Spontaneous Rotation in Trapped One-dimensional Bose Gases

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A. Tokuno and MS, Phys. Rev. A 78, 13623 (2008).

Outline

1, Trapped one-dimensional cold atoms

2, Model

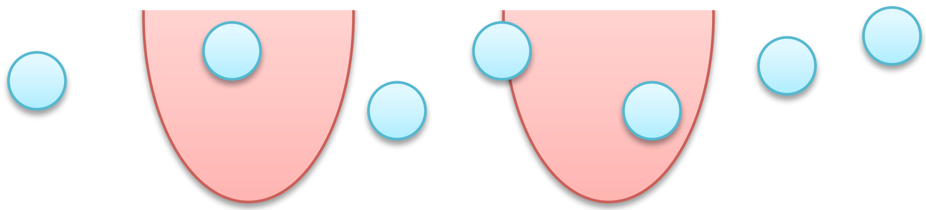
3, Analysis

4, Related systems

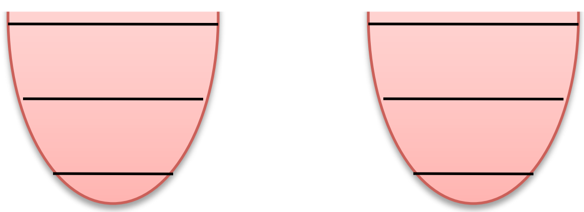
5, Summary

1, Trapped one-dimensional cold atoms

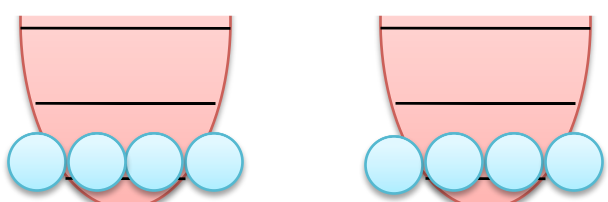
2D magnetic or optical potential



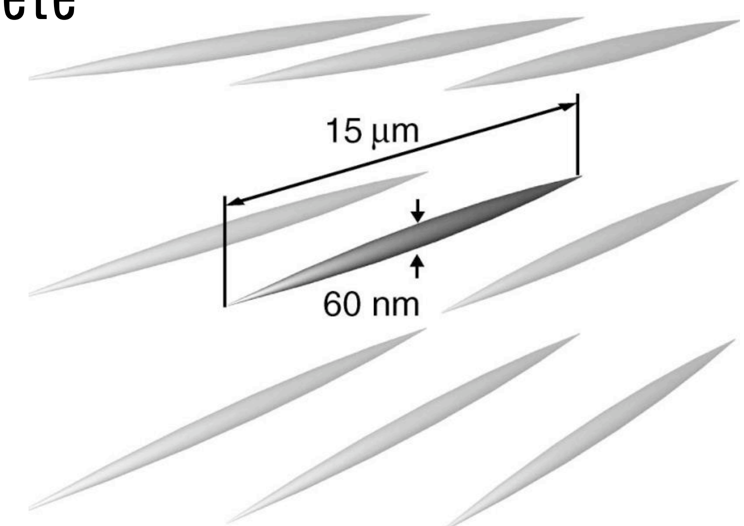
Energy spectrum in 2D plane : discrete



Almost all atoms : the lowest band



1D cold atom system



Moritz, *et al*, PRL91 (2003)

Theoretical studies of 1D cold-atom systems

Lieb-Liniger model

Bose-Hubbard chains

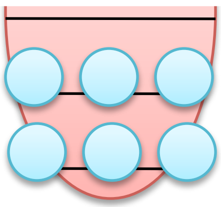
Bose-Fermi mixtures

(Partially) polarized Fermi gases

Multi-component cold-atom gases

, and so on.

If we vary the strength of the trap potential, the total number of cold atoms, etc., higher-energy bands are occupied by the atoms.



What happens when atoms occupy not only lowest band but also higher ones ?

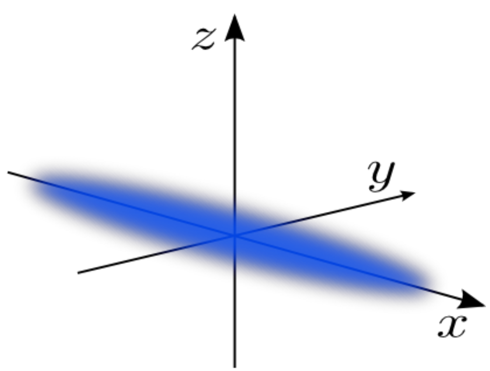
Any novel phenomenon induced by higher-band atoms ?

2, Model

We begin with the Hamiltonian for 3D bosonic atoms.

$$H_{3D} = \int d\vec{r} \left[\psi^\dagger \left(-\frac{\hbar^2 \nabla^2}{2m} - \mu + V(\vec{r}) \right) \psi + \frac{U}{2} \mathcal{D}^2 + \cdots \right]$$

- $\psi(\vec{r})$ Boson annihilation operator $\vec{r} = (x, y, z)$
- $\mathcal{D}(\vec{r}) = \psi^\dagger \psi$ Boson density operator
- $V(\vec{r}) = \frac{1}{2} m \omega_0^2 (y^2 + z^2) - \hbar \omega_0$ 2D-harmonic-type potential
- $U = 4\pi\hbar^2 a/m > 0$ Repulsive interaction
- $a > 0$ 3D inter-atom scattering length
- μ Chemical potential



One particle under the 2D harmonic potential

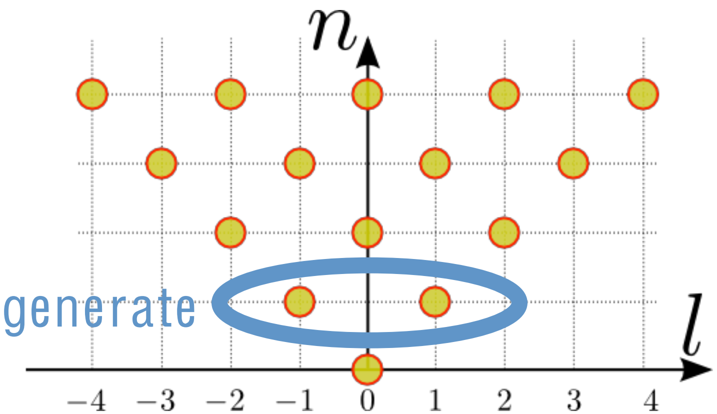
$$\left[-\frac{\hbar^2}{2m}(\partial_y^2+\partial_z^2)+\frac{1}{2}m\omega_0(y^2+z^2)-\hbar\omega_0\right]u_{n,l}(y,z)=\epsilon_{n,l}u_{n,l}(y,z)$$

$$\epsilon_{n,l}=n\hbar\omega_0$$

$$l=n,n-2,\cdots,-n$$

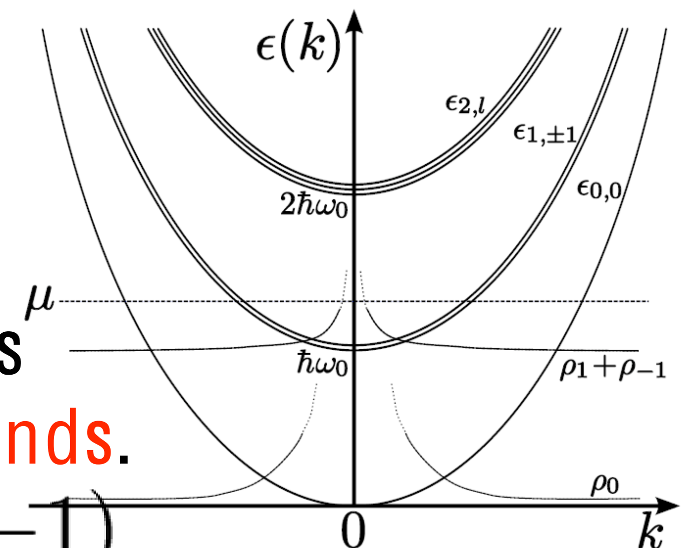
Angular momentum

Doubly degenerate



3D one-particle energy spectrum

$$\epsilon_{n,l}(k)=\frac{\hbar^2k^2}{2m^2}+n\hbar\omega_0$$



We focus on the case where atoms occupied only the lowest three bands.

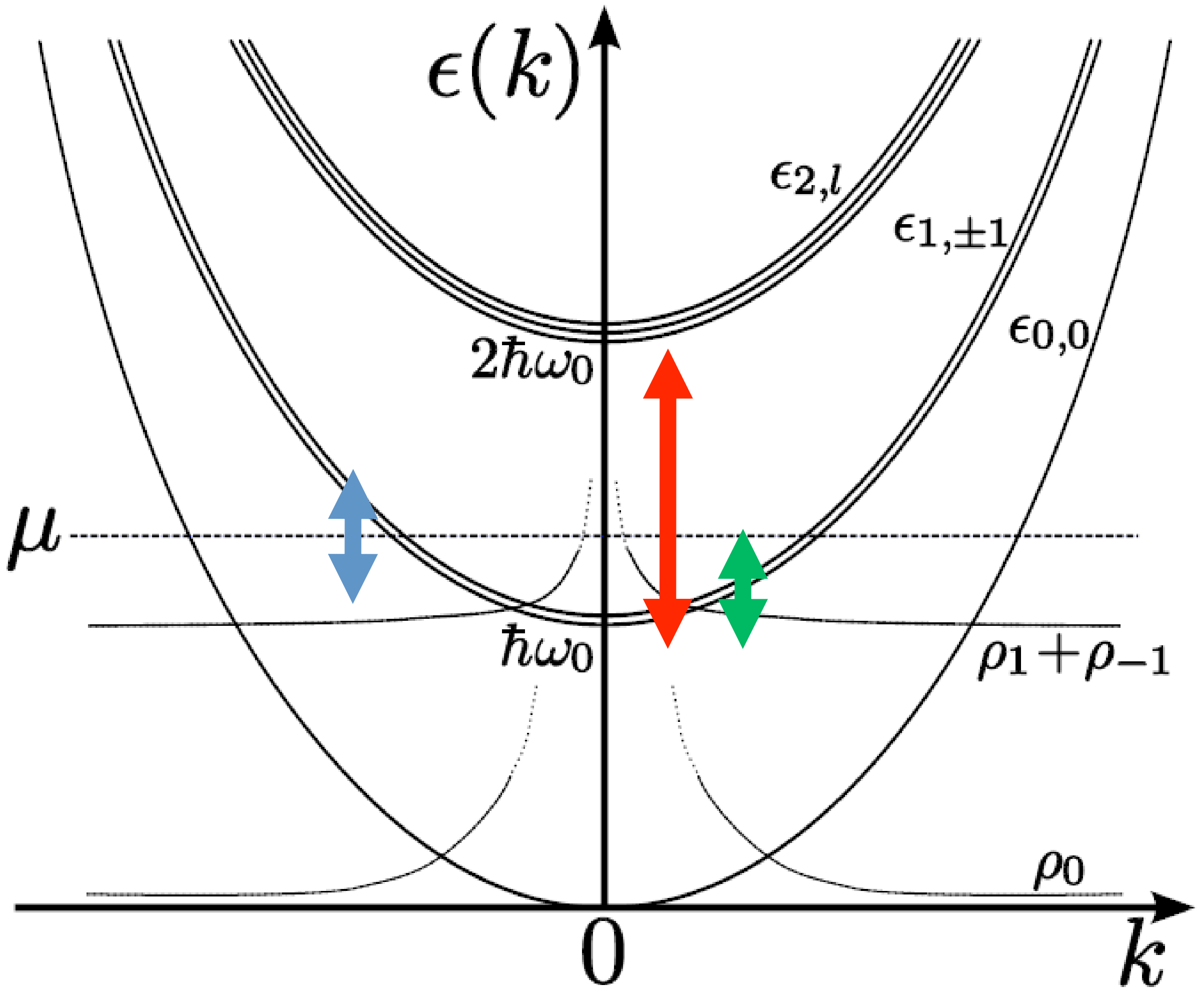
$$(n,l)=(0,0),(1,1)\text{ and }(1,-1)$$

Assumption

(1) $\hbar\omega_0 < \mu < 2\hbar\omega_0$ (2) $\mu - \hbar\omega_0 \ll \hbar\omega_0$ (3) $U/a_\perp^3 \ll \hbar\omega_0$

$(a \ll a_\perp)$

$a_\perp = \sqrt{\hbar/m\omega_0}$: Width of wave function $u_{n,l}(y,z)$



Derivation of effective theory for 1D confined cold atoms

Expansion of Bose field operator

$$\psi(\vec{r}) = \sum_{n=0}^{\infty} \sum_{l=-n}^{n} u_{n,l}(y,z)\phi_{n,l}(x)$$

$\phi_{0,0},\ \phi_{1,\pm 1}$ occupied

$\phi_{n>2,l}$ unoccupied

Integrating out all the gapped boson fields in unoccupied bands

$$Z = \int \mathcal{D}\phi_{0,0}\mathcal{D}\phi_{1,\pm1}\mathcal{D}\phi_{n>2,l} \exp(-S_{3D}) \propto \int \mathcal{D}\phi_{0,0}\mathcal{D}\phi_{1,\pm1} \exp(-S_{1D})$$

$$S_{3D} = S_0 + S_1 + S_{\rm int}$$

S_0 Part including only occupied bosons

S_1 Kinetic part of gapped bosons

$S_{\rm int}$ Interaction part

Cumulant expansion

$$S_{1D} = S_0 + \langle S_{\rm int}\rangle_1 - \frac{1}{2}(\langle S_{\rm int}^2\rangle_1 - \langle S_{\rm int}\rangle_1^2) + \cdots$$

$\langle \cdots \rangle_1$ Expectation value w.r.t S_1

$$\phi_{0,0} \to \phi_0 \quad \phi_{1,\pm 1} \to \phi_{\pm 1} \quad \rho_\alpha = \phi_\alpha^\dagger \phi_\alpha$$

The resulting Hamiltonian $H_{1D} = \int dx\ \mathcal{H}_0 + \mathcal{H}_{\text{int}}$

Kinetic (free boson) part

$$\mathcal{H}_0 = \frac{\hbar^2}{2m} \sum_{\alpha=0,\pm 1} \partial_x \phi_\alpha^\dagger \partial_x \phi_\alpha - \mu_0 \rho_0 - \mu_1 (\rho_1 + \rho_{-1})$$

Interaction part

Intra-band interactions

$$\mathcal{H}_{\text{int}} = \frac{U}{8\pi a_\perp^2} \Big[2\Gamma_0 \rho_0^2 + \Gamma_1 (\rho_1^2 + \rho_{-1}^2) + 4\Gamma_{01} \rho_0 (\rho_1 + \rho_{-1})$$

Inter-band interactions
between the lowest and the second lowest bands

$$+ 4\Gamma_{\pm 1} \rho_1 \rho_{-1} + 2\Gamma_{0\pm 1} (\phi_0^\dagger \phi_0^\dagger \phi_1 \phi_{-1} + h.c.) \Big]$$

Inter-band interaction
between the second lowest degenerate bands

Tunneling
between the lowest and the second lowest bands

$$\Gamma_\alpha \approx 1 + \sum_{n=1}^{\infty} c_{\alpha,n} \left(\frac{a}{a_\perp} \right)^n \longrightarrow 1 \quad (a \ll a_\perp)$$

3, Analysis

3-1, Ginzburg-Landau (mean-field) argument

$$\mathcal{F}_{GL} = -\mu_0\rho_0 - \mu_1\rho_s + \frac{U}{8\pi a_\perp^2}\Big[2\Gamma_0\rho_0^2 + \Gamma_s\rho_s^2 + \Gamma_a\rho_a^2$$

$$+4\Gamma_{01}\rho_0\rho_s + 2\Gamma_{0\pm1}\rho_0\sqrt{\rho_s^2-\rho_a^2}\cos(2\theta_0 - 2\theta_s)\Big]$$

$$\phi_\alpha = \rho_\alpha^{1/2} e^{i\theta_\alpha}$$ Amplitude and phase

$$\rho_{s,a} = \rho_1 \pm \rho_{-1}$$

$$\theta_{s,a} = (\theta_1 \pm \theta_{-1})/2$$

$$\Gamma_{s,a} = (\Gamma_1 \pm 2\Gamma_{\pm1})/2$$

ρ_s Total boson density of the second lowest bands

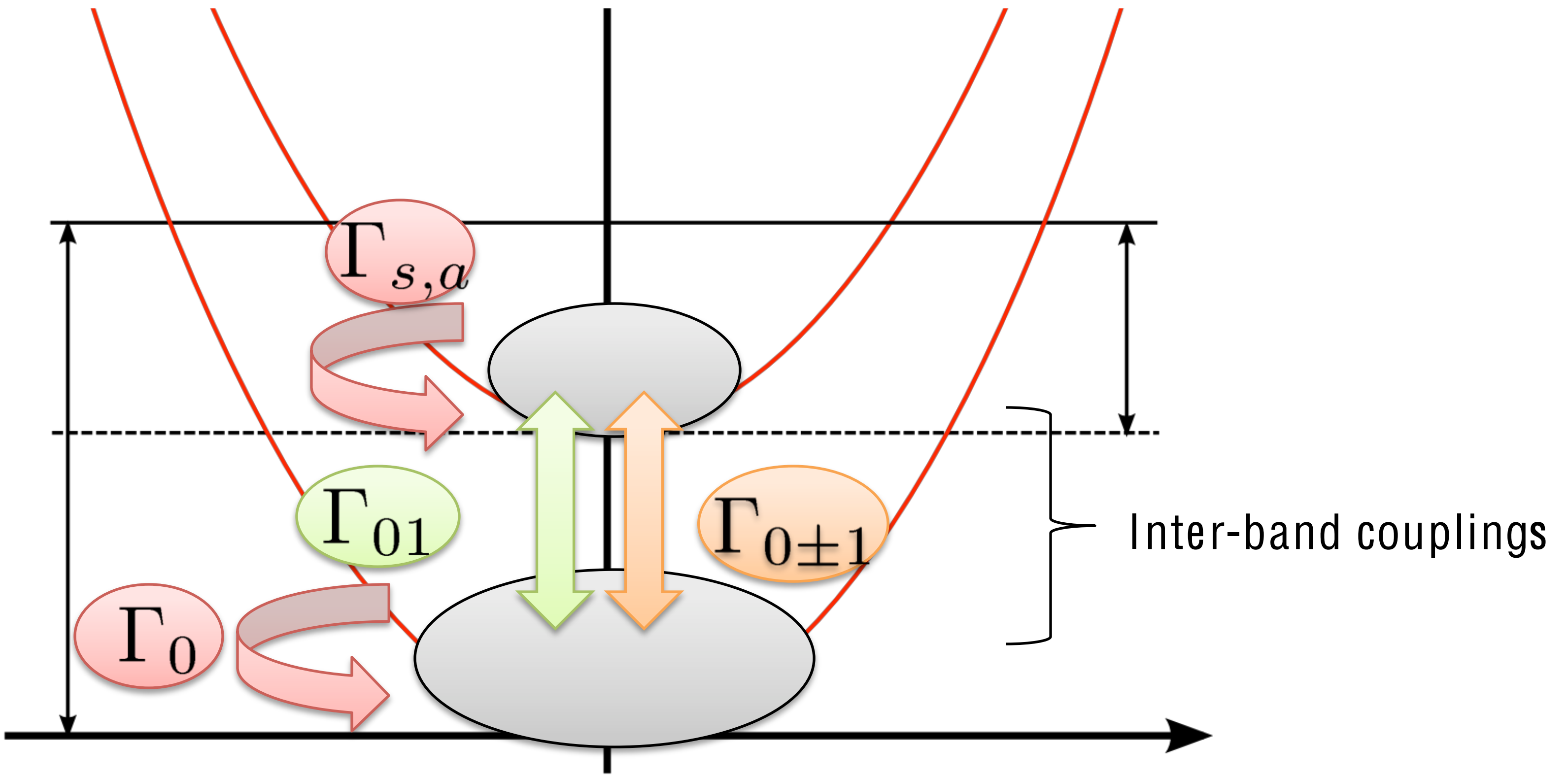
ρ_a Angular momentum density of them

$\Gamma_a < 0$ $\Gamma_s > 0$

Inter-band interaction > Intra-band interaction

$4\Gamma_{\pm1}\rho_1\rho_{-1}$ > $\Gamma_1(\rho_1^2 + \rho_{-1}^2)$

Physical meaning of coupling constants



$\Gamma_{01},\ \Gamma_{0\pm1}$ would be overestimated since we now consider the weak-coupling region $(a \ll a_\perp)$.

We should replace them with effective small constants $\tilde{\Gamma}_{01},\ \tilde{\Gamma}_{0\pm1}$.

First we omit the tunneling term $\tilde{\Gamma}_{0\pm1}$

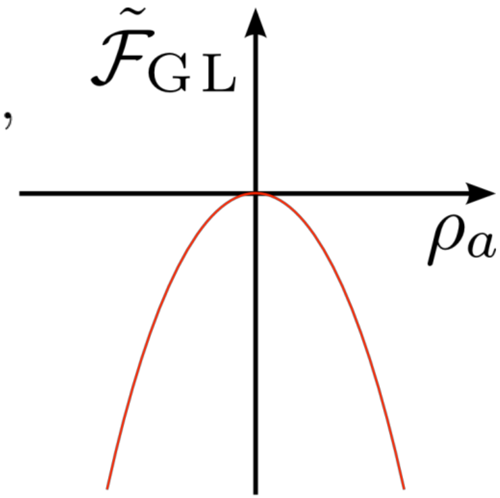
$$
\begin{aligned}
\tilde{\mathcal{F}}_{GL} &= -\mu_0\rho_0 - \mu_1\rho_s \\
&\quad + \frac{U}{8\pi a_\perp^2}\left[2\Gamma_0\rho_0^2 + \Gamma_s\rho_s^2 + \Gamma_a\rho_a^2 + 4\tilde{\Gamma}_{01}\rho_0\rho_s\right]
\end{aligned}
$$

GL solution

$\partial\tilde{\mathcal{F}}_{GL}/\partial\rho_\alpha = 0$

$$
\begin{aligned}
\bar{\rho}_0 &= \frac{2\pi a_\perp^2}{U}\frac{\Gamma_s\mu_0 - 2\tilde{\Gamma}_{01}\mu_1}{\Gamma_0\Gamma_s - 2\tilde{\Gamma}_{01}}, \\
\bar{\rho}_s &= \frac{4\pi a_\perp^2}{U}\frac{\Gamma_0\mu_1 - \tilde{\Gamma}_{01}\mu_0}{\Gamma_0\Gamma_s - 2\tilde{\Gamma}_{01}}, \\
\bar{\rho}_a &= 0.
\end{aligned}
$$

Unstable



$\rho_a \to \pm\infty$ minimizes the GL free energy !

Unphysical solution

To get a physical solution,
we may introduce a density-quartic interaction !

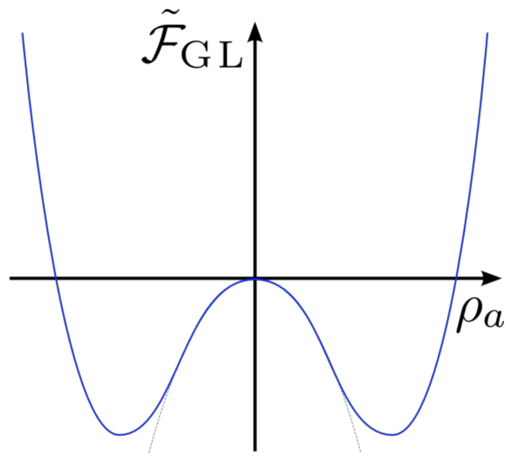
(More sophisticated approaches are necessary to obtain accurate results.)

$$\tilde{\mathcal{F}}_{GL} \rightarrow \tilde{\mathcal{F}}_{GL} + \frac{U}{8\pi a_\perp^2} \frac{\Gamma_a^{(4)}}{4} \rho_a^4 \qquad \Gamma_a^{(4)} > 0$$

4-body interaction in the original 3D system

Higher-order terms in the cumulant expansion

$$\bar{\rho}_a = \pm(-2\Gamma_a/\Gamma_a^{(4)})^{1/2}$$



GL solution indicates a finite angular momentum.

3-2, bosonization (hydrodynamic) approach

We should take into account quantum-fluctuation effects for the G L solution. To this end, we use the bosonization approach for H_{1D} recovering the tunneling term $\tilde{\Gamma}_{0\pm1}$.

$$\rho_\alpha \approx \bar{\rho}_\alpha + \frac{1}{\pi}\partial_x\varphi_\alpha + \cdots \qquad \phi_\alpha \sim \sqrt{\bar{\rho}_\alpha}\, e^{-i(\bar{\theta}_\alpha + \theta_\alpha)} + \cdots$$

Fluctuation around mean value — Amplitude — Phase

$$\alpha = 0, \pm 1$$

$$[\varphi_\alpha(x), \partial_{x'}\theta_{\alpha'}(x')] = i\pi\delta_{\alpha,\alpha'}\delta(x - x')$$

3-component Tomonaga-Luttinger liquid + perturbations

$$\mathcal{H}_{1D} \approx \int dx \sum_{\alpha=0,s,a} \frac{v_\alpha}{2\pi}\left\{K_\alpha(\partial_x\theta_\alpha)^2 + K_\alpha^{-1}(\partial_x\varphi_\alpha)^2\right\}$$

$$+\tilde{g}_{sa}\partial_x\theta_s\partial_x\theta_a + \tilde{g}_{0s}\partial_x\varphi_0\partial_x\varphi_s + \tilde{g}\cos(2\theta_0 - 2\theta_s) + \cdots$$

$$(\phi_{s,a}, \theta_{s,a}) = (\phi_1 \pm \phi_{-1}, \tfrac{1}{2}(\theta_1 \pm \theta_{-1})) \qquad \rho_{s,a} \approx \bar{\rho}_{s,a} + \frac{1}{\pi}\partial_x\varphi_{s,a} + \cdots$$

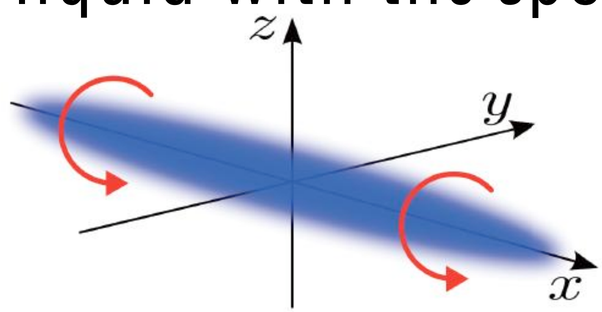
$\cos(2\theta_0 - 2\theta_s)$ relevant : open a gap

3-component TLL → 2-component TLL

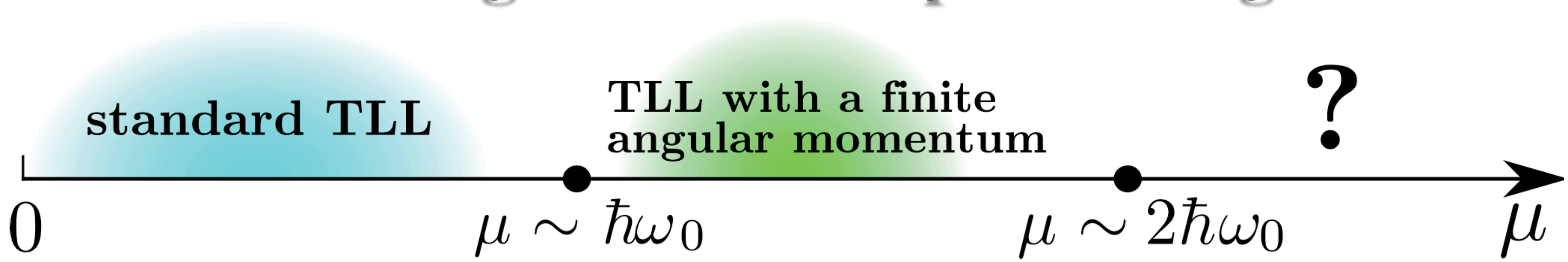
Any vertex operator with (ϕ_a, θ_a) **doesn't appear** due to symmetries.

→ The GL solution $\rho_a \neq 0$ survives even after introducing quantum fluctuations !

2-component TL liquid with the spontaneous rotation



Predicted ground-state phase diagram



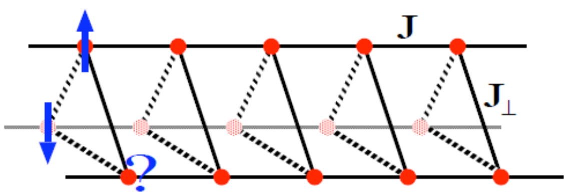
4, Related systems

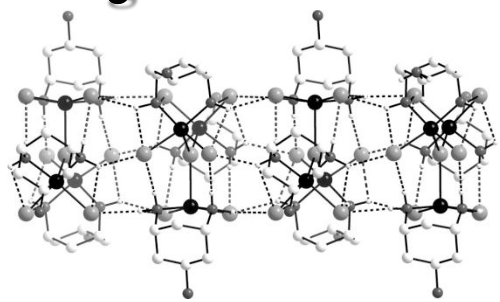
Mechanism of the spontaneous rotation

Inter-band interaction > Intra-band interaction

This situation also occurs in other physical systems.

(1) 3-leg spin tubes in magnetic field ⇨ Vector spin chiral order

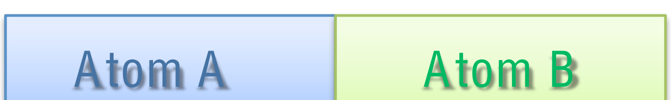




$$\kappa^z = (\vec{S}_i \times \vec{S}_j)^z$$

M.S, and T.Sakai, PRB (2007).

(2) Multi-component cold atoms (mixture) ⇨ Phase separation



Cazallila and Ho, PRL (2003).

(3) U(1)-symmetric itinerant electrons ⇨ Spontaneous magnetization (Ferromagnetism)

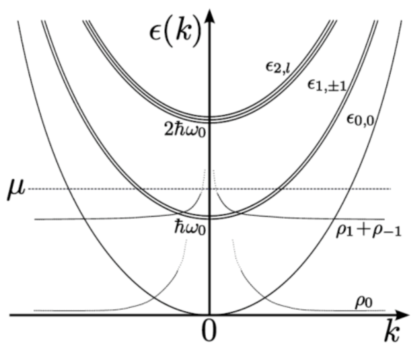
K. Yang, PRL (2004).

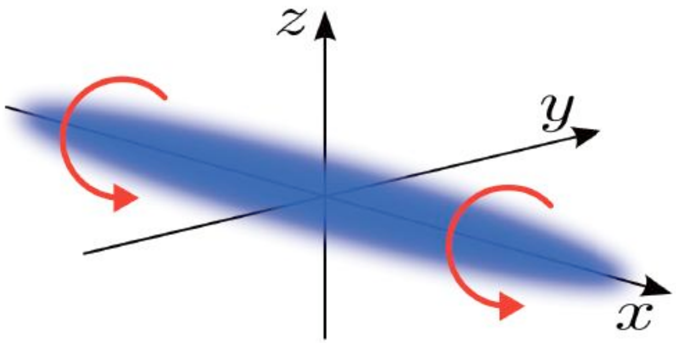
5, Summary

We study effects of higher bands in 1D Bose gas.

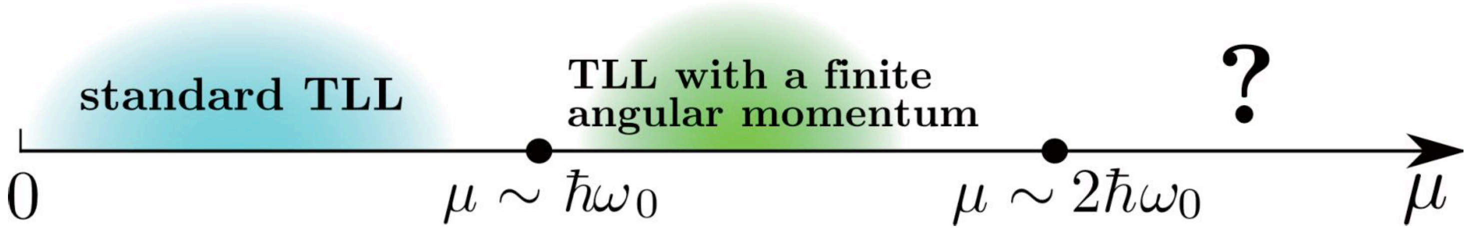
- A mechanism of the spontaneous rotation

Inter-band interaction > Intra-band interaction





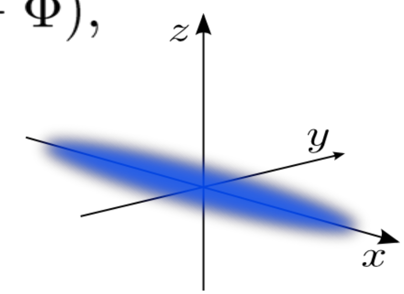
- Structure of the low-energy excitations
- Ground-state phase diagram of 1D Bose gas



A. Tokuno and MS, Phys. Rev. A 78, 13623 (2008).

Symmetries in the bosonization language

Translation for *x* axis	$(\varphi_a(x), \theta_a(x)) \rightarrow (\varphi_a(x+\delta) + \pi\bar{\rho}_a\delta, \theta_a(x+\delta)),$
Parity for *x* axis	$(\varphi_a(x), \theta_a(x)) \rightarrow (-\varphi_a(-x), \theta_a(-x)),$
Reflection in *y-z* plane	$(\varphi_a(x), \theta_a(x), \bar{\rho}_a) \rightarrow (-\varphi_a(x), -\theta_a(x), -\bar{\rho}_a),$
SO(2) rotation in *y-z* plane	$(\varphi_a(x), \theta_a(x)) \rightarrow (\varphi_a(x), \theta_a(x) - \Phi),$
U(1) gauge transformation	$(\varphi_a(x), \theta_a(x)) \rightarrow (\varphi_a(x), \theta_a(x)),$



$e^{in\varphi_a}$ forbidden by the translational symmetry.

$e^{in\theta_a}$ forbidden by the rotational symmetry around the *y-z* plane.

No instability of restoring $\rho_a \rightarrow 0$

Only $\cos[2n(\theta_0 - \theta_s)]$ is allowed in all the vertex operators.

Effective Interaction in 1D confined systems

M.Olshanii, PRL 81(1998) Girardeau, Nuguyen, Olshanii, (2004)

$$g = \frac{4\pi\hbar^2 a}{m}$$ Strength of 3D inter-atom interaction

a 3D s-wave scattering length

2D confinement potential

$$g_{1D} = -\frac{2\hbar^2}{m a_{1D}}$$

$$a_{1D} = -\frac{a_\perp^2}{2a}\left(1 - C\frac{a}{a_\perp}\right)$$ 1D scattering length

$$a_\perp = \sqrt{\hbar/(m\omega_\perp)}$$ **"Radius "of wave function** in 2D plane

$\omega_\perp$ Frequency of confinement potential

$$a/a_\perp \gg 1 \qquad g_{1D} \rightarrow -\frac{4\hbar^2}{mCa_\perp}$$

Attractive interaction

$$a/a_\perp \ll 1 \qquad g_{1D} \rightarrow \frac{4\hbar^2 a}{m a_\perp^2}$$

Repulsive interaction