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# Quantum Hall hierarchy wave functions from Conformal Field Theory

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# Outline

- FQHE: basics, hierarchy and composite fermions
- Introduction to CFT description of QH wave functions
- Motivation and main results of our work
- CFT description of QH quasielectrons
- ‘Condensates’ of quasielectrons: Jain sequence & hierarchy
- Justification of results (consistency checks)
- Explicit hierarchical form of Jain wave functions
- Quasihole condensates and ‘mixed’ states
- Summary and outlook

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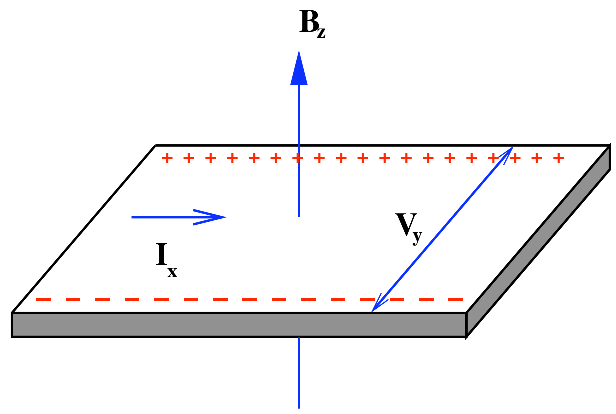
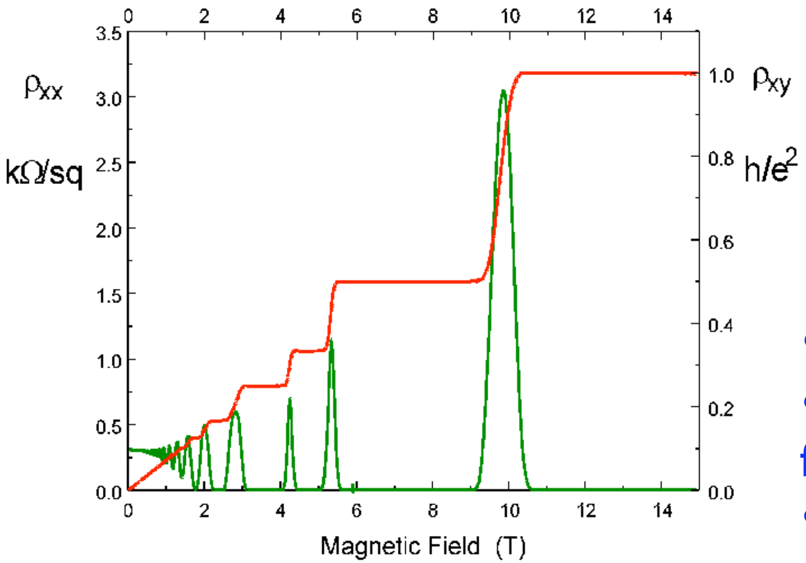
# The quantum Hall effect

2-dimensional electron gas in a strong perpendicular magnetic field at low temperatures




Transverse (Hall) resistance is quantized

$$R_{xy} = \frac{V_y}{I_x} = \frac{1}{\nu}\frac{h}{e^2}$$

- *ν*: Landau level filling at center of plateau.
- Incompressible ground states at integer and fractional *ν* (e.g. 1/3, 2/5, ...).
- Fractionally charged quasiparticle excitations

# FQHE and many-body wave functions

- 2-dimensional electron gas in a strong perpendicular magnetic field at low T
- Electrons residing (mainly) in the lowest Landau level (LLL).

Single particle basis states: $|l\rangle = N_l z^l e^{-|z|^2/4}, \quad z = x + iy$

N-particle (trial) wave functions constructed as *antisymmetric combinations* of these, i.e. homogeneous polynomials. Total angular momentum = degree of polynomial.

The construction of trial many-body wave functions, believed to capture the essential physics, has proven fruitful in the study of the quantum Hall system. Laughlin, hierachy, composite fermions...

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# Laughlin state and its excitations

Laughlin's famous wave function, ground state at $\nu$=1/m:

$$\psi_N(z_1, ...z_N) = \prod_{i<j}(z_i - z_j)^m e^{-\sum |z_i|^2/4}$$

Quasihole at $z_0$: Multiply Laughlin wf by $\prod_i (z_i - z_0)$

Fundamental excitation at $\nu$=*1/m. Local depletion of charge*, with fractional charge *e/m* and obeying fractional statistics.

Quasielectron: Opposite of a quasihole. Local contraction of the QH liquid, with fractional charge and statistics. Form of wave function less obvious...

# Beyond Laughlin I: Hierarchy picture

- Idea: A quantum Hall state can give rise to a sequence of 'daughter states' as successive 'condensates' of quasielectrons and/or quasiholes:
- As the B-field is changed away from its value at the center of the plateau, the number of quasiparticles increases.
- These quasiparticles eventually may form a strongly correlated state, in much the same way as the electrons form the Laughlin state.
- Result: New incompressible *ground state* at a different filling fraction. Quasielectrons *or* quasiholes of this state may again condense to form the next 'daughter' etc.......

$$\Psi_{n+1}(z_1 \ldots z_N) = \int d^2\vec{R}_1 \ldots \int d^2\vec{R}_M \, \Phi^\star(\vec{R}_1 \ldots \vec{R}_M) \Psi_n(\vec{R}_1 \ldots \vec{R}_M; z_1 \ldots z_N)$$

***Hard integrals!***

Multi quasi particle "pseudo" wave function for the quasi particles at $\mathbf{R}_\mathbf{i}$

Electronic wave function parametrically depending on the quasi particle positions $\mathbf{R}_\mathbf{i}$

# Beyond Laughlin II: Composite fermions

- Jain: Composite fermion = Electron with 2p flux quanta attached (thus effectively moving in a reduced magnetic field). Weakly (non-)interacting.

$$\psi(z_1, ..., z_N) = \mathcal{P}\left[\Phi_{CF}(z_i, \bar{z}_i)\prod_{k<l}(z_k - z_l)^{2p}\right]$$

- Ground states of the *Jain sequence* $\nu = n/(2np+1)$ as $\nu$ = n integer quantum Hall states of composite fermions. Similar construction for $\nu = n/(2np-1)$.
- Quasiholes and -electrons constructed as holes/single CFs in effective CF Landau levels
- The CF quasielectron wave function does much better than the one originally proposed by Laughlin [Kjønsberg&Leinaas]
- Machinery that provides numerically excellent wave functions in a systematic way. Served as a guide for CFT construction.

# CFT description of QH wave functions

- Basic observation (early 90s): QH wave functions (Laughlin, quasiholes, Pfaffians...) can be expressed as correlators of RCFT vertex operators.

$$\nu = 1/m : \quad \psi_L(z_1, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^m = \langle V_1(z_1) V_1(z_2) \ldots V_1(z_N) \rangle$$

where

$$V_1(z) = e^{i\sqrt{m}\varphi_1(z)}$$

- $\varphi_1(z)$: Free massless boson field compactified on radius $R^2=m$,

$$\langle \varphi_1(z) \varphi_1(z) \rangle = -ln(z-w) \,, \qquad \mathcal{Q} = \frac{1}{\sqrt{m}} \frac{1}{2\pi} \oint dz \, \partial_z \varphi_1(z)$$

Charge/
vorticity

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# CFT description of QH wave functions

- The CFT giving relevant bulk WFs defines 1D edge theory
- The braiding properties (monodromies) of the conformal blocks reflect the expected fractional statistics of the QH quasiparticles.
- Quasiholes created by insertions of $H_{1/m}(\eta) = e^{(i/\sqrt{m})\varphi_1(\eta)}$

$$\langle H_{1/m}(\eta) \prod_i V_1(z_i) \rangle = \prod_i (z_i - \eta)\, \psi_L(z_1, ....z_N)$$

- Pushes all electrons away from point $\eta$. Fractional charge/statistics.
- How to write quasi*electrons??* In particular, how to *contract* the electron liquid without running into trouble with the Pauli principle?

# Our approach and main results

- Inspired by previous literature, we extend the CFT description of the FQHE, to express hierarchical QH wave functions and their qp excitations as CFT correlators.
- Inspiration from composite fermions (*n* different electron operators at level *n* of the hierarchy)
- *Explicit* candidate wave functions for all Abelian hierarchy states: qe condensates, qh condensates, mixed states. Exactly reproduces CF wave functions of positive and negative Jain sequences as special cases.
- Consistency checks: Topological classification, TT limit, numerics
- Explicit proof of equivalence of composite fermions and hierarchy.
- Candidate wave functions for non-Abelian quasielectrons (Pfaffian)

# The quasielectron

- Quasielectron constructed by ***modifying one of the electron operators,*** corresponding to a local contraction of the electron liquid (charge density), with excess charge *-e/m:*

$$P_{1/m}(z) = \partial\, e^{i(\sqrt{m}-\frac{1}{\sqrt{m}})\varphi_1(z)}$$

- Essentially inverse quasihole operator combined with electron operator. Descendant. (Derivative necessary to get non-zero wave functions).
- Physical interpretation: *Shrinking of the correlation hole* around one of the electrons.
- Note: Gives quasielectrons in *angular momentum states*, rather than localized

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# One-quasielectron wave function

- One-qe wave function constructed from Laughlin state by replacing one of the electronic vertex operators by *P* and antisymmetrizing over particle coordinates. *Identical to the wave function obtained from composite fermions.* (But no projection needed!)

$$\Psi_{1qp}^{(l)}(z_i) = \mathcal{A}\langle P_{1/m}(z_1)V_1(z_2)\ldots\ldots V_1(Z_N)\rangle$$

$$= \sum_i (-1)^i \prod_{j<k}^{(i)} (z_j - z_k)^m \partial_i \prod_{l\neq i} (z_l - z_i)^{m-1}$$

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# Two quasielectrons (m=3)

- Two-qe wave function (minimum relative angular momentum):

$$\psi_{2qp} = \mathcal{A}\{(z_1 - z_2)^{5/3} \langle P_{1/3}(z_1) P_{1/3}(z_2) \prod_{i=3}^{N} V_1(z_i) \rangle \}$$

- Excellent overlap with exact state. Numerical and analytical arguments that qes have fractional charge and statistics as expected

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# ‘Condensates’ of many quasiparticles

- Many-quasielectron wave function must be analytic and antisymmetric. For MDD of N/2 electrons and N/2 quasielectrons, this is achieved by introducing a second Bose field (m=3):

$$V_2(z) = \partial e^{\frac{2i}{\sqrt{3}}\varphi_1(z)} e^{i\sqrt{\frac{5}{3}}\varphi_2(z)}$$

Charge e, fermionic. Corresponds to a CF in the second CF Landau level.

leading to the next-level ground state

$$\psi_{2/5} = \mathcal{A}\{ \langle \prod_{i=1}^{N/2} V_1(z_i) \prod_{j=N/2+1}^{N} V_2(z_j) \rangle \}$$

- *Exactly* reproduces the 2/5 CF wave function!
- Indicates that the 2/5 state can be viewed as a quasielectron condensate. (More later!)

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# Condensates of quasiparticles: hierarchy

- Other consistent coefficients of $\varphi_2$ : Non-Jain states such as 4/11:

$$V_2(z) = \partial e^{\frac{2i}{\sqrt{3}}\varphi_1(z)+i\sqrt{\frac{11}{3}}\varphi_2(z)}$$

$$\Psi_{4/11} = \mathcal{A}\{(1-1)^3\partial_2(2-2)^5(1-2)^2\}$$

- Less dense. Can be interpreted as a *fractional (ν = 1 + 1/3)* state of composite fermions.
- Promising numerics

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# Condensates of quasiparticles: hierarchy

- At level *n* of the hierarchy: *n* electronic vertex operators (*n* Bose fields), *n* hole operators. Recursive construction:

$$V_{n+1} \sim \left(\partial H_n^{-1} V_n\right) e^{i\alpha_{n+1}\varphi_{n+1}}$$

- Reproduces *exactly* the *v* = n/(2np+1) CF wave functions. Generates candidate WFs for all (quasielectron condensate) hierarchy states and their qh/qe excitations,

$$\nu_n = \cfrac{1}{t_1 - \cfrac{1}{t_2 - \cfrac{1}{\ddots \cfrac{}{t_{n-1} - \cfrac{1}{t_n}}}}} = p/q$$

$$t_1 = (1), 3, 5, \dots;\ t_{i>1} = 2, 4, \dots$$

# Consistency checks...

- The quantum Hall problem is exactly solvable in the *thin cylinder limit* (Bergholtz & Karlhede), where it is reduced to an electrostatic problem, with gapped crystal-like ground states (TT states). These solutions are believed to be adiabatically connected to bulk QH states.
- Hierarchy construction is manifest in this limit.
- *Taking the 'thin limit' of our proposed hierarchy wave functions exactly reproduces the correct TT states.*
- *Also: Wave functions fit into Wen's topological classification scheme for abelian QH fluids, in terms of K-matrix, t- and l-vectors.*
- *Exactly reproduce Jain's wave functions for $\nu = n/(2np+1)$. Promising numerics for $\nu = 4/11$.*

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# Hierarchical form of CF wave functions

- Recall: The operator *P(z)* gave qe states with good *angular momentum*. Can get *localized* states by coherent superposition.
- We have constructed an operator $\mathcal{P}(\eta)$ which *directly* creates a (quasi)localized qe at position $\eta$. (Hans Hansson's talk)

Using $$\Psi(\vec{R}_1 \dots \vec{R}_M; \vec{r}_1 \dots \vec{r}_N) = \langle \mathcal{P}(\vec{R}_1) \dots \mathcal{P}(\vec{R}_M) V_e(z_1) \dots V_e(z_N) \mathcal{O}_{bg} \rangle$$

in $$\Psi_{n+1}(z_1 \dots z_N) = \int d^2\vec{R}_1 \dots \int d^2\vec{R}_M \, \Phi^\star(\vec{R}_1 \dots \vec{R}_M) \Psi_n(\vec{R}_1 \dots \vec{R}_M; z_1 \dots z_N)$$

with $$\Phi_k(\vec{R}_1 \dots \vec{R}_M) = \prod_{i<j}^{M} (\bar{\eta}_i - \bar{\eta}_j)^{2k} e^{-\frac{1}{4m\ell^2} \sum_{i=1}^{M} |\eta_i|^2}$$

(pseudo wave function for charge e/m qe's in bosonic representation),

***identically reproduces the level 2 composite fermion WFs, including e.g. ν=2/5.***

# Quasihole condensates and 'mixed' states

- Procedure outlined so far only applies to pure qe condensates. What about quasi*hole* condensates (including the negative Jain sequence $\nu = n/(2np-1)$ ) and combinations of qe & qh condensates?

- We have generalized our construction to describe these cases. Introduce *antiholomorphic* fields for qh condensates.

- Exactly reproduce Jain's WFs for the negative sequence, e.g. 2/3:

$$\Psi_{2/3} = \mathcal{A}\left\{(\partial_{(1)} - \partial_{(1)})\, z_{(2)} (\partial_{(2)} - \partial_{(2)})\right\} \prod_{i<j} (z_i - z_j)^2$$

- Explicit candidate WFs for more general qh candidates and for mixed states. Remain to be tested...

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# Summary and open questions

- CFT description of quasi*electrons* (Abelian and non-Abelian) and their condensates, including Jain states. Explicit trial wave functions.
- Equivalence of hierarchy and composite fermion pictures
- Condensates of quasi*holes* (incl. negative Jain sequence) and 'mixed' states, ie rest of Abelian hierarchy. (In preparation)
- Open: description of the state at $\nu=1/2$.
- Next talk (Hansson): Details on quasilocal quasielectron operator, quasielectrons of Moore-Read Pfaffian state.

References:
Phys. Rev. Lett. **98**, 076801 (2007);
Phys. Rev. B **76**, 075347(2007);
Phys. Rev. B **77**, 165325 (2008).
arXiv:0810.0636