Splitting electrons into quasiparticles with fractional edge-state interferometers

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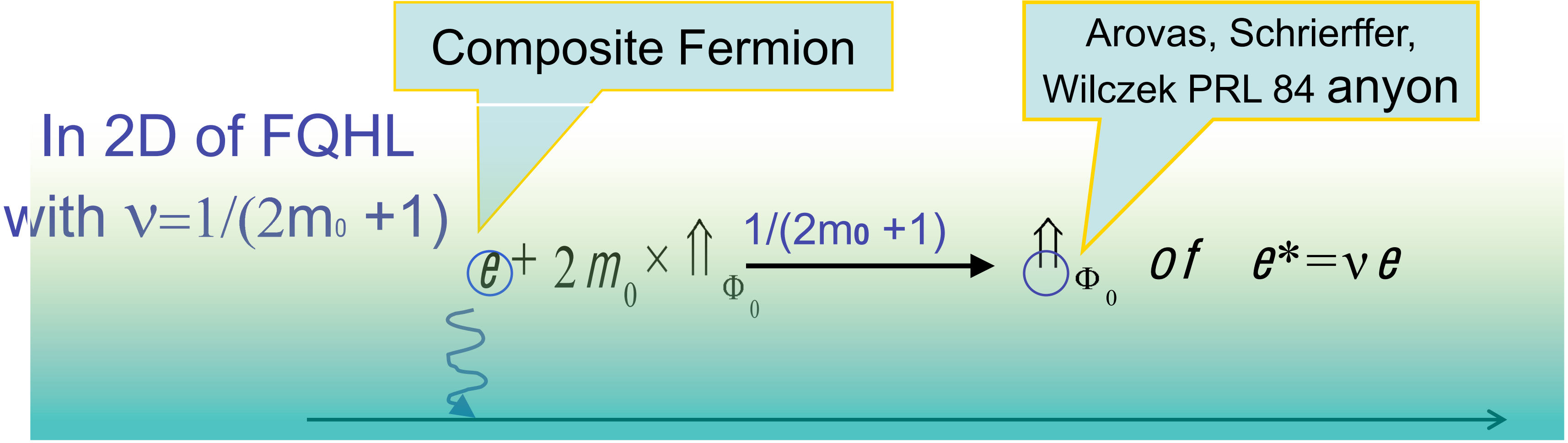
Acknowledgment: Dmitri V. Averin, Stony Brook University, USA

From anyons to edge excitations



On the edge: $\psi(x,t)$

$$<\psi(x,t)\psi^{+}(0,0)> \propto (x - vt)^{-(2m_0+1)}$$

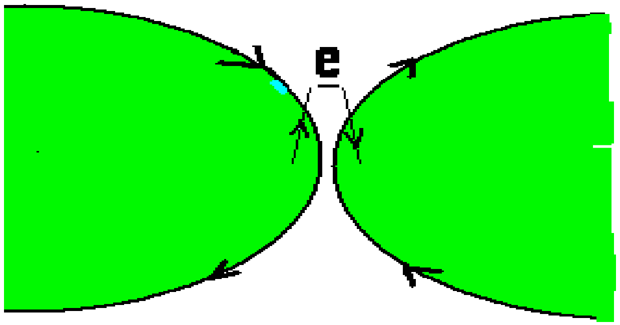
$$\psi \propto \exp\{i\frac{1}{\sqrt{\nu}}\varphi\}, \quad [\psi(x),\rho(y)] = \delta(x-y), \quad \rho(x) = \frac{\sqrt{\nu}}{2\pi}\partial_x\varphi(x)$$

No quantization of charge propagating along the edge:

$$L_{chiral} = \frac{1}{8\pi}\int dx\, \partial_x\varphi(t,x)(\partial_t - v\partial_x)\varphi(t,x)$$

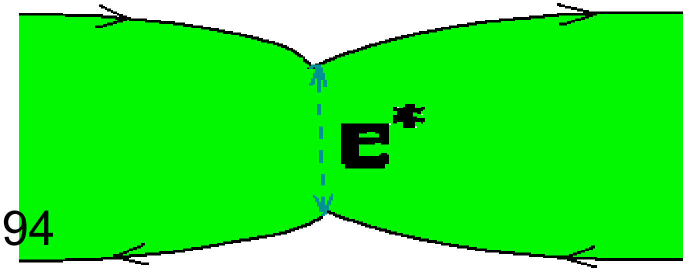
Quantization through injection in the contacts $\Longrightarrow$
- Two-terminal conductance,
- "contact" fractional charge

Single-point contact



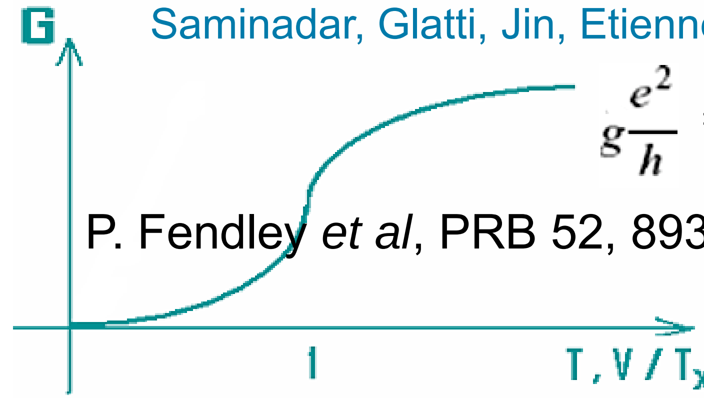
$e^* = \nu e$

Wen PRB 91;
Kane, Fisher PRL 94

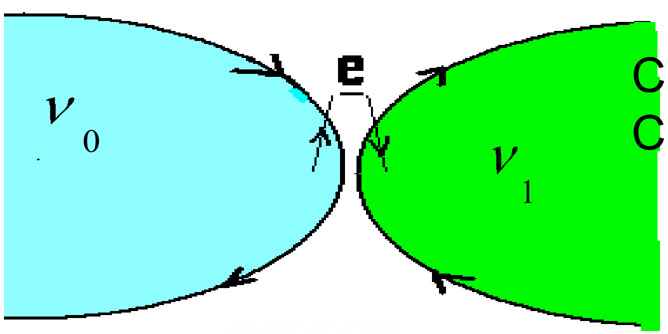


de Piccioto,Reznikov, Heiblum et al Nature 97;
Saminadar, Glatti, Jin, Etienne PRL97

$$G_t = g\frac{e^2}{h}\times\begin{cases}\sum_{n=1}^{\infty} c_n(1/g)\left(\frac{V}{2T_K}\right)^{2n(1/g-1)}, & \frac{V}{2T_K}<e^{\delta}\\ 1-\sum_{n=1}^{\infty} c_n(g)\left(\frac{V}{2T_K}\right)^{2n(g-1)}, & \frac{V}{2T_K}>e^{\delta},\end{cases}$$

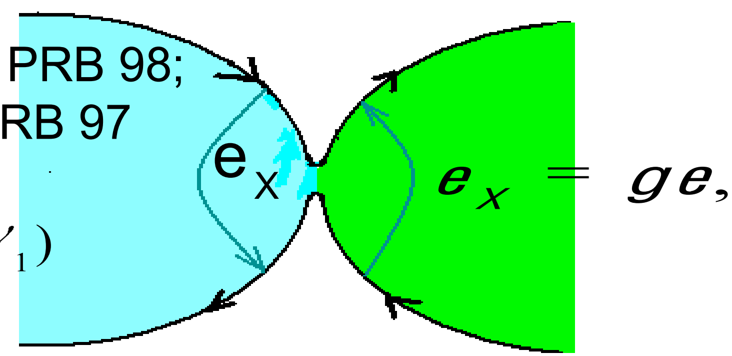


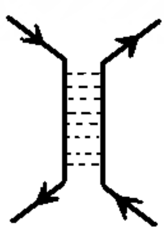
P. Fendley *et al*, PRB 52, 8934 (1995))



Chklovskii, Halperin PRB 98;
Chamon, Fradkin PRB 97

$g = 2\nu_0\nu_1/(\nu_0+\nu_1)$





VP, D.Averin PRL 97, 159701 (06)

Physical motivation

1. *Interference and quantum transport through interfaces between different FQHLs or IQH Fermi liquid and FQHL:*

   reaching equilibrium conductance;
   and stability of "contact" quasiparticles.

2. *Realization of qubits in Topological Quantum Computation*

k^g *degeneracy of ground state of* $\nu = 1/k$ *Laughlin FQHL on genus g surface*

$$W_a W_b = W_b W_a e^{i(2\pi/k)}$$

For each couple of Wilson loop exponentials $W_{a,b} = e^{i\phi_{a,b}}$.

E. Witten, Commun. Math. Phys. 117 (1988), 353.
A. P. Polychronakos, Ann. Phys. 203 (1990), 231.

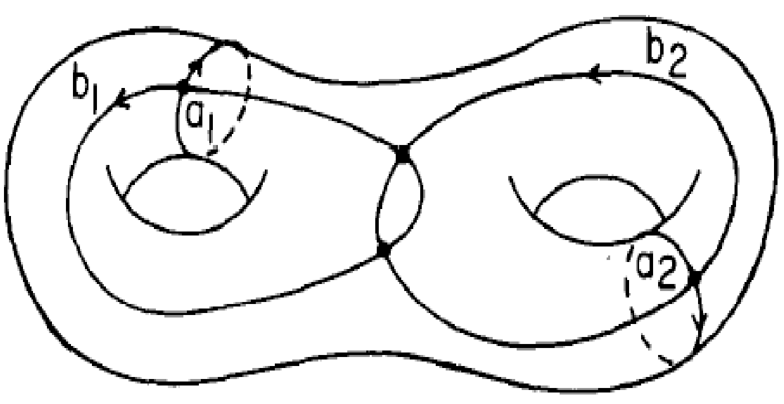


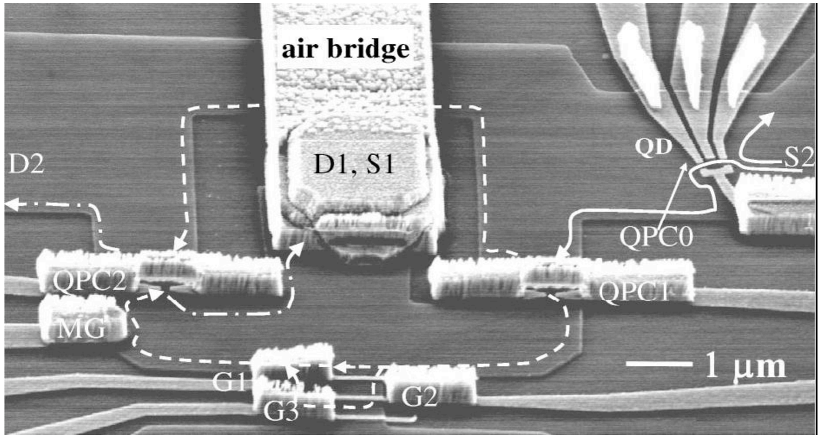
FIG. 1. A genus two surface is depicted with the standard basis of nontrivial loops

Contents

- Mach-Zehnder interferometer
- Antidot interferometer

Electron tunneling regime;
Strong coupling dual model of quasiparticle tunneling ;
Exactly solvable model for resonances;

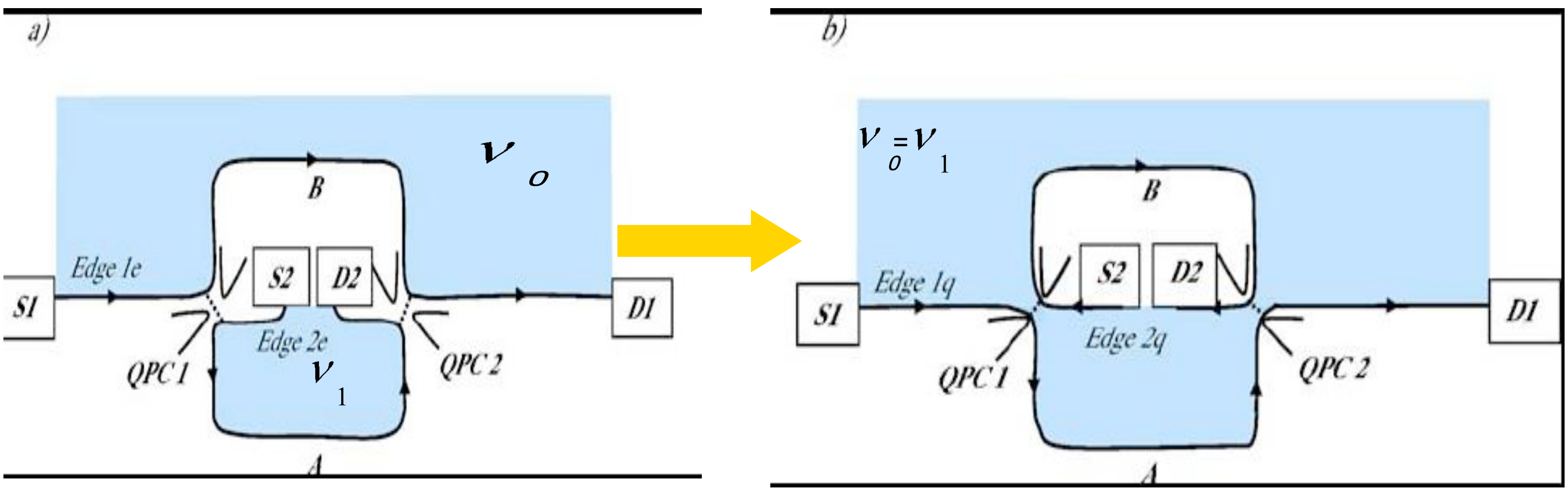
Mach-Zehnder Interferometer



Practical realization in the integer Quantum Hall regime:

Y. Ji *et al.*, Nature (London) **422**, 415 (03);
I. Neder *et al.*, PRL 96, 16804 (06);
L.V. Litvin *et al.*, PRB 75, 033315 (07).

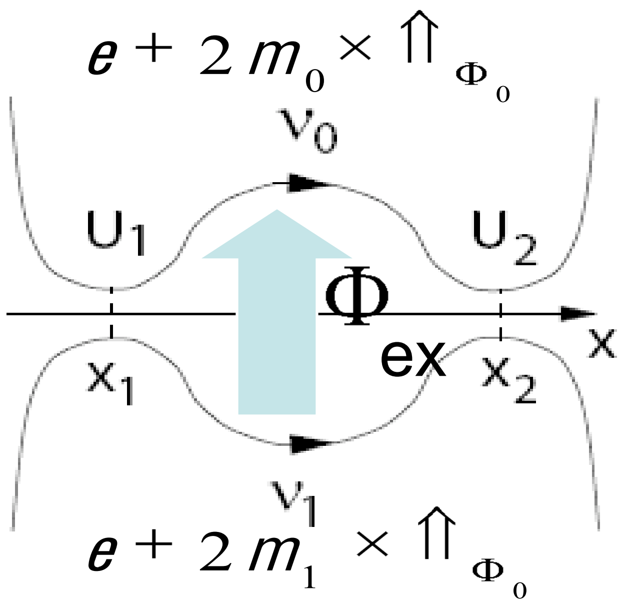
Theory expectation in the fractional QH regime:



VP and D.Averin, PRL 99, 66803 (07);
cond-mat/08093588

C.L.Kane PRL 90, 226802 (03)
T. Jonckheere, P.Devillard, A.Crepieux, T.Martin, PRB (05)
K. T. Law, D. E. Feldman, and Y. Gefen, PRB 74, 45319 (06)

Electron tunneling model 1.



QH liquids of Laughlin series $\nu_l = 1/(2m_l + 1)$, $l = 0, 1$ with integers $m_0 \geq m_1 \geq 0$

$m = m_0 + m_1 + 1$ characteristic parameter

Tunneling electrons:

$$\mathcal{L}_t = \sum_{j=1,2} [\sqrt{v_0 v_1} U_j e^{i\kappa_j} \psi_0^+(x_j, t)\psi_1(x_j, t) + h.c.]$$

propagate along the edges of the liquids with velocities $v_{0,1}$ as chiral low energy density excitations below some common energy cut-off D.

The propagation times between the contacts $t_{0,1} = x_{0,1}/v_{0,1} = \bar{t} \pm \Delta t$

The phases κ_j are defined as

$$\kappa_2 - \kappa_1 = 2\pi[(\Phi_{ex}/\Phi_0) + (N_0/\nu_0) - (N_1/\nu_1)] + \text{const} \equiv \kappa(V)$$

by the external magnetic flux Φ_{ex} through the interferometer and average electron numbers $N_{0,1}$ on both edges, which depend on voltage V.

Its chiral form $\kappa_V \equiv \kappa(V) - V\bar{t}$.

Electron tunneling model 2.

The operator ψ_l of electron propagating along the l edge is

$$\psi_l = (D/2\pi v_l)^{1/2}\xi_l e^{i(\phi_l(x,t)/\sqrt{\nu_l}+k_{F_l}x)},$$

Bosonic form of the tunneling Lagrangian:

$$\mathcal{L}_t = \sum_{j=1,2}[\frac{DU_j}{2\pi}e^{i\kappa_j}e^{i\lambda\varphi_j - iVt} + h.c.] \equiv \sum_{j=1,2}\sum_{\pm} T_j^{\pm}e^{\mp iVt},$$

$$\lambda\varphi_j(t) \equiv \frac{\phi_0(x_j,t)}{\sqrt{\nu_0}} - \frac{\phi_1(x_j,t)}{\sqrt{\nu_1}}, \quad \lambda = \left[\frac{\nu_0+\nu_1}{\nu_0\nu_1}\right]^{1/2} = \sqrt{2m}$$

ϕ_l a free chiral bosonic field at temperature T:

$$< T_\tau\phi_l\phi_l > \equiv g(\frac{x}{v_l},\tau) = -ln\{\delta \mathrm{sgn}\tau \sinh(\pi T(i\tau + \frac{x}{v_l}))\},$$

$$[\phi_l(x),\phi_p(0)] = i\pi \mathrm{sgn}(x)\,\delta_{lp}$$

Perturbative electron current

$$I^e = i[\int dx \rho_0(x), \mathcal{H}] = \frac{\delta}{\delta\phi_0}\mathcal{L}_t = i \sum_{j=1,2} \sum_{\pm} (\pm) T_j^{\pm} e^{\mp iVt}$$

In the second order $I = i\int_{-\infty}^{0} dt < [I^e(0), \mathcal{L}_t(t)] >= \bar{I}^e + \Delta I^e(\kappa)$

$$\begin{aligned}\Delta I^e &= \int_{-\infty}^{\infty} dt e^{iVt} \langle [T_2^-(t), T_1^+(0)] \rangle + \text{h. c.} = \\ &= (\frac{U_1 U_2 D}{\pi^2})(\frac{\pi T}{D})^{\lambda^2-1} \text{Im}\{ \int_{-\infty}^{\infty} ds \sin(\kappa(V) - V\bar{t} - \frac{sV}{\pi T}) \\ &\quad \cdot \prod_{l=0,1} [i \sinh(s - (-1)^l \Delta t \pi T - i0)]^{-1/\nu_l} \}\end{aligned}$$

In general, by residues for integer $1/\nu_l$

$$\Delta I^e \approx \frac{2U_1 U_2 D \sin(V\Delta t) \cos(\kappa_V)(V/D)^{\lambda^2-1}}{\pi i^{1/\nu_1+1}(1/\nu_0 - 1)!(2V\Delta t)^{1/\nu_1}} \quad \text{for} \quad \nu_0 = \nu_1$$

suppressed as a power of $V\Delta t \gg 1$ or an exponent of $T\Delta t \gg 1$

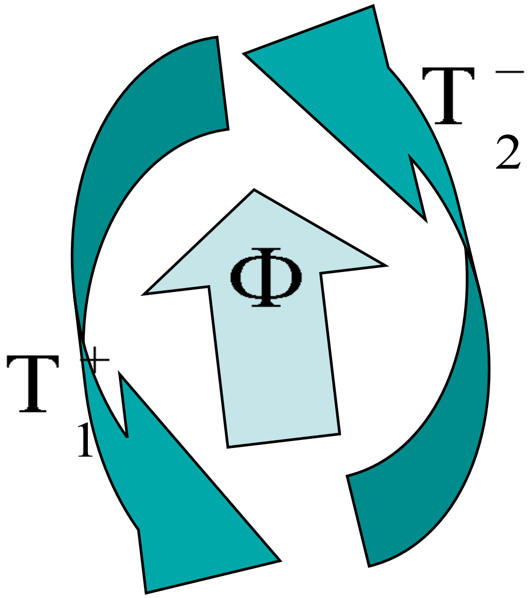
in comparison with ΔI for $T, V < \Delta t^{-1}$

$$\begin{aligned} I &\approx (|U_1 + U_2 e^{i\kappa_V}|^2 D/2\pi)(2\pi T/D)^{\lambda^2-1} C_{\lambda^2}(V/2\pi T) \\ &\propto V[V^2 + (2\pi T)^2]^{\lambda^2/2-1}, \quad C_g(v) = \frac{v}{\Gamma(g)} \prod_{n=1}^{g/2-1} (n^2 + v^2) \end{aligned}$$

behaves as a single point-contact in the lowest order.

Electron interference and effective flux Φ

$T_2^{\pm} T_1^{+} = e^{\mp 2\pi m i} T_1^{+} T_2^{\pm}$ follows from $[\varphi_2, \varphi_1] = i\pi$.



Then $\{T_2^- T_1^+\} T_{1,2}^+ = e^{2\pi m i} T_{1,2}^+ \{T_2^- T_1^+\}$

$$\kappa_2 - \kappa_1 = 2\pi \frac{\Phi}{\Phi_0} \qquad \Phi \to \Phi + m\Phi_0$$

Interferometer in m quantum states :

$|n>$, n=0,…,m-1

specified by values of flux

$$\Phi \, | n > \equiv | n > (\Phi + n\Phi_0 + \text{integer X } m\,\Phi_0)$$

The states are not mixed, if only e-tunneling occurs.

In the strong coupling limit the states are mixed: $\Phi \longrightarrow \Phi + \Phi_0$

The tunneling charge $e \longrightarrow e/m$

Dual model of tunneling quasiparticle

Calculation of the partition function $\mathcal{Z}$ in the limit $U_{1,2} \to \infty$.

1) Search for the strong coupling ground states:

$$T_j^{\pm} \to T_j^{\pm} \exp\{\pm i\sqrt{2\gamma}\eta_j\}, < T_\tau \eta_i(\tau)\eta_j(0) >= i\pi\Theta((j-i)\tau)(1-\delta_{ij})$$

For any integer γ, these Klein factors do not change the perturbation expansion of $\mathcal{Z}$ in $\mathcal{L}_t$ in any order. From minimization of the energy under conditions

VP and D. Averin, Phys. Rev. B 67, 35314 (2003)

$$\lambda\varphi_j + \sqrt{2\gamma}\eta_j + \kappa_j \equiv \Phi_j + \kappa_j = 2\pi n_j \ ,$$

we find $\gamma = m$ and the infinite set of the ground states labeled with integers $n_{1,2}$.

2) The instanton expansion $\Phi_j(\tau) = \Phi_{n_j} + \sum_l 2\pi e_{l,j}\theta(\tau - \tau_{l,j})$, $e_{l,j} = \pm$ can be summed up with the dual tunneling Lagrangian:

$$\bar{\mathcal{L}}_t = \sum_{j=1,2}[\frac{W_j D}{2\pi}\bar{F}_j e^{i(\frac{2}{\lambda}\vartheta_j(t)+\frac{\kappa_j}{m})} + h.c.] \ \text{after} \ \ e_{l,j} \to \bar{F}_j^{e_{l,j}} e^{ie_{l,j}\frac{2}{\lambda}\vartheta_j(\tau_{l,j})} \, .$$

The Klein factors: $\bar{F}_2^{(+)}\bar{F}_1 = e^{\frac{(-)2\pi i}{m}}\bar{F}_1\bar{F}_2^{(+)}$, $< \Pi_{j=1,2}\, \bar{F}_j^{l_j}(\bar{F}_j^{+})^{l'_j} >= \Pi_j\, \delta_{l_j,l'_j}(\mathrm{mod}\, m)$

The dual fields kinetics $< \vartheta_j^2 >= g(0,\omega)$, $< \vartheta_2\vartheta_1 >= \frac{g(t_0,\omega)}{\nu_0\lambda^2} + \frac{g(t_1,\omega)}{\nu_1\lambda^2}$.

Each instanton tunneling of the amplitudes $W_{1,2}$ changes the effective flux by $\pm\Phi_0$.

Dynamics of the dual boson fields 1

If $\Delta t = 0$, $\varphi_j = \phi_-(x_j)$, where $\phi_- \equiv \frac{\sqrt{\nu_1}\phi_0 - \sqrt{\nu_0}\phi_1}{\sqrt{\nu_0+\nu_1}}$, $\quad \phi_+ \equiv \frac{\sqrt{\nu_0}\phi_0 + \sqrt{\nu_1}\phi_1}{\sqrt{\nu_0+\nu_1}}$

"Unfolded" Dirichlet boundary condition:

for a single $\mathcal{L}_{t,j} = DU_j \cos(\lambda\phi_-(x_j) + \kappa_j)/\pi$, $U_j \to \infty$, free chiral propagation of $\theta_{j,-}(x) \equiv \mathrm{sgn}(x - x_j)(\phi_-(x) + \kappa_j/\lambda)$ perturbed by the dual tunnelings $\bar{\mathcal{L}}_{t,j} = DW_j \cos(\frac{2}{\lambda}\theta_{-,j}(x_j))/\pi$.

Successive application of two Dirichlet boundary conditions:

free propagation of the dual chiral field

$$\vartheta_-(x) = \phi_-(x)\theta(x_1 - x) + (\phi_-(x) - 2\frac{\kappa}{\lambda})\theta(x - x_2) - (\phi_-(x) + 2\frac{\kappa_1}{\lambda})\theta(x - x_1)\theta(x_2 - x)$$

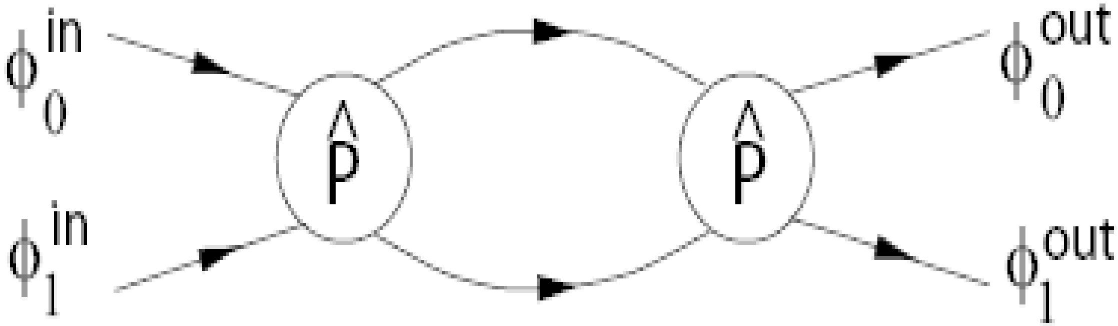
with dual tunnelings $\bar{\mathcal{L}}_{t,j} = DW_j \cos(\frac{2}{\lambda}(\vartheta_-(x_j) + \kappa_j/\lambda))/\pi$.

From comparison with $\bar{\mathcal{L}}_t \;\to\; \vartheta_j = \vartheta_-(x_j)$ and $\bar{\mathcal{L}}_t \simeq \sum_j \bar{\mathcal{L}}_{t,j} \times \bar{F}_j$

Application of voltage $\phi_0^{in} \to \phi_0^{in} - \sqrt{\nu_0}Vt$ leads to $\vartheta_- - Vt/\lambda$.

Dynamics of the dual boson fields 2

For $\Delta t \neq 0$ the fields $\phi_{1,2}$ propagation is determined from recombination of $\phi_{\pm}$



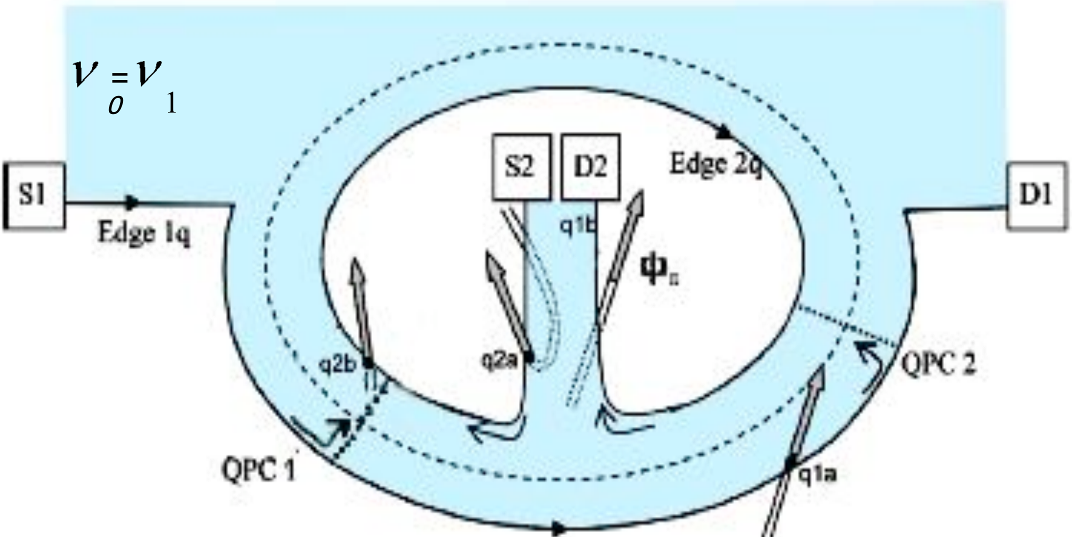
$$P_{00} = -P_{11} = \frac{\nu_0 - \nu_1}{\nu_0 + \nu_1}\ ;$$

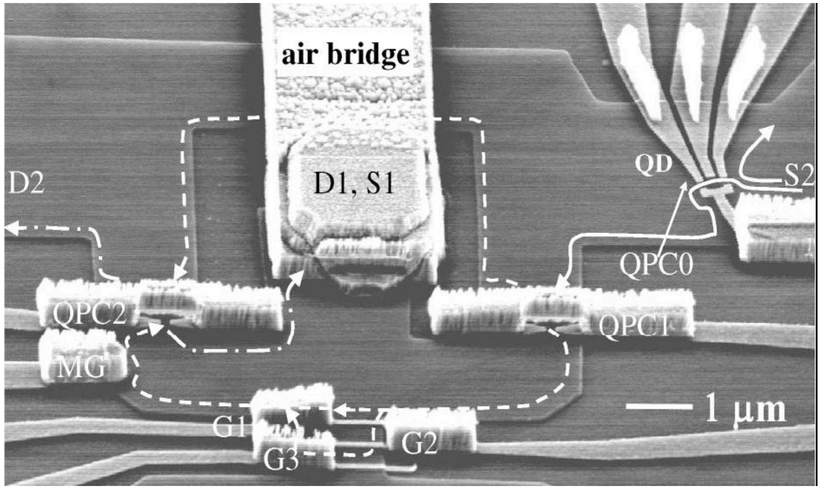
$$P_{01} = P_{10} = -\frac{2\sqrt{\nu_0 \nu_1}}{\nu_0 + \nu_1}$$

for $\nu_0 = \nu_1$ interchange of the edge modes in both contacts.
The S-matrix is always diagonal $\boldsymbol{\phi}^{out} = \hat{S}\boldsymbol{\phi}^{in}, \ \hat{S} = \hat{P}^2 = \mathbf{1}$

The tunneling current $I^q = (i/m)\,\Sigma_{j=1,2}\,\Sigma_{\pm} \pm T_j^{\pm} e^{\mp iVt/m}$ is carried by quasiparticles of fractional charge $e^* = \frac{e}{m} = e_X$ in accordance with $\pm m\mathbf{\Phi}_0 \to \pm\mathbf{\Phi}_0$ reduction

Physical picture of quasiparticle tunneling in the MZI





The Klein factors: $\bar{F}_2^{(+)} \bar{F}_1 = e^{\frac{(-)2\pi i}{m}} \bar{F}_1 \bar{F}_2^{(+)}$, $< \Pi_{j=1,2} \bar{F}_j^{l_j} (\bar{F}_j^{+})^{l'_j} >= \Pi_j \, \delta_{l_j, l'_j} (\mathrm{mod}\ m)$

⇩

$\{\bar{T}_2^{-} \bar{T}_1^{+}\} \bar{F}_{1,2} = e^{-i\frac{2\pi}{m}} \bar{F}_{1,2} \{\bar{T}_2^{-} \bar{T}_1^{+}\}$

- any quasiparticle tunneling from the internal edge decreases Φ→Φ-h/e=Φ-Φ_0;
- qp-interference distinguishes Φ modulo $\Phi_0 \nu$.

m-dimensional matrix irreducible representation:

$X_{l,j} = \delta_{l+1,j}\ (\mathrm{mod}\ m),\ Y_{l,j} = \delta_{l,j} e^{-i\frac{2\pi}{m} l}$ In flux-diagonal basis:

$$\bar{F}_1 = X,\ \bar{F}_2 = -e^{\pm i\pi/m} XY$$

for m=2: $\bar{F}_{1,2} = \sigma_{X,Y}$ - Pauli matrices

1. Exact solution for V,T<<1/Δt

For $m = 2$ ($\nu_0 = 1/3$ and $\nu_1 = 1$) through fermionization.

The Klein factors are two Pauli matrices $\bar{F}_j = i\xi_j\xi_0$ in terms of three Majorana fermions $\{\xi_n, \xi_{n'}\}_+ = 2\delta_{n,n'}$. With chiral fermion field $\psi = \xi_0\sqrt{D/(2\pi v)}\exp(i\theta_-)$ the Hamiltonian reads

$$\mathcal{H} = \{\frac{v}{i}\int dx\, \psi^+\partial_x\psi\} - \sqrt{\frac{Dv}{2\pi}}[\sum_j W_j i\xi_j\psi(x_j)e^{\frac{i}{2}\kappa_{j,V}} + h.c.]$$

Free chiral propagation $\psi(x,t) = \int dk\psi_k \exp\{ik(x - vt)\}/(2\pi)$ and scattering at the two points described with

$$i\psi(x)|_{x_j-0}^{x_j+0} = w_j\xi_j \;,\; \partial_t\xi_j(t) = 2iv[w_j\psi^+(x_j,t) - w_j^*\psi(x_j,t)]\,,$$

$$\text{where} \qquad w_j = i\sqrt{\frac{D}{2\pi v}}W_j e^{-\frac{i}{2}\kappa_{j,V}}.$$

Each of these *disentangled* matching conditions defines a (2×2) scattering matrix $\hat{\mathcal{S}}_{j,k}$ of particle and hole (ψ_k, ψ^+_{-k}).

2 Exact solution for V,T<<1/Δt

In general, for $\lambda^2 = 2m$ with the thermodynamic Bethe ansatz solution (P. Fendley *et al*, Phys. Rev. B **52**, 8934 (1995))

$$I = \int_0^\infty \frac{vdk}{2\pi} |(\hat{\mathcal{S}}_2\hat{\mathcal{S}}_1)^{-,+}|^2 n[f_+ - f_-]\,.$$

Two impurity scattering matrices

$$\mathcal{S}_{j,k}^{\pm\pm} = \frac{(k/T_{jB})^{m-1}e^{i\alpha_k}}{1+i(k/T_{jB})^{m-1}}\,, \mathcal{S}_{j,k}^{-+} = \frac{e^{i(\alpha_k - \kappa_j V)}}{1+i(k/T_{jB})^{m-1}}$$

do not change the density of states $n(k,V)$ and the distribution functions $f_\pm$ for kinks and antikinks defined by the "bulk".

$$\frac{I}{V} = \frac{|T_{1B}^{m-1}e^{i\kappa V} + T_{2B}^{m-1}|^2}{T_{1B}^{2(m-1)} - T_{2B}^{2(m-1)}}\left(G_{1/m}\left(\frac{V,T}{T_{2B}}\right) - G_{1/m}\left(\frac{V,T}{T_{1B}}\right)\right),$$

$G_{1/m}$ is conductance of a single point contact between two $\nu = 1/m$ edges.

The interference is not suppressed by temperature or voltage.

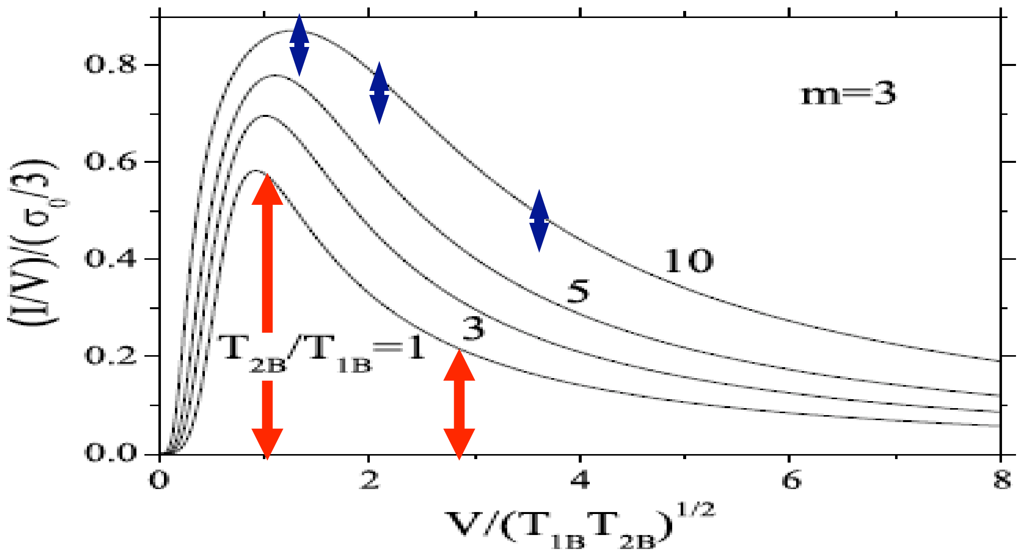
The interference pattern is the same for electrons and quasiparticles.

Asymptotics of electron tunneling for $V < T_{jB}$ and $U_j \propto T_{jB}^{1-m}$

$$\frac{I}{V} \approx \frac{\sigma_0}{m} c_1(m) V^{2(m-1)} \left| \sum_{j=1}^{2} T_{jB}^{1-m} e^{i\kappa_j V} \right|^2 = \sigma_0 \frac{|U_1 + U_2 e^{i\kappa_V}|^2}{\Gamma(2m)} \left(\frac{V}{D}\right)^{2(m-1)},$$

and quasi particle tunneling for $V > T_{jB}$ and $W_j^m \propto T_{jB}^{m-1}$

$$\frac{I}{V} \approx \frac{|\Sigma_{j=1}^2 W_j^m e^{-i\kappa_j V}|^2}{W_1^{2m} - W_2^{2m}} \frac{W_1^2 - W_2^2}{2\pi\Gamma(\frac{2}{m})} \left(\frac{V}{mD}\right)^{\frac{2}{m}-2} = \frac{\sigma_0 (V/mD)^{2/m-2} m}{\Gamma(\frac{2}{m}) \Sigma_{n=1}^m |W_1 + W_2 e^{-i[\kappa_V + 2\pi n]/m}|^{-2}}$$



$$\frac{I}{V} \approx \left[1 + 2\cos\kappa_V \left(\frac{T_{2B}}{T_{1B}}\right)^{m-1}\right] \left(G_{1/m}\left(\frac{V}{T_{2B}}\right) - G_{1/m}\left(\frac{V}{T_{1B}}\right)\right) \quad \text{for} \quad T_{2B} \ll T_{1B}$$

$$\frac{I}{V} \approx \frac{\cos^2(\kappa_V/2)}{m-1} V \, \partial_V G_{1/m}\left(\frac{V}{T_B}\right) \quad \text{for} \quad T_{2B} = T_{1B}$$

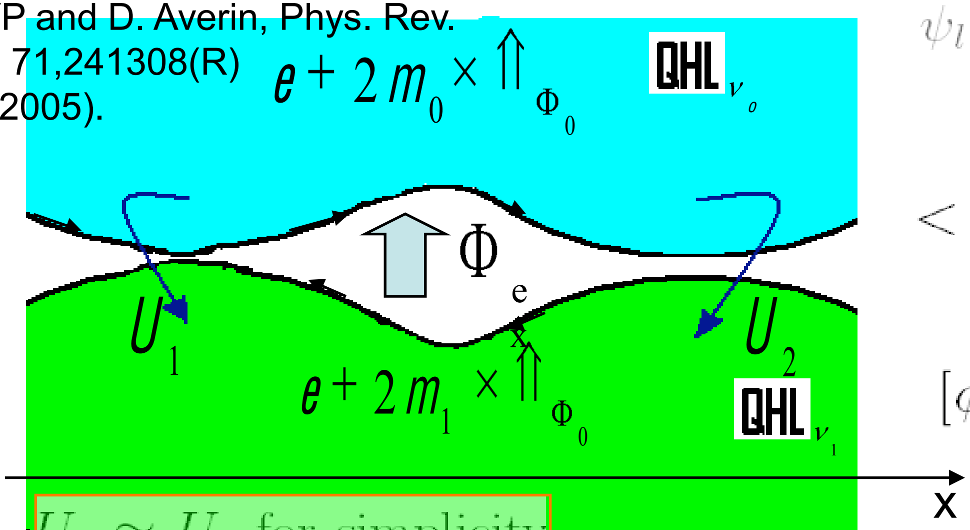
Conclusion about Mach-Zehnder interferometer

- In general, the tunneling conductance G vanishes in both limits of high and low energy (temperature T or voltage V) as $G_H \propto T^{4/\lambda^2-2}$ and $G_L \propto T^{\lambda^2-2}$, $\lambda^2 = 2m = \frac{1}{\nu_0} + \frac{1}{\nu_1}$.
  a periodical function of the external flux Φ_{ex} with its regular period $\Phi_0 = \frac{hc}{e}$.
- Fractional edge-state MZI has m-state quantum dynamics with:
  - reduction of effective flux variation mΦ_0 to Φ_0 restore Φ_0 periodicity and defines fractional charge e/m of quasiparticles at high energy.
- For ΔtV, ΔtT⇨0, the MZI model allows an exact solution, which describes crossover from electron to quasiparticle tunneling

Antidot tunneling model

Electrons propagate in opposite directions along the $l = 0, 1$ edges

VP and D. Averin, Phys. Rev. B 71,241308(R) (2005).



$U_1 \approx U_2$ for simplicity

$$\psi_l = (D/2\pi v_l)^{1/2} \xi_l e^{i(-)^l(\phi_l(x,t)/\sqrt{\nu_l}+k_{F_l}x)}$$

$$< T_\tau \phi_l \phi_l >= g((-)^l t_l, \tau) \approx -ln\{\mathrm{sgn}\tau(\tau \mp it_l)\}$$

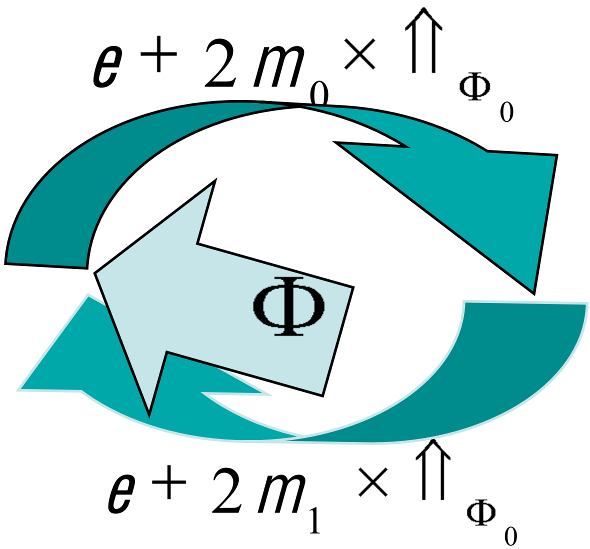
$$[\phi_l(x), \phi_p(0)] = i\pi \mathrm{sgn}(x)\, \delta_{lp}(-)^l$$

$$\mathcal{L}_t = \sum_{j=1,2} [\frac{DU_j}{2\pi} e^{i\kappa_j} e^{i\lambda\varphi_j - iVt} + h.c.] \equiv \sum_{j=1,2} \sum_{\pm} T_j^{\pm} e^{\mp iVt},$$

$$< \varphi_2 \varphi_1 >= \frac{\nu_1 g(t_0,\tau) + \nu_0 g(-t_1,\tau)}{\nu_0 + \nu_1}$$

Electron interference and effective flux Φ

$$T_2^{\pm}T_1^{\mp} = e^{2\pi i m_D}T_1^{\mp}T_2^{\pm} \longrightarrow \Phi \to \Phi \pm m_D\Phi_0 \quad \textit{for} \quad \kappa_2 - \kappa_1 = 2\pi\frac{\Phi}{\Phi_0}$$



antidot in m quantum states : $|n>$, n=0,…,m_D-1

specified by values of flux

$$\Phi_{|n>} \equiv \Phi + n\Phi_0 + \text{integer X } m_D\Phi_0$$

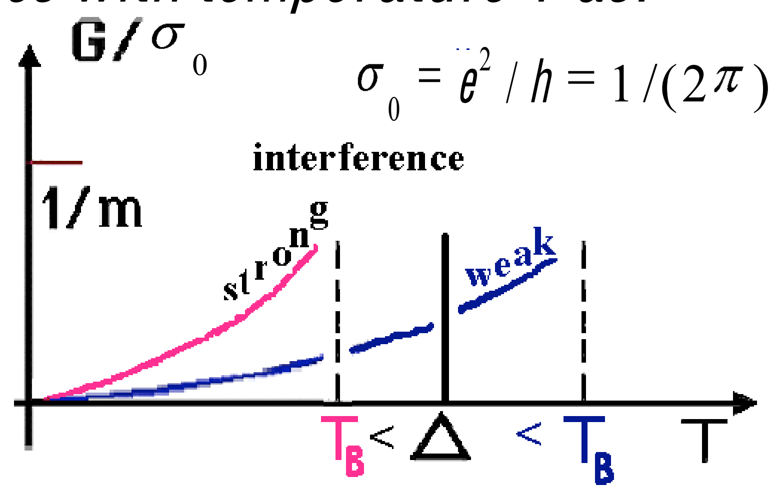
The states are not mixed, if only e-tunneling occurs :

$$(t_0+t_1)^{-1} \equiv \Delta \;, \mathrm{T} < T_B \;: \quad D\,U_{1,2}^{-2/(\lambda^2-2)}, \;\; \lambda = \sqrt{\frac{\nu_0+\nu_1}{\nu_0\nu_1}}$$

Linear bias conductance G increases with temperature T as:

$$G \approx \left[\sum_{j=1}^{2}\frac{U_j}{2\pi}e^{i\kappa_j}\right]^2\left(\frac{T}{D}\right)^{\lambda^2-2} \text{ for } T < \Delta$$

$$G \approx \sum_{j=1}^{2}\left[\frac{U_j}{2\pi}\right]^2\left(\frac{T}{D}\right)^{\lambda^2-2} \text{ for } \Delta < T$$



Dual model of tunneling quasiparticle

1) In the limit $U_{1,2} \to \infty$, the strong coupling ground states from

$$T_j^{\pm} \to T_j^{\pm} \exp\{\pm i\sqrt{2m_D}\eta_j\}, \text{ where } < T_\tau\{\eta_1(\tau)\eta_2(0)\} >= i\pi\theta(\tau)$$

are defined by two conditions :

$$\lambda\varphi_j + \sqrt{2m_D}\eta_j + \kappa_j = 2\pi n_j$$

labeled with integers $n_{1,2}$.

2) The instanton expansion can be summed up with dual tunneling Lagrangian:

$$\bar{\mathcal{L}}_t = \sum_{j=1,2} [\frac{W_j D}{2\pi} \bar{F}_j \exp\{i(\frac{\kappa_j}{m_D} + \frac{2\theta_j}{\lambda} - \frac{Vt}{m_D})\} + h.c.] \equiv \sum_{j=1,2} \sum_{\pm} \bar{T}_j^{\pm} e^{\mp iVt/m_D},$$

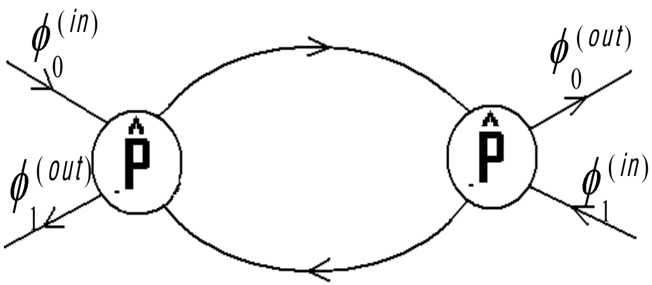
where $W_{1,2}$ are some amplitude of the instanton tunneling.
The new Klein factors $\bar{F}_j$ satisfy:

$$\bar{F}_1 \bar{F}_2 = e^{\frac{2\pi i}{m_D}} \bar{F}_2 \bar{F}_1 \ , < \bar{F}_1^{l_1} (\bar{F}_1^+)^{l_1'} \bar{F}_2^{l_2} (\bar{F}_2^+)^{l_2'} >= \prod_j \delta_{l_j, l_j'} (\text{mod } m_D) \ ,$$

where the delta function is defined modulo m_D.

Dynamics of the dual charge fields

In the strong coupling limit $U_{1,2} \to \infty$
dynamics of the charge densities $\rho_j = \sqrt{\nu_j}\partial_x \phi_j/(2\pi)$, $j = 1, 2$ from application of *two Dirihlet boundary conditions successively*



$$\hat{P}_{0,0} = -\hat{P}_{1,1} = -\cos(2\gamma) = \frac{\nu_0 - \nu_1}{\nu_0 + \nu_1}$$

$$\hat{P}_{0,1} = \hat{P}_{1,0} = \sin(2\gamma) = -\frac{2\sqrt{\nu_0\nu_1}}{\nu_0 + \nu_1}$$

The complete S-matrix remains diagonal $\boldsymbol{\phi}^{out} = \hat{S}\boldsymbol{\phi}^{in}, \ \hat{S} = \mathbf{1}$
the tunneling current in due to quasiparticle tunneling only.

The bosonic field correlators follow from the propagation diagram as:

$$< \theta_j \theta_j >= \quad g(0,\omega) + \frac{1}{2}\sum_{n=1}^{\infty} \sin^{2n}(2\gamma)[g(nt_\Sigma, \omega) + g(-nt_\Sigma, \omega)]$$

$$< \theta_2 \theta_1 >= \quad \sum_{n=0}^{\infty} \sin^{2n}(2\gamma) \times [\cos^2(\gamma) g(t_0 + nt_\Sigma, \omega) + \sin^2(\gamma) g(-t_1 - nt_\Sigma, \omega)] \ .$$

2. Dynamics of the dual charge fields

short-time exponent of qp tunneling for $t < t_{0,1}$: $\frac{4}{\lambda^2} = \frac{4\nu_0\nu_1}{\nu_0+\nu_1} \leq 2$

long-time exponent of qp tunneling for $t > t_\Sigma$:

$$\frac{4}{\lambda^2}\left(\frac{\nu_0+\nu_1}{\nu_0-\nu_1}\right)^2 = \left(\frac{\lambda}{m_D}\right)^2 \begin{cases} > 2, m_D = 1 \\ < 2, m_D \geq 2 \end{cases}$$

Quasiparticle tunneling current.

$$I^{qp} = \frac{i}{m_D}\sum_{j=1,2}\left[\frac{W_j D}{2\pi}\bar{F}_j \exp\{i\left(\frac{2}{\lambda}\theta_j(t) - \frac{Vt}{m_D} + \frac{\kappa_j}{m_D}\right)\} - h.c.\right]$$

The qp-model is convergent in the high energy limit:

$$< I^{qp} > \quad \approx V \sum_{j=1}^{2}\left[\frac{W_j}{2\pi}\right]^2 \left(\frac{|V|}{D}\right)^{4/\lambda^2-2} \quad \text{for } \Delta, T_B \ll |V| \ ,$$

$$< \{I^{qp} \ , \ I^{qp}\}_+ > |_{\omega\to 0} = 2\frac{e}{m_D} < I^{qp} > \ \text{ and } T \ll |V| \ .$$

The qp-tunneling current is quantized in the fractions of charge $e^* = \frac{e}{m_D} \neq e_X$.

The quasiparticle tunneling corresponds to $\Phi \pm \mathbf{\Phi_0}$ variation of the effective flux.

Anti-dot tunneling for $m_D = 1$.

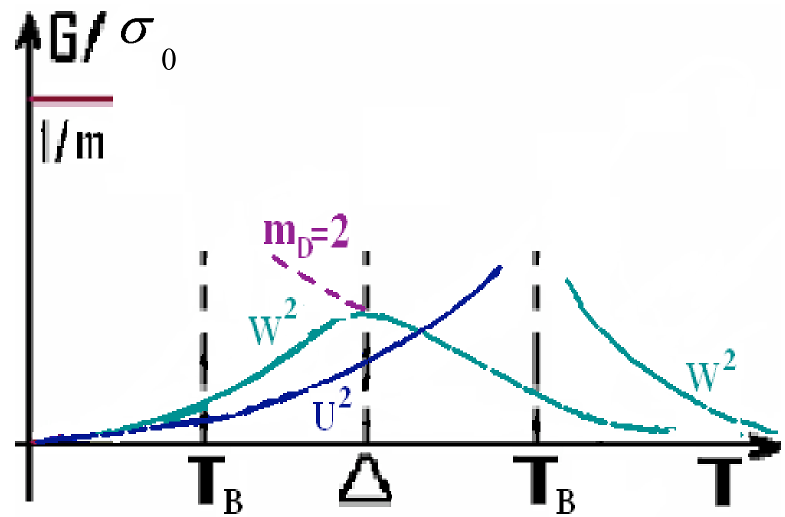
The Klein factors commute and drop out.

In the **strong interference regime**:
$T_B \leq \Delta$ and $W_{1,2}$ are small, the qp-model converges at any energy.

$$G \approx \left[\sum_{j=1}^{2} \frac{W_j}{2\pi} e^{i\kappa_j}\right]^2 \left(\frac{T}{D}\right)^{\lambda^2-2} \quad \text{for } T < \Delta$$

$$G \approx \sum_{j=1}^{2} \left[\frac{W_j}{2\pi}\right]^2 \left(\frac{T}{D}\right)^{4/\lambda^2-2} \quad \text{for } \Delta < T$$

and reaches its maximum $G \ll \frac{\sigma_0}{m}$ at $T \approx \Delta$.



In the **weak interference regime**,
when $\Delta < T_B$ and $U_{1,2}$ are small,
the perturbative qp-solution is physical only if V or T is larger than T_B. It matches at T_B with the perturbative solution of the electron tunneling model, which is relevant below T_B and non-perturbative in $W_{1,2}$. $G(T)$ has maxima at $T \approx T_B$.

Two-state antidot Q-bit

m_D =2 if tunneling between ν_0=1/5 FQHL and ν_1=1 IQHL

Flux through the antidot takes two values $\Phi|\uparrow\rangle=\Phi+\Phi_0$ *and* $\Phi|\downarrow\rangle=\Phi$

The operators of qp-tunneling flip the flux as two Pauli matrices $F_{1,2}=\sigma_{x,y}$

In the **strong interference regime**, $T_B < \Delta$

at low energy $(T, V < \Delta)$ the two qp-tunneling operators are combined into the Lagrangian:

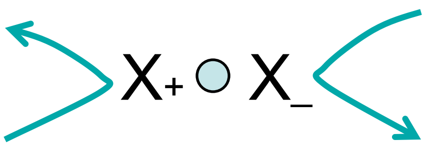
$$\hat{U} = \exp\{i\sigma_3\sqrt{\frac{3}{8}}\tilde{\vartheta}(0)\}$$

$$\bar{\mathcal{L}}_{tunn} = [\sum_{\pm}\frac{\Delta}{2\pi}X_{\pm}e^{i\varphi_{\pm}}\hat{\sigma}_{\pm}]e^{i\sqrt{\frac{3}{2}}\vartheta(t)} + H.c.$$

$$X_{\pm} = \tilde{c}\quad(\alpha\Delta)^{2/\lambda^2-1}\sqrt{W_1^2+W_2^2\mp 2W_1W_2\sin(\kappa/2)}$$

$$e^{i2\varphi_{\pm}} = \frac{W_1\mp iW_2e^{-i\kappa/2}}{W_1\pm iW_2e^{i\kappa/2}}, \quad \kappa = 2\pi\frac{\Phi_{ex}}{\Phi_0}$$

where $\hat{\sigma}_{\pm}$ are rising and lowering flux, respectively, and bosonic field ϑ is defined by $<\vartheta(t)\vartheta(0)>\approx -\ln(t)$.

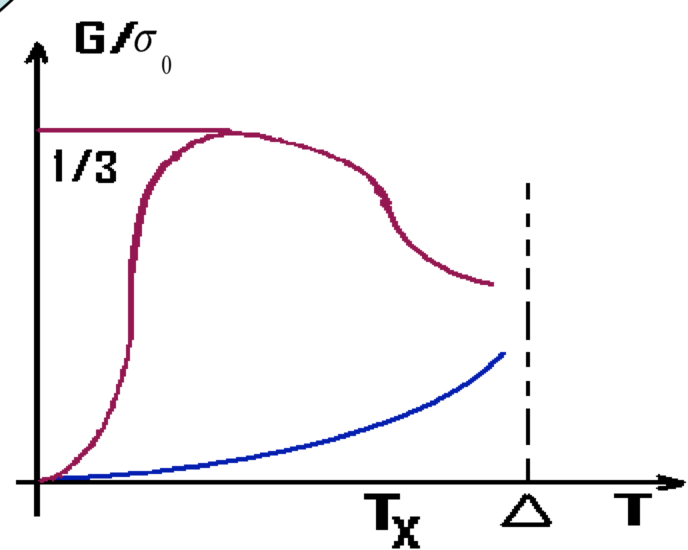


Model of a resonant level between two $g = 1/3$ TLL

Linear bias tunneling conductance on temperature:

The conductance reaches its maximum $\frac{\sigma_0}{3}$ at low T in exact resonances at $\Phi_{ex} = integer \times \Phi_0$.

goes to zero at zero T everywhere except for exact resonances as a function of $\sin^2(\frac{\kappa_1-\kappa_2}{2})/T^{2/3}$.



Conclusion about antidot interferometer

1. For $\nu_0 = 1/(2m_0+1) \neq \nu_1 = 1/(2m_1+1)$ the antidot tunneling conductance G vanishes in the both limits of high and low energy (temperature T or voltage V) as $G_H \propto T^{4/\lambda^2-2}$ and $G_L \propto T^{\lambda^2-2}$, $\lambda^2 = \frac{1}{\nu_0} + \frac{1}{\nu_1} = 2m$ (except for resonances; is a periodical function of the flux Φ with its period equal to $\Phi_0 = \frac{hc}{e}$.

2. The antidot interferometer possesses m_D-state quantum dynamics: reduction of the effective fluxes Φ variations $m_D\Phi_0 \to \Phi_0$ introduces fractional charge $e/m_D \neq e_X = e/m$ observable in the high energy shot-noise.

3. In the strong interference regime, $T_B \leq \Delta$, G is non-perturbative in electron tunnelings;
for $\frac{m}{m_D^2} < 1$ (the antidot tunneling between $\nu_0 = 1/5$ and $\nu_1 = 1$) features zero-energy resonances $G = \frac{\sigma_0}{m}$, where backscattering is quantized in the fractional charge e_X of point-contact quasiparticles.