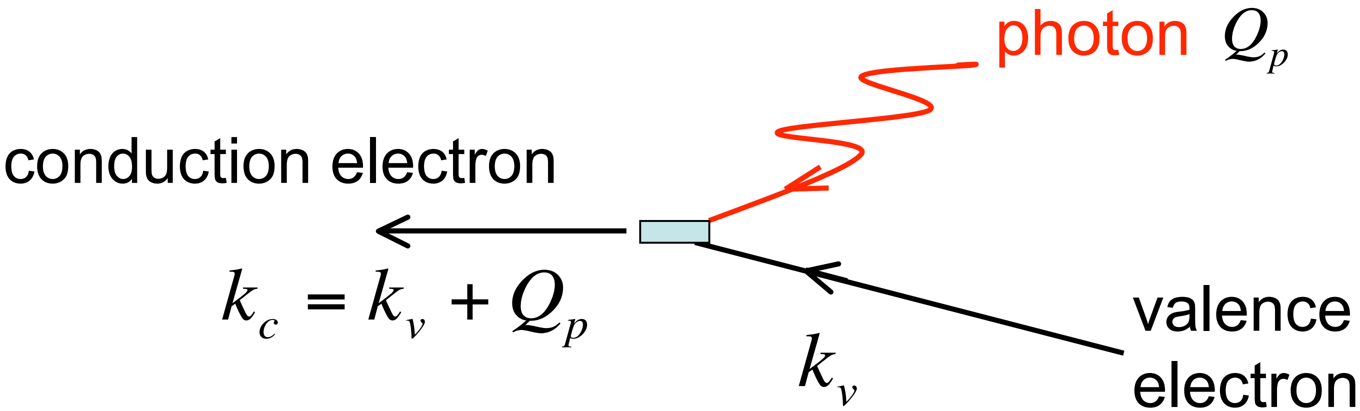
Part 1

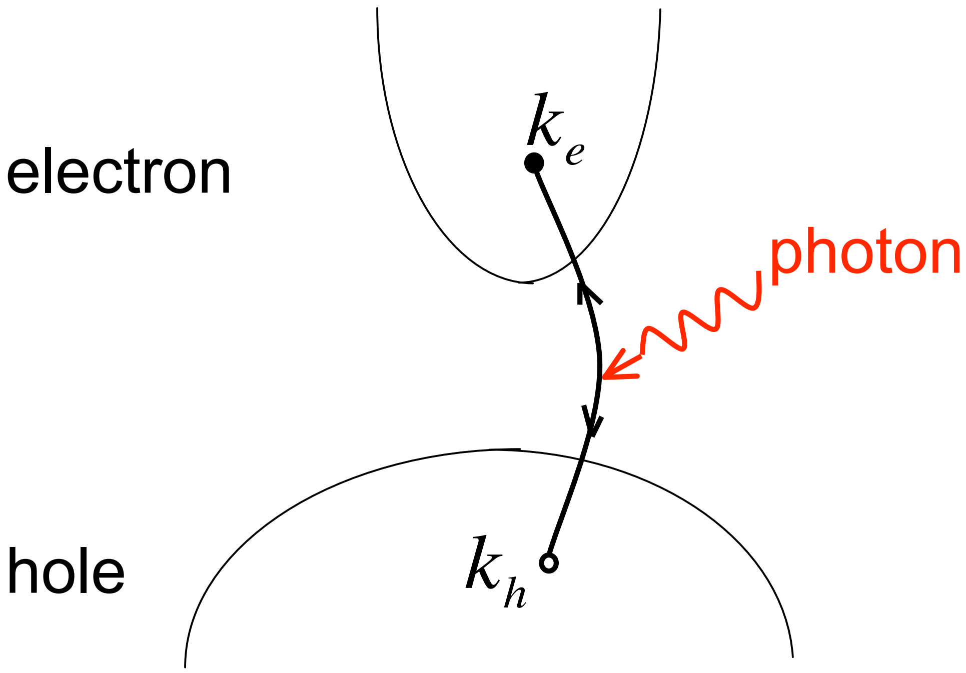
dilute limit

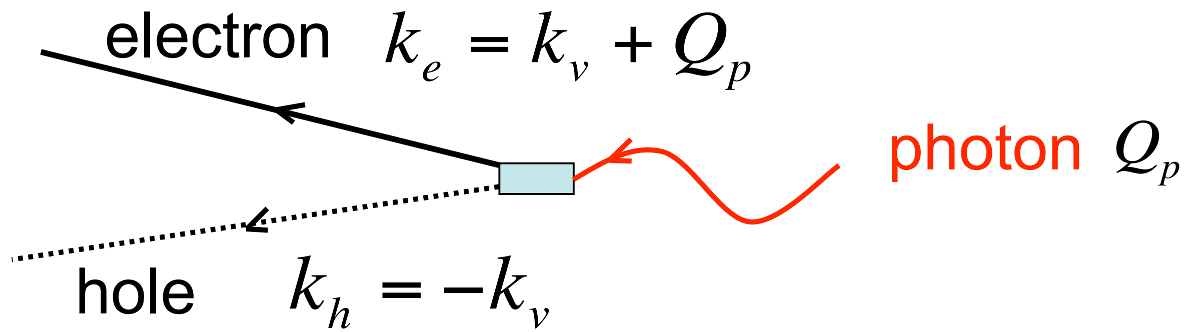
In the dilute limit,
electrons and holes
form excitons.
At low T, they undergo a
Bose-Einstein condensation
in a linearly polarized
dark state

Semiconductor





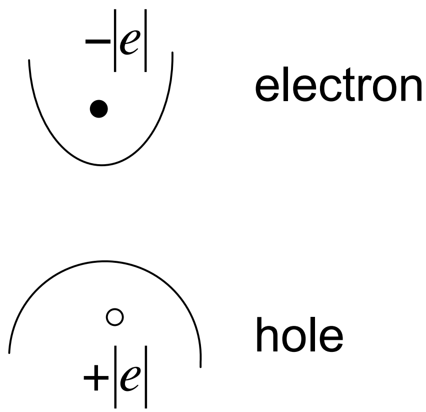

electron
k_e
photon
hole
k_h


electron $k_e = k_v + Q_p$
photon Q_p
hole $k_h = -k_v$

hole: full valence band minus one electron

formidable reduction of the many-body problem !

Naive view of an exciton



exciton similar to Hydrogen atom

light effective mass $m_e^* \approx 0.1 m_e$

dielectric constant $\varepsilon_{sc} \approx 10$

$$R_X = \frac{me^4}{2\hbar^2 \varepsilon_{sc}^2} \approx 13.6eV \left(\frac{0.1}{10^2} \right)$$

$$R_X \approx 10meV$$

$$a_X = \frac{\hbar^2 \varepsilon_{sc}}{me^2} \approx 0.53A \left(\frac{10}{0.1} \right)$$

$$a_X \approx 50A$$

The true story

is

far more complex

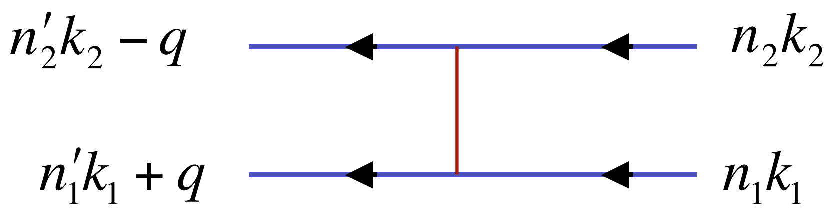
Coulomb interaction in a periodic lattice

Bloch states

$$\langle r | nk \rangle = \frac{e^{ik.r}}{L^{3/2}} u_{nk}(r)$$

$$u_{nk}(r) = u_{nk}(r + a)$$

$$| nk \rangle = a^{+}_{nks} | v \rangle$$



$$V_{Coul} = V_{e-e} - \overline{V}_{e-e}$$

$$= \frac{1}{2} \sum \ldots\ldots\ldots\ldots\ldots a^{+}_{n_1' k_1 + q} a^{+}_{n_2' k_2 - q} a_{n_2 k_2} a_{n_1 k_1}$$

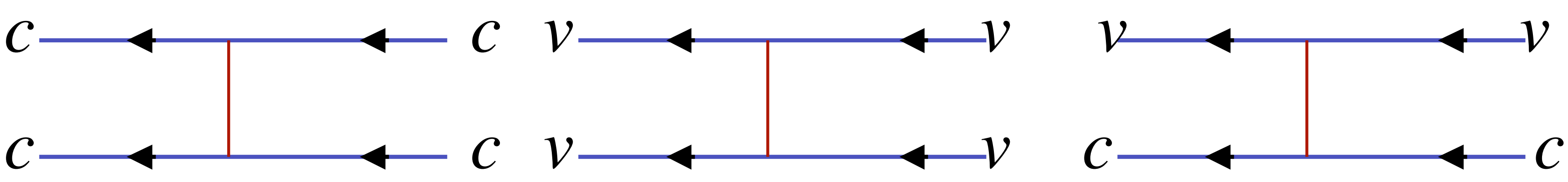
$$v_q(n_1', n_1) v_{-q}(n_2', n_2)$$

$$v_q(n,n) = \sqrt{\frac{4\pi e^2}{L^3 q^2}} \qquad v_q(n' \neq n) = O(q^0)$$

repulsive scattering between conduction and valence electrons

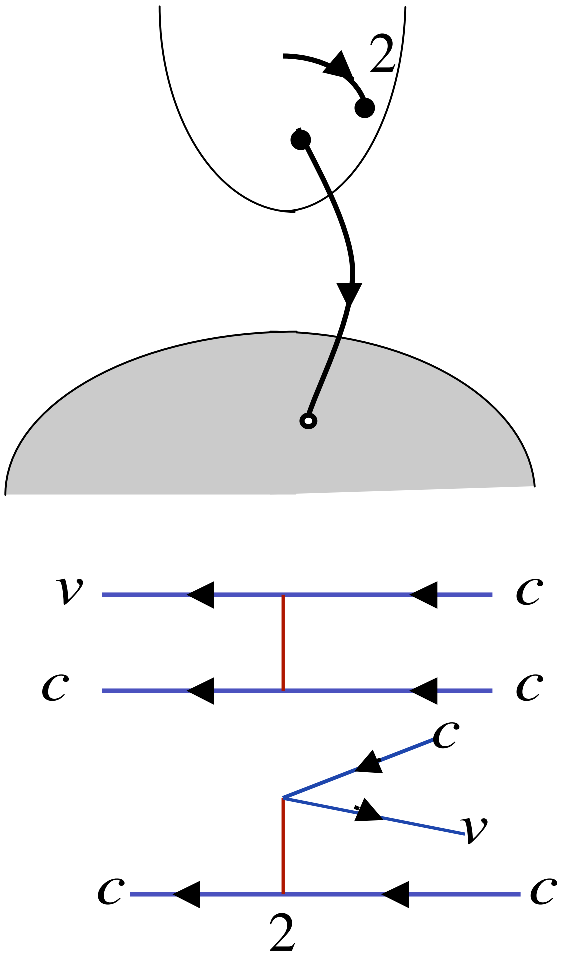
Intraband Coulomb processes

$$\frac{4\pi e^2}{L^3 q^2}$$



dressed by a dielectric constant

which results from many-body effects induced by
« boiling valence band »


2
v
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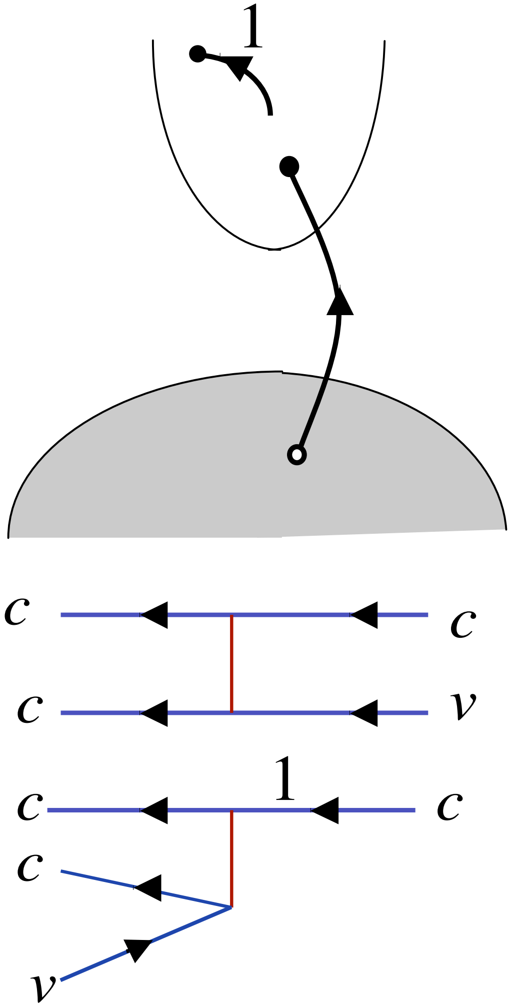
2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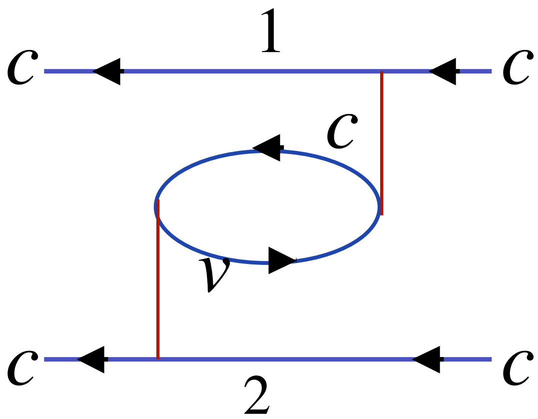
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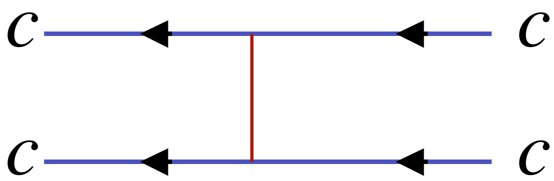
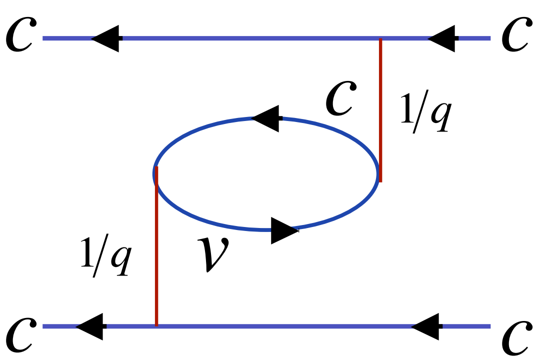


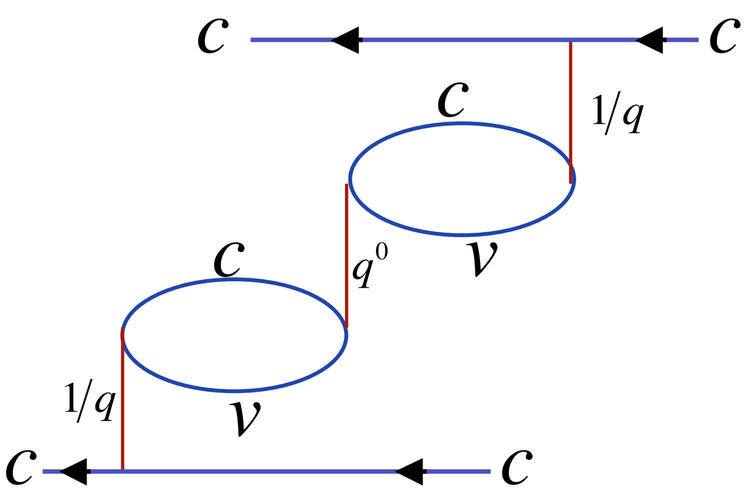
$$\frac{4\pi e^2}{L^3 q^2}$$

$$v_q(n,n) = \sqrt{\frac{4\pi e^2}{L^3 q^2}}$$

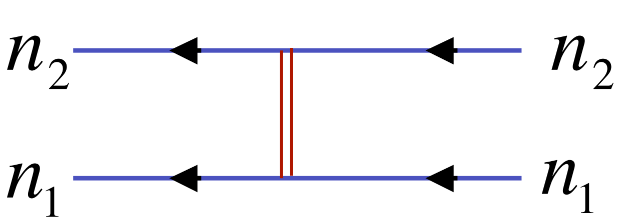
$$v_q(n' \neq n) = O(q^0)$$



$$\frac{\cdots}{q^2}$$



$$\frac{\cdots}{q^2}$$



$$\frac{4\pi e^2}{\varepsilon_{sc} L^3 q^2}$$

Direct scatterings between valence and/or conduction electrons:

repulsive as between free electrons

reduced by the semiconductor dielectric constant repulsion between conduction and valence electrons

→ attraction between electrons and holes

$$a^{+}_{ck_1+q}\,a^{+}_{vk_2-q}\,a_{vk_2}\,a_{ck_1}$$

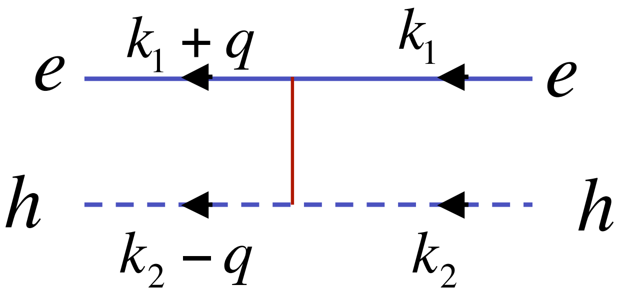
$$-a_{vk_2}\,a^{+}_{vk_2-q}$$

We turn from conduction/valence electrons to electrons/holes

$$a^{+}_{ck} = a^{+}_{k} \qquad a_{vk} = b^{+}_{-k}$$

$$V_{vc} = \sum_{q\neq 0}\frac{-4\pi e^{2}}{\varepsilon_{sc}L^{3}q^{2}}\sum_{k_1k_2} a^{+}_{k_1+q}\,b^{+}_{-k_2}\,b_{-k_2+q}\,a_{k_1}$$

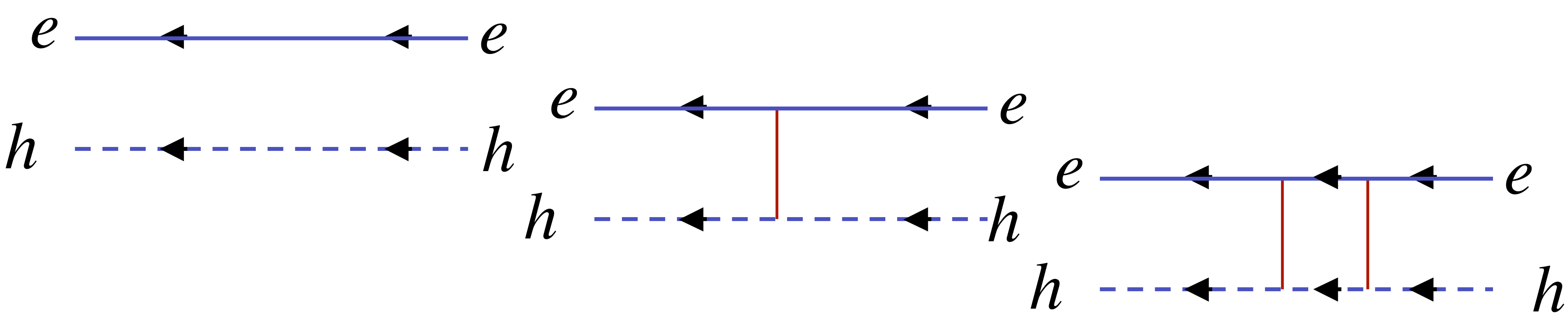
$$-k_2 + q = k_2'$$



$$-\frac{4\pi e^2}{\varepsilon_{sc} L^3 q^2}$$

Attraction between one electron and one hole reduced by dielectric constant

One Wannier exciton



$k_e + k_h = Q$ conserved through Coulomb scatterings

Wannier exciton $\quad |\nu, Q\rangle = \sum |k_e, k_h\rangle \ldots\ldots\ldots$

$\langle k_h, k_e | \nu, Q\rangle$

energy $\quad \varepsilon_\nu + \frac{Q^2}{2(m_e + m_h)}$

wave function $\quad \langle r_e, r_h | \nu, Q\rangle = \frac{e^{iQ.R}}{L^{3/2}} \varphi_\nu(r)$

$$R = \frac{m_e r_e + m_h r_h}{m_e + m_h}$$

$$r = r_e - r_h$$

Exciton creation operator

$$\left|\nu,Q\right\rangle = \sum \left|k_e,k_h\right\rangle\left\langle k_h,k_e\middle|\nu,Q\right\rangle$$

$$B^+_{\nu Q}\left|0\right\rangle \qquad\qquad a^+_{k_e} b^+_{k_h}\left|0\right\rangle$$

$$B^+_{\nu Q} = \sum a^+_{k_e} b^+_{k_h} \left\langle k_h,k_e\middle|\nu,Q\right\rangle$$

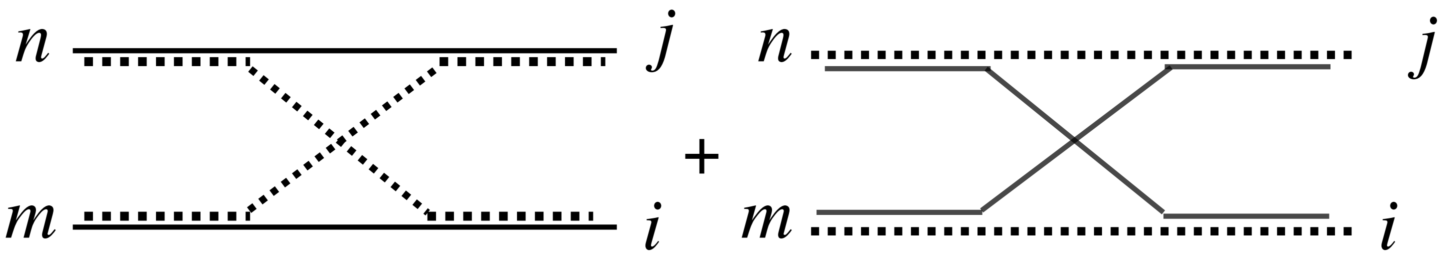
Excitons are boson-like particles

$$i = (\nu_i, Q_i)$$

$$\left[B_j^+, B_i^+\right]_- = B_j^+ B_i^+ - B_i^+ B_j^+ = 0$$ as bosons

$$\left[B_m, B_i^+\right]_- = \delta_{mi} - D_{mi}$$ but not exactly

$$\left[D_{mi}, B_j^+\right]_- = \sum_n \{ \;\ldots\ldots\ldots\ldots\ldots\ldots\; \} B_n^+$$



Being boson-like particles,

excitons, as cold atoms, must undergo

Bose-Einstein condensation

Yet, the precise nature of composite boson condensate

is not known !

close to $(B_0^+)^N |0\rangle + \ldots$

but surely not exactly $(B^+)^N |0\rangle$

Exciton BEC searched for decades ….
but never evidenced !

Reason: the condensate must be dark …
and search has been done through
photoluminescence experiments

conduction band $l = 0$

$s = 1/2$

$$j_e^z = \pm \frac{1}{2}$$

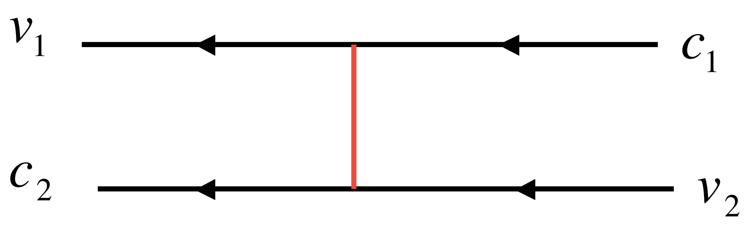
valence band $l = 1$ $\quad l_h^z = \pm 1, 0$

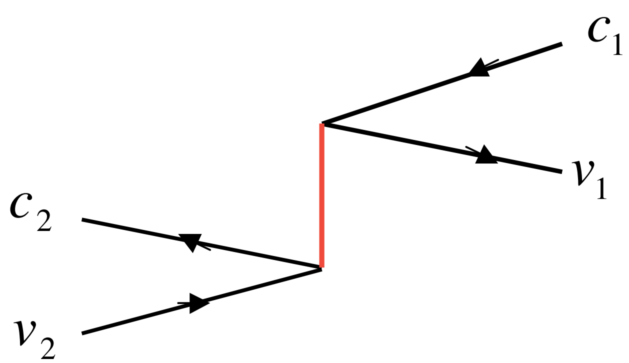
$s = 1/2$ $\quad s_h^z = \pm \frac{1}{2}$

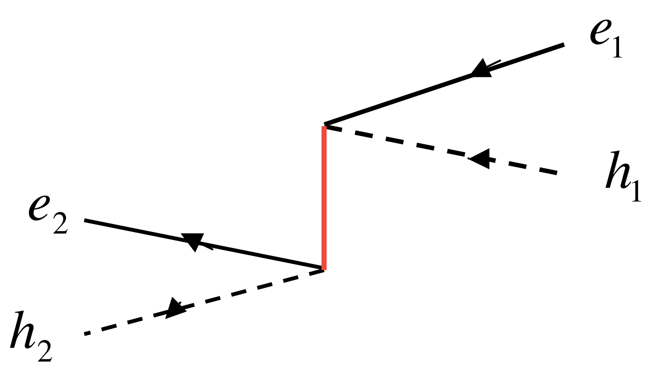
$$j_h^z = \pm \frac{3}{2}, \pm \frac{1}{2}$$

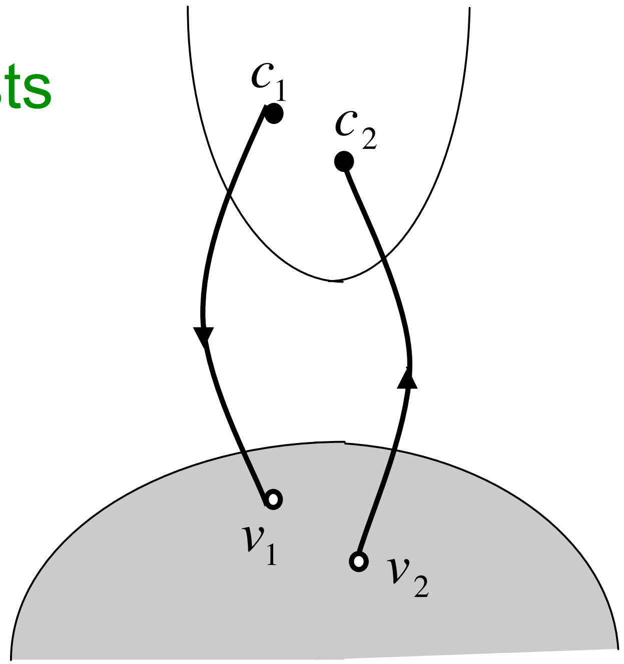
pair $$J_{eh}^z = j_e^z + j_h^z = \pm 2, \pm 1, 0$$

Although small, interband Coulomb interaction also exists

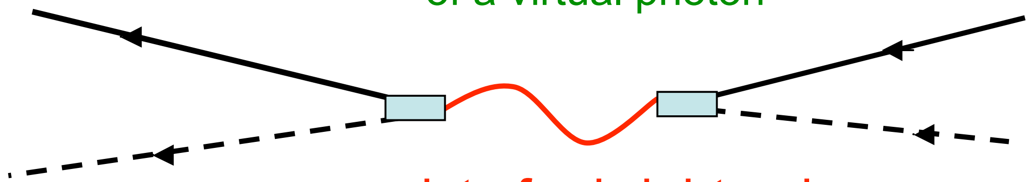








emission and reabsorption of a virtual photon



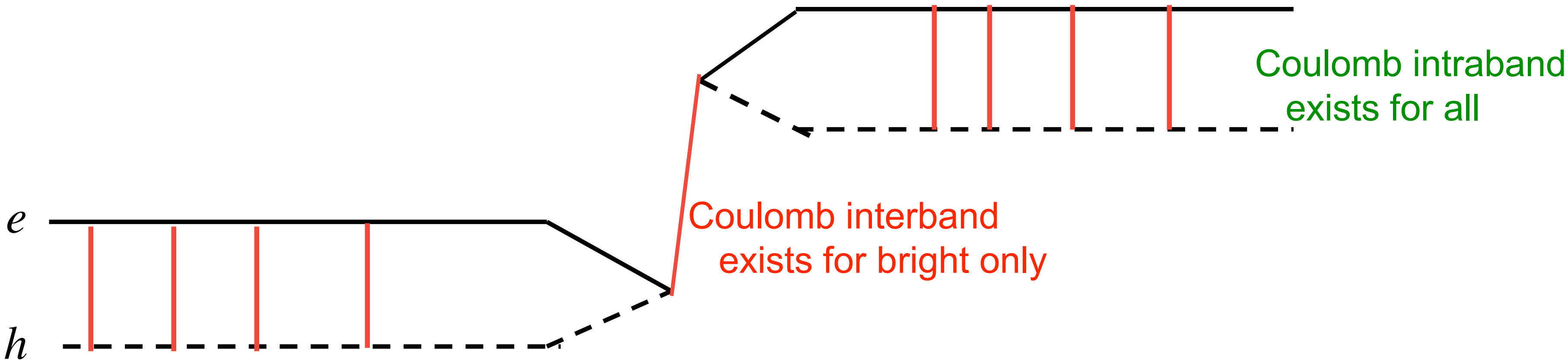
exists for bright pairs
J= (+1, 0, -1) only

$J_{eh}^{z} = \pm 1, 0$ bright coupled to $\sigma_{\pm}, \pi$ photons

$J_{eh}^{z} = \pm 2$ dark

→ uncoupled to photons

→ no valence-conduction Coulomb processes



(repulsive) interband Coulomb processes push
bright exciton above dark exciton

Dark excitons have the lowest energy

just because they are dark !

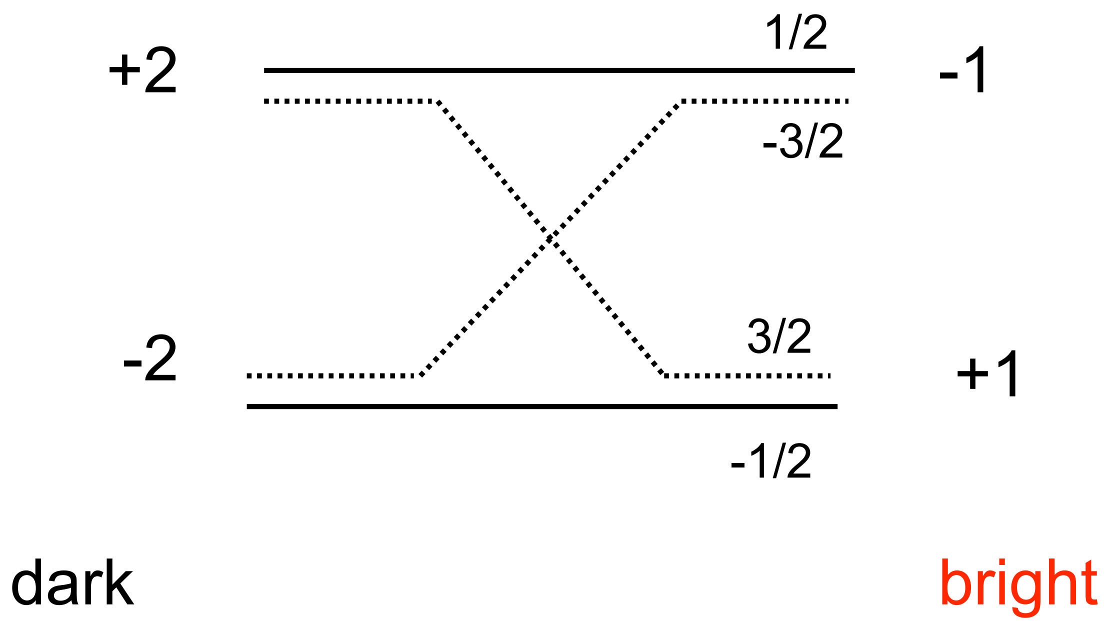
Exciton BEC must be dark …

No hope to see it (directly) with
photoluminescence experiments

Better to know where condensation takes place !

Pairs are created in bright states by photon absorption

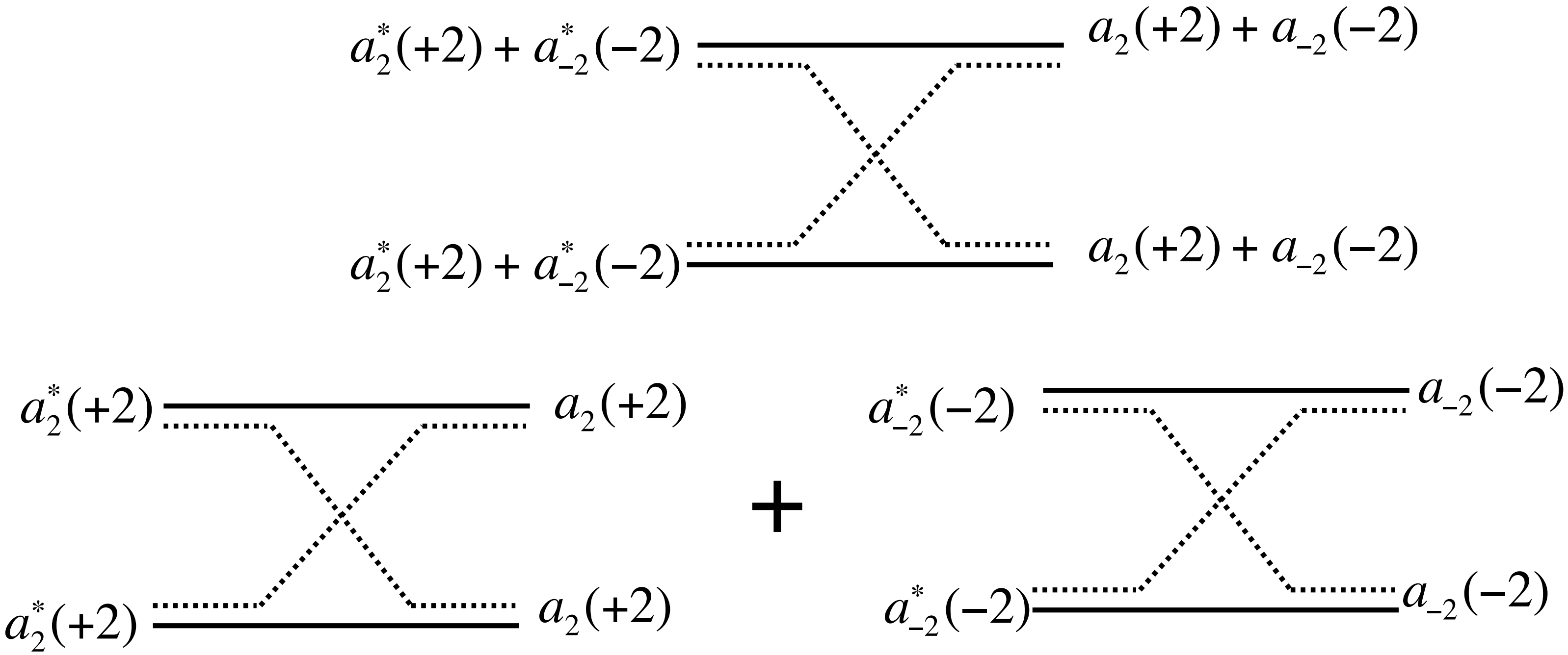
However, dark / bright excitons are linked by carrier exchange



Dark condensate must have a linear polarization

$$\langle H \rangle_N = \frac{\langle 0 | B^N H B^{+N} | 0 \rangle}{\langle 0 | B^N B^{+N} | 0 \rangle}$$

$$B^+ = a_2 B_2^+ + a_{-2} B_{-2}^+$$



$\langle H \rangle_N$ minimum for $|a_2|^2 = |a_{-2}|^2 = 1/2$

linear polarization

So,

in the dilute limit,

electrons and holes

form excitons.

At low T, they undergo a

Bose-Einstein condensation

in a linearly polarized

dark state

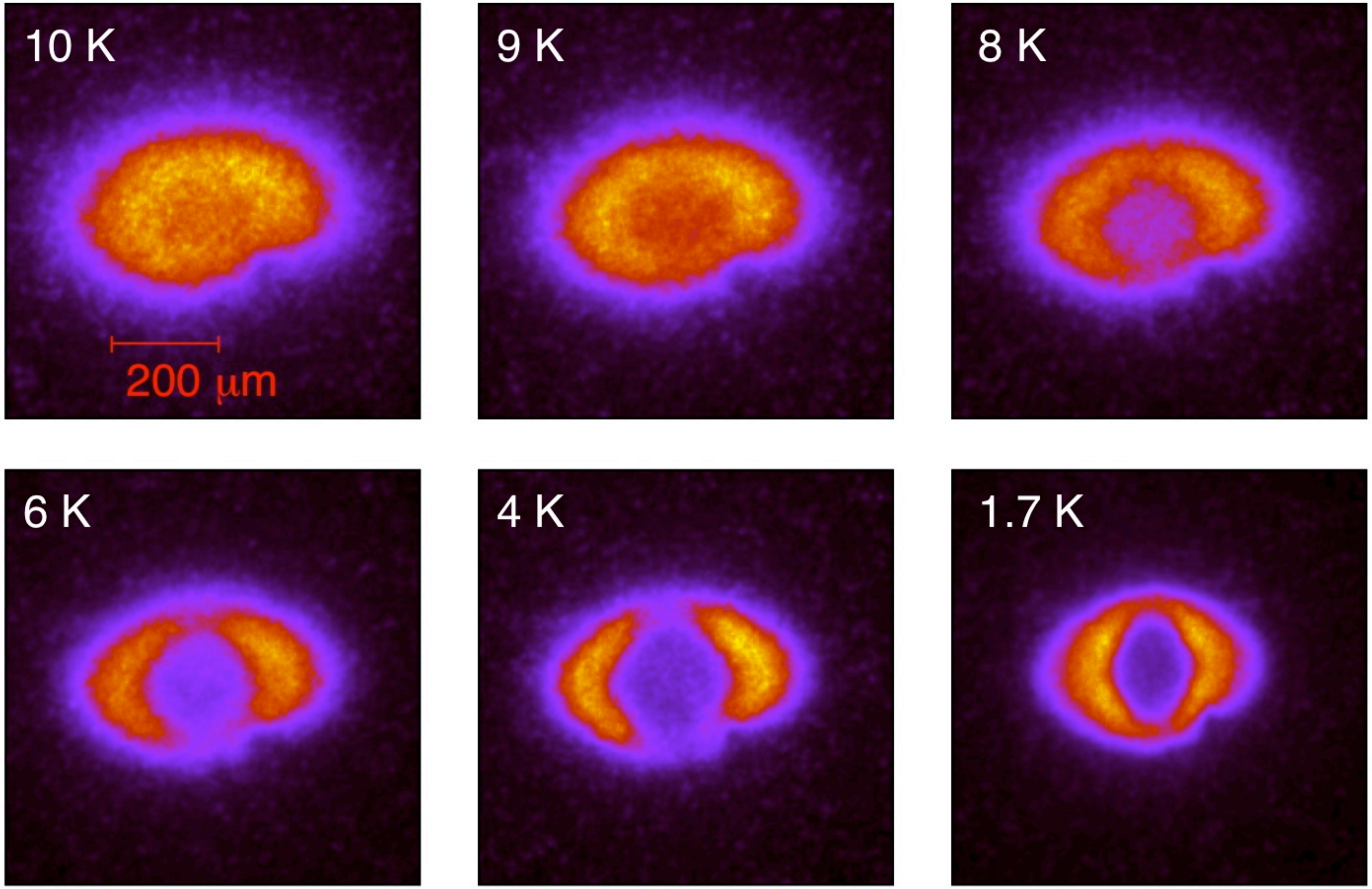
How can we see a dark condensate ?

parabolic trap

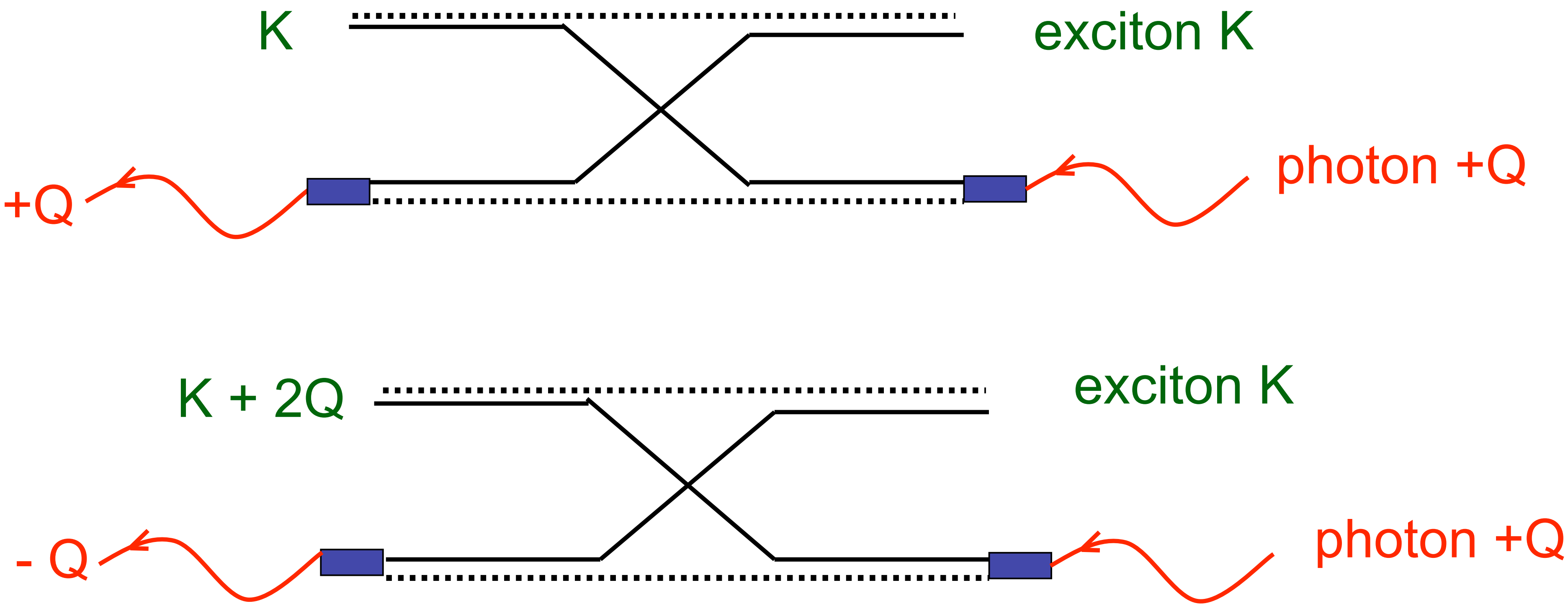
lowest state → dark / lowest trap level

we predict the appearance of a dark spot
at the center of the trap
when BEC takes place

400 μW



Optical trap: standing wave (+Q, -Q)



K exciton becomes superposition of (K, K +2Q, K- 2Q)

so, it is trapped. *Potential depth: a fraction of meV*

Part 2

Under a density increase

(A)

a bright component appears in the condensate

dark excitons D_k^+

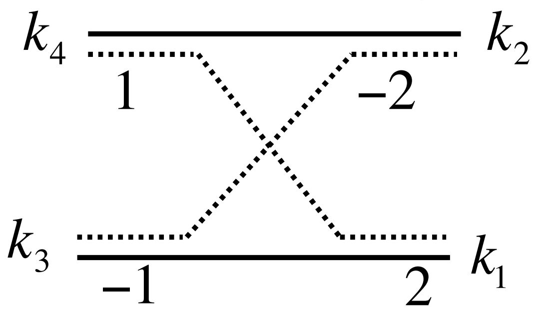
bright excitons B_k^+

$$H_{kin} = \sum \frac{k^2}{2M} D_k^+ D_k + \sum (\varepsilon_0 + \frac{k^2}{2M}) B_k^+ B_k$$

lowest state $(D_0^+)^N |0\rangle$

dark condensate

When the density increases, we must include interactions



the Hamiltonian then reads

$$H = \sum \frac{k^2}{2M} D_k^+ D_k + \sum (\varepsilon_0 + \frac{k^2}{2M}) B_k^+ B_k + g \sum B_{k_4}^+ B_{k_3}^+ D_{k_2} D_{k_1} + h.c.$$

mean field treatment

$$D^+ D \approx N_d \rightarrow D \approx \sqrt{N_d}\, e^{i\varphi_d}$$

$$B^+ B \approx N_b \rightarrow B \approx \sqrt{N_b}\, e^{i\varphi_b}$$

$$H \approx \varepsilon_0 N_b + 2g\, N_b N_d \cos 2(\varphi_b - \varphi_d)$$

minimum $-|g|$

$$H \approx \varepsilon_0 N_b - 2|g| N_b N_d$$

N_d ← $N - N_b$

minimum $\dfrac{\partial H}{\partial N_b} = 0$

$$N_b^{(0)} = \frac{2|g|N - \varepsilon_0}{4|g|}$$

threshold ⟹

$$N_{th} = \frac{\varepsilon_0}{2|g|} \approx \frac{\varepsilon_0}{\xi^{exch}\binom{00}{00}} \approx \frac{\varepsilon_0}{R_X}\left(\frac{L}{a_X}\right)^D$$

$\varepsilon_0 \approx 10 \mu eV$

$m \approx 0.1$ ⟹ $\dfrac{N_{th}}{L^2} \cong 10^9 cm^{-2}$

for $N > N_{th}$ the lowest state $(D_0^+)^{N-N_b^{(0)}} (B_0^+)^{N_b^{(0)}} |0\rangle$

condensate has a bright component

Josephson oscillations

$\varphi = \varphi_b - \varphi_d$ and $\delta N = (N_b - N_d)/2$ are conjugate variables

Hamilton equations
$$\frac{d(\delta N)}{dt} = \frac{\partial H}{\partial \varphi}$$
$$\frac{d\varphi}{dt} = -\frac{\partial H}{\partial(\delta N)}$$

close to equilibrium $\delta N \cong \delta N^{(0)} + ...\cos\omega_J t$

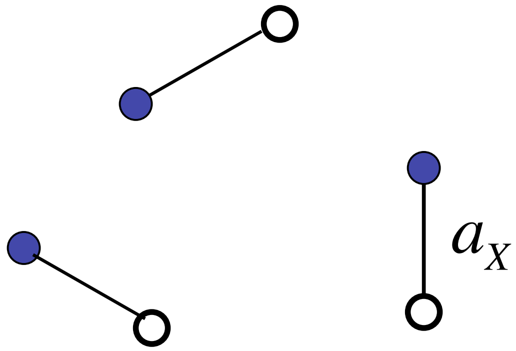
$$\hbar^2\omega_J^2 = 32 g_{bd}^2 N_b^{(0)} N_d^{(0)} = 2\varepsilon_0^2\left(\frac{N^2}{N_{th}^2} - 1\right)$$

$\varepsilon_0 \approx 10\mu eV$ ⟹ $\omega_J \approx 10^{10} N/N_{th}$

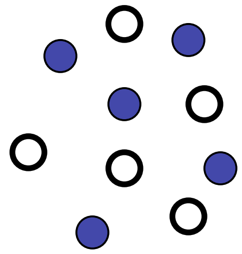
(B)

a phase separation takes place between exciton gas and electron-hole plasma

Mott dissociation



Exciton gas



Electron-hole plasma

$$\eta = N\left(\frac{a_X}{L}\right)^D$$

Excitons dissociate into an electron-hole plasma for

$$\eta \approx 1$$

dilute exciton gas

$$\eta = N\left(\frac{a_X}{L}\right)^D$$

$$E_N = NR_X\left[-1 + (\ldots)\eta + \ldots\right] = N\varepsilon_X(\eta)$$

positive to avoid collapse

dense electron-hole plasma

$$E_N = N\left[(\ldots)\frac{k_F^2}{2m} - (\ldots)e^2 k_F + \ldots\right] = N\varepsilon_{eh}(\eta)$$

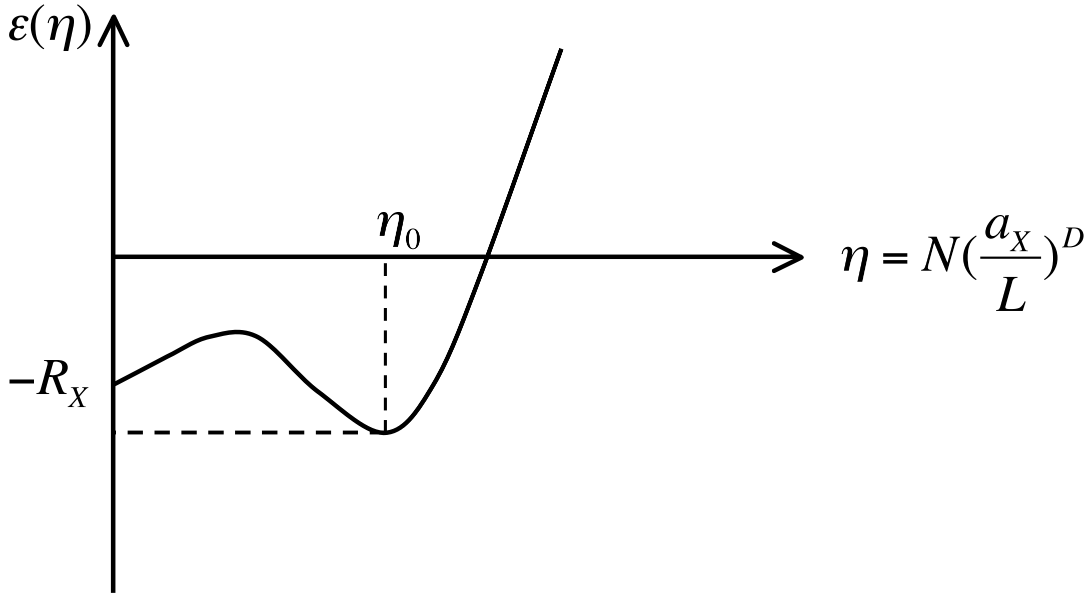
$$N \approx (k_F L)^D$$

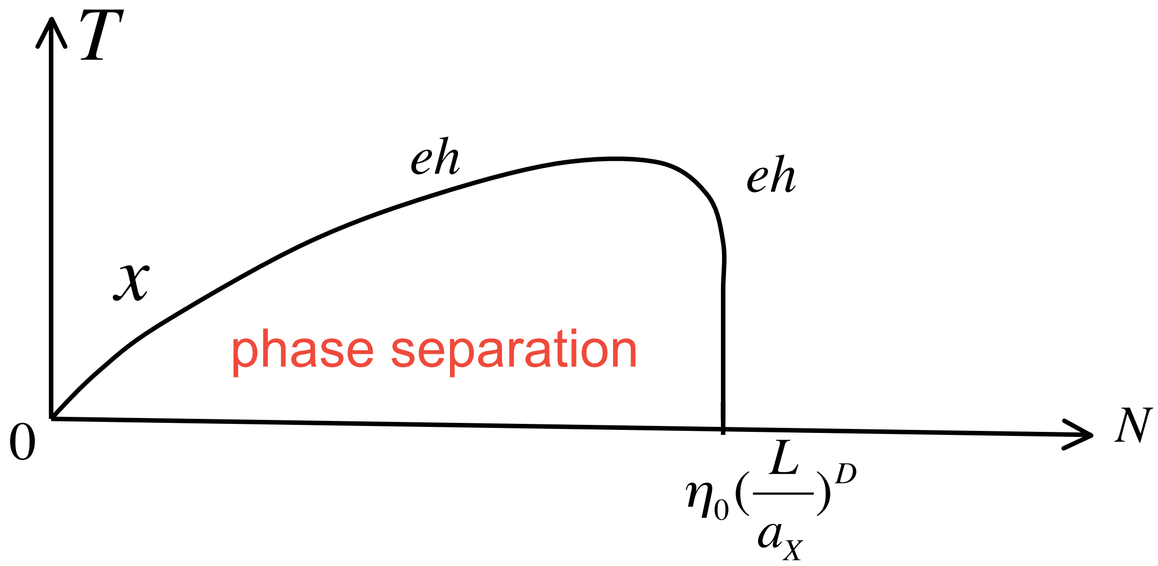
$$\varepsilon_{eh}(\eta) \approx \alpha n^{2/3} - \beta n^{1/3} \quad \text{in 3D}$$

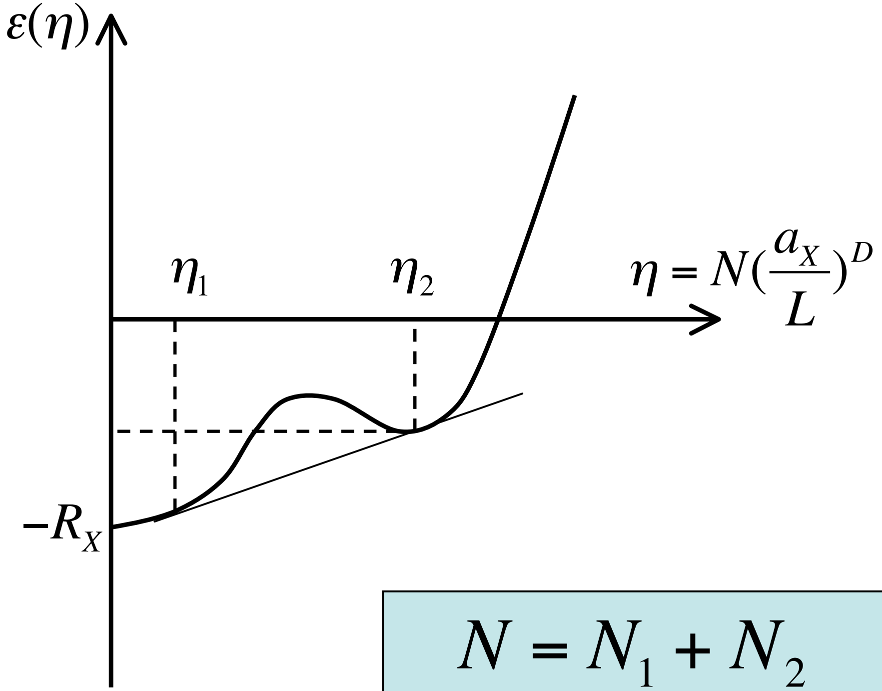
$$\approx \alpha n - \beta n^{1/2} \quad \text{in 2D}$$

the average pair energy has a minimum in the dense limit

« electron-hole droplets »
in Ge and Si







$$N = N_1 + N_2$$

$$E_N = N_1\varepsilon(\eta_1) + N_2\varepsilon(\eta_2)$$

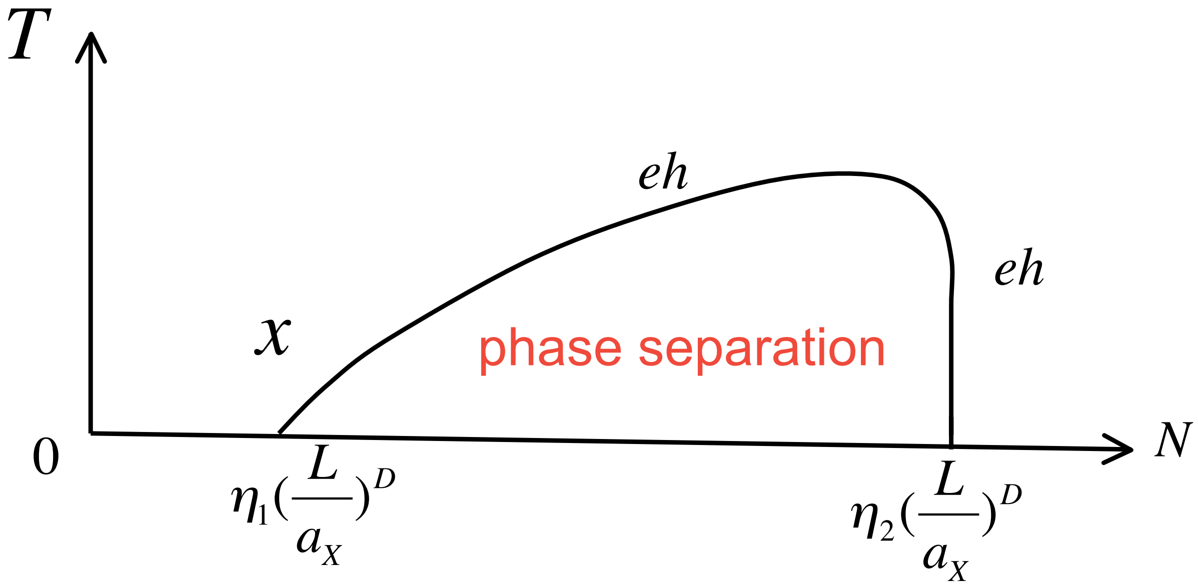
$$dN_1 = -dN_2$$

$$0 = \frac{dE_N}{dN_1} = \varepsilon(\eta_1) + \eta_1\varepsilon'(\eta_1) - \varepsilon(\eta_2) - \eta_2\varepsilon'(\eta_2)$$

for $\varepsilon'(\eta_1) = \varepsilon'(\eta_2)$

$$\frac{\varepsilon(\eta_2) - \varepsilon(\eta_1)}{\eta_2 - \eta_1} = \varepsilon'(\eta_1) = \varepsilon'(\eta_2)$$

⇨ Phase separation along double tangente



T
eh
eh
x
phase separation
0
N
$\eta_1(\frac{L}{a_X})^D$
$\eta_2(\frac{L}{a_X})^D$

(C)

a BCS condensation

occurs in the

dense electron-hole plasma

Even if strongly screened

electrons and holes still attract each other

$$\sum ......a_k^+ b_{-k}^+ = B_{Cooper}^+$$

Formation of « electron-hole Cooper pairs »

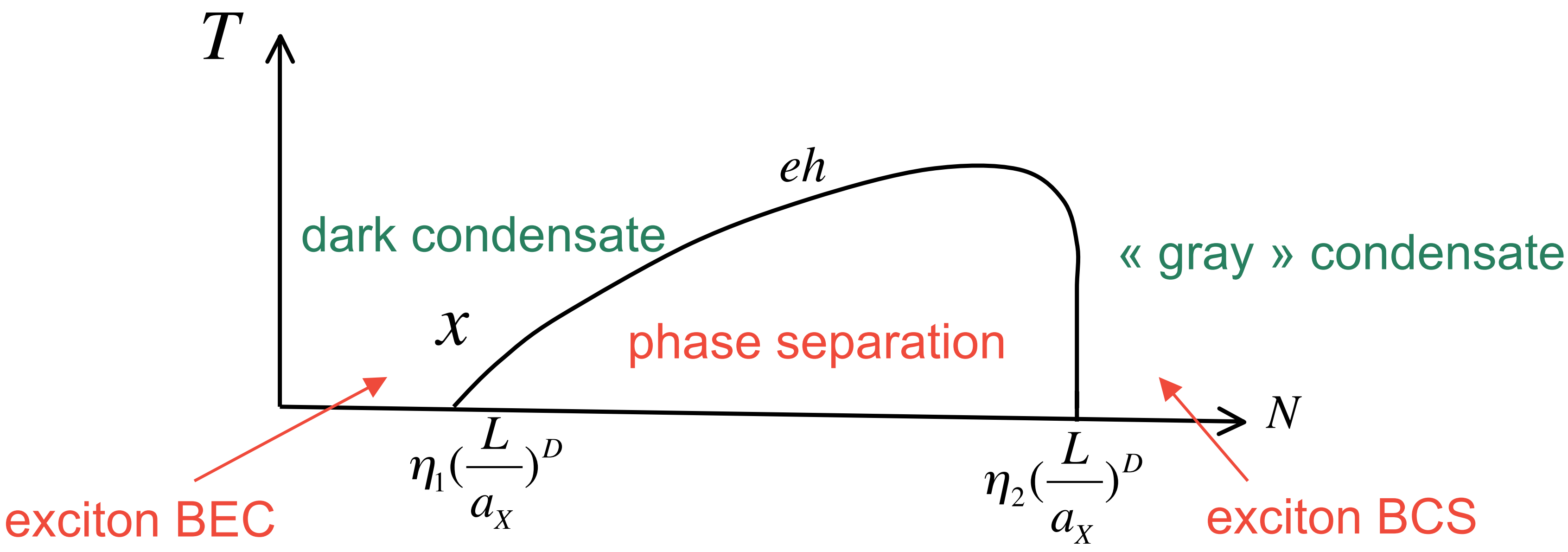
neutral, so not superconducting
but still superfluid

Again, dark pairs have the lowest energy

Exchange coupling must however bring
a bright component to the condensate since it is dense

Moreover,
degeneracy between the two dark and the two bright pairs
must lead to a linear polarisation
as optimum state

$$(B_2^+ + B_{-2}^+)^{N-N_1^{(0)}} (B_1^+ + B_{-1}^+)^{N_1^{(0)}} \left|0\right\rangle$$



T
eh
dark condensate
« gray » condensate
x
phase separation
N
$\eta_1(\frac{L}{a_X})^D$
$\eta_2(\frac{L}{a_X})^D$
exciton BEC
exciton BCS

So,

under a density increase,

-a bright component appears in the condensate.

-a phase separation occurs between exciton gas and e-h plasma

-a BCS condensation of excitonic Cooper pairs takes place in the e-h plasma

Conclusion

Electron-hole systems are quite rich !

1) At low density, excitons are formed.
They suffer BEC condensation into a dark state
with linear polarisation

2) Under a density increase,
- a bright component appears in the condensate
- a phase separation occurs between
exiton gas and electron-hole plasma
- a BCS condensation of excitonic Cooper pairs takes
place in the dense plasma

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