Cats, Decoherence and Quantum Measurement

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Outline

Basic Quantum Mechanics : Stern – Gerlach

Entanglement

Cats (not the ordinary ones)

Dissipation and decoherence: old results but not so old realizations

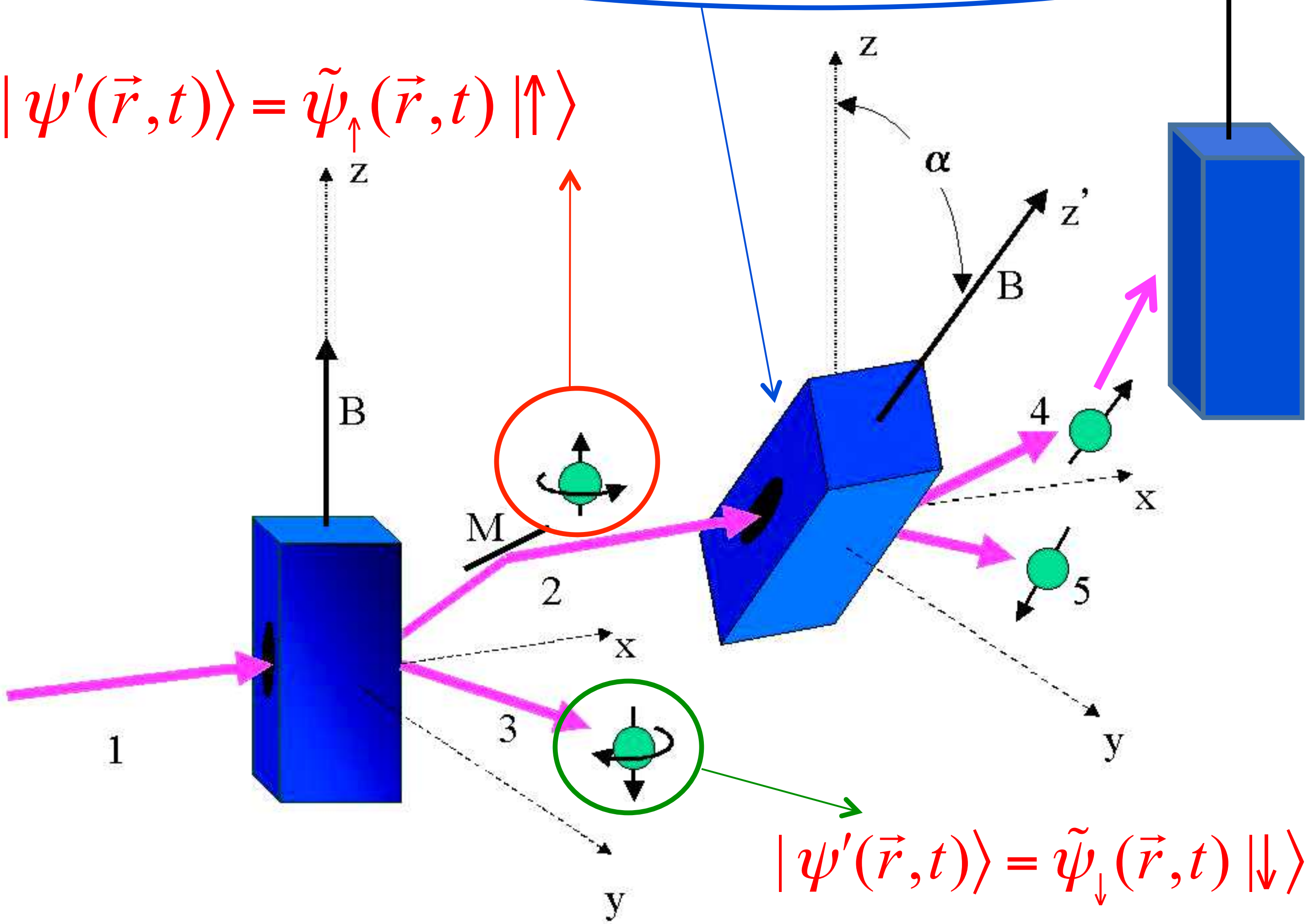
Quantum measurement

The Stern-Gerlach Experiment

Compatible and
incompatible variables

$$|\psi'(\vec r,t)\rangle = \psi'_{\uparrow}(\vec r,t)\,|(+)_{z'}\rangle + \psi'_{\downarrow}(\vec r,t)\,|(-)_{z'}\rangle$$

$$|\psi'(\vec r,t)\rangle = \tilde\psi_{\uparrow}(\vec r,t)\,|\!\uparrow\rangle$$



$$|\psi'(\vec r,t)\rangle = \tilde\psi_{\downarrow}(\vec r,t)\,|\!\downarrow\rangle$$

The Stern-Gerlach Experiment

Initial state of the magnetic moment

$$|(+)_x\rangle = \frac{(|\uparrow\rangle + |\downarrow\rangle)}{\sqrt{2}}$$

Total initial state

$$|\psi(\vec{r},t)\rangle = \psi_0(\vec{r},t)\frac{(|\uparrow\rangle + |\downarrow\rangle)}{\sqrt{2}}$$

$$|\psi(\vec{r},t)\rangle = \frac{\psi_0(\vec{r},t)\,|\uparrow\rangle + \psi_0(\vec{r},t)\,|\downarrow\rangle)}{\sqrt{2}}$$

Total state along the magnet

$$|\psi(\vec{r},t)\rangle = \psi_\uparrow(\vec{r},t)\,|\uparrow\rangle + \psi_\downarrow(\vec{r},t)\,|\downarrow\rangle$$

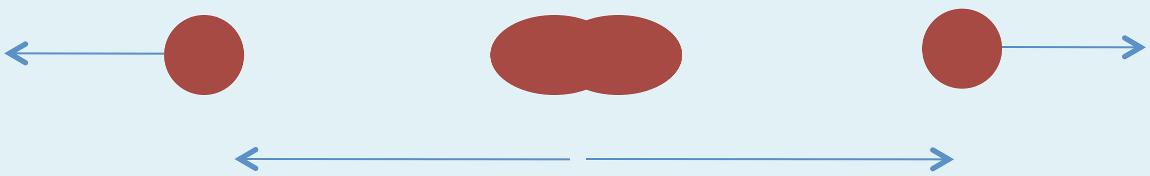
Entangled States

Two-particle state $$|\phi(\vec{r}_1,\vec{r}_2,t)\rangle$$

Separable two-particle state

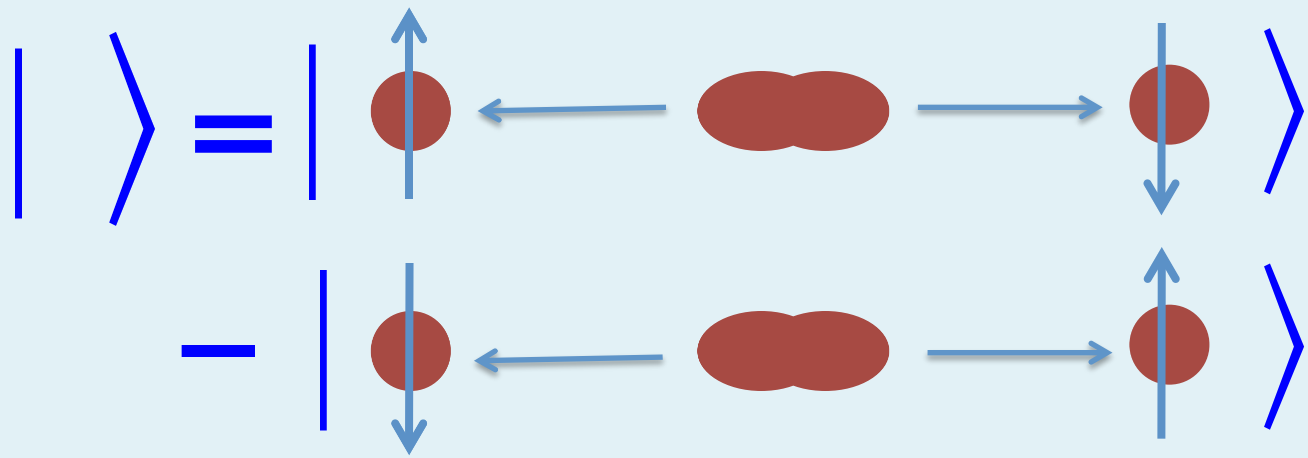
$$|\phi(\vec{r}_1,\vec{r}_2,t)\rangle = \psi(\vec{r}_1,t)\varphi(\vec{r}_2,t)|\sigma_1\sigma_2\rangle \; ; \; \sigma_i = \uparrow \text{or} \downarrow$$

Entangled (in spin) two-particle state



$$\psi_L(\vec{r},t) = \psi_0(\vec{r}+\vec{r}_0(t)) \qquad \psi_R(\vec{r},t) = \psi_0(\vec{r}-\vec{r}_0(t))$$

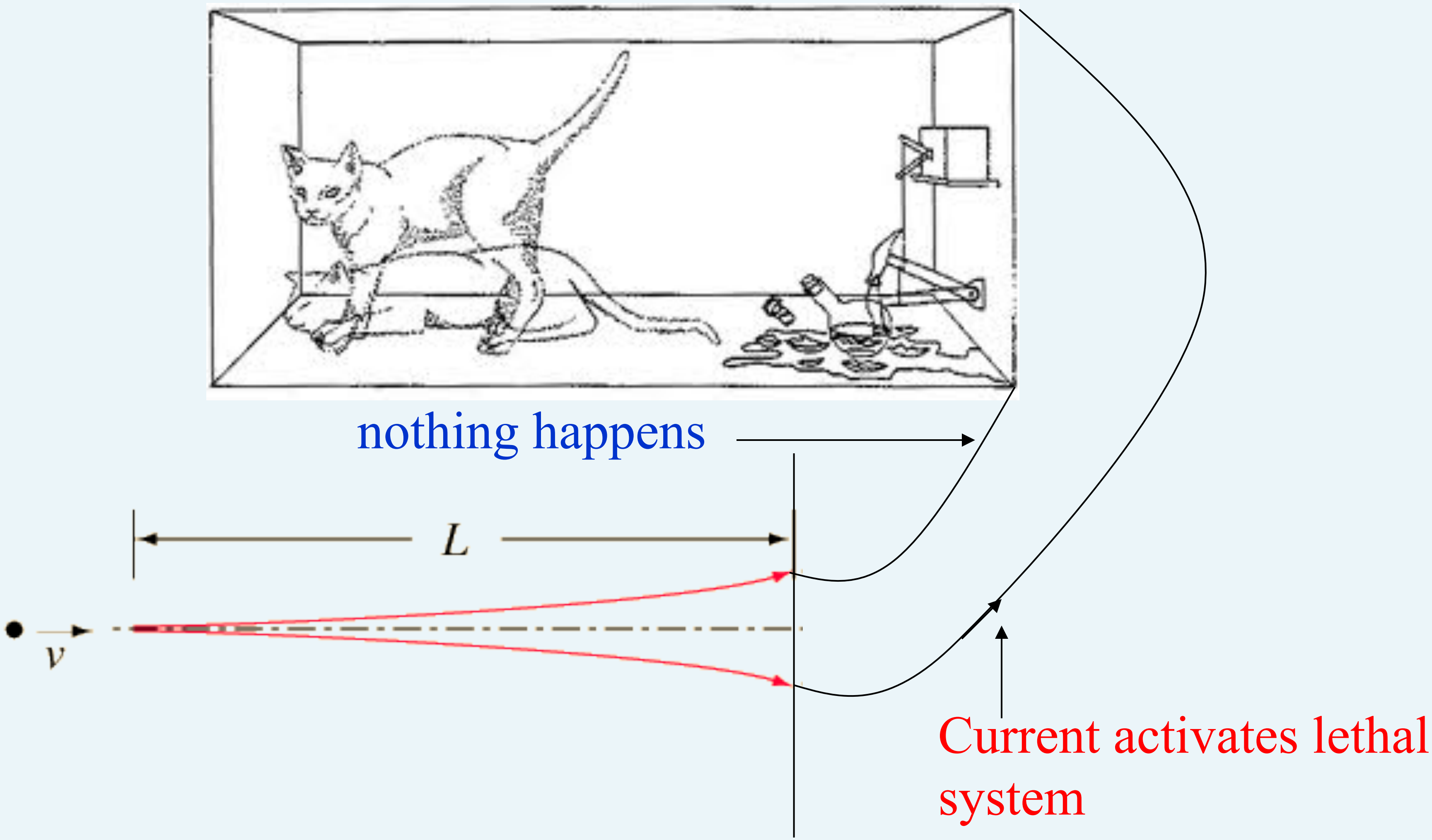
Entangled States



$$|\phi(\vec{r}_1,\vec{r}_2,t)\rangle = \psi_L(\vec{r}_1,t)\,\psi_R(\vec{r}_2,t)\left(\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right)$$

EPR-like states

The Schrödinger`s cat



The Schrödinger`s cat

$$|\varphi\rangle = (a|\uparrow\rangle + b|\downarrow\rangle)\,\psi_A(\vec{r}_1,\ldots,\vec{r}_N)$$
$$= a|\uparrow\rangle\psi_A(\vec{r}_1,\ldots,\vec{r}_N) + b|\downarrow\rangle\psi_A(\vec{r}_1,\ldots,\vec{r}_N)$$

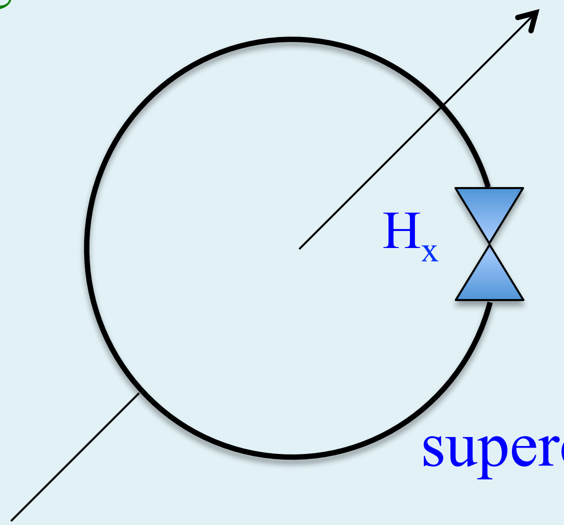
Interaction entangles two alternatives

$$|\tilde{\varphi}\rangle = a|\uparrow\rangle\psi_A(\vec{r}_1,\ldots,\vec{r}_N) + b|\downarrow\rangle\psi_D(\vec{r}_1,\ldots,\vec{r}_N)$$

Superposition of macroscopically distinct configurations

More about cats

The SQUID (superconducting quantum interference device): a paradigm



The equation of motion for the total flux

$$C\ddot{\phi} + \frac{1}{R}\dot{\phi} + \frac{dU}{d\phi} = 0 \quad \text{where}$$

$$U(\phi) = \frac{(\phi - \phi_x)^2}{2L} - \frac{i_c \phi_0}{2\pi}\cos\frac{2\pi\phi}{\phi_0}$$

C is the capacitance of the junction

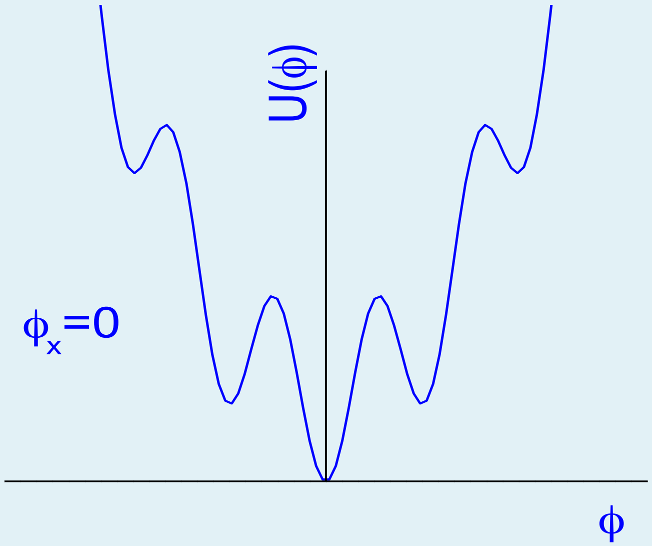
i_c is the critical current

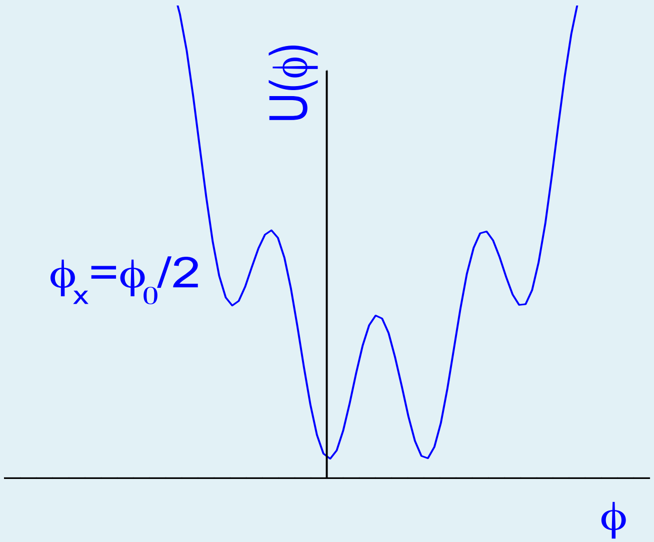
R is the normal state resistance

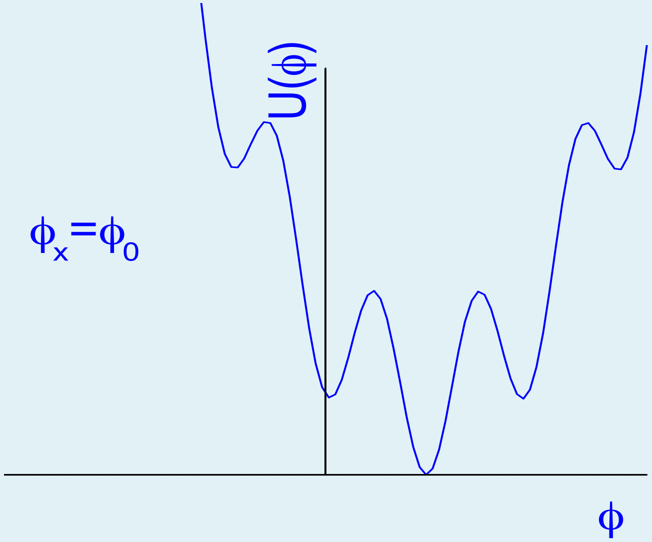
ϕ_0 is the flux quantum

ϕ_x is the external flux (H_x x area)

More about cats







More about cats

For a SQUID, in the case $\frac{2\pi L i_c}{\phi_0} >> 1$,

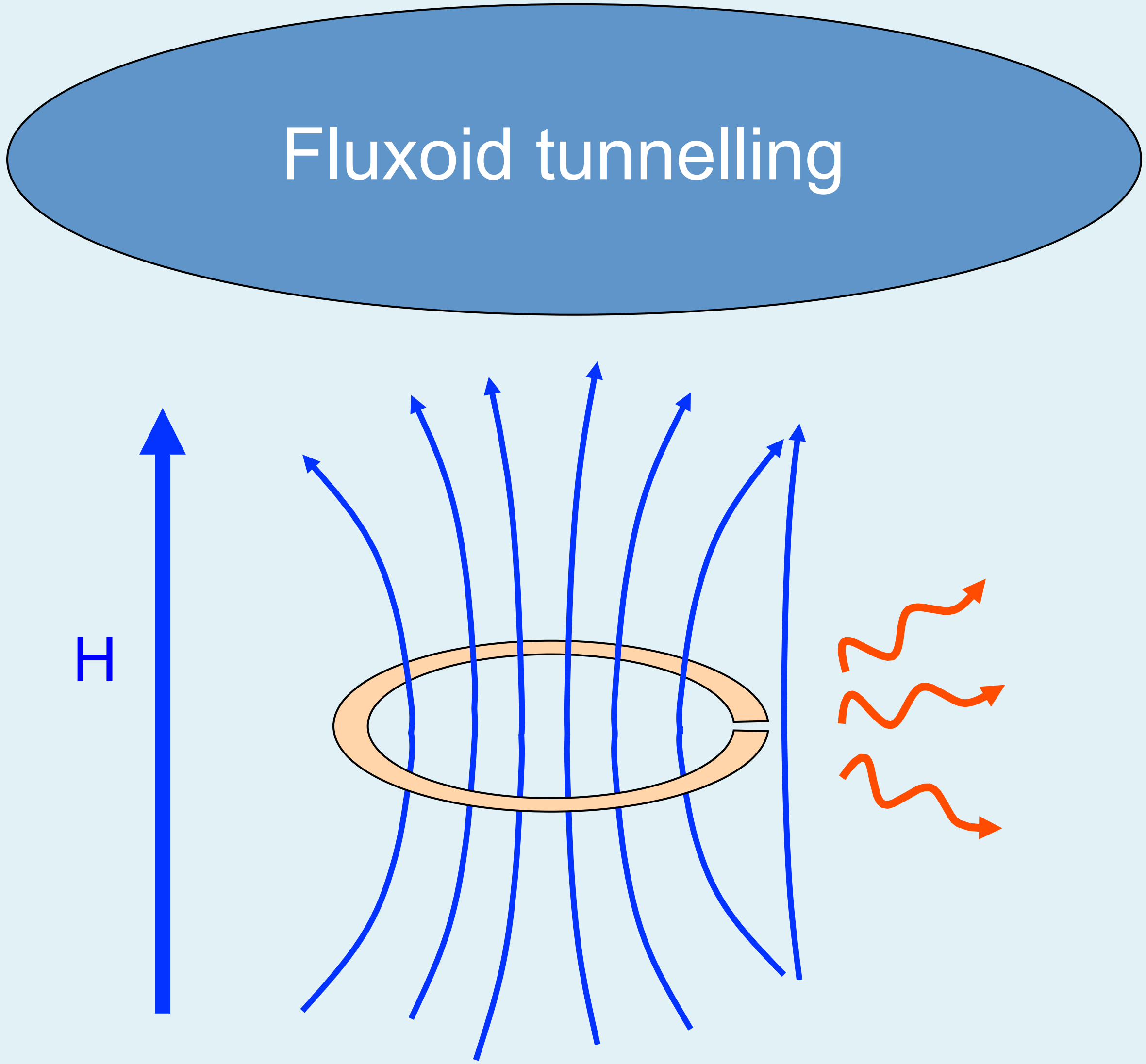
the minima of $U(\phi)$ are at $\phi \approx n\phi_0$

$$\Downarrow$$

transitions from $n\phi_0 \rightarrow (n \pm 1)\phi_0$ imply that superpositions of macroscopically distinguishable quantum mechanical states are taking place (N-body states)

$$a\Psi_A(x_1, x_2, \ldots, x_N) + b\Psi_B(x_1, x_2, \ldots, x_N)$$

More about cats



More about cats

$$a\Psi_A(x_1, x_2, \ldots, x_N) + b\Psi_B(x_1, x_2, \ldots, x_N)$$

In this specific example these two states correspond to the superconducting condensate carrying distinct macroscopic currents

$$\Downarrow$$

a) Experimental realization of the "Schrödinger's cat"
b) Fundamental questions of the quantum theory of measurement
c) Dissipation and macroscopic quantum phenomena seem to be always related

Dissipation

The phenomenological approach

Dissipative systems are such that

$$H = H_S + H_{\text{int}} + H_R$$

$$H_S = \frac{p^2}{2m} + V(q) \;\; ; \;\; H_{\text{int}} = \sum_k C_k q q_k ; \;\; H_R = \sum_k \frac{p_k^2}{2m_k} + \frac{1}{2} m_k \omega_k^2 q_k^2 \;\; ;$$

$$H_{CT} = -\sum_k \frac{C_k^2 q^2}{2m_k \omega_k^2}$$

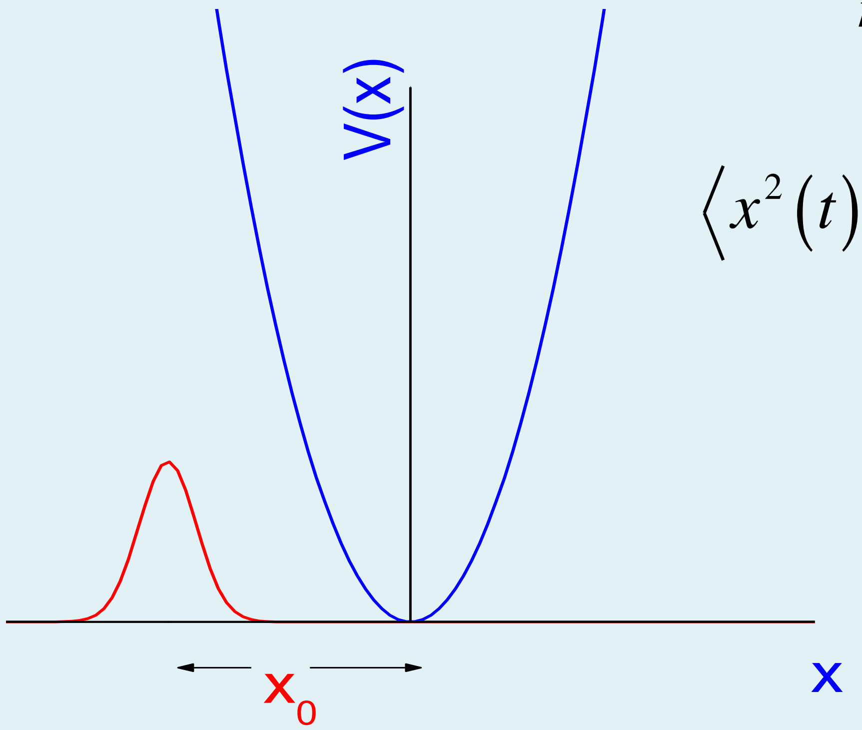
Defining the spectral function for ohmic dissipation $(\eta \dot{q})$ as

$$J(\omega) \equiv \sum_k \frac{\pi C_k^2}{2m_k \omega_k} \delta(\omega - \omega_k) = \begin{cases} \eta\omega & \text{if} \quad \omega < \Omega \\ 0 & \text{if} \quad \omega > \Omega \end{cases}$$

Dissipation

A few problems of interest

i) Motion of a wavepacket in the harmonic potential



$$m\langle \ddot{x}(t)\rangle + \eta\langle \dot{x}(t)\rangle + m\omega_0^2\langle x(t)\rangle = 0$$

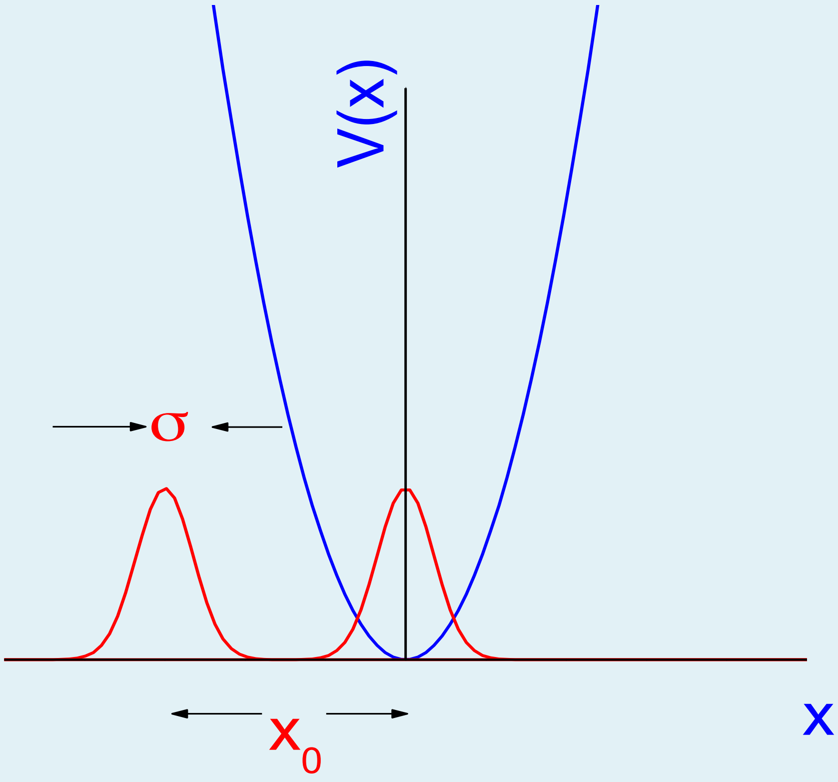
$$\langle x^2(t)\rangle = \frac{\hbar}{\pi}\int_0^\infty \chi''(\omega)\coth\left(\frac{\hbar\omega}{2k_BT}\right)\cos\omega t\, d\omega$$

$$\chi''(\omega) = \frac{1}{m}\frac{2\gamma\omega}{\left(\omega^2-\omega_0^2\right)^2 + 4\gamma^2\omega^2}$$

$$\gamma = \frac{\eta}{2m}$$

Dissipation

ii) Interference of wavepackets in the harmonic potential



Initial state of S

$$\psi(x,0) = \psi_1(x,0) + \psi_2(x,0)$$

Dissipation

Diagonal element of the reduced density operator for S at t is

$$\rho(x,t) = \rho_1(x,t) + \rho_2(x,t) + \\ + \; 2\sqrt{\rho_1(x,t)\,\rho_2(x,t)}\;\cos\varphi(x,t)\;\; e^{-\Gamma t}$$

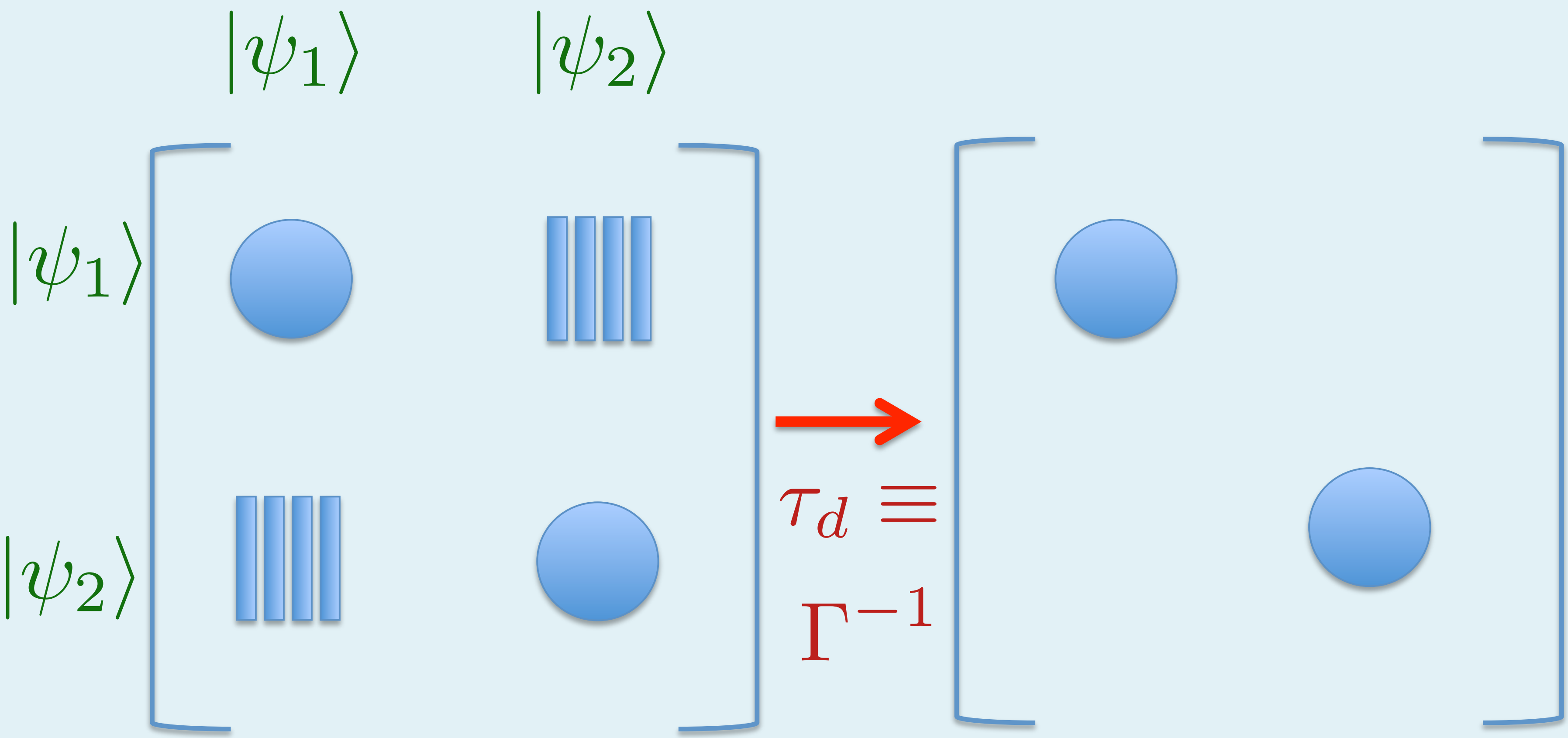
center of $\rho_1(x,t) \sim e^{-\gamma t}$ but for the underdamped case, for instance,

$$\Gamma = \begin{cases} N\gamma & if \quad T = 0 \\ \dfrac{2N\gamma\, k_B T}{\hbar\omega_0} & if \quad k_B T >> \hbar\omega_0 \end{cases}$$

where $N = (x_0/\sigma)^2$. For the strongly overdamped case,

$$\gamma \rightarrow \frac{\omega^2}{2\gamma}$$

Dissipation



Within a very short time interval the pure state becomes a statistical mixture : decoherence

Dissipation

Subtle procedures of preparation of quantum mechanical states allow us to build the following "Schrödinger cat" state either in optical cavities where one has the superposition of two coherent states of the electromagnetic field

$$|\psi\rangle = |\alpha\rangle + |-\alpha\rangle$$

Brune M. et al PRL 77, 4887 (1996)
Haroche

or in atomic traps where

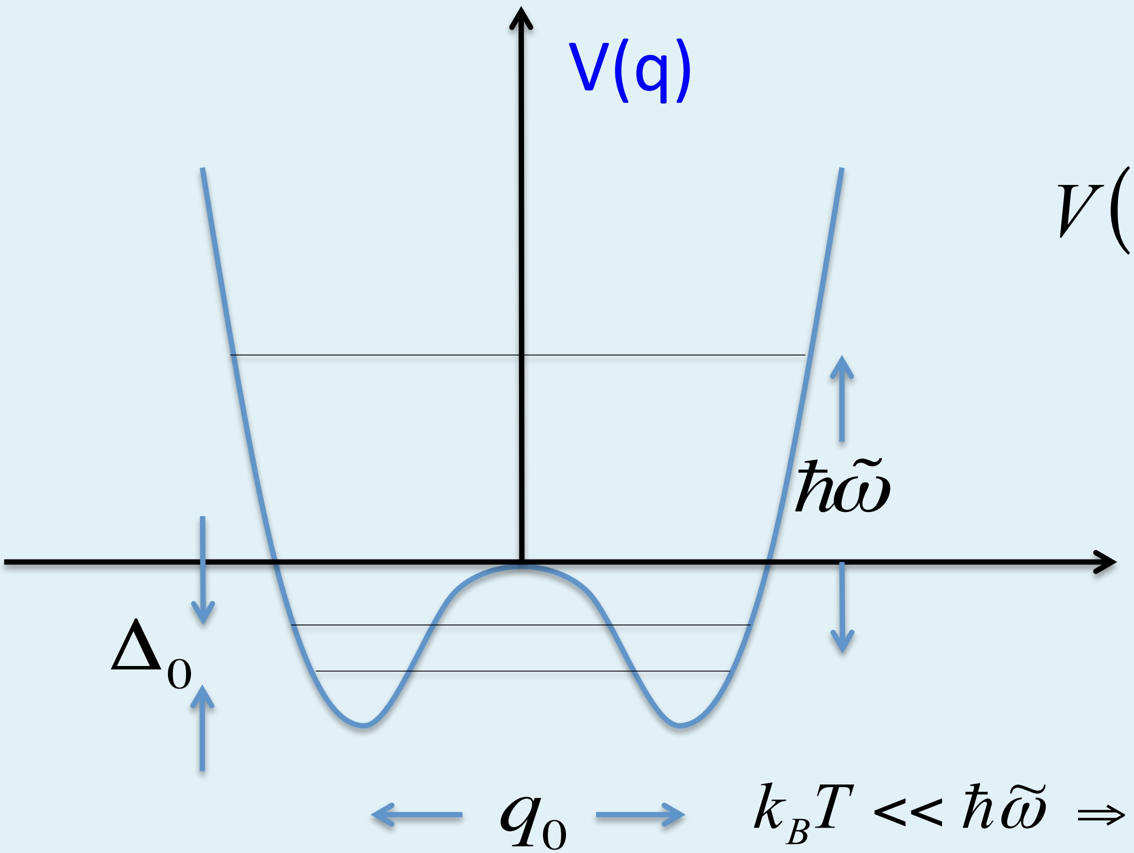
$$|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$$

Monroe C. et al Science 272, 1131 (1996)
Wineland

represents a superposition of two spatially separated states of the ion, exactly like we proposed in the case of the harmonic oscillator

Dissipation

iii) Coherent tunnelling



$$V(q) = -\frac{1}{2} m\omega_0^2 q^2 + \frac{\lambda}{4} q^4$$

$k_B T << \hbar\tilde{\omega} \Rightarrow$ two level system + damping

$$\langle q(t) \rangle = \frac{q_0}{2} \cos \frac{\Delta_0 t}{\hbar} \qquad \text{if} \qquad \gamma = 0$$

Dissipation

Quartic Hamiltonian

$$H = \frac{p^2}{2m} - \frac{1}{2} m\omega_0^2 q^2 + \frac{\lambda}{4} q^4$$

truncation

Spin – boson Hamiltonian

$$H = -\frac{1}{2}\hbar\Delta\sigma_x + \frac{1}{2}\epsilon\,\sigma_z + \frac{1}{2} q_0\,\sigma_z \sum_k C_k\, q_k + \sum_k \frac{p_k^2}{2m_k} + \sum_k \frac{1}{2} m_k\,\omega_k^2\, q_k^2$$

Dissipation

Two important models

Spin – boson:
Amplitude damping

$$H = -\frac{1}{2}\hbar\Delta\sigma_x + \frac{1}{2}\, q_0\, \sigma_z \sum_k C_k\, q_k + \sum_k \frac{p_k^2}{2m_k} + \sum_k \frac{1}{2}\, m_k\, \omega_k^2\, q_k^2$$

Phase damping

$$H = -\frac{1}{2}\hbar\Delta\sigma_z + \frac{1}{2}\, q_0\, \sigma_z \sum_k C_k\, q_k + \sum_k \frac{p_k^2}{2m_k} + \sum_k \frac{1}{2}\, m_k\, \omega_k^2\, q_k^2$$

Dissipation

Maps: an alternative

Time parametrization $p = (1 - e^{-\gamma t})$

Map of phase damping for 1-qubit

$$|g\rangle_B \otimes |0\rangle_E \to |g\rangle_B \otimes |0\rangle_E$$

$$|e\rangle_B \otimes |0\rangle_E \to \sqrt{1-p}\,|e\rangle_B \otimes |0\rangle_E + \sqrt{p}\,|e\rangle_B \otimes |1\rangle_E$$

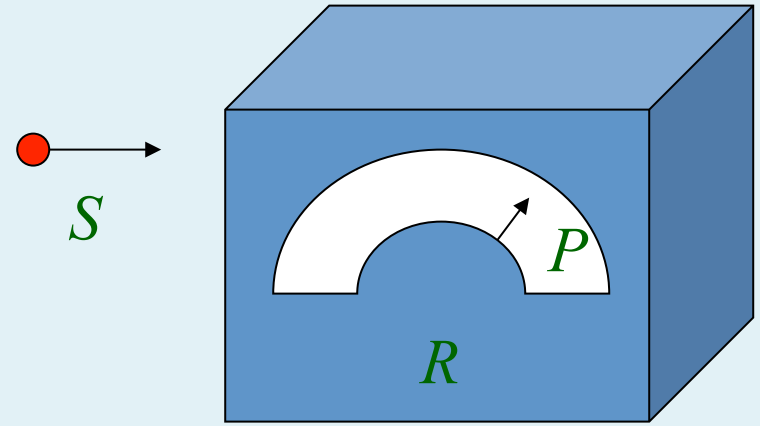
Map of amplitude damping for 1-qubit

$$|g\rangle_B \otimes |0\rangle_E \to |g\rangle_B \otimes |0\rangle_E$$

$$|e\rangle_B \otimes |0\rangle_E \to \sqrt{1-p}\,|e\rangle_B \otimes |0\rangle_E + \sqrt{p}\,|g\rangle_B \otimes |1\rangle_E$$

Quantum measurement theory

Apparatus



system, pointer:

$$|\varphi^{(i)}\rangle = \underbrace{\left(\alpha|\uparrow\rangle + \beta|\downarrow\rangle\right)}_{\text{system}}\underbrace{|p_\downarrow\rangle}_{\text{pointer}}$$

Appropriate coupling

$$|\phi^{(c)}\rangle = \alpha|\uparrow, p_\uparrow\rangle + \beta|\downarrow, p_\downarrow\rangle$$ ← System + pointer entangled

reservoir:

$$|\phi^{(c)}\rangle\underbrace{\psi_0(\vec{r}_1,\ldots,\vec{r}_N)}_{\text{reservoir}} = \left(\alpha|\uparrow, p_\uparrow\rangle + \beta|\downarrow, p_\downarrow\rangle\right)\psi_0(\vec{r}_1,\ldots,\vec{r}_N)$$

Quantum measurement theory

General scheme

Entangled state $$|\psi_s\rangle|A_0\rangle|E_0\rangle \to \sum_i c_i|s_i\rangle|A_i\rangle|E_i(t)\rangle$$

Tracing procedure

$$\rho_{sa} = \sum_{i,j} c_i c_j^* \langle E_j(t)|E_i(t)\rangle |s_i\rangle|A_i\rangle\langle s_j|\langle A_j|$$

$$\langle E_j(t)|E_i(t)\rangle \longrightarrow 0$$

Very fast decay:
Decoherence time

Statistical mixture $$\rho_{sa} = \sum_i |c_i|^2|s_i\rangle|A_i\rangle\langle s_i|\langle A_i|,$$

Quantum information theory

Classical definitions

Shannon entropy

$$H(X) \equiv H(p_1, \ldots, p_n) \equiv$$
$$\equiv -\sum_x p(x) \log p(x)$$

Joint entropy

$$H(X,Y) \equiv -\sum_{x,y} p(x,y) \log p(x,y)$$

Conditional entropy

$$H(X|Y) \equiv H(X,Y) - H(Y)$$

Mutual Information (classical)

$$J(X:Y) \equiv H(X) + H(Y) - H(X,Y) =$$
$$= H(X) - H(X|Y)$$

Quantum information theory

Quantum definitions

Von Neumann (vN) entropy $$S(\rho) \equiv -\mathrm{tr}(\rho \log \rho)$$

Two possible choices for the mutual information

$$\begin{cases} I(\rho_{sa}) = S(\rho_s) - S(\rho_{s|a}) \\ J_{s|\{\Gamma_i^a\}}(\rho_{sa}) = S(\rho_s) - \sum_i p_i S(\rho_s^i|\Gamma_i^a) \end{cases}$$

Classical correlations

$$J_{s|a}^{\max}(\rho_{sa}) = \max_{\{\Gamma_i^a\}} \left[S(\rho_s) - \sum_i p_i S(\rho_s^i|\Gamma_i^a) \right]$$

Quantum discord

$$\delta_{s|a}(\rho_{sa}) = I(\rho_{sa}) - J_{s|a}^{\max}(\rho_{sa})$$

Quantum information theory

Theorem 1: Let ρ_{sa} be the state of system-apparatus at a given moment. If the apparatus is subject to a decoherence process that leads to the projectors on the pointer basis $\{\Pi_i^a\}$, then $J_{s|\{\Pi_i^a\}}$ is constant throughout the entire evolution.

Theorem 2: Let ρ_{sa} be the state of the system-apparatus at a given moment. If the apparatus is subject to a decoherence process leading to the projectors on the pointer basis $\{\Pi_i^a\}$ and $J_{s|\{\Pi_i^a\}} > 0$, then either (i) $J_{s|a}^{\max}$ is constant and equal to $J_{s|\{\Pi_i^a\}}$, or (ii) $J_{s|a}^{\max}$ decays monotonically to value $J_{s|\{\Pi_i^a\}}$ in a finite time, remaining constant and equal to $J_{s|\{\Pi_i^a\}}$ for the rest of the evolution.

M. F. Cornelio et al
http://arxiv.org/abs/1203.5068
and accepted PRL

Quantum information theory

Initial state of a 2-qubit system $\rho_{sa} = \begin{pmatrix} c & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z & b & 0 \\ w & 0 & 0 & c \end{pmatrix}$

$$\begin{aligned} \rho_{sa} = c(&|00\rangle\langle 00| + |11\rangle\langle 11|) \\ +w(&|00\rangle\langle 11| + |11\rangle\langle 00|) \\ +b(&|01\rangle\langle 01| + |10\rangle\langle 10|) \\ +z(&|01\rangle\langle 10| + |10\rangle\langle 10|) \end{aligned}$$

Initial condition 1: c = 0.4, b = 0.1, z = 0.1 and w = 0.4

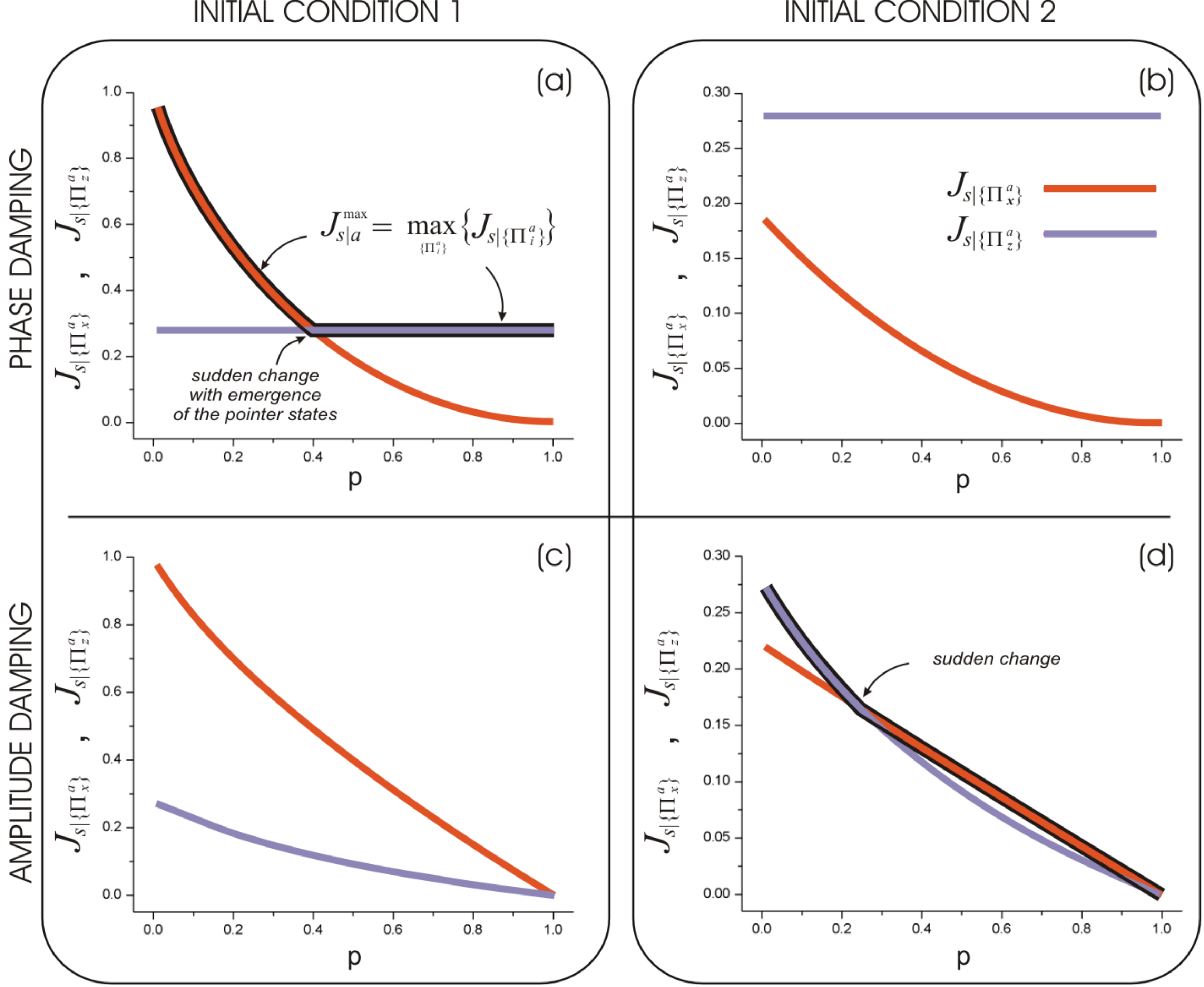
Initial condition 2: c = 0.4, b = 0.1, z = 0.1 and w = 0.15

Quantum information theory

Quantum to classical transition $$J_{s|a}^{\max} = J_{s|\{\Pi_z^a\}}$$

Emergence of the pointer basis $$\tau_E = \frac{1}{\gamma} \ln \left| \frac{z+w}{c-b} \right|$$

Time parametrization $$p = (1 - e^{-\gamma t})$$



INITIAL CONDITION 1
INITIAL CONDITION 2
PHASE DAMPING
AMPLITUDE DAMPING
(a)
(b)
(c)
(d)
$J_{s|\{\Pi_x^a\}}$, $J_{s|\{\Pi_z^a\}}$
$J_{s|a}^{\max} = \max_{\{\Pi_i^a\}} \{J_{s|\{\Pi_i^a\}}\}$
sudden change
with emergence
of the pointer states
sudden change
p

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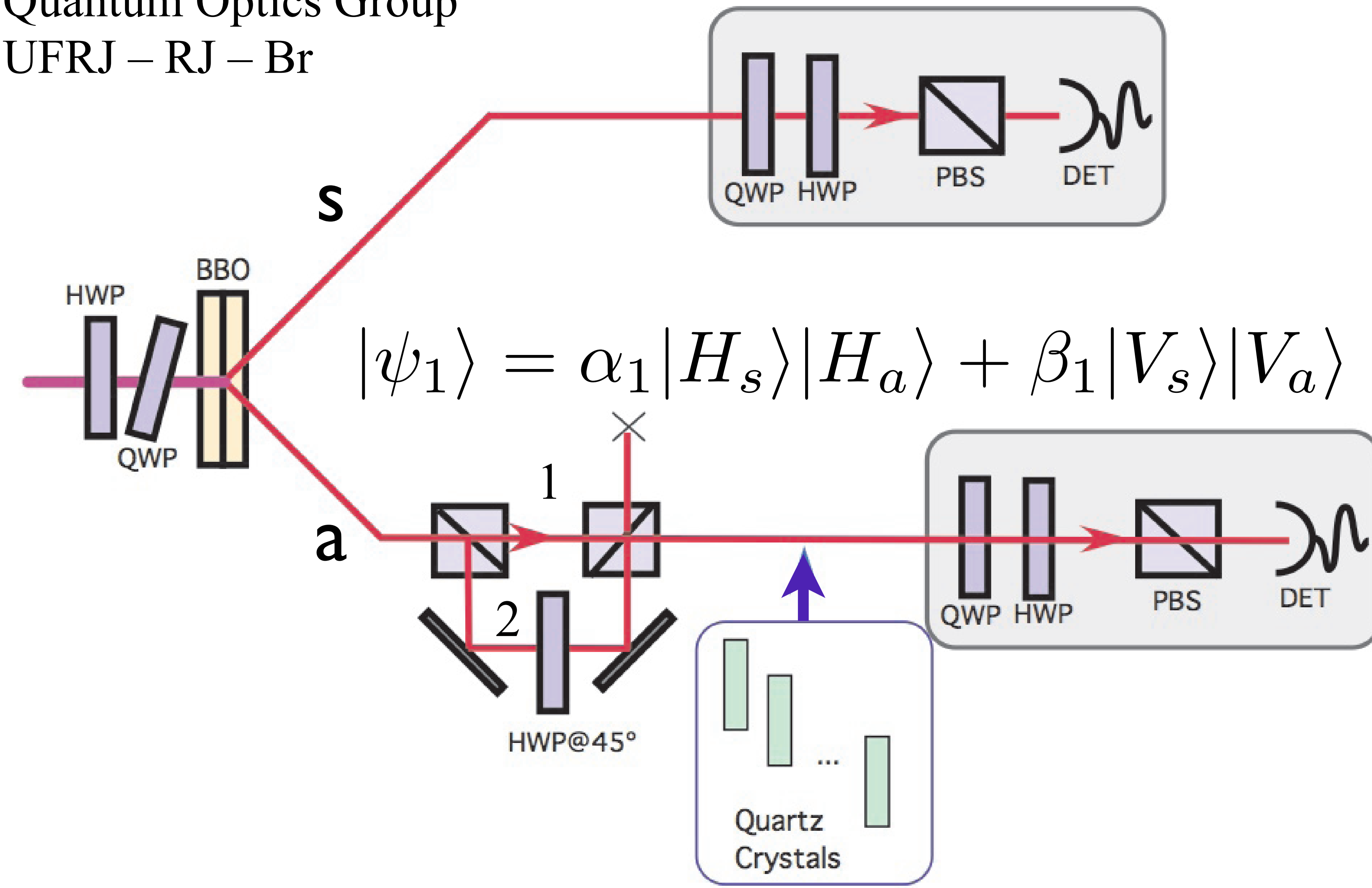
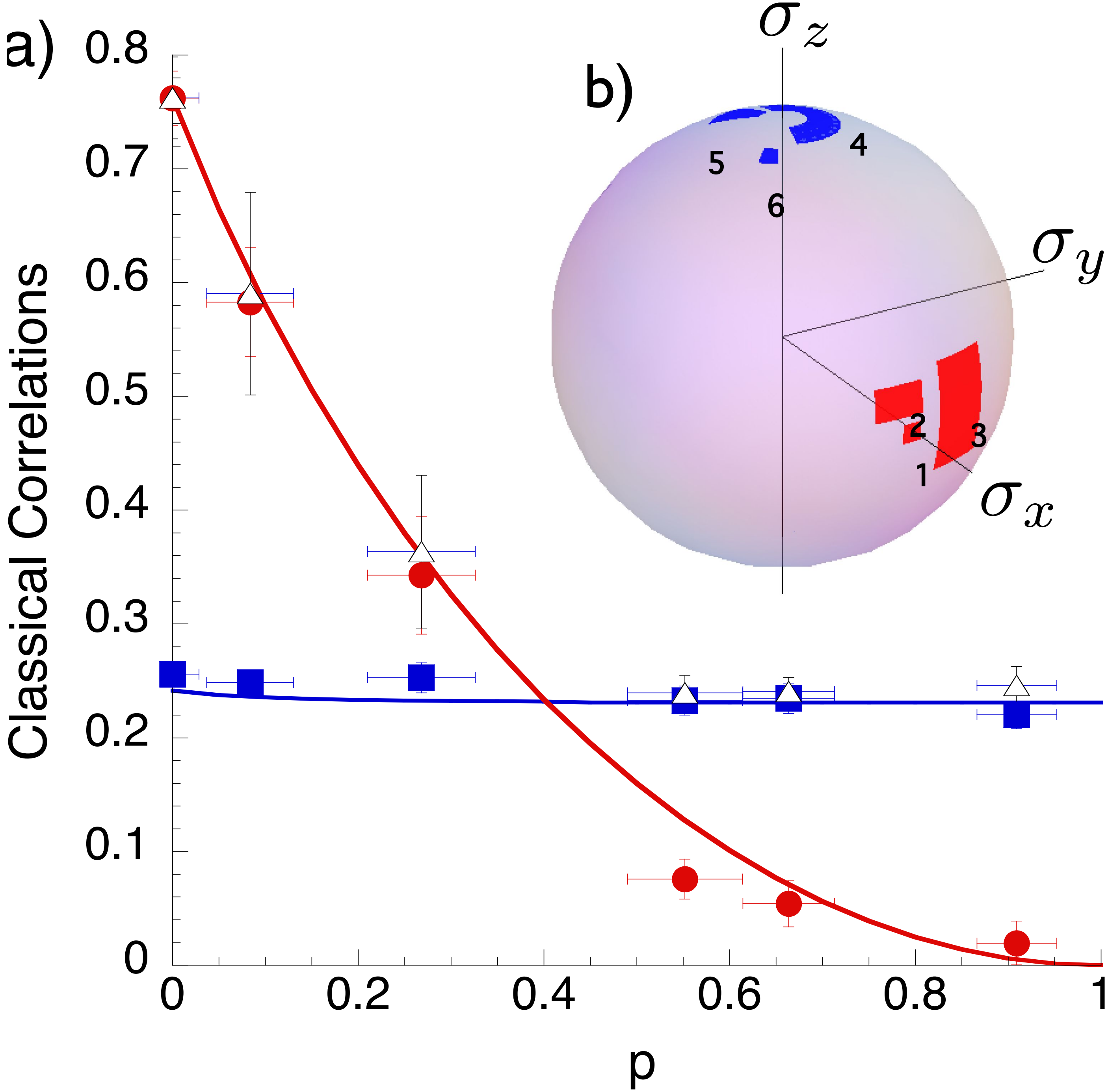


$$|\psi_1\rangle = \alpha_1|H_s\rangle|H_a\rangle + \beta_1|V_s\rangle|V_a\rangle$$

$$|\psi_2\rangle = \alpha_2|H_s\rangle|V_{a2}\rangle + \beta_2|V_s\rangle|H_2\rangle$$



a)
Classical Correlations
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
p
b)
σ_z
σ_y
σ_x
1
2
3
4
5
6

Conclusions

Cat-like states can be built , at least meso or nanoscopic ones, but …

The dreadful decoherent effects of environments can drastically shorten their lifetimes and consequently severely limit their utility

Bad for superposition states but could explain classical pointers, moreover …

Classical correlations can be even more efficient for the emergence of the pointer basis. Would this shine some light on the quantum measurement problem?