Interferometry in the quantum Hall effect

How to observe non-abelian statistics

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J.Stat. Mech, 08

Nordita

Outline

- The ν=5/2 quantum Hall effect
- Recent experiments
- Edge theory
- Interferometry
- Experimental signatures of non-abelian statistics

Non-abelian statistics

A non-abelian topological phase with n excitations has $d > 1$ ground states:

$$\Psi(x_1, \cdots, x_n) = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_d \end{pmatrix}$$

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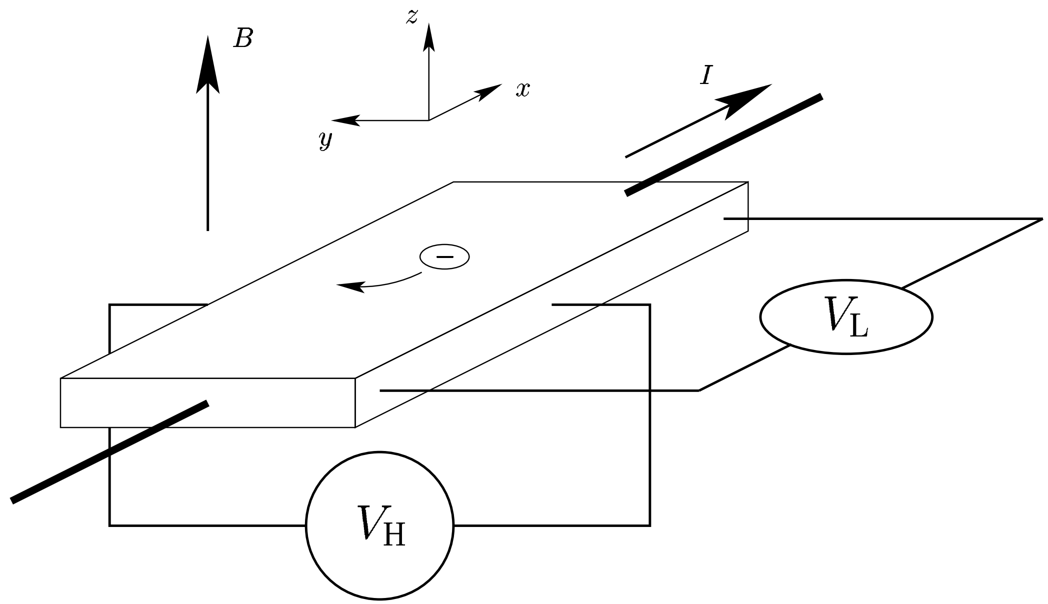
In general, M and N do not commute!

‘Experimental’ setup

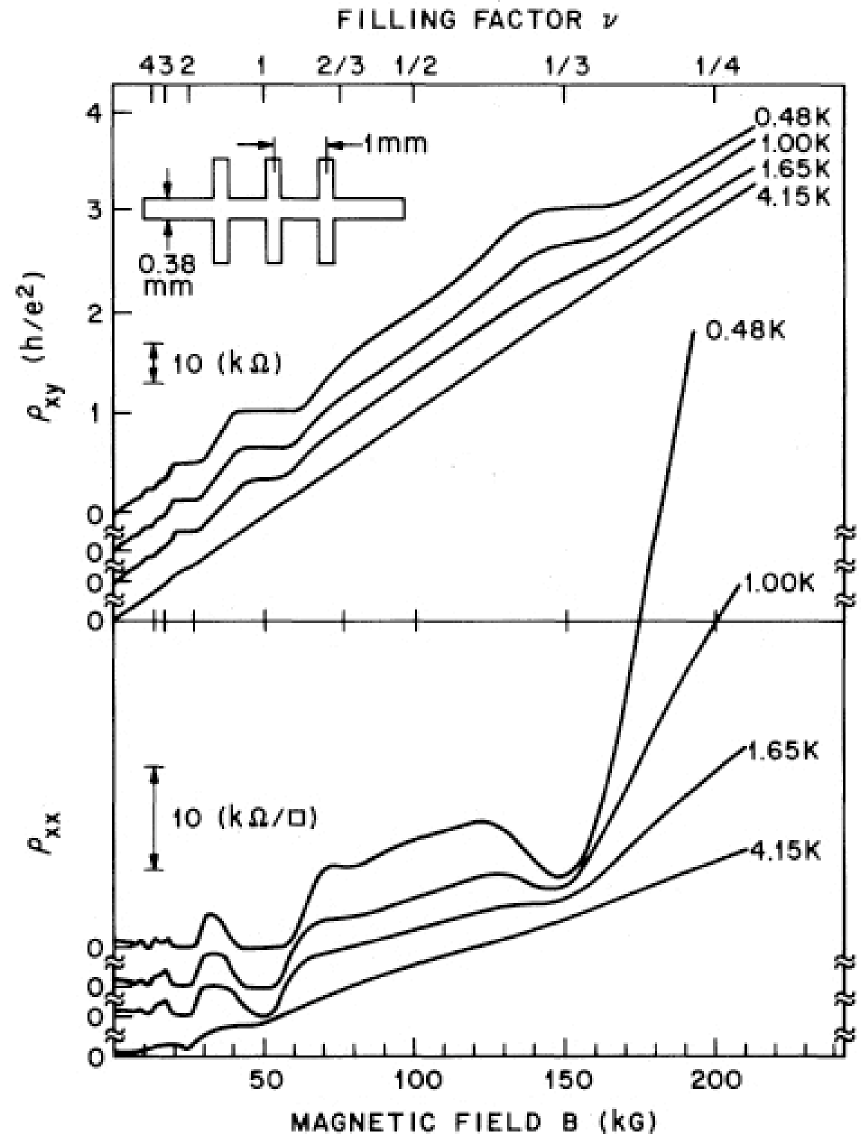


- (Fractional) quantum Hall effect occurs in a 2-dimensional electron gas (GaAs/AlGaAs heterojunctions)

- $T \sim 0.1K \qquad B_\perp = 10T$

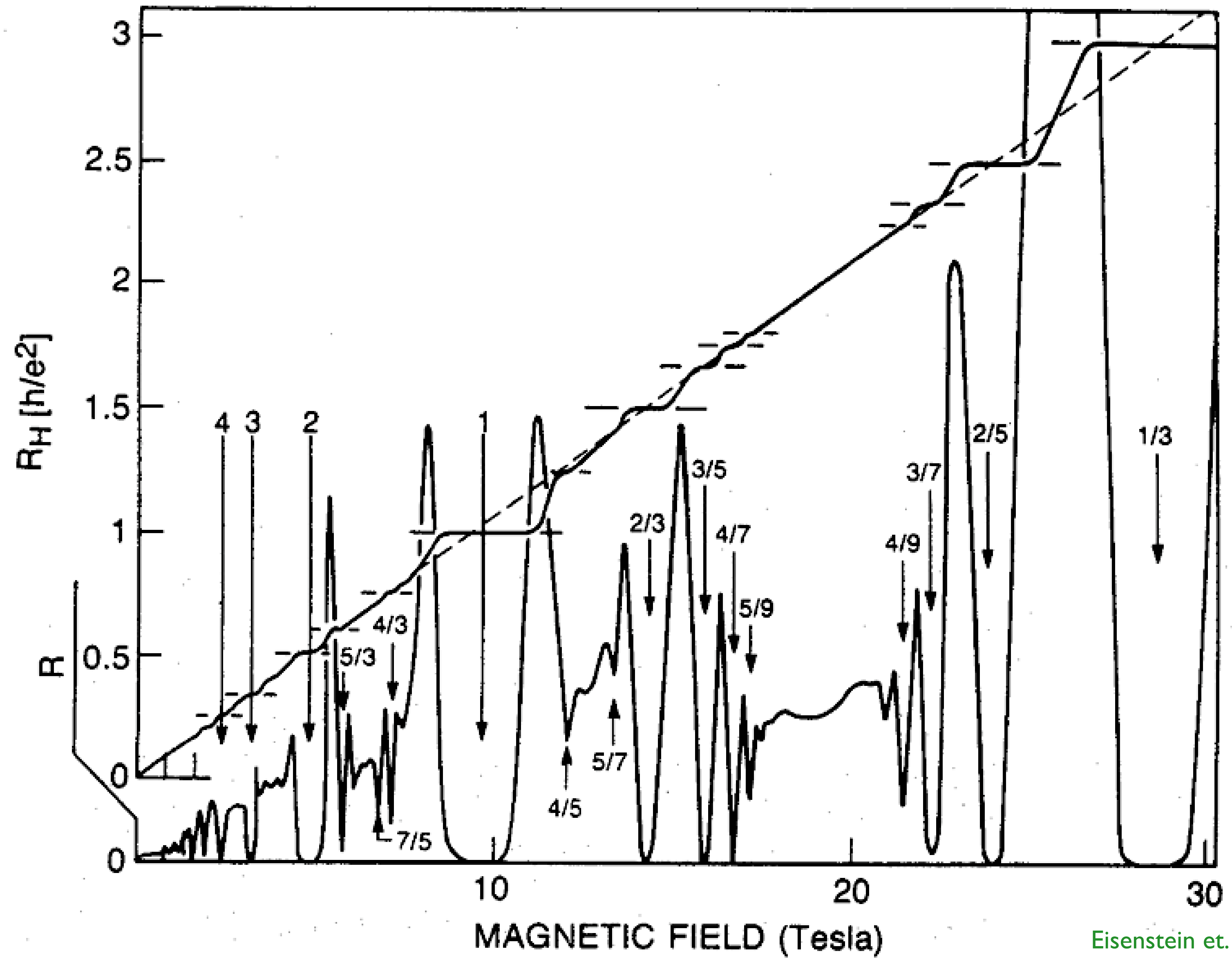
- Signatures $\quad \rho_{xx} = 0 \qquad \sigma_{xy} = \nu \frac{e^2}{h}$

Fractional quantum Hall effect



Tsui et. al, 1982

Topological phases exist!

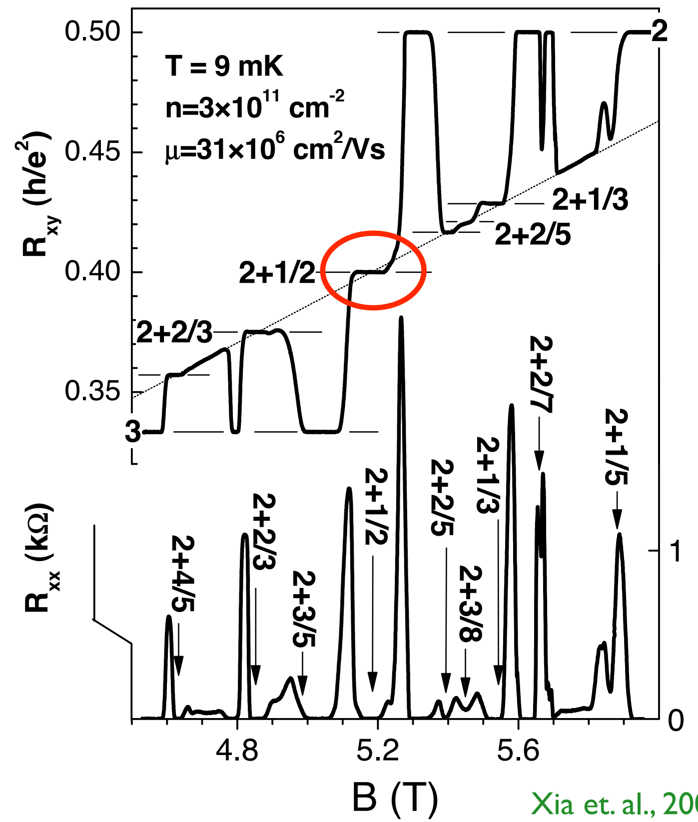


Eisenstein et. al., 1990

Non-abelian candidate

The fractional quantum Hall effect at $\nu=5/2$ was discovered in 1987 (Willet et.al. 1987)

- Numerics suggests that the Moore-Read (a.k.a. pfaffian) state describes this quantum Hall effect (Morf, 1998, Rezayi et.al. 2001)
- Excitations of the Moore-Read state have charge e/4
- Four excitations lead to a two-fold topological degeneracy: non-abelian statistics!

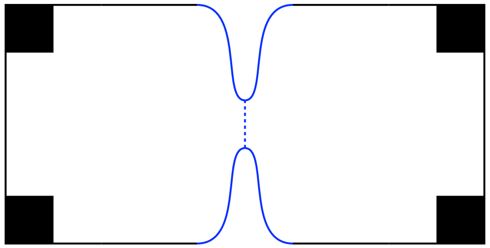


Xia et. al., 2004

Probing quantum Hall states

Charge is transported by the edge modes

In a point contact, excitations can tunnel from one edge to the other



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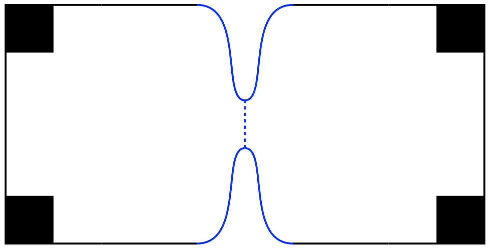

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The (shot) noise in the tunneling current reveals properties of the tunneling particles

$$S = e^* \langle I_t \rangle$$

charge of the tunneling particles

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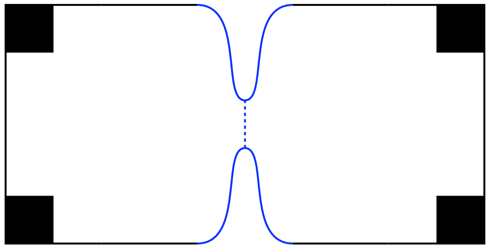

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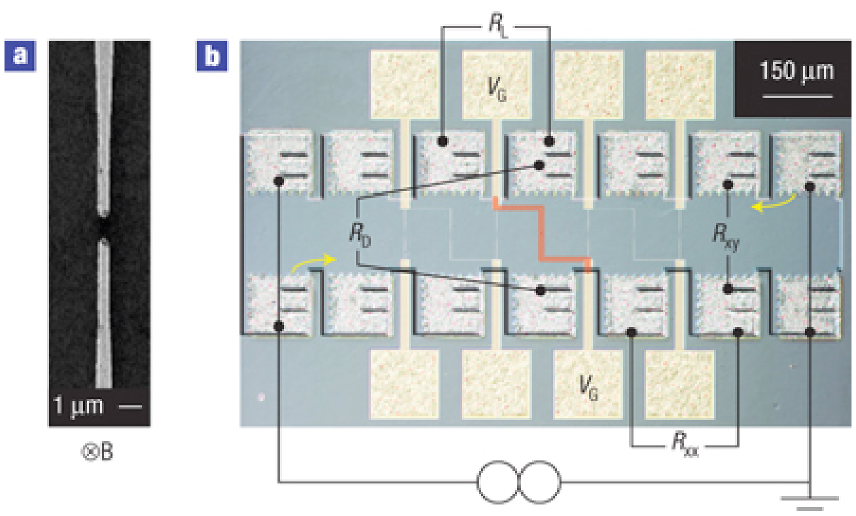
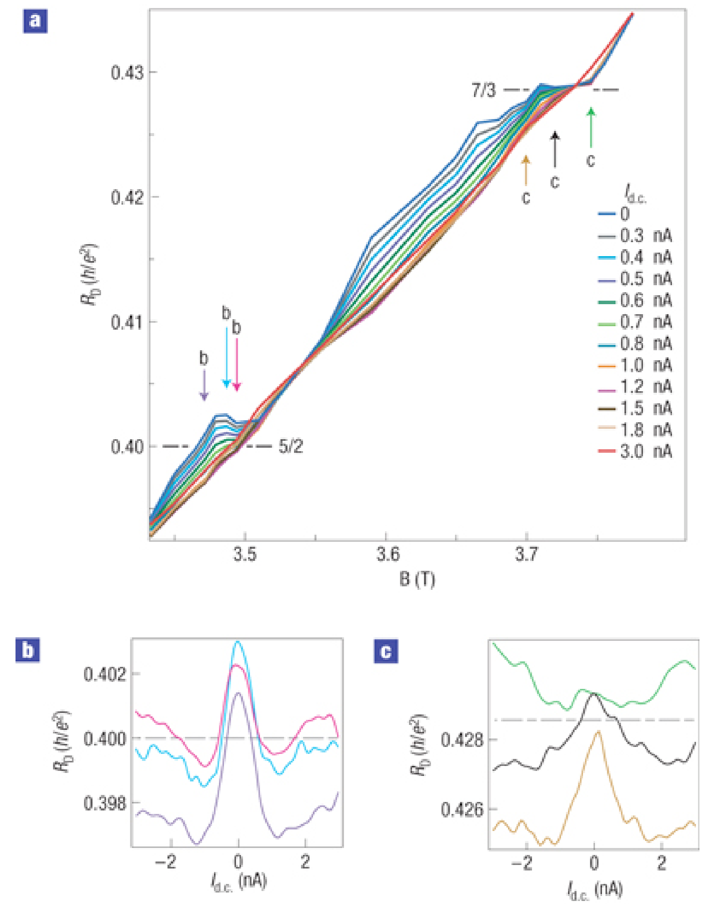
Shot noise measurements have confirmed the fractional charge of excitations (Saminadayar et.al. & de Picciotto et.al., 1997)

Experiments at ν=5/2

First progress: a point contact was shown to work at ν=5/2

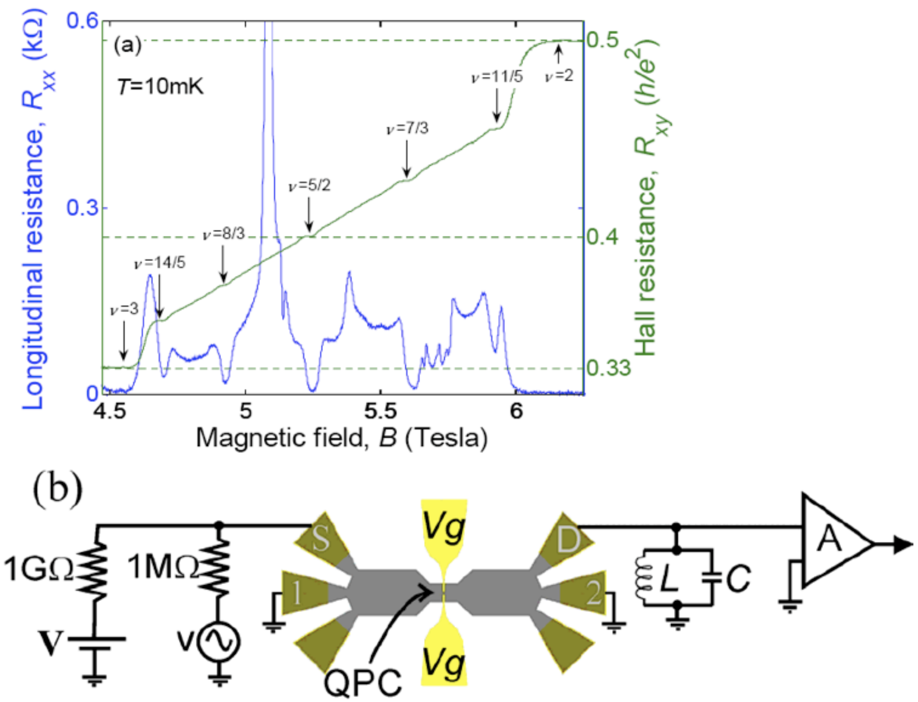
Miller et. al., Nature Physics, 2007

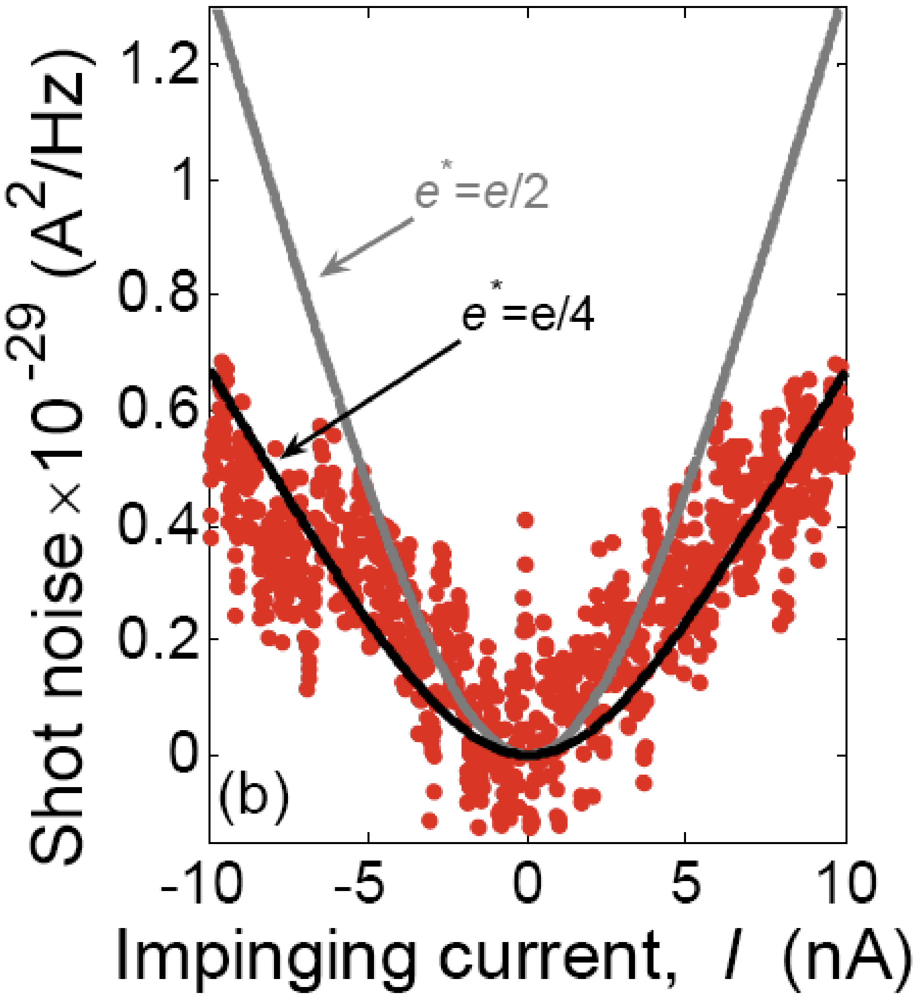




Shot-noise at $\nu=5/2$

Shot-noise experiments confirmed the charge e/4.

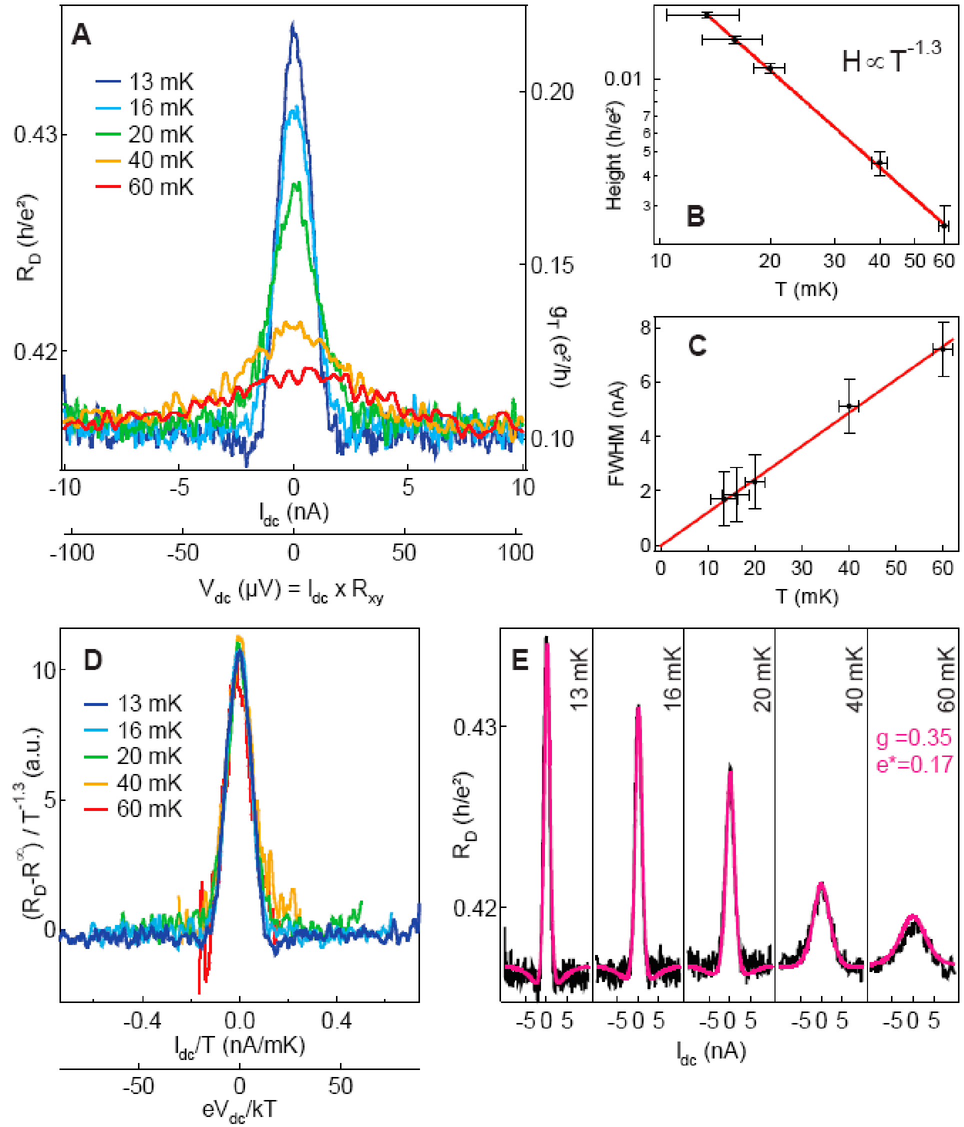




M.Dolev et.al, 0802.0930

Recent measurements

In recent measurements, the scaling exponents were measured:



I.P. Radu et.al, 0803.3530

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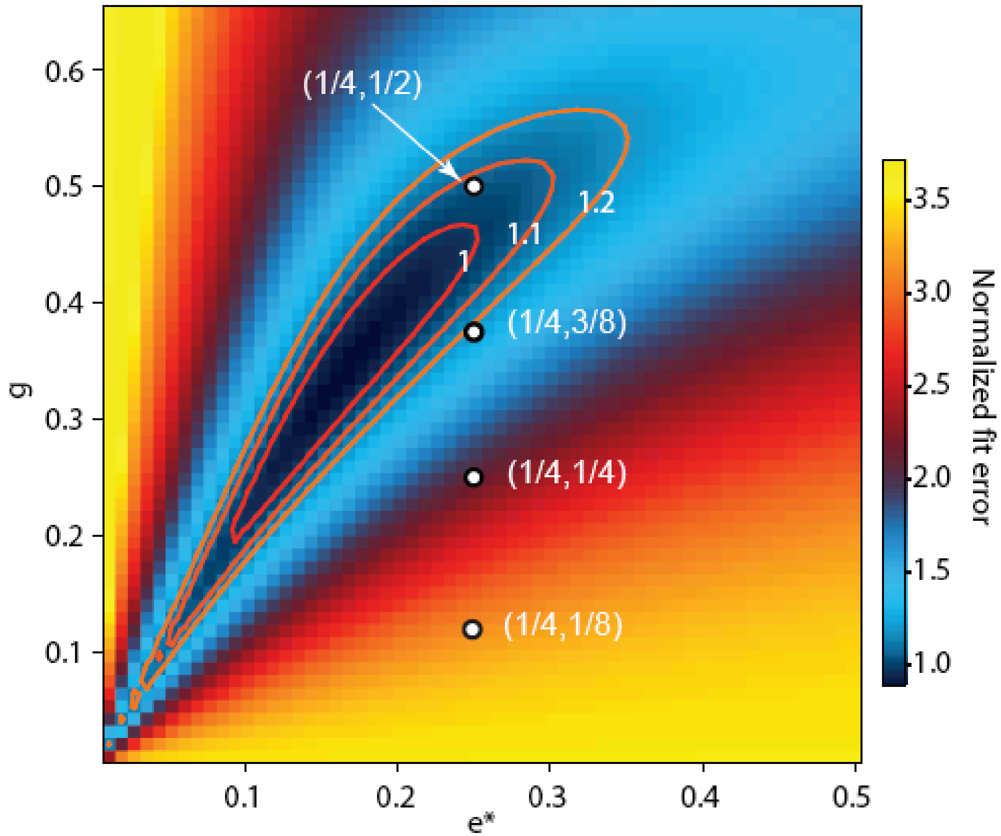
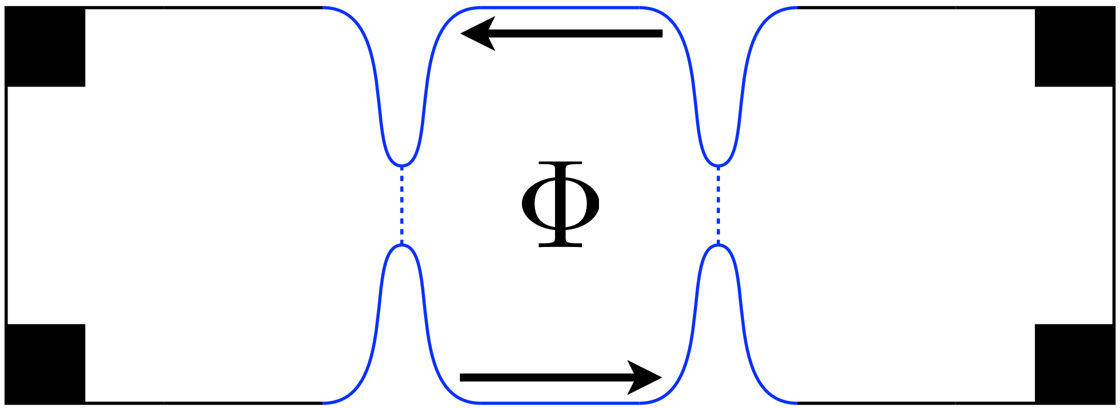


FIG. 5: Map of the fit quality. Normalized fit error is the residual from the least-squares fit, divided by the number of points and by the noise of the measurement. Also marked on the map are proposed theoretical pairs (e^*, g).

Interference in the quantum Hall effect



In the quantum Hall effect, the particles move along the edge. In the constrictions, particles can tunnel from one edge to the other.

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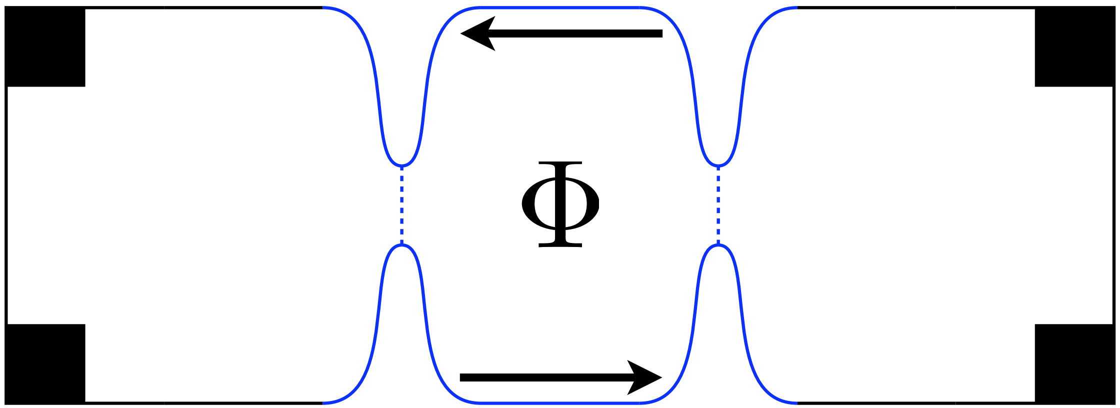
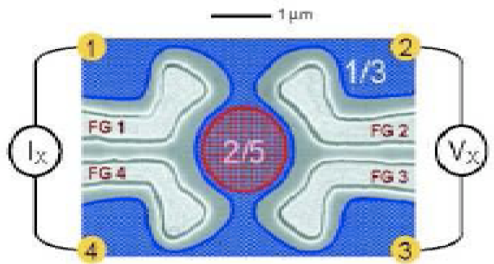
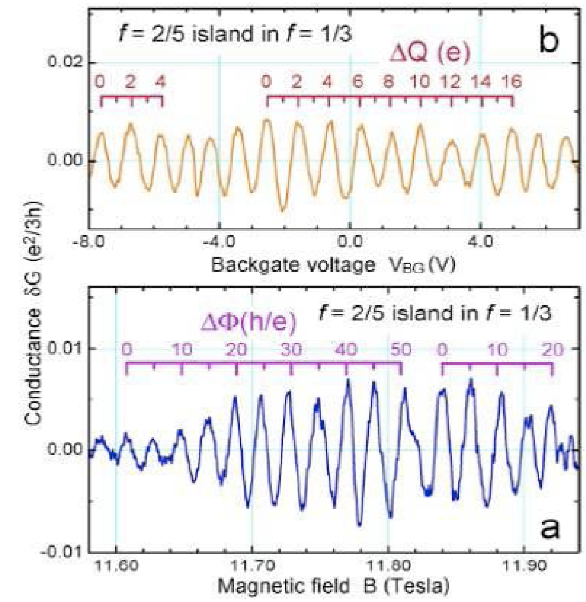


In the quantum Hall effect, the particles move along the edge. In the constrictions, particles can tunnel from one edge to the other.

To probe statistics, a double point contact is an ideal setup, because the two path can interfere with each other, leading to Aharonov-Bohm oscillations

Double point contact





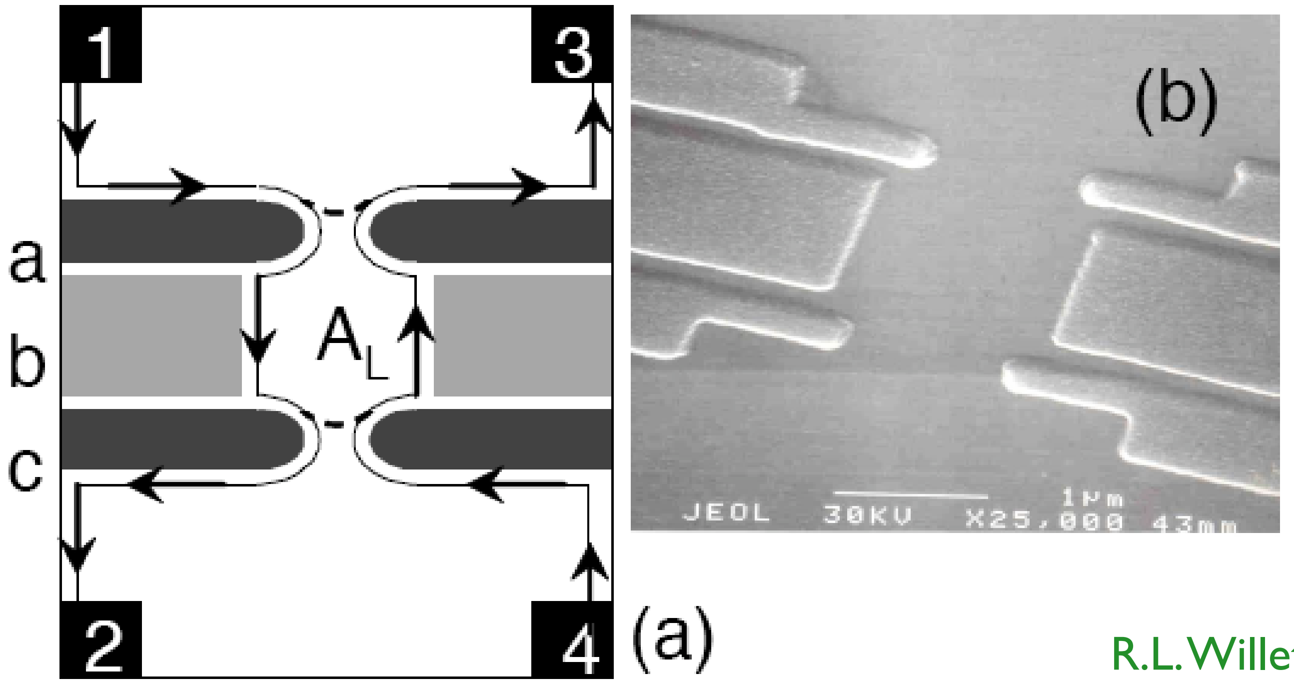
Experiment: V. Goldman (2005)

The two tunneling paths lead to interference oscillations in the current and noise.

Consistent with fractional statistics, but not undisputed.

The double point contact probes a **four** point function!

Interferometry @ ν=5/2

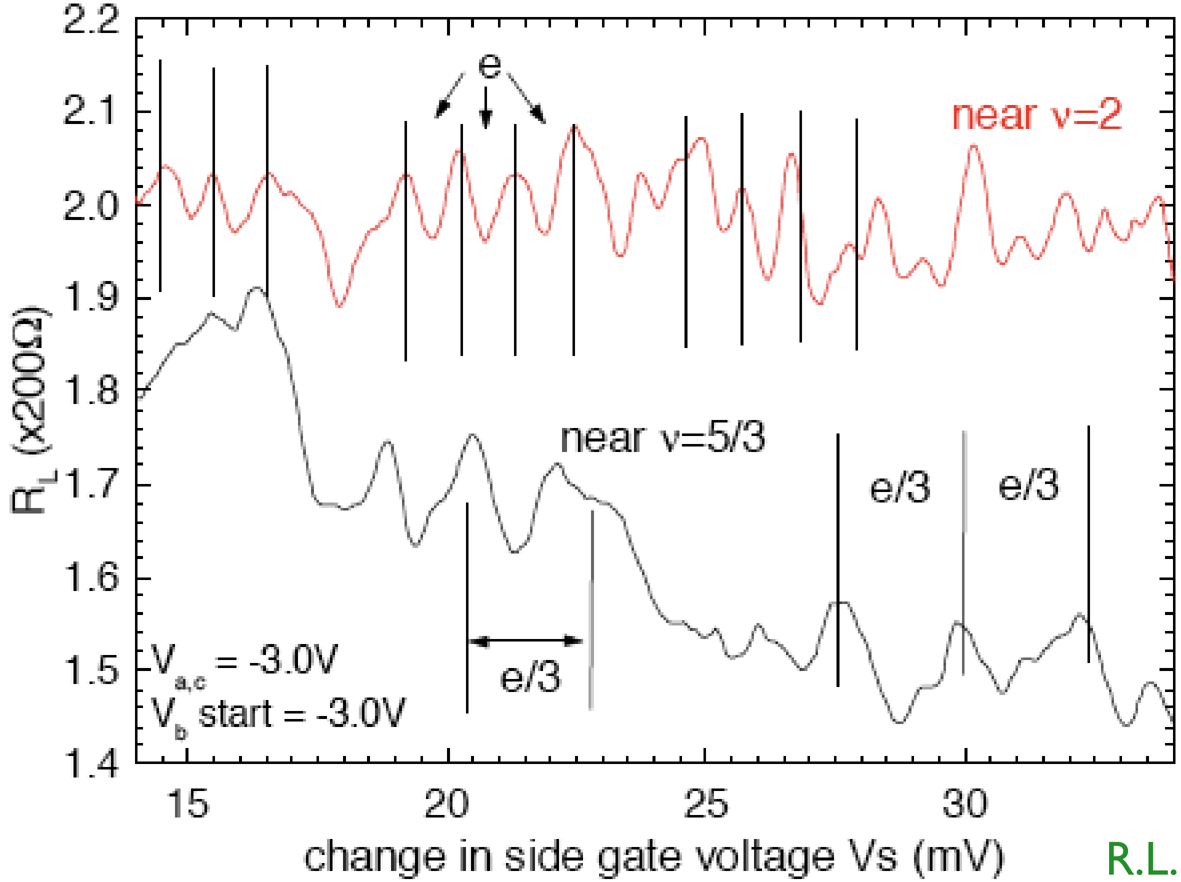


R.L. Willet et.al., 0807.0221

Changing the magnetic field: $\Delta B = hA/e^*$

Changing the area: $\Delta A = h/(e^* B)$

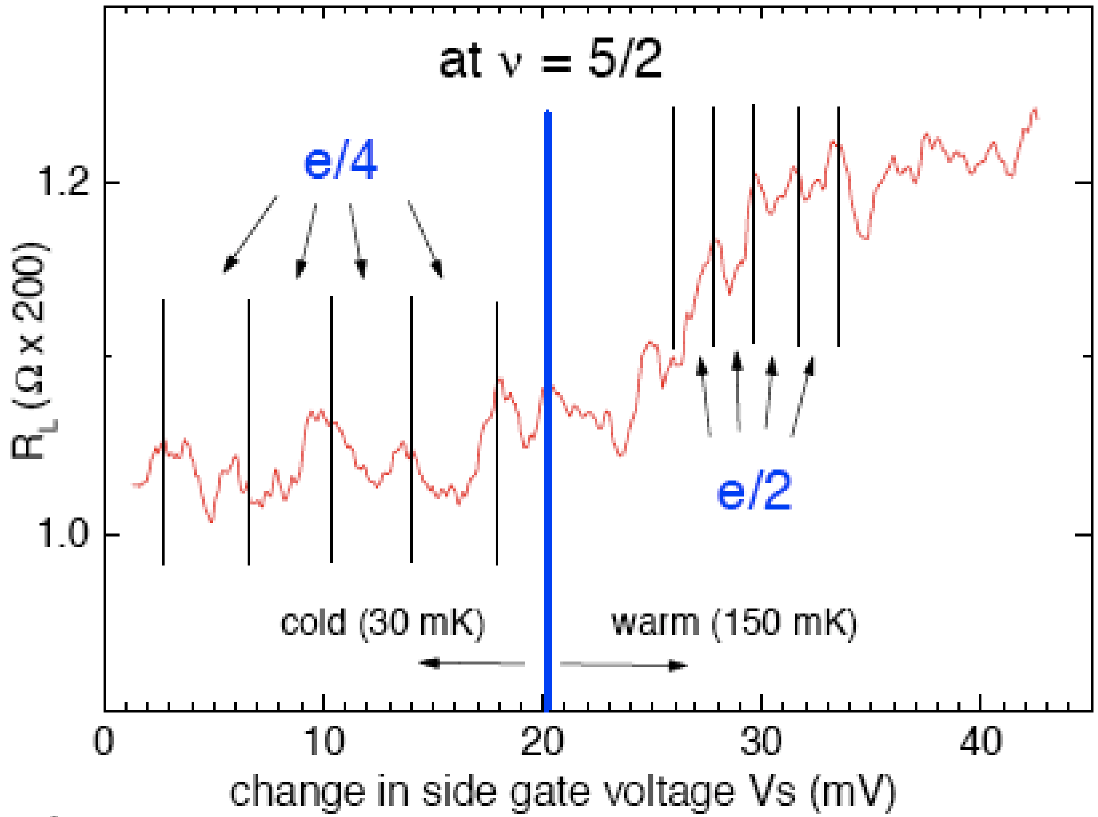
Interferometry @ ν=5/2



R.L. Willet et.al., 0807.0221

Use ν=2 to calibrate the system, and find e/3 at filling ν=5/3

Interferometry @ ν=5/2



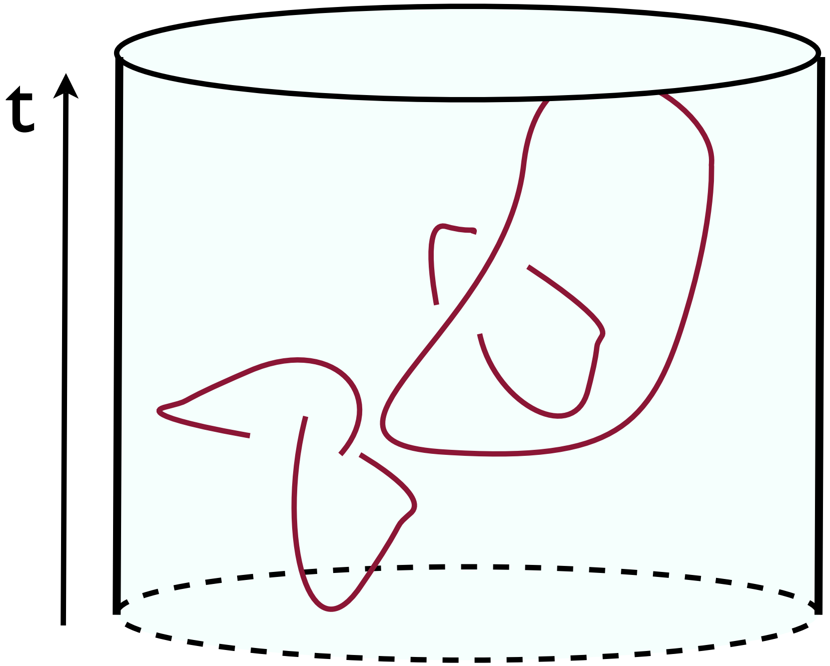
R.L. Willet et.al., 0807.0221

At low T, one finds e/4 at ν=5/2, while for 'high' T, on observes e/2.

Topological Theory

Chern Simons Theory:

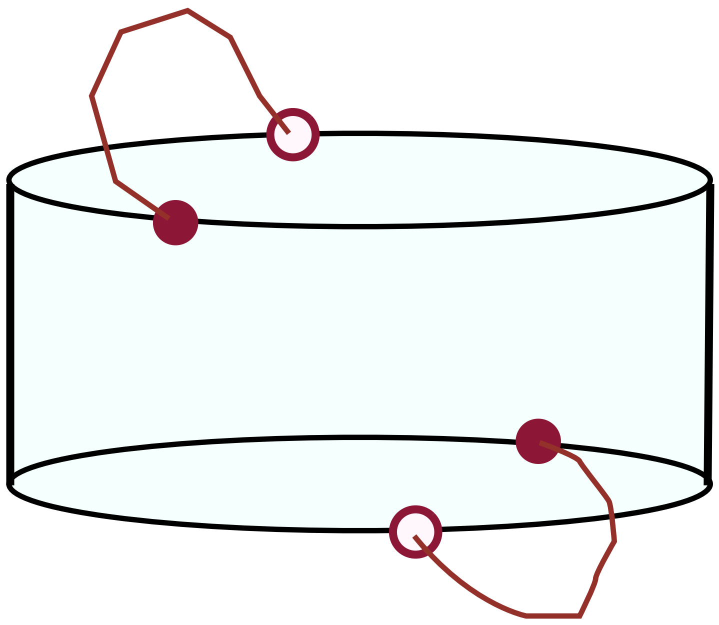
- 2+1 dimensional gauge theory
- Topologically invariant
- H=0
- Observables are Wilson loops describing the bulk particles.



Boundaries

In the presence of a boundary, the Wilson lines are cut along the boundary, 'marking' points.

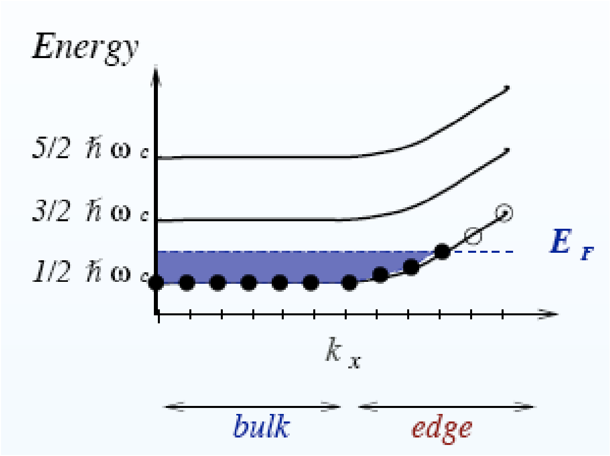
The theory describing the end points is a conformal field theory on the (1+1) dimensional boundary.

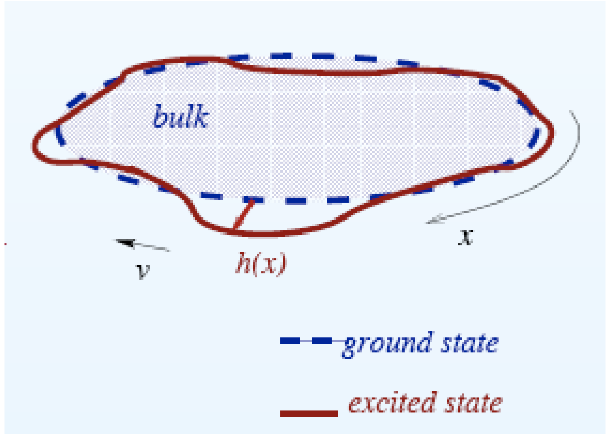


Witten 1989

Edge states

- Gapless excitations form ‘surface waves’
- 1+1 dimensional relativistic theory
- Measurements probe the edge states
- Gives rise to a CFT description of the edge





X.-G. Wen 1989

Edge theory of the MR state

A chiral boson φ_c describes the charge:

$$\mathcal{L} = \frac{g}{2\pi}\partial_x\phi_c(\partial_t + v\partial_x)\phi_c$$

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$$\begin{aligned} V_{\mathrm{el}}(z) &= \psi(z)e^{i\sqrt{2}\varphi_c}(z) \\ V_{\mathrm{qh}}(z) &= \sigma(z)e^{i/\sqrt{8}\varphi_c}(z) \end{aligned}$$

$$\begin{aligned} \psi \times \psi &= \mathbf{1} \\ \sigma \times \sigma &= \mathbf{1} + \psi \\ \sigma \times \psi &= \sigma \end{aligned}$$

$$z = i(vt - x)$$

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Origin of the two states

CFT correlators satisfy a neutrality condition:

The overall topological ‘charge’ is zero.

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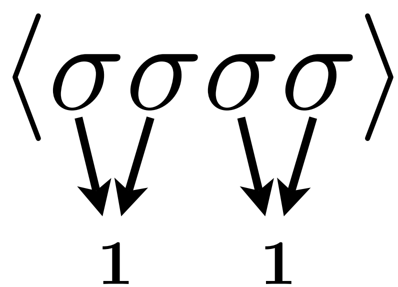

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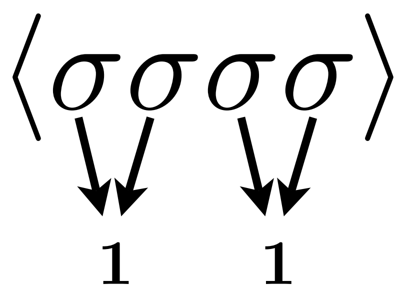

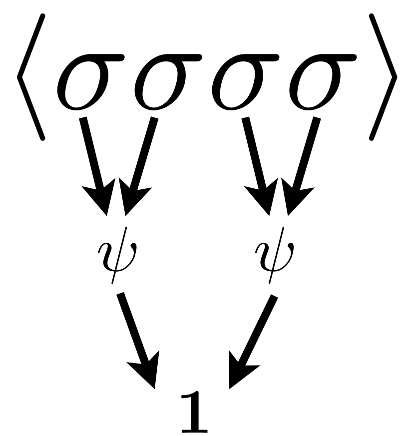

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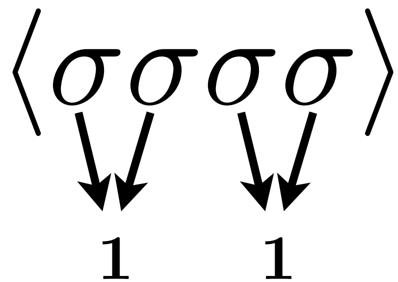

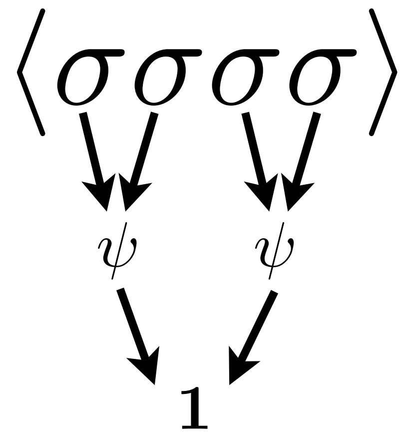

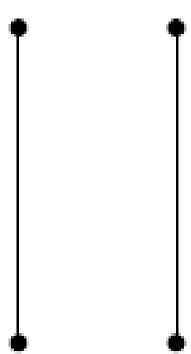

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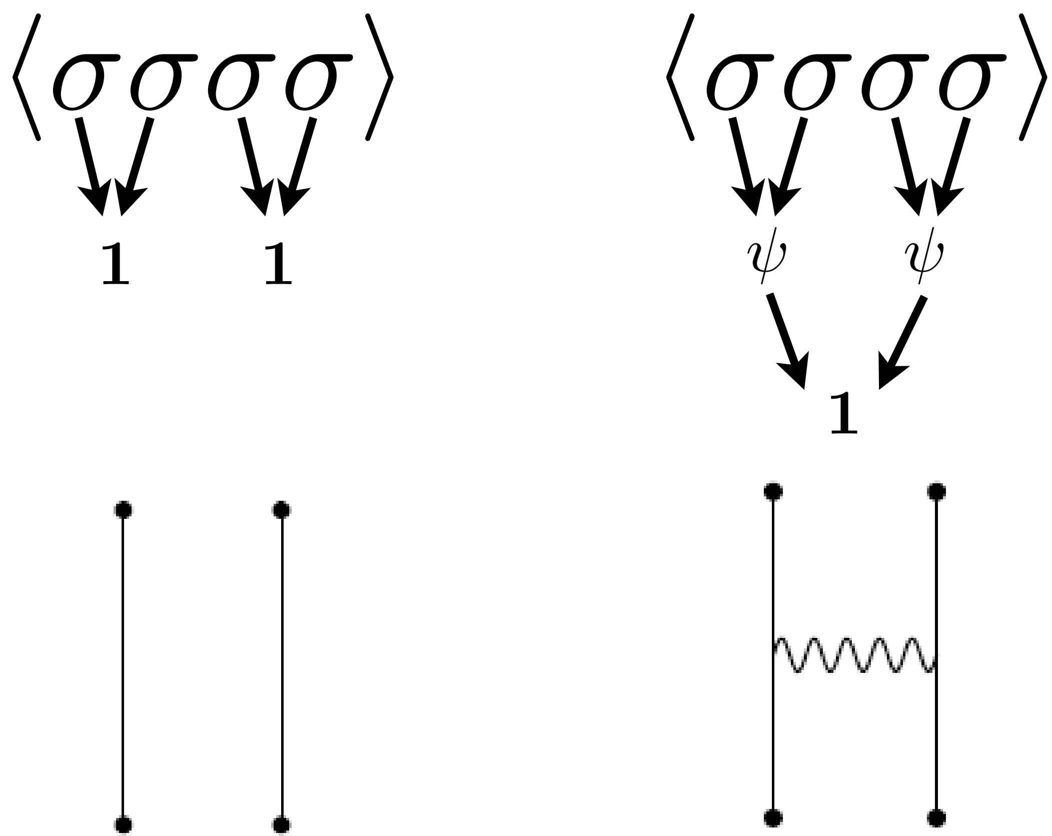
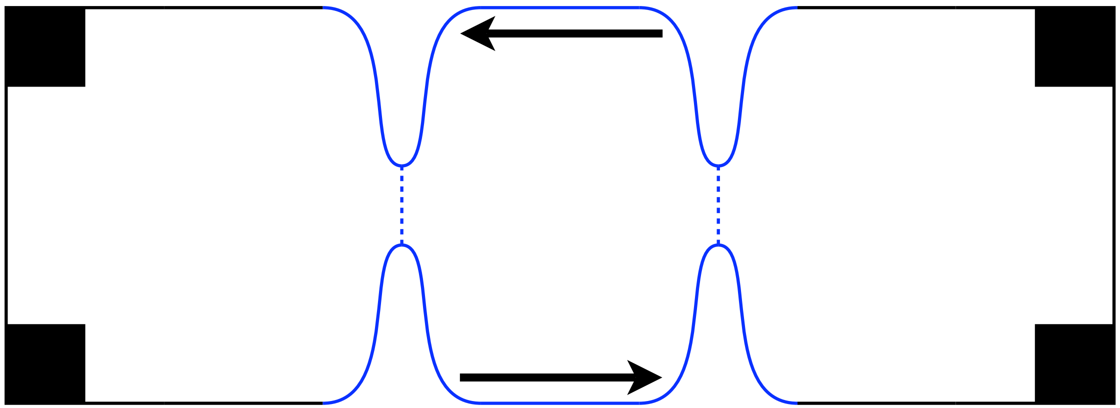


The even-odd effect @ ν=5/2 (topology)



When there are no (or an even number) of excitations in the interferometer, interference should be observed.

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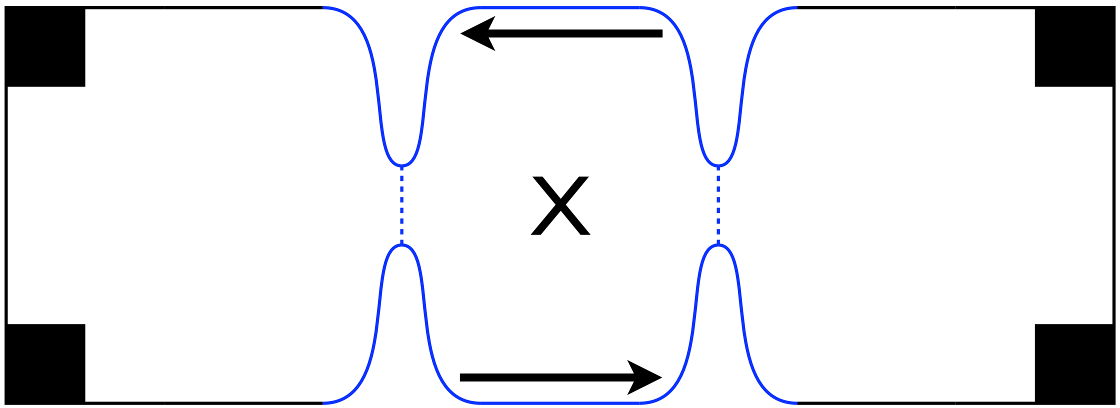

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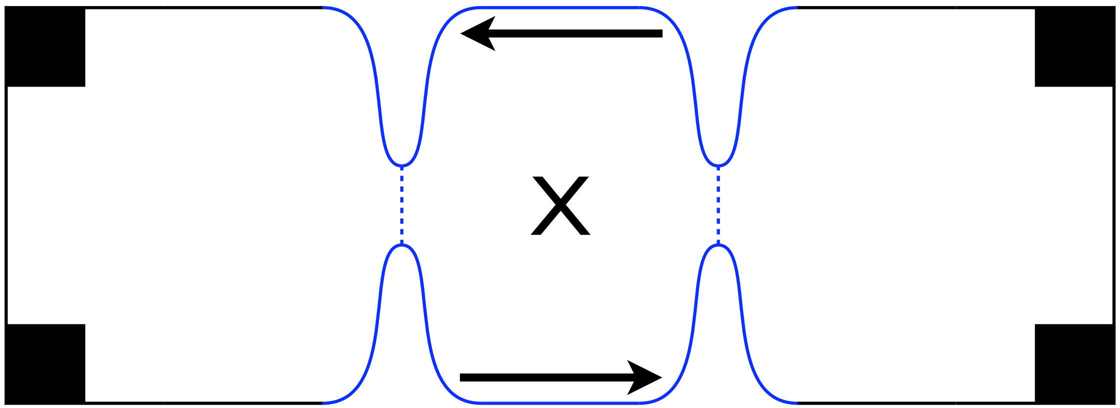
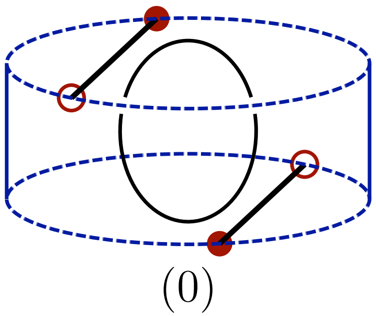


When there are no (or an even number) of excitations in the interferometer, interference should be observed.

For an odd number of excitations, the interference vanishes.

Bonderson et.al, Stern et.al,2006

The edge-bulk connection



Tunneling corresponds to a Wilson line ending on the boundary

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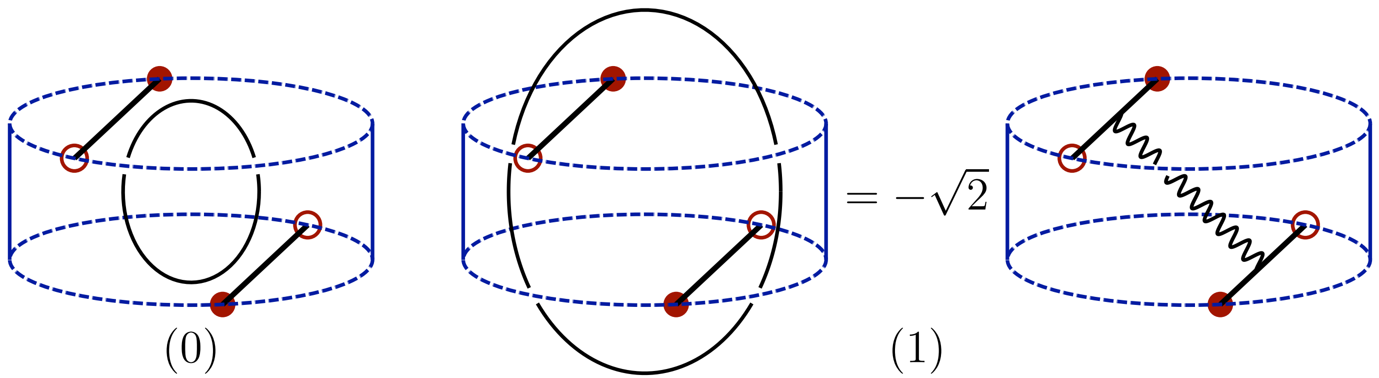

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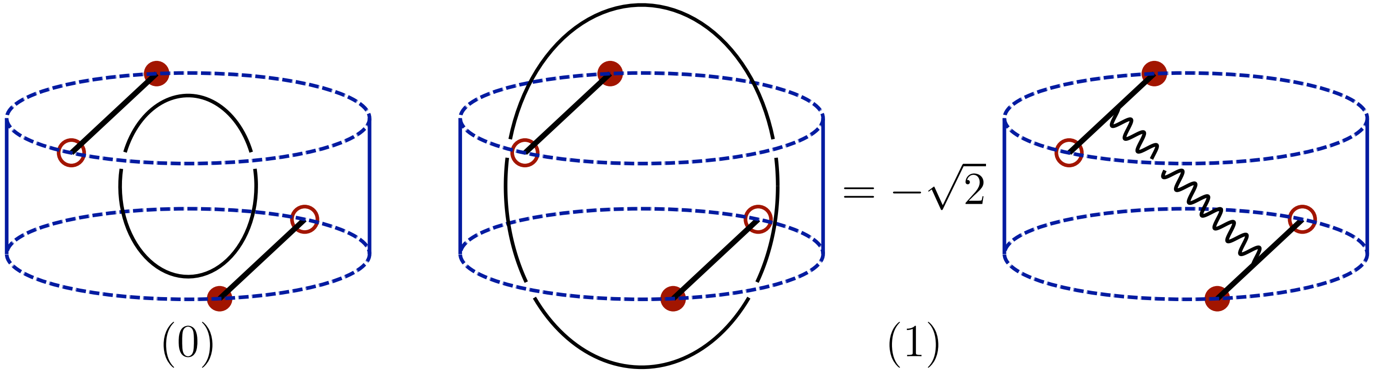
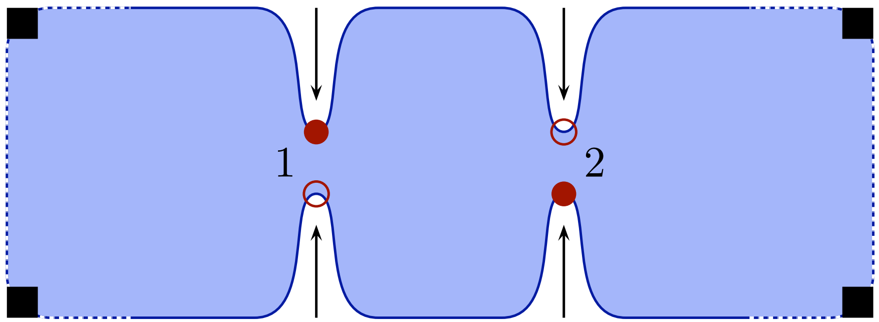


Tunneling corresponds to a Wilson line ending on the boundary

The 'history' of the bulk (beyond our control!) will determine which state is measured.

Only noise measurements can tell the difference!

Setup of the calculation



(following Chamon et.al.)

Charge transfer across the point contacts:

$$\hat{V}_1(t) = \sigma(x_1,t)\sigma(x_2,t)e^{i/\sqrt{8}\varphi_c(x_1,t)}e^{-i/\sqrt{8}\varphi_c(x_2,t)}$$
$$\hat{V}_2(t) = \sigma(x_3,t)\sigma(x_4,t)e^{i/\sqrt{8}\varphi_c(x_3,t)}e^{-i/\sqrt{8}\varphi_c(x_4,t)}$$

Tunneling Hamiltonian:

$$\hat{H}_{\rm tun}(t) = \sum_j \Gamma_j(t)\hat{V}_j(t) + {\rm h.c.} \qquad \Gamma_j(t) = \Gamma_j e^{i\omega_0 t} \qquad \omega_0 = \frac{e^* V}{\hbar}$$

$$\hat{I}(t) = ie^* \sum_j \Gamma_j(t)\hat{V}_j(t) + {\rm h.c.}$$

Linear response theory

The (causal) current is given by a commutator, while the noise is given by an anti-commutator

$$\langle \hat{I}(t) \rangle = -i \int_{-\infty}^{t} dt' \langle [\hat{I}(t), \hat{H}_{\text{tun}}(t')] \rangle$$

$$S(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} dt' e^{i\omega t'} \langle \{\hat{I}(t), \hat{I}(t')\} \rangle$$

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Both the current and the noise probe the four σ correlator:

$$\langle \sigma(z_1)\sigma(z_2)\sigma(z_3)\sigma(z_4) \rangle^{(p)} = \frac{1}{\sqrt{2}} (z_1 - z_2)^{-\frac{1}{8}} (z_3 - z_4)^{-\frac{1}{8}} (1-\xi)^{-1/8} \times \sqrt{1 + (-1)^p \sqrt{1-\xi}}$$

$$\xi = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)}$$

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$$\langle \hat{I} \rangle^{(p)} \equiv \langle \hat{I} \rangle_d^{(p)} + \cos(\tfrac{\phi}{\Phi_0}) \langle \hat{I} \rangle_{\mathrm{osc}}^{(p)} =$$

$$\Re \int_0^{\infty} dt \sum_{j,k=1}^{2} \Gamma_j \Gamma_k^* \left[e^{-i\omega_0 t} \left(\langle \hat{V}_j \hat{V}_k^\dagger \rangle^{(p)}(t) - \langle \hat{V}_k^\dagger \hat{V}_j \rangle^{(p)}(t) \right) \right]$$

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$$\langle \hat{I}(t) \rangle = -i \int_{-\infty}^{t} dt' \langle [\hat{I}(t), \hat{H}_{\rm tun}(t')] \rangle$$

$$S(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} dt' e^{i\omega t'} \langle \{ \hat{I}(t), \hat{I}(t') \} \rangle$$

$$\langle S(\omega) \rangle^{(p)} \equiv \langle S(\omega) \rangle_d^{(p)} + \cos(\frac{\phi}{\Phi_0}) \langle S(\omega) \rangle_{\rm osc}^{(p)} = \Re \int_{-\infty}^{\infty} dt \sum_{j,k=1}^{2} \Gamma_j \Gamma_k^* \Big[e^{i(\omega-\omega_0)t} \big(\langle \hat{V}_j \hat{V}_k^\dagger \rangle^{(p)}(t) + \langle \hat{V}_k^\dagger \hat{V}_j \rangle^{(p)}(t) \big) \Big]$$

The role of causality

In the non-causal region, tunneling operators commute: $(t < a)$

$$\langle \hat{V}_k^\dagger \hat{V}_j \rangle^{(p)} = \langle \hat{V}_j \hat{V}_k^\dagger \rangle^{(p)}$$

In the causal region, they do not: $(t > a)$

$$\langle \hat{V}_k^\dagger \hat{V}_j \rangle^{(p)} = (\langle \hat{V}_j \hat{V}_k^\dagger \rangle^{(p)})^*$$

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Only the non-causal region shows a channel dependence, so only the noise depends on the channel!

Results for current & noise

The result for the current oscillations:

$$\langle \hat{I} \rangle_{\text{osc}}^{(p)} = 4e^* \sqrt{\pi T} |\Gamma_1 \Gamma_2| \int_a^\infty dt \frac{\sin(\omega_0 t)}{\sinh(\pi T(t-a))^{1/4} \sinh(\pi T(t+a))^{1/4}}$$

The result for the noise oscillations:

$$\begin{aligned} \langle S(\omega) \rangle_{\text{osc}}^{(p)} &= 4(e^*)^2 \sqrt{\pi T} |\Gamma_1 \Gamma_2| \times \\ & \Big(\int_a^\infty dt \frac{\cos((\omega+\omega_0)t) + \cos((\omega-\omega_0)t)}{\sinh(\pi T(t-a))^{1/4} \sinh(\pi T(t+a))^{1/4}} + \\ & (-1)^p \int_0^a dt \frac{\sqrt{2}(\cos((\omega+\omega_0)t) + \cos((\omega-\omega_0)t))}{\sinh(\pi T(t-a))^{1/4} \sinh(\pi T(t+a))^{1/4}} \Big) \,. \end{aligned}$$

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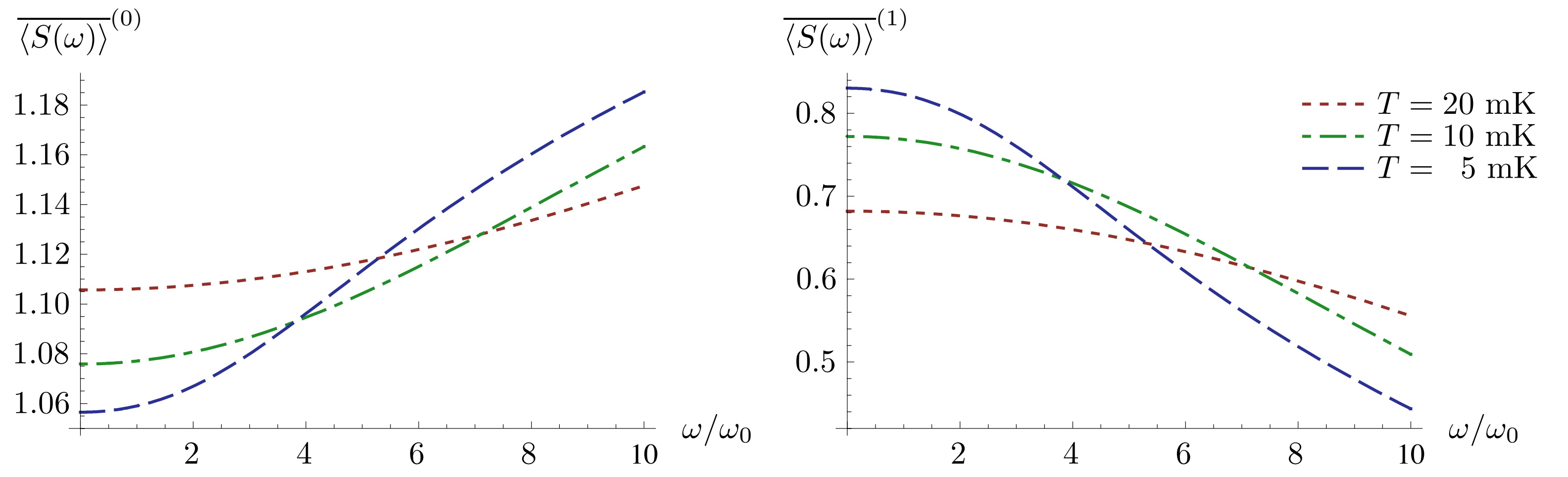
The interference noise

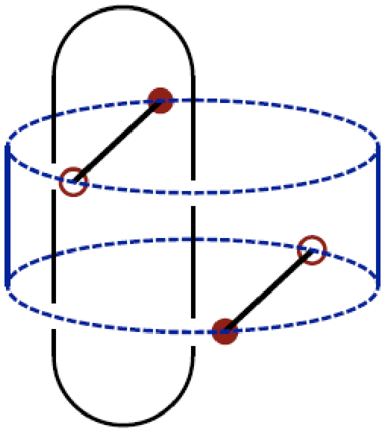


$$a = 5\mu\mathrm{m},\ V = .38\mu V\ (\omega_0 = 100\ \mathrm{MHz}),\ v = 1.10^7 m/s$$

The (dimensionless) amplitude of the Aharonov-Bohm oscillations shows a clear channel dependence!

The even-odd effect

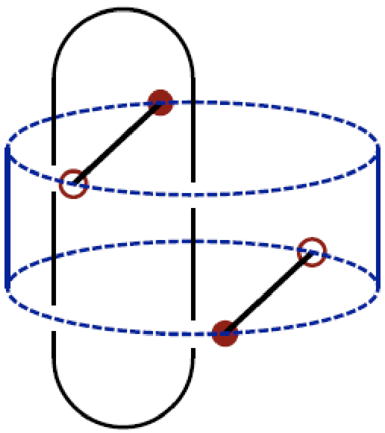
The corresponding diagram is zero:



$$= 0$$

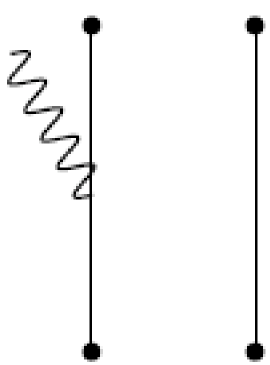
The even-odd effect

The corresponding diagram is zero:



$= 0$

In terms of the edge-theory we also get zero:



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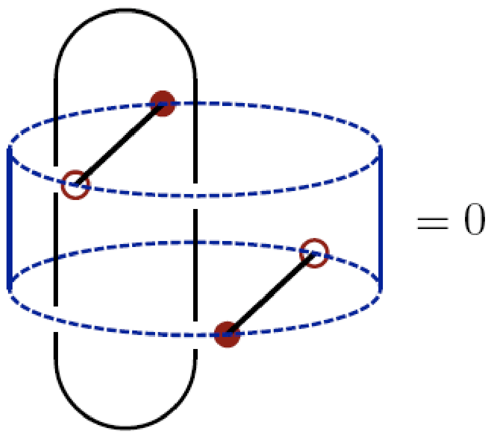

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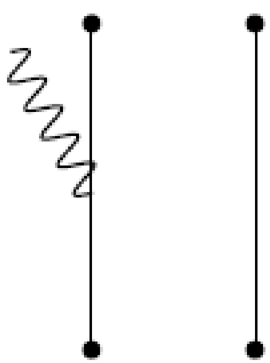
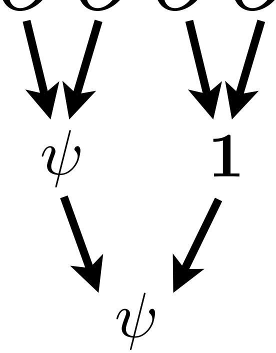


$$\langle \sigma\sigma\sigma\sigma \rangle = 0$$



Conclusion & outlook

- We calculated the noise in a double point contact for the first time
- We predict a clear signature of non-abelian statistics in the interference noise of a double point contact
- These noise measurements are accessible with current technology
- Improvement of the calculation
  - different edge velocities & edge reconstruction
  - different filling in the point contacts
- The Anti-Pfaffian (counter flowing neutral modes)
- Connection between edge & bulk needs better understanding

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