CFT approach to QH quasi electrons - Abelian and non-Abelian

T. H. Hansson and

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Nicolas Regnault	ENS, Paris
Susanne Viefers	Oslo Univ.
Jainendra Jain	Penn. State Univ.
Chiachen Chang	College of W&M

Outline:

- **Localized quasielectron states and CFT operators**
- **Fractional statistics - monodromies and Berry phases**
- **Quasielectrons in the Moore-Reade pfaffian state**
- **Genuine non-Abelian hierarchy states?**

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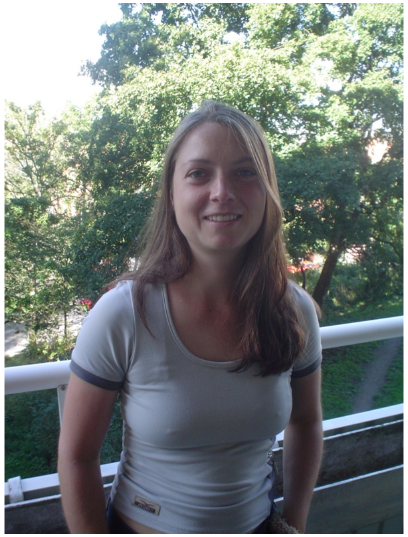

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Main points of the talk:

- ★ States of localized quasielectrons can be created by inserting a quasi-local operator in CFT correlators.
- ★ The statistics of quasiparticles is encoded both in the algebraic properties of the operators - the monodromies - and the properties of the electronic states - the Berry phases.
- ★ There are explicit candidate wave functions for quasi-electron states in the Moore-Read pfaffian state.

From Susanne Viefers' talk:

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Electron operator: $V_e(z) = e^{i\sqrt{m}\varphi(z)}$, $m = 2p + 1$

$$\Psi_L(z_1, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^m e^{-\frac{1}{4}\sum_i |z_i|^2} = \langle \prod_i V_e(z_i) O_{back} \rangle$$

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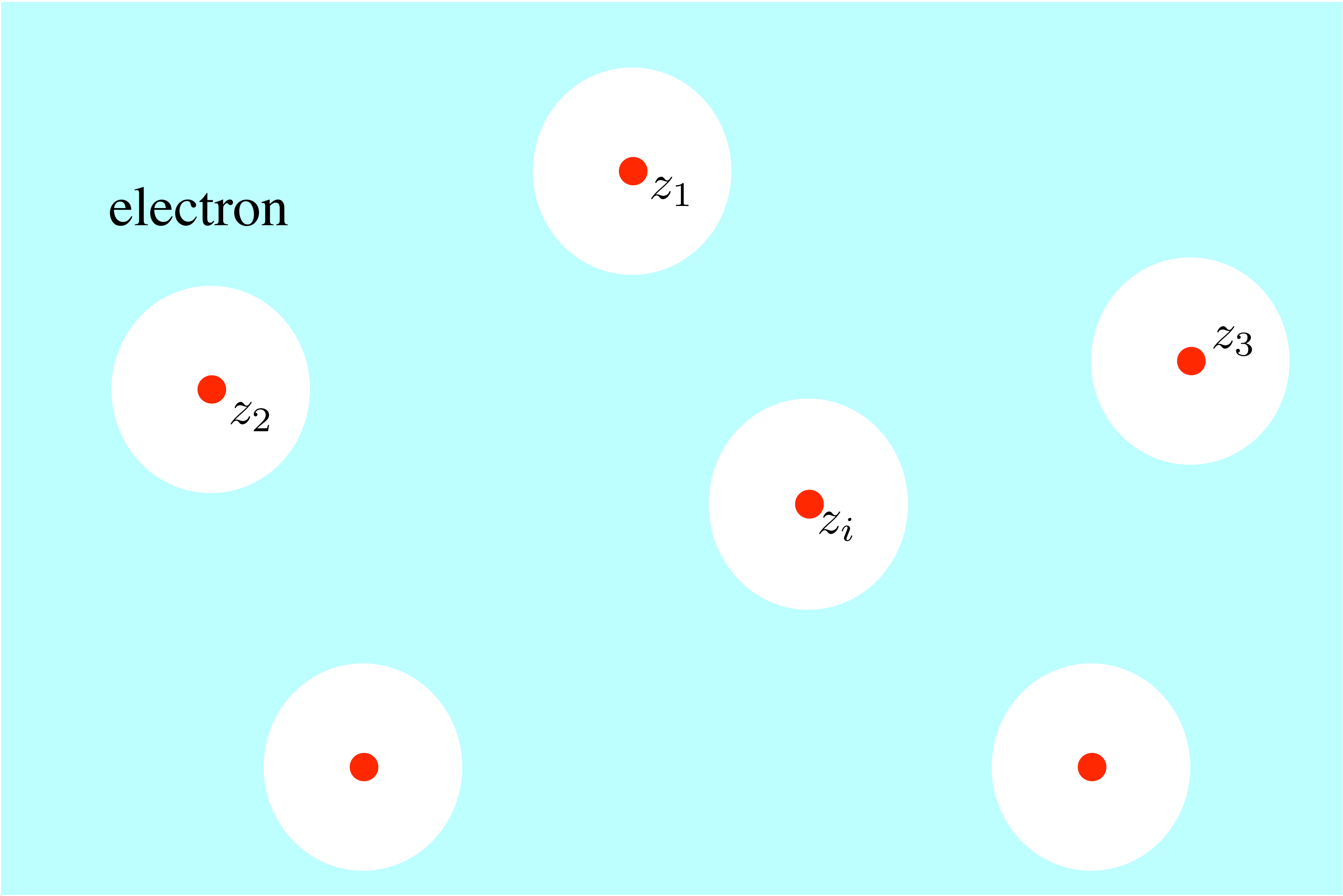
No, but almost!

The QH quasielectron



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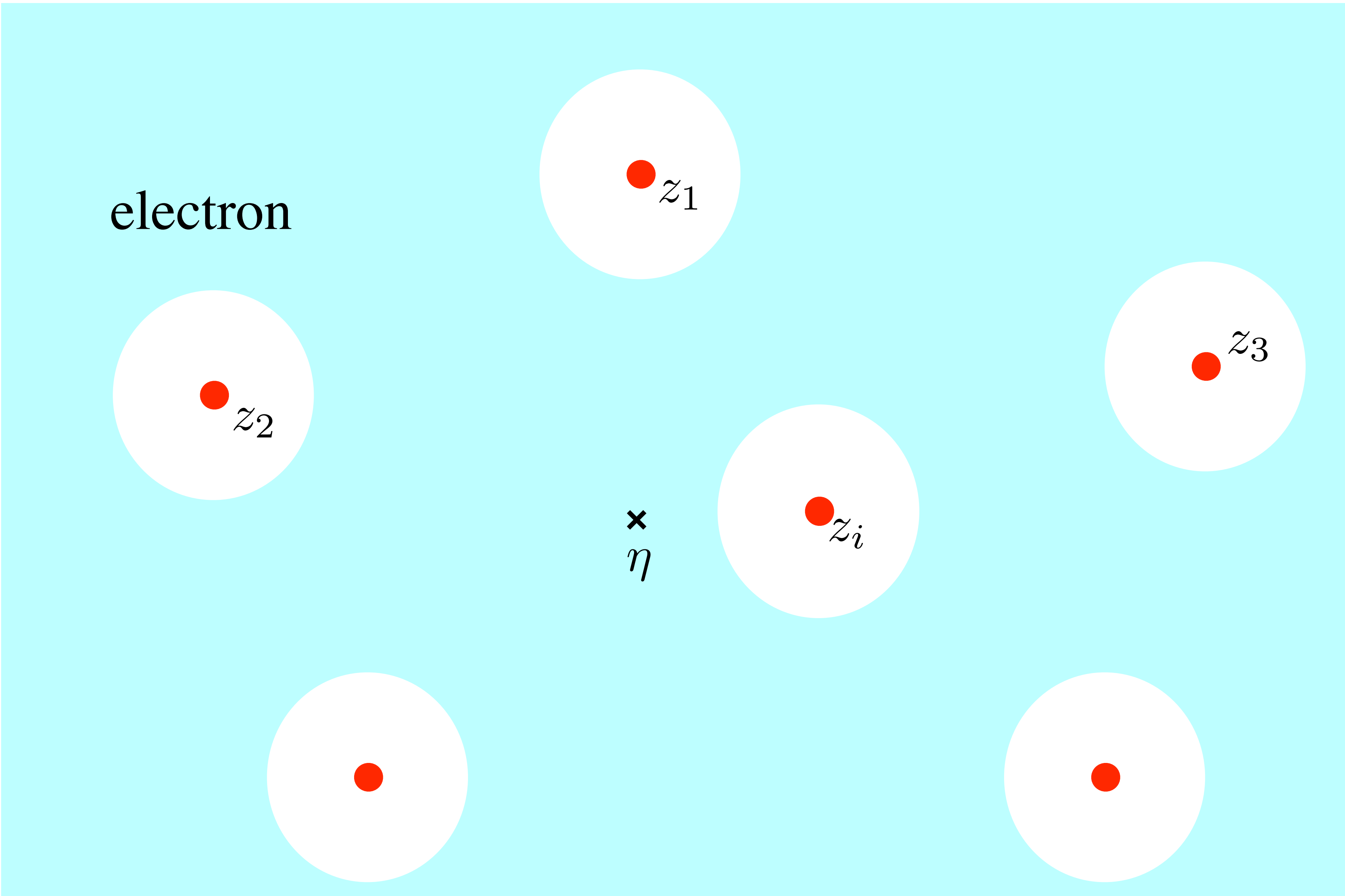

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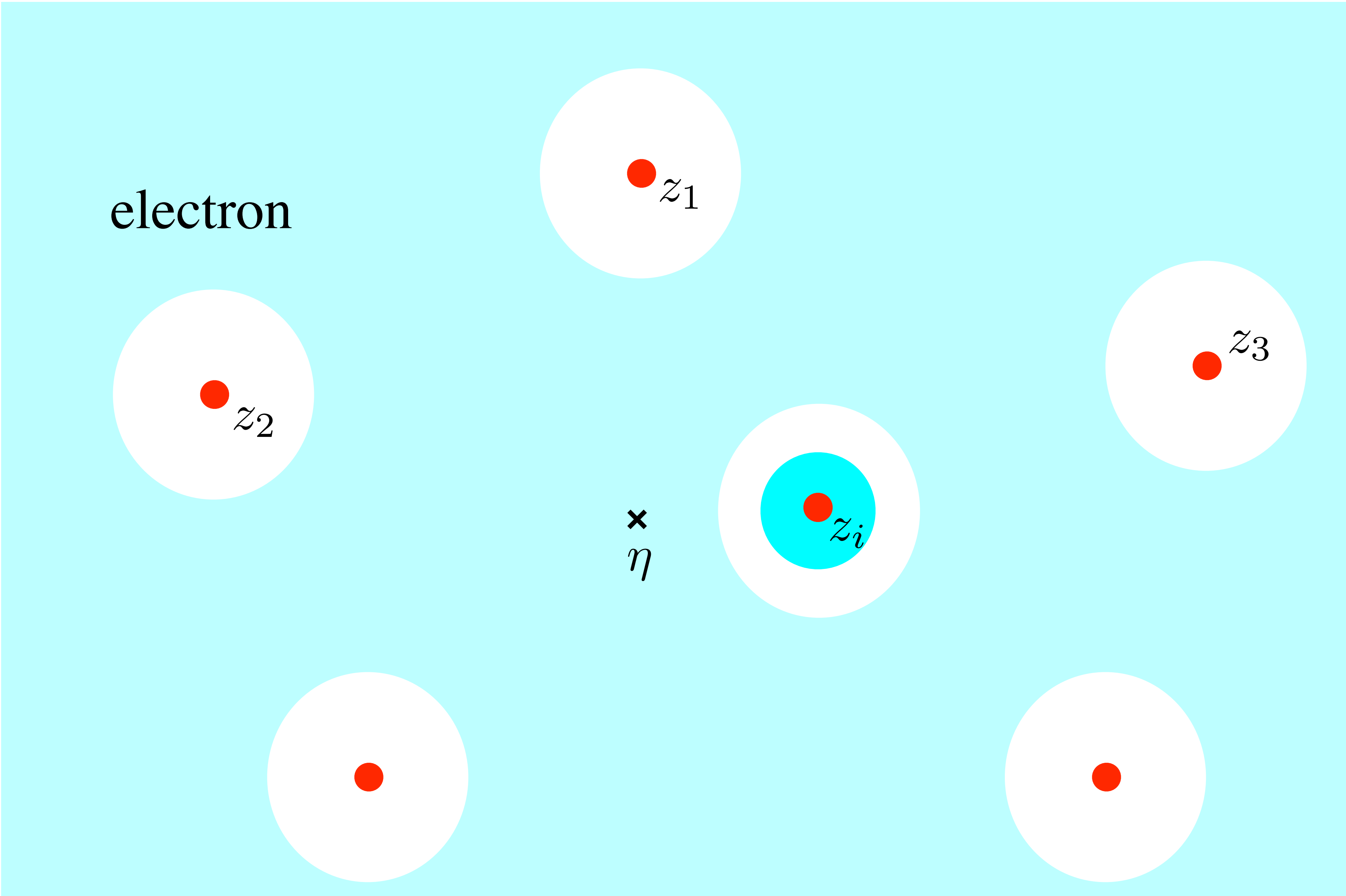

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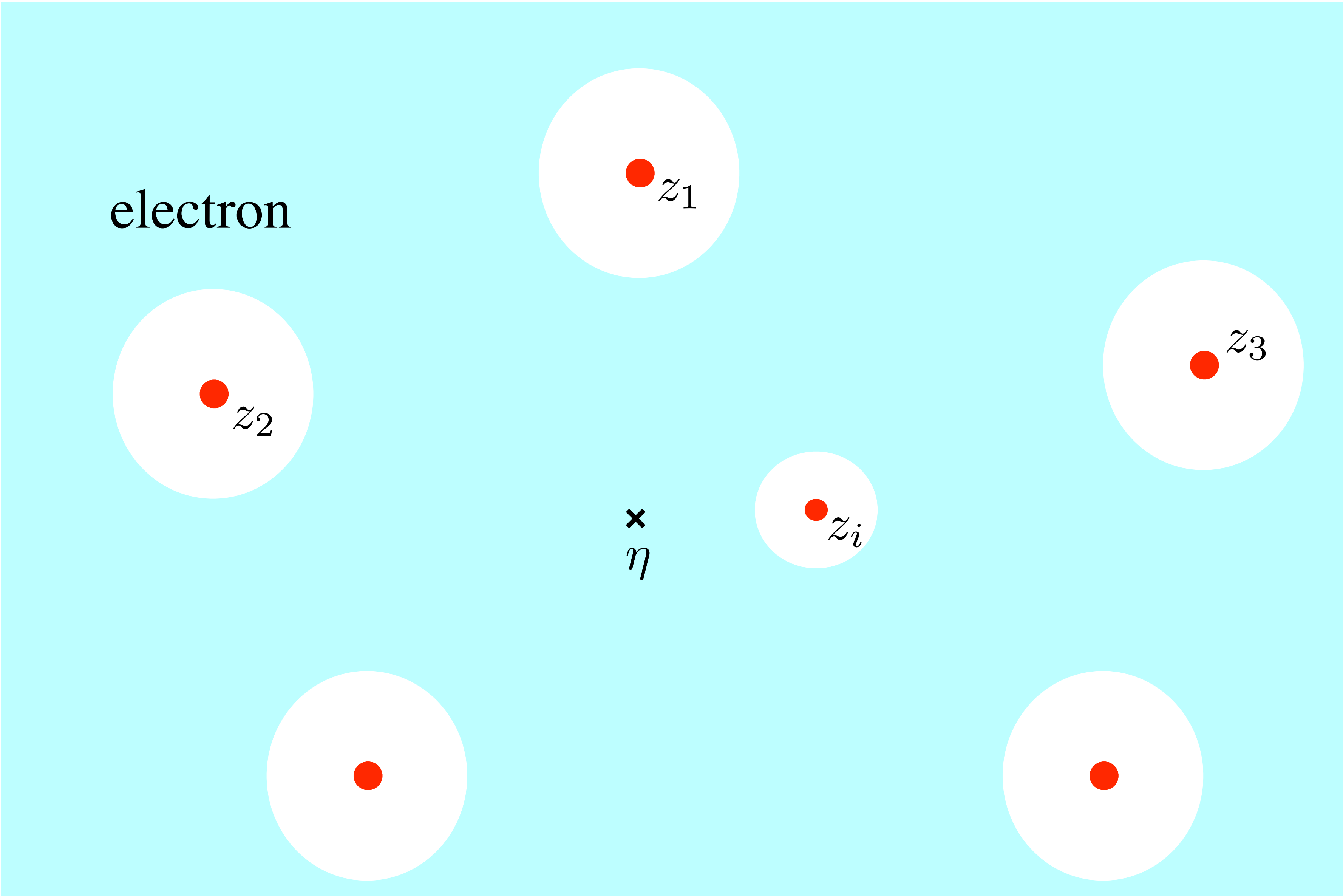

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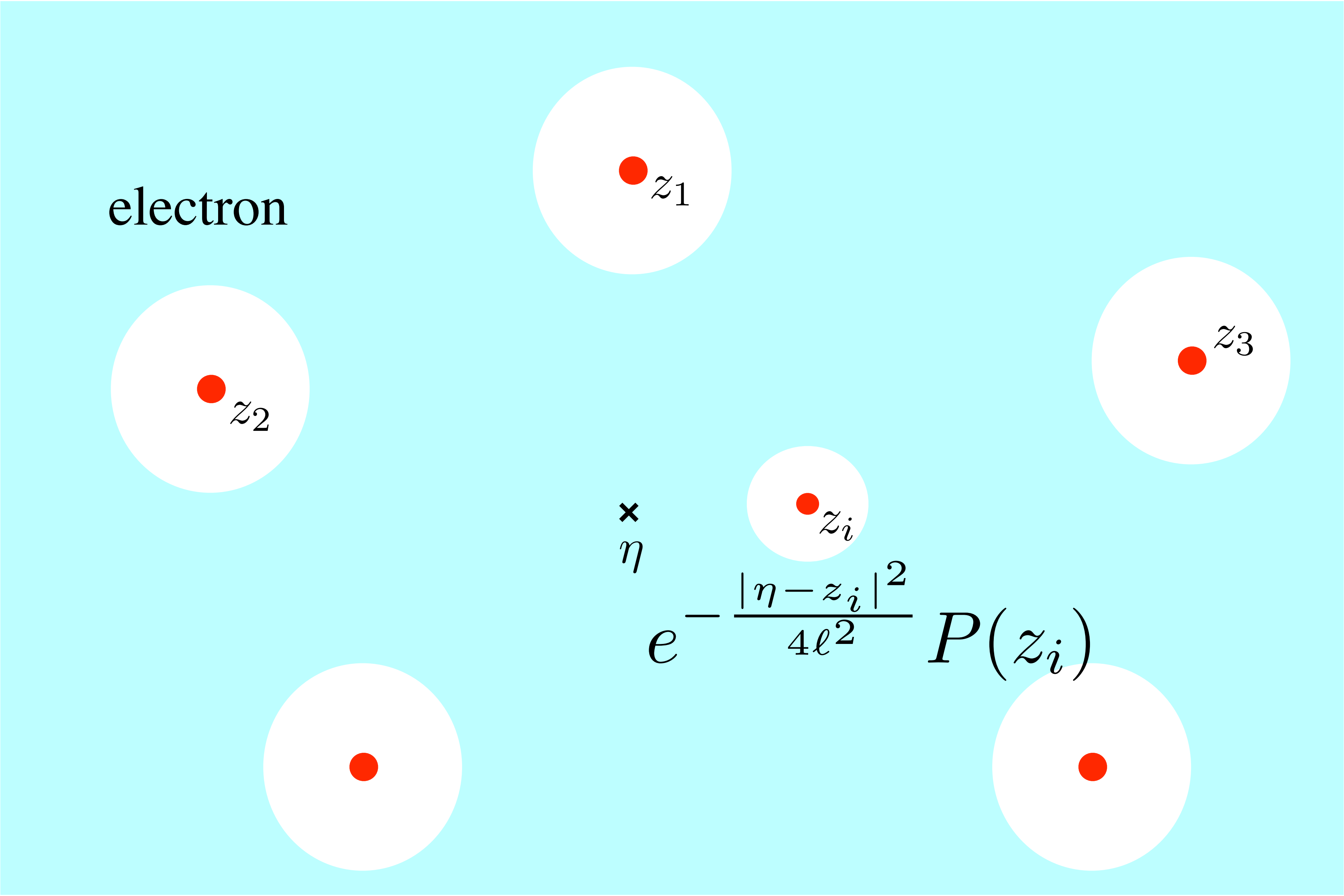

The quasielectron operator

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$$\Psi^{(m)}_{1qe}(\vec{R}\,; z_1 \ldots z_N) = \langle \mathcal{P}(\vec{R}) V_e(z_1) \ldots\ldots V_e(z_N) \rangle$$

$$\begin{aligned} \mathcal{P}(\vec{R}) &= e^{-\frac{|\eta|^2}{4m}} \mathcal{P}(\bar{\eta}) \qquad ; \qquad \eta = X + iY \\ &= e^{-\frac{|\eta|^2}{4m}} \int d^2w \, e^{\frac{1}{2m}\bar{\eta} w} \left(\tilde{H}^{-1}(z)\, \bar{\partial}_w J_p(w) \right) \end{aligned}$$

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- $\mathcal{P}(\bar\eta)$ **is quasi-local on the magnetic length scale**
- $\mathcal{P}(\bar\eta)$ **has same charge and same conformal dim. as** $H^{-1}(z)$
- **Multiple insertions of** $\mathcal{P}(\bar\eta)$ **gives multi-quasielectron states**

Fractional statistics: Two Laughlin holes

$$\Psi_L^{2qh}(\eta_1, \eta_2; z_1 \dots z_N) = \prod_i (z_i - \eta_1)(z_i - \eta_2)\Psi_L(z_1 \dots z_N)$$

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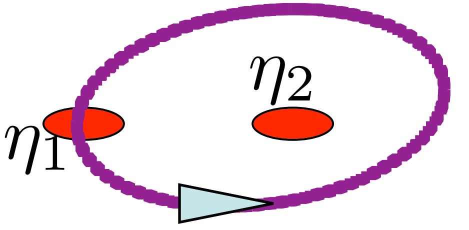

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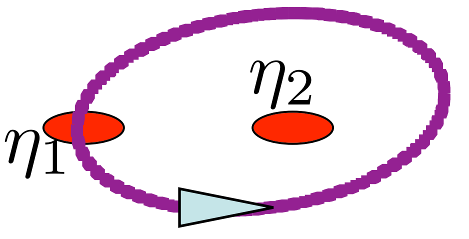

Arovas *et.al*. derived the Berry phase:

$$\gamma_B=\frac{\Phi}{\phi_0}-\frac{2\pi}{3}$$

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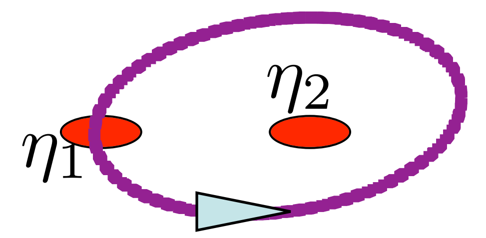

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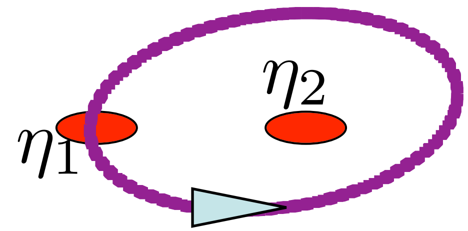

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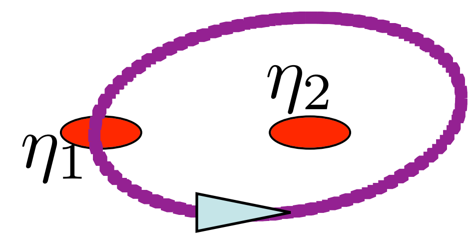

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Statistical angle: $\theta = -\frac{1}{2}\gamma_B^{top} = \frac{\pi}{3}$ *i.e.* anyons!

Fractional statistics without pain:

$$\Psi_L^{2qh}(\eta_1, \eta_2; z_1 \dots z_N) = \langle H(\eta_1) H(\eta_2) \prod_{i=1}^{N} V(z_i) \rangle$$

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Non-anlytic, the cut gives a monodromy

- Simple expressions as a CFT conformal block.
- Charge and statistics coded in the algebraic properties of the electron and hole operators.
- By plasma analogy, no Berry phases so exchange statistics can be read directly from the monodromies!

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We have no general way to determine whether or not the Berry phase vanish!

BUT, the freedom to choose the statistics of the quasiparticle operators is important for,

- ★ **The hierarcy states** (S. Viefers talk)
- ★ **Non-Abelian quasielectron states**

Quasielectrons in the Moore-Read state

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Different ordering, gives different wave functions:

$$\langle \mathcal{P}_+(\eta_1)\mathcal{P}_+(\eta_2)\mathcal{P}_-(\eta_3)\mathcal{P}_-(\eta_4) V(z_1)\dots V(z_N)\rangle = \Psi_{(\eta_1,\eta_2)(\eta_3,\eta_4)}(z_1\dots z_N)$$

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$$\langle \mathcal{P}_+(\eta_1)\mathcal{P}_+(\eta_2)\mathcal{P}_-(\eta_3)\mathcal{P}_-(\eta_4)V(z_1)\ldots V(z_N)\rangle = \Psi_{(\eta_1,\eta_2)(\eta_3,\eta_4)}(z_1\ldots z_N)$$

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Which (unfortunately) comes out like:

Quasielectrons in the Moore-Read state

$$\mathcal{P}(\bar{\eta}) = \int d^2w\, e^{\frac{1}{2m}\bar{\eta}w} \left(\tilde{H}^{-1}(z)\,\bar{\partial}_w J_p(w)\right)^{\star}$$

Generalized normal ordering

Ising representation

$$V(z) = \psi(z)e^{i\sqrt{2}\varphi(z)}$$

$$H^{-1}(z) = \sigma(z)e^{-\frac{i}{2\sqrt{2}}\varphi(z)}$$

Does not (directly) give holomorphic wf:s...

Bosonic representation

$$V(z) = \cos(\phi(z))e^{i\sqrt{2}\varphi(z)}$$

$$H_{\pm}(\eta) = e^{\pm i\phi(\eta)/2}e^{\frac{i}{2\sqrt{2}}\varphi(z)}$$

$$H_{\pm}^{(b)}(\eta) = e^{\frac{i}{\sqrt{8}}\varphi}e^{\pm\frac{i}{2}\phi}e^{i\sqrt{\frac{3}{8}}\phi_1 \pm \frac{i}{2}\varphi_2}$$

$$\Psi_{(\eta_1,\eta_2)(\eta_3,\eta_4)} = \sum_I (-1)^{\sigma(I)} e^{(\bar{\eta}_1 z_\alpha + \bar{\eta}_2 z_\beta + \bar{\eta}_3 z_\gamma + \bar{\eta}_4 z_\delta)/8} \partial_\alpha\partial_\beta\partial_\gamma\partial_\delta$$
$$\times \left[\psi_{(\alpha,\beta)(\gamma,\delta)} \prod_{i<j\notin I}(z_i - z_j)^2 \prod_{\substack{a\in I\\ j\notin I}}(z_a - z_j)(z_\alpha - z_\beta)(z_\gamma - z_\delta)\right]$$

$$\psi_{(\alpha,\beta)(\gamma,\delta)} = \mathrm{Pf}\left(\frac{(z_i - z_\alpha)(z_i - z_\beta)(z_j - z_\gamma)(z_j - z_\delta) + (i \leftrightarrow j)}{z_i - z_j}\right)$$

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We expect these functions to span the expected 2-dim Hilbert space corresponding to 4 non-Abelian quasielectrons. The NA statistical matrix is however coded in the Berry matrix rather than in the monodromies.

Two Moore-Read quasielectrons

Two Moore-Read quasielectrons

$$\Psi_{2qe}(\eta_1,\eta_2) = \langle \mathcal{P}(\bar\eta_1)\mathcal{P}(\bar\eta_2)\prod_{i=1}^{N} V(z_i)\rangle$$

$$= \sum_{a<b}(-1)^{a+b} e^{\bar N Z_{ab}} \cosh(\bar n(z_a - z_b)/16) e^{-\frac{1}{4\ell^2}\sum_i |z_i|^2} e^{\bar N Z_{ab}}$$

$$\left[\partial_a\partial_b \mathrm{Pf}\left(\frac{(z_i-z_a)(z_j-z_b)+(a\leftrightarrow b)}{z_i-z_j}\right)(z_a-z_b)\prod_{i<j}^{(a,b)}(z_i-z_j)^2\prod_i^{(a,b)}(z_i-z_a)(z_i-z_b)\right]$$

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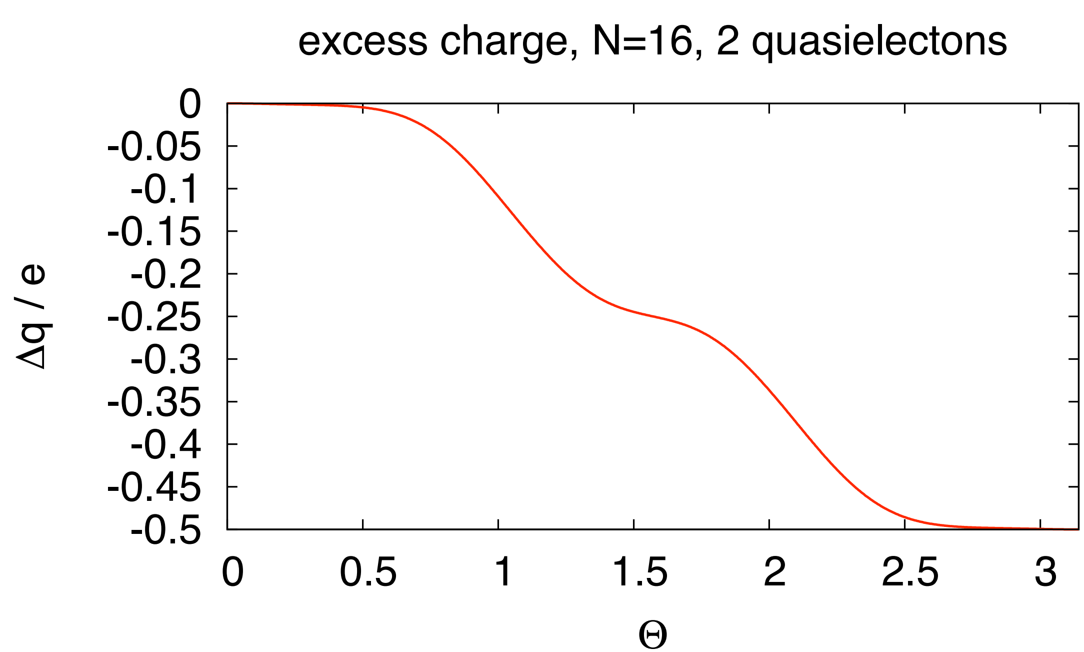
Clear indication of charge e/4 particles!

Related work by Bernevig and Haldane, arXiv:08102366



Hierarchical NA-states?

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Using the above methods, we can do something that is potentially more interesting - we can construct condensate of non-Abelian, charge 1/4 quasi electrons, by considering the correlators:

$$\Psi_\xi = \langle \mathcal{P}_{\xi(1)}(\eta_1)\mathcal{P}_{\xi(2)}(\eta_2)\ldots\mathcal{P}_{\xi(n)}(\eta_n)\prod_{i=1}^{N} V(z_j)\rangle$$

where ξ is a string of equally many + and - , multiply with an appropriate pseudo wave function and integrate over η_i to get holomorphic hierarchy states.

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Will this yield new interesting non-Abelian states?

More details in these references:

Conformal Field Theory of Composite Fermions
T.H. Hansson, C.-C. Chang, J.K. Jain and S. Viefers.
Phys. Rev. Lett. **98**, 076801 (2007); arXiv:cond-mat/0603125.

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M. Hermanns, J. Suorsa, E.J. Bergholtz, T.H. Hansson and A. Karlhede
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E.J. Bergholtz, T. H. Hansson, M. Hermanns, A. Karlhede, and S. Viefers
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T. H. Hansson, M. Hermanns and S. Viefers
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