# Quantum spin systems far from equilibrium
# –
# Dynamics of antiferromagnetic order in the XXZ-chain prepared in a Néel state

**Peter Barmettler (University of Fribourg)**

**In collaboration with:**

**- Matthias Punk (TU München, Germany)**

**- Ehud Altman (Weizmann Institute, Israel)**

**- Eugene Demler,  Vladimir Gritsev (Harvard University)**

# Outline

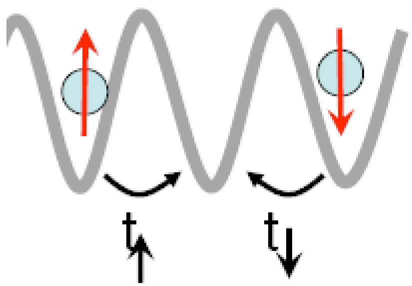
- Introduction
- Analytically tractable special cases
  - 2 spins
  - XX-limit
- Numerical study of the XXZ-chain
- Conclusion

# From two-species Bose gas to spin-superexchange

Example: $^{87}\mathrm{Rb}$ *Mandel et al., Nature 425: 937 (2003).*

$$V(\mathbf{r}) = \sum_{\mu=x,y,z} V_\mu \sin^2 k r_\mu$$

$$E_r = \frac{\hbar^2 k^2}{2m} \ll V_\mu$$




$$|\uparrow\rangle = |F=1, m_F=-1\rangle$$

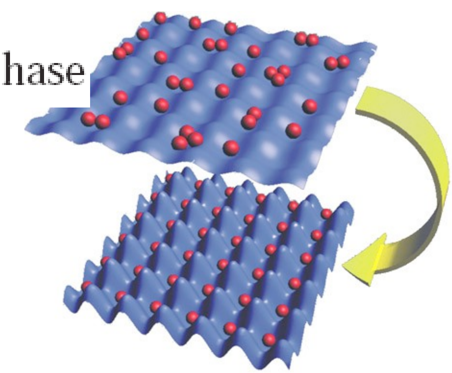
$$|\downarrow\rangle = |F=2, m_F=-2\rangle$$

$$H = \sum_{ij\sigma}(t_{ij\sigma} a^\dagger_{i\sigma} a_{j\sigma} + \mathrm{H.c.}) + \sum_{i\sigma} \frac{U_{\uparrow\downarrow}}{2} n_{i\sigma} n_{i-\sigma} + U_\sigma (n_{i\sigma}-1) n_{j\sigma} + \mu_{i\sigma} n_{i\sigma}$$

Mott insulating phase

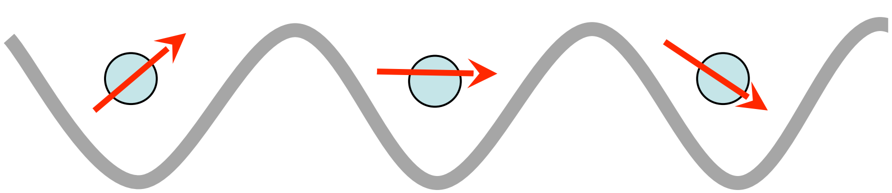
$$t_{ij\sigma} \ll U_\sigma, U_{\uparrow\downarrow}$$

$$\langle n_{i\uparrow}\rangle + \langle n_{i\downarrow}\rangle \simeq 1$$



$$H_{XXZ} = \sum_{ij} \{ J_\perp^{ij} (S_i^x S_j^x + S_i^y S_j^y) + J_z^{ij} S_i^z S_j^z \}$$

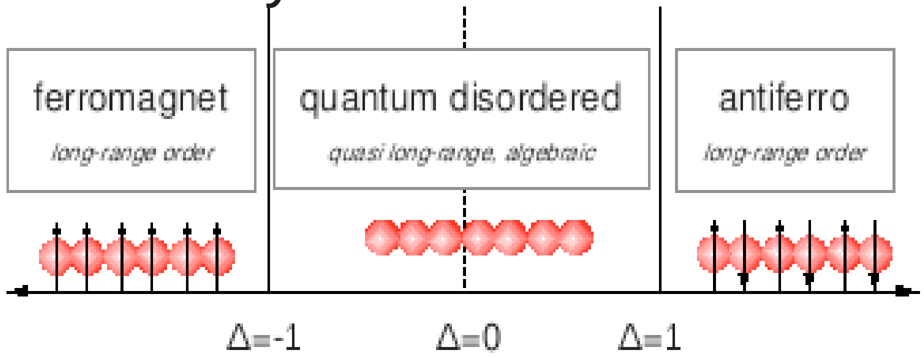
$$J_z^{ij} = \frac{t_{ij\uparrow}^2 + t_{ij\downarrow}^2}{2U_{\uparrow\downarrow}} - \frac{t_{ij\uparrow}^2}{U_\uparrow} - \frac{t_{ij\downarrow}^2}{U_\downarrow}, \quad J_\perp^{ij} = \frac{t_{ij\uparrow} t_{ij\downarrow}}{U_{\uparrow\downarrow}}$$



Spin interactions can be tuned:
AF vs ferro, easy plane vs axis.
*Duan et al., PRL 91: 94514 (2003)*

# Dynamics of an antiferromagnetic state

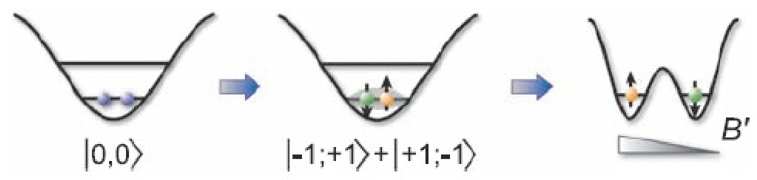
- Magnetically ordered initial state $|\psi_0\rangle = |\uparrow\downarrow\uparrow \ldots \downarrow\uparrow\downarrow\rangle$
- Dynamics of the order parameter $m_s(t) = \frac{1}{N}\sum_j (-1)^j \langle\psi_0|S_j^z(t)|\psi_0\rangle$
- Highly non-trivial relaxation process, conserved energy and an infinite number of conserved quantities
- New perspective on quantum magnetism beyond low-energy effective theories
- Role of quantum criticality




$$H_{\mathrm{XXZ}} = J\sum_j \left\{S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\right\}$$

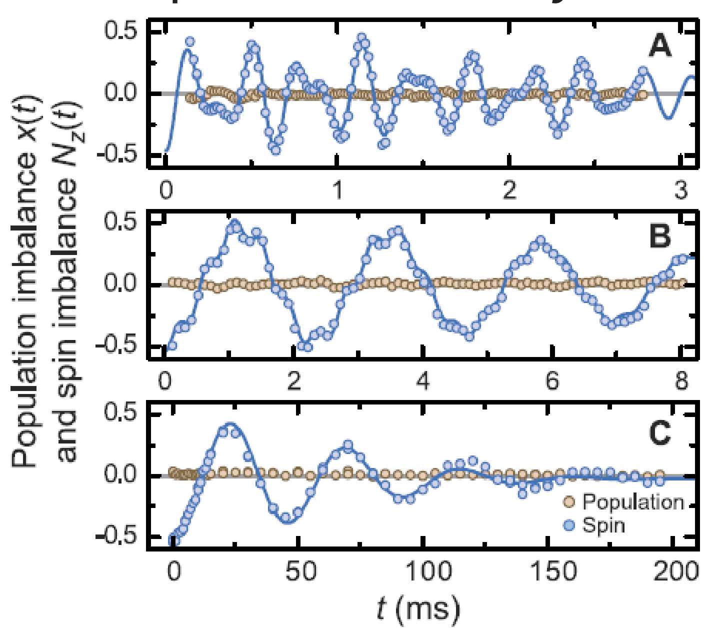
# 2-spin model *Trotzky et al. Science 319 (2008)*

- Array of double-well potentials prepared in a Néel state $|\psi_0\rangle = |\uparrow\downarrow\rangle$




- $H = J(S_1^x S_2^x + S_1^y S_2^y + \Delta S_1^z S_2^z)$
- $|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{-iJt/2\hbar}|s\rangle + e^{iJt/2\hbar}|t^z\rangle)$ $\quad |s\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), |t^z\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle).$
- Rabi oscillations with frequency J $\quad m_s(t) = \frac{1}{2}\cos(\frac{Jt}{\hbar})$

Experiment, *Trotzky et al*.




**U/t =0.7, fast oscillations, strong effects of holes**

**U/t =4, still effects of holes**

**U/t =20, holes suppressed, but slow oscillations, dephasing due to gaussian shape of laser beams**

# XX-chain, non-interacting fermions

- Δ=0, free fermion representation $H = -\frac{J}{2}\sum_j c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j$
- Diagonal in Fourier space $\tilde{H} = \sum_k \epsilon_k \tilde{c}_k^\dagger \tilde{c}_k \qquad \tilde{\epsilon}_k = \tilde{J}\cos k.$
- Rabi oscillations in the charge-density wave

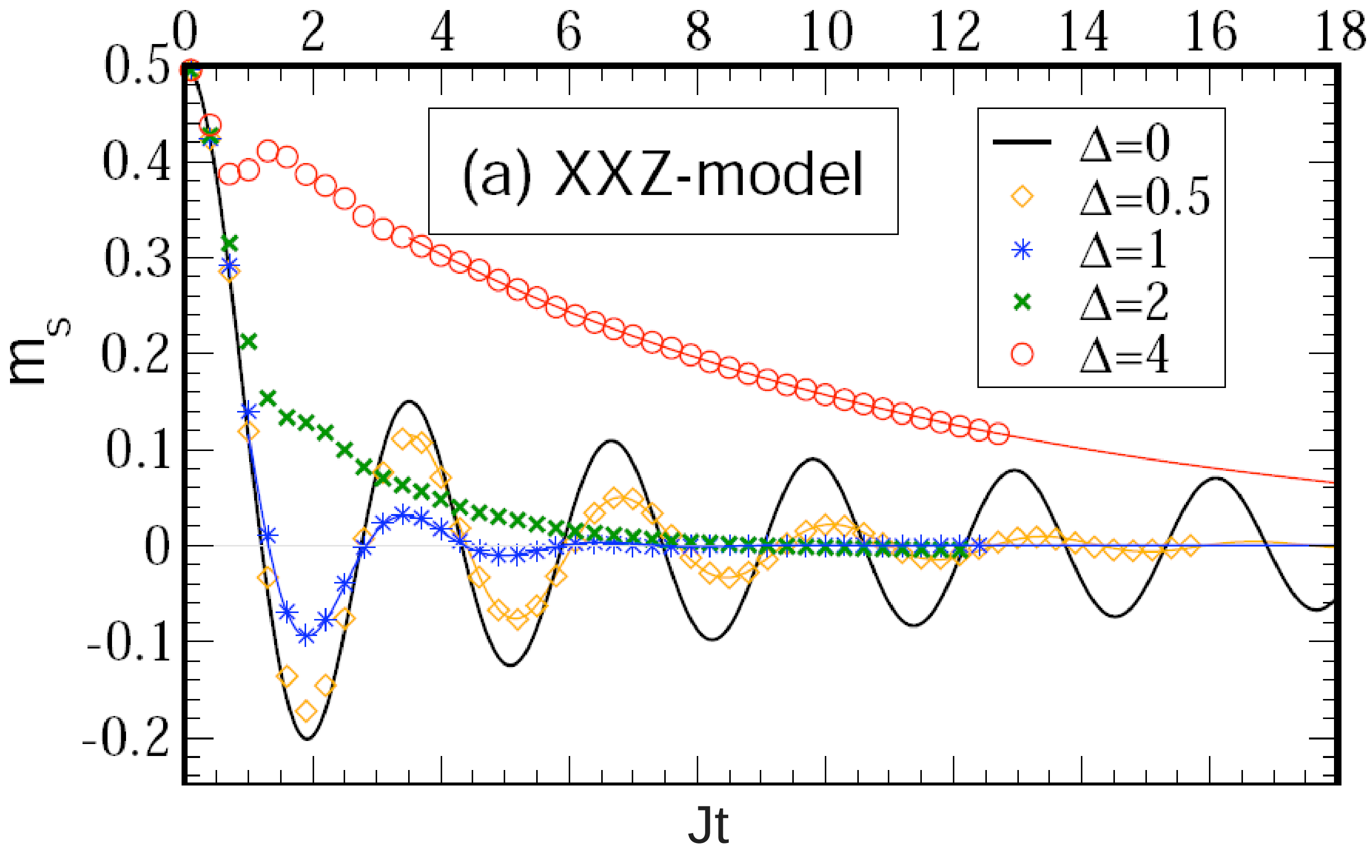
$$|\psi(t)\rangle = \prod_{\frac{-\pi}{2}<k\leq\frac{\pi}{2}} (e^{i\epsilon_k t/\hbar} c_k^\dagger + e^{-i\epsilon_k t/\hbar} c_{k+\pi}^\dagger)\,|0\rangle$$

$$m_s(t) = \int_{-J}^{0} d\epsilon \frac{\cos(2\epsilon t/\hbar)}{\sqrt{1-(\frac{\epsilon}{J})^2}} \xrightarrow{Jt\gg\hbar} \sqrt{\frac{\hbar}{4\pi Jt}}\cos(2Jt/\hbar - \frac{\pi}{4})$$

- Rabi oscillations from the edge of the band with diverging density of states
- Quadratic dispersion at the edge origin of $1/\sqrt{t}$ dephasing
- Low energy part is ineffective for Jt>>1

# Numerical study of the XXZ Heisenberg model

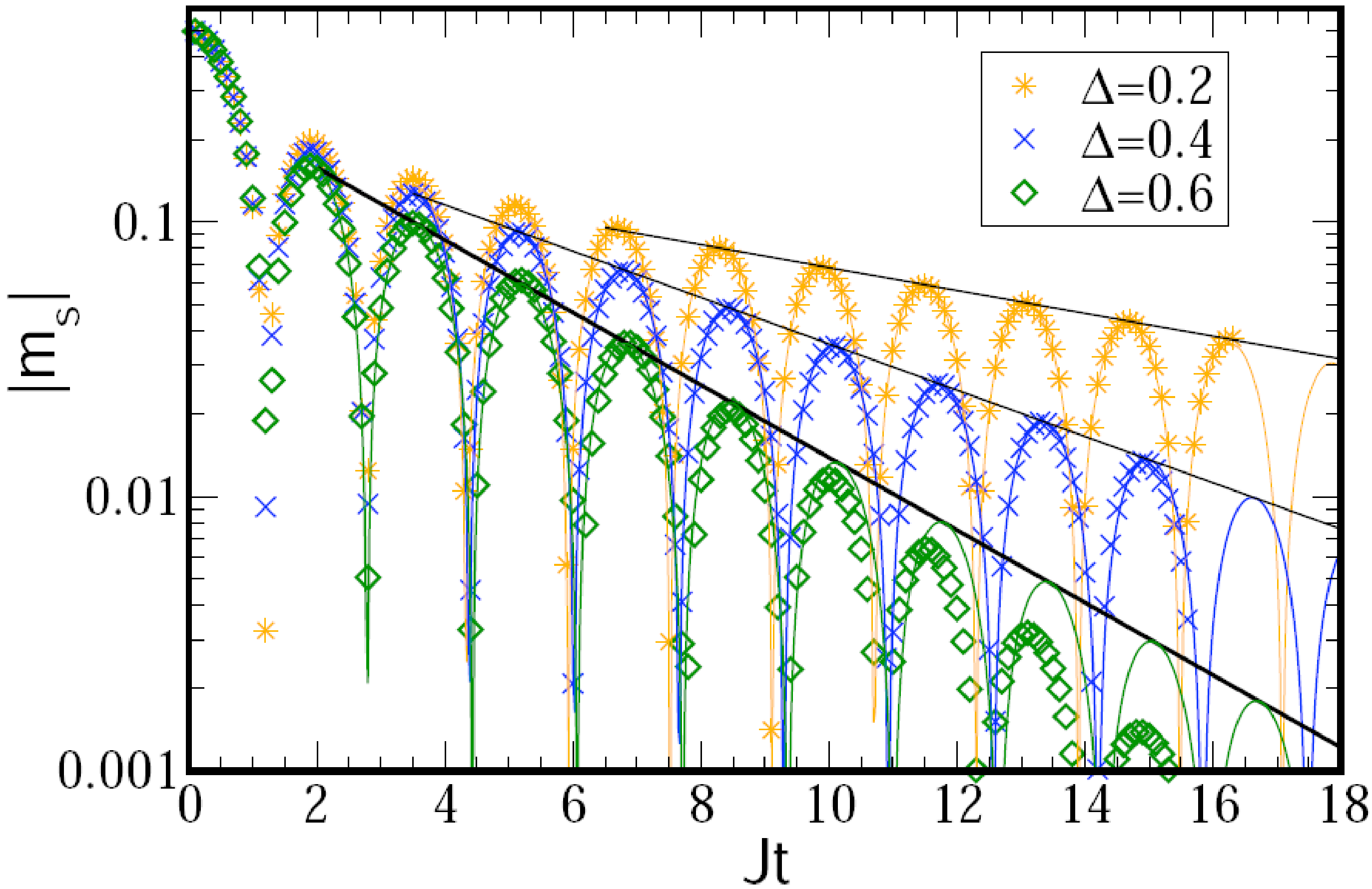
- Numerical matrix product method for an infinite lattice, iTEBD, *(Vidal, PRL 98, 2007),* efficient but limited in time Jt<16, with up to 7000 states.




- From oscillatory to non-oscillatory relaxation

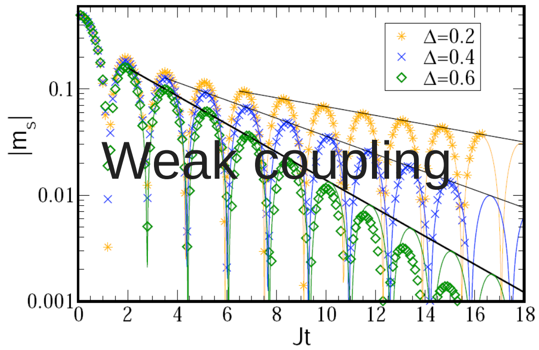
# Weak anisotropies

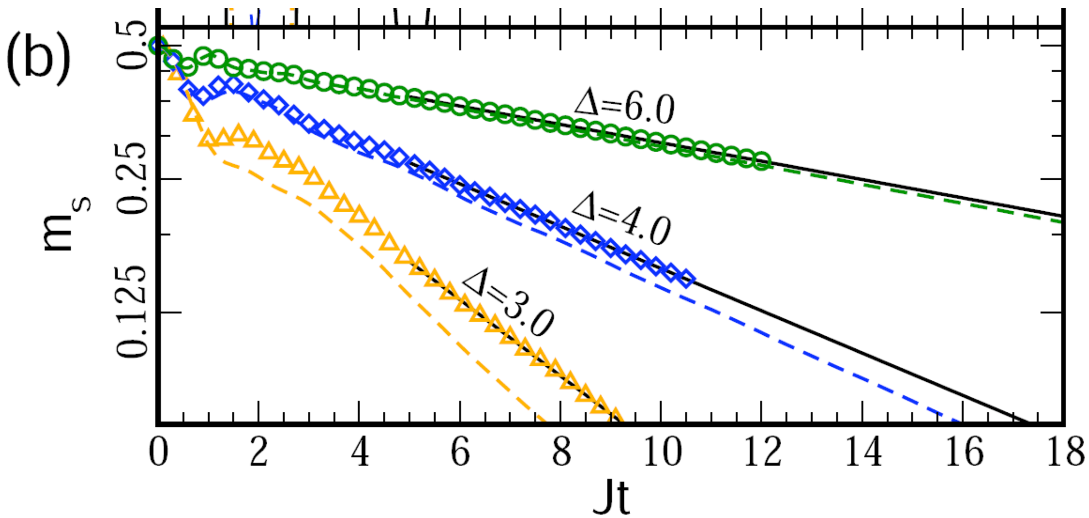
- *Exponential* decay, as opposed to algebraic decay of oscillations for Δ=0 $m_s(t) \propto e^{-t/\tau}\cos(\omega t + \phi)$



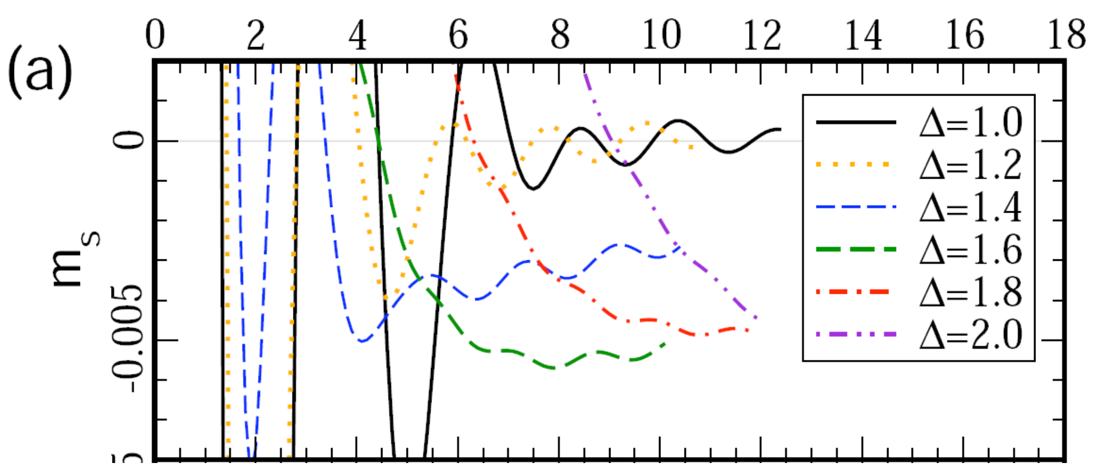

- The point Δ=0 is singular and is not a good representative for the easy-plane regime of the XXZ-Hamiltonian

# Strong and intermediate anisotropies



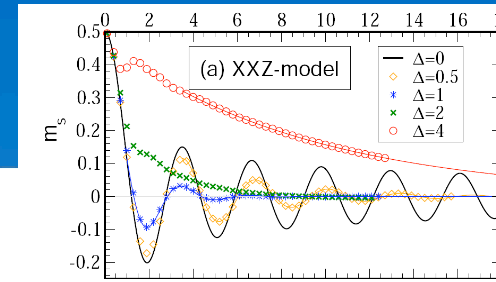




- Exponential decay for Δ>>1, $m_s(t) \propto e^{-t/\tau}$

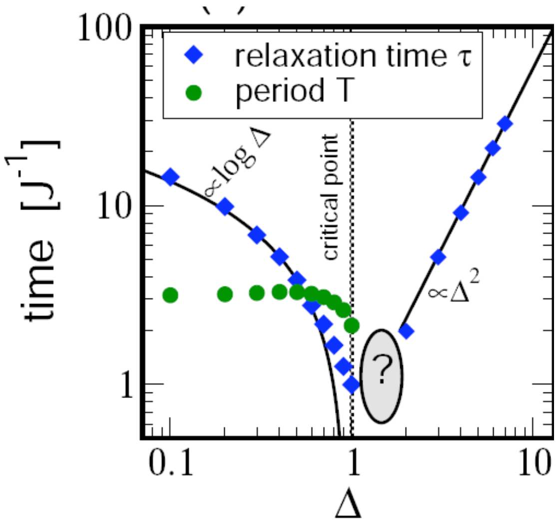



- Non-generic behavior in the vicinity of the critical point

# 'Phase diagram'




- An exponential relaxation is found for all Δ>0
- Relaxation time shows unusual scaling
- Minimal value of the relaxation time close to the critical point, opposite behavior to critical slowing down




- Explanation of exponential mode (heuristic):
  - Small Δ, fermionic modes not sharp
  - Large Δ, gap reduces phase-space
- Possible analytical approaches
  - ~~Luttinger-Liquid, Mean field,~~ Sine-Gordon (??)
  - Include curvature

# Conclusion

- Far from equilibrium is different
- P. Barmettler, M. Punk, V. Gritsev, E. Demler, E. Altman, a*rXiv:0810.4845* **.**