ETH
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מכון
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למדע

Characterizing a Mott Insulator by Dynamically Generating Double Occupancy

arXiv:0808.2350

MaNEP
SWITZERLAND

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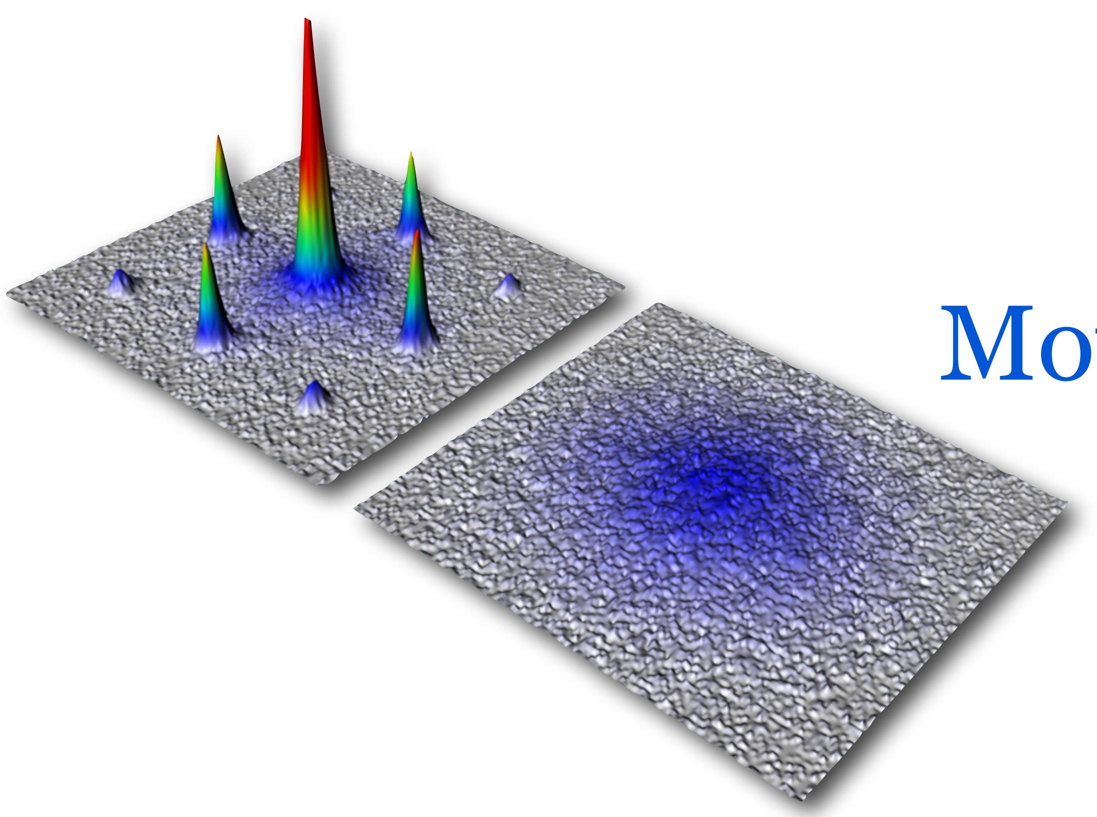
Andreas Rüegg

cold atoms

Bosons

$$H = -t \sum_{\langle i,j \rangle} (a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2} \sum_i a_i^\dagger a_i (a_i^\dagger a_i - 1)$$

^{87}Rb filling between 1-3 per site



- cold atoms in artificial lattices
- single bands realizable
- interactions tunable
- filling tunable

cold atoms

Fermions

$$H = -t \sum_{\langle i,j \rangle, \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + \text{h.c.}) + U \sum_i c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow}$$

^{40}K filling between 1-2 per site in the hyperfine states:

$|\downarrow\rangle \rightarrow |m_F = -9/2\rangle$ $|\uparrow\rangle \rightarrow |m_F = -5/2\rangle$

- cold atoms in artificial lattices
- single bands realizable
- interactions tunable
- filling tunable

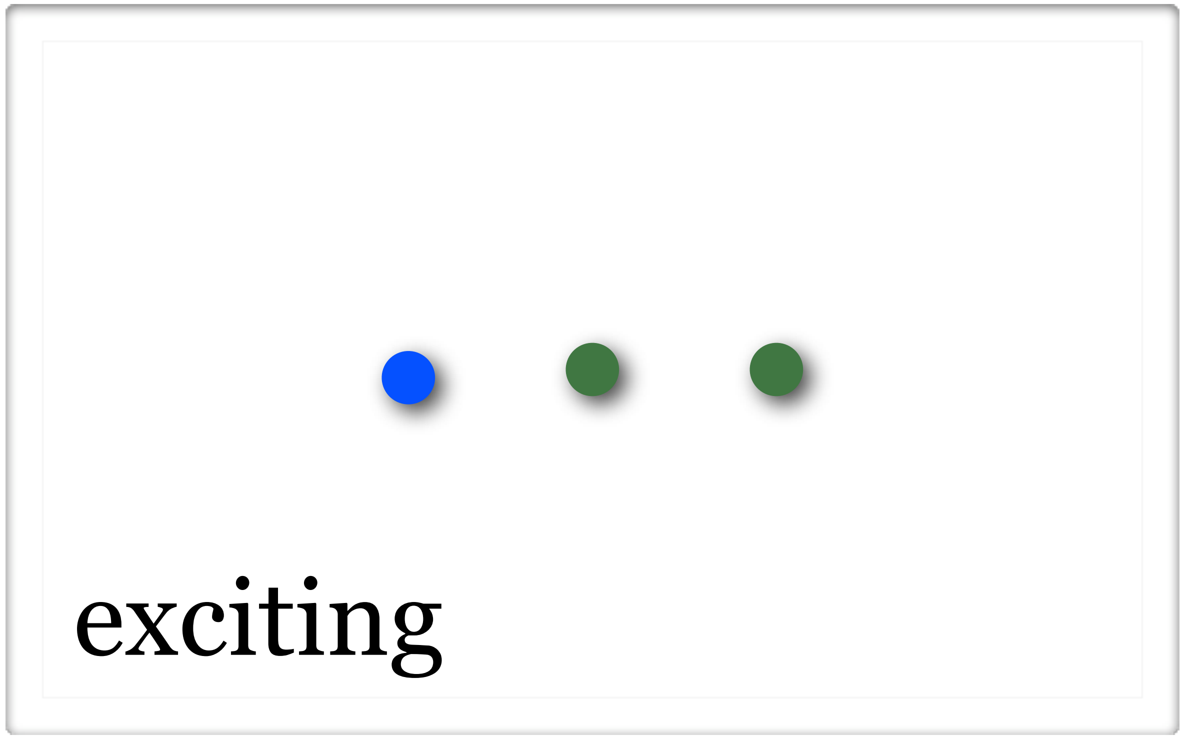
?

ground state

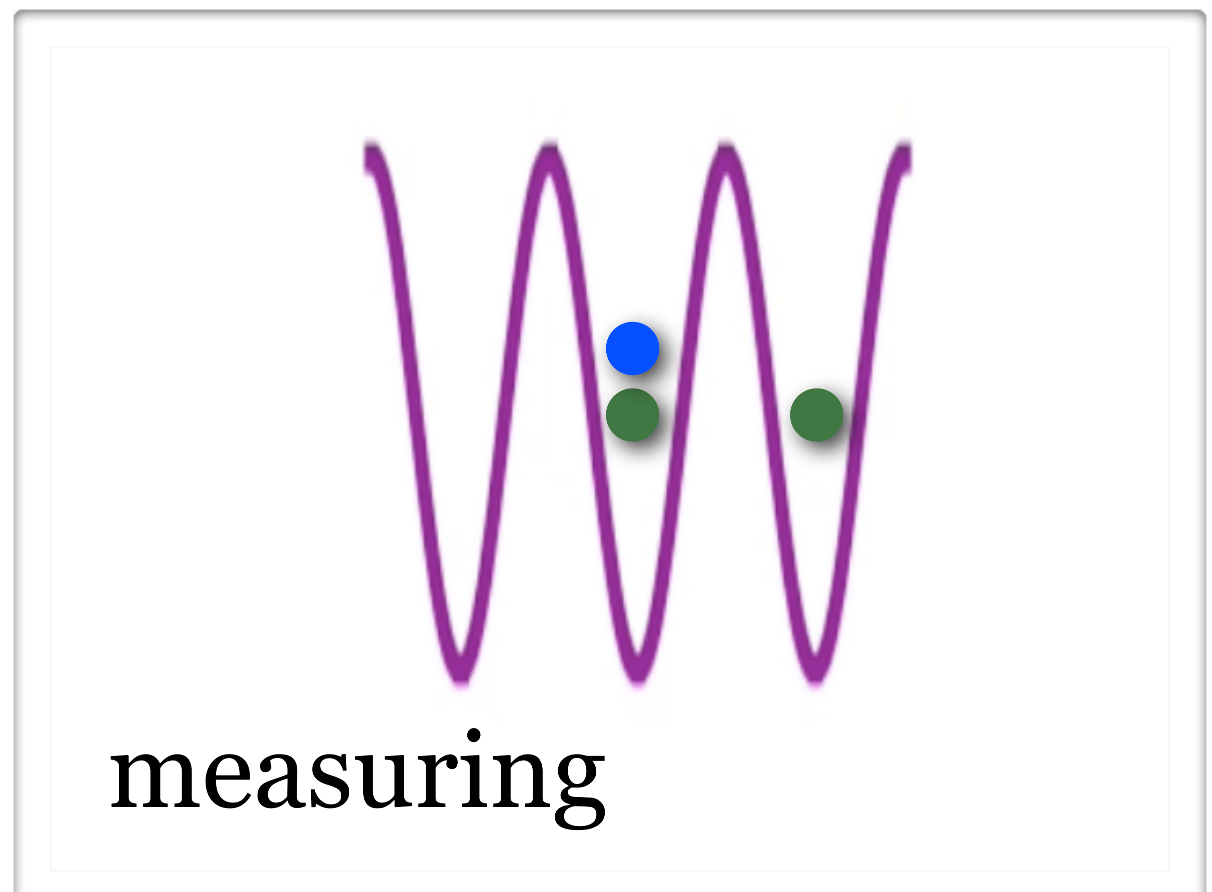
spectral measurement

DGDO, experiment

$$K = \delta t \cos(\omega\tau) \sum_{\langle i,j\rangle,\sigma} c^\dagger_{i\sigma} c_{j\sigma} + \text{h.c.}$$



$$D = \sum_i n_{i\uparrow} n_{i\downarrow}$$



Kollath et al., PRA **74**, 041604(R) (2006)
Jördens et al., Nature **455**, 204 (2008)

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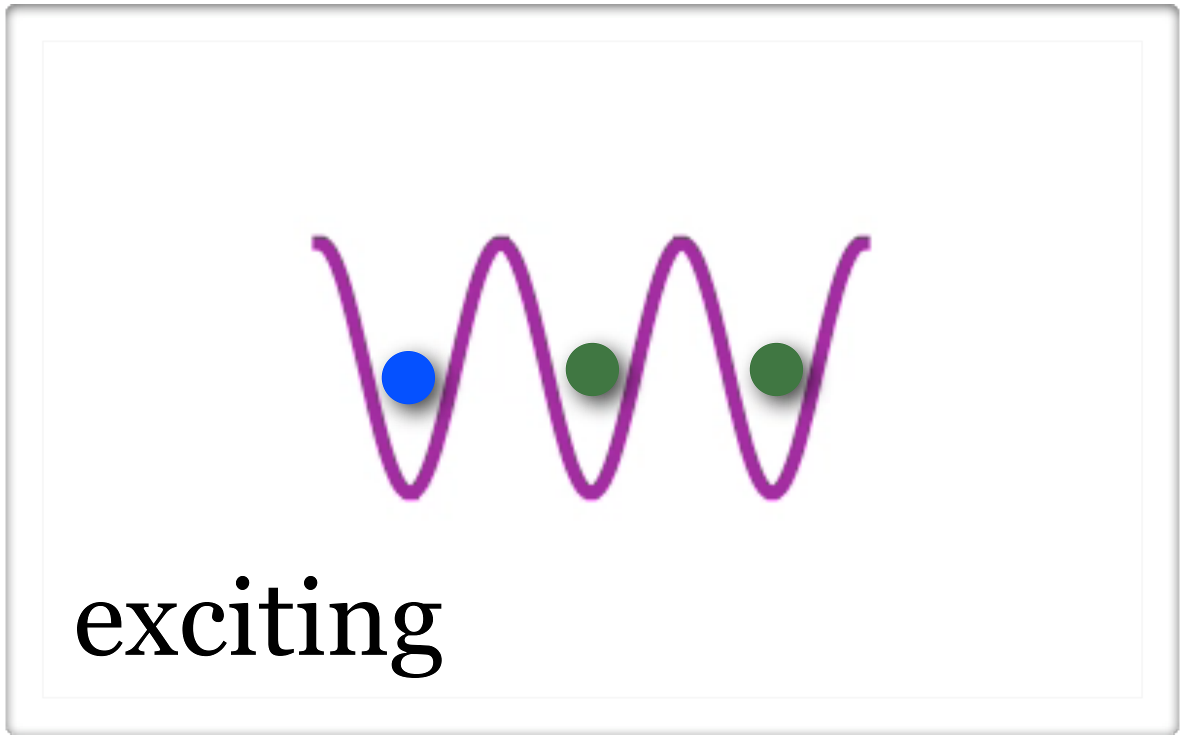

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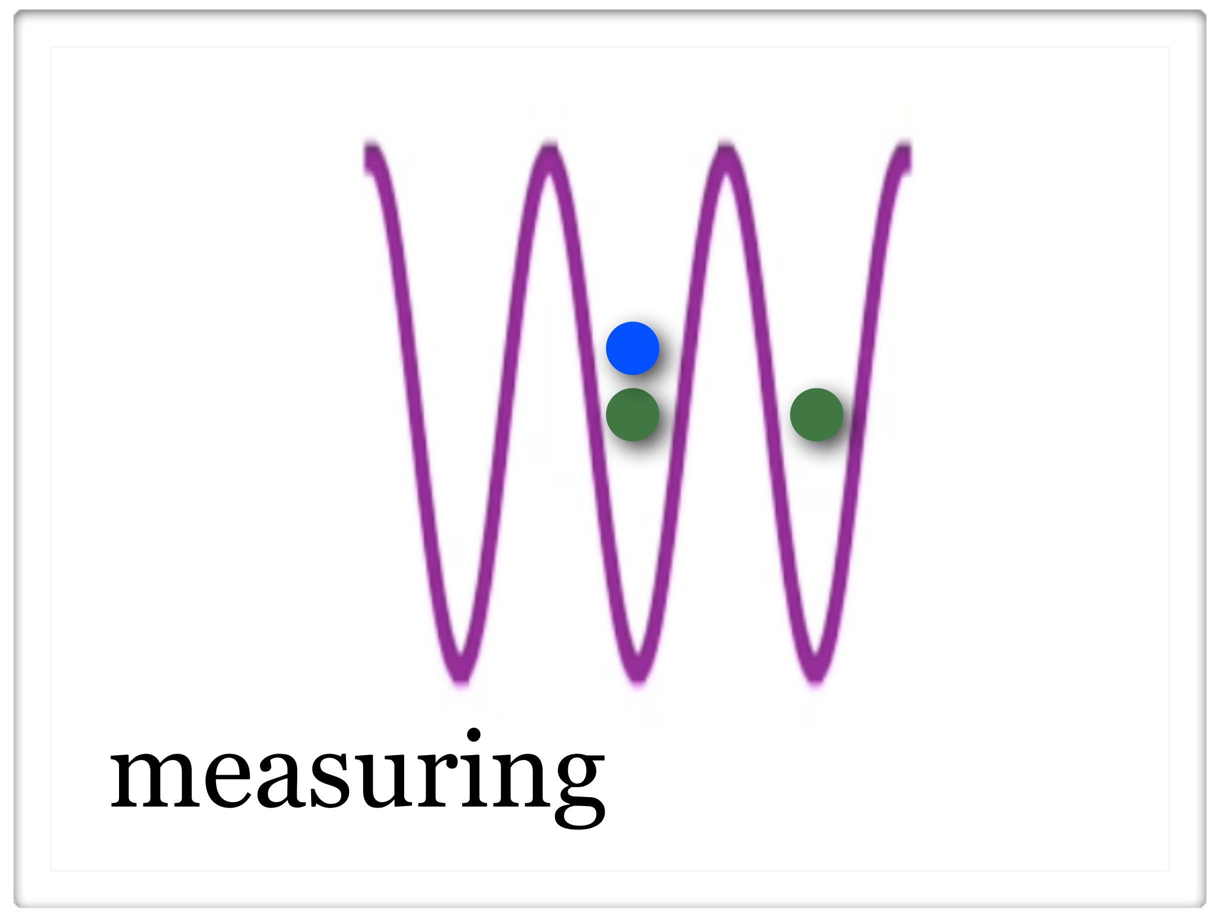

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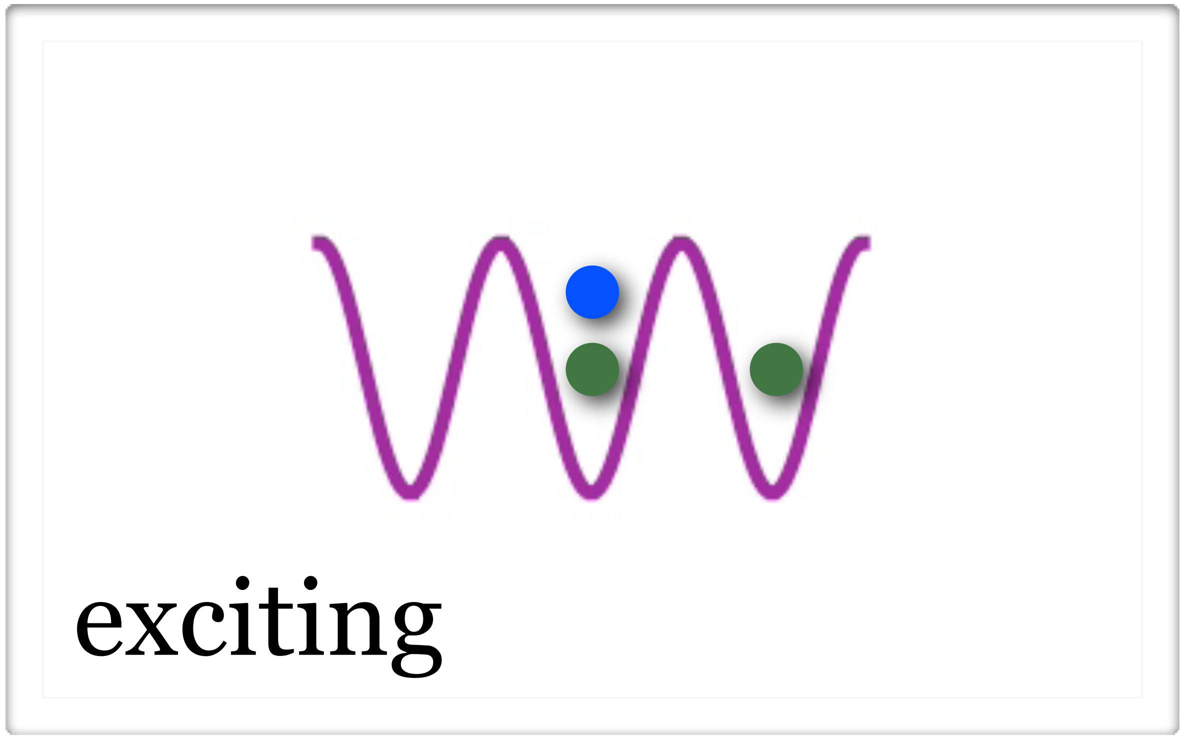

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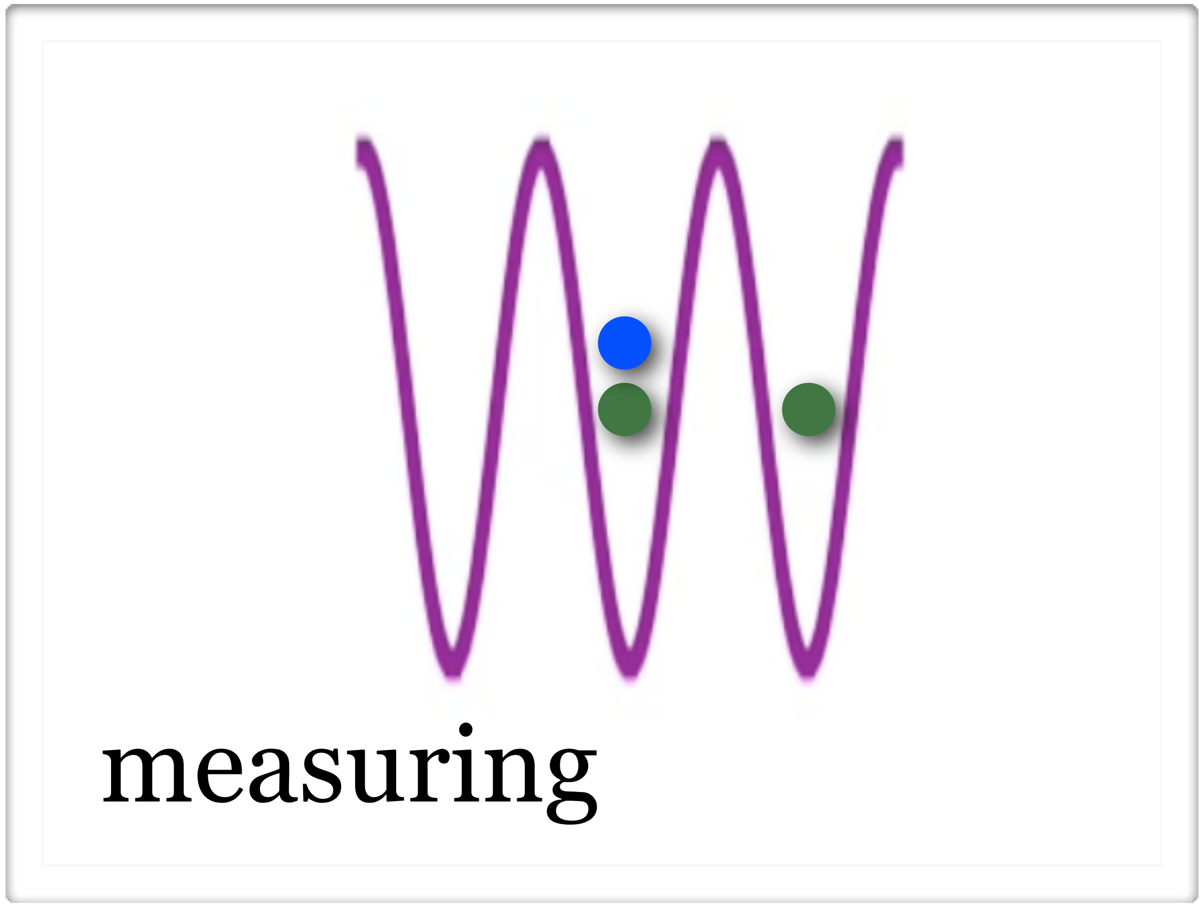

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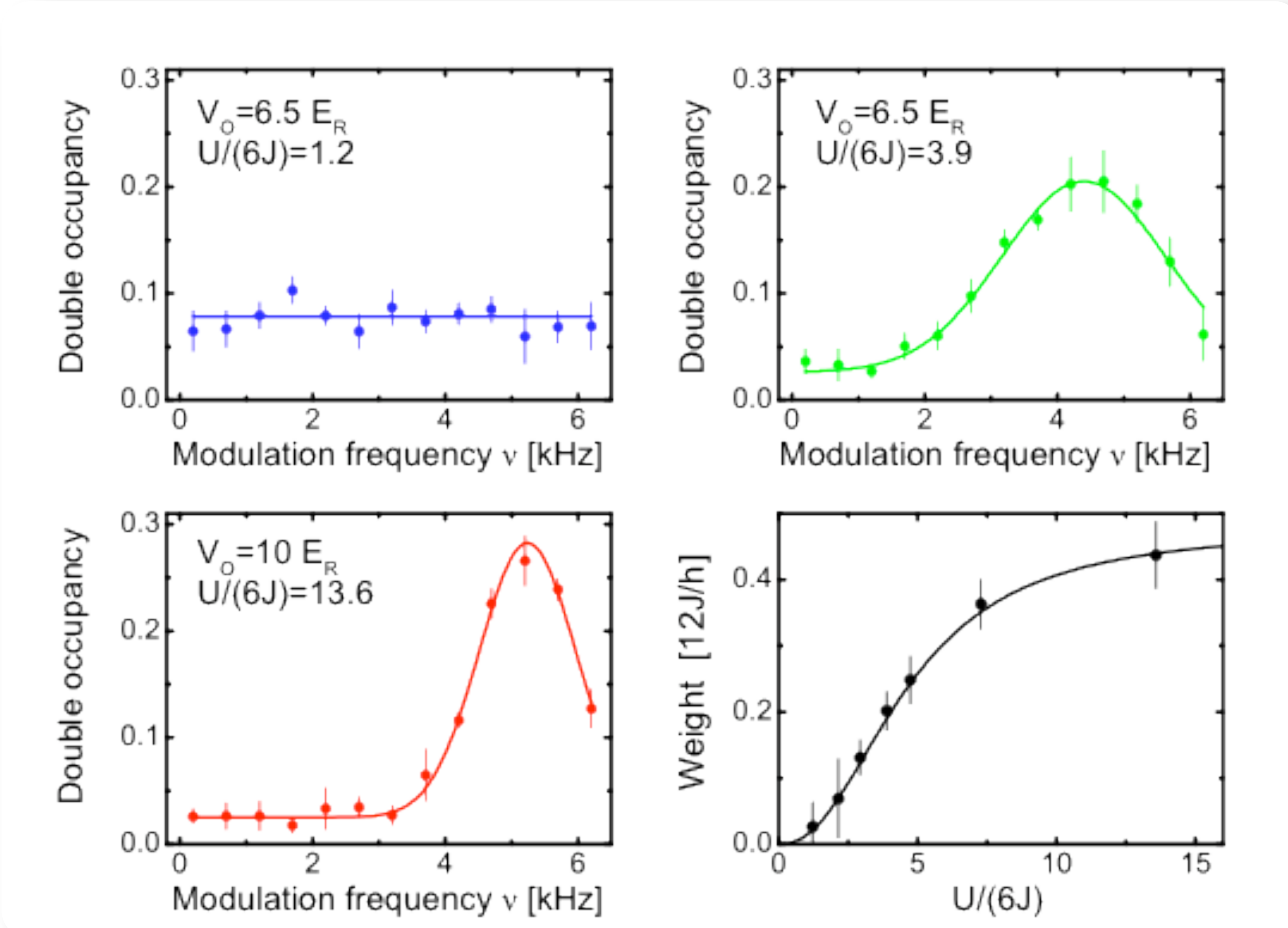
results

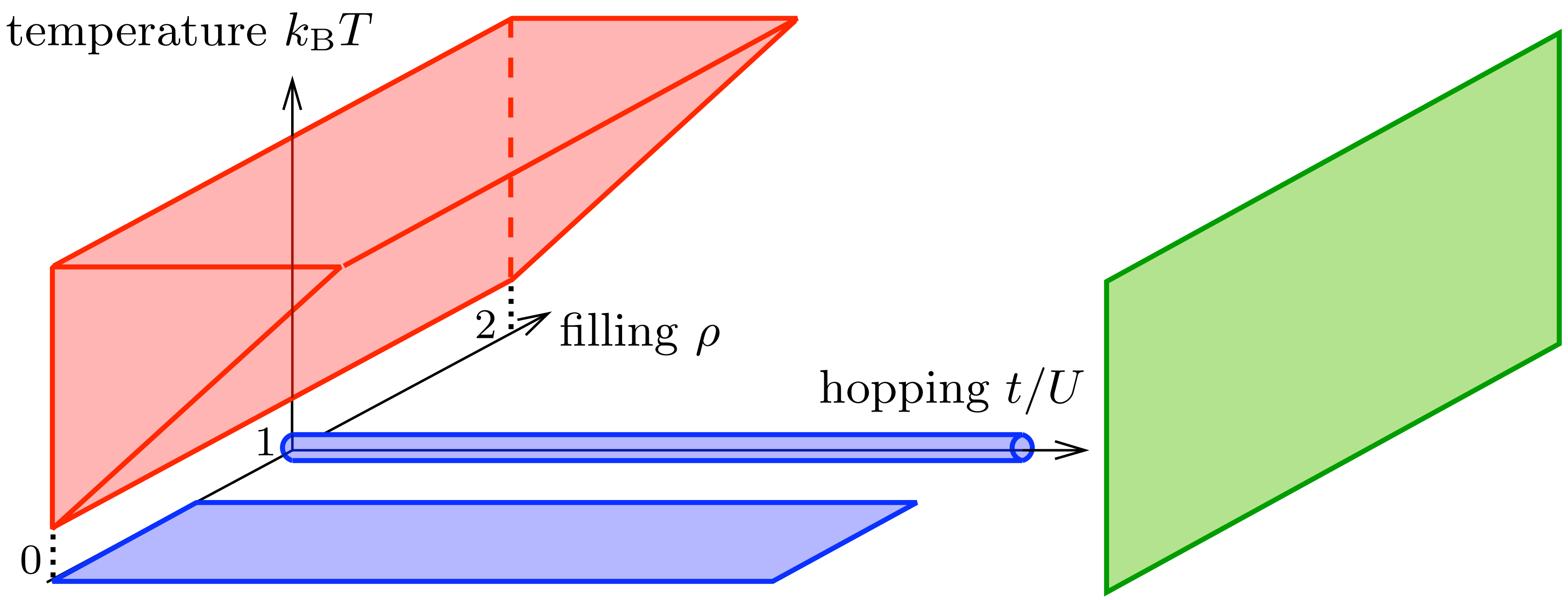


V_0=6.5 E_R
U/(6J)=1.2
V_0=6.5 E_R
U/(6J)=3.9
V_0=10 E_R
U/(6J)=13.6
Double occupancy
Modulation frequency ν [kHz]
Weight [12J/h]
U/(6J)
0.0
0.1
0.2
0.3
0.4
0
2
4
6
5
10
15

regimes

$$\delta\mathcal{D}(\omega) = \sum_n \langle\langle n|D|n\rangle\rangle |\langle n|K|0\rangle|^2 \delta(\hbar\omega - \hbar\omega_{n0})$$

$$\delta\mathcal{D}_{\text{tot}} = \frac{2}{Nz}\int d\omega\, \delta\mathcal{D}(\omega)$$ total DGDO



- non-interacting -- trivial
- at high temperatures, $k_\mathrm{B}T > t$, we solve the atomic limit ($t = 0$)
- at low filling $\rho < 1$ we solve the two particle problem
- at half filling: Mott transition, captured within slave-spin scheme

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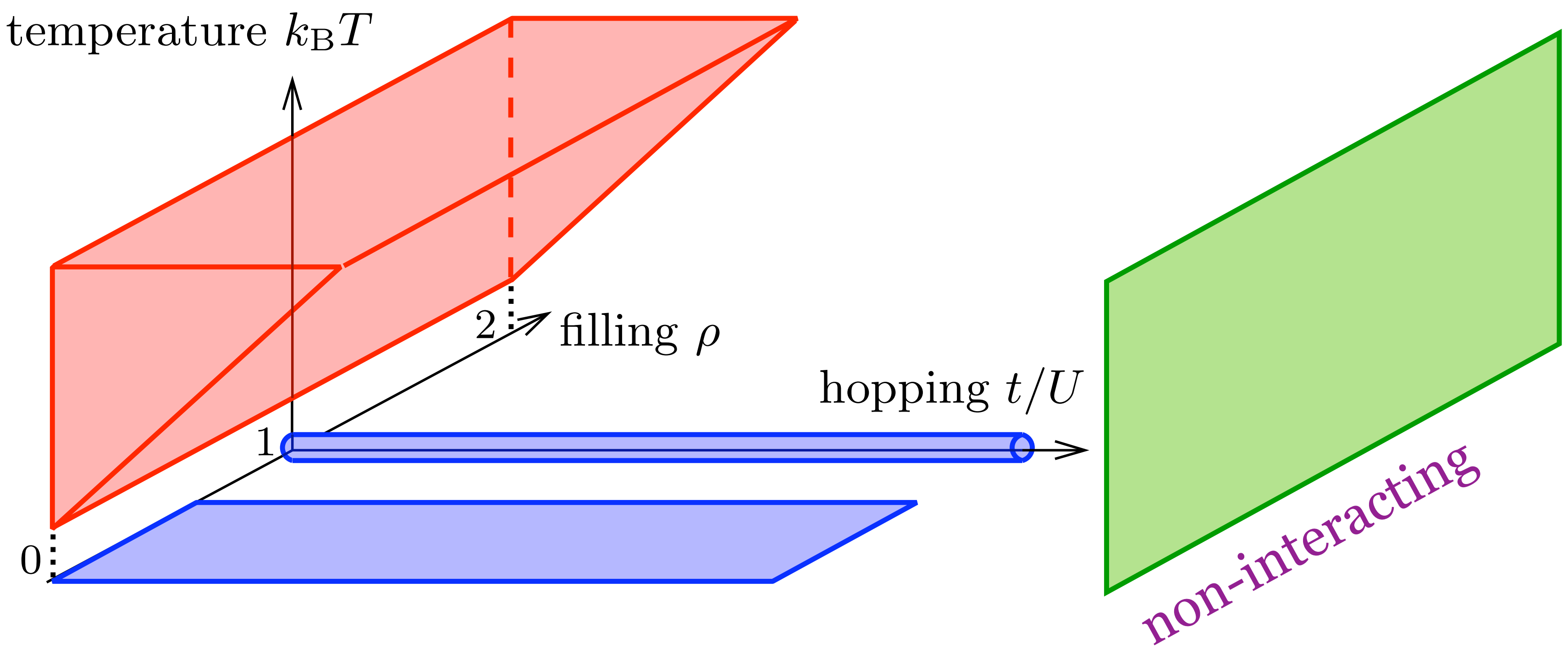

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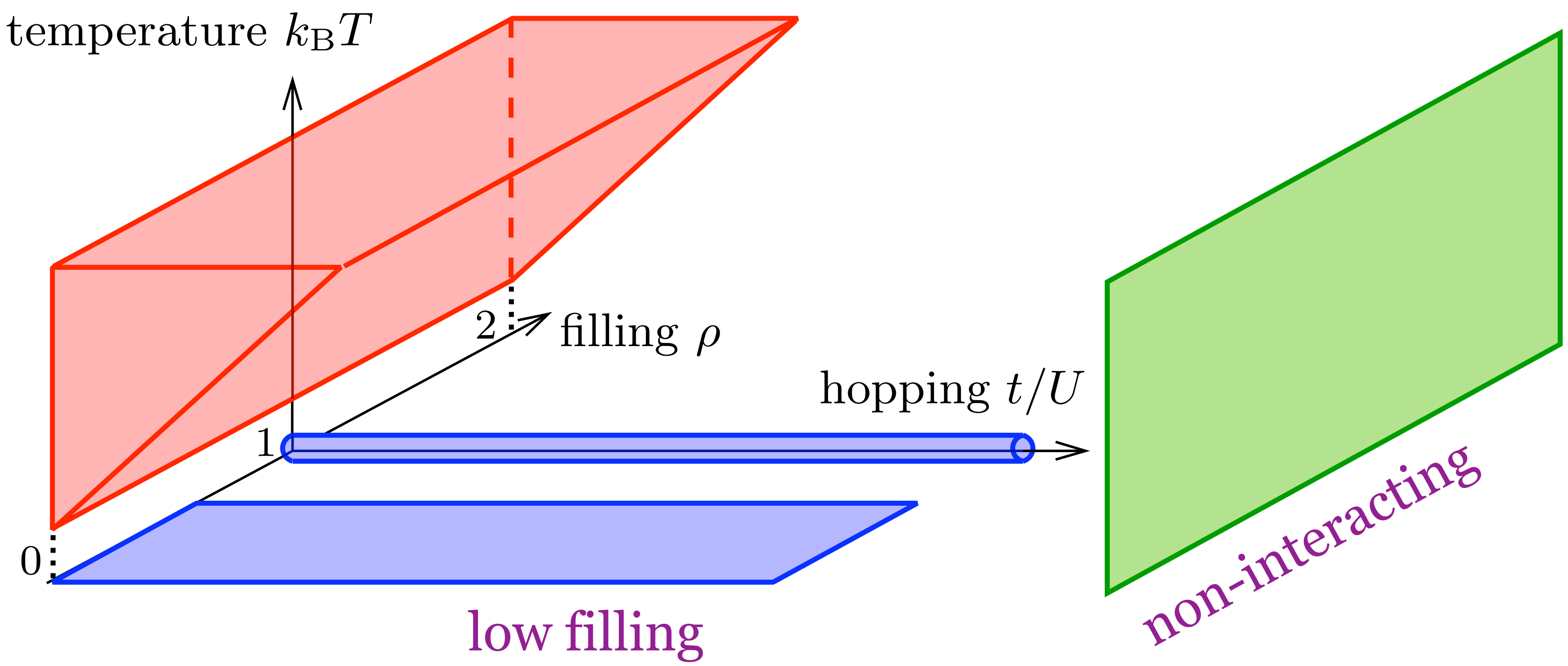

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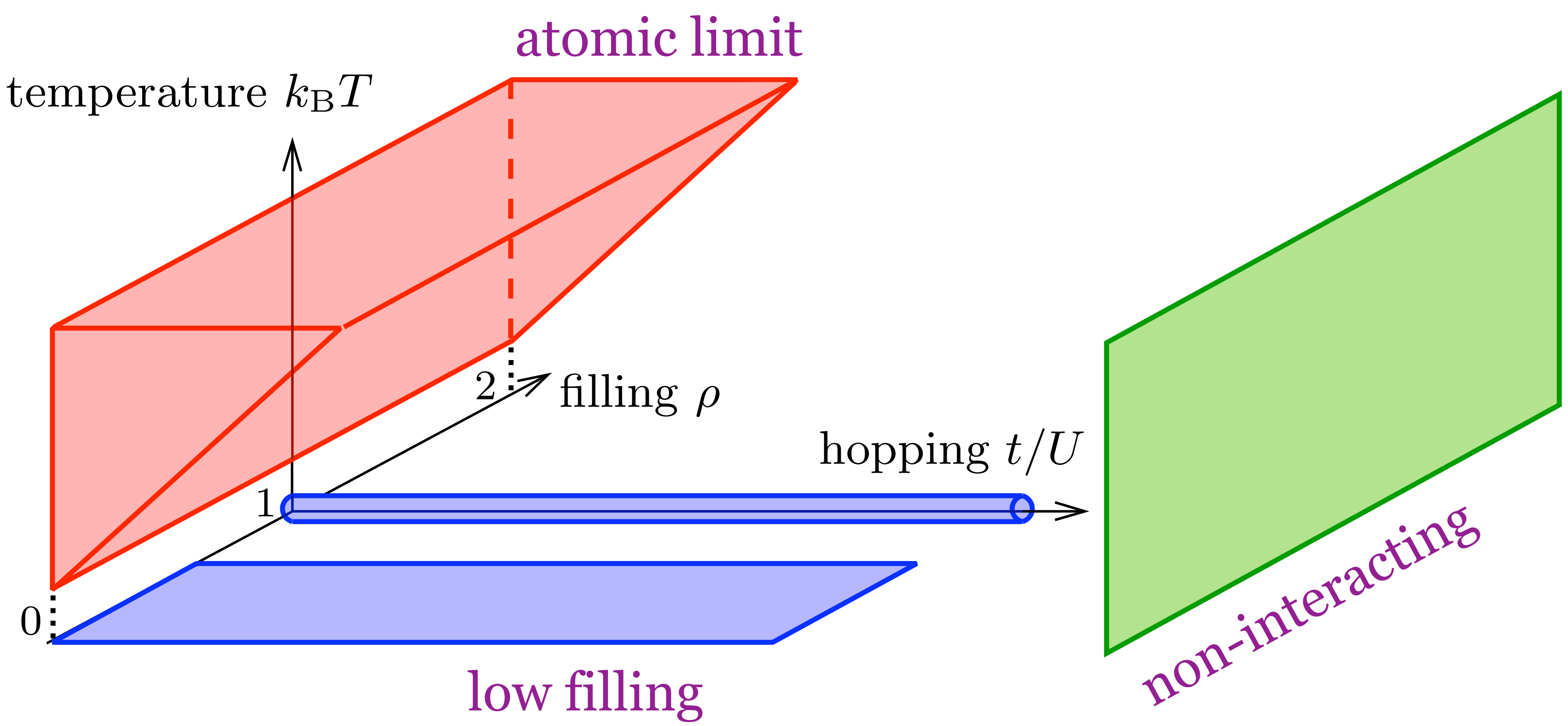
$$\delta\mathcal{D}_{\mathrm{tot}} = \frac{2}{Nz}\int d\omega\, \color{red}{\delta\mathcal{D}(\omega)}$$ total DGDO



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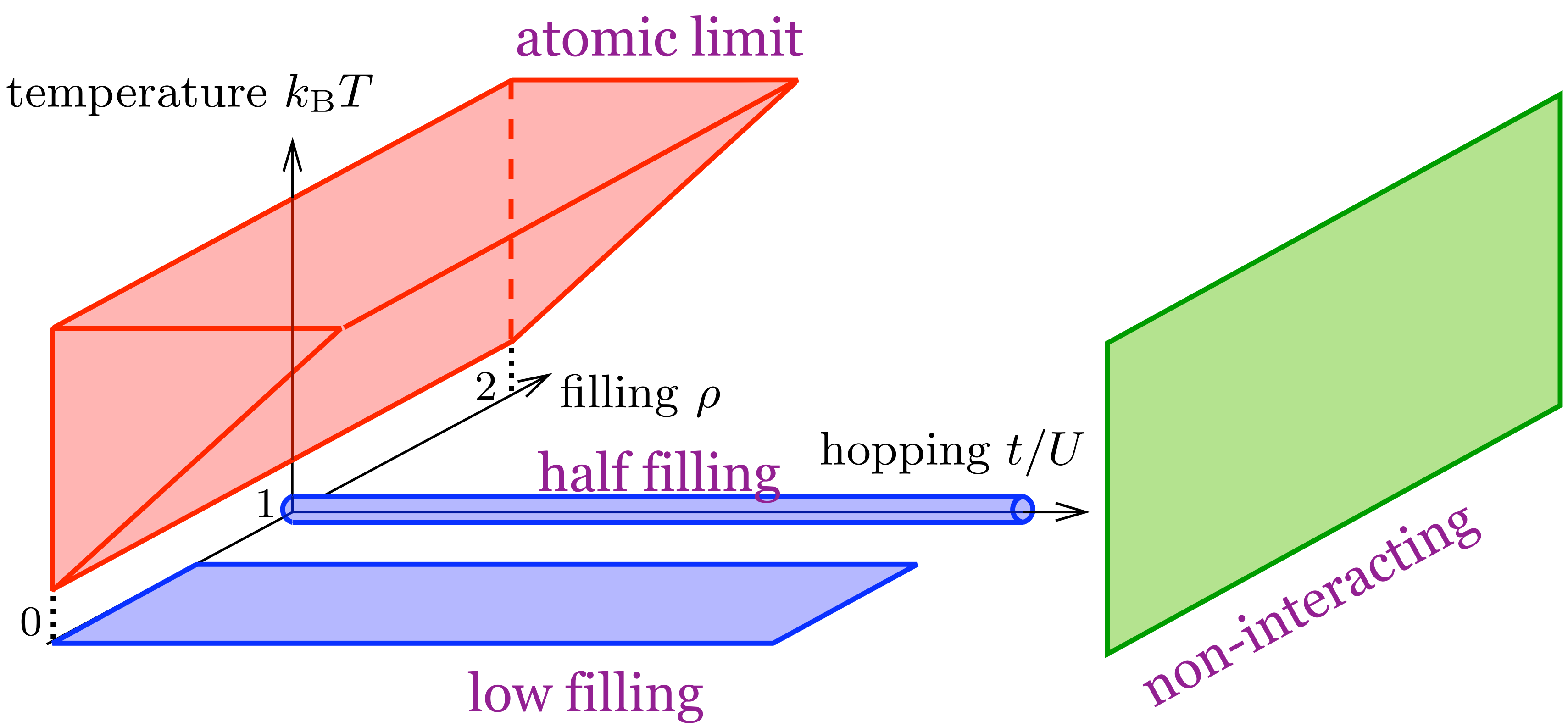
$$\delta\mathcal{D}_{\rm tot} = \frac{2}{Nz}\int d\omega\, \delta\mathcal{D}(\omega)$$ total DGDO



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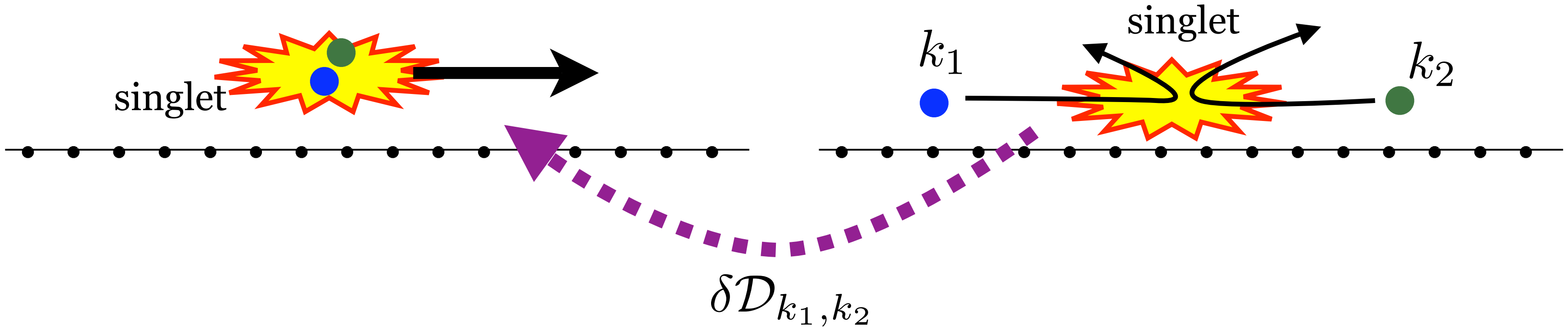
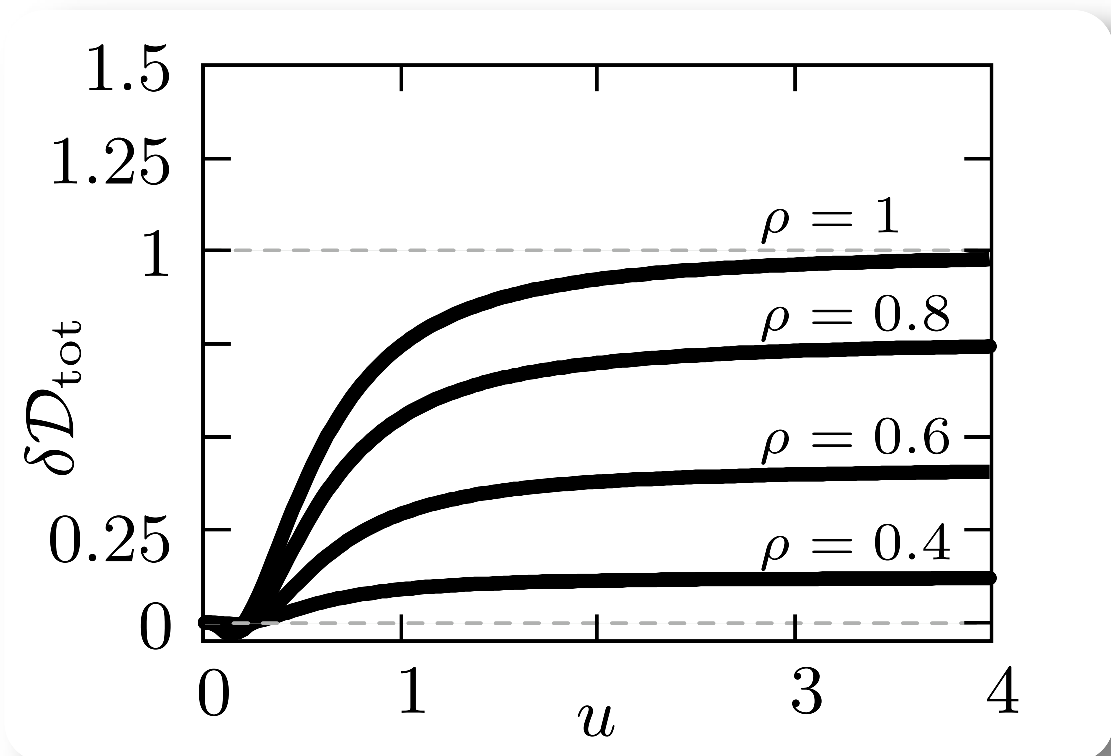
low density

repulsively bound pair



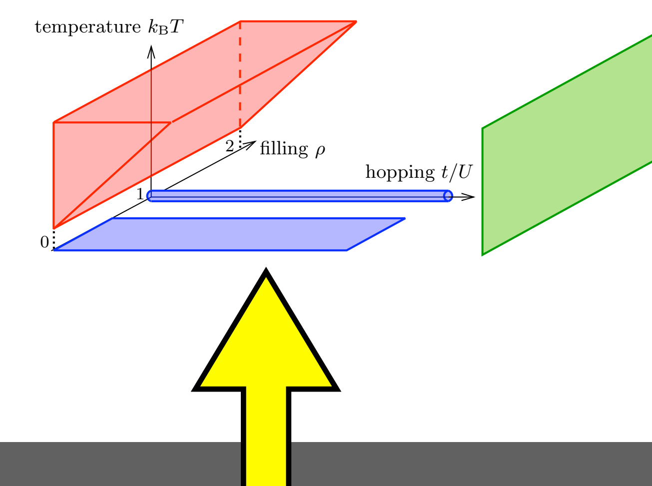


- two-particle problem solvable
- construct many body wave function
- saturation at large values of $U > t$

Lippmann-Schwinger equation

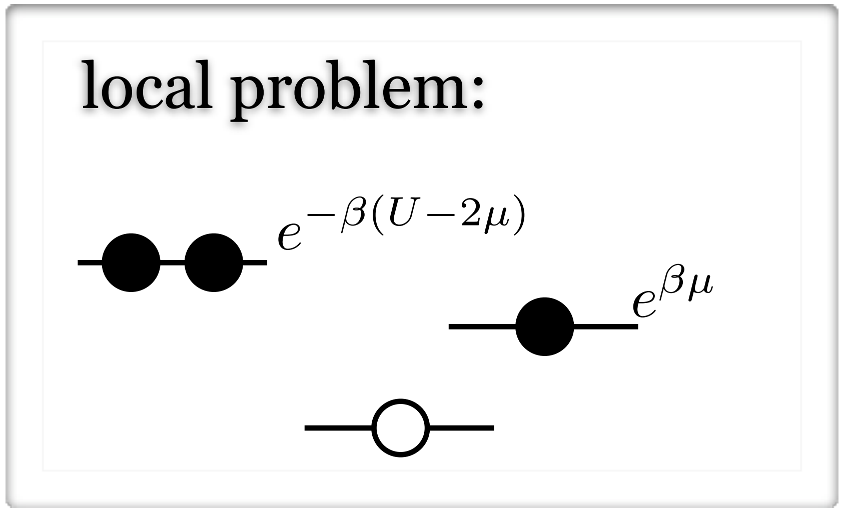
$U\delta_{r',0}$

$$\psi_k(r) = e^{ikr} + \int dr'\, G(r - r', E(k))\psi_k(r')V_{\mathrm{int}}(r')$$

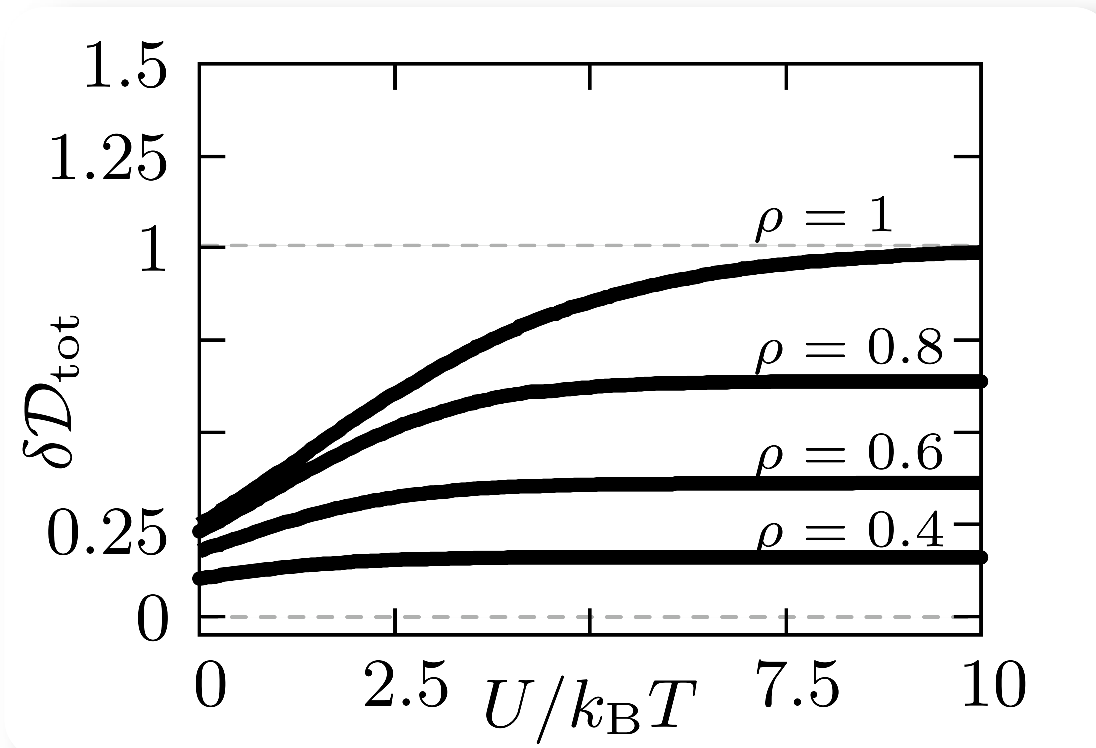


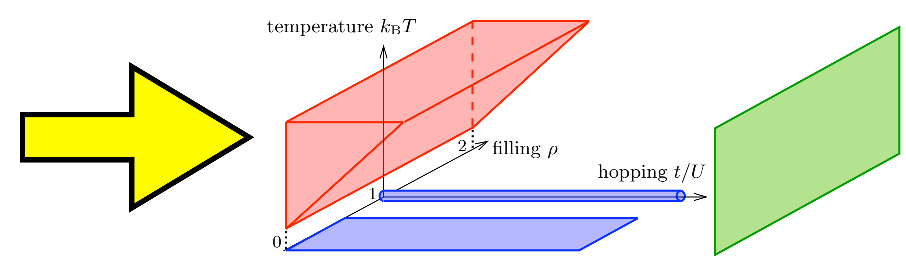
Maldague, PRB **16,** 2427 (1977)

atomic limit



- single site problem solvable
- response reduces to probability to find singly occupied sites
- saturation for $U > k_\mathrm{B}T$





$$\delta\mathcal{D}(\omega) = \sum_{n>m} \frac{e^{-\beta E_m}}{Z} \left(\langle n|D|n\rangle - \langle m|D|m\rangle\right) |\langle n|K|m\rangle|^2 \delta(\hbar\omega - \hbar\omega_{nm})$$

Bari et al., PRB **6,** 4623 (1972)

half-filling

slave-spins

$$c_{i\sigma}^\dagger \to 2S_i^x f_{i\sigma}^\dagger$$

mean-field decoupling

$$|\Psi\rangle \approx |\text{Spin}\rangle \otimes |\text{Fermions}\rangle$$

$$H_F = -g_t t \sum_{\langle i,j\rangle,\sigma} \left(f_{i\sigma}^\dagger f_{j\sigma} + \text{h.c.} \right)$$

$$g_t = 4\langle S_i^x S_j^x \rangle$$

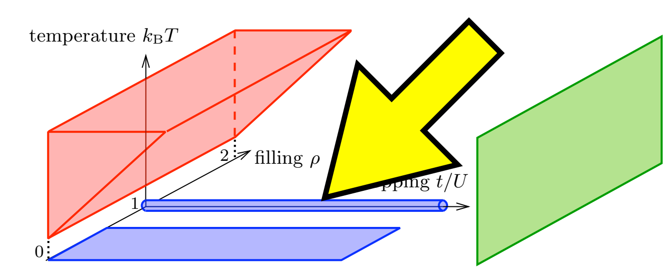
renormalized Fermi sea, leading to Brinkman-Rice transition

$$H_{\text{TIM}} = -J \sum_{\langle i,j\rangle} S_i^x S_j^x + h \sum_i S_i^z$$

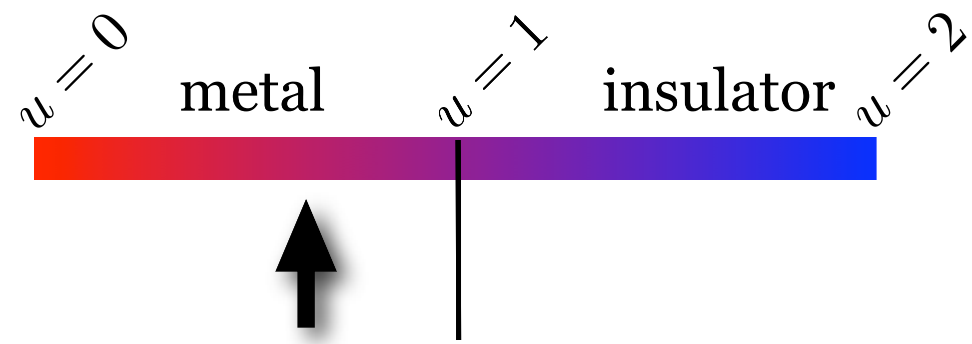
$$J = 4t \sum_\sigma (\langle f_{i\sigma}^\dagger f_{j\sigma} \rangle + \text{c.c}) \qquad h = U/2$$

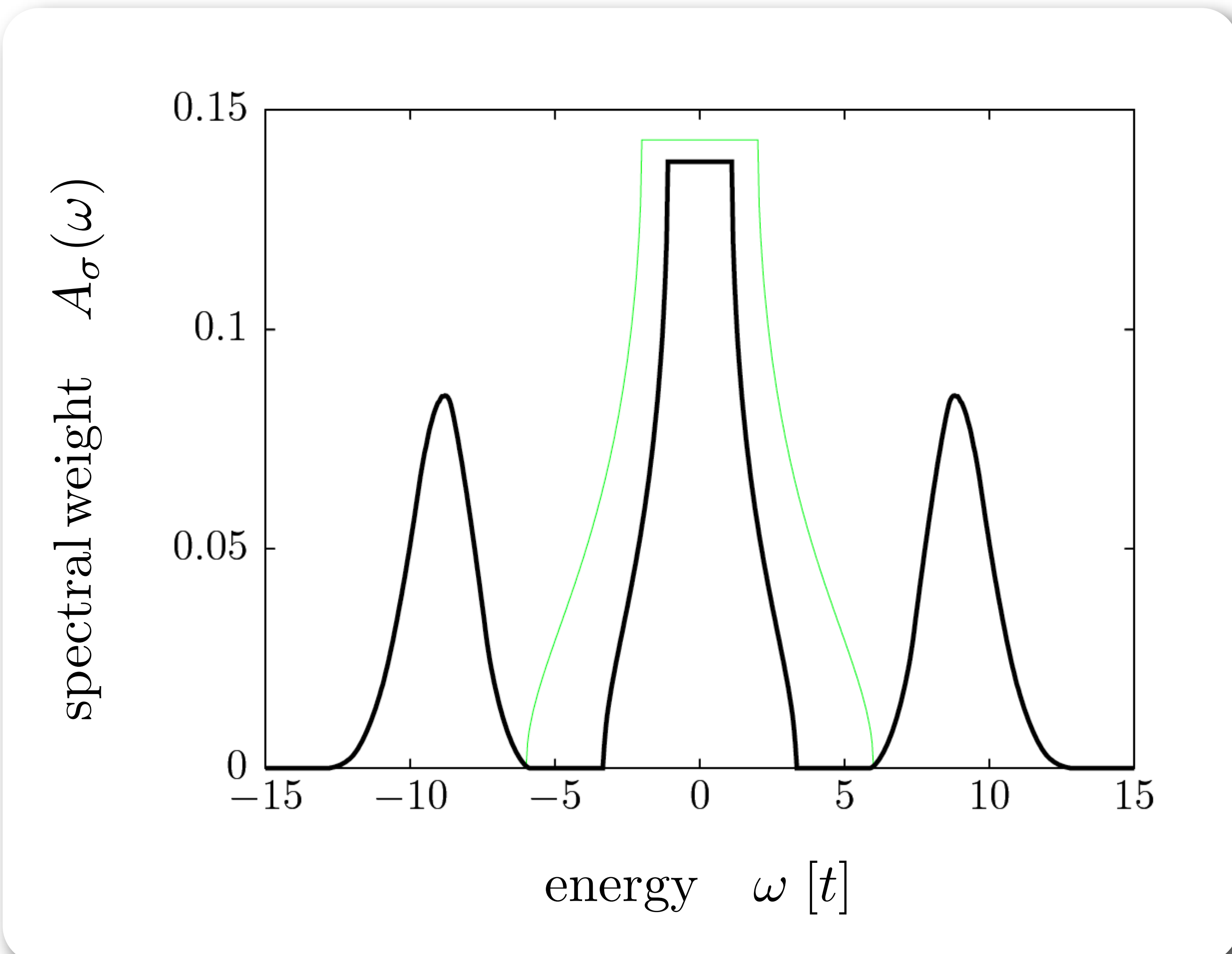
transverse-field Ising model, displaying phase transition

- product wave function in different sectors
- “static” quantities link the two sectors
- renormalized Fermi surface
- transverse Ising model for the spins



DGDO processes I

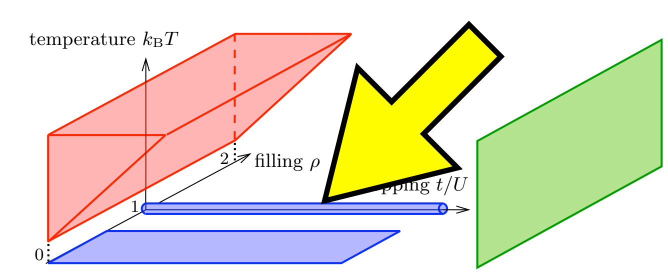




$$A_\sigma(\omega) = -\sum_k \mathrm{Im}\, G^R_\sigma(k,\omega)/N\pi$$

$$\delta\mathcal{D}(\omega) = \sum_n \langle\langle n|D|n\rangle\rangle |\langle n|K|0\rangle|^2 \delta(\hbar\omega - \hbar\omega_{n0})$$

- No transitions within Gutzwiller band
- Transition from the coherent peak to the upper Hubbard band at $\omega = U_c/2$
- Transition between the preformed Hubbard bands at $\omega = U_c$



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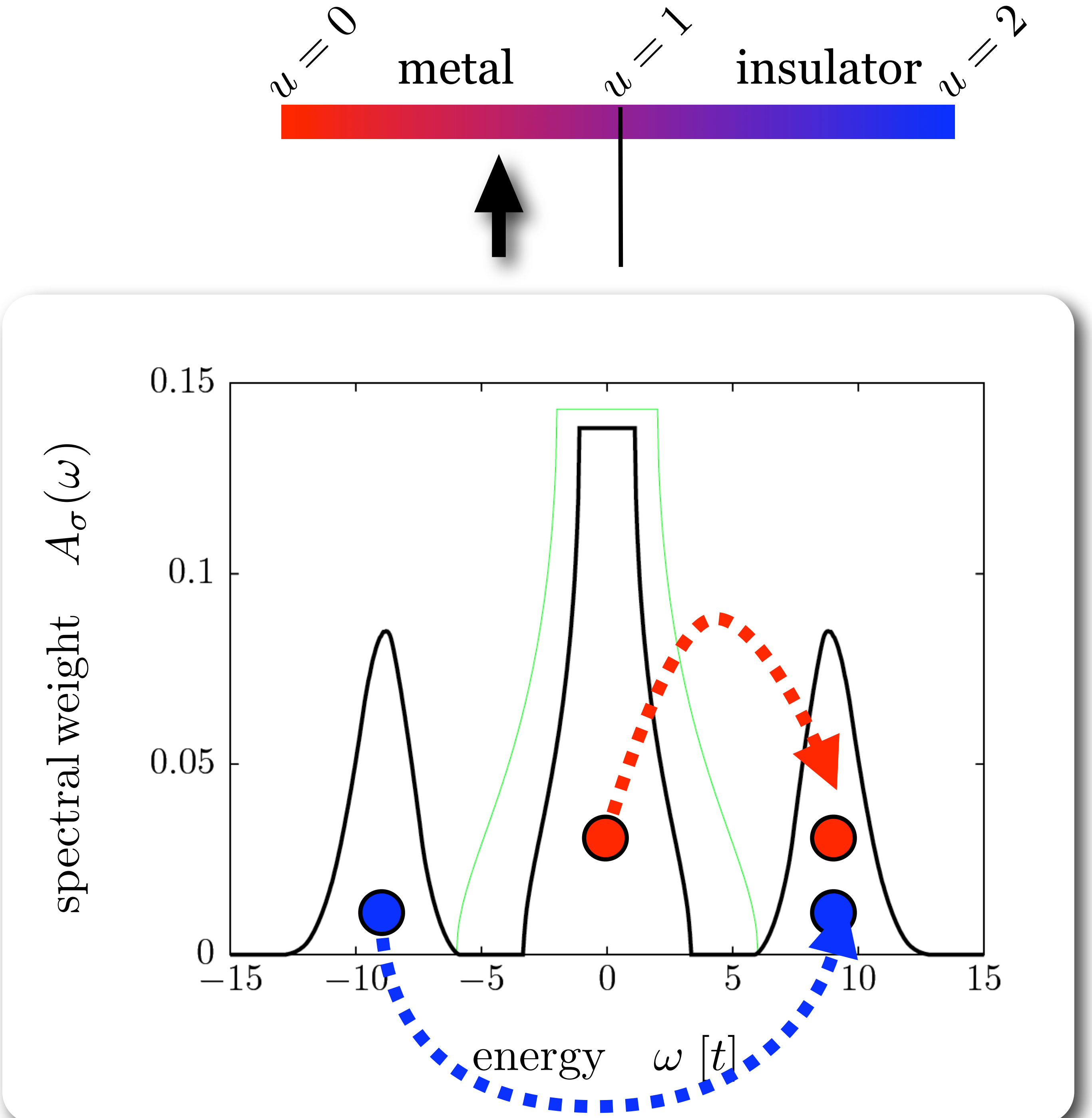

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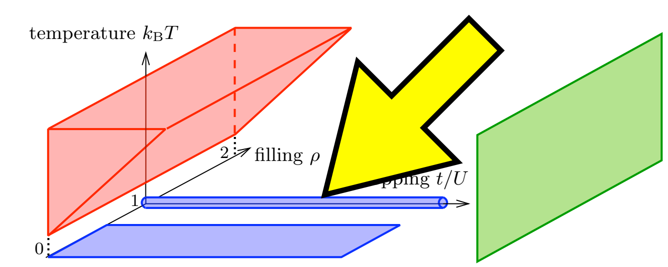
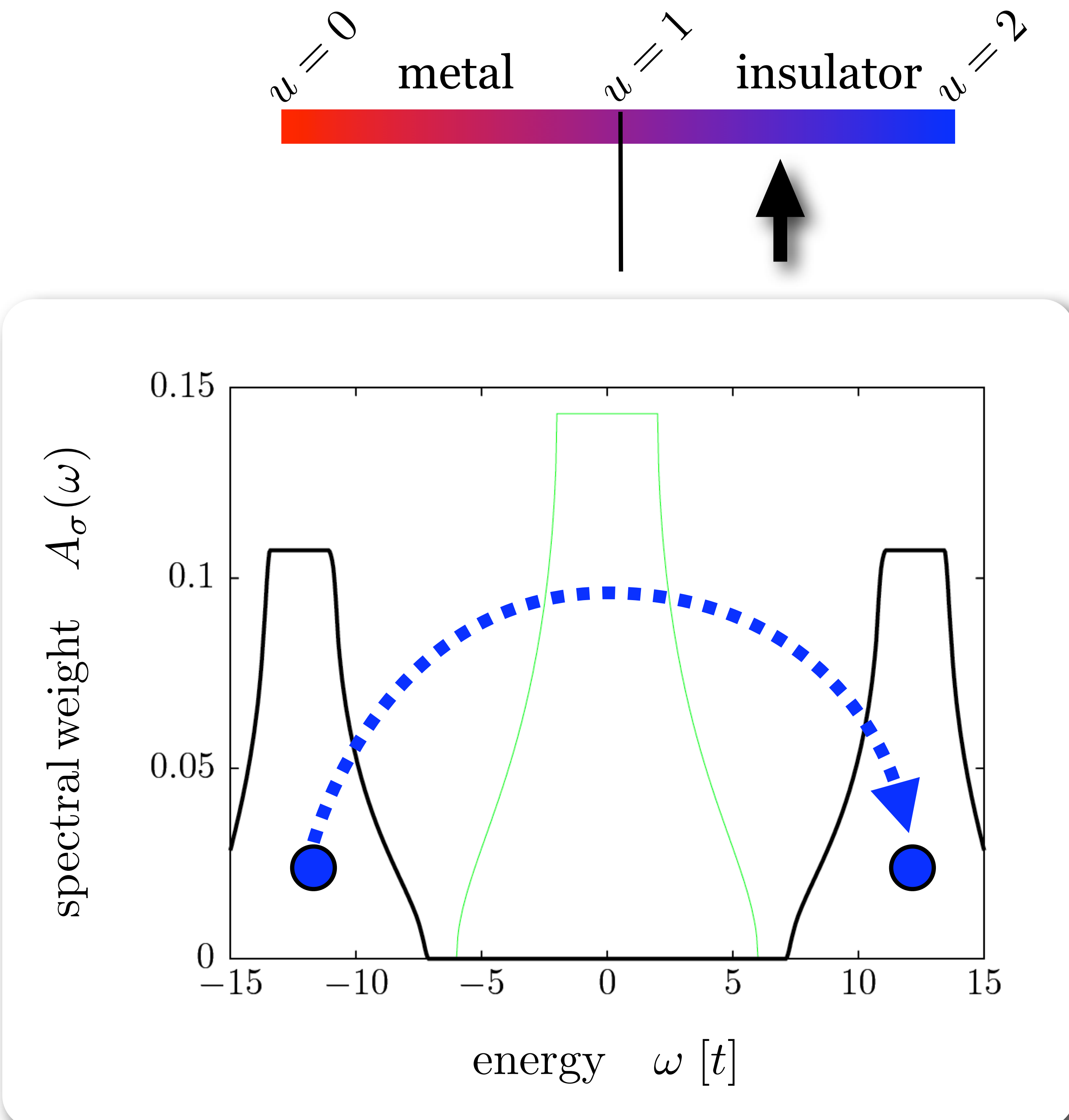


DGDO processes II



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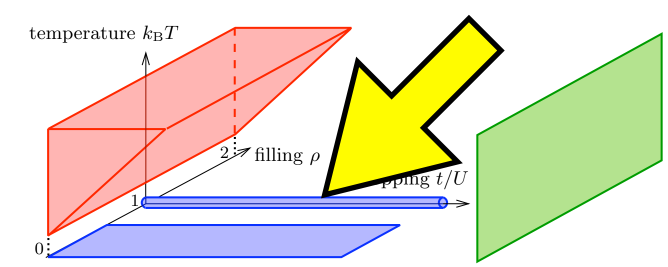
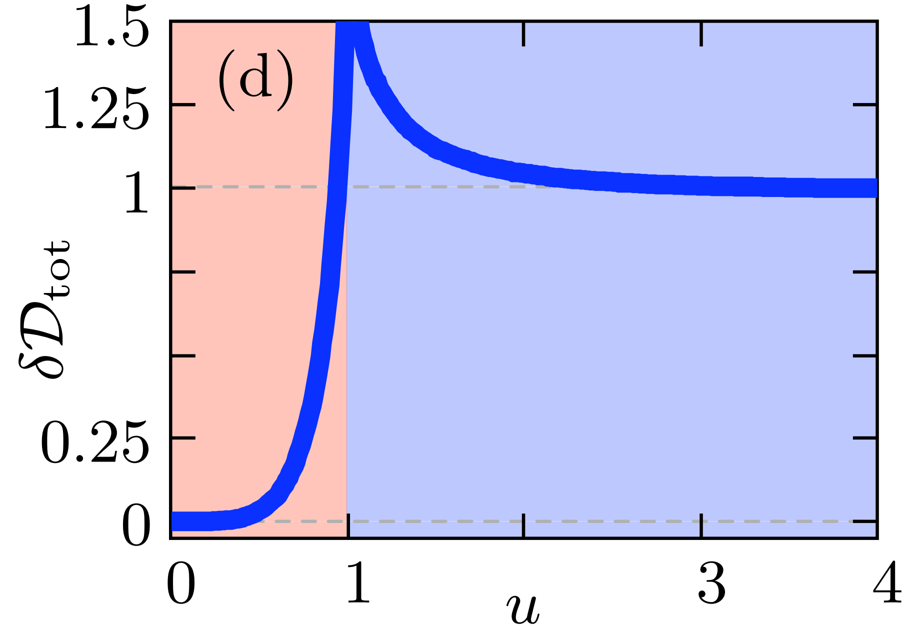
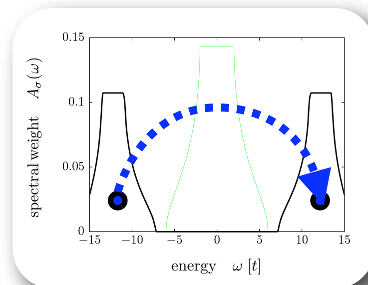
- Transition between the Hubbard bands at ***ω=U***
- No coherent feature

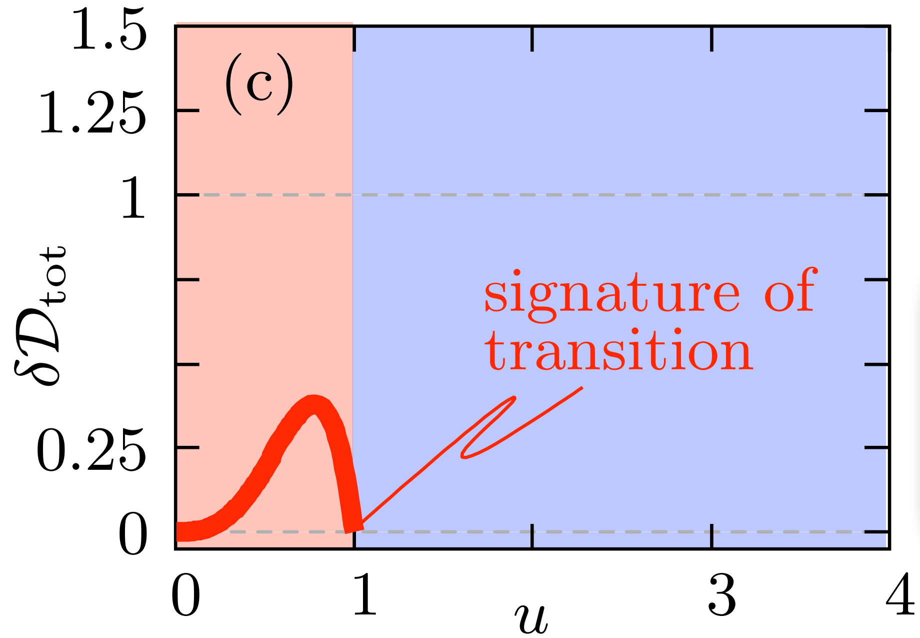


DGDO at half-filling



$\hbar\omega \approx U_c \; (U)$





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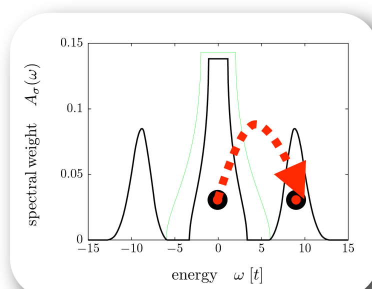
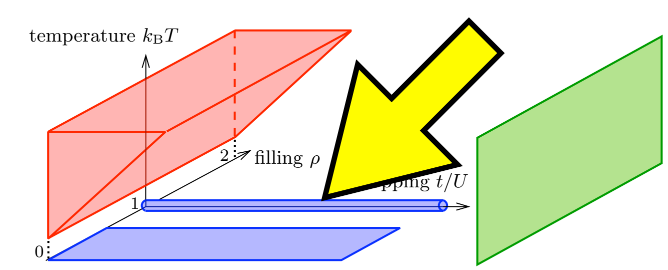
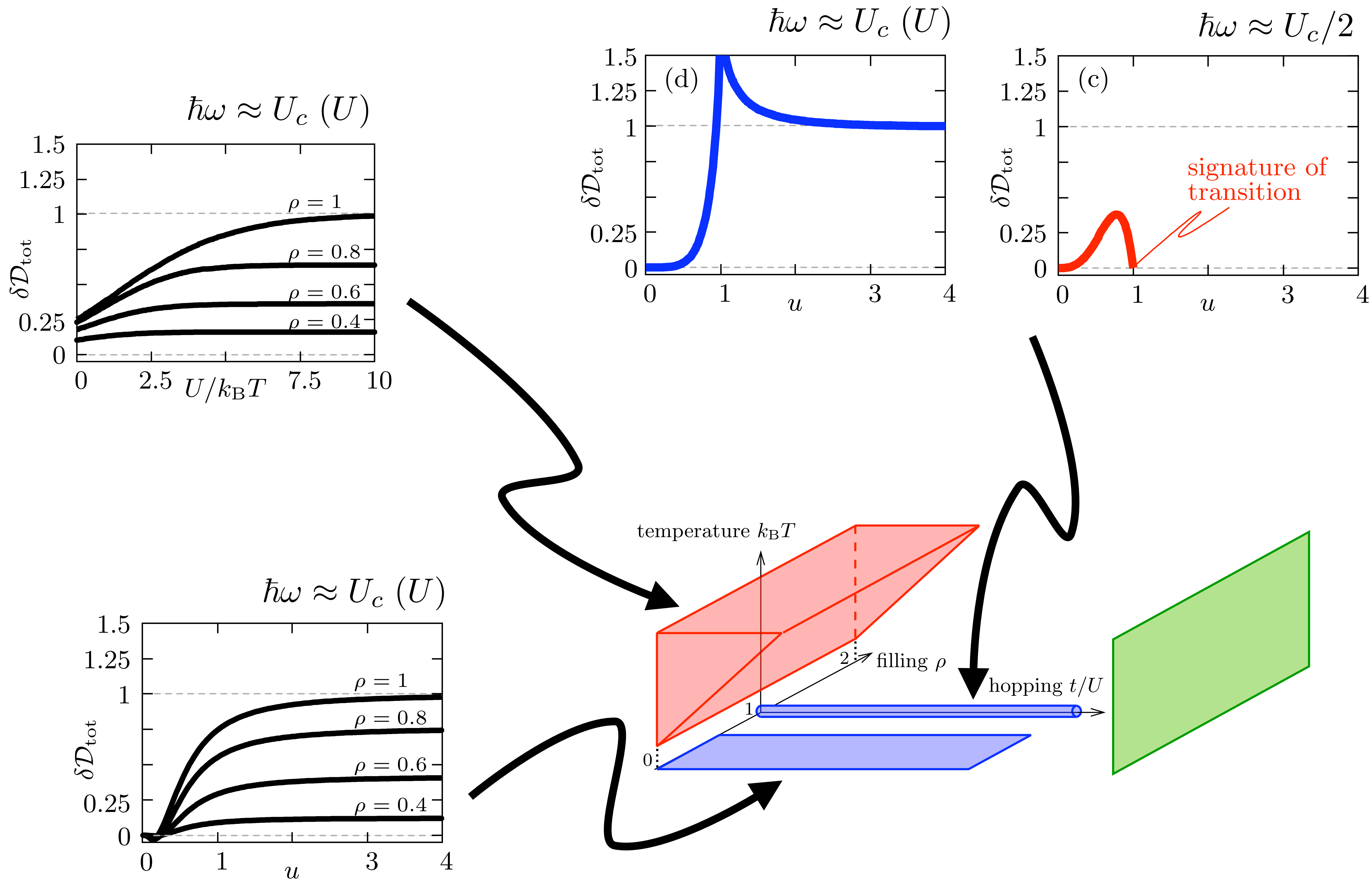


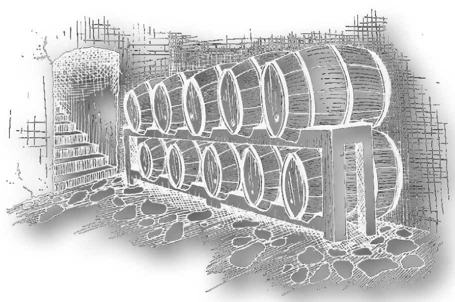
- Transition between the Hubbard bands at ***ω=U***
- No coherent feature for the Mott insulator
- Coherent peak disappears at the transition



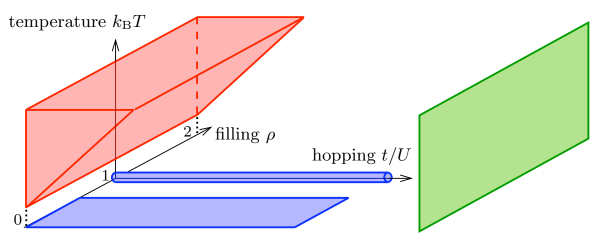
summarizing



conclusions/outlook

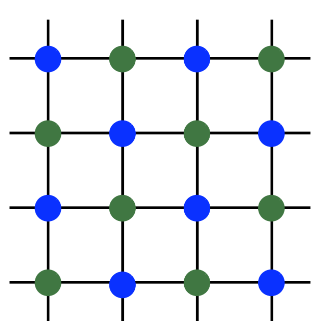


- **a “new” probing tool**
  - DGDO measures spectral properties, biased with the double-occupancy character of the excited states



- **what can one learn from it?**
  - at high temperatures: no clear distinction between the half-filled and a doped situation
  - density dependent saturation value
  - at low temperatures: disappearance of the quasiparticles visible at intermediate energies --> signature for Mott transition

- **slave spins**
  - preformed Hubbard bands
  - “correct” Hubbard gap and bandwidth of the doublon



- **where do we go from here?**
  - AFM order?
  - full trapping potential --> line shapes