

Correlations and quenches in integrable systems



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P. Calabrese, N. Slavnov, A. Faribault

Plan of the talk

Correlation dynamics

Systems which can be treated using ‘exact’ methods

Short overview of the methods used

Example 1: quantum spin chains

3 approaches: lattice, q group, field theory

Example 2: one-d Bose gas

repulsive, attractive

Example 3: the Richardson model

Quantum quenches

General comments

... quenches in the Richardson model

The contract: calculate dynamical correlation functions of local operators in interacting models

General form:

$$\langle \mathcal{O}(1) \mathcal{O}^\dagger(2) \rangle$$

some local operator

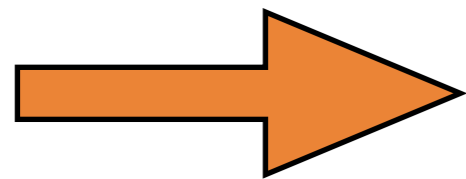
Ground state, prepared state, thermal average, ...

- Difficulty: multiparticle eigenstates are not obtained as simple products of single-particle states (alt., not created using simple products of local ops)
- Exact methods (integrability): traditionally restricted to equilibrium thermodynamics

Correlation functions: elements

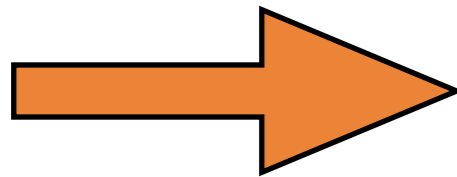
$$S^{a,\bar{a}}(q, \omega) = 2\pi \sum_{\mu} |\langle 0 | \mathcal{O}_q^a | \mu \rangle|^2 \delta(\omega - E_{\mu} + E_0)$$

1) Eigenstates basis: energies, states (+ norms)



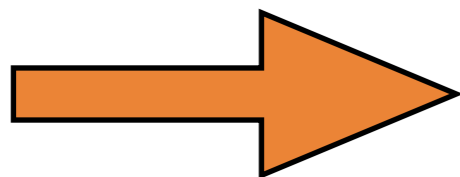
Bethe Ansatz; quantum groups

2) We need to be able to compute the matrix elements of the operators we're interested in



Algebraic Bethe Ansatz; q. groups

3) We need to be able to perform the sum over intermediate states



Numerics, analytics

Models which we can treat:

● Heisenberg spin-1/2 chain

$$H = \sum_{j=1}^N [J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z]$$

(finite lattice BA; quantum groups)

● Interacting Bose gas (Lieb-Liniger)

$$\mathcal{H}_N = - \sum_{j=1}^N \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1 \leq j < l \leq N} \delta(x_j - x_l)$$

(BA for finite particle numbers)

● Richardson model (+ Gaudin magnets)

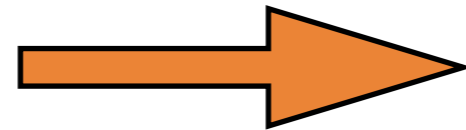
$$H_{BCS} = \sum_{\substack{\alpha=1 \\ \sigma=+,-}}^N \frac{\epsilon_\alpha}{2} c_{\alpha\sigma}^\dagger c_{\alpha\sigma} - g \sum_{\alpha,\beta=1}^N c_{\alpha+}^\dagger c_{\alpha-}^\dagger c_{\beta-} c_{\beta+}$$

What we can calculate:

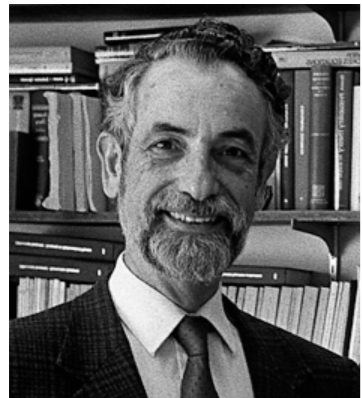


● DYNAMICAL STRUCTURE FACTOR

$$S^{a\bar{a}}(q, \omega) = \frac{1}{N} \sum_{j, j'=1}^N e^{iq(j-j')} \int_{-\infty}^{\infty} dt e^{i\omega t} \langle S_j^a(t) S_{j'}^{\bar{a}}(0) \rangle_c$$



inelastic neutron scattering

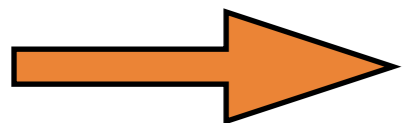


● DENSITY-DENSITY FUNCTION

$$S(k, \omega) = \int dx \int dt e^{-ikx + i\omega t} \langle \rho(x, t) \rho(0, 0) \rangle$$

● ONE-BODY FN

$$G_2(x, t) = \langle \Psi^\dagger(x, t) \Psi(0, 0) \rangle$$



Bragg spectroscopy, interference experiments, ...

(zero temperature only (for now !))

Specific cases treated:



General XXZ AFM, H_z
XXX gapless AFM, $H_z=0$
XXZ gapped AFM, $H_z=0$
XXZ gapless AFM, $H_z=0$

Finite BA
numerics

done

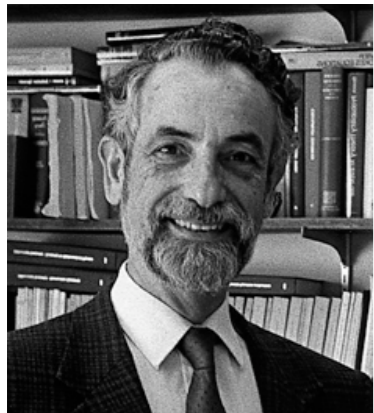
Analytics
(q groups)

no go

done: (2)+4 sp

done: 2 sp

done: 2 sp (t.b.p.)



Repulsive
Attractive

Finite BA
numerics

done

Analytics
(brute force)

no go

done



Richardson

Finite BA
numerics

done

Quenches

done (t.b.p.)

Method I: Bethe Ansatz

Hans Bethe, 1931



July 2, 1906 – March 6, 2005

$$H = \sum_{j=1}^N [J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z]$$

Exact solution through Bethe Ansatz:

Eigenstate with M down spins
fully characterized by set of
rapidities $\{\lambda_j\}$, $j = 1, \dots, M$

$$\Psi(j_1, \dots, j_M | \lambda_1, \dots, \lambda_M) = \sum_P^{M!} A(P | \{\lambda\}) e^{i \sum_{a=1}^M k(\lambda_{P_a}) j_a}$$

Known amplitudes



The M rapidities are solutions of the Bethe equations

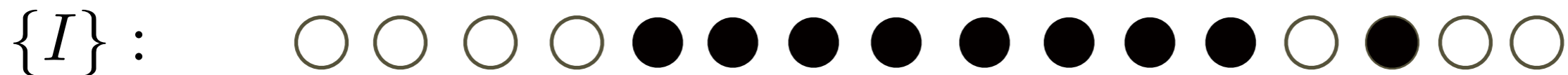
$$\theta_1(\lambda_j) - \frac{1}{N} \sum_{l=1}^M \theta_2(\lambda_j - \lambda_l) = 2\pi \frac{I_j}{N}, \quad j = 1, \dots, M$$

Eigenstates: labeled by set of quantum numbers

Ground state:



Simple excitations:



Solving the BE: nontrivial in general, still: it's feasible

Deformed strings (XXX)

M = 2: Bethe, Vladimirov, Essler Korepin Schoutens, ...

Wide pairs, narrow pairs, extra real solutions for $N > 22$

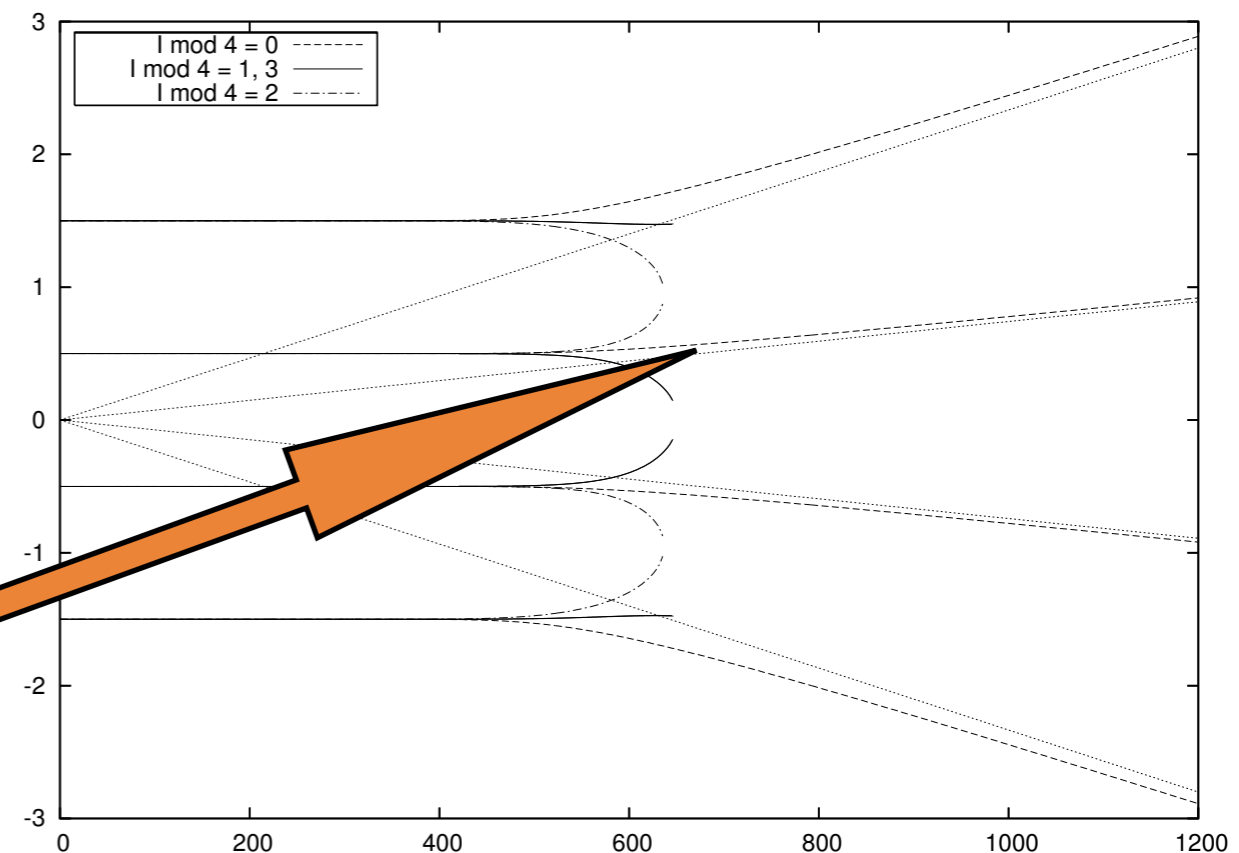
Higher M

(R. Hagemans & JSC, JPA 2007)

Here: 4-strings on
a chain of $1M$ sites

Asymptotes:

$$\Im \lambda = \frac{\Re \lambda}{\sqrt{N - 3 \pm \sqrt{\frac{2}{3}(N - 3)(N - 2)}}}$$



Beyond strings: important for completeness, finite size, ...

Full understanding of solutions to BE
remains to be obtained

Algebraic Bethe Ansatz

Like '2nd quantization' for Bethe Ansatz

Introduce family $A(\lambda), B(\lambda), C(\lambda), D(\lambda)$ of nonlocal operators which act in Hilbert space of model

$B(\lambda)$ **creation operator**, increasing particle number by 1

Wavefunctions: $|\Psi(\{\lambda\})\rangle = \prod_j B(\lambda_j)|0\rangle$

provided the **rapidities satisfy Bethe equations**

Mapping ABA ops to local ops: quantum inverse problem (Maillet 1999)

For spin chains: $A(\lambda), B(\lambda), C(\lambda), D(\lambda) \longleftrightarrow S_j^a$

State norms: Gaudin-Korepin formula

Form factors: Slavnov's theorem

Det representation of form factors from the Algebraic Bethe Ansatz

$$|\langle \{\mu\} | S_q^z | \{\lambda\} \rangle|^2 = \frac{|F_M^z(\{\mu\}, \{\lambda\})|^2}{|N_M(\{\mu\})N_M(\{\lambda\})|} = \prod_{j=1}^M \left| \frac{\sinh(\mu_j - i\zeta/2)}{\sinh(\lambda_j - i\zeta/2)} \right|^2 \prod_{j>k=1}^M |\sinh^2(\mu_j - \mu_k) + \sin^2 \zeta|^{-1} \times \\ \times \prod_{j>k=1}^M |\sinh^2(\lambda_j - \lambda_k) + \sin^2 \zeta|^{-1} \frac{|\det[\mathbf{H}(\{\mu\}, \{\lambda\}) - 2\mathbf{P}(\{\mu\}, \{\lambda\})]|^2}{|\det \Phi(\{\mu\}) \det \Phi(\{\lambda\})|}$$

All matrices are given explicitly as functions of the rapidities of the eigenstates involved:

$$\mathbf{H}_{ab}(\{\mu\}, \{\lambda\}) = \frac{1}{\sinh(\mu_a - \lambda_b)} \left[\prod_{j \neq a} \sinh(\mu_j - \lambda_b - i\zeta) - \left[\frac{\sinh(\lambda_b + i\zeta/2)}{\sinh(\lambda_b - i\zeta/2)} \right]^N \prod_{j \neq a} \sinh(\mu_j - \lambda_b + i\zeta) \right],$$

(Kitanine, Maillet, Slavnov, Terras 1999 & 2000)

String states: need modified determinants

(Caux, Hagemans, Maillet 2005)

Integrability for correlations: generic features

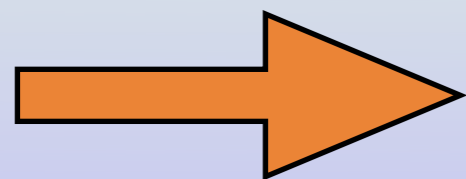
- Exact realization of ground state, taking all 'entanglement' into account
- Exact realization of excited states (spinons, Lieb types I, II, Gaudinos,...), *irrespective of their energy*
- Action of local operators: accurately captured by using only a handful of BA excitations

 ***incredibly efficient basis for many physically relevant correlations***

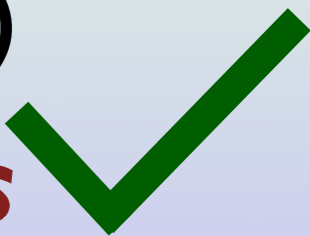
Correlation functions: elements

$$S^{a,\bar{a}}(q, \omega) = 2\pi \sum_{\mu} |\langle 0 | \mathcal{O}_q^a | \mu \rangle|^2 \delta(\omega - E_{\mu} + E_0)$$

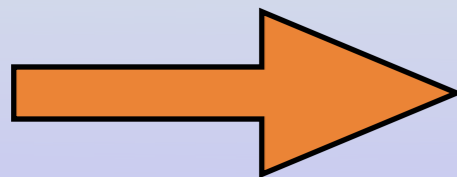
1) Eigenstates basis: energies, states (+ norms)



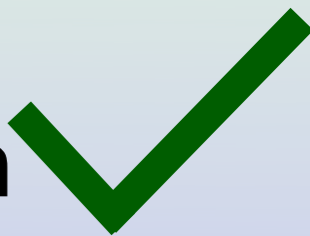
Bethe Ansatz; quantum groups



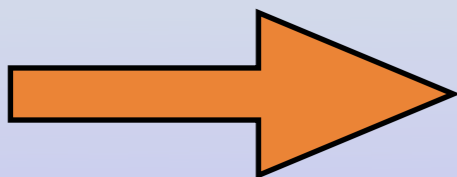
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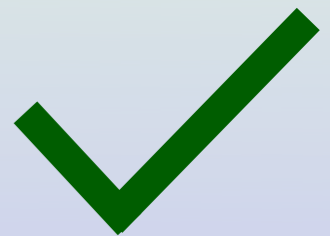
Algebraic Bethe Ansatz; q. groups



3) We need to be able to perform the sum over intermediate states

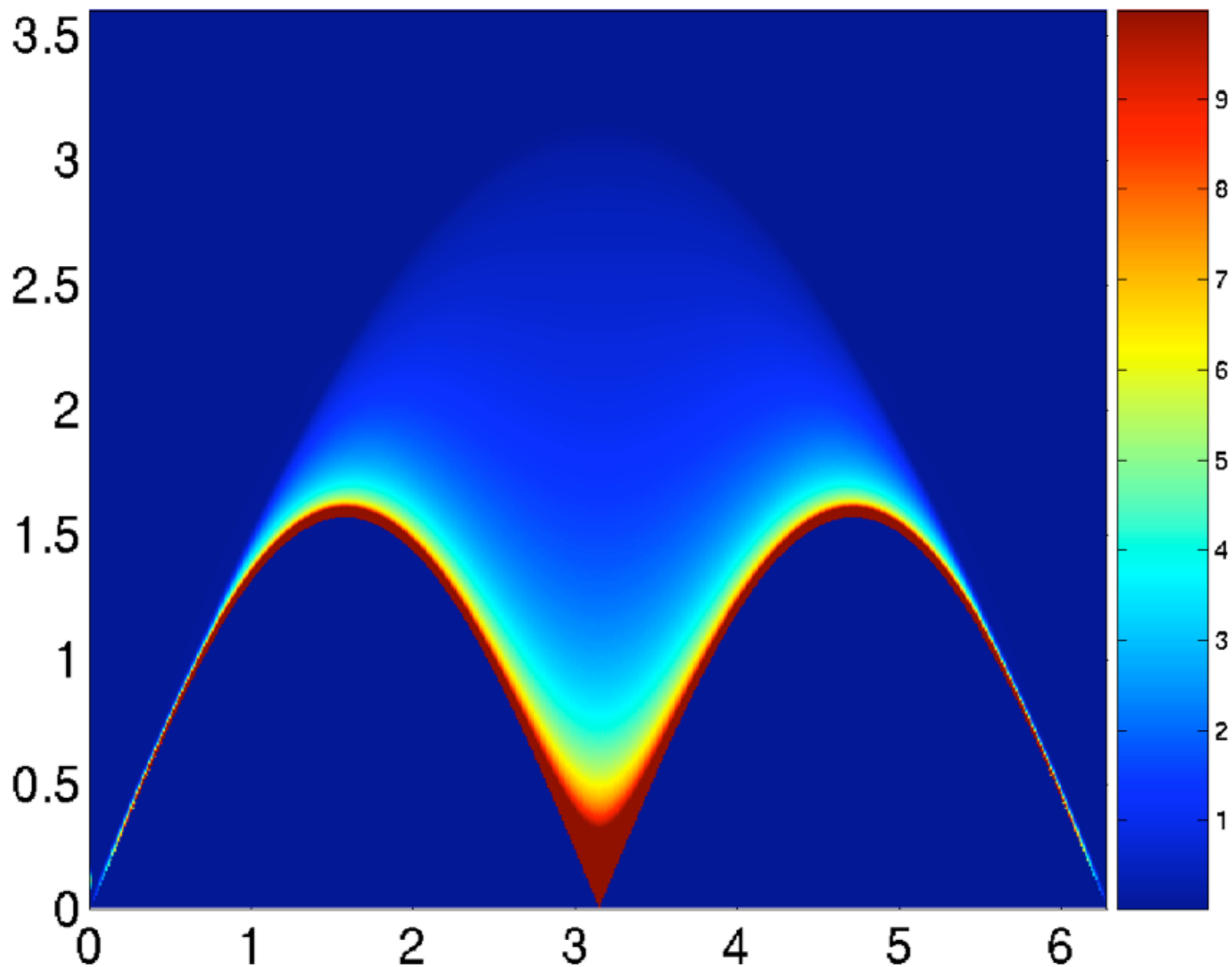


Numerics, analytics

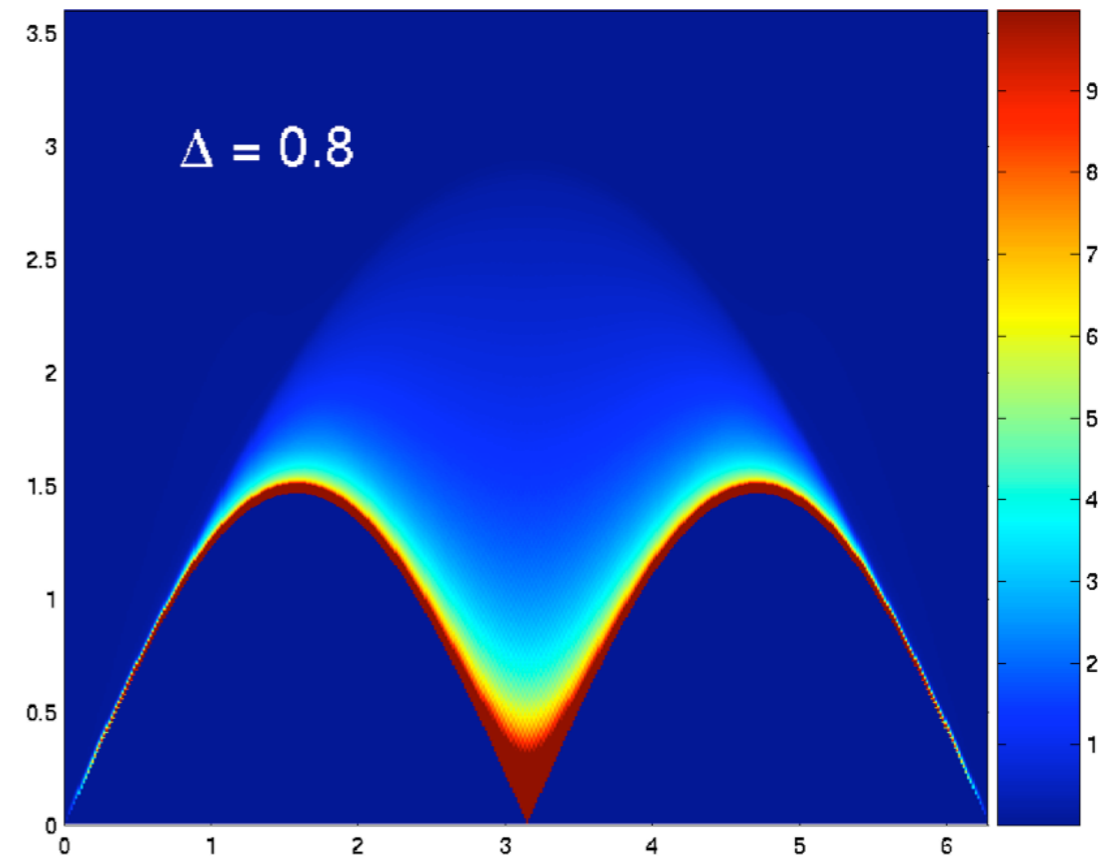
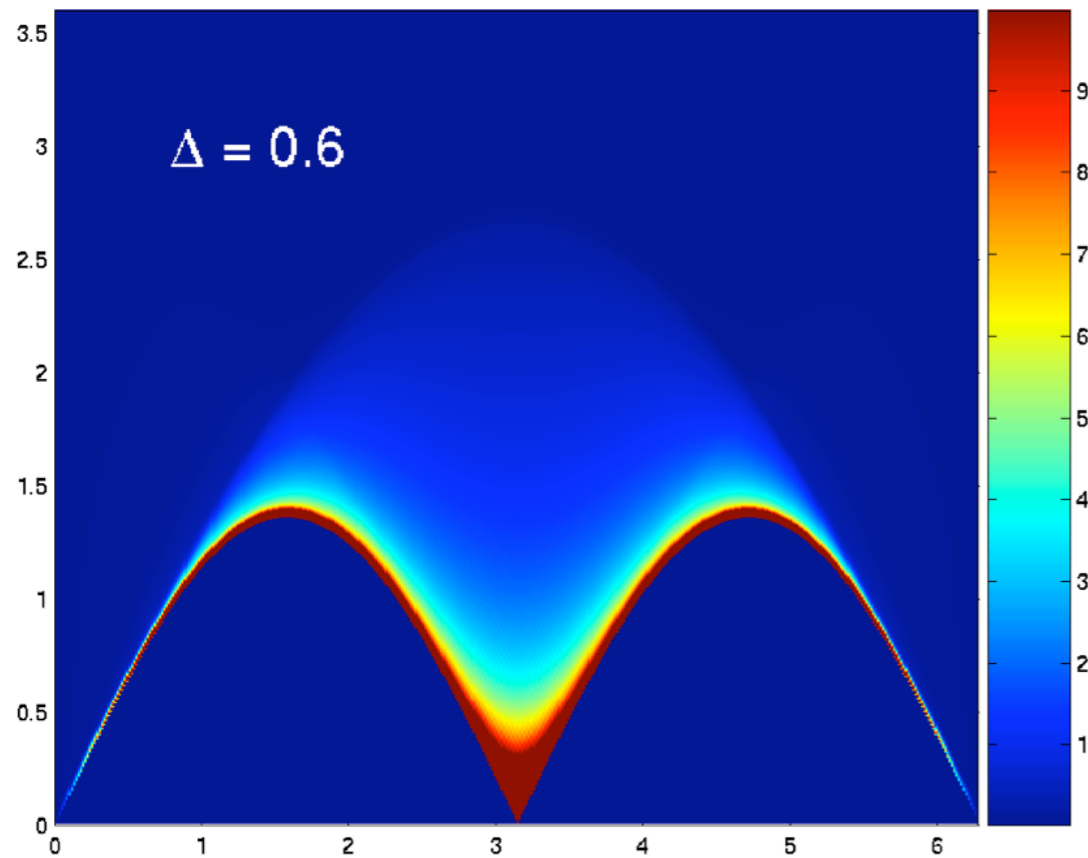
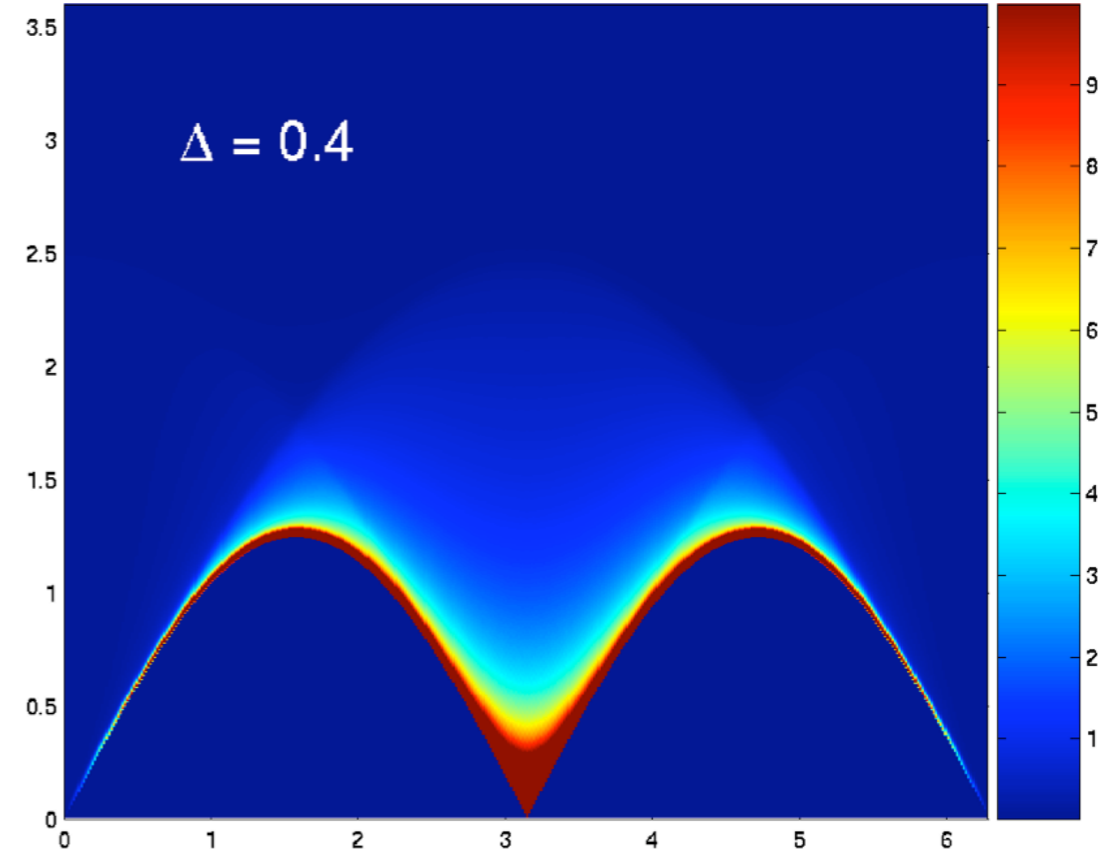
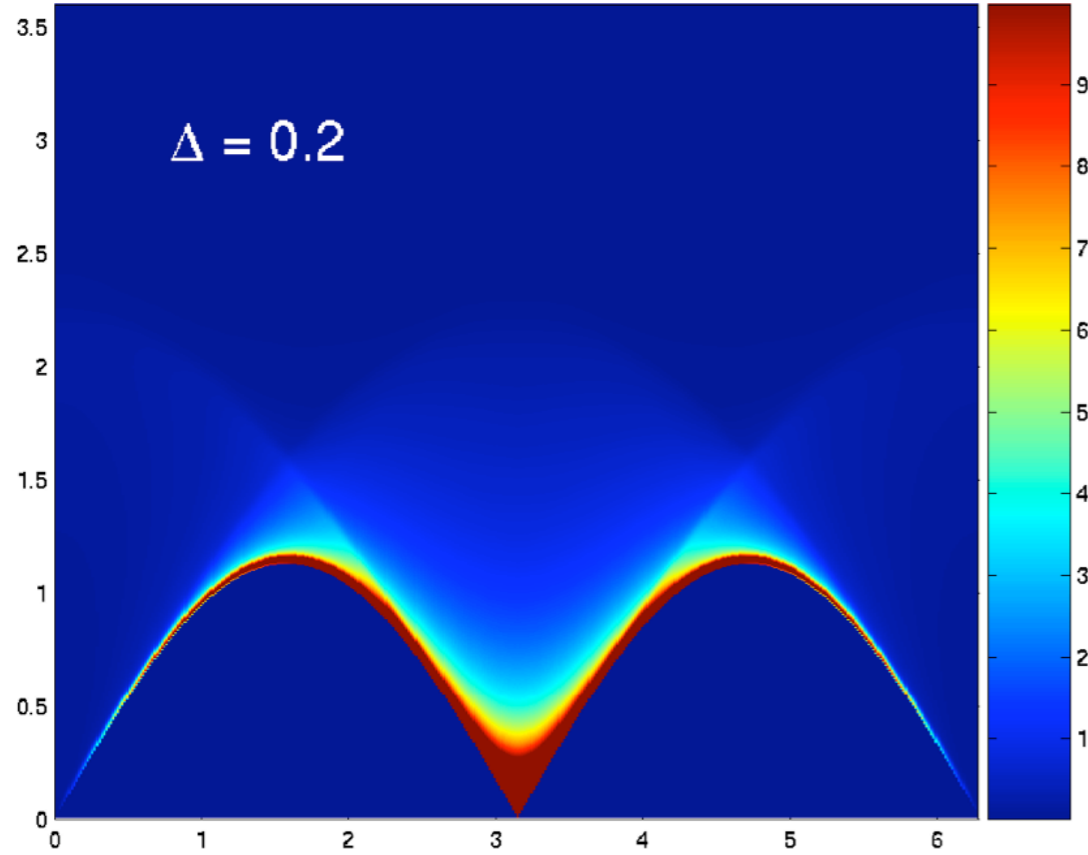


Method I: ABACUS

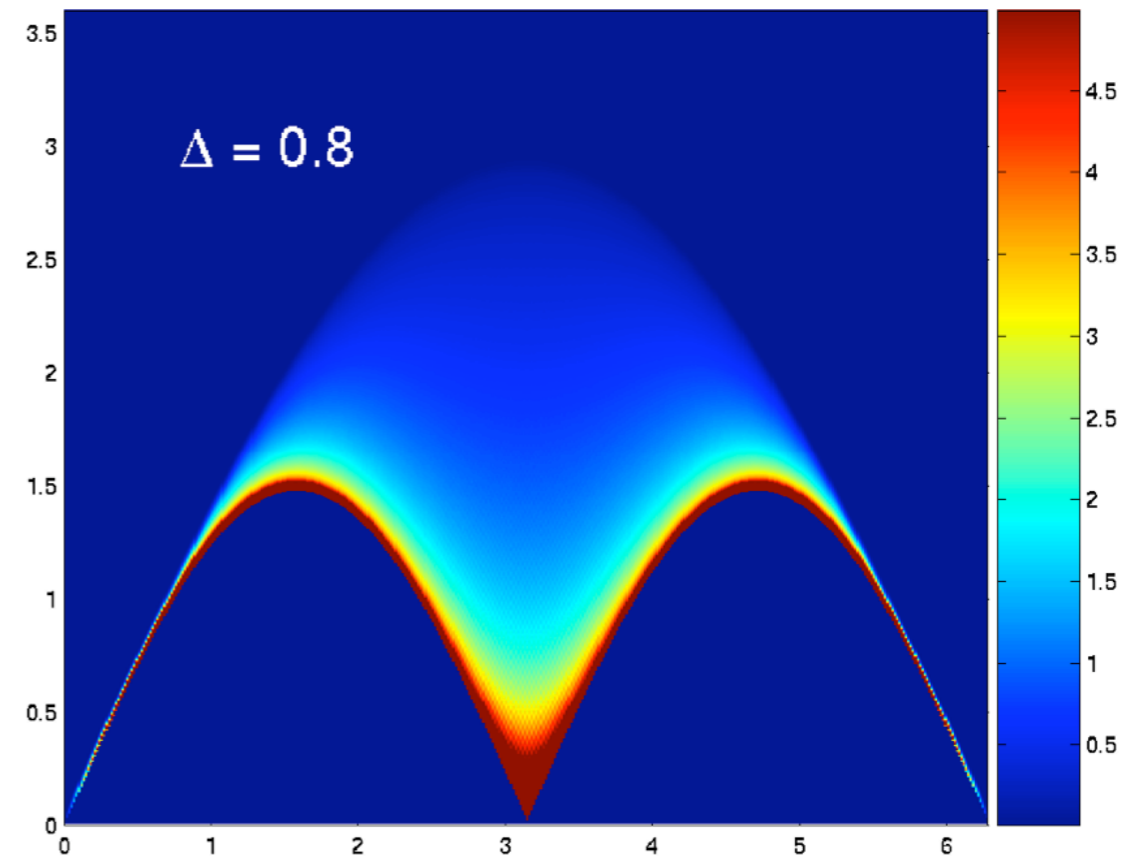
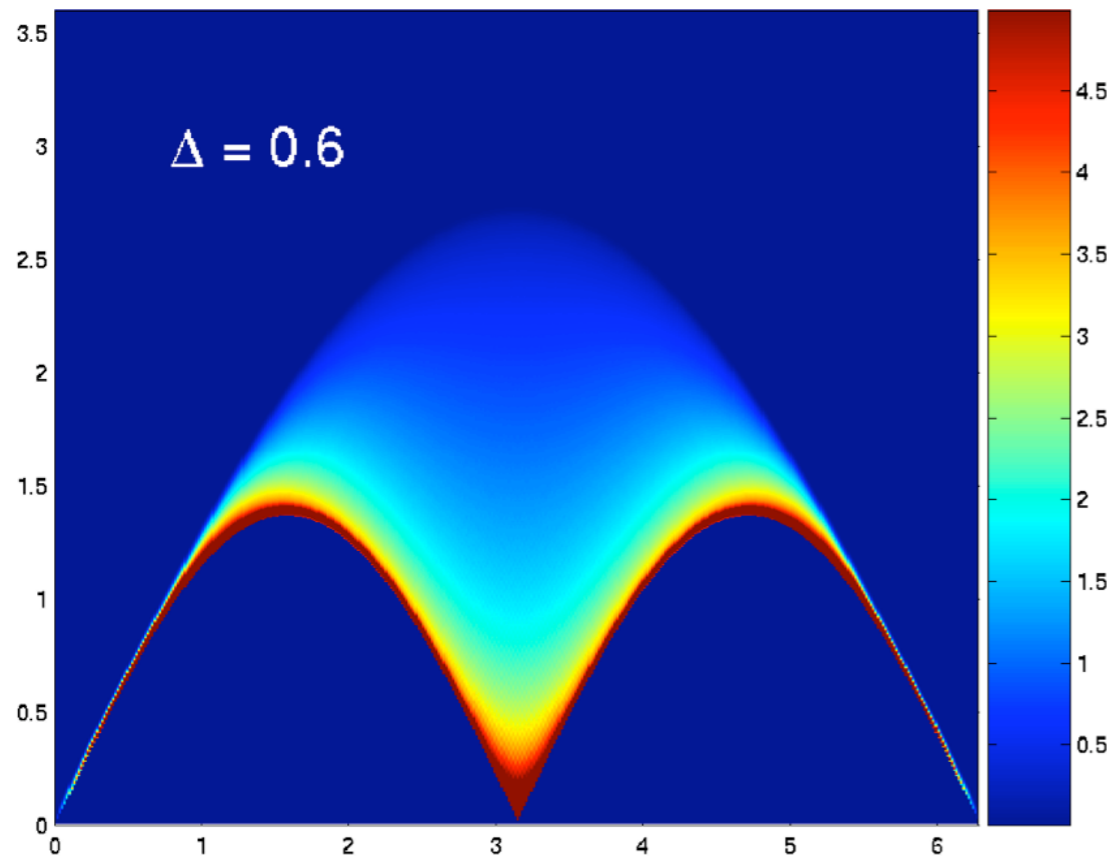
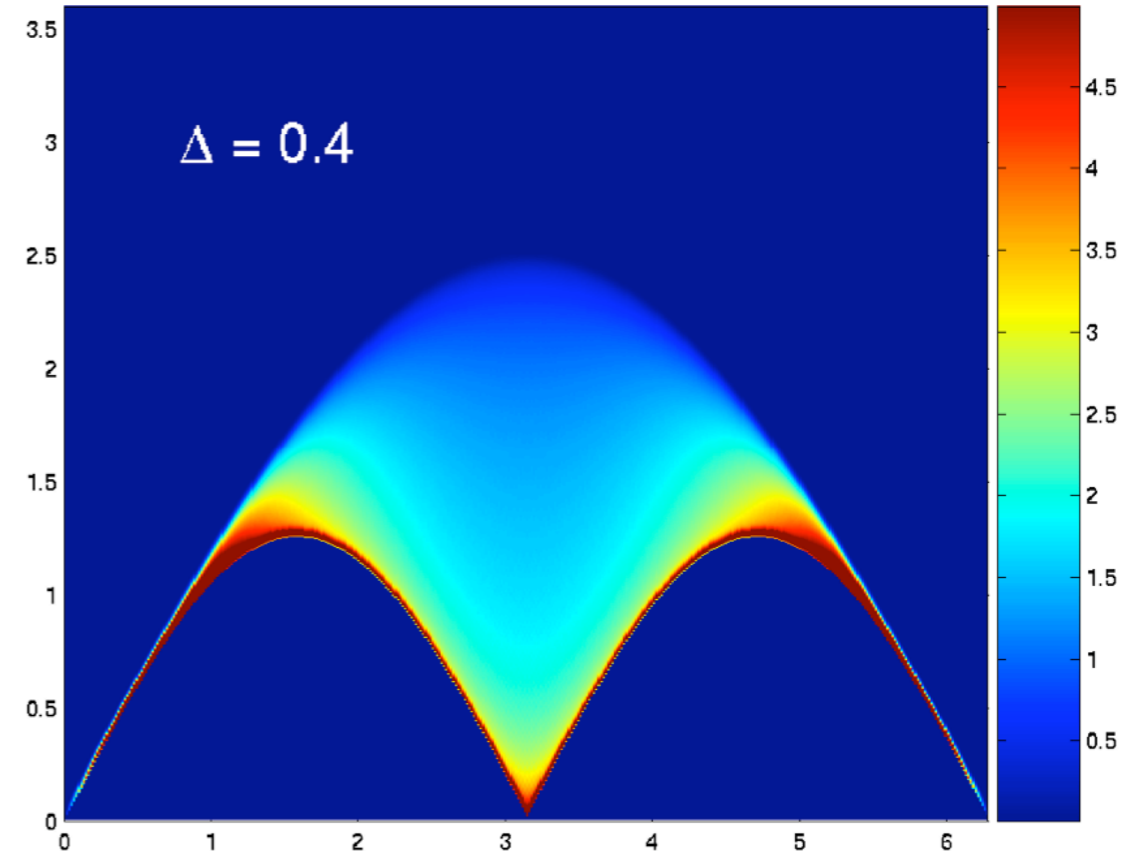
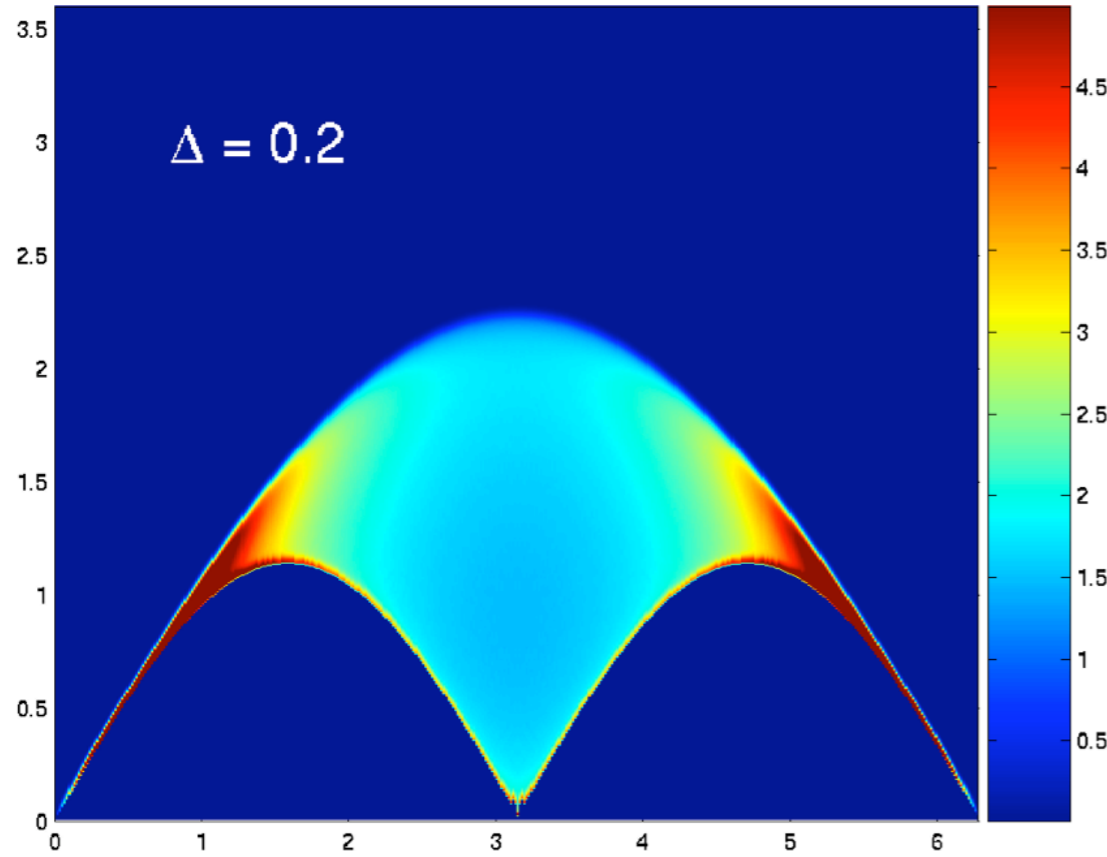
$$S(k, \omega), \quad \Delta = 1, \quad h = 0$$



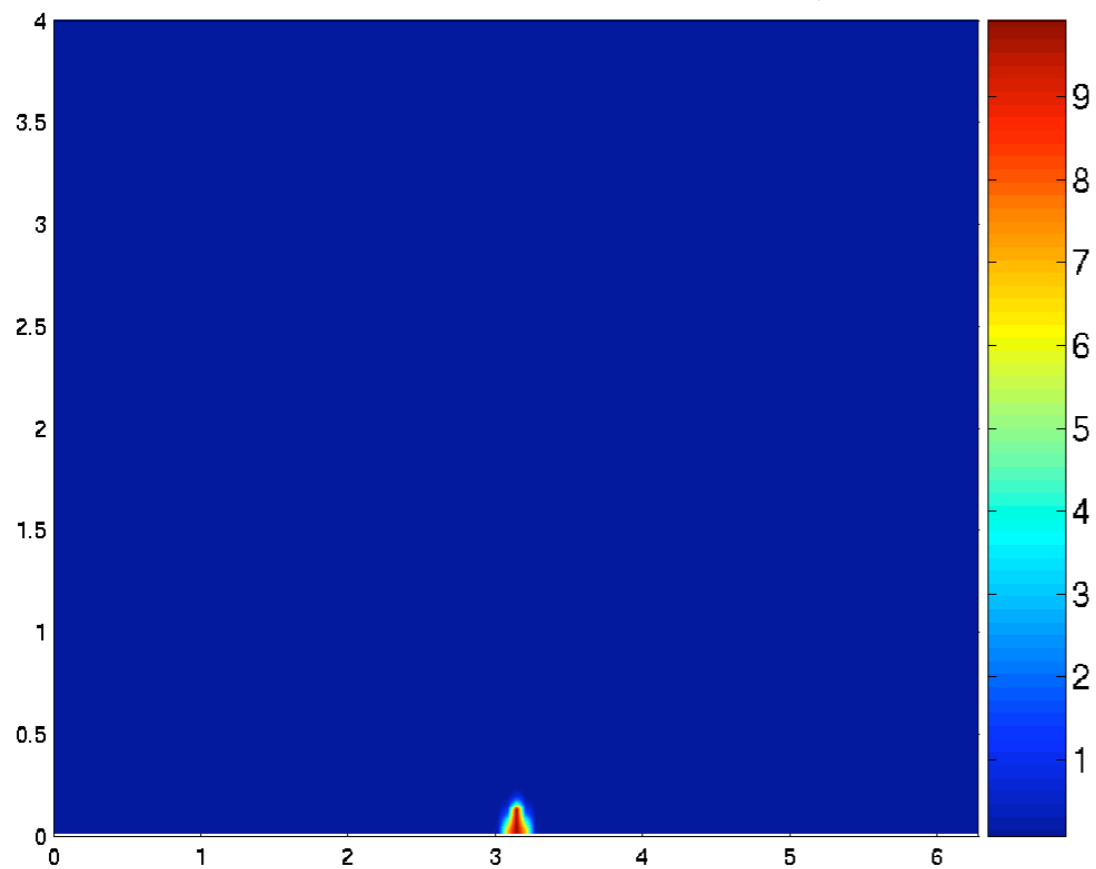
Zero field chain: transverse SF



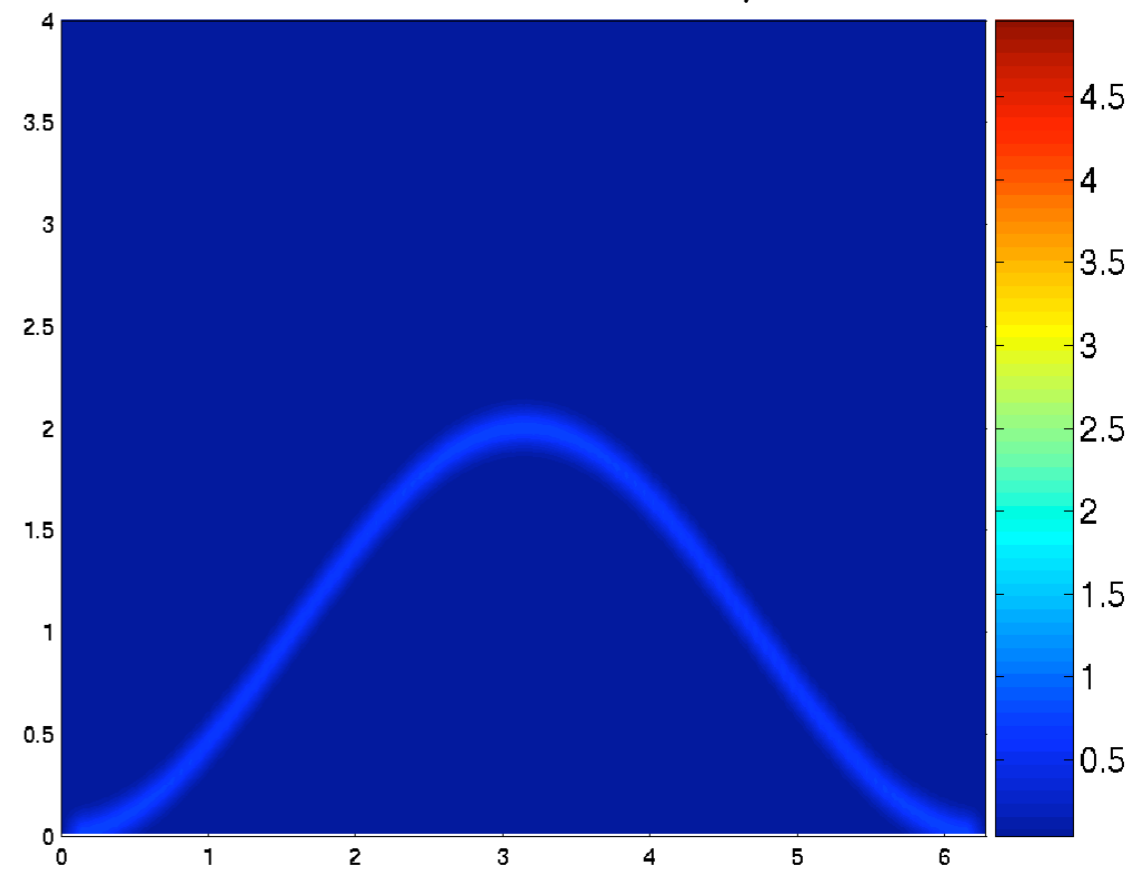
Zero field chain: longitudinal SF



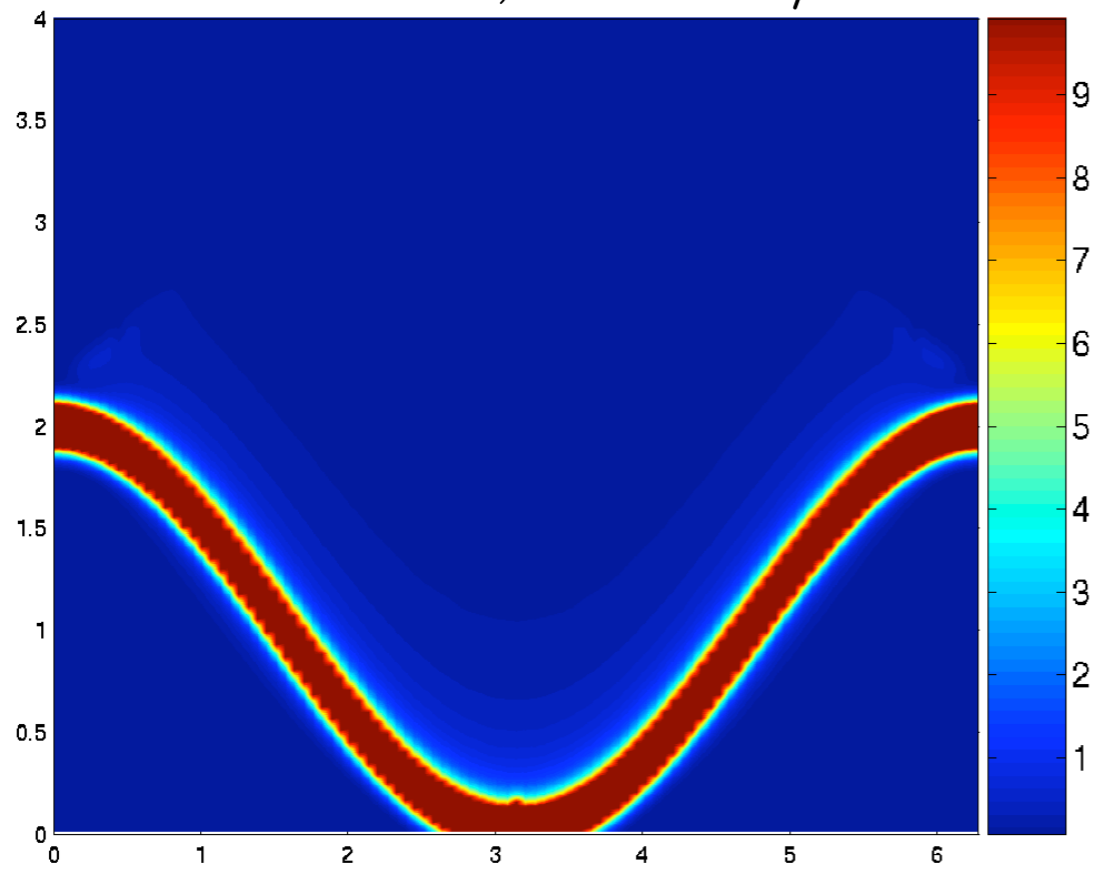
$$S^{-+}, \Delta = 1/4$$



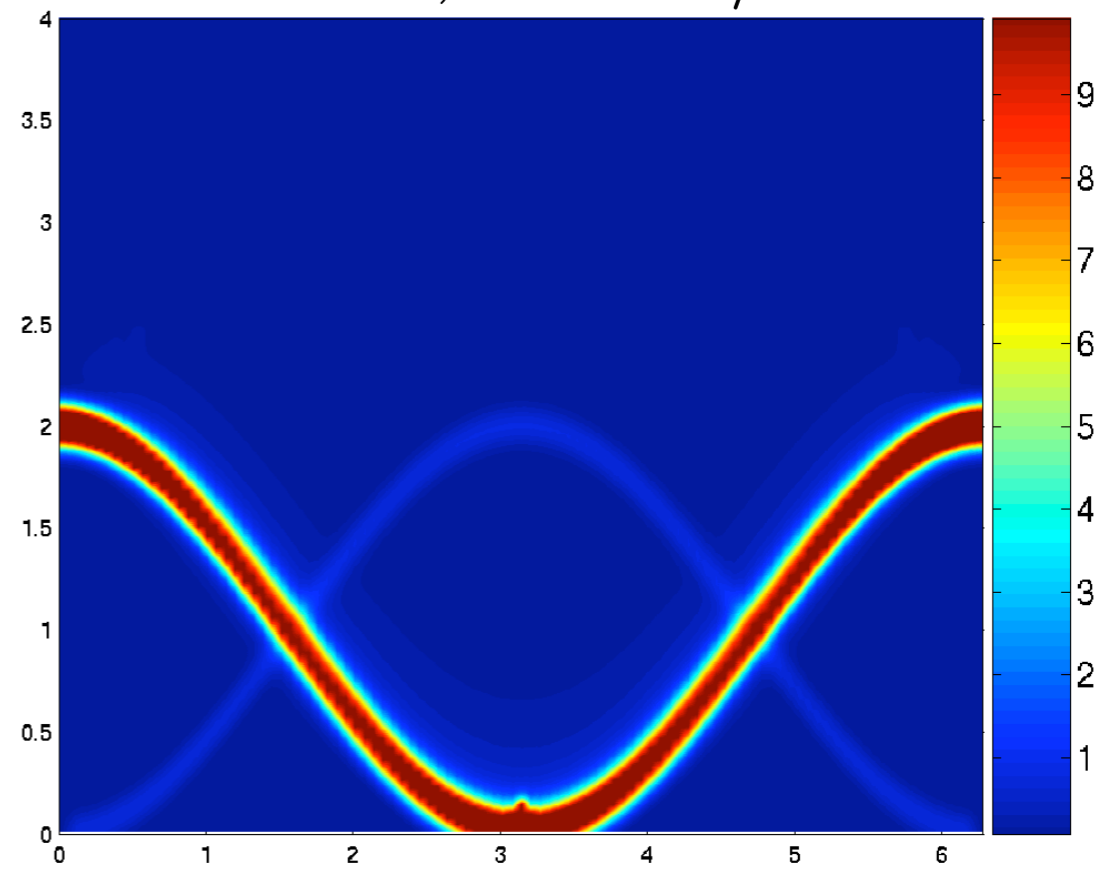
$$S^{zz}, \Delta = 1/4$$

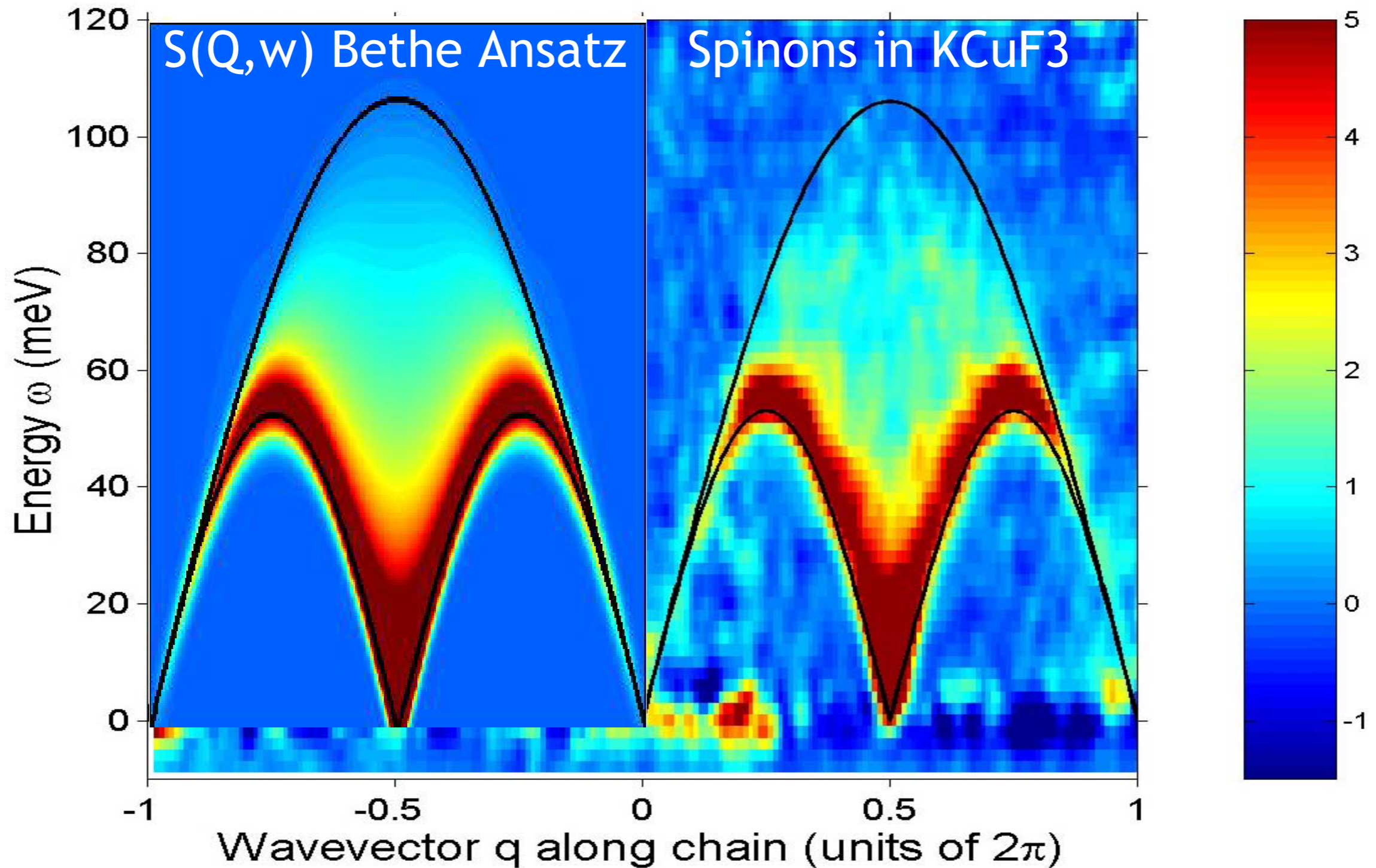
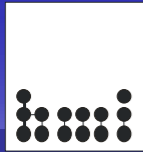


$$S^{+-}, \Delta = 1/4$$



$$S^{tot}, \Delta = 1/4$$





Method 2: analytics (XXX, $h = 0$)

● Infinite model, zero field: possesses $U_q(\hat{sl}_2)$
quantum group symmetry

● Representation theory of q group

➔ eigenstates and form factors (Jimbo, Miwa, ...)

Excitations: built up of even numbers of spinons

Two spinon part of the structure factor:

Bougourzi, Couture, Kacir 1996; Karbach, Müller, B., Fledderjohann, Mütter 1997

➔ **Two spinon states carry 72.89%** of integrated intensity (71.30% of first frequency moment)

Remarkable: measure 0 set in Hilbert space carries majority of correlation weight !

Missing part: **higher spinon numbers**

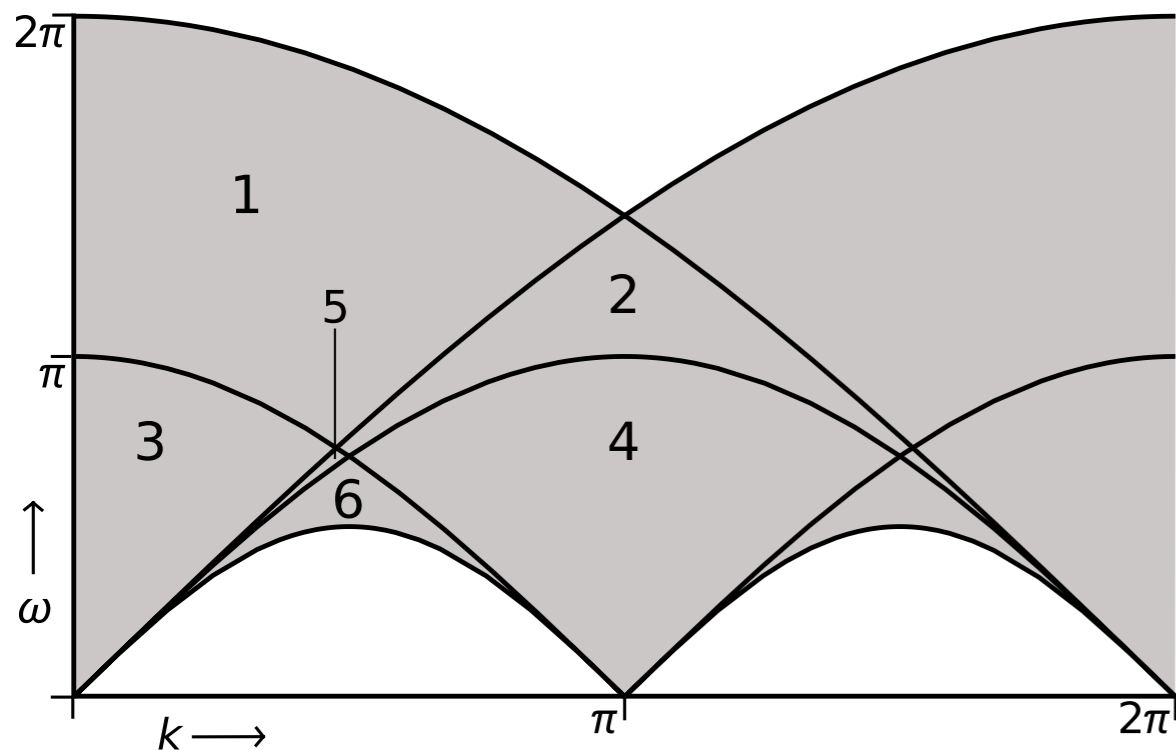
Four spinon part of zero-field structure factor in the thermodynamic limit

(Abada, Bougourzi, Si-Lakhal 1997, revised in JSC & R. Hagemans JSTAT 2006)

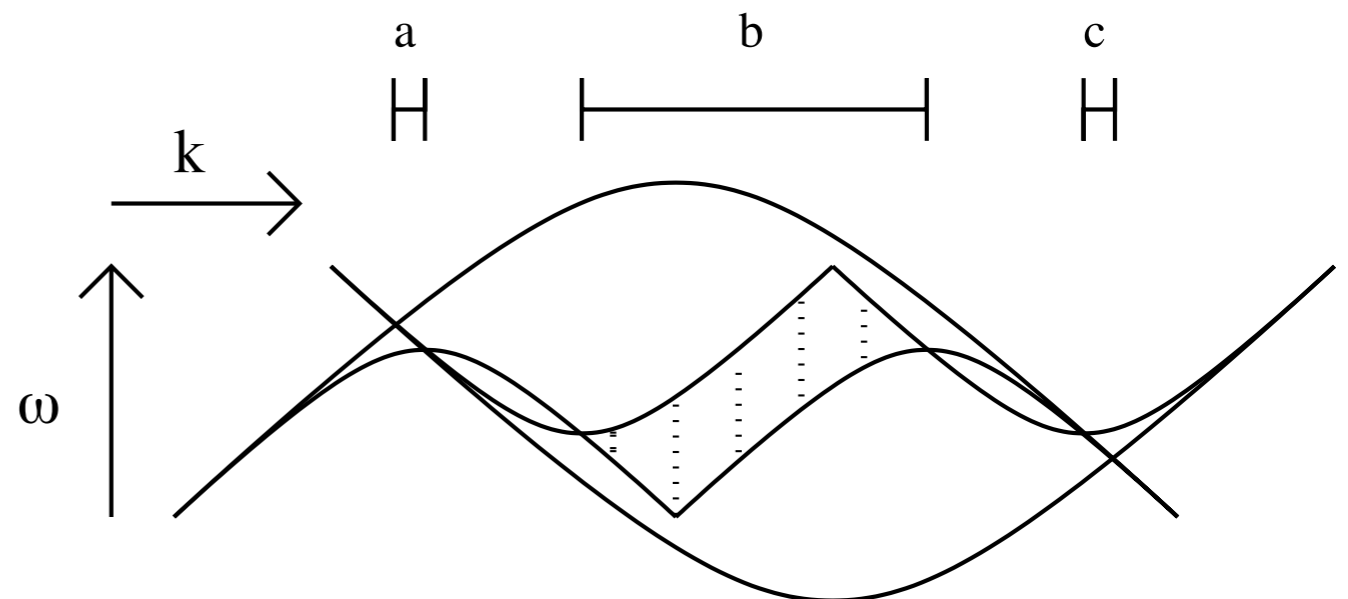
At each point, 4 spinon SF is two-fold integral:

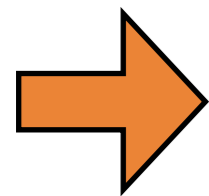
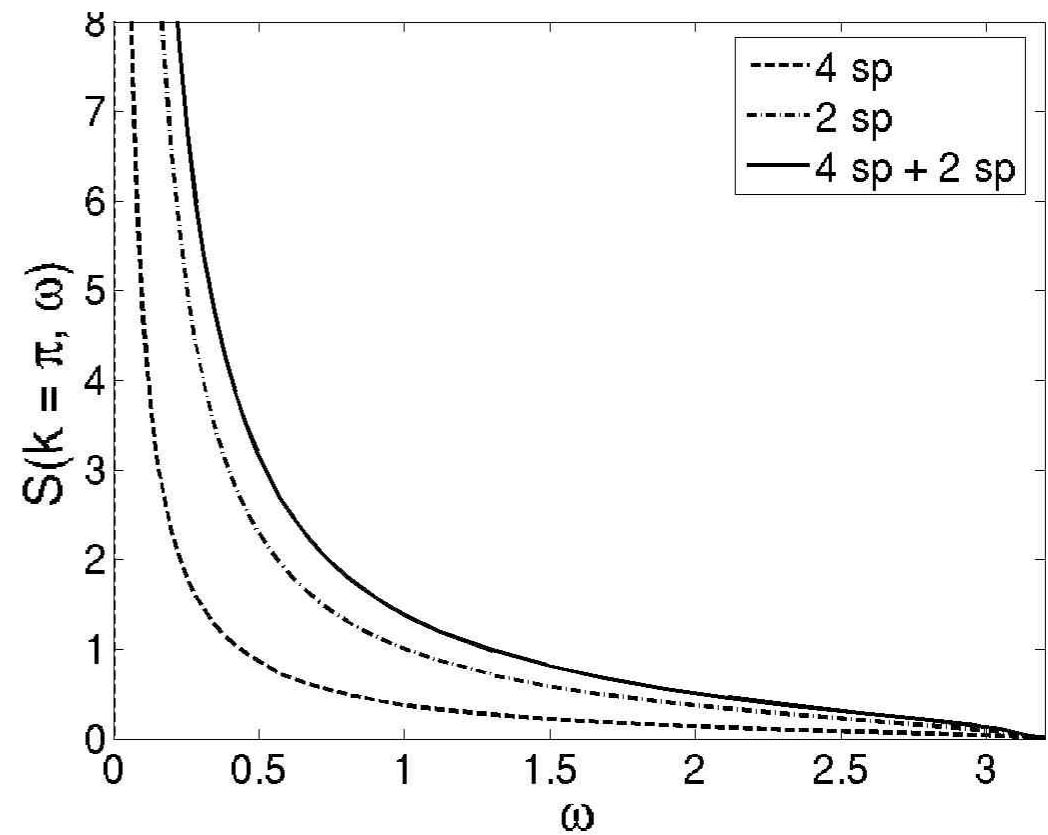
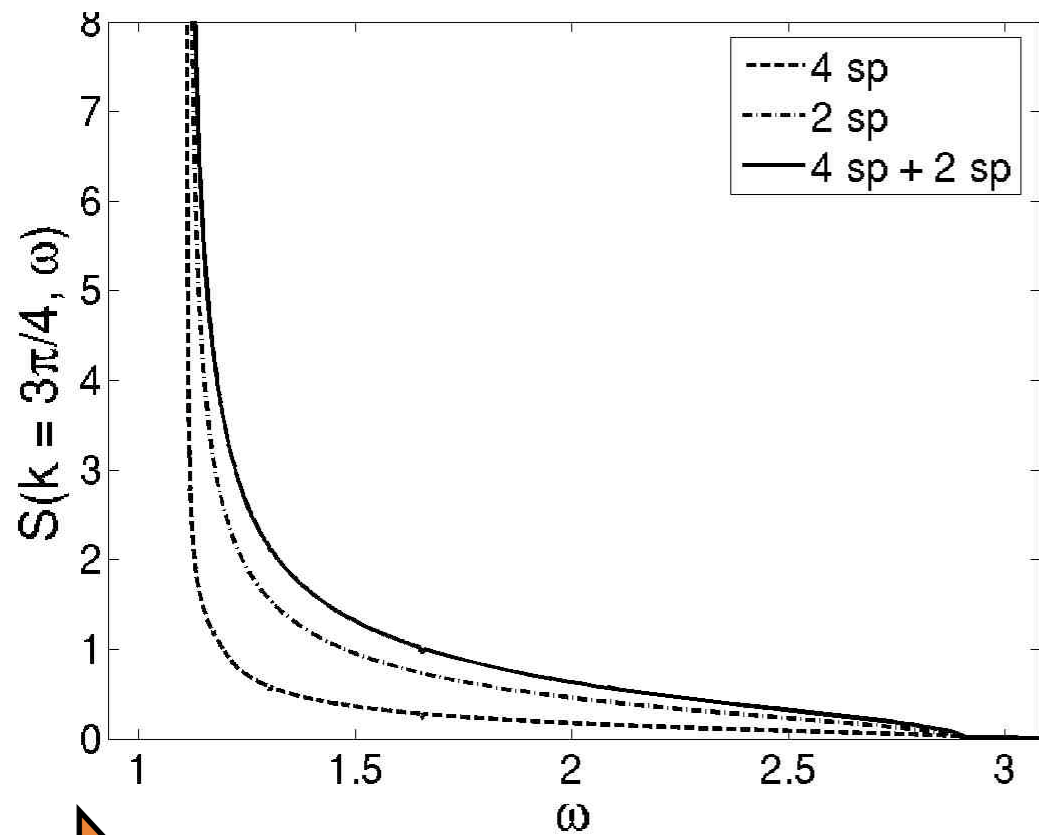
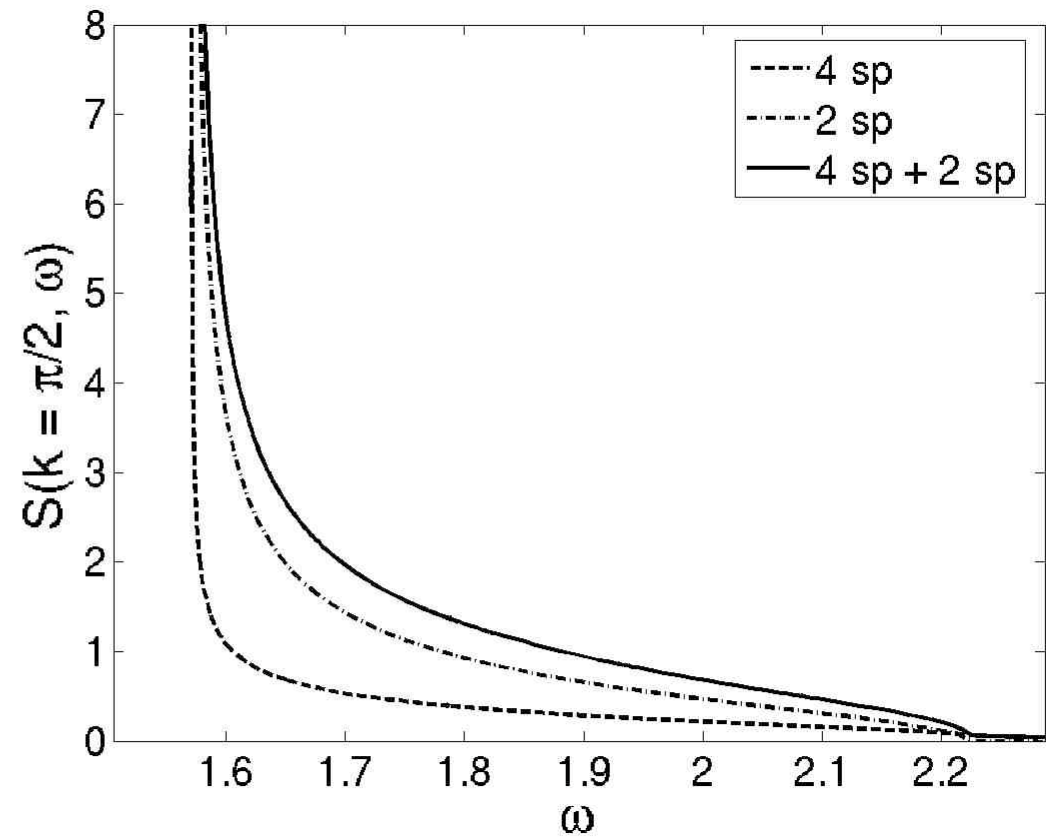
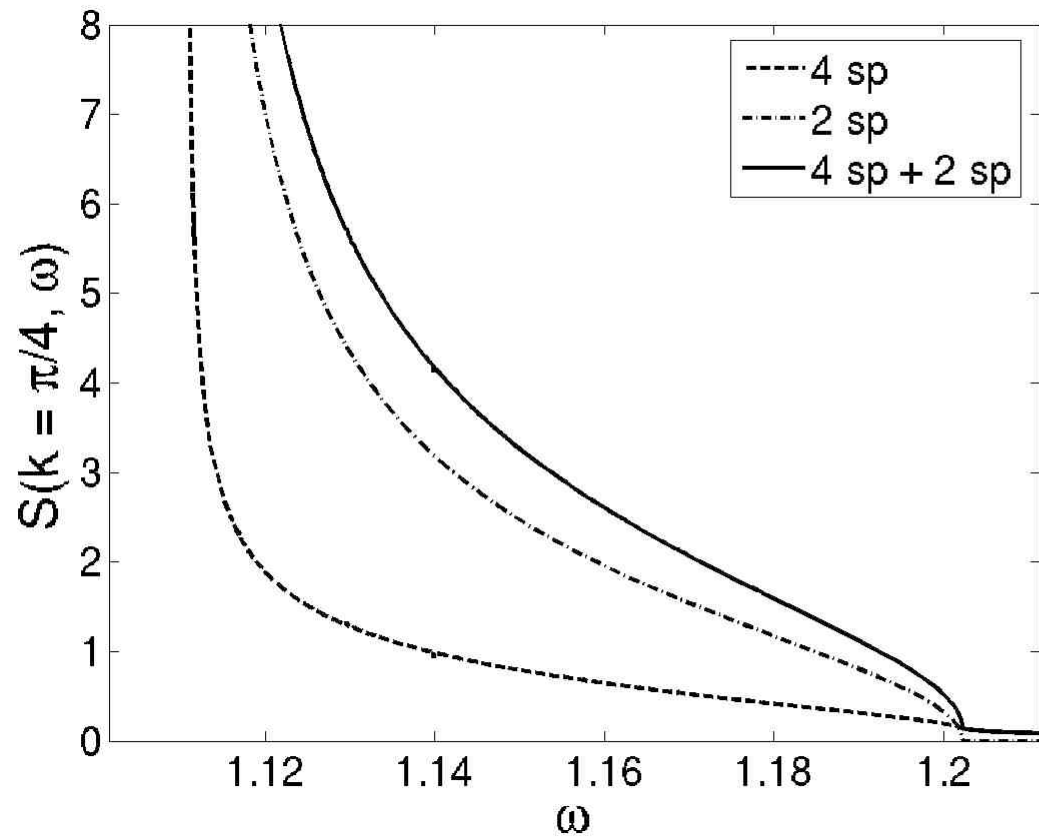
$$S_4(k, \omega) = C_4 \int_{\mathcal{D}_K} dK \int_{\Omega_l(k, \omega, K)}^{\Omega_u(k, \omega, K)} d\Omega \frac{J(k, \omega, K, \Omega)}{\left\{ [\omega_{2,u}^2(K) - \Omega^2] [\omega_{2,u}^2(k - K) - (\omega - \Omega)^2] \right\}^{1/2}}$$

4-spinon continuum:



Integration regions: intersection of two 2-spinon continua





4-spinon states carry about 27% of full intensity
 2 + 4 spinons: approx 98% of correlations !

Analytics (II): gapped XXZ, $h = 0$

(Bougourzi, Karbach, Müller 1998, revisited in JSC, Mossel & Pérez Castillo, JSTAT 2008)

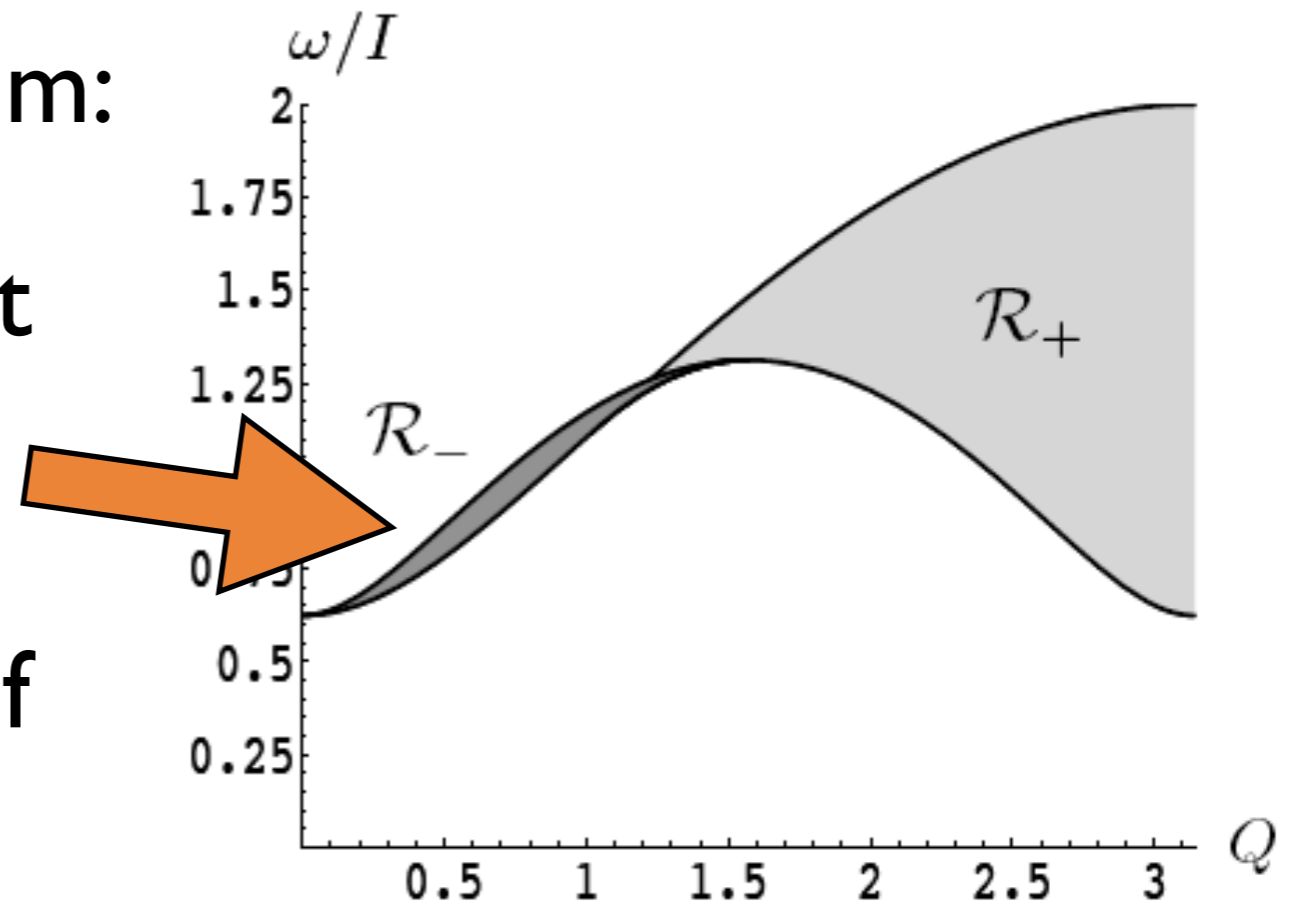
Spinon excitations:

$$e(\beta) = I \operatorname{dn}(\beta), \quad p(\beta) = \operatorname{am}(\beta) + \frac{\pi}{2}, \quad I \equiv \frac{JK}{\pi} \sinh\left(\frac{\pi K'}{K}\right)$$

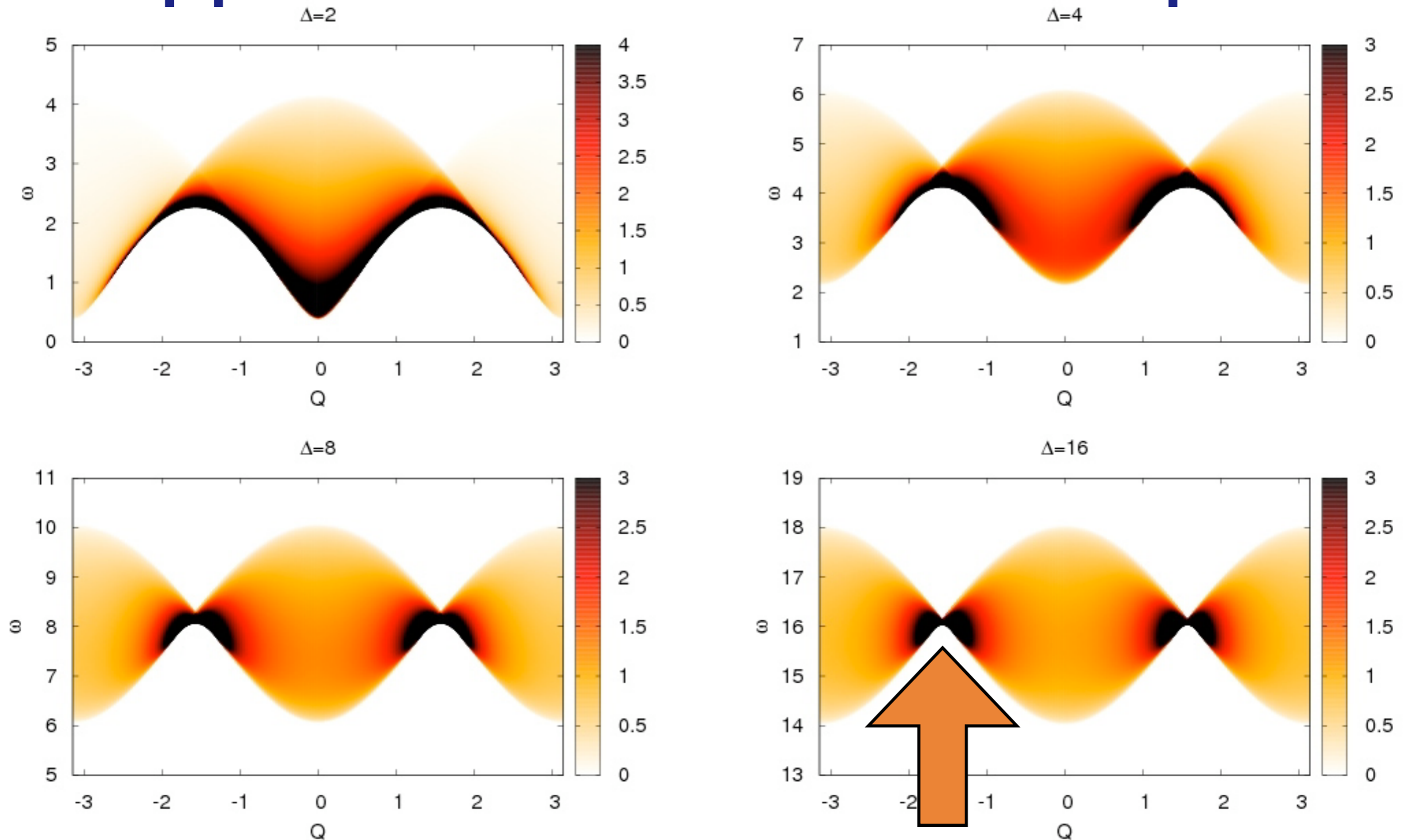
Dispersion relation: $e_1(p) = I \sqrt{1 - k^2 \cos^2(p)}$, $0 \leq p \leq \pi$

Nontrivial 2-spinon continuum:

‘Folding up’ of continuum at small momentum transfer
(curvature of dispersion relation changes sign as fn of momentum)



Gapped XXZ AFM, $h = 0$, 2 spinons



EXACT density matrix correlator in the thermodynamic limit for
A periodic chain recovered in the hydrodynamic limit for
energies below twice the gap

Method 3: Field theory approach (small-q limit) / DMRG / BA for longitudinal structure factor

(Pereira, Sirker, Caux, Hagemans, Maillet, Affleck, White: PRL 2006, JSTAT 2007)

Straight free boson: $\mathcal{H}_{LL} = \frac{v}{2} \left[\Pi^2 + (\partial_x \phi)^2 \right]$ **nonzero field**

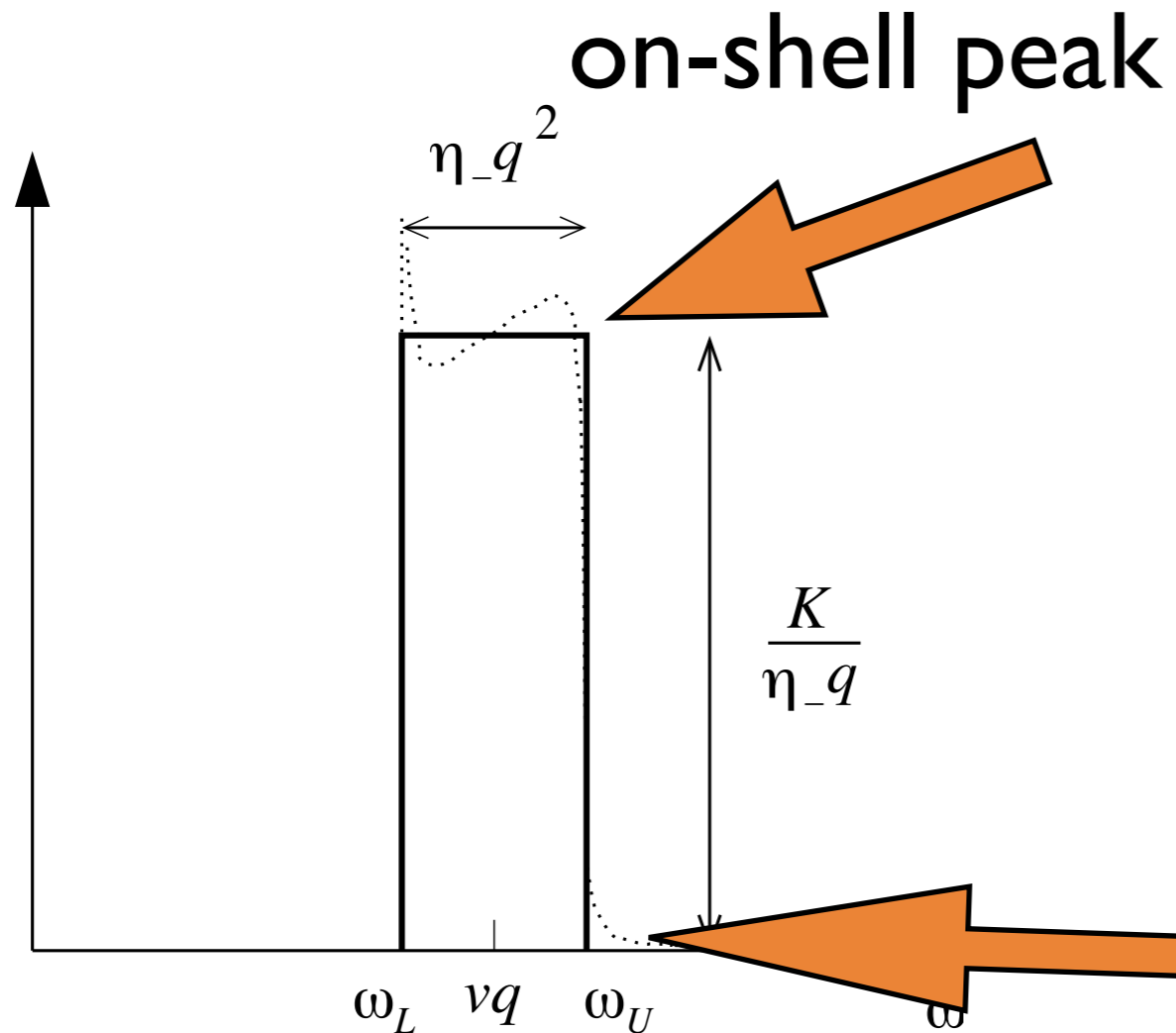
simply gives $S^{zz}(q, \omega) = K |q| \delta(\omega - v |q|)$

$$\delta\mathcal{H}(x) = \left[\eta_- \left[(\partial_x \phi_L)^3 - (\partial_x \phi_R)^3 \right] + \eta_+ \left[(\partial_x \phi_L)^2 \partial_x \phi_R - (\partial_x \phi_R)^2 \partial_x \phi_L \right] \right] \\ + \zeta_- \left[(\partial_x \phi_L)^4 + (\partial_x \phi_R)^4 \right] + \zeta_+ (\partial_x \phi_L)^2 (\partial_x \phi_R)^2 \\ + \zeta_3 \left[\partial_x \phi_L (\partial_x \phi_R)^3 + \partial_x \phi_R (\partial_x \phi_L)^3 \right] + \lambda \cos \left(4\sqrt{\pi K} \phi + 4k_F x \right)$$

non-integrable

zero field

General appearance of the small q lineshape:



Field theory: resummation of series in η_- leads to box-like lineshape of width

$$W^{(2)} = \eta_- = \sqrt{\frac{v}{\pi\chi}} \left[\frac{3}{2} \frac{\partial v}{\partial h} + \frac{1}{2} \frac{v}{\chi} \frac{\partial \chi}{\partial h} \right]$$

(proof from BA: 2p width)

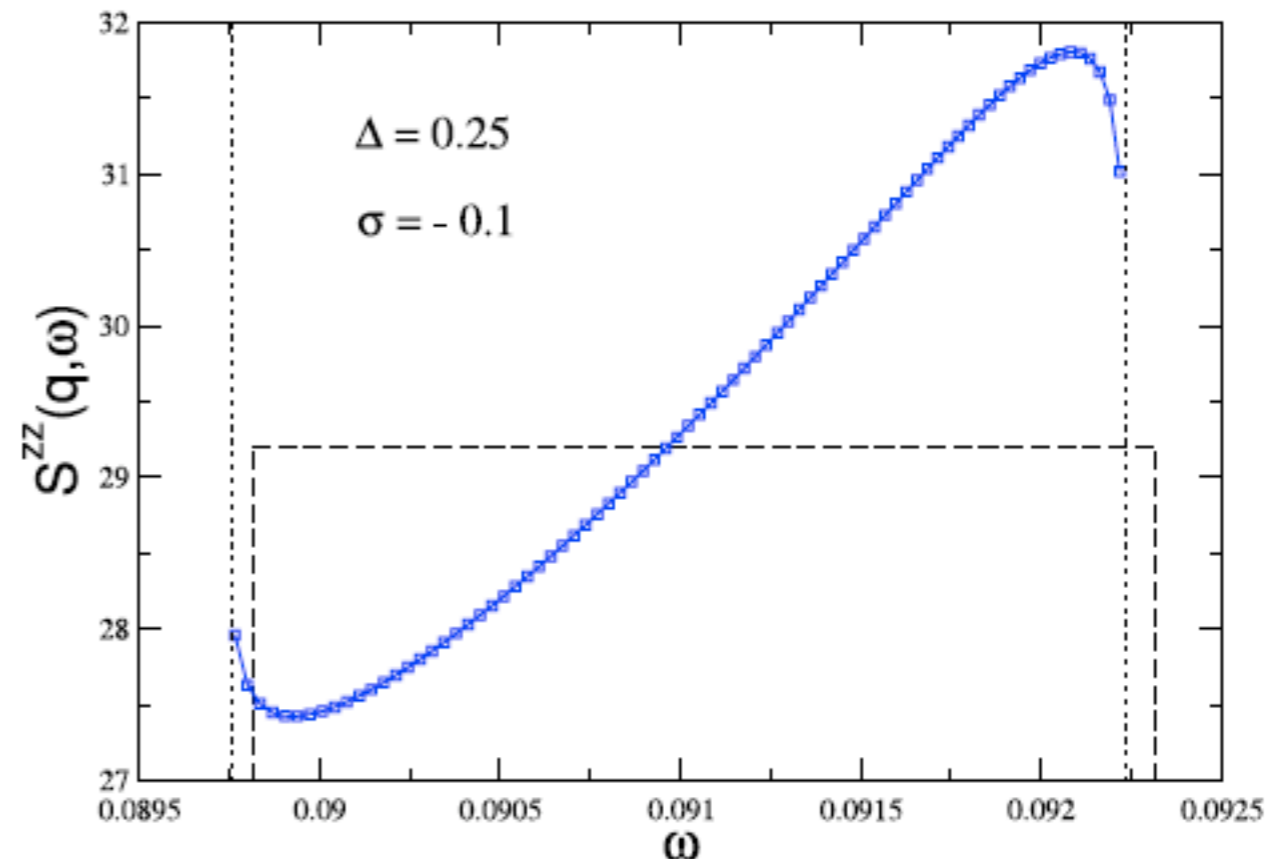
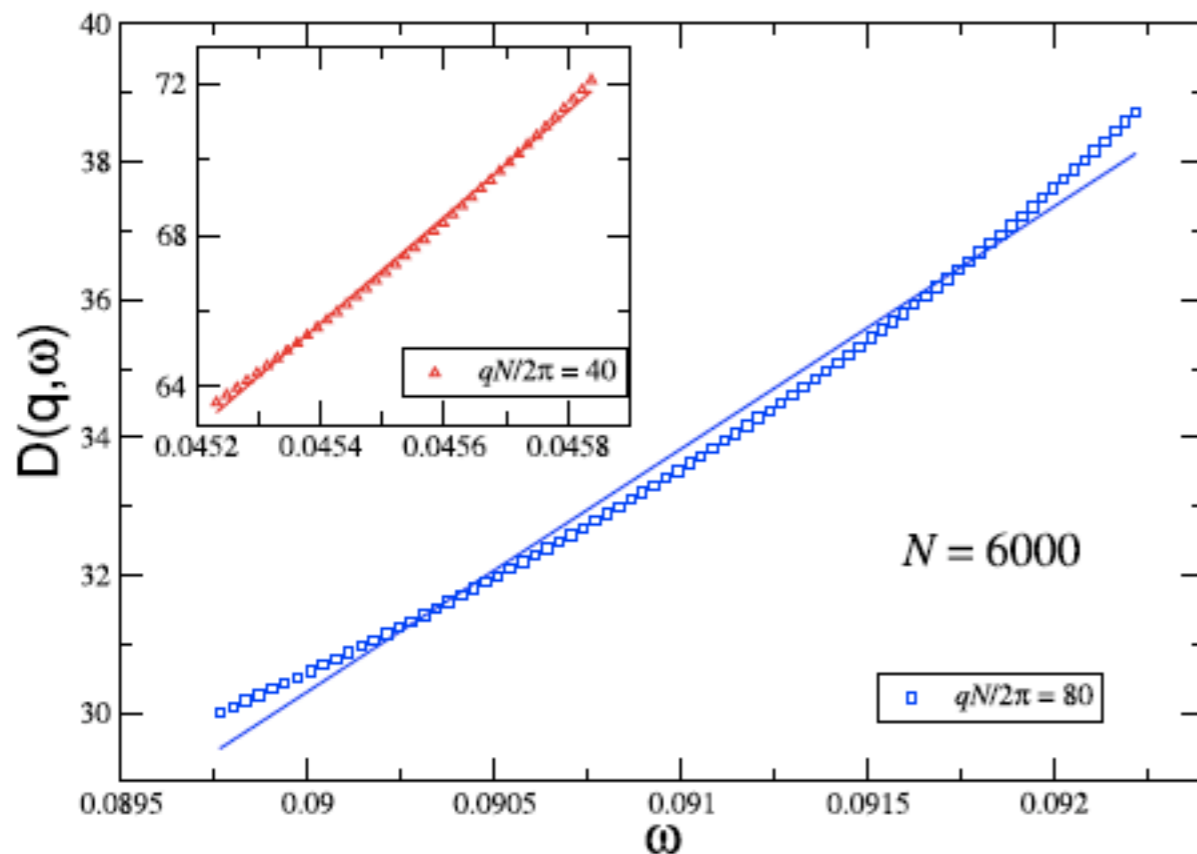
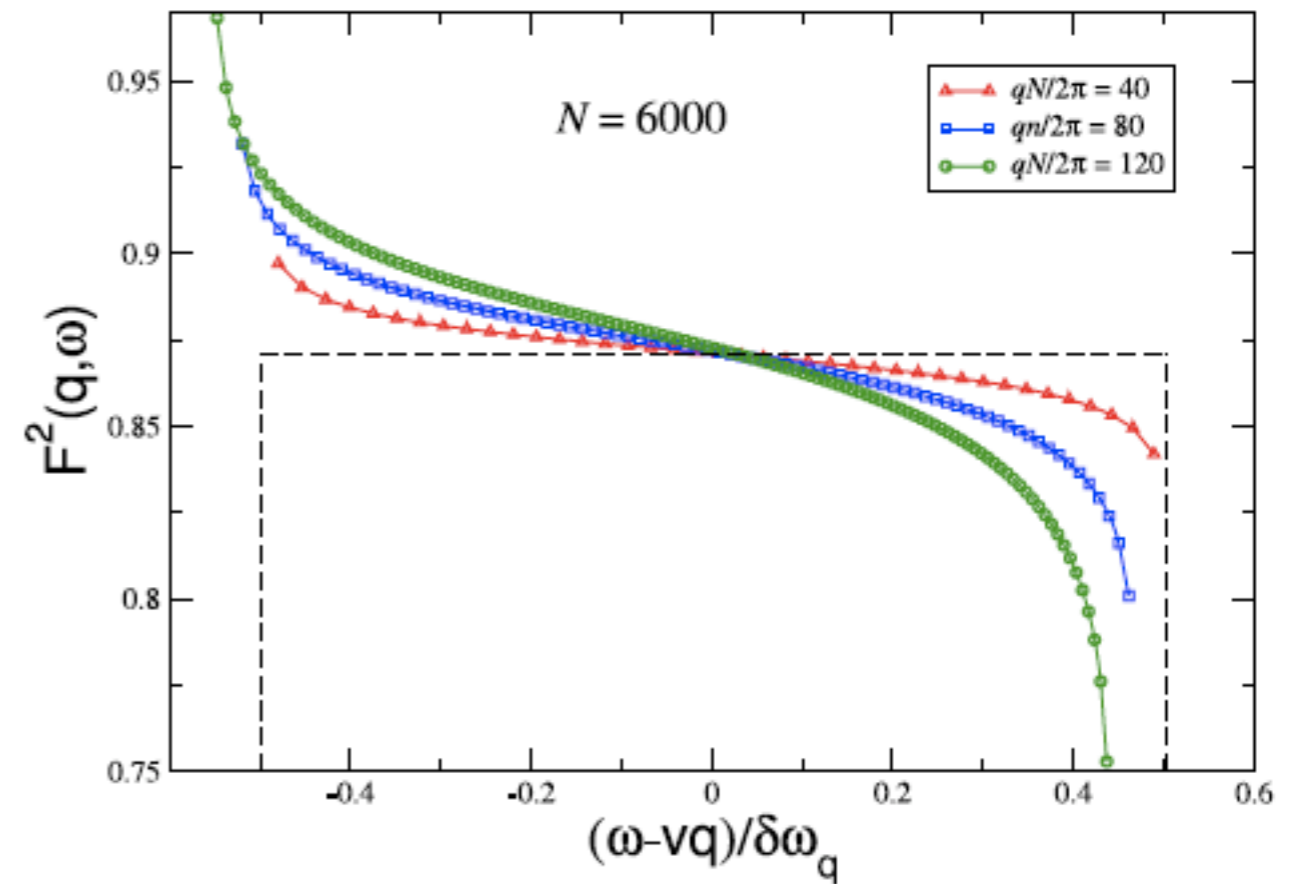
high-energy tail carrying much lower weight

As for interacting fermions (Pustilnik, Glazman & al.)

Finer structure for XXZ: can be investigated using BA

Peak region: $2p$

Combining:
frequency-dependent
form factors
& density of states,
a nontrivial lineshape
is obtained



Advantages/ disadvantages of the 3 approaches presented here

- Quantum group

Exact result

Zero field only

2 & 4 sp only

Finite T

- ABACUS

Any integrable chain

Any field

Accurate at any energy

Finite N

Finite T

- FIELD THEORY

Not only for integrable cases

Small window of q

Fine structure in w : tough

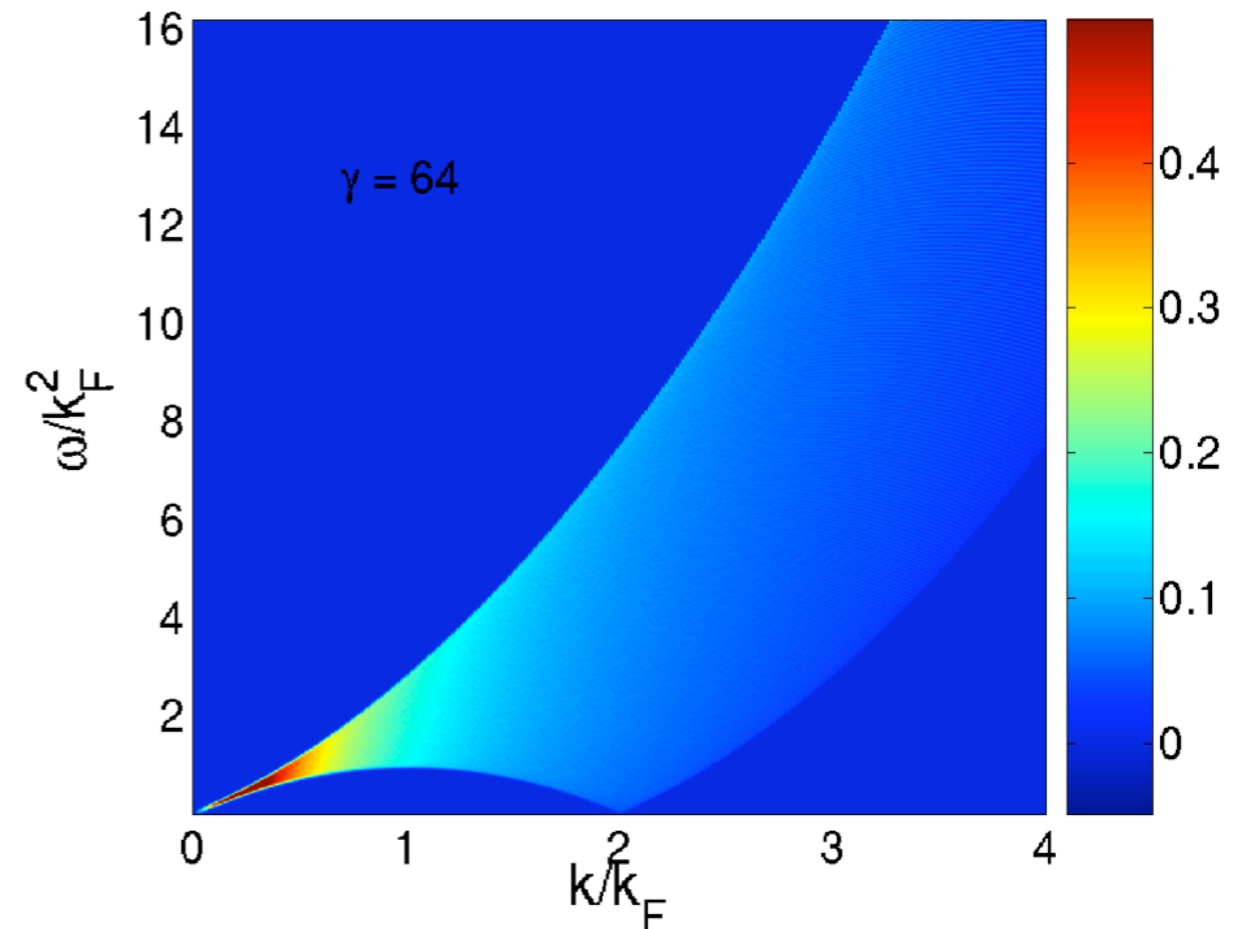
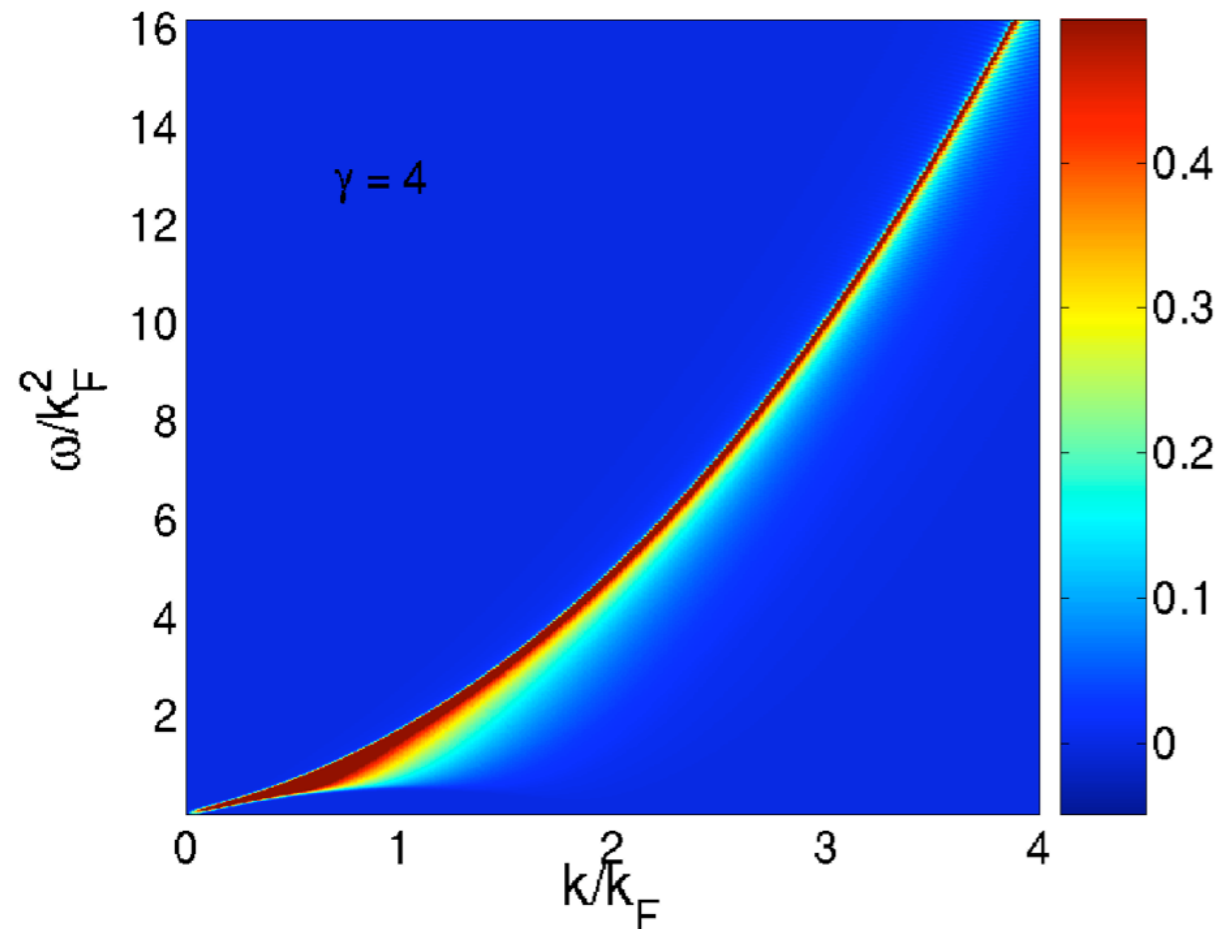
Finite T

Lieb-Liniger Bose gas

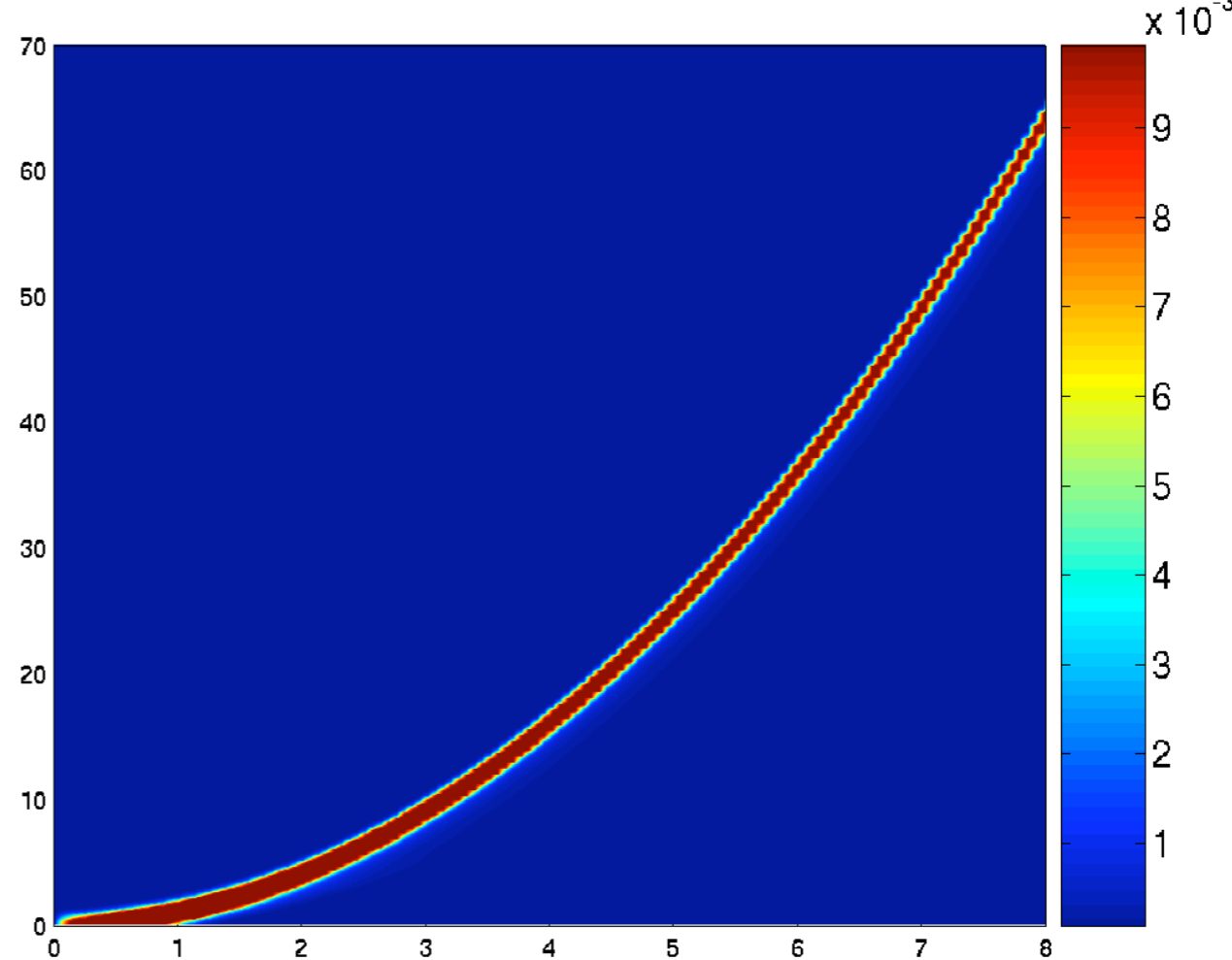
Density-density (dynamical SF)

(J-S C & P Calabrese, PRA 2006)

$$S(k, \omega) = \frac{2\pi}{L} \sum_{\alpha} |\langle 0 | \rho_k | \alpha \rangle|^2 \delta(\omega - E_{\alpha} + E_0)$$



Correspondence with excitations



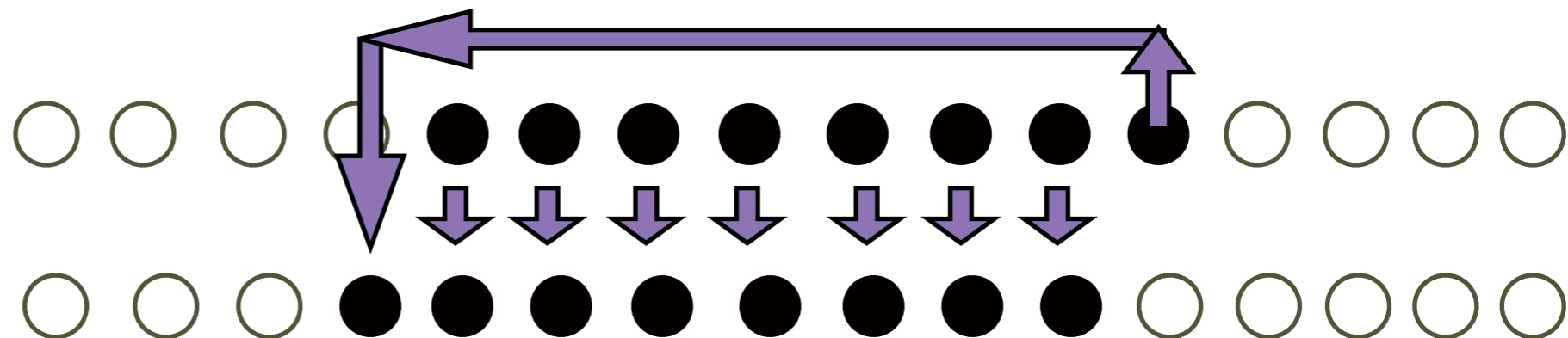
Particle-like



Hole-like



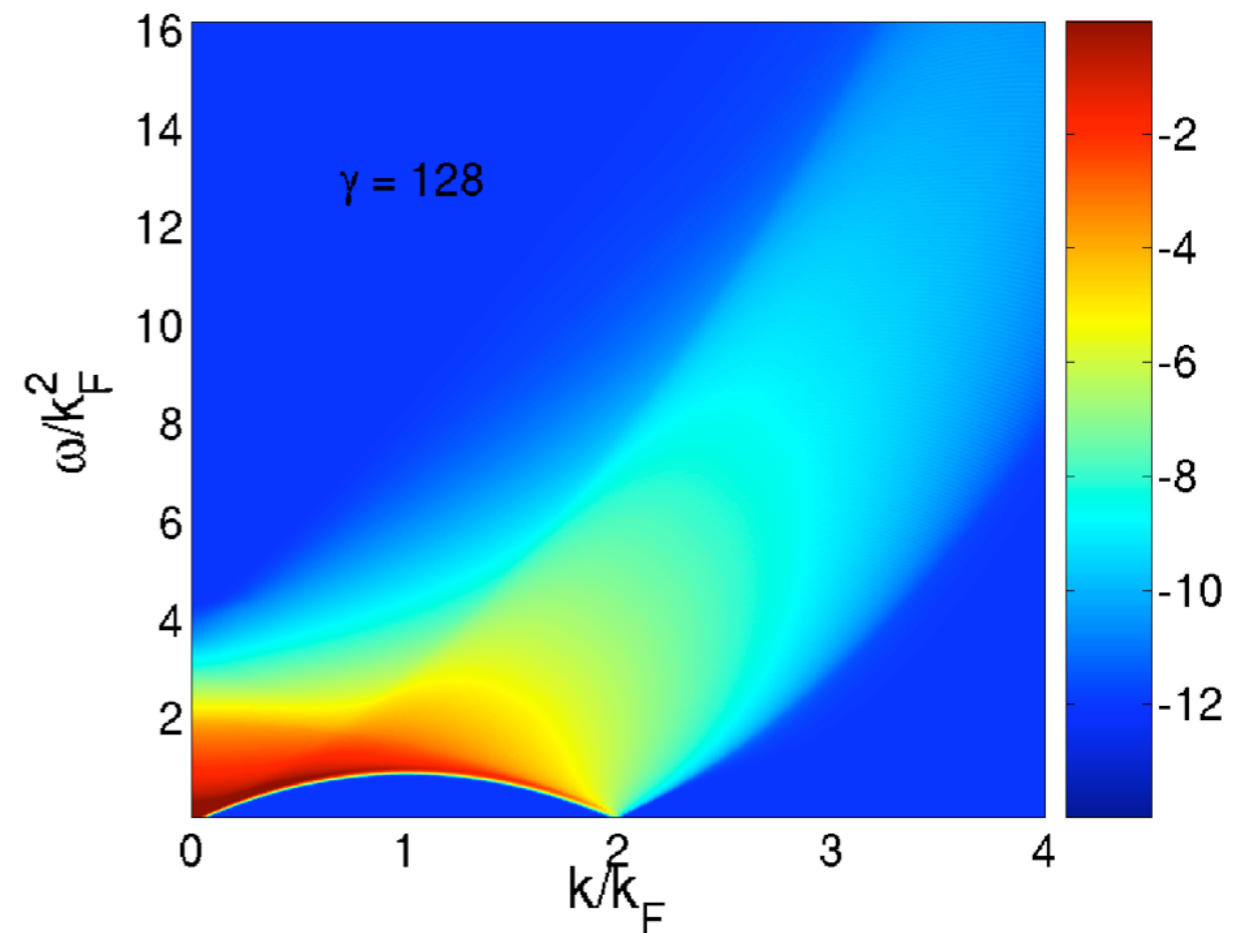
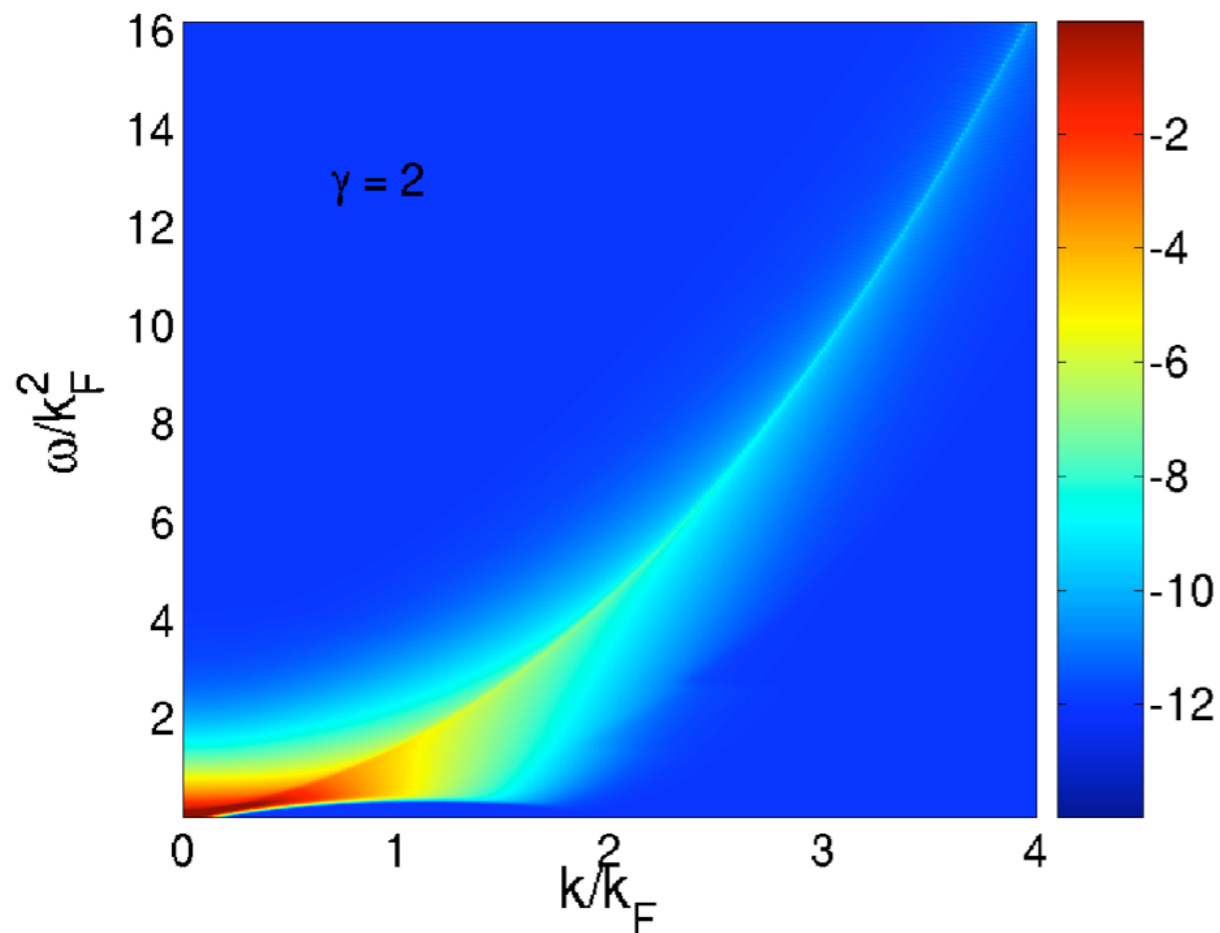
Umklapp



One-particle dynamical function


$$G_2(x, t) = \langle \Psi^\dagger(x, t) \Psi(0, 0) \rangle_N$$

(J-S C, P Calabrese & N Slavnov, JSTAT 2007)

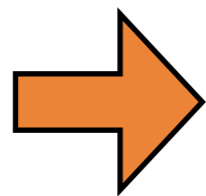


The attractive Lieb-Liniger model: analytical solution

$$H = -\frac{\hbar^2}{2m} \sum_{j=1}^N \frac{\partial^2}{\partial x_j^2} - 2\bar{c} \sum_{\langle i,j \rangle} \delta(x_i - x_j)$$

!!! 

Bethe eqns: $e^{i\lambda_a L} = \prod_{a \neq b} \frac{\lambda_a - \lambda_b - i\bar{c}}{\lambda_a - \lambda_b + i\bar{c}}, \quad a = 1, \dots, N$



bound state solutions: strings

$$\lambda_{\alpha}^{j,a} = \lambda_{\alpha}^j + \frac{i\bar{c}}{2} (j + 1 - 2a) + i\delta_{\alpha}^{j,a}.$$

(J. B. McGuire, 1964; F. Calogero & A. DeGasperis, 1975; Y. Castin & C. Herzog, 2001)

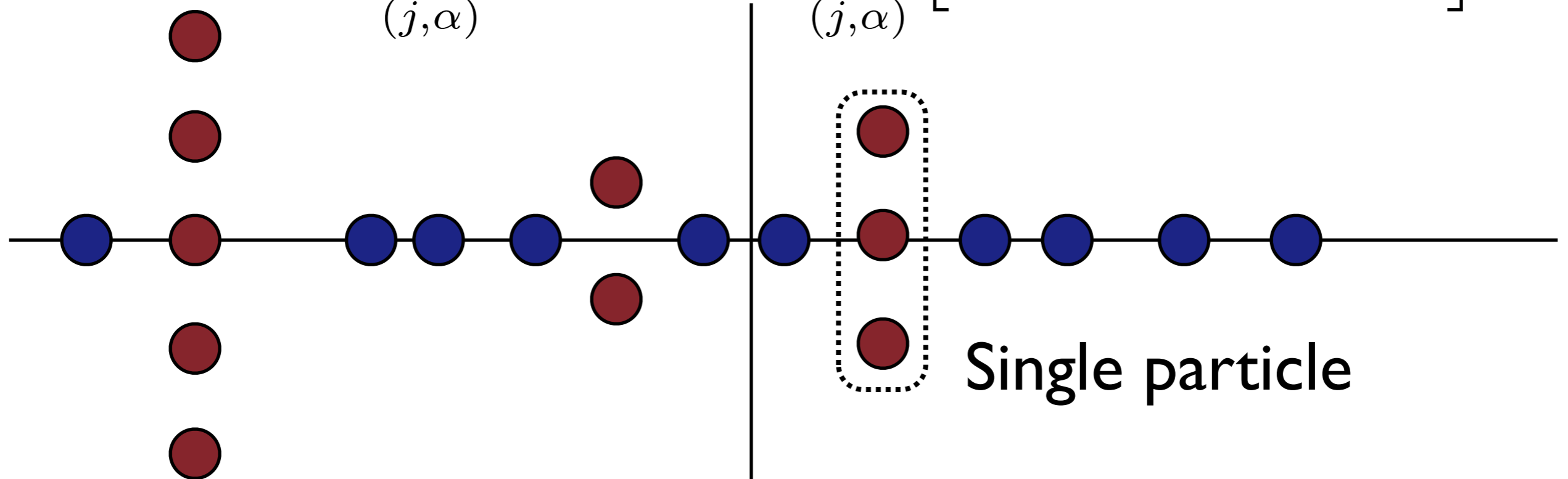
The attractive Lieb-Liniger model

(J. B. McGuire, 1964; F. Calogero & A. DeGasperis, 1975; Y. Castin & C. Herzog, 2001)

Ground state: single N string with zero momentum

Excitations: 'partition' N atoms into bound states

$$P = \sum_{(j,\alpha)} j \lambda_{\alpha}^j, \quad E = \sum_{(j,\alpha)} \left[j \lambda_{\alpha}^{j^2} - \frac{\bar{c}^2}{12} j(j^2 - 1) \right]$$



Bethe equations for GS solved to exponential accuracy:
determinants can be calculated explicitly !!

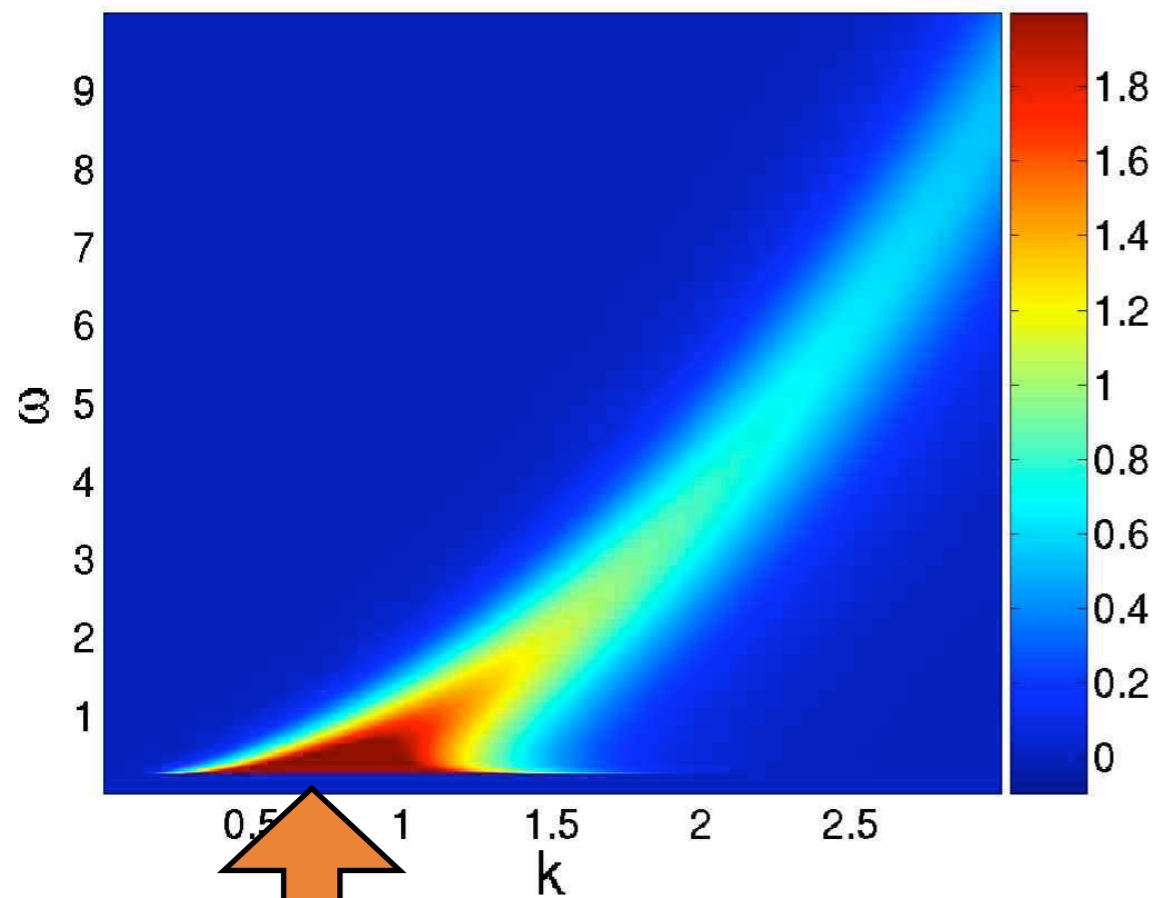
(J.-S.C & P. Calabrese PRL 2007; JSTAT 2007)

Analytical solution for CFs

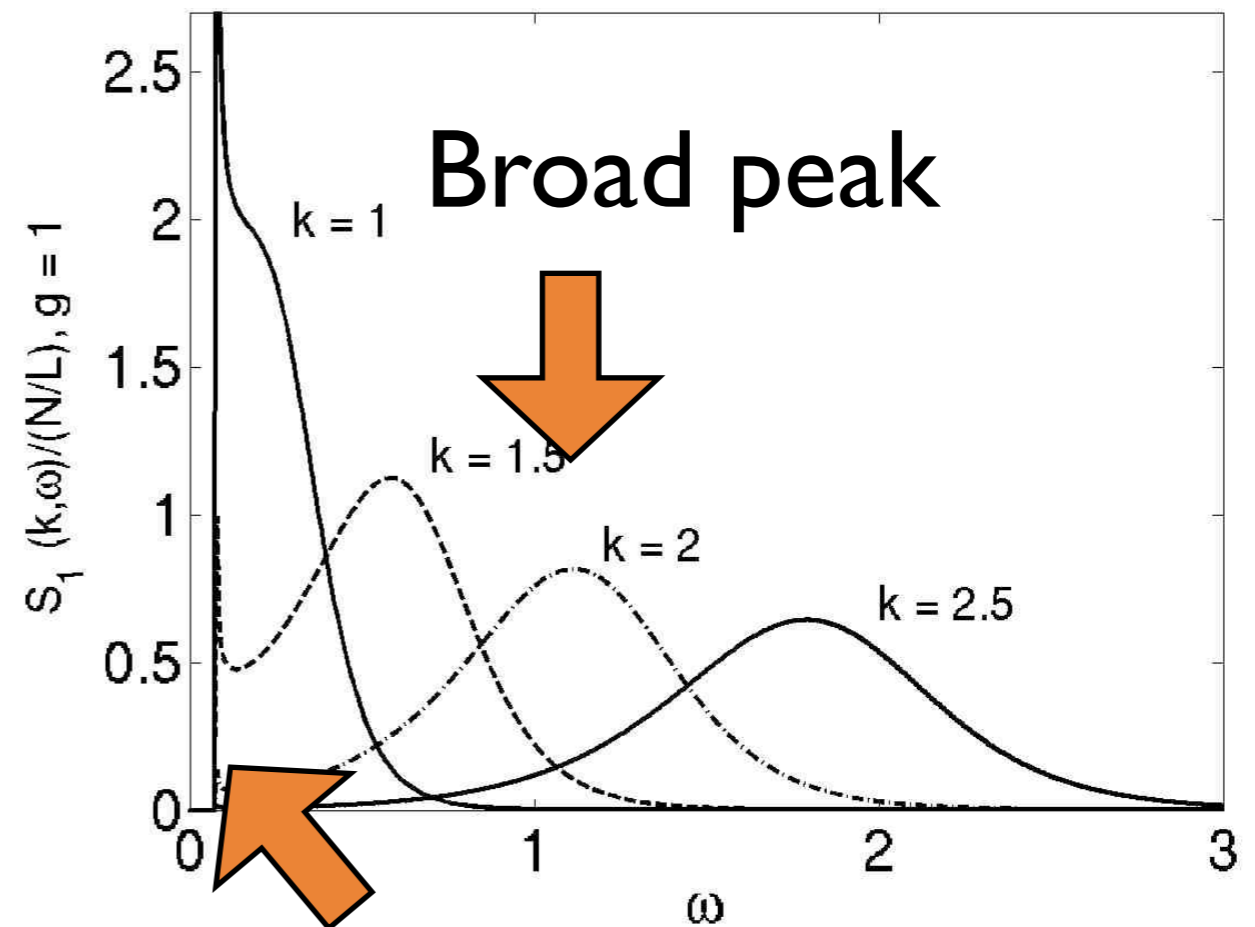
(J.-S.C & P. Calabrese PRL 2007; JSTAT 2007)

Single-particle coherent part + two-particle continuum

$$S_1^p(k, \omega)/(N/L), g = 1$$



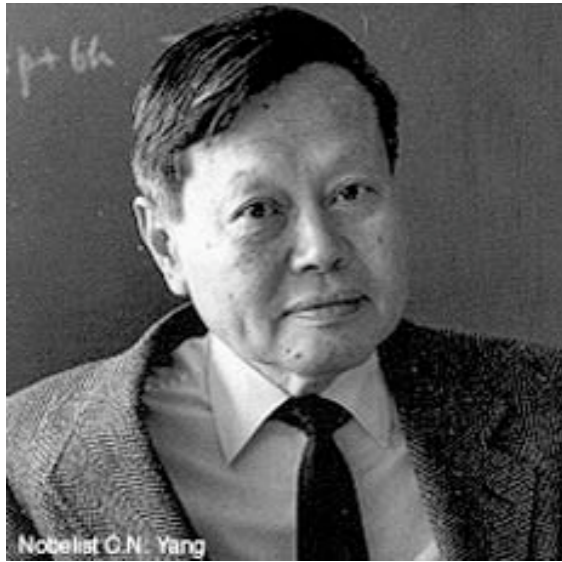
Finite threshold



Square-root singularity

Single-particle part: leads to Mössbauer-like effect
(gas reacts like a single massive particle)

The 2-component Bose gas (special case of Yang permutation model)



$$H = - \sum_{a=1}^{N_C} \sum_{i=1}^{N_a} \frac{\partial^2}{\partial x_{a,i}^2} + 2c \sum_{(a,i) < (b,j)} \delta(x_{a,i} - x_{b,j})$$

● Dynamics: hum... nested BA

● Equilibrium thermodynamics: OK !

$$\epsilon(\lambda) = \lambda^2 - \mu - \Omega - a_2 * T \ln(1 + e^{-\epsilon(\lambda)/T}) - \sum_{n=1}^{\infty} a_n * T \ln(1 + e^{-\epsilon_n(\lambda)/T})$$

$$\epsilon_1(\lambda) = f * T \ln(1 + e^{-\epsilon(\lambda)/T}) + f * T \ln(1 + e^{\epsilon_2(\lambda)/T})$$

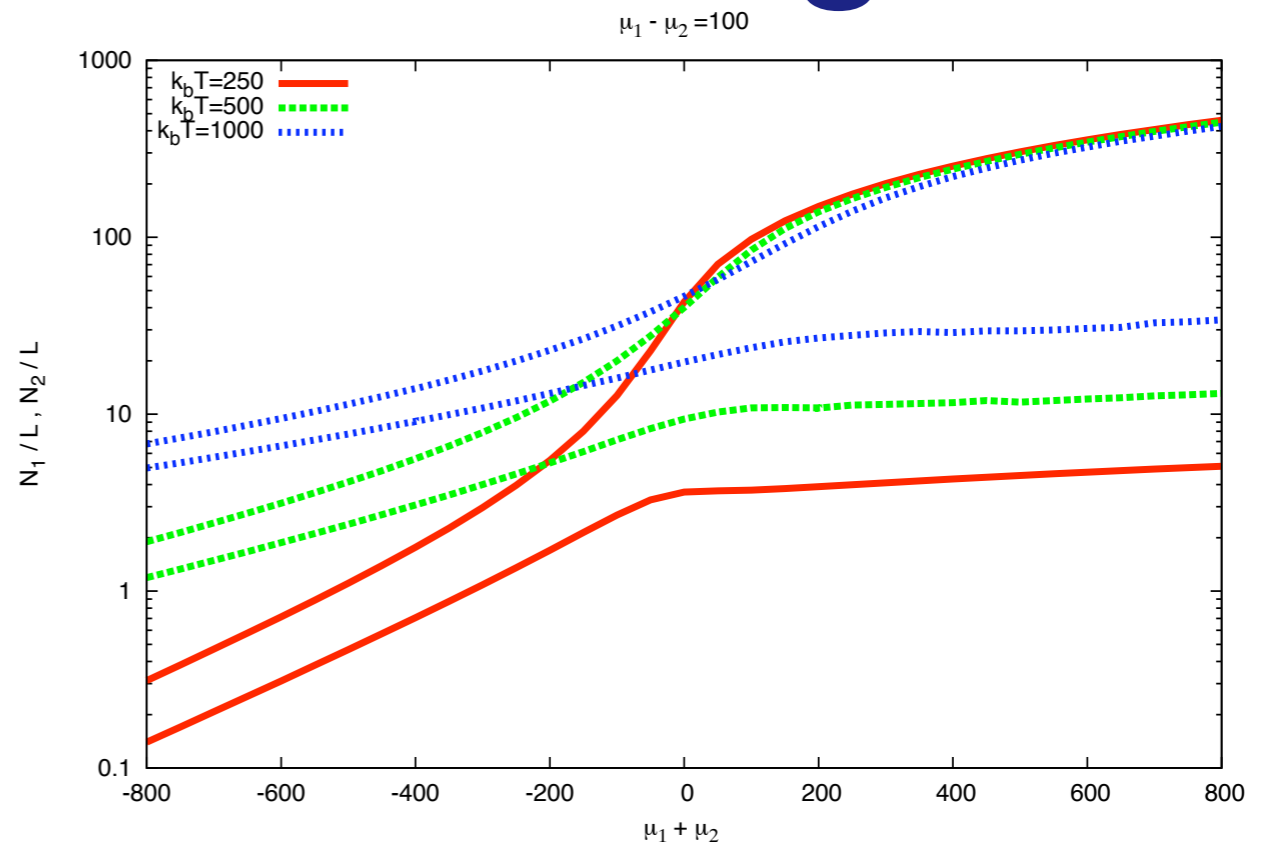
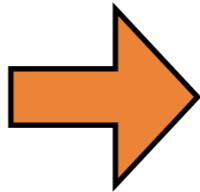
$$\epsilon_n(\lambda) = f * T \ln(1 + e^{\epsilon_{n-1}(\lambda)/T}) + f * T \ln(1 + e^{\epsilon_{n+1}(\lambda)/T})$$

$$\lim_{n \rightarrow \infty} \frac{\epsilon_n(\lambda)}{n} = 2\Omega$$

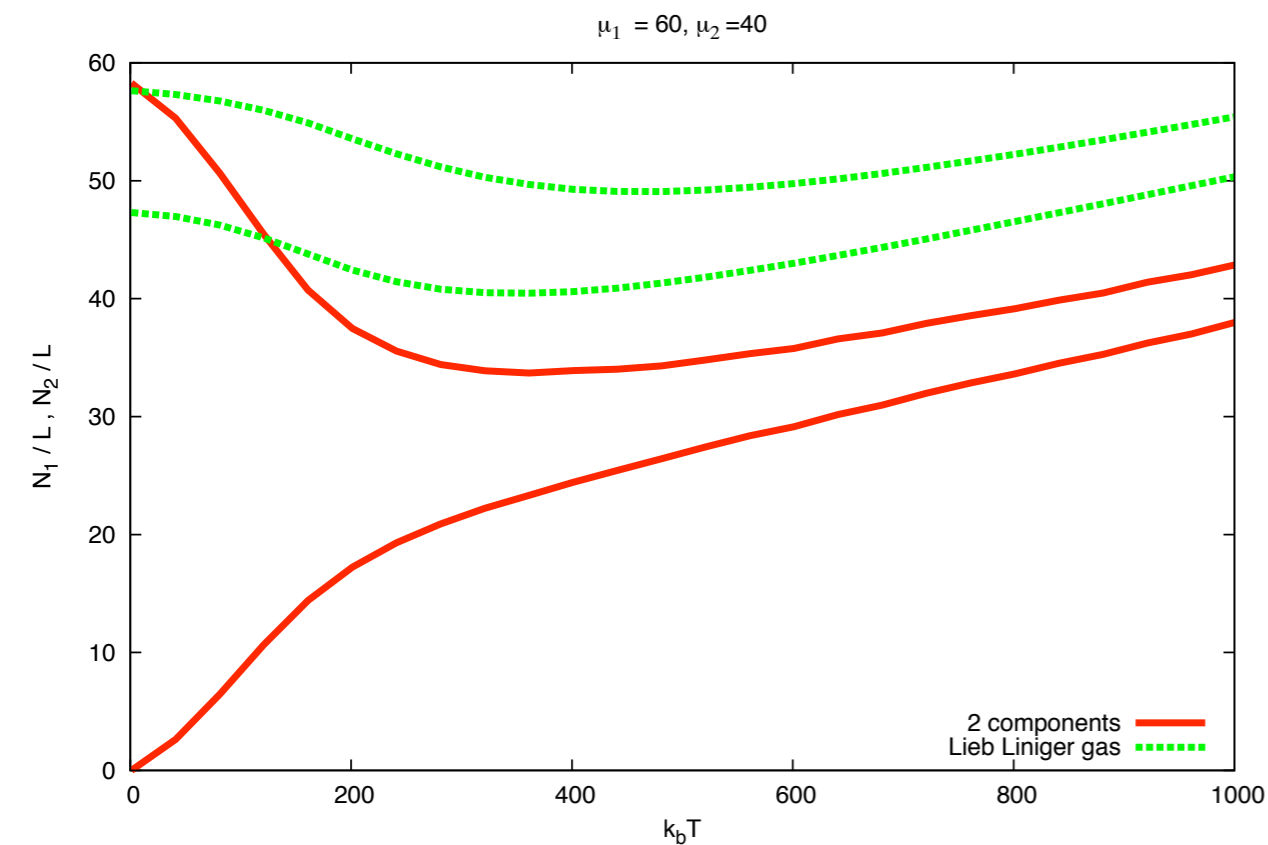
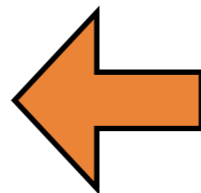
➔ Numerical solution

The 2-component Bose gas

Populations as a function of total chemical potential



Populations as a function of temperature: contrast with single component case



Waiting for experimental data...

The Richardson model

$$H_{BCS} = \sum_{\substack{\alpha=1 \\ \sigma=+,-}}^N \frac{\varepsilon_\alpha}{2} c_{\alpha\sigma}^\dagger c_{\alpha\sigma} - g \sum_{\alpha,\beta=1}^N c_{\alpha+}^\dagger c_{\alpha-}^\dagger c_{\beta-} c_{\beta+}$$

(R.W. Richardson, 1963; R.W. Richardson & N. Sherman, 1964)

“Reduced BCS”: ground state is BCS in th. limit, grand-canonical.
Exactly solvable in canonical ensemble.

Eigenstates are Bethe, Rapidities: (Bethe) Richardson equations

$$|\{w_j\}\rangle = \prod_{k=1}^{N_r} \mathcal{B}(w_k) |0\rangle$$

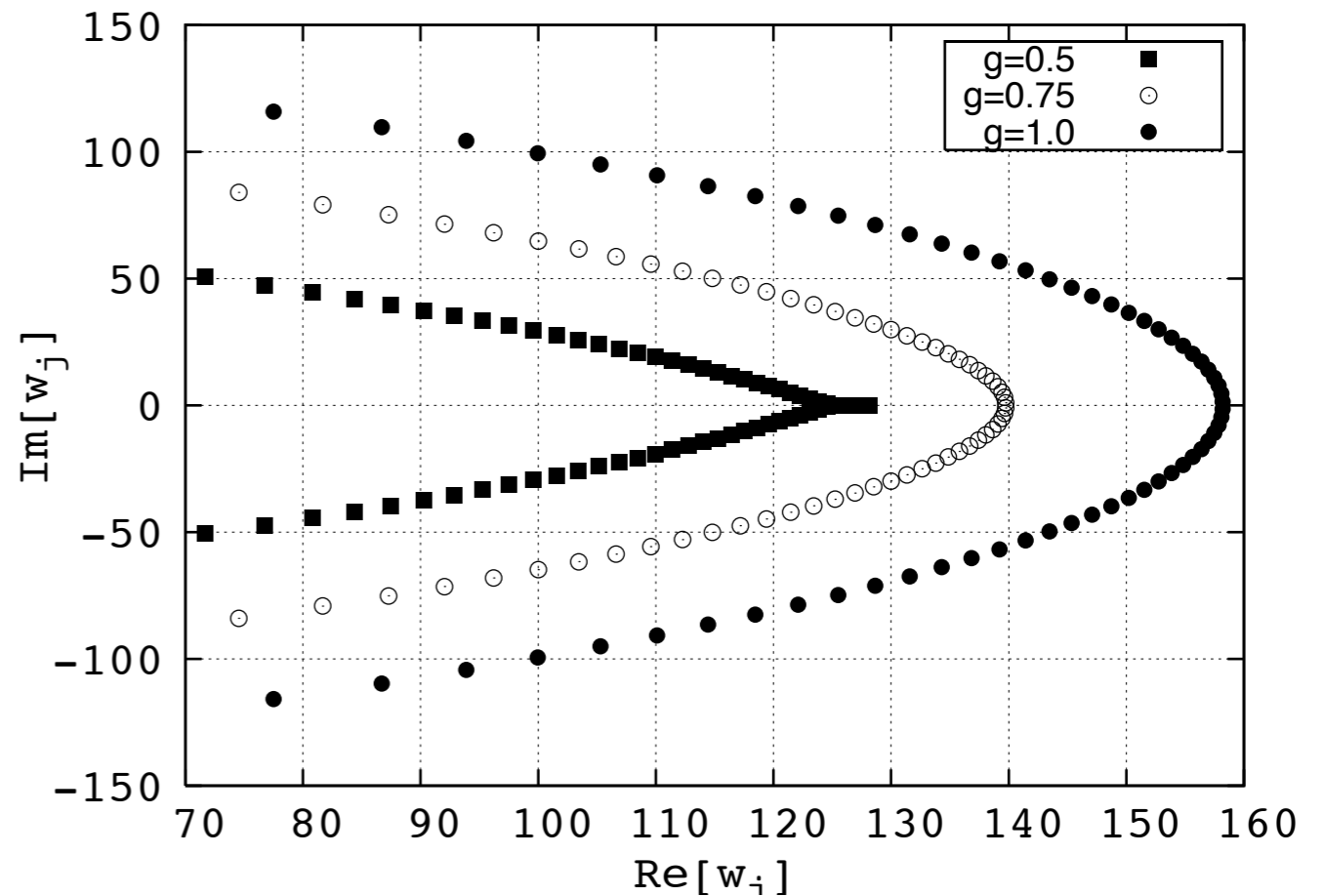
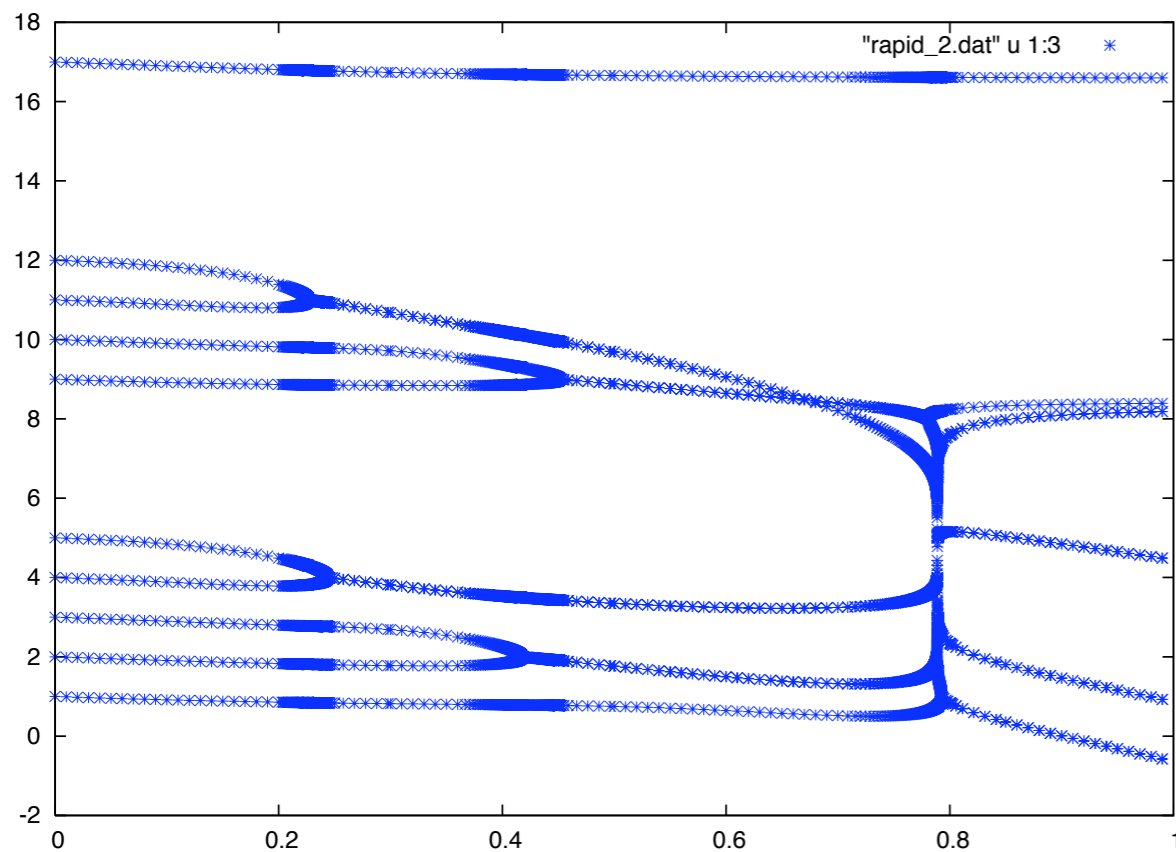
$$\frac{1}{g} = \sum_{\alpha=1}^N \frac{1}{w_j - \varepsilon_\alpha} - \sum_{k \neq j}^{N_r} \frac{2}{w_j - w_k}, \quad j = 1, \dots, N_r$$

Pseudospin representation: $S_\alpha^z = b_\alpha^\dagger b_\alpha - 1/2$, $S_\alpha^- = b_\alpha$, $S_\alpha^+ = b_\alpha^\dagger$

$$b_\alpha = c_{\alpha-} c_{\alpha+}, \quad b_\alpha^\dagger = c_{\alpha+}^\dagger c_{\alpha-}^\dagger \quad H = \sum_{\alpha=1}^N \varepsilon_\alpha S_\alpha^z - g \sum_{\alpha,\beta=1}^N S_\alpha^+ S_\beta^-$$

Solving the Richardson equations

(relatively)
straightforward for
the ground state



For excited states:
can become a real
challenge !!

(Richardson, 1964; Schechter, Imry, Levinson & von Delft, 2001; von Delft & Ralph, 2001; Yuzbashyan, Baytin & Altshuler, 2003; Roman, Sierra & Dukelsky, 2003; Snyman & Geyer, 2006; Sambataro, 2007)

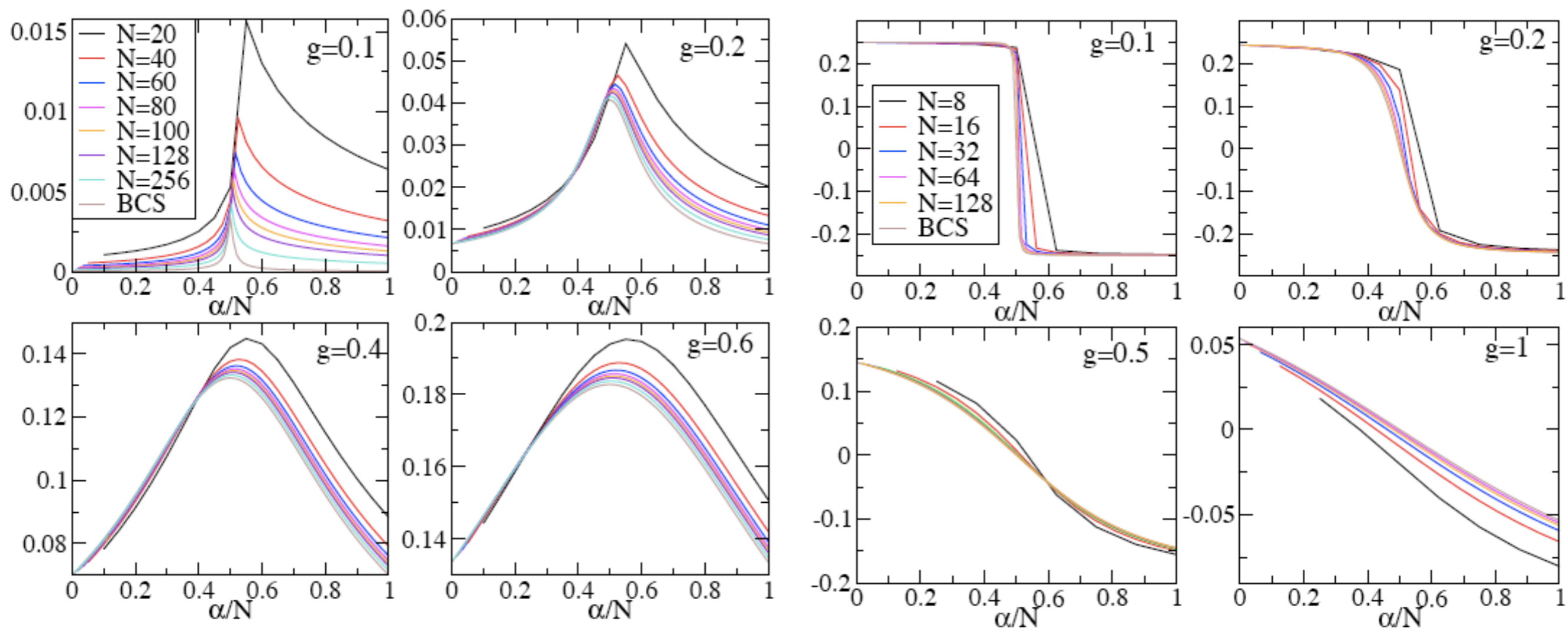
The Richardson model: (static) correlation functions

(A. Faribault, P. Calabrese & J-S C, PRB 2008)

(Following up on ABA work by J. von Delft & R. Poghossian, 2002
and H.-Q. Zhou, J. Links, R. H. McKenzie & M. D. Gould, 2002-3)

$$\langle S_1^- S_\alpha^+ \rangle$$

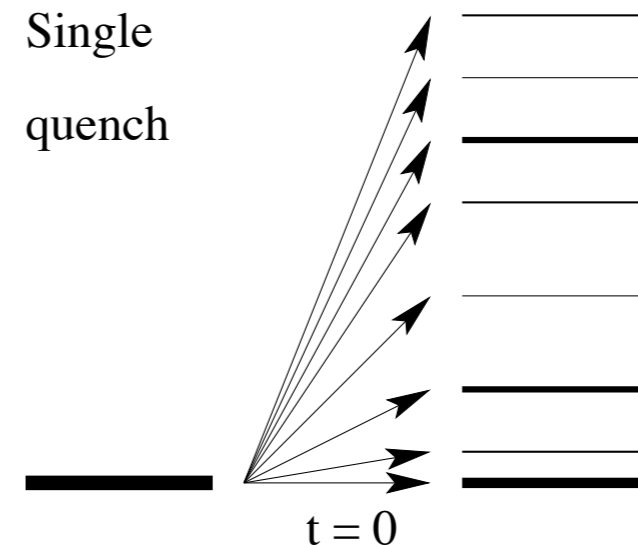
$$\langle S_1^z S_\alpha^z \rangle$$



Quenches: some trivialities

Sudden change of interaction parameter

(Barouch & McCoy, ..., Calabrese & Cardy, ...
Cazalilla, Lamacraft, Klich, Lannert & Refael, ...)



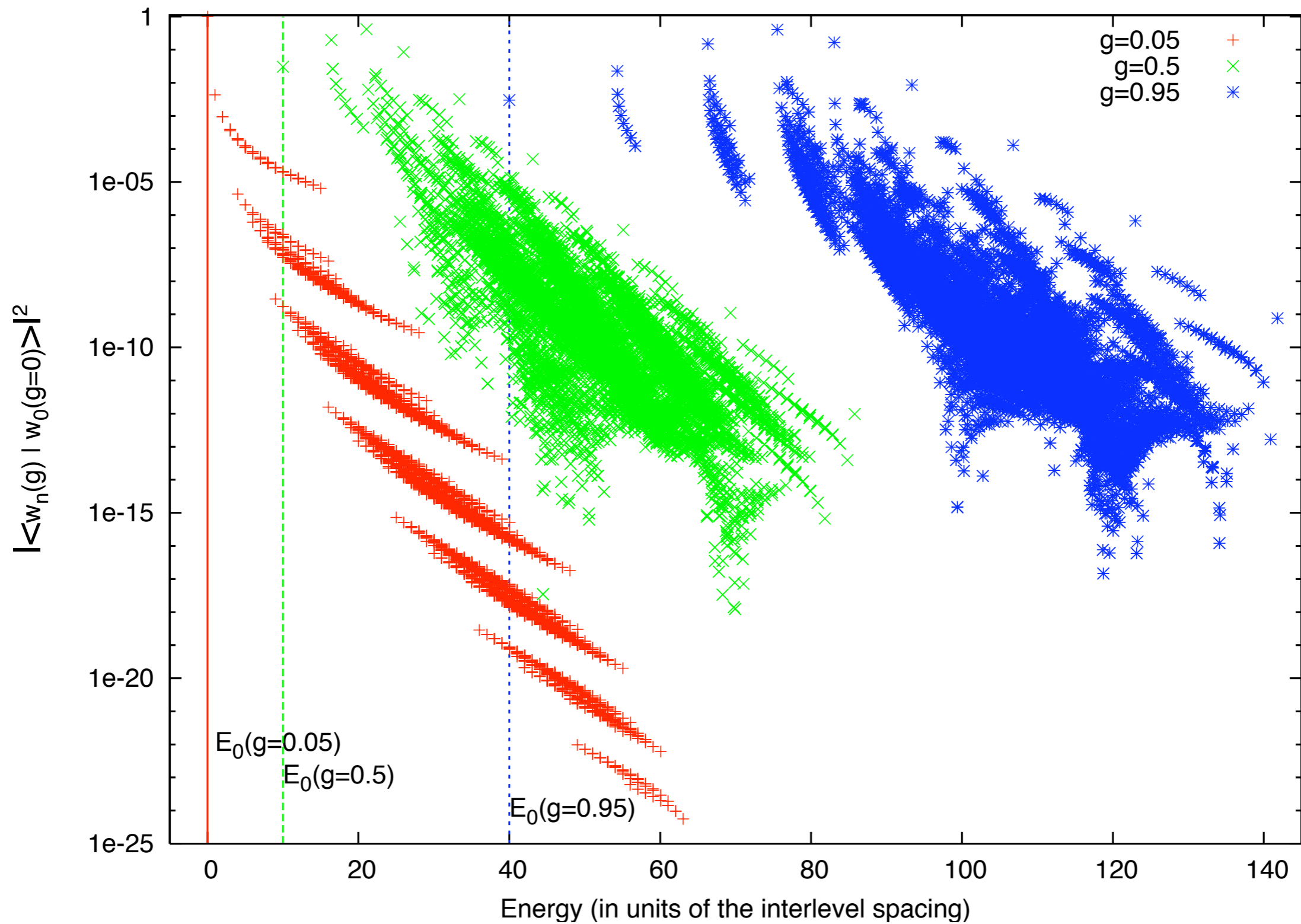
At quench time:
$$|\Psi_g^0\rangle = \sum_{\alpha} |\Psi_{g'}^{\alpha}\rangle \langle \Psi_{g'}^{\alpha} | \Psi_g^0\rangle \equiv \sum_{\alpha} M_{g'g}^{\alpha 0} |\Psi_{g'}^{\alpha}\rangle$$

Subsequent time evolution:
$$|\Psi(t)\rangle = \sum_{\alpha} M_{g'g}^{\alpha 0} e^{-i\omega_{g'}^{\alpha} t} |\Psi_{g'}^{\alpha}\rangle$$

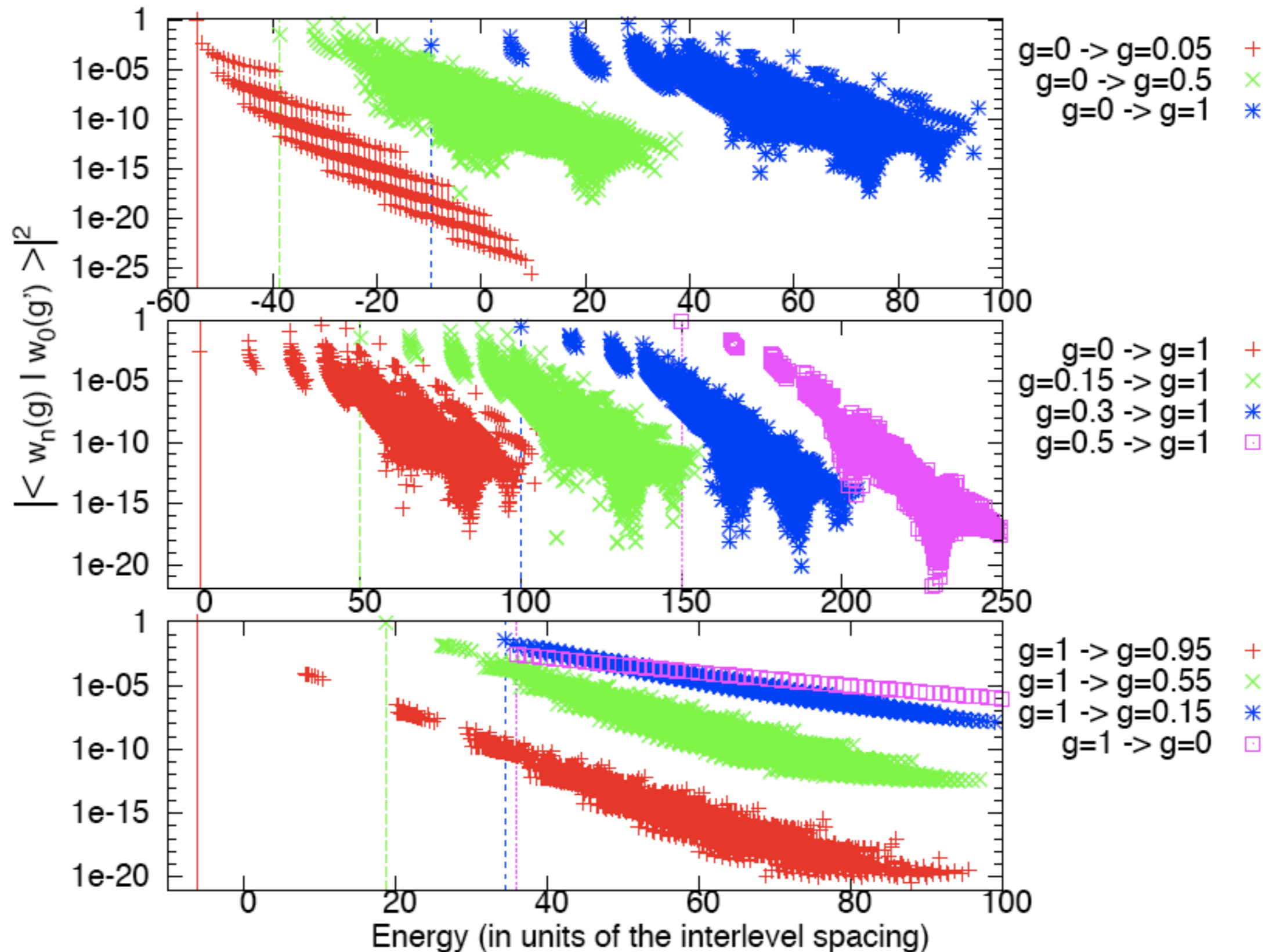
Crucial building block:
$$\langle \Psi_{g'}^{\alpha} | \Psi_g^{\beta} \rangle \equiv M_{g'g}^{\alpha\beta}$$

We know how to calculate this for Richardson !!

Quench matrix elements

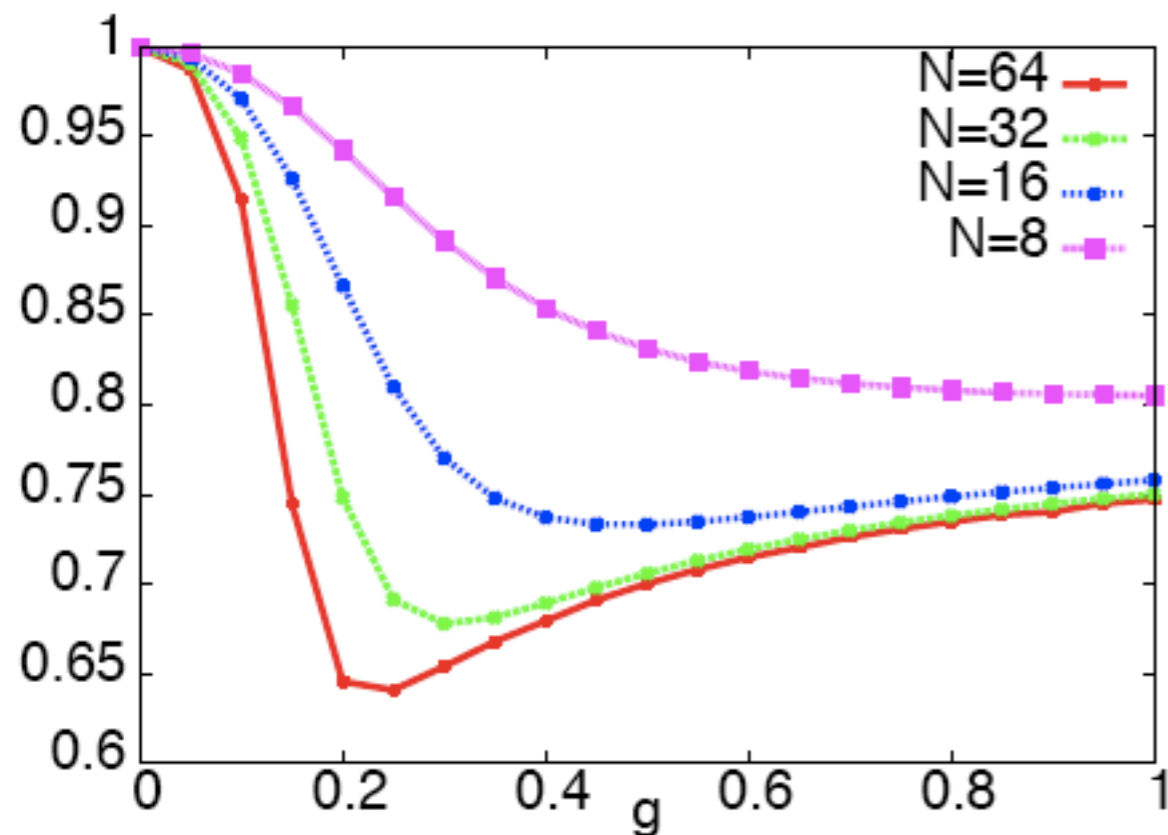
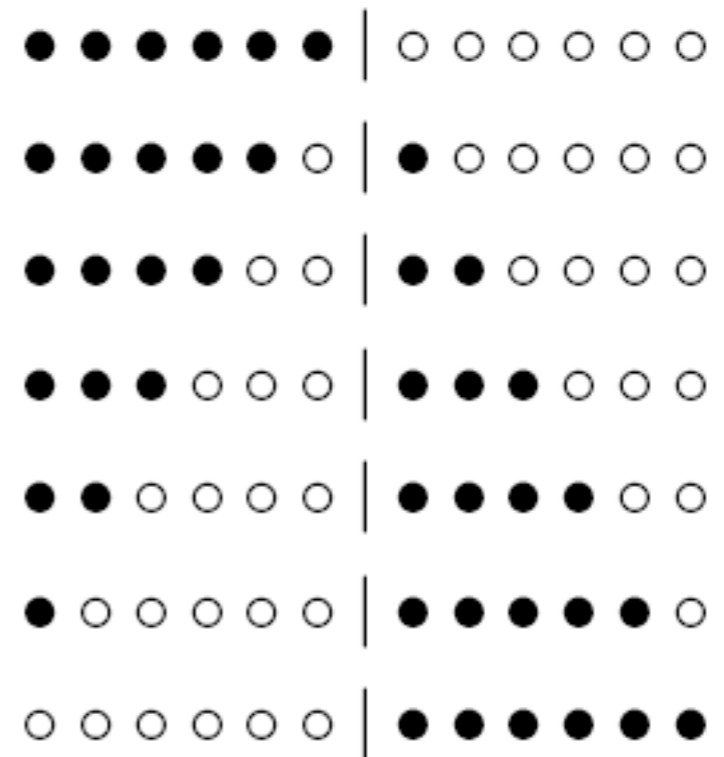


Quench matrix elements



Quench: dominant excitations

Promoting 'blocks' of spins from under to above the Fermi level



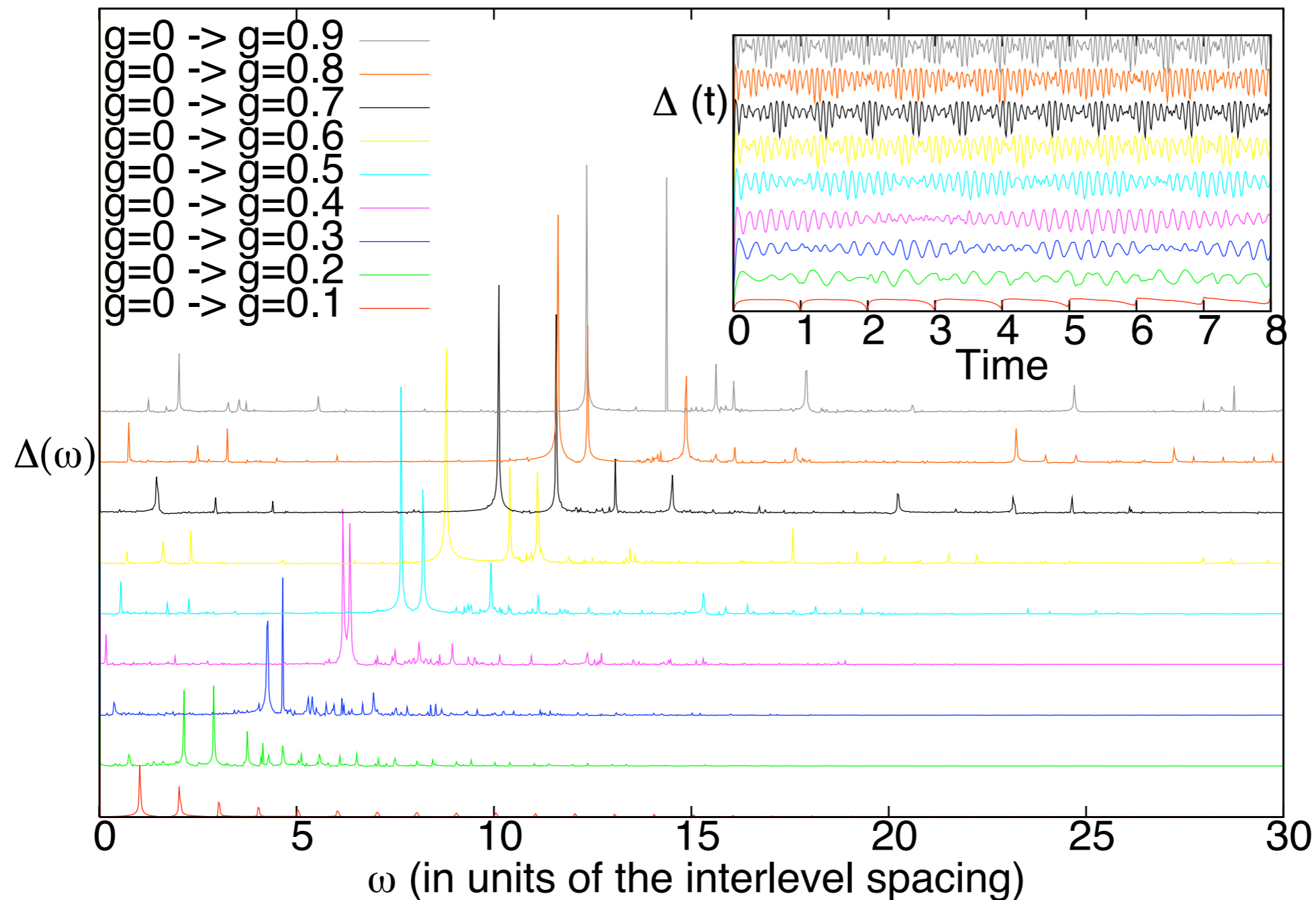
Contribution to square amplitude of resulting state

Importance of states: not same logic as for correlations, but BA basis still pretty good

Time dependence of observables

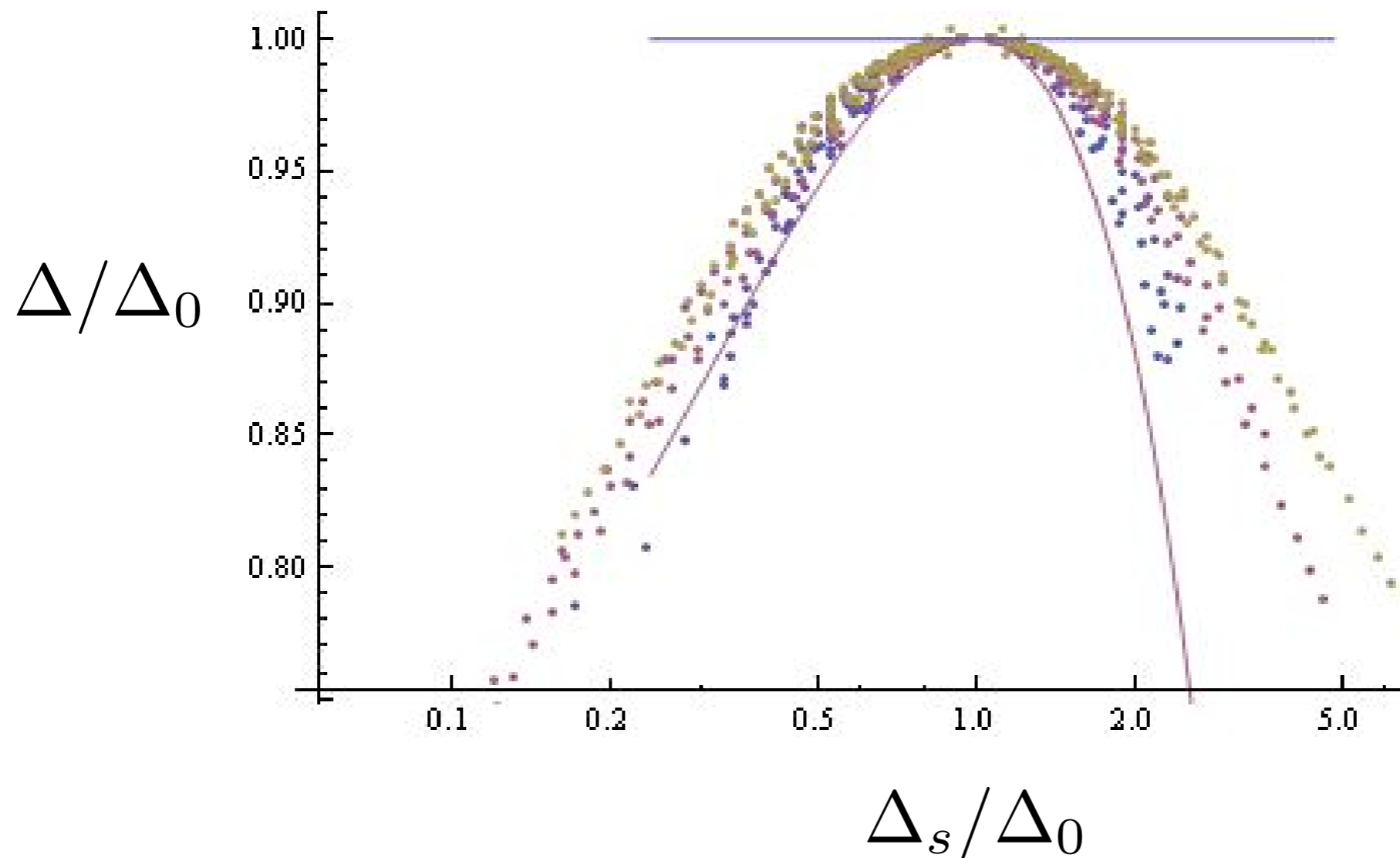
‘Canonical order parameter’

$$\Psi(t) = \sum_{\alpha=1}^N \sqrt{\frac{1}{4} - \langle S_{\alpha}^z(t) \rangle^2}$$



Asymptotic pairing order parameter

Plotted against mean-field prediction (Barankov & Levitov, PRL 2006)



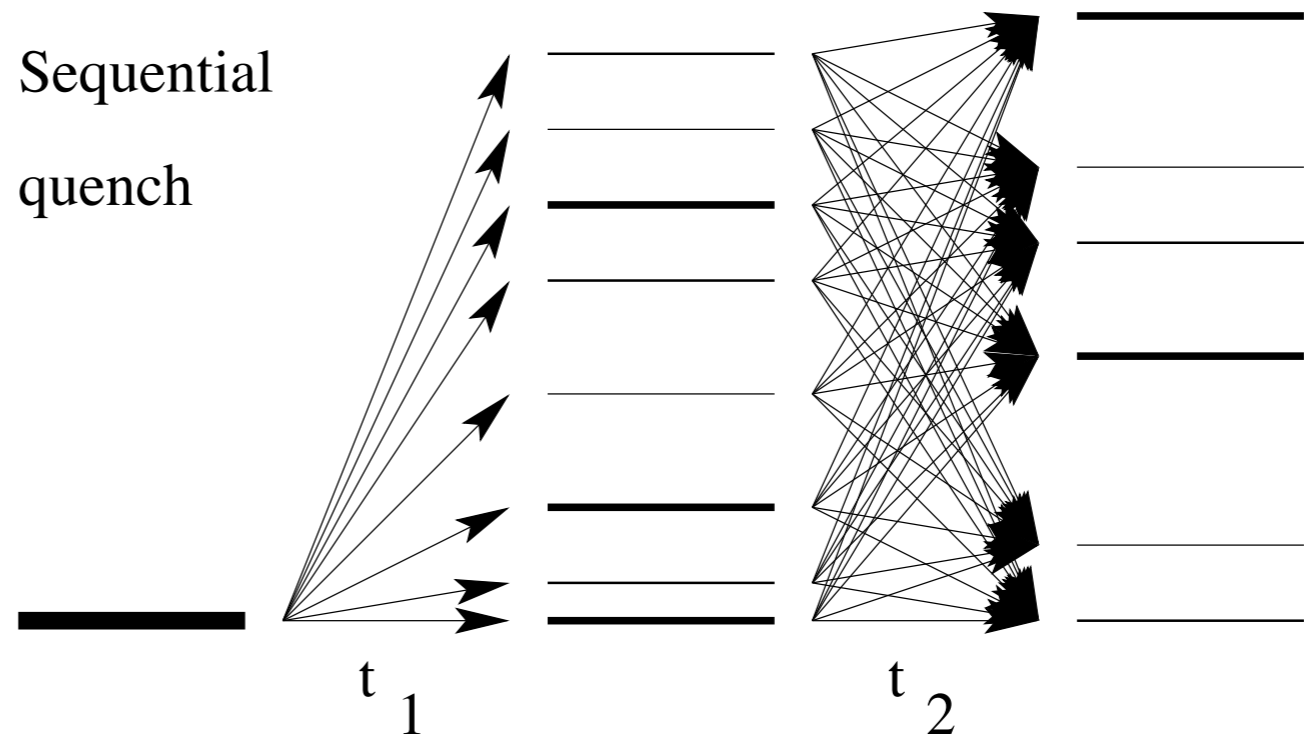
Δ_s BCS gap for initial g

Δ_0 BCS gap for final g

Δ actual OP
after quench

Sequential quenches

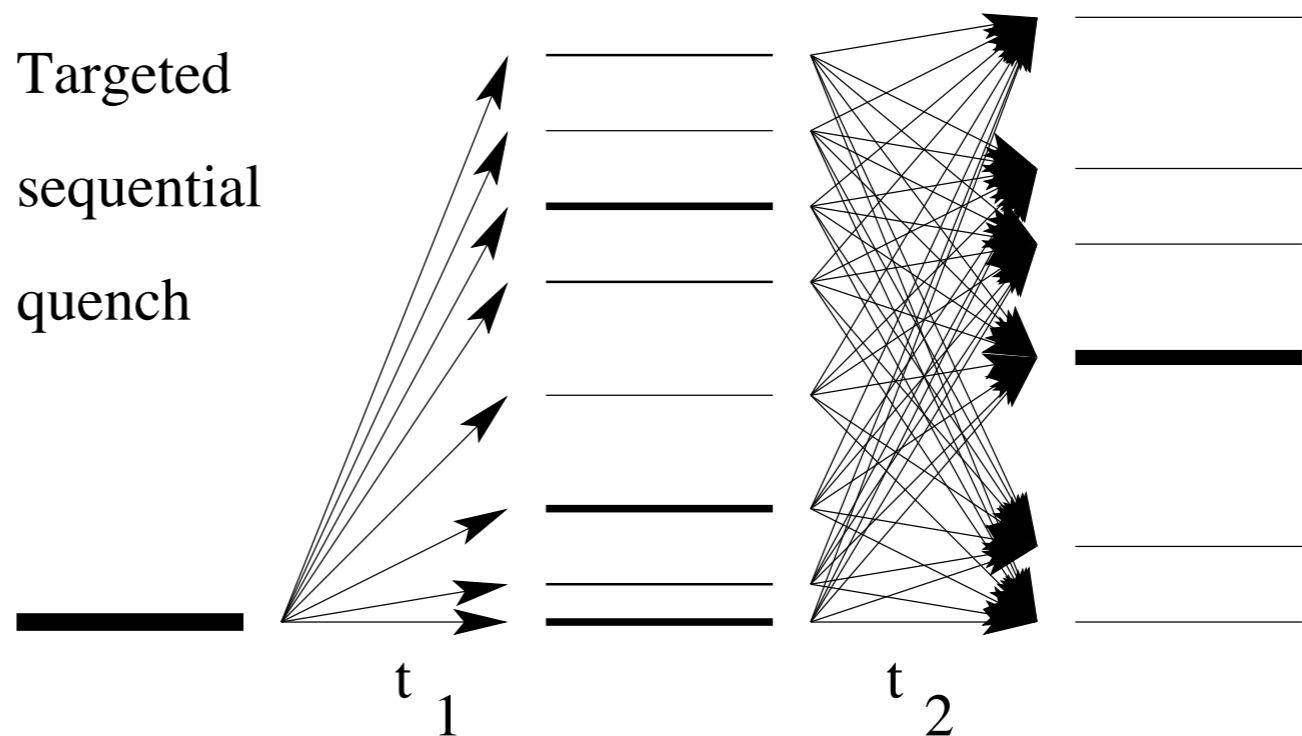
Generic situation, here for 2 quenches:



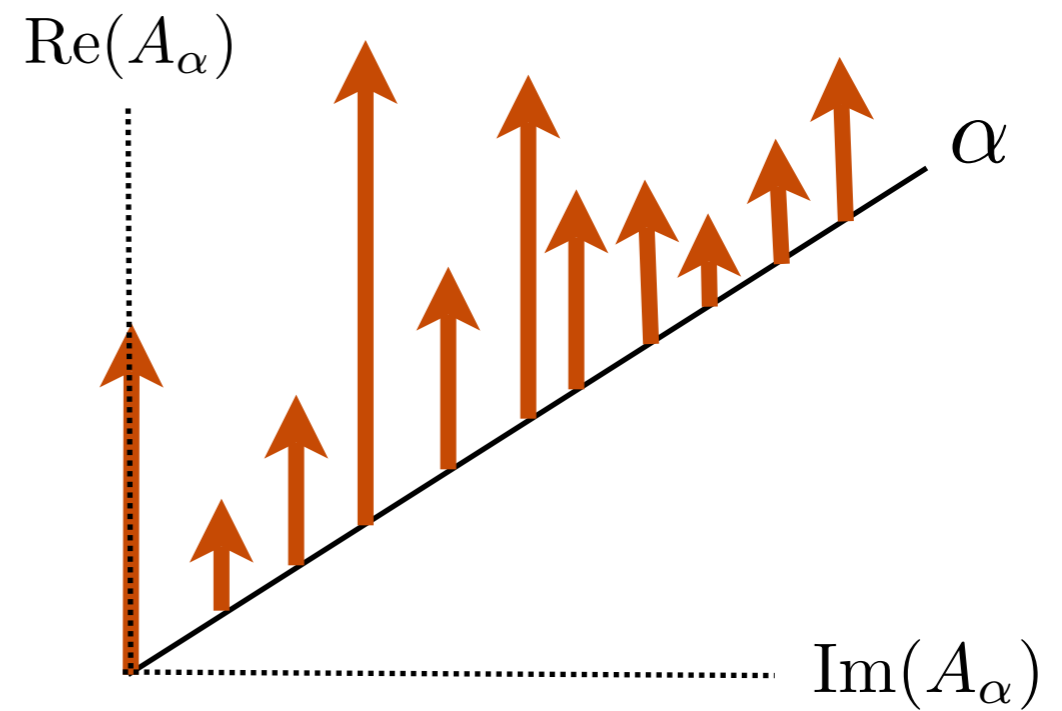
‘Quench propagator’ for quench-dequench

$$Q_{\beta\alpha}(t_q) = \sum_{\gamma \in \mathcal{H}_{g_1}} M_{g_0 g_1}^{\beta\gamma} M_{g_1 g_0}^{\gamma\alpha} e^{-i\omega_\gamma t_q}$$

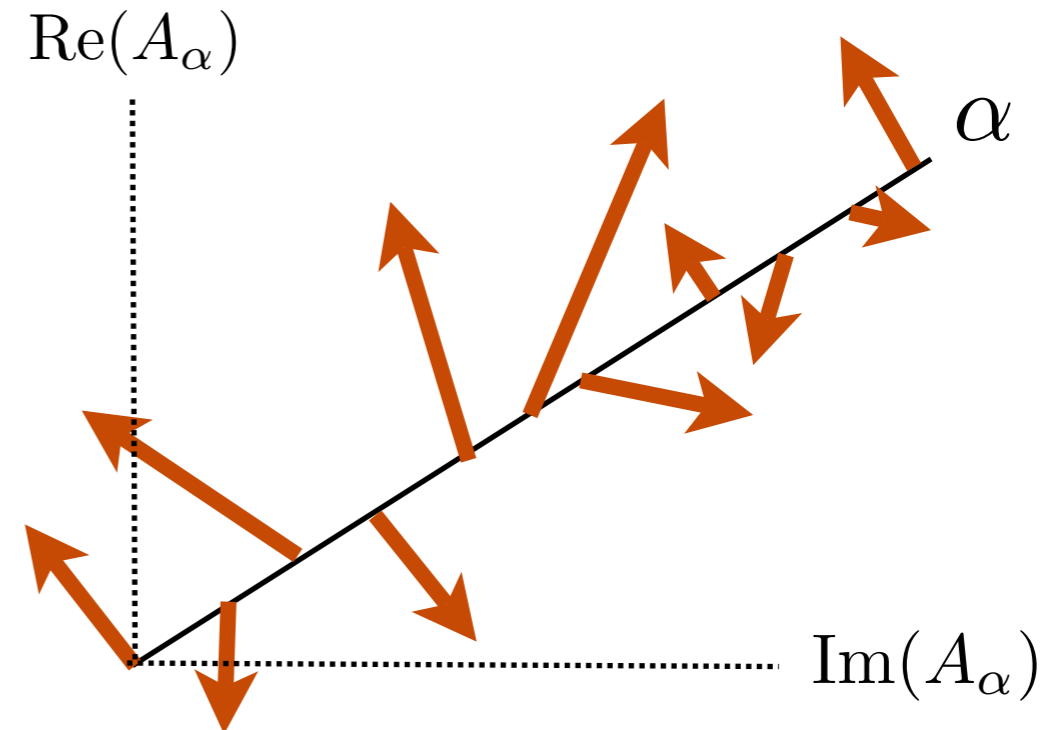
Possible to focus on specific excited states ?



At $t = 0$, the initial quench populates excited states of H_g



As the quench lasts, each 'arrow' rotates at the appropriate frequency



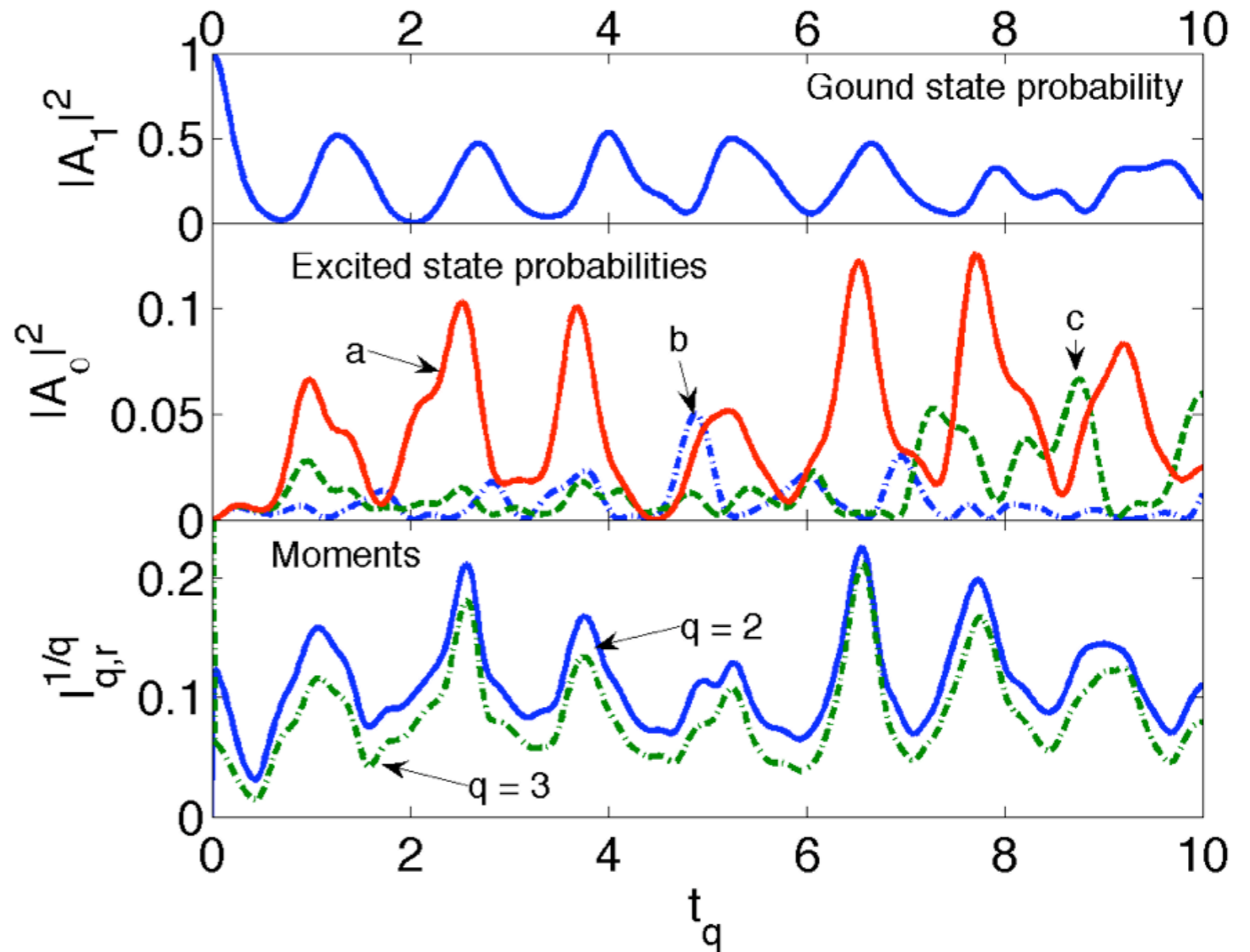
The dequench repopulates states of original Hamiltonian

When arrows 'add up to zero': state destruction

When arrows realign: state reconstruction

State occupation probabilities after double quench (quench-dequench)

Ground state disappears and reappears ('collapse and revival'); excited states nontrivially weighted



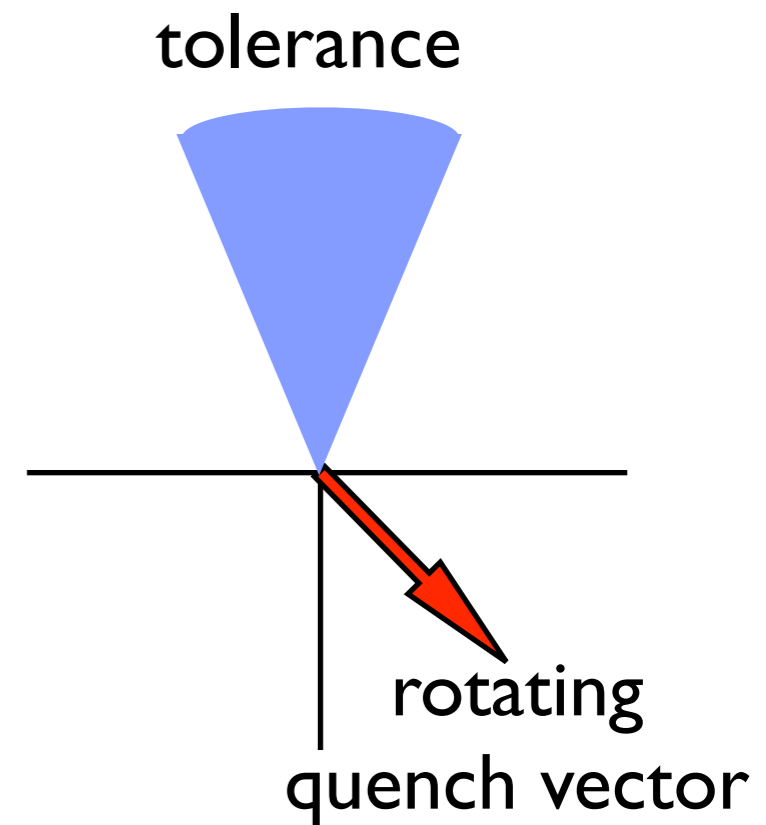
Weight distribution among excited states: look at IPRs

$$I_{q,r} = \frac{\sum_{\alpha>0} |A_{\alpha}|^{2q}}{(\sum_{\alpha>0} |A_{\alpha}|^2)^q}$$

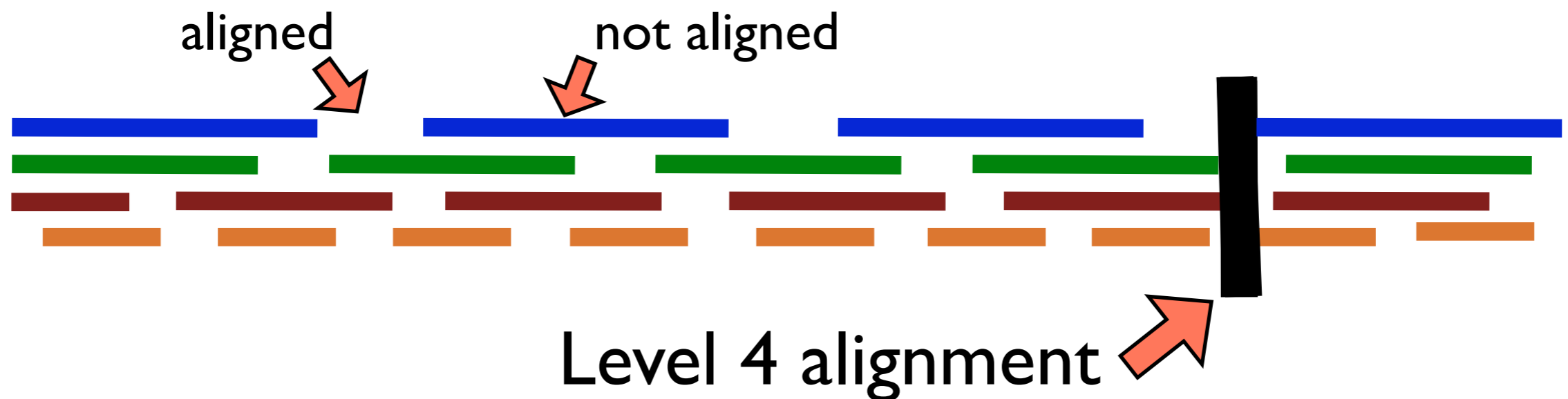
Predicting state occurrences

‘Continuous’ sieve of Eratosthenes:

- States ordered in decreasing quench weight
- Times kept: such that phases align to a specified tolerance
- High level alignment = constructive interference for that state



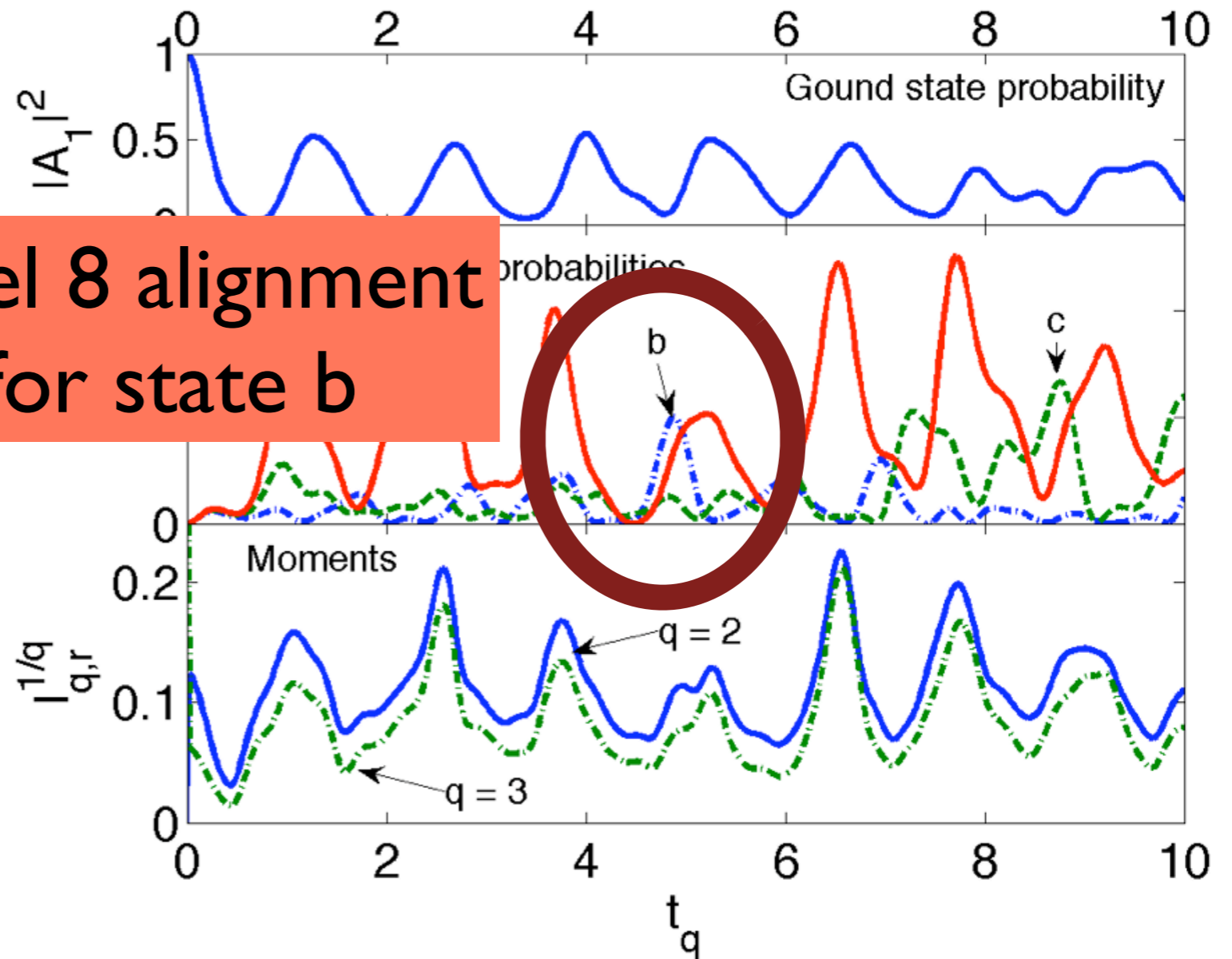
Phase timelines: progressively erase non-aligned times



State occupation probabilities after double quench (quench-dequench)

Ground state disappears and reappears; excited states nontrivially weighted

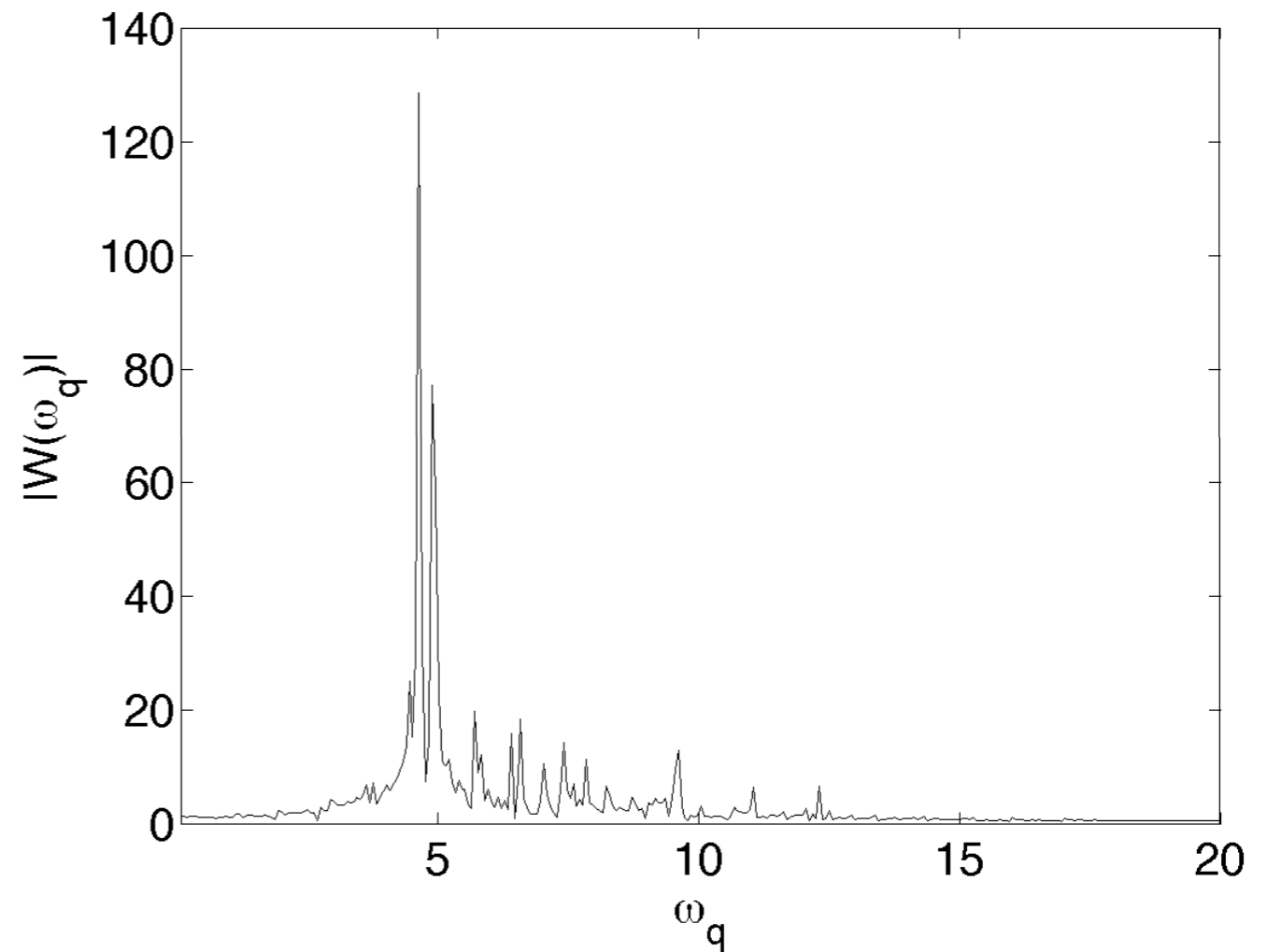
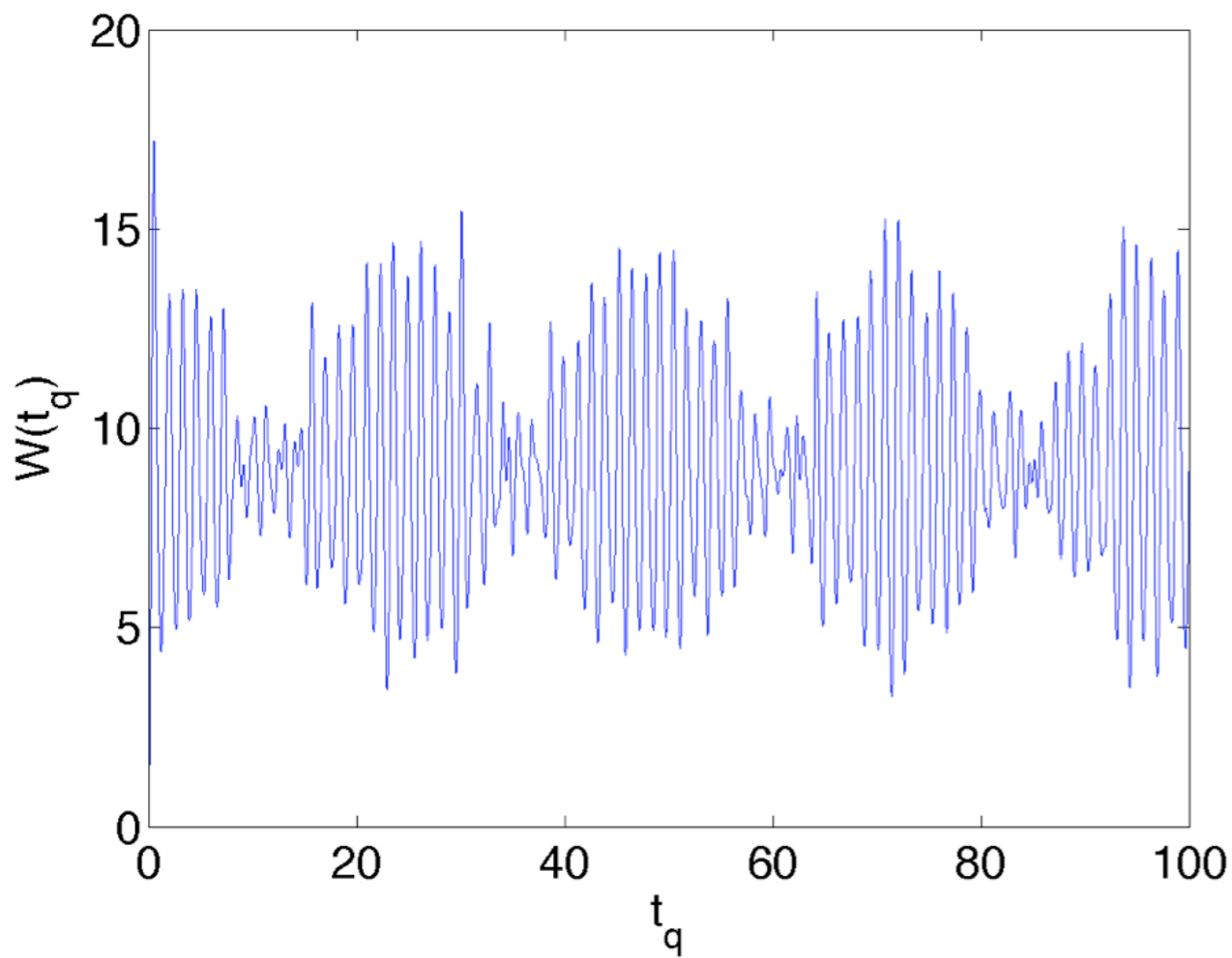
Level 8 alignment for state b



Weight distribution among excited states: look at IPRs

$$I_{q,r} = \sum_{\alpha>0} |A_\alpha|^{2q} / \left(\sum_{\alpha>0} |A_\alpha|^2 \right)^q$$

Work and its Fourier transform for quench/ dequench sequence



Conclusions & open problems

- Dynamical correlations in integrable models: now accessible from ABACUS, q groups
- Provides extensive, quantitative predictions for experiments

To do list/work in progress:

- Better classification of solutions to Bethe eqns
- Ferromagnetic spin chains
- Q group approach: other regimes/polarizations
- Finite temperatures
- Nested systems **Postdoc positions available !!**
- Quenches from integrability: other cases
- Non-integrable deformations: RG using integrability (TSA, NRG)