LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS

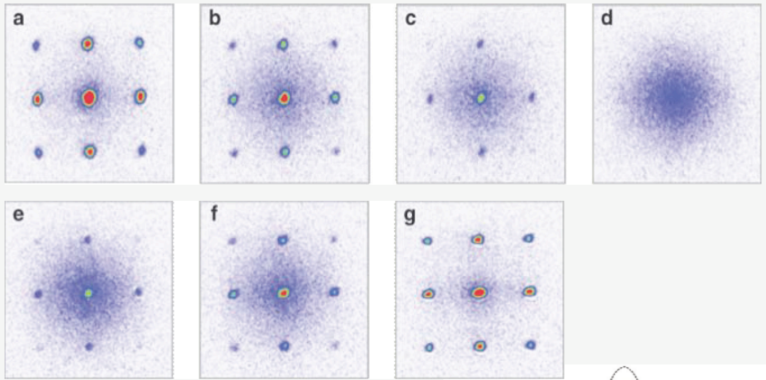
Michael Moeckel, Stefan Kehrein

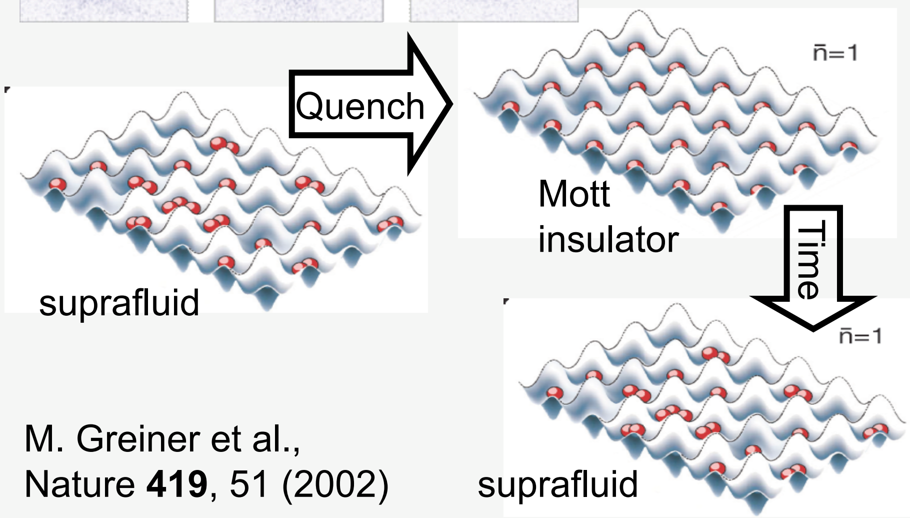
Interaction Quench in the Hubbard Model

Phys. Rev. Lett. **100**, 175702 (2008)

Experiments with ultracold bosons

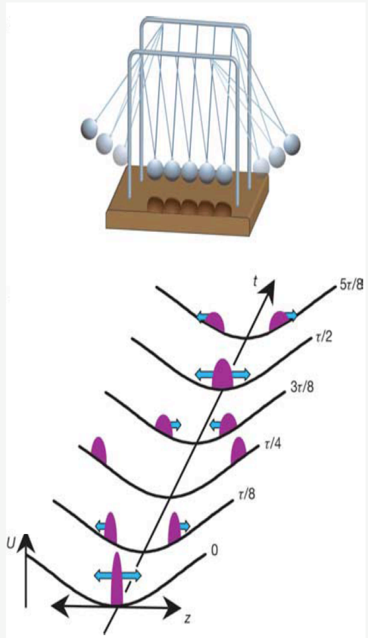
Collapse and revival of a many-body quantum state

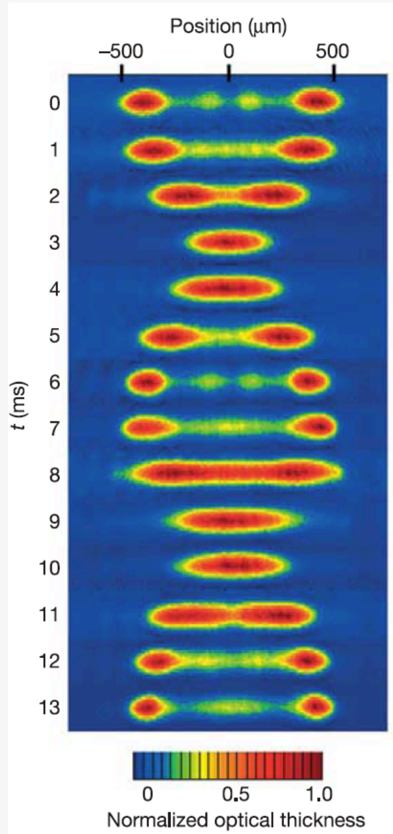




M. Greiner et al.,
Nature **419**, 51 (2002)

Persistent oscillations for 1D hard-core bosons





Kinoshita et al.,
Nature (London)
440, 7086 (2006)

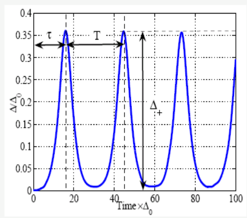
How do closed excited many-body quantum systems relax?

Thermalization debate

Linearity of Schroedinger eqn (deterministic) ⇔ Probabilistic description

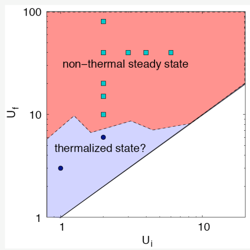
Role of integrability of the model

- Conserved integrals of motion
- Memory of initial state



Intrinsic restrictions of the model

Delayed or avoided relaxation



Deferred for later study!

Restriction to particular observables:

Kinetic energy

Momentum distribution

validity of **Quantum Boltzmann** equation

Hubbard model in the Fermi liquid phase (nonintegrable)

intermediate nonequilibrium states

Interaction quench within the Fermi liquid phase

Test of nonequilibrium properties of a Fermi liquid

Challenge of Fermi liquid theory

Fermi liquid theory in equilibrium:

Noninteracting Fermi gas

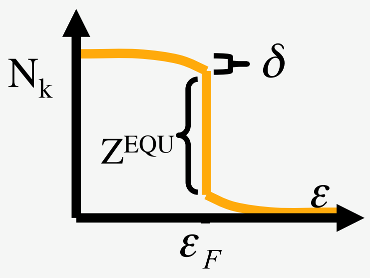
Adiabatic switching-on of interaction

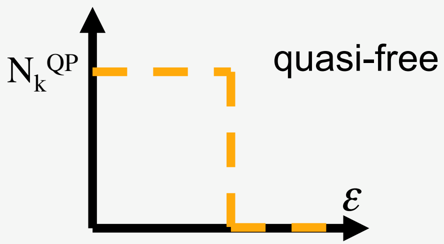
1:1 of quantum numbers

Interacting physical fermions

Two equivalent descriptions

Landau quasiparticles





residual quasiparticle interaction

In nonequilibrium:

Interaction quench

Rapid build-up of a quasiparticle picture

Nonequilibrium transient Fermi liquid state

Thermalization by Quantum Boltzmann equation

Interaction quench of Hubbard model

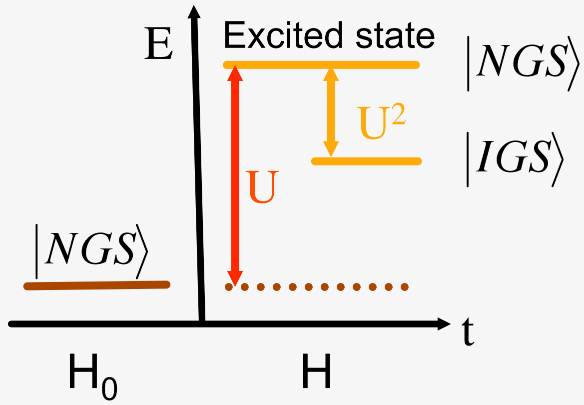
Hubbard model in more than 1D and at zero temperature

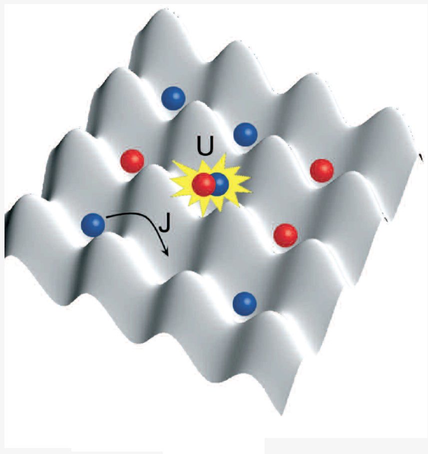
$$H = \sum_{k,\sigma} \varepsilon_k c^{+}_{k\sigma} c_{k\sigma} + \Theta(t)\, U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right)$$

Kinetic term (Hopping J)

Sudden switch

Local onsite repulsion U





Displays rich physics, but we only work in the

Fermi-liquid phase U << J

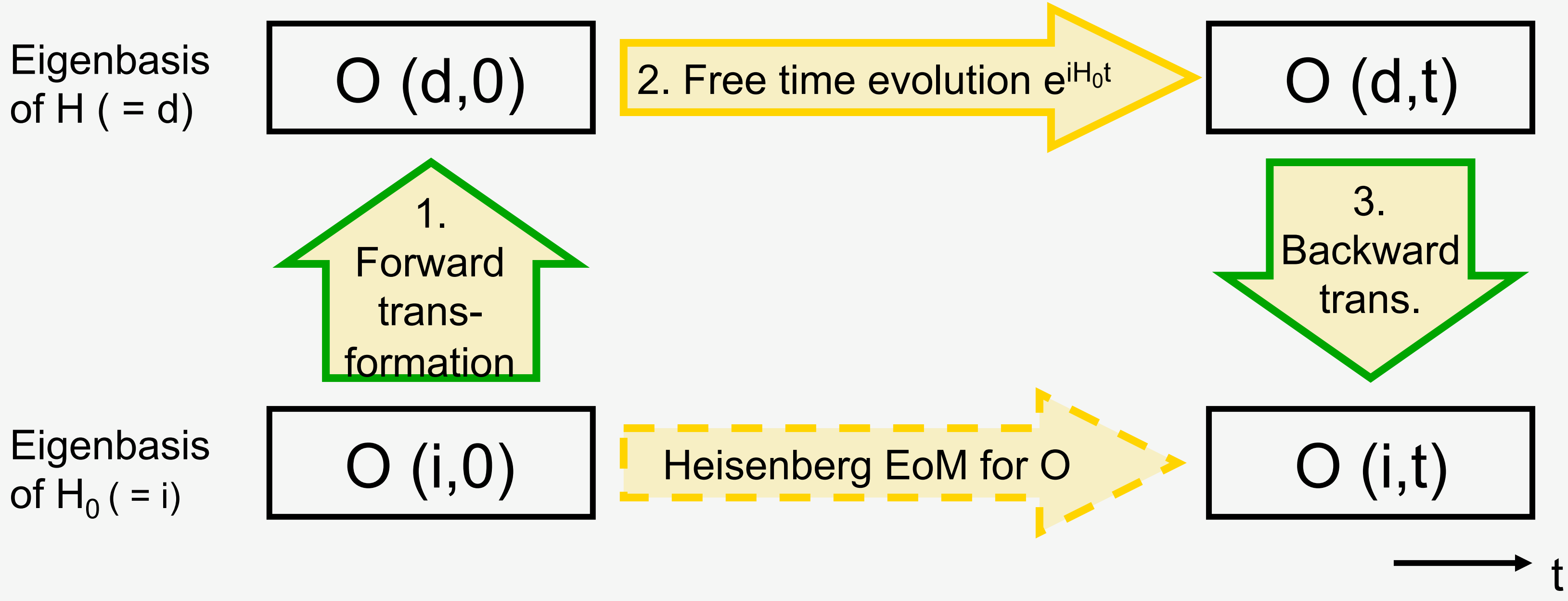
Time evolution of excited state w.r.t interacting Hamiltonian

Implemented in optical lattices a.o. by Tilman Esslinger et al., Europhys. News **37**,2 (2006)

Unitary perturbation theory

Solution of the Heisenberg equations of motion in the interacting eigenbasis

- Motivated by canonical perturbation theory in classical mechanics
- Avoids the emergence of secular terms



A. Hackl and S. Kehrein, PRB (accepted for publication)

Flow equation method (following Wegner)

Continuous sequence of infinitesimal unitary transformations

Parametrized by flow parameter B

$$H(B) = :H_0(B): + :H_{\text{int}}(B):$$

$$\eta(B) = \left[H_0(B), H_{\text{int}}(B)\right]$$ Canonical generator

$$\Rightarrow \quad H(\infty) \approx \tilde{H}_0(\infty)$$

Transformation of observables

$$\frac{\partial O(B)}{\partial B} = \left[\eta(B), O(B)\right]$$

Generates higher order terms

Truncated ansatz:

$$c_{k\uparrow}^{+}(B) = h_{k\uparrow}(B) c_{k\uparrow}^{+} + \sum_{1'2'2} M_{1'2'2\uparrow\downarrow\downarrow}^{k\uparrow}(B)\, c_{1'\uparrow}^{+} c_{2'\downarrow}^{+} c_{2\downarrow}$$

'Quasiparticle' residue — Incoherent multi-particle background

- Set of coupled differential flow equations for parameters h(B) and M(B)
- Second order in interaction U

F. Wegner, Ann. Physik (Leipzig) **3**, 77 (1994)
S. Kehrein, Springer, 2006

Results for the Hubbard model

1. Transient nonequilibrium state

by perturbative calculation of

$$\lim_{t\to\infty} N_k^{NEQ:U^2}(t)$$

relaxation of average energies E_{KIN}, E_{POT} to final values
Pre-thermalization

Avoided relaxation of the momentum distribution:
Fermi liquid like transient state

2. Thermalization

by argument:
Quantum Boltzmann eqn

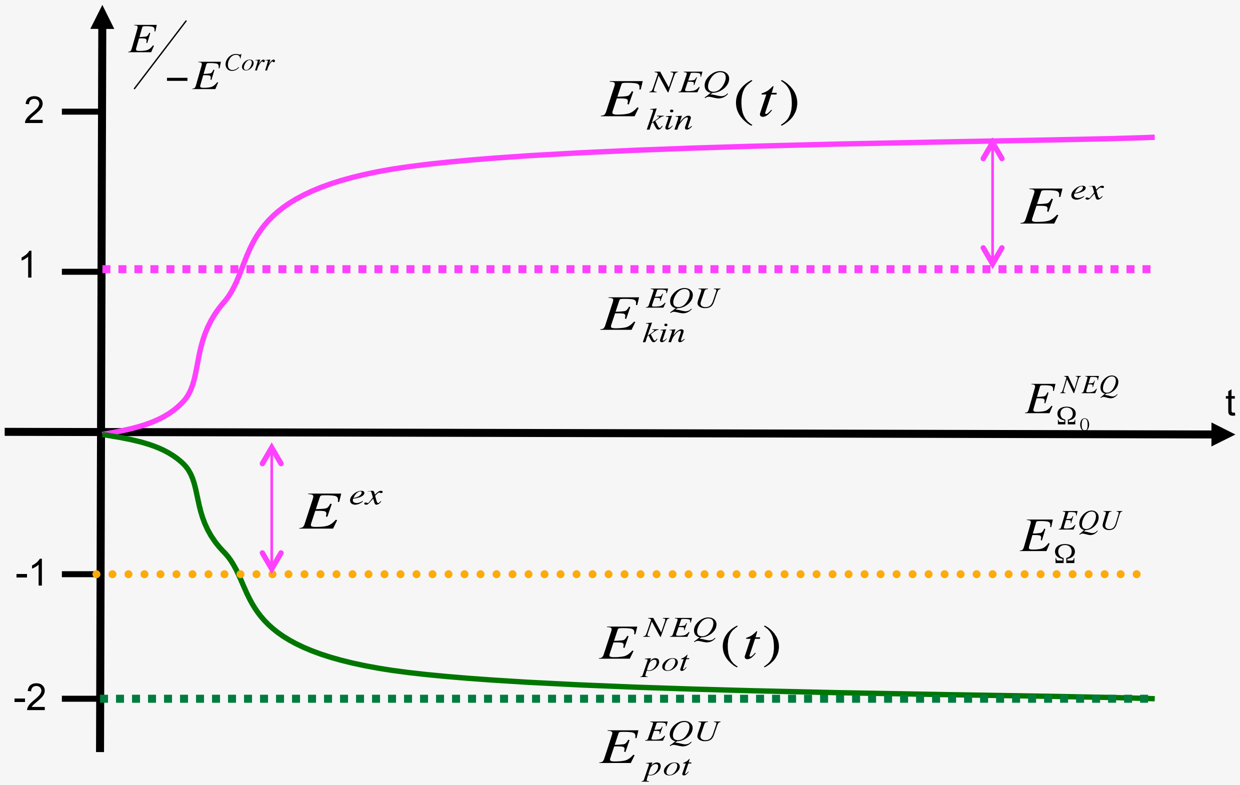
relaxation of momentum distribution function

Temperature T~U

$\rho_F^{-1} U^{-2}$ **time-scale** $\rho_F^{-3} U^{-4}$

Energy relaxation after quenches

Equilibrium and nonequilibrium energies (sketch)



Up to second order in U:

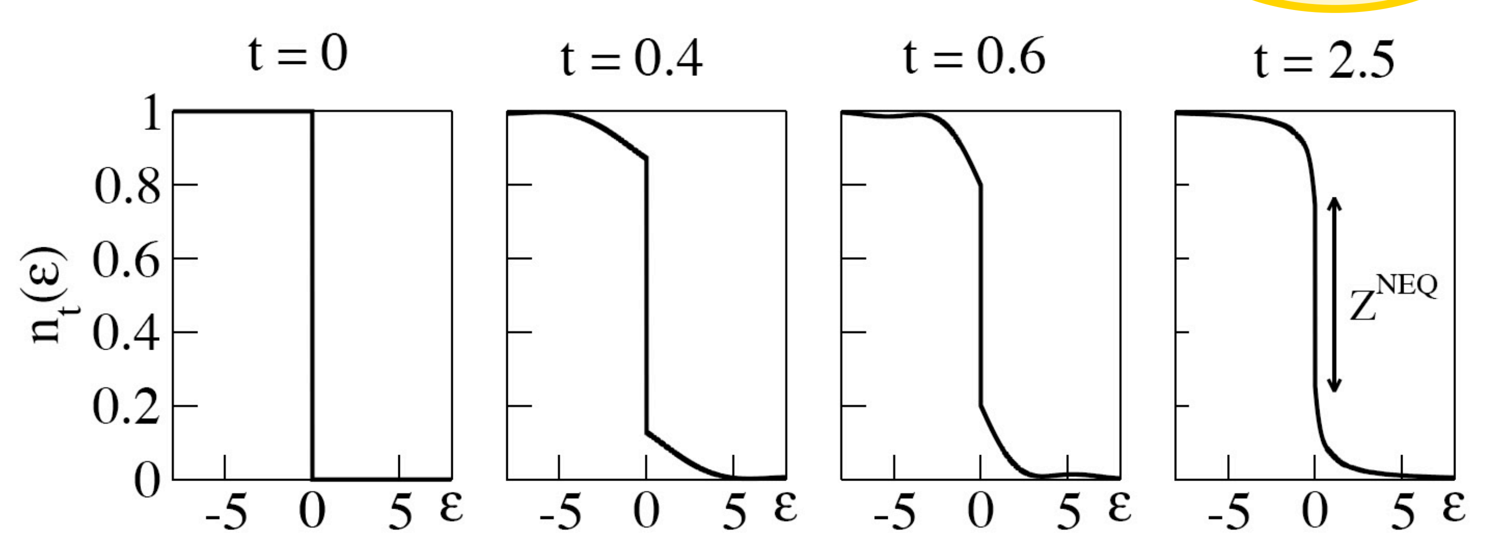
All excitation energy is transferred into kinetic energy

Correlation buildup and transient state

Buildup of an interacting Fermi liquid description

- Nonvanishing quasiparticle residue Z
- Timescale set by the interaction $t \propto \rho_F^{-1} U^{-2}$

Transient nonequilibrium state



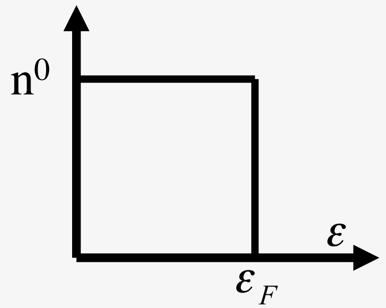
U=1.5

Explicit calculations in the limit of infinite dimension
Metzner and Vollhardt, PRL **62**, 324 (1989)

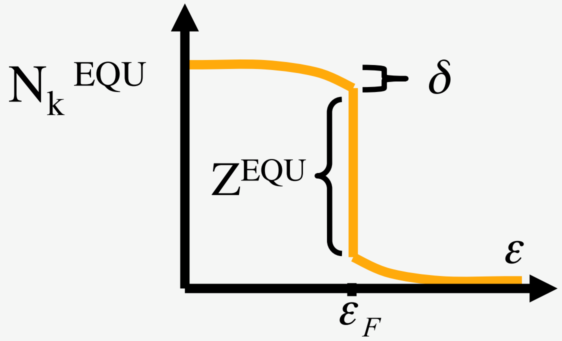
Explaining the momentum distribution

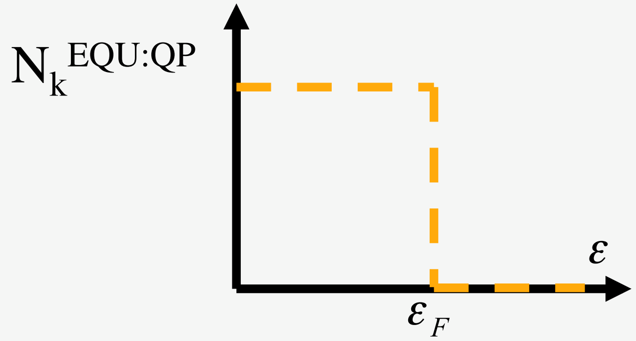
Sketch for physical fermions and related quasiparticles

EQUILIBRIUM
(adiabatic switch)



Initial state





Explaining the momentum distribution

Sketch for physical fermions and related quasiparticles

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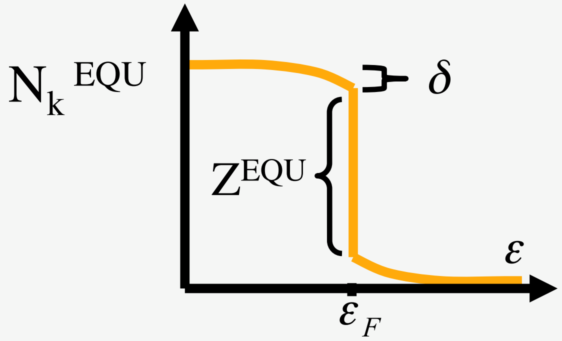
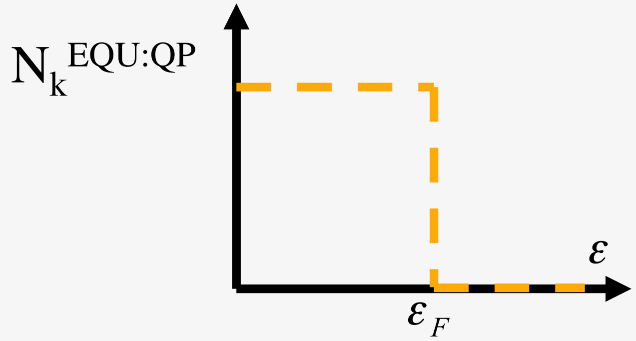
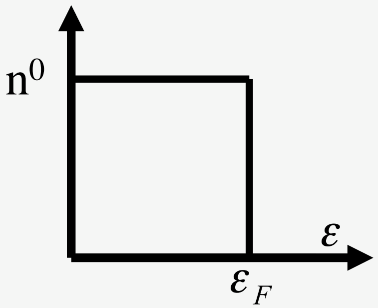


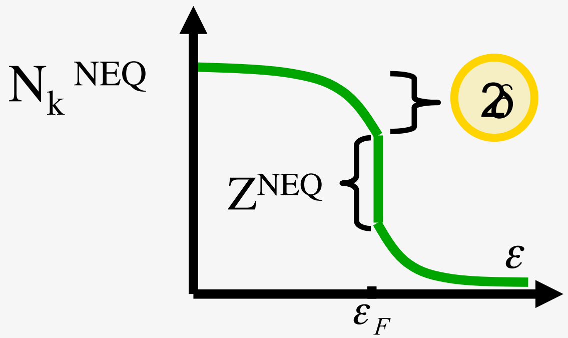




Initial state

Factor of 2

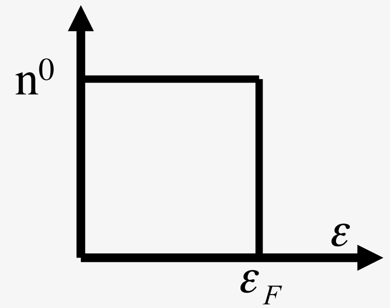
NONEQUILIBRIUM
(sudden switch)



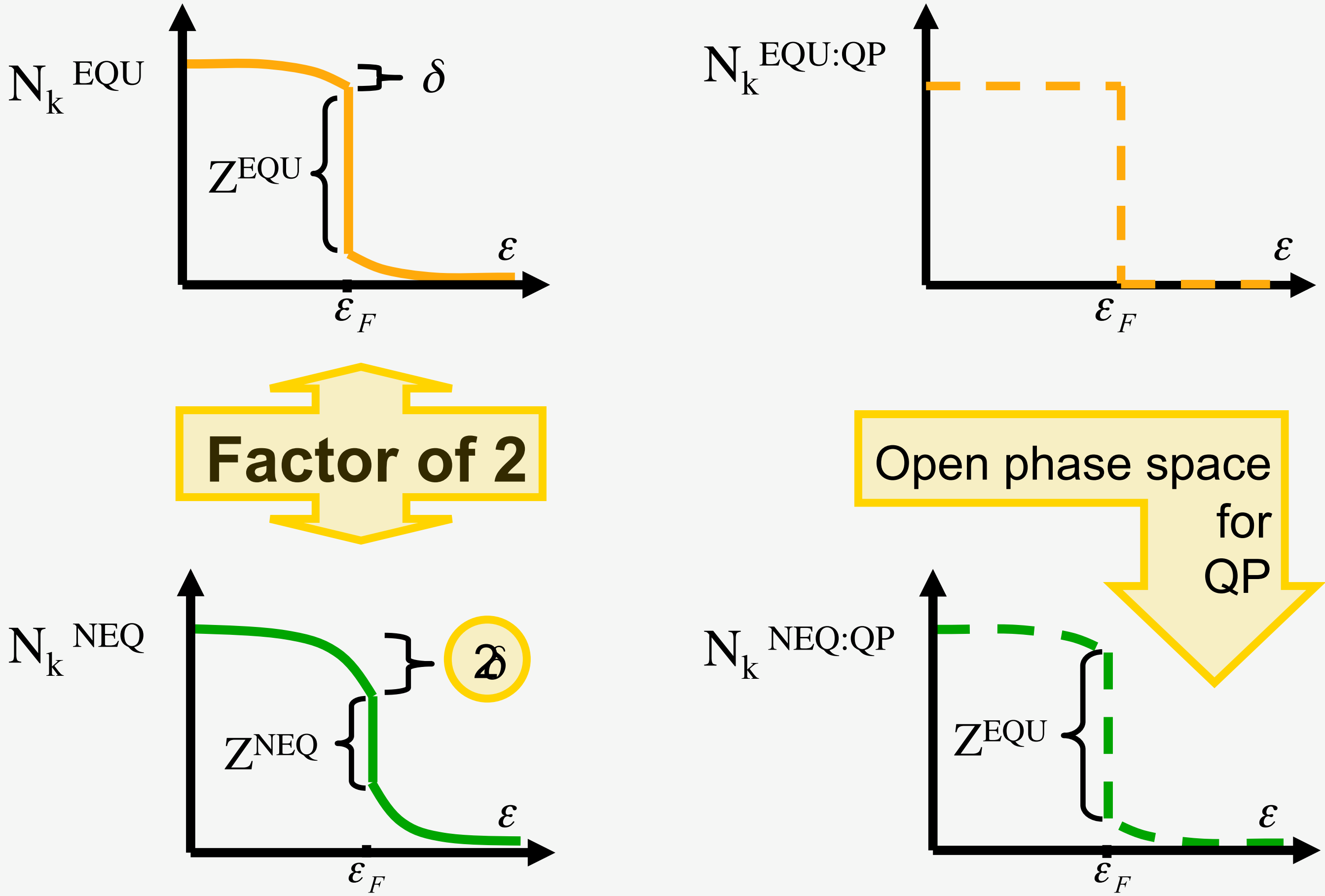
Explaining the momentum distribution

Sketch for physical fermions and related quasiparticles

EQUILIBRIUM
(adiabatic switch)



Initial state



NONEQUILIBRIUM
(sudden switch)

Quantum Boltzmann equation

Effective kinetic description for momentum distribution

Includes free evolution of a system and a random Markovian collision mechanism

$$\frac{\partial n_{k_1}(X,t)}{\partial t} = -4\pi U^2 \int dk_2 dk_3 dk_4 \; \delta_{k_3+k_4}^{k_1+k_2} \; \delta\left(E_1 + E_2 - E_3 - E_4\right) \times \left[n_{k_1} n_{k_2}(1-n_{k_3})(1-n_{k_4}) - n_{k_4} n_{k_3}(1-n_{k_2})(1-n_{k_1})\right]$$

Prerequisites for its application:

- No memory effects, i.e. not for short time processes
- Quasiparticle picture holds

Fixed point argument

- Thermal distributions are fixed points of QBE dynamics $n(\varepsilon_k) = \frac{1}{1+e^{-\beta\varepsilon_k}}$

Nonthermal distributions evolve into thermal ones

Bottleneck picture of delayed relaxation

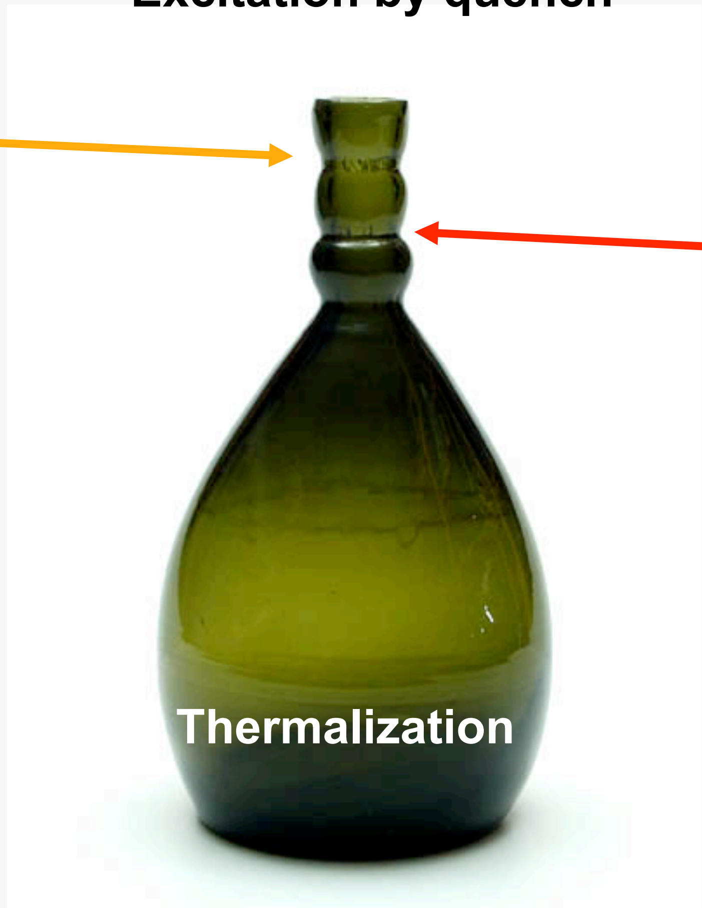
Excitation by quench

Translation invariance of Hubbard model

Momentum conservation

Momentum relaxation only by scattering events

Quantum Boltzmann



zero temp.

Pauli principle

Restricted phase space for two particle scattering processes

Characteristic for a zero temperature Fermi liquid

Long-lasting transient Fermi liquid state

Conclusions

3-step dynamics of the Hubbard model (weak U)

0 QUENCH

1. **Buildup of correlations**
Quasiparticles are formed

2. $t \sim \rho_F^{-1} U^{-2}$
Intermediate NEQ T=0 Fermi liquid state
Nonequilibrium quasi-particle distribution

3. $t \sim \rho_F^{-3} U^{-4}$
Thermalization
Quantum Boltzmann Eq$^{\underline{n}}$

t

momentum distribution

Mismatch of quasiparticle residue Z^{NEQ} by a Factor 2

Relaxation to thermal distribution

Energy

Excitation energy fully transferred to kinetic energy

Heating temperature T~U