Topological phases driven by electron interactions in certain 2D lattices

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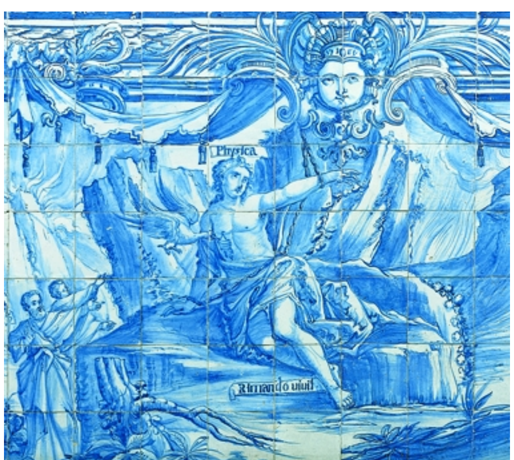


Correlations and coherence in quantum systems

Évora, Portugal, 8 – 12 October, 2012

Content

- *Motivation*
- *From trivial to non-trivial through interactions*
  - Topological Fermi liquids from Coulomb interactions in the doped honeycomb lattice

    Adolfo Grushin, Belén Valenzuela, María Vozmediano, Alberto Cortijo, Fernando de Juan

    Phys. Rev. Lett. **107**, 106402 (2011)
- *From non-trivial to non-trivial through interactions*
  - Change of an insulator's topological properties by a Hubbard interaction

    Miguel Araújo, Pedro Sacramento

    ArXiv:1208.1289 (2012)
- *Conclusions*

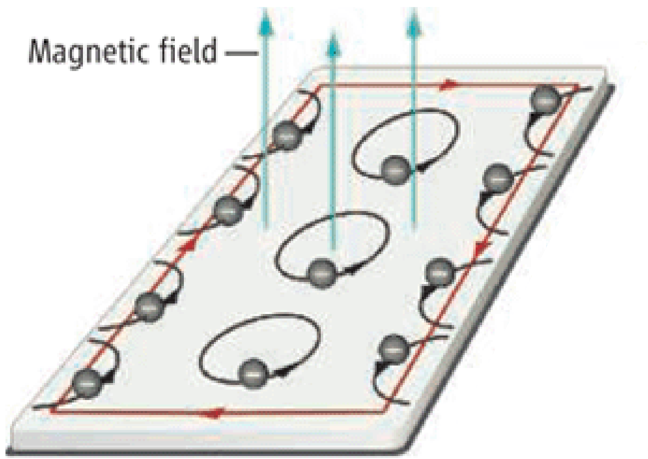
Motivation

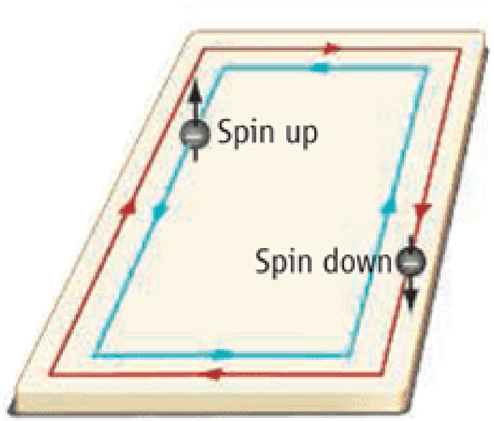
Quantum Hall insulators

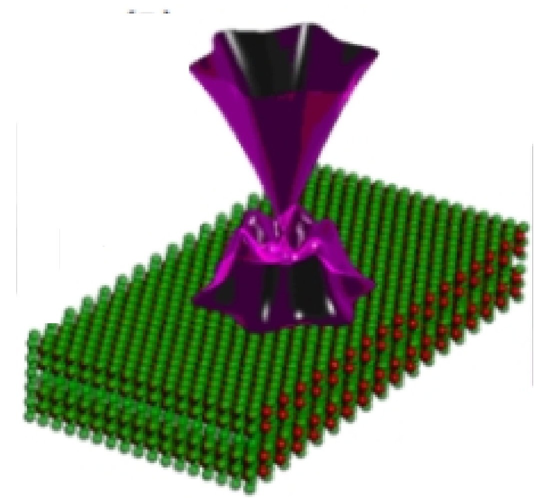
Topological insulators

2D

3D





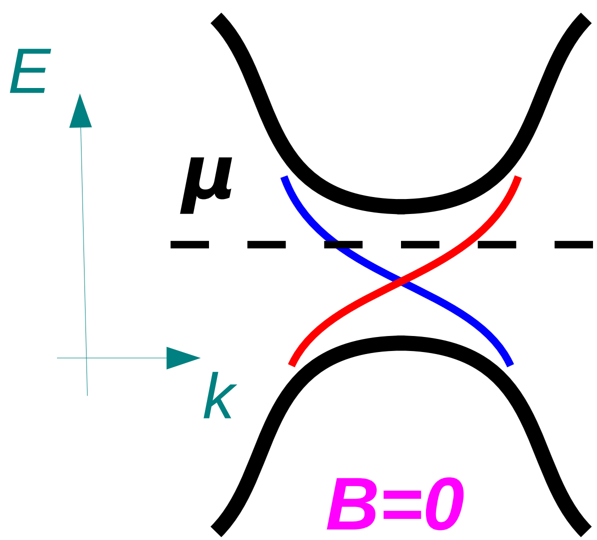


- *Well understood in the non-interacting picture*
- *What about correlations?*
  - From non-trivial to trivial
  - **From trivial to non-trivial**
  - **From non-trivial to non-trivial**

Pesin & Balents, Nat. Phys.(2010)
Varney, Sun, Rigol, Galitski PRB (2010)
Varney, Sun, Rigol, Galitski PRBr (2011)
Rachel & Le Hur, PRB (2010)
Zheng, Zhang, Wu, PRB (2011)
Hohenadler, Lang, Assaad, PRL (2011)
Yu, Xie, Li, PRL (2011)
Wu, Rachel, Liu, Le Hur, PRB (2012)
Hohenadler, Meng, Lang, Wessel, Muramatsu, Assaad, PRB (2012)

Topological phases in 2D (no spin)

- *Quantum Anomalous Hall phases*
  - Like QH insulators without magnetic field



<u>*Key ingredients:*</u>

- Time reversal symmetry $(\mathcal{T})$ broken
- Non-trivial band topology

Quantitatively:

$$\sigma_0^{ab}(\mu) = \frac{e^2}{\hbar}\frac{1}{\Omega N}\sum_{kn}\mathcal{F}_n^{ab} n_n^0(\mathbf{k},\mu)$$

Fermi-Dirac

Berry curvature

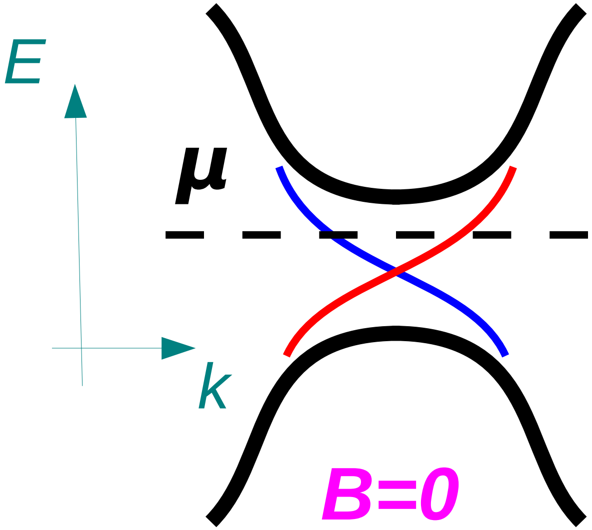
$$\mathcal{F}_n^{ab}(\boldsymbol{k}) = \nabla_k^a \mathcal{A}_n^b(\boldsymbol{k}) - \nabla_k^b \mathcal{A}_n^a(\boldsymbol{k}) \qquad \mathcal{A}_n^a(\boldsymbol{k}) = -i\langle\psi_n(\boldsymbol{k})|\nabla_k^a\psi_n(\boldsymbol{k})\rangle$$

Berry connection

Bloch wave function

Topological phases in 2D (no spin)

- *Quantum Anomalous Hall phases*
  - Like QH insulators without magnetic field



Key ingredients:

- Time reversal symmetry $(\mathcal{T})$ broken
- Non-trivial band topology

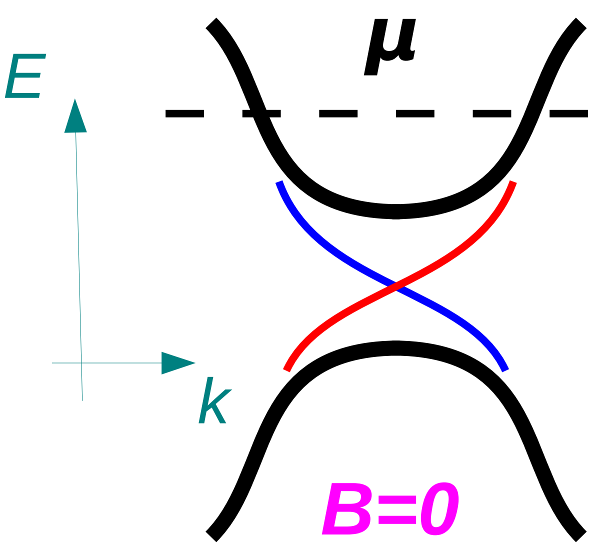
Quantitatively:

$$\sigma_0^{ab}(\mu) = \frac{e^2}{h}\sum_n C_n$$

Chern number

Topological phases in 2D (no spin)

- *Anomalous Hall (metallic) phases*
  - Classical Hall effect without magnetic field



<u>*Key ingredients:*</u>

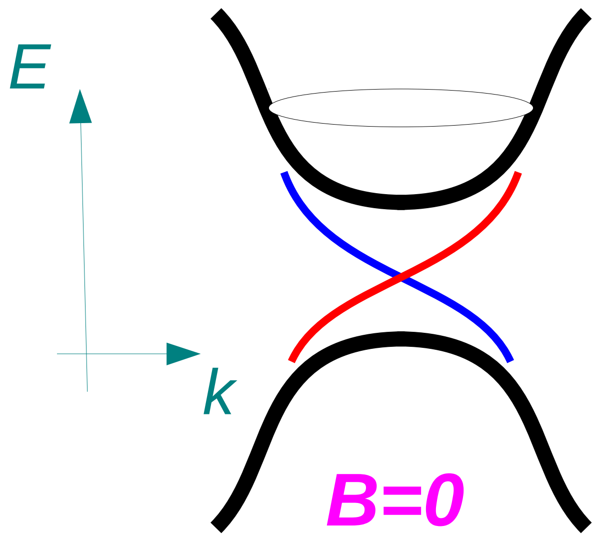
- Time reversal symmetry $(\mathcal{T})$ broken
- Non-trivial band topology

Quantitatively:
$$\sigma_0^{ab}(\mu) = \frac{e^2}{\hbar}\frac{1}{\Omega N}\sum_{kn}\mathcal{F}_n^{ab} n_n^0(\mathbf{k},\mu)$$

Non quantized part**:**
$$\sigma_0^{ab}(\mu) = \frac{e^2}{h}\nu_n$$

Topological phases in 2D (no spin)

- *Anomalous Hall (metallic) phases*
  - Classical Hall effect without magnetic field



Key ingredients:

- Time reversal symmetry $(\mathcal{T})$ broken
- Non-trivial band topology

Quantitatively:

$$\nu_n = \frac{1}{2\pi}\oint \mathcal{A}_n^a(\boldsymbol{k}_F)d\boldsymbol{k}_{Fa} = \frac{\phi_F}{2\pi}$$

Haldane
PRL, (2004)

Topological Fermi liquid

How to break T?

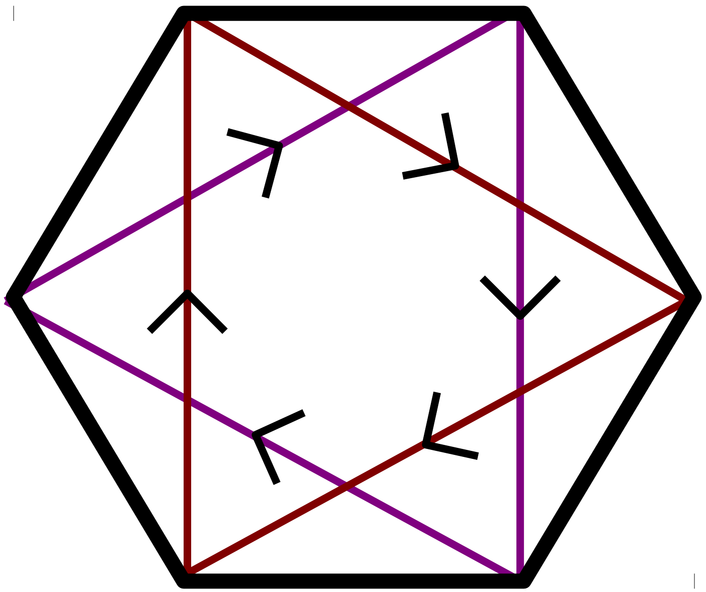
- *Historical example*
  - Add imaginary hoppings in a clever way

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

**Model for a Quantum Hall Effect without Landau Levels:
Condensed-Matter Realization of the "Parity Anomaly"**

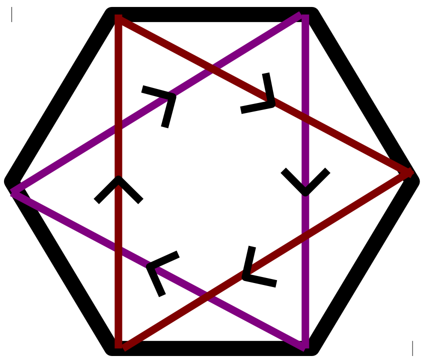
F. D. M. Haldane

- **QAH phase at half filling**
- **Anomalous Hall phase when doped**



Alternative route: Interactions

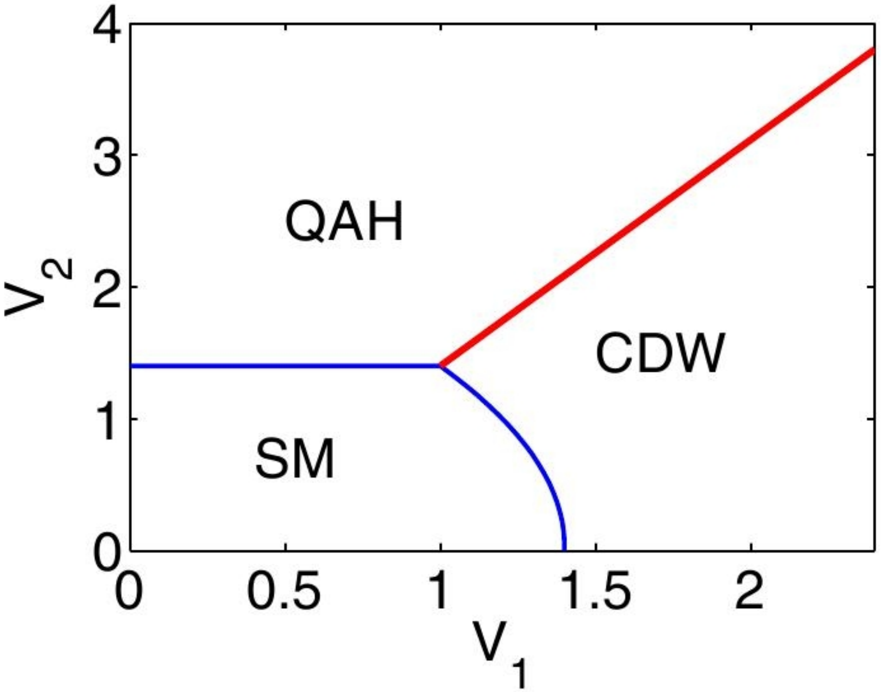
- *NN + NNN interactions*
  - Raghu *et al.* PRL, (2008)



$$\mathcal{H} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V_1 \sum_{\langle i,j \rangle} n_i n_j + V_2 \sum_{\langle\langle i,j \rangle\rangle} n_i n_j$$

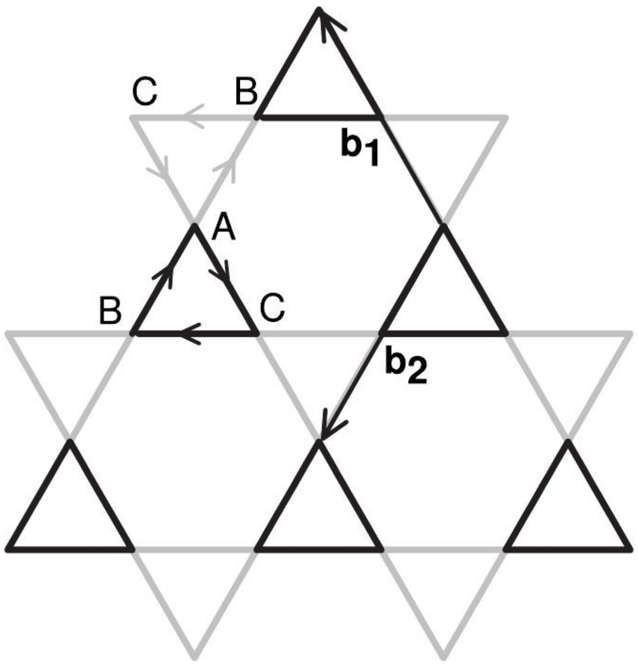
- Mean field
- Half filling

V2 > V1

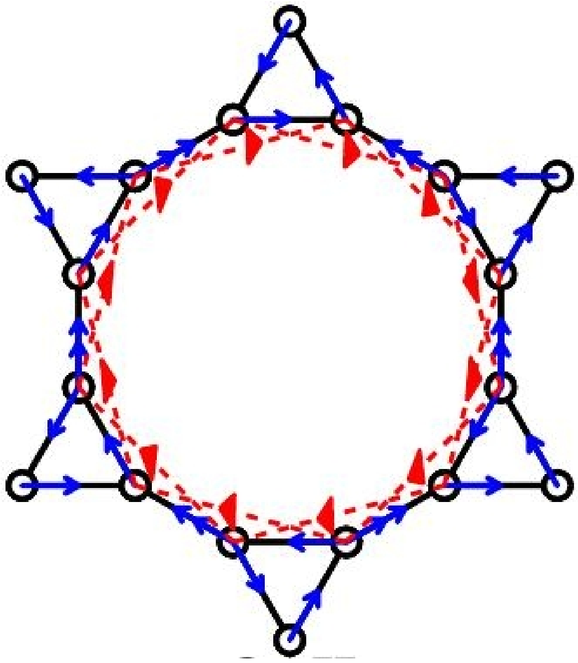


Interactions and "sofisticated" lattices

- *Possible with only NN interactions?*
  - Take lattices that naturally allow for intracell currents with zero net magnetic field



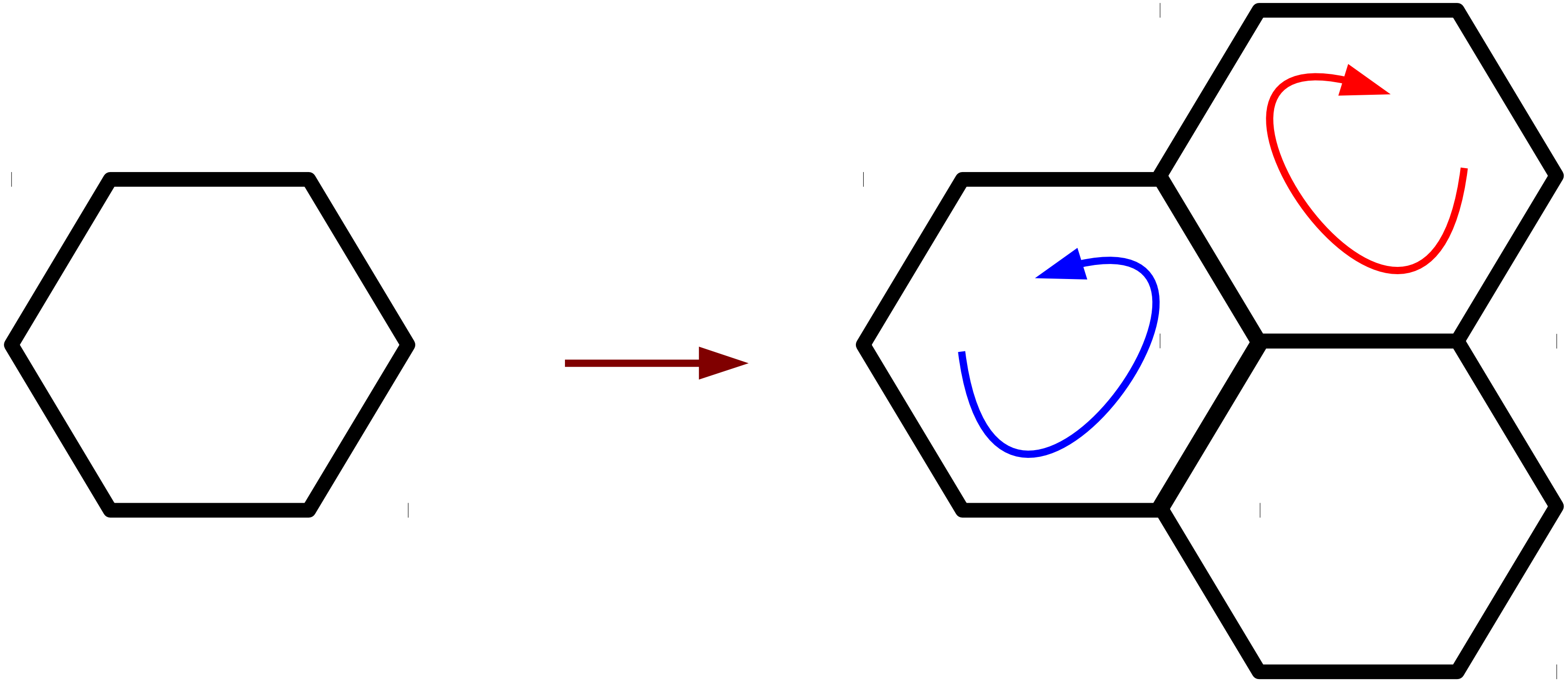
Liu *et al.* PRB, (2010)



Wen *et al.* PRB, (2010)

Interactions and enlarged unit cells

- *There is yet another possibility*
  - Take your favorite lattice, and just ***enlarge the unit cell*** to allow for intracell currents with zero net magnetic field



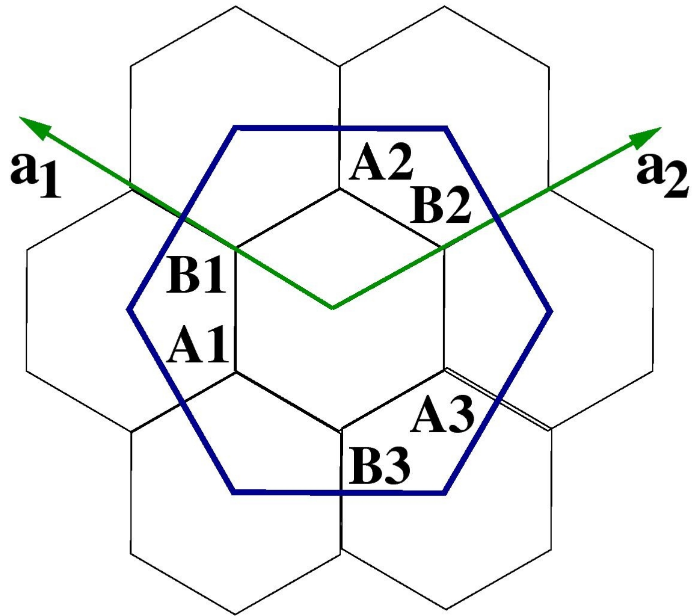
Phys. Rev. Lett. **107**, 106402 (2011)

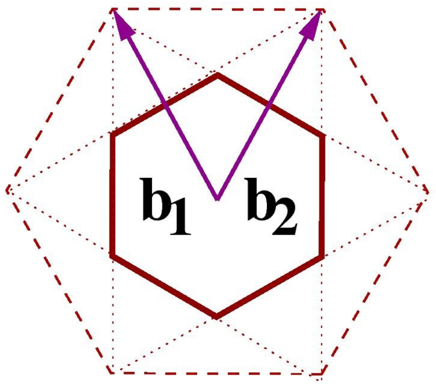
Case study: the honeycomb lattice

- *Simplest (spinless) Hamiltonian*
  - (For fermions on a lattice with Coulomb interaction)

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V \sum_{\langle i,j \rangle} n_i n_j$$

- *From 2 to 6 atom unit cell*





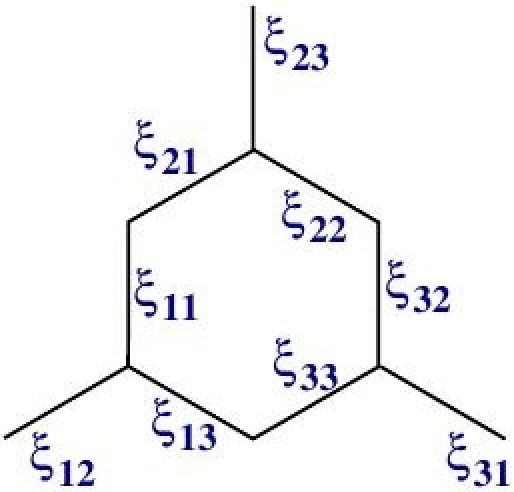
(not dynamical) Mean field

- *Variational mean field method*
  - Bogoliubov – Feynmann – Gibbs inequality

$$\Omega \leq \Omega_{MF} + \langle \mathcal{H} - \mathcal{H}_{MF} \rangle_{MF}$$

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V \sum_{\langle i,j \rangle} n_i n_j$$

$$\mathcal{H}_{MF} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V \sum_{\langle i,j \rangle} \xi_{ij} c_i^\dagger c_j$$



- *nine complexed valued order parameters*
- *ignore charge decoupling for the moment*

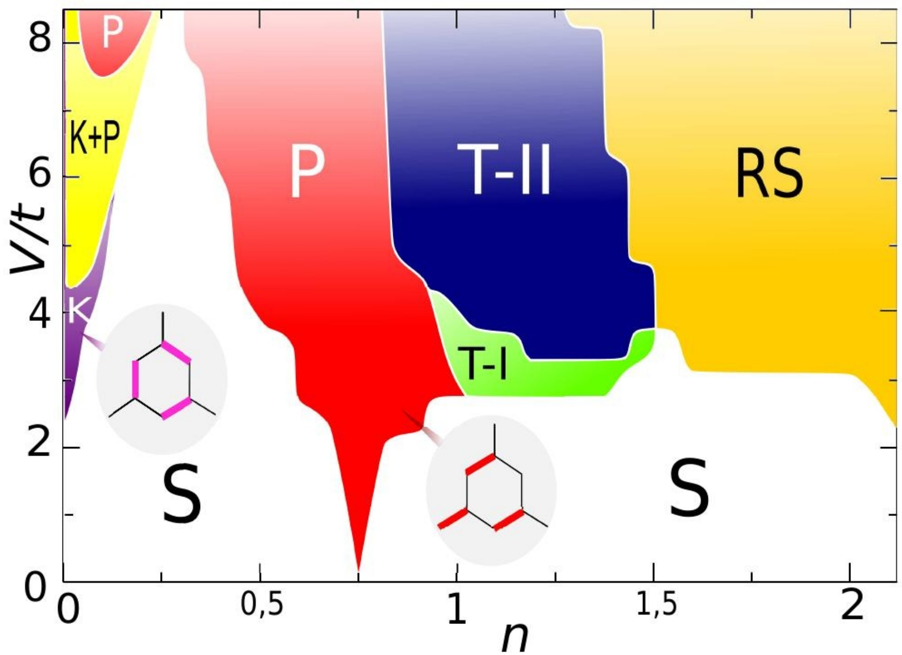
Phase diagram

- *Mean field equation + Luttinger's theorem*

$$\xi_{ij} = -\frac{2}{N}\sum_{\mathbf{k}} \gamma_{\mathbf{k}}^{ij} \langle c_j^\dagger(\mathbf{k}) c_i(\mathbf{k}) \rangle_{MF}$$

$$\frac{N_e}{N} = 3 + n = \frac{1}{N}\sum_{\mathbf{k},l} n_l^0(\mathbf{k},\mu)$$

- *n vs V phase diagram:*

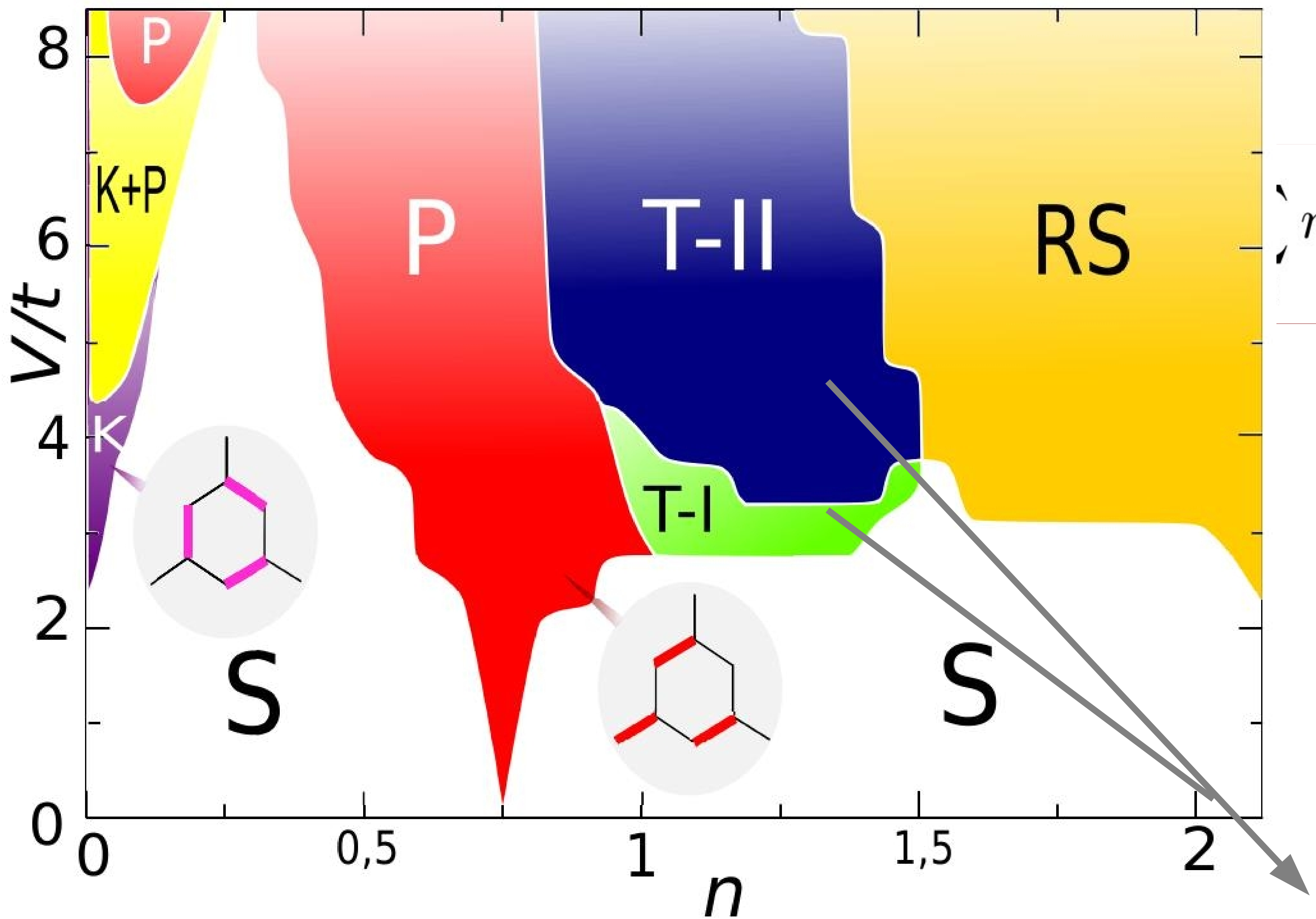


Phase diagram

- *Me*

$\xi_{ij} =$ … $n_l^0(\mathbf{k}, \mu)$

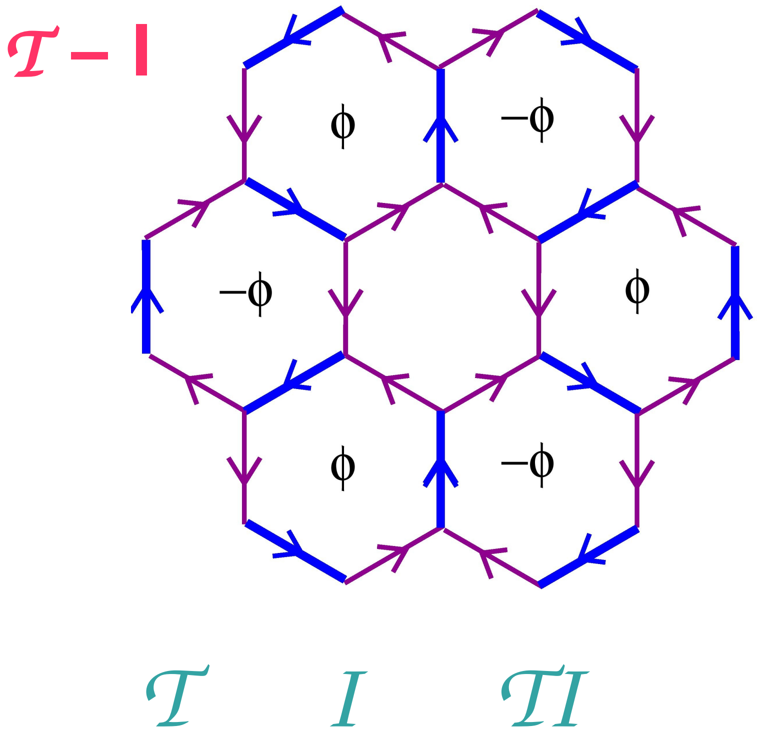
- *n v*

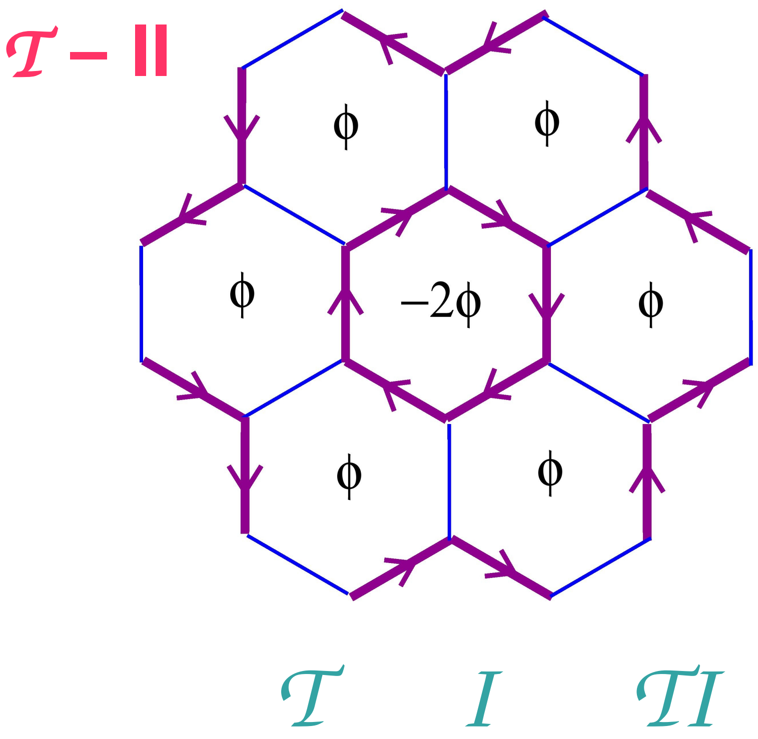


$\mathcal{T}$ – broken phases

The $\mathcal{T}$ – broken phases

- *Different flux (orbital current) patterns*



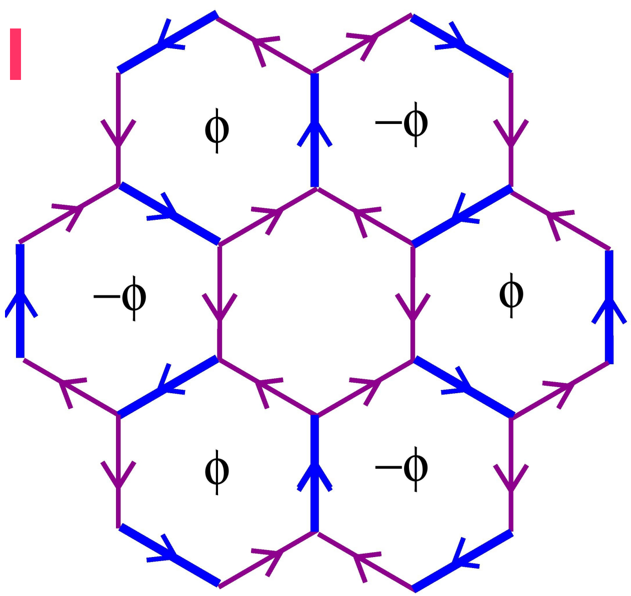


Symmetry	Berry curvature	σ_{xy}
$\mathcal{T}$	$\mathbf{\Omega}_n(\mathbf{k}) = -\mathbf{\Omega}_n(-\mathbf{k})$	0
$\mathcal{I}$	$\mathbf{\Omega}_n(\mathbf{k}) = \mathbf{\Omega}_n(-\mathbf{k})$	$\neq 0$
$\mathcal{T}\mathcal{I}$	$\mathbf{\Omega}_n(\mathbf{k}) = -\mathbf{\Omega}_n(\mathbf{k}) = 0$	0

The $\mathcal{T}$ - broken phases

- *Different flux (orbital current) patterns*

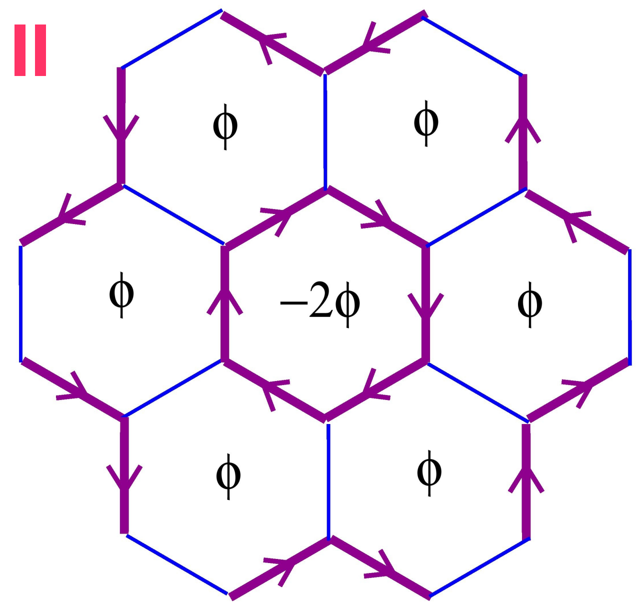
$\mathcal{T}$ – I



~~$\mathcal{T}$~~ ~~$\mathcal{I}$~~ $\mathcal{TI}$

$$\sigma^{xy}(\mu) = 0$$

$\mathcal{T}$ – II



~~$\mathcal{T}$~~ $\mathcal{I}$ ~~$\mathcal{TI}$~~

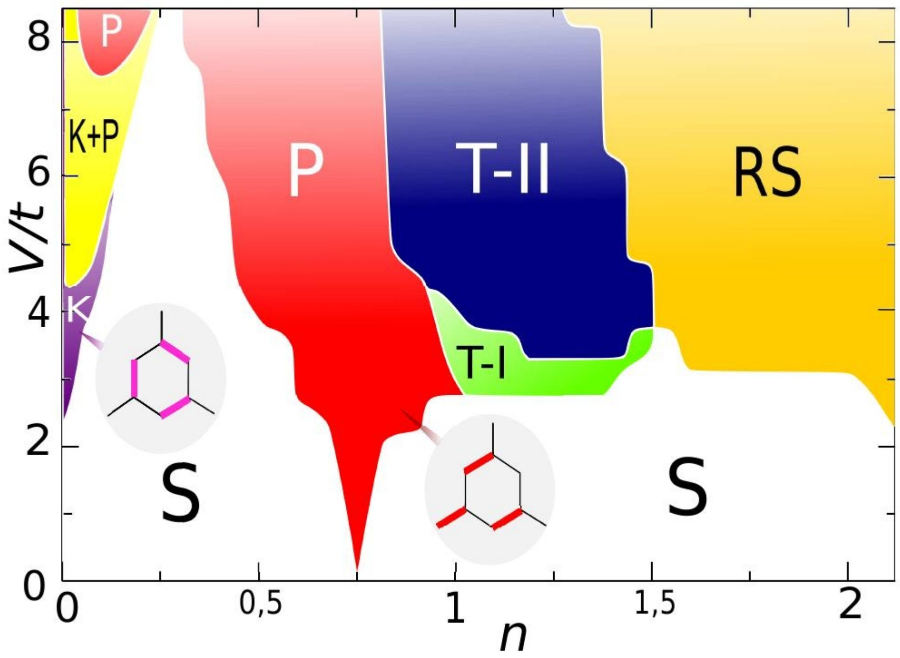
$$\sigma^{xy}(\mu) \neq 0$$

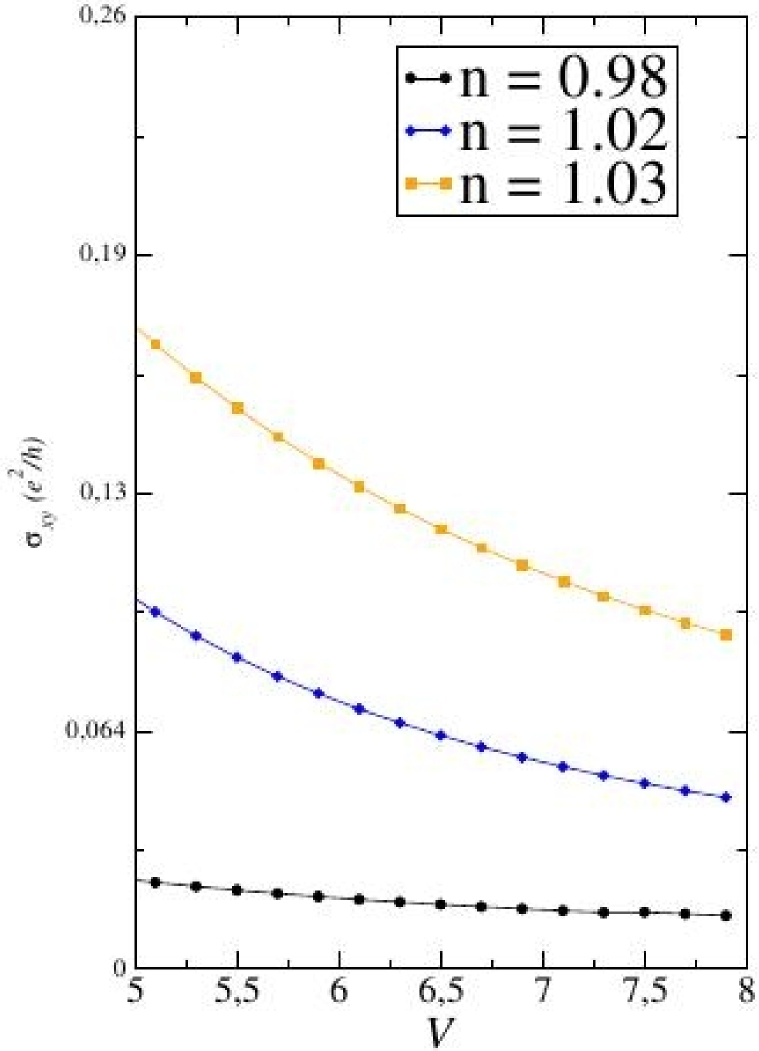
Symmetry	Berry curvature	σ_{xy}
$\mathcal{T}$	$\mathbf{\Omega}_n(\mathbf{k}) = -\mathbf{\Omega}_n(-\mathbf{k})$	0
$\mathcal{I}$	$\mathbf{\Omega}_n(\mathbf{k}) = \mathbf{\Omega}_n(-\mathbf{k})$	$\neq 0$
$\mathcal{TI}$	$\mathbf{\Omega}_n(\mathbf{k}) = -\mathbf{\Omega}_n(\mathbf{k}) = 0$	0

The Anomalous Hall phase

- *Finite AH conductivity in* $\mathcal{T}$ **– II**

$$\sigma_0^{ab}(\mu) = \frac{e^2}{\hbar}\frac{1}{\Omega N}\sum_{kn}\mathcal{F}_n^{ab} n_n^0(\mathbf{k},\, \mu)$$



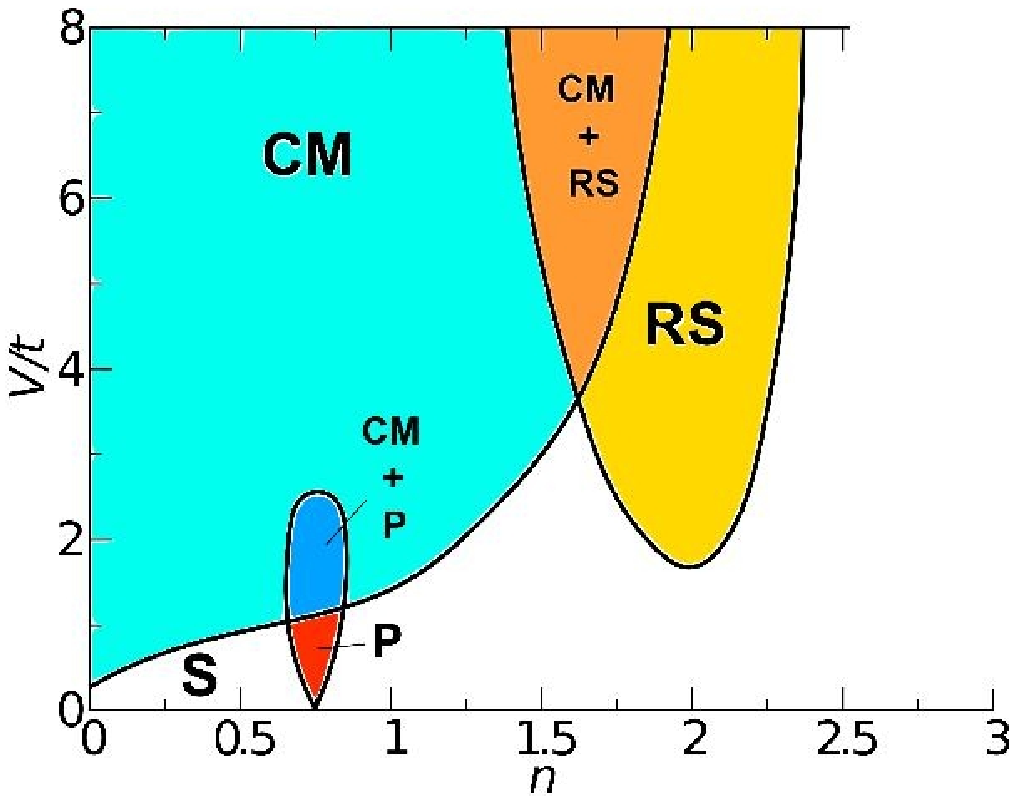


$\mathcal{T}$ **– II** is a topological Fermi liquid

Charge inhomogeneous phases

- *Include charge decoupling*

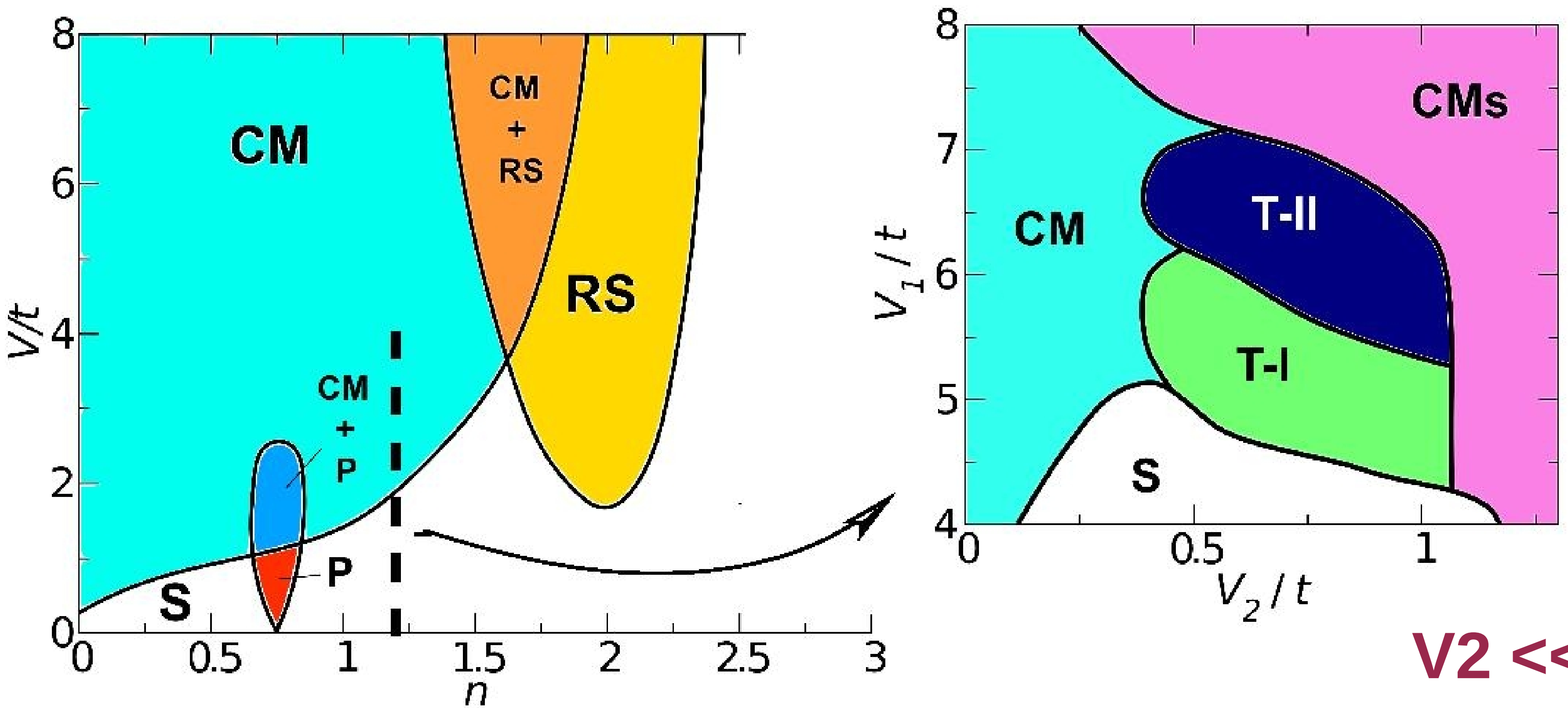
$$\mathcal{H}_{MF} = -t\sum_{\langle i,j\rangle} c_i^\dagger c_j + V\sum_{\langle i,j\rangle} \xi_{ij} c_i^\dagger c_j + V\sum_i \rho_i n_i$$



Charge inhomogeneous phases

- *Include NNN interaction (frustration)*

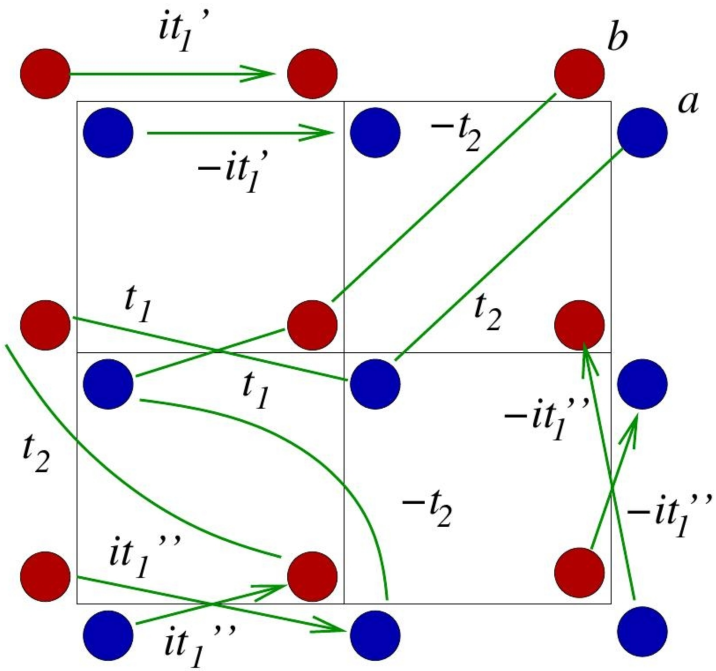
$$\mathcal{H} = -t\sum_{\langle i,j\rangle} c_i^\dagger c_j + V_1 \sum_{\langle i,j\rangle} n_i n_j + V_2 \sum_{\langle\langle i,j\rangle\rangle} n_i n_j$$



V2 << V1

Changing the Chern number through interactions

- *Square lattice, two-orbital per site, spinless fermions*



$$H_0 = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \boldsymbol{\sigma} \cdot \boldsymbol{h} c_{\mathbf{k}},$$

$$\begin{aligned} h_x &= \sqrt{2} t_1 (\cos k_x + \cos k_y) \\ h_y &= \sqrt{2} t_1 (\cos k_x - \cos k_y) \\ h_z &= 4t_2 \sin k_x \sin k_y + 2t_1' (\sin k_x + \sin k_y) + \delta \end{aligned}$$

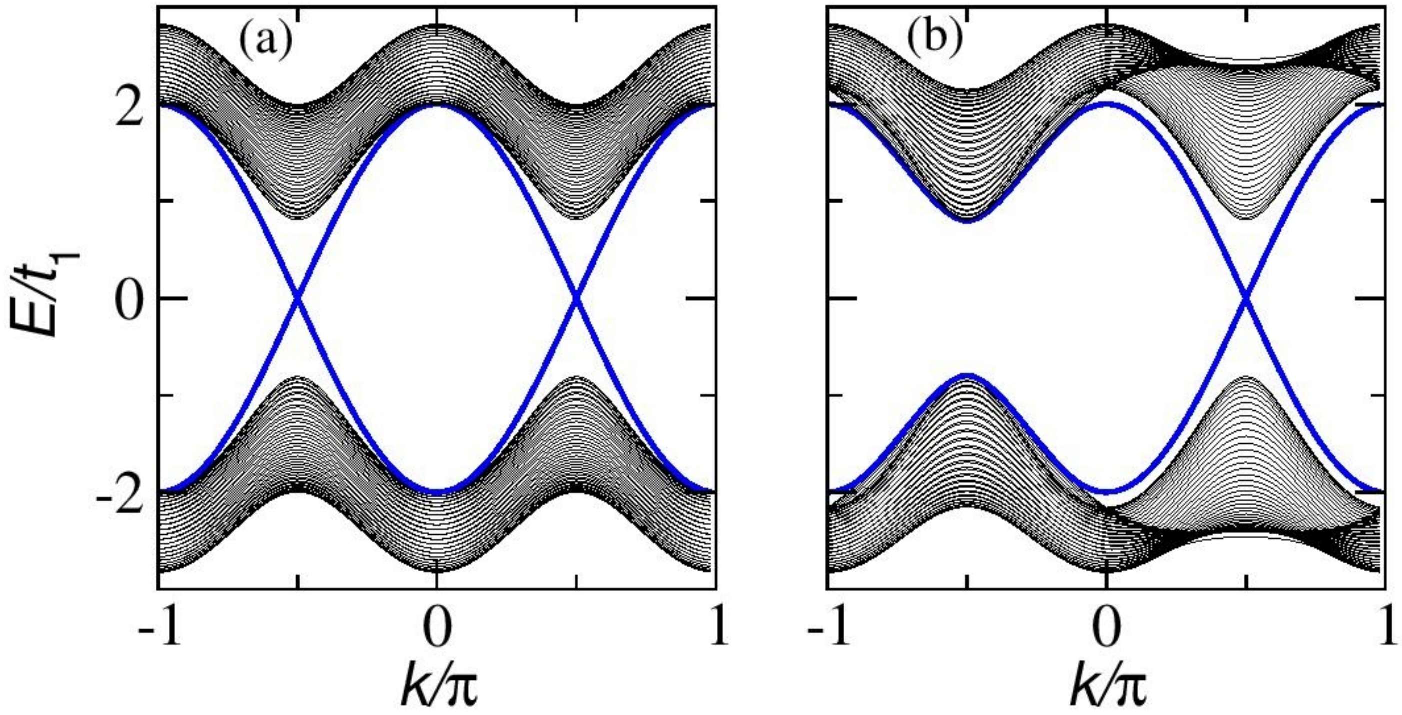
ArXiv:1208.1289 (2012)

$$t_2 > t_1' - \delta/4 \;\rightarrow\; C = 2$$

$$t_2 < t_1' - \delta/4 \;\rightarrow\; C = 1$$

Changing the Chern number through interactions

- *Square lattice, two-orbital per site, spinless fermions*



ArXiv:1208.1289 (2012)

$$t_2 > t_1' - \delta/4 \;\rightarrow\; C = 2$$

$$t_2 < t_1' - \delta/4 \;\rightarrow\; C = 1$$

Changing the Chern number through interactions

- *Slave rotor approach*

$$\hat{c}_j = \hat{f}_j e^{-i\theta_j} \qquad \hat{H}_{int} = \frac{U}{2}\sum_j \hat{L}_j^2$$

- *Mean field decoupling*

$$H(\bar{f}X, fX^*) = H_f(\bar{f}, f) + H_X(X, X^*)$$

$$e^{i\theta_j} = X_j \qquad \boldsymbol{h}_{ij} \rightarrow \boldsymbol{h}_{ij}\langle X_i X_j^* \rangle$$

- NNN X-boson correlation functions decays faster:

$$\delta = 0,\ t_1 = 1,\ t_2 = 0.7,\ t_1' = 0.8t_2$$

$$\bar{U} \approx 1.4 \quad \begin{cases} U < \bar{U} & C = 2 \\ U > \bar{U} & C = 1 \end{cases}$$

$$U_c \approx 2.9$$

Conclusions

- *Topologically non-trivial phases via electronic (Coulomb) correlations are possible*

- *Relaxing translational symmetry by enlarging the unit cell enables for T – broken states*
    - Tologically non-trivial AH phases may be stabilized even in simple lattices

- *Case study: honeycomb lattice*
    - Topological Fermi liquid is dominant T – broken state

- *Possible to change Chern number through interactions*