Shortcuts To Adiabaticity: Theory and Application

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eman ta zabal zazu

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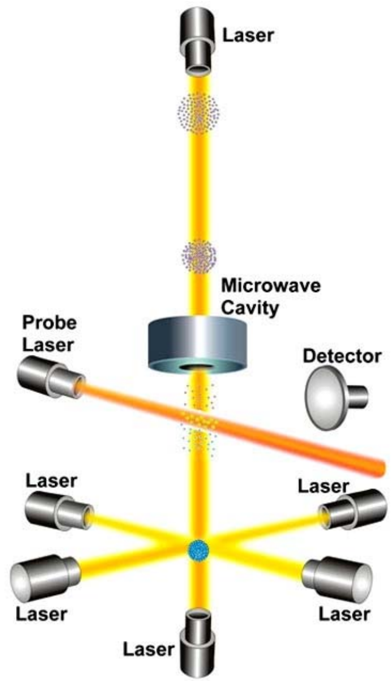
Outline

- Introduction
- Techniques of shortcuts to adiabaticity
- Applications (1) trap expansion

  (2) atomic transport

  * (3) population transfer
- Other results

Motivation- accelerate adiabatic process

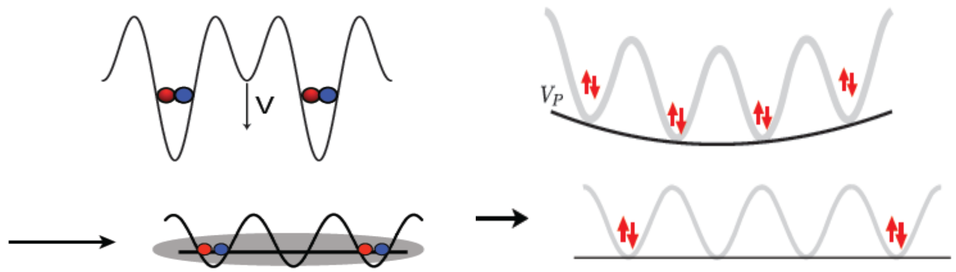


(1) Adiabatic adjustment for cold atoms
(expansions, contractions, rotation, transport)

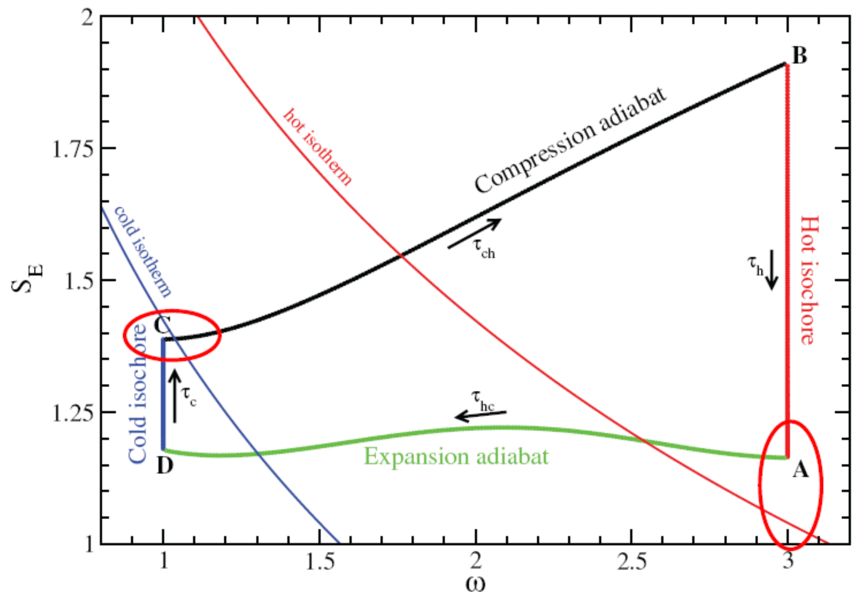
i. e. Prepare atoms on a lattice
Reach very low T
Reduce Dv in spectroscopy& metrology

(2) Bottleneck step in a "quantum refrigerator cycle"

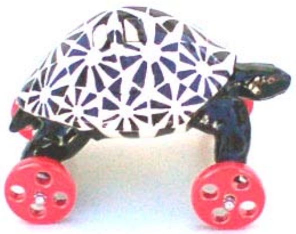
Y. Rezek, et al., EPL 85, 30008 (2009).



optical lattice
(for example, adiabatic ramping schemes)



Techniques of Shortcuts to Adiabaticity



Invariant-based inverse engineering

J. G. Muga et al. J. Phys. B: At. Mol. Opt. Phys. 42, 241001 (2009).

X. Chen et al. Phys. Rev. Lett. 104, 063002 (2010).

Trasitionless quantum driving (counter-diabatic protocols)

M. Demirplak and S. A. Rice, J. Phys. Chem. A 107, 9937 (2003).

M. V. Berry, J. Phys. A 42, 365303 (2009).

X. Chen et al. Phys. Rev. Lett. 105, 123003 (2010).

Fast-forward (FF) scaling approach

S. Masuda and K. Nakamura, Proc. R. Soc. A 466, 1135 (2010).

Optimal control theory etc. ….

Inverse engineering

general idea: design the dynamics first
then deduce Hamiltonian H(t)

$$U = \sum_n e^{i\alpha_n(t)} |\phi_n(t)\rangle\langle\phi_n(0)| \quad \Longrightarrow \quad H(t) = i\hbar(\partial_t U)U^\dagger$$

Relation: **X. Chen et al. Phys. Rev. A 83, 062116 (2011)**

two routes: (a) dynamical invariants $I(t)|\phi_n(t)\rangle = \lambda_n|\phi_n(t)\rangle$

(b) transitionless tracking $\xi_n(t) = \alpha_n(t) \qquad |n_0(t)\rangle = |\phi_n(t)\rangle$

if we choose
$$\xi_n(t) = -\frac{1}{\hbar}\int_0^t dt' E_n^{(0)}(t') + i\int_0^t dt' \langle n_0(t')|\partial_{t'} n_0(t')\rangle.$$

counter-diabatic term

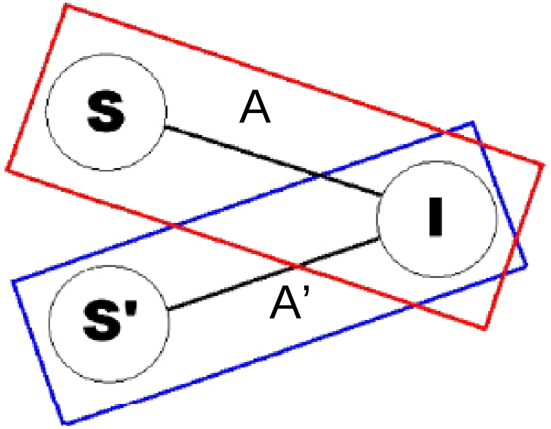
$$H(t) = H_0(t) + H_1(t),$$

M. V. Berry, J. Phys. A 42, 365303 (2009)

$$H_1(t) = i\hbar\sum_n \left(|\partial_t n_0(t)\rangle\langle n_0(t)| - \langle n_0(t)|\partial_t n_0(t)\rangle |n_0(t)\rangle\langle n_0(t)| \right),$$

Third route: related to FF approach in Torrontegui et al. PRA 86 (2012)

multiple Schödinger pictures



$$|\psi_S\rangle = A|\psi_I\rangle$$

$$\begin{aligned} H &= AH_IA^\dagger + K \\ K &= i\hbar\dot{A}A^\dagger \end{aligned}$$

$$|\psi'_S\rangle = A'|\psi_I\rangle$$

$$\begin{aligned} H' &= A'H_IA'^\dagger + K' \\ K' &= i\hbar\dot{A}'A'^\dagger \end{aligned}$$

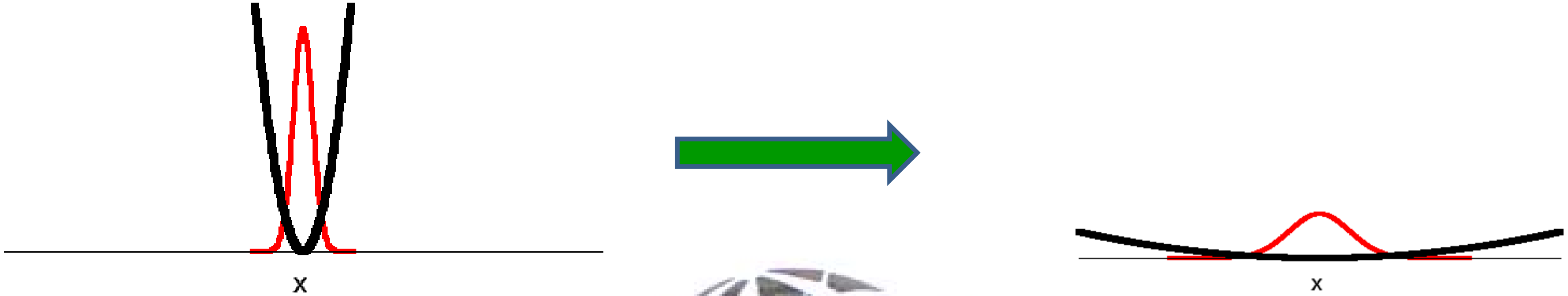
Application in shortcuts to adiabaiticty

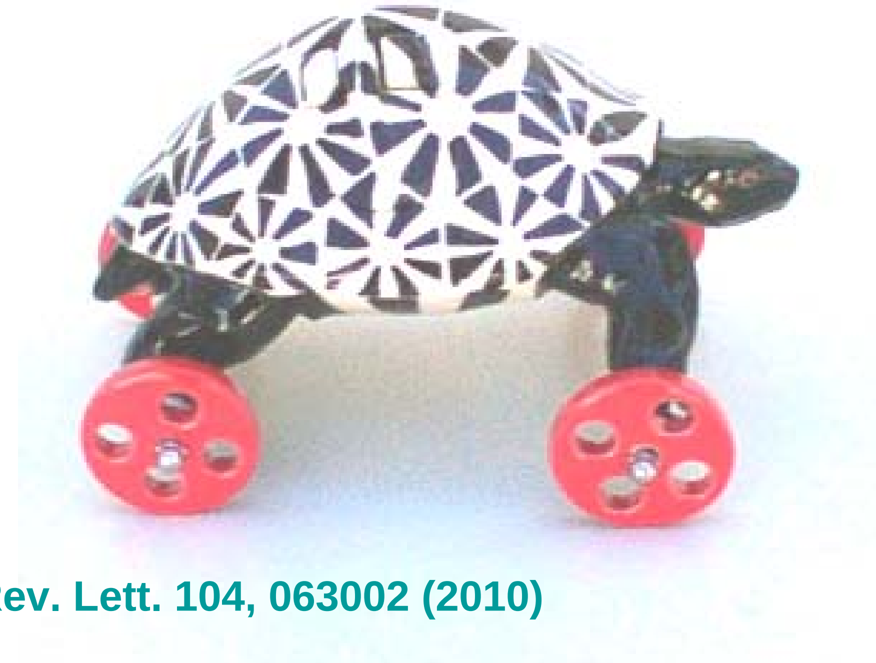
- A simple case is A'=1
- Then the IP is directly interpreted as an alternative physical reality
- If A(0)=A(t_f)=1 initial and final states for S & S' are equal
- Observables that commute with A are also equal for S & S'

to overcome the difficulties to implement "counterdiabatic" terms

S. Ibáñez et al. Phys. Rev. Lett.109, 100403 (2012)

1: fast trap expansions



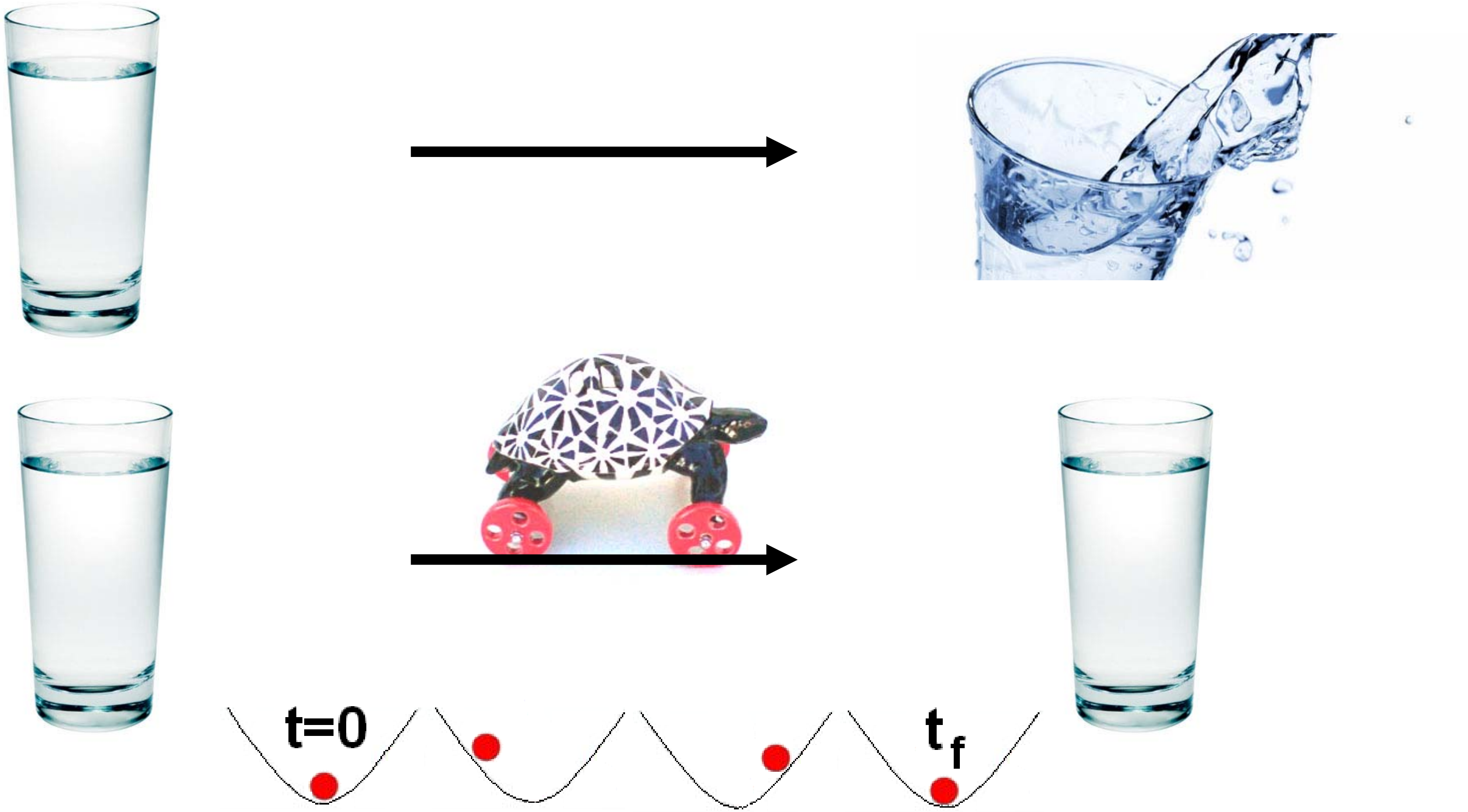


X. Chen et al. Phys. Rev. Lett. 104, 063002 (2010)

BECs J. G. Muga et al. J. Phys. B: At. Mol. Opt. Phys. 42, 241001 (2009)

Experiments：J. F. Schaff et al. Phys. Rev. A 82, 033430 (2010)
Europhys. Lett. 93, 23 001 (2011).

2: fast atomic transport



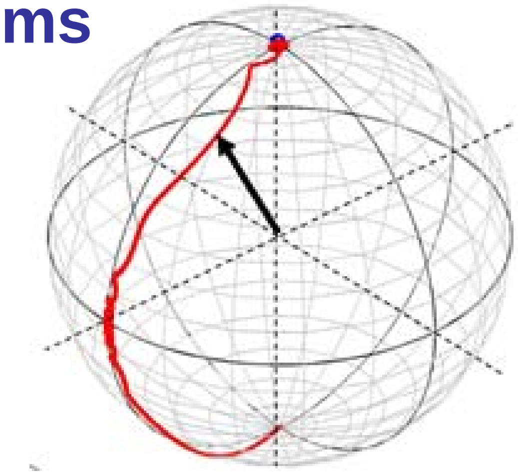
E. Torrontegui et al, Phys. Rev. A 83, 013415 (2011)

BECS: Torrontegui et al, New J. Phys. 14, 013031 (2012)

Optimal Control: X. Chen et al. Phys. Rev. A 84, 043415 (2011)

3: population transfer

Population Transfer in 2- and 3-level systems



Two major routes to control internal state

- **resonant pulses (pi, pi/2, etc…)**
  fast but unstable
- **adiabatic passage methods (RAP, STIRAP..)**
  robust but slow

In general we would like *fast & robust* processes

- Composite pulses and Optimal control

Shortcuts To Adiabaticity:

X. Chen et al., Phys. Rev. Lett. 105, 123003 (2010)

3-level: X. Chen and J. G. Muga, Phys. Rev. A 86, 033405 (2012)

(I) Transitionless quantum driving

X. Chen et al. Phys. Rev. Lett. 105, 123003 (2010)

reference Hamiltonian

$$H_0^I(t) = \frac{\hbar}{2}\begin{pmatrix} -\Delta(t) & \Omega_R(t) \\ \Omega_R(t) & \Delta(t) \end{pmatrix}$$

- with same frequency as H0
- orthogonal polarization
- time-dependent (shaped) intensity

counter-diabatic term

$$H_1(t) = i\hbar \sum_n |\partial_t \lambda_n\rangle\langle\lambda_n| \quad \Longrightarrow \quad H_1^I(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & -i\Omega_a \\ i\Omega_a & 0 \end{pmatrix}$$

where $\Omega_a \equiv \dot{\theta} = (\Omega_R \dot{\Delta} - \dot{\Omega}_R \Delta)/\Omega^2.$ $\quad \Omega(t) = \sqrt{\Delta^2 + \Omega_R^2}.$

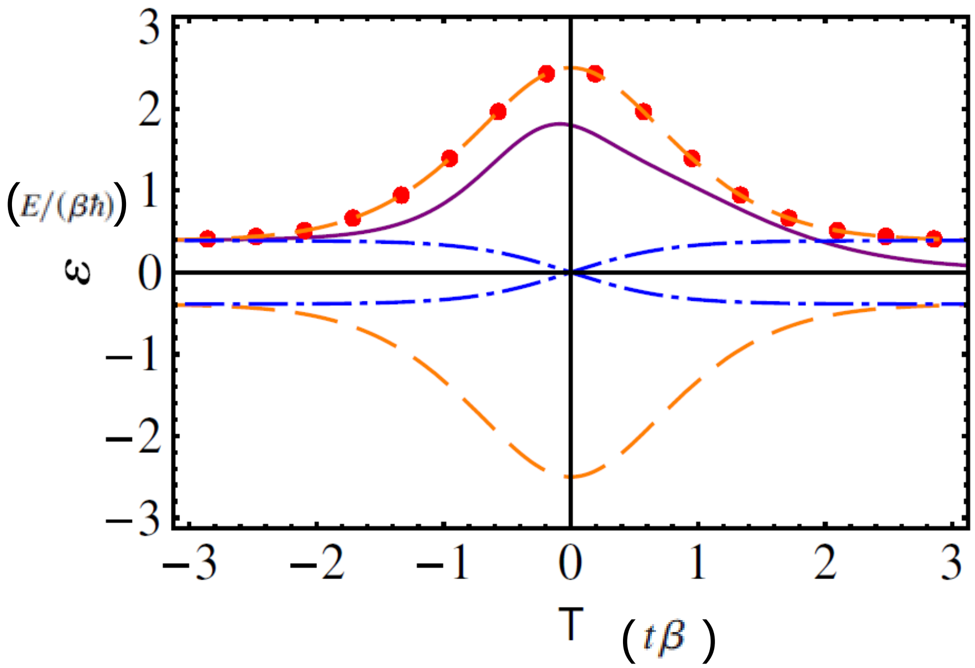
We will use *$H_1(t)$* to make Rapid Adiabatic Passage (RAP) *really* RAPID

As far as populations are concerned (the adiabatic phase is not needed), we can

$H_1(t)$ (substitute $H_0(t)$) or $H_0(t)$ + $H_1(t)$ (supplement $H_0(t)$)

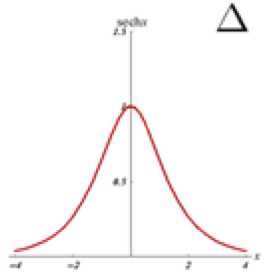
transfer populations in 2- level systems

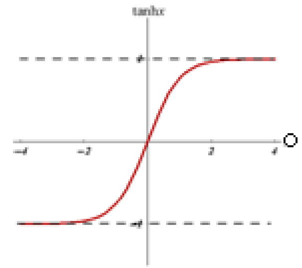
RAP is a standard robust way to invert the population



Allen-Eberly (AE) scheme
more adiabatic than LZ scheme

$$\Delta = (2\beta^2 t_0/\pi)\tanh(\pi t/2t_0)$$





$$\Omega_R = \Omega_0\,\mathrm{sech}(\pi t/2t_0)$$

$$\tau = t_0\beta \text{ and } \omega = \Omega_0/\beta$$

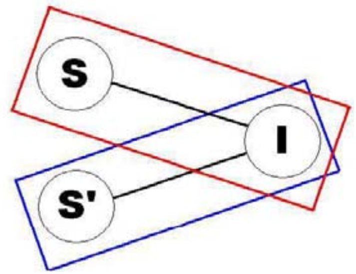
$$|\psi^I_{\pm}(t)\rangle = \exp\{-\tfrac{i}{\hbar}\int_{t_i}^{t} dt' E_{\pm}(t')\}|\lambda_{\pm}(t)\rangle$$

Parameters: $\omega = 5$ and $\tau = 1.22$.

adiabatic approximation $\frac{1}{2}|\Omega_a| \ll |\Omega(t)|$

which is non-adiabatic

(II) Multiple Schödinger pictures



$$H_0 + H_{cd}^{(0)} = \begin{pmatrix} Z_0 & X_0 - i\hbar\dot{\Theta}_0/2 \\ X_0 + i\hbar\dot{\Theta}_0/2 & -Z_0 \end{pmatrix} = \begin{pmatrix} Z_0 & Pe^{-i\phi} \\ Pe^{i\phi} & -Z_0 \end{pmatrix}$$

where $H_{cd}^{(0)} = \hbar(\dot{\Theta}_0/2)\sigma_y$ **counter-diabatic term**

$$P = [X_0^2 + (\hbar\dot{\Theta}_0/2)^2]^{1/2} \qquad \phi = \arctan(\hbar\dot{\Theta}_0/2X_0)$$

Define new IP based on

$$A_z = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} \qquad K_z = i\hbar\dot{A}_z A_z^\dagger$$

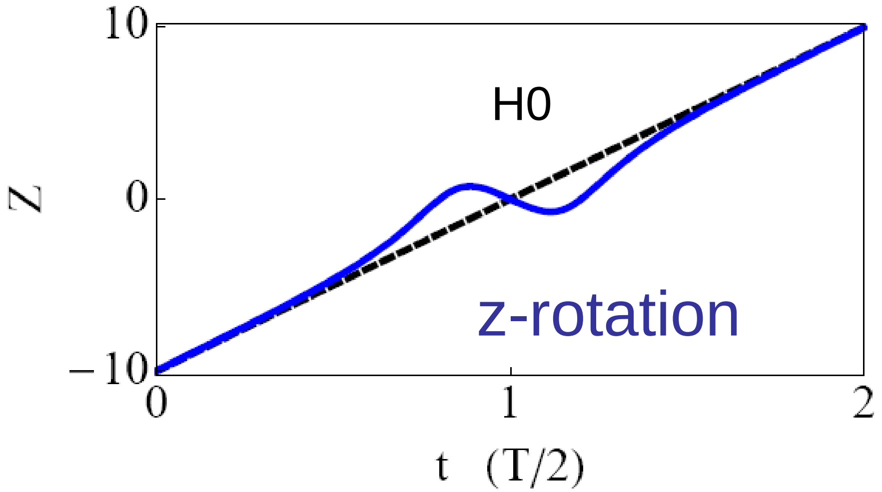
Z-rotation

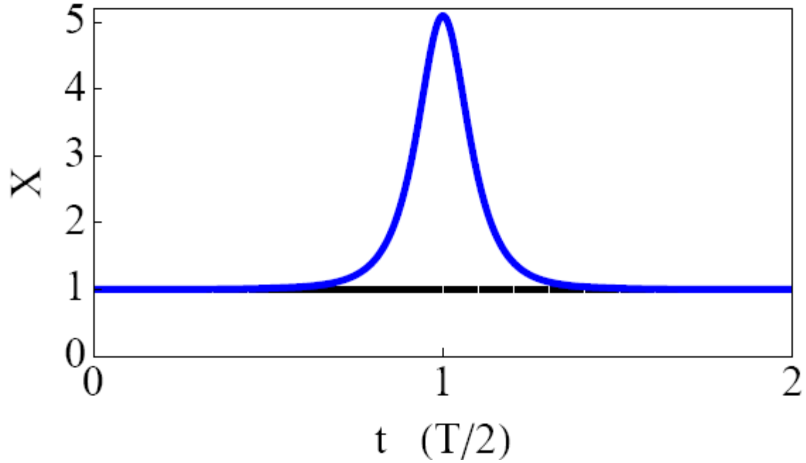
$$A_z^\dagger(H_0 + H_{cd}^{(0)} - K_z)A_z = \begin{pmatrix} Z_0 - \hbar\dot{\phi}/2 & P \\ P & -Z_0 + \hbar\dot{\phi}/2 \end{pmatrix}$$

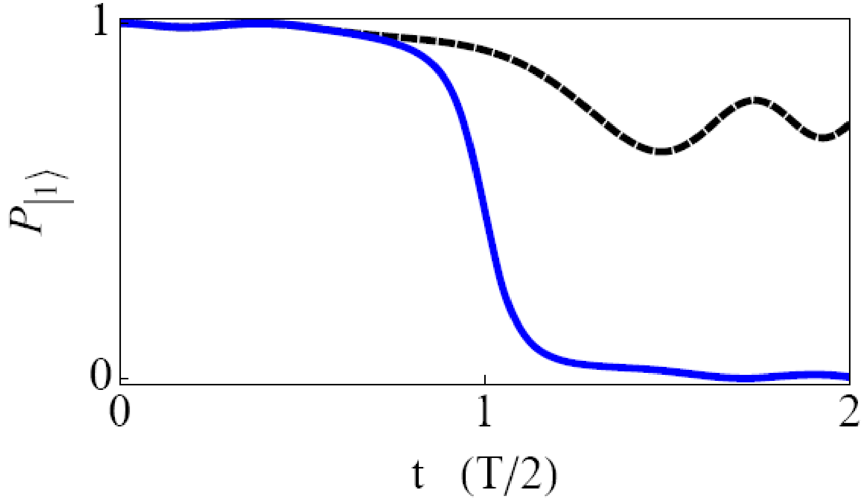
Reinterpret as a SP (A'=1)

S. Ibáñez et al. Phys. Rev. Lett.109, 100403 (2012)

example and experiment Landau–Zener Scheme



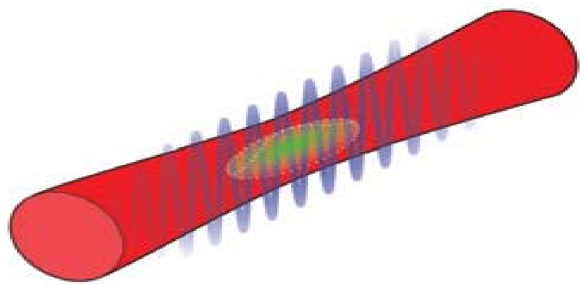




nature physics ARTICLES
PUBLISHED ONLINE: 18 DECEMBER 2011 | DOI:10.1038/NPHYS2170

High-fidelity quantum driving

Mark G. Bason[1], Matthieu Viteau[1], Nicola Malossi[2], Paul Huillery[1,3], Ennio Arimondo[1,2,4], Donatella Ciampini[1,2,4], Rosario Fazio[5], Vittorio Giovannetti[5], Riccardo Mannella[4] and Oliver Morsch[1]*



…use z-rotation
X controlled by trap depth
Z by lattice acceleration

(III) invariant-based inverse engineering

$$H(t) = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega_R \\ \Omega_R & -\Delta \end{pmatrix}$$

the Rabi frequency and detuning can be constructed by two parameters:

$$i\hbar\frac{\partial I(t)}{\partial t} - [H(t), I(t)] = 0,$$

$$\Omega_R = \dot{\gamma} / \sin\beta,$$

$$\Delta = \Omega_R \cot\gamma \cos\beta - \dot{\beta}.$$

which results from the invariant

X. Chen et al. Phys. Rev. A 83, 062116 (2011)

$$I(t) = \frac{\hbar}{2}\Omega_0 \begin{pmatrix} \cos\gamma & \sin\gamma e^{i\beta} \\ \sin\gamma e^{-i\beta} & -\cos\gamma \end{pmatrix}$$

$$|\phi_+(t)\rangle = \cos\left(\frac{\gamma}{2}\right)e^{i\beta}|2\rangle + \sin\left(\frac{\gamma}{2}\right)|1\rangle,$$

$$|\phi_-(t)\rangle = \sin\left(\frac{\gamma}{2}\right)|2\rangle - \cos\left(\frac{\gamma}{2}\right)e^{-i\beta}|1\rangle,$$

design the trajectory

stability amplitude noise and systematic errors

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -\frac{\mathrm{i}}{\hbar}[H_0 + \beta H_1, \rho] - \frac{\lambda^2}{2\hbar^2}[H_2, [H_2, \rho]]$$

amplitude noise

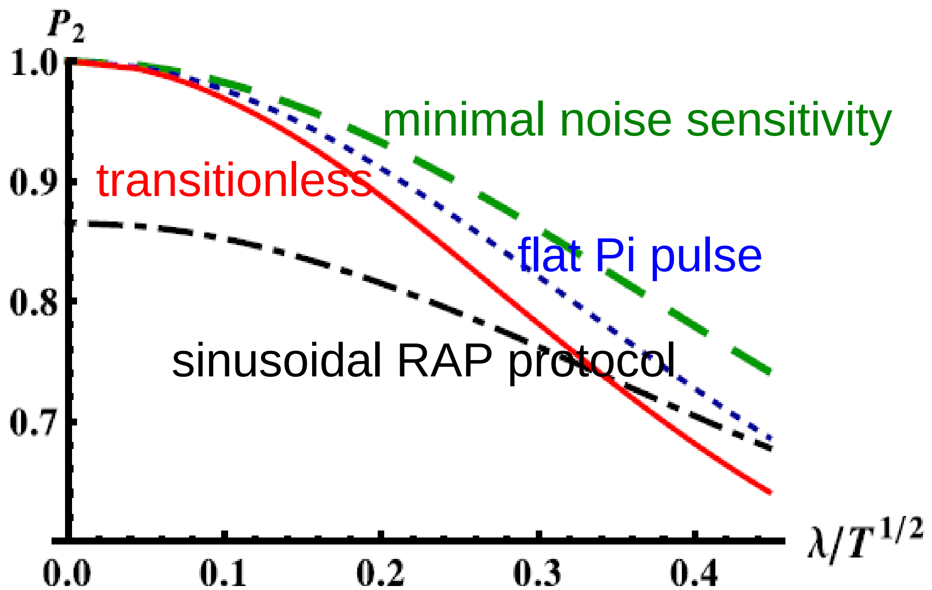
systematic errors

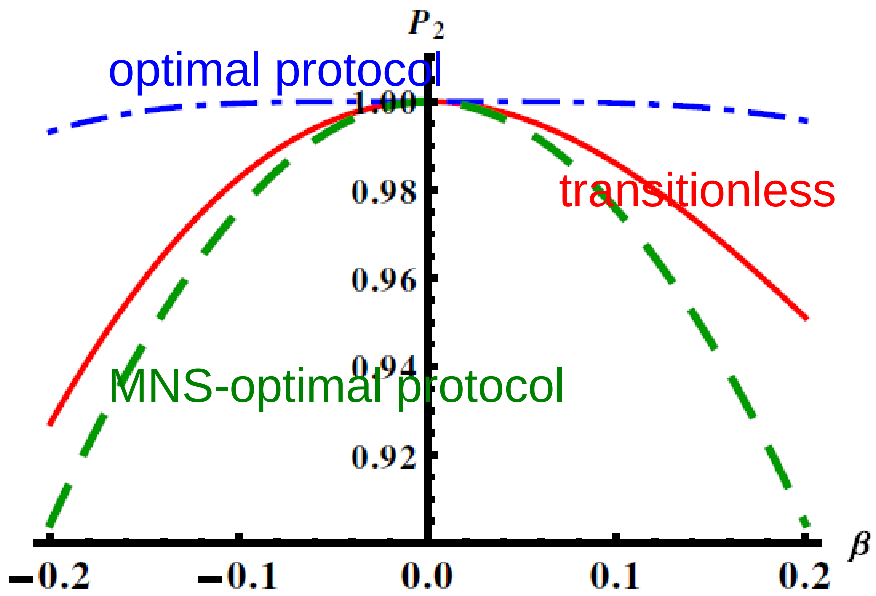
noise sensitivity:

$$q_N := -\frac{1}{2}\left.\frac{\partial^2 P_2}{\partial \lambda^2}\right|_{\lambda=0} = -\left.\frac{\partial P_2}{\partial(\lambda^2)}\right|_{\lambda=0}$$

systematic error sensitivity

$$q_S := -\frac{1}{2}\left.\frac{\partial^2 P_2}{\partial \beta^2}\right|_{\beta=0} = -\left.\frac{\partial P_2}{\partial(\beta^2)}\right|_{\beta=0}$$





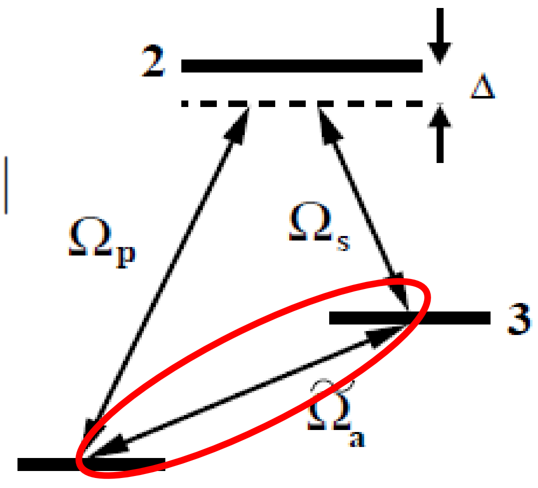
A Ruschhaupt et al., New J. Phys. 14 093040 (2012)

Stimulated Raman Adiabatic Passage (STIRAP)

(I) Transitionless quantum driving

Reference Hamiltonian H_0 (two-photon resonance)

$$H_0(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & 2\Delta_p & \Omega_s(t) \\ 0 & \Omega_s(t) & 0 \end{pmatrix}$$



Couterdiabatic term H_1 $H_1(t) = i\hbar \sum_n |\partial_t n\rangle\langle n|$

M. V. Berry, J. Phys. A 42, 365303 (2009)

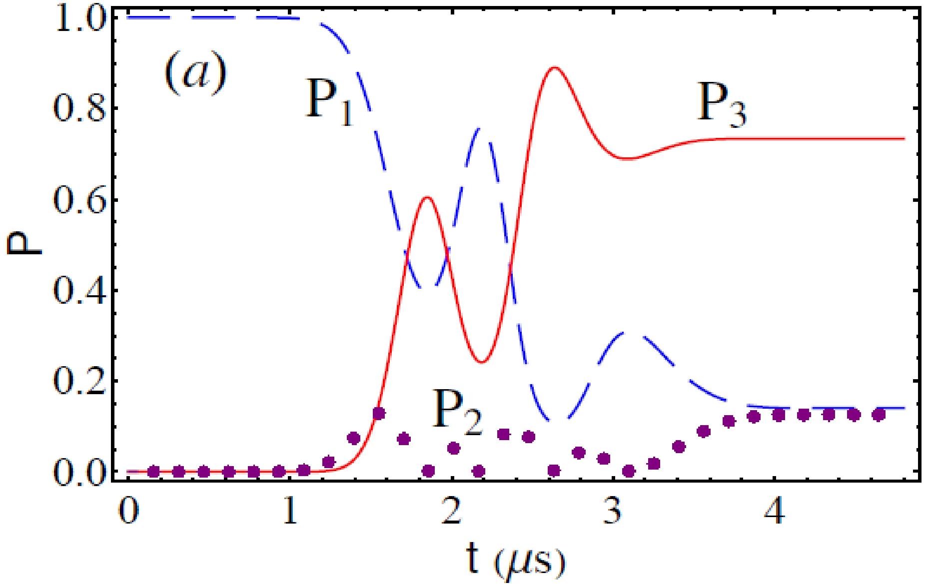
X. Chen et al. Phys. Rev. Lett. 105, 123003 (2010)

|1〉 → |3〉 H_1 is simplied as

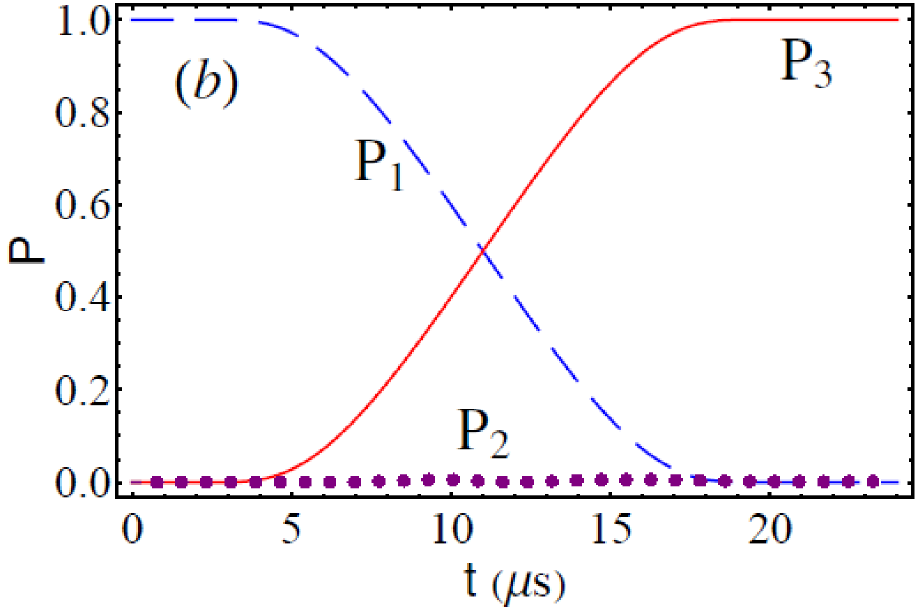
$$\tilde{H}_1(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & i\tilde{\Omega}_a \\ 0 & 0 & 0 \\ -i\tilde{\Omega}_a & 0 & 0 \end{pmatrix}$$

where $\tan\theta = \Omega_p(t)/\Omega_s(t)$ $\tan(2\phi) = \Omega/\Delta_p$ $\tilde{\Omega}_a = 2\dot{\theta}$

H_0



$H_0 + H_1$ OR H_1



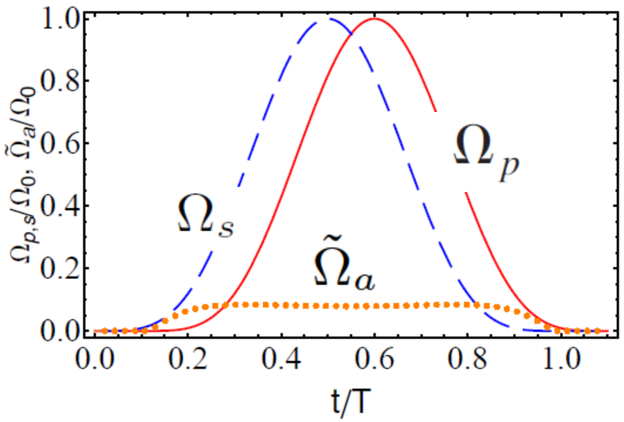
Parameters: $\Omega_0 = 2\pi \times 2\ \mathrm{MHz} \quad \Delta_p = 2\pi \times 0.1\ \mathrm{MHz}$

$\mathrm{T} = 4\ \mu s \qquad \tau = 0.1\mathrm{T}$

Here weak magnetic diople transition may be used !

$\tilde{\Omega}_{max}/2\pi \ = \ 159\ \ \mathrm{kHz}$ OK !

BUT this limits the ability to shorten the times



(II) Invariant-based inverse engineering

we consider the simplest case of one-photon resonance

$$H(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & 0 & \Omega_s(t) \\ 0 & \Omega_s(t) & 0 \end{pmatrix} = H(t) = \frac{\hbar}{2}(\Omega_p(t)\hat{K}_1 + \Omega_s(t)\hat{K}_2)$$

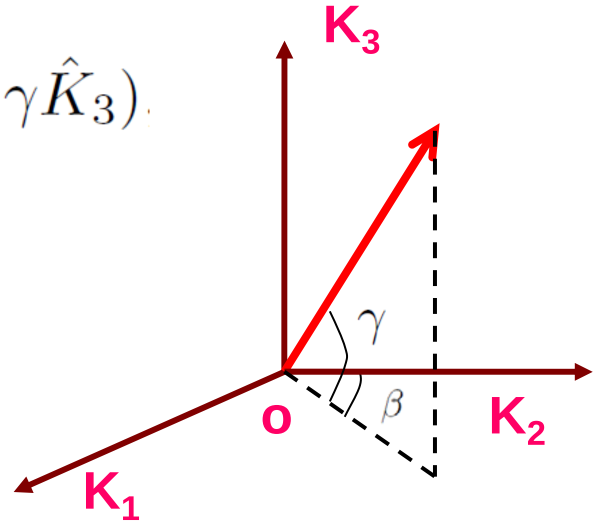
Carroll & Hioe, JOSA B 5, 1335 (1988)

invariant can be constructed as

$$I(t) = \frac{\hbar}{2}\Omega_0(\cos\gamma\sin\beta\hat{K}_1 + \cos\gamma\cos\beta\hat{K}_2 + \sin\gamma\hat{K}_3).$$

$$\hat{K}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \hat{K}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \hat{K}_3 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

SU(2)



$$\Omega_s = 2(\dot{\beta}\cot\gamma\cos\beta - \dot{\gamma}\sin\beta),$$
$$\Omega_p = 2(\dot{\beta}\cot\gamma\sin\beta + \dot{\gamma}\cos\beta).$$

$$i\hbar\frac{\partial I(t)}{\partial t} - [H(t), I(t)] = 0,$$

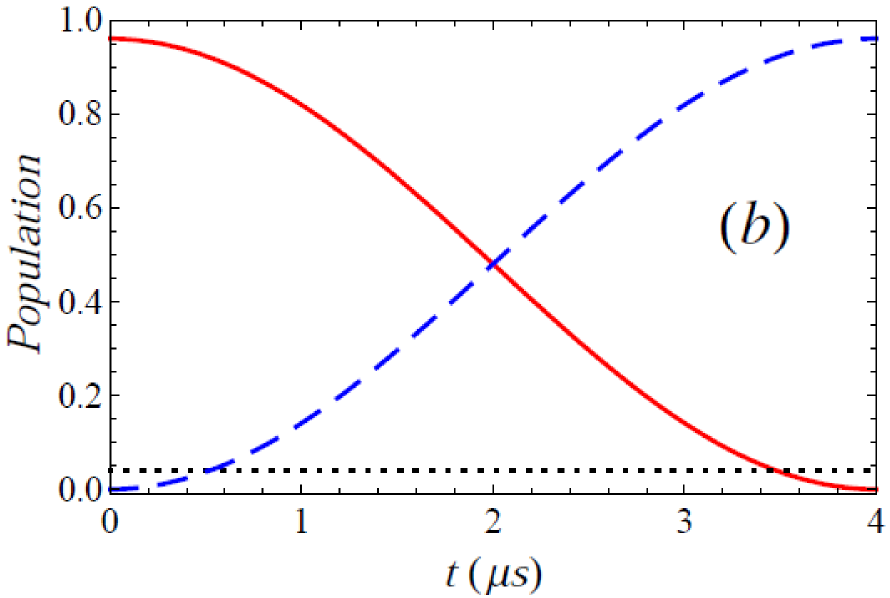
single-mode driving $\gamma(0) = \epsilon, \quad \dot{\gamma}(0) = 0, \quad \gamma(t_f) = \epsilon, \quad \dot{\gamma}(t_f) = 0,$

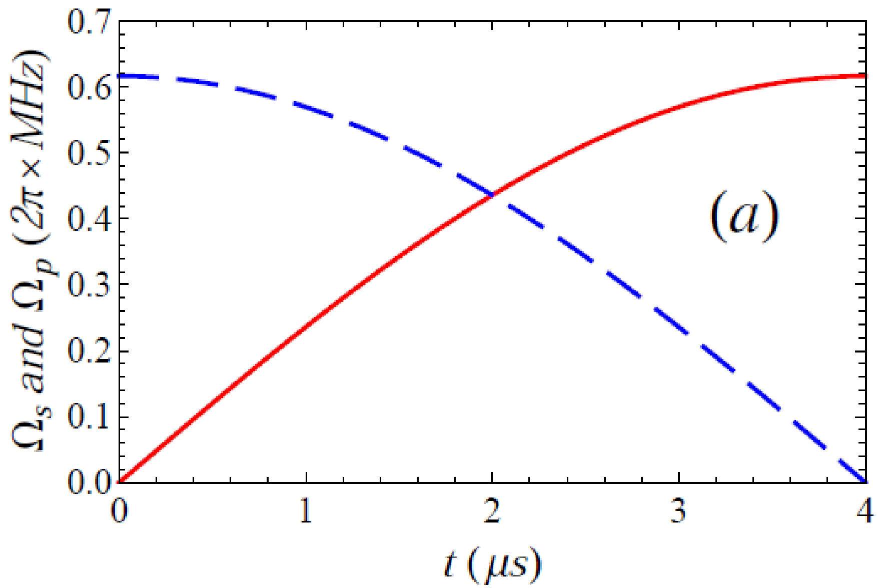
$$\beta(0) = 0, \quad \beta(t_f) = \pi/2.$$

we choose the simple $\gamma(t) = \epsilon, \quad \beta(t) = \pi t/2t_f,$

$$\Omega_s(t) = (\pi/t_f) \cot\epsilon \cos(\pi t/2t_f),$$

$$\Omega_p(t) = (\pi/t_f) \cot\epsilon \sin(\pi t/2t_f).$$





Parameters: $t_f = 4\mu s, \ \epsilon = 0.2.$

$|\phi_0(0)\rangle \longrightarrow |\phi_0(t_f)\rangle$

multi-mode driving

Initial state |1⟩ → final state |-3⟩

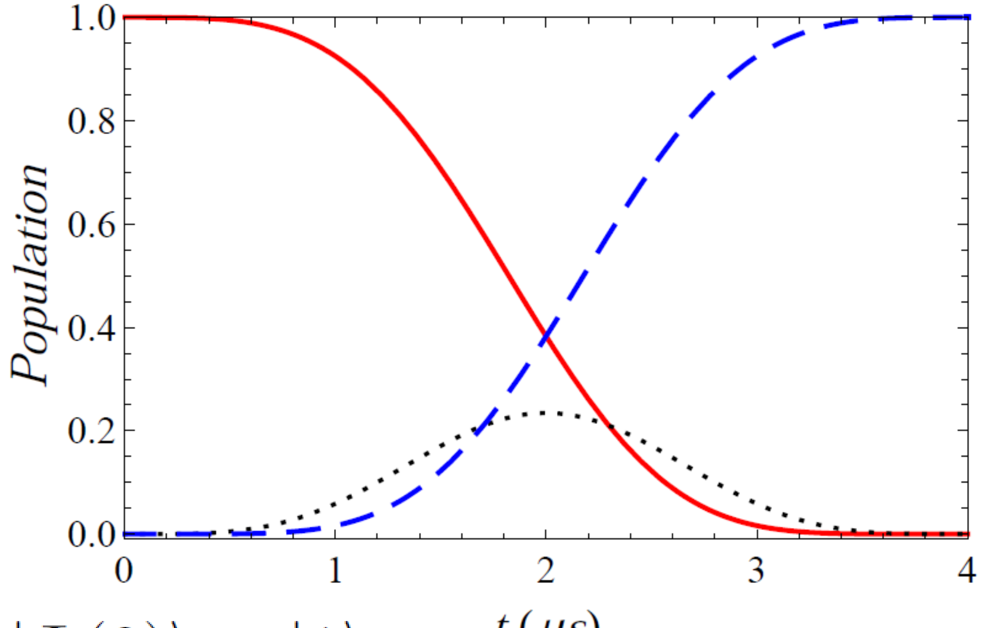
$$\Omega_s(t) = (\pi/t_f)\cot\epsilon\cos(\pi t/2t_f),$$

$$\Omega_p(t) = (\pi/t_f)\cot\epsilon\sin(\pi t/2t_f).$$

F=1 with less intensities !

$$\epsilon = 0.2527.$$

$$\overline{E}/\hbar = 37.00 \text{ MHz}$$



$$|\Psi(0)\rangle = |1\rangle$$

$$t_f = 4\mu s.$$

Protocol 1 F=0.9682

$$\epsilon = 0.2$$

$$\overline{E}/\hbar = 60.04 \text{ MHz}$$

F=0.9998

$$\epsilon = 0.02$$

$$\overline{E}/\hbar = 6166.86 \text{ MHz}$$

Explanation- multi-mode driving

the final state is calculated as $\Psi(t_f) = \sum_n C_n e^{i\alpha_n} |\phi_n(t_f)\rangle$

where $C_n = \langle \phi_n(0)|1\rangle$ $|\Psi(0)\rangle = |1\rangle$

Using eigenstates of invariant at t=0 and $t=t_f$

we finally obtain the **fidelity**

$$F \equiv \langle -3|\Psi(t_f)\rangle = e^{i\alpha_0} \cos^2\epsilon + \frac{1}{2}(e^{i\alpha_+} + e^{i\alpha_-}) \sin^2\epsilon.$$

resulting from different modes

In the first protocol

$$\alpha_0 = 0, \quad \alpha_\pm = \mp \frac{\pi}{2\sin\epsilon},$$

$$F = 1 - \sin^2\epsilon \left\{1 - \cos\left(\frac{\pi}{2\sin\epsilon}\right)\right\}.$$

$$(\sin\epsilon)^{-1} = 4N, \quad (N = 1, 2, 3 ...)$$

F=1 !

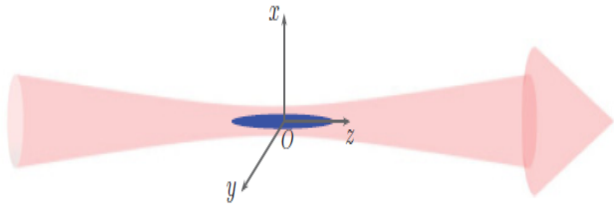
Other results & applications

Transient energy and 3rd principle, maximal cooling rates

X. Chen and J. G. Muga, Phys. Rev. A 82, 053403 (2010).

3D effects in fast expansions

E. Torróntegui et al., Phys. Rev. A 85, 033605 (2012).

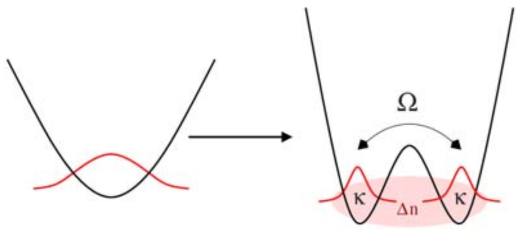


Complex potentials

S. Ibáñez et al., Phys. Rev. A 84, 023415 (2011). +86 (2012)(E)

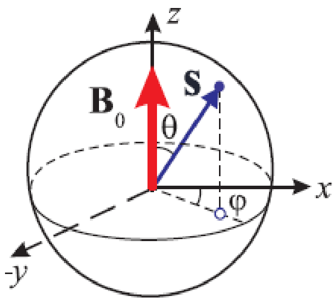
Fast splitting of matter waves (interferometry)

E. Torrontegui et al., arXiv:1207.3184



Spintronics [fast spin-flip in quantum dot]

Y. Ban et al,, arXiv:1208.4890 see poster!



Thank you for your attention!

STA working team and collaborators

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