Collective processes in the ultra-cold gas

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Outline:

1. Laser cooling forces;
2. From wave equation to kinetic equation;
3. Collective oscillations of a cold atom gas;
4. Plasma-acoustic hybrid mode;
5. Dynamical analogue of CARL;
6. Dipole (Mie) and Tonks-Dattner resonances;
7. Coulomb-like explosions;
8. Trivelpiece-Gould solitons;
9. Rydberg plasmas;
10. Conclusions.

Laser cooling forces

1) Induced light pressure force [Ashkin, PRL (1970)]

$$F = F_0 - \beta v + O(v^2)$$

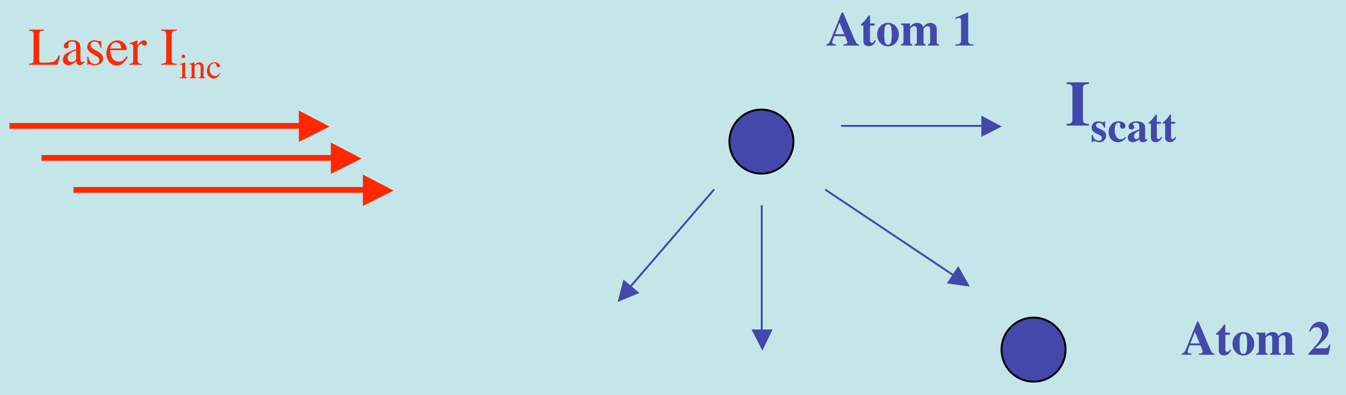
2) Shadow effect or absorption force [Dalibard, Opt.Commun. (1988)]

$$\vec{\nabla} \cdot [\vec{F}_A(\vec{r})] = -\sigma_L^2 \frac{I}{c} n(\vec{r})$$

3) Repulsive effect or radiation trapping force [Sesko et al., JOSA B (1990)]

$$\vec{\nabla} \cdot [\vec{F}_R(\vec{r})] = \sigma_R \sigma_L \frac{I}{c} n(\vec{r})$$

Basic principle of the repulsive force



Atomic repulsion results from radiation pressure of the scattered radiation ($I_{scatt} \sim 1 / r^2$)

$$\nabla \cdot \left[\vec{F}(\vec{r})\right] = Qn(\vec{r}), \qquad Q = (\sigma_R - \sigma_L)\sigma_L I / c$$

Competing effect: repulsive force dominates over shadow effect

From the Wave Equation ...

Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} | \vec{r}, \vec{R} >= H | \vec{r}, \vec{R} >$$

$$H \equiv H(\vec{r}, \vec{R}, t) = \frac{1}{2m}(\vec{p} + e\vec{A})^2 + \frac{1}{2M}(\vec{P} - Ze\vec{A})^2 + V(r)$$

Potentials

$$V(r) = -\frac{Z_{eff} e^2}{4\pi\varepsilon_0 r},$$

$$\vec{A}(\vec{R}, t) = \vec{A}_0 \exp(i\vec{k} \cdot \vec{R} - i\omega t) + \vec{A}_S(\vec{R}, t) + \vec{A}_C(\vec{R})$$

Laser cooling beam

Scattered radiation

Confining static field

... to Wave Kinetic equation

Wigner matrix

$$W_{nk}(\vec{R},\vec{q},t) = \int \Phi_n^*(\vec{R}+\vec{s}/2,t)\Phi_k(\vec{R}-\vec{s}/2,t)\exp(-i\vec{q}\cdot\vec{s})d\vec{s}$$

Wigner-Moyal equation

$$\left(\frac{\partial}{\partial t}+\frac{\hbar\vec{q}}{M}\cdot\frac{\partial}{\partial\vec{R}}\right)W_{nn} = \sum_k h_{nk}(\omega)[W_{nk}^{(-)} - W_{nk}^{(+)}]\exp(i\vec{k}\cdot\vec{R}-i\Delta\omega t)$$

$$\Delta\omega = \omega-\omega_{nk}, \qquad h_{nk}(\omega) = \frac{\omega}{\hbar}A_0 p_{nk}, \qquad W_{nk}^{(\pm)} = W_{nk}(\vec{R},\vec{q}\pm\vec{k}/2,t)$$

Quasi-classical approximation

$$W_{nk}^{(\pm)} \approx W_{nk} \pm \frac{\vec{k}}{2}\cdot\frac{\partial}{\partial\vec{q}}W_{nk} + \frac{\vec{k}\vec{k}}{2^3}\cdot\frac{\partial^2}{\partial\vec{q}^2}W_{nk}$$

Force term

Diffusion term

Collective forces in cold atom gas

Wave kinetic equation in the quasi-classical limit

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} + \frac{1}{M}\left(\vec{F}_{conf} + \vec{F}\right)\cdot\frac{\partial}{\partial \vec{v}}\right]W = 0$$

+ viscous term
+ diffusion term

Collective (shadow - repulsive) force

$$\nabla\cdot\vec{F} = Qn(\vec{r},t) \equiv Q\int W(\vec{v})d\vec{v}$$

Coulomb-like atom-atom interaction

$$Q = (\sigma_R - \sigma_L)\sigma_L I/c$$

Equilibrium

$$\vec{F}_{conf} + \vec{F}_0 = 0, \qquad \nabla \cdot \vec{F}_0 = Q n_0(\vec{r})$$

Perturbation

$$\delta\vec{F} = \vec{F}_{conf} + \vec{F} \propto \exp(i\vec{k}\cdot\vec{r} - i\omega t),$$

$$W(\vec{r},\vec{v},t) = W_0(\vec{v}) + \tilde{W}(\vec{v})\exp(i\vec{k}\cdot\vec{r} - i\omega t)$$

Linearized evolution equations

$$\tilde{W} = -\frac{i}{M}\frac{\delta\vec{F}\cdot\partial W_0/\partial\vec{v}}{(\omega - \vec{k}\cdot\vec{v})}$$

$$i\vec{k}\cdot\delta\vec{F} = Q\int \tilde{W}(\vec{v})d\vec{v}$$

Dispersion relation for cold atom gas (infinite geometry)

$$1 + \frac{Q}{Mk^2}\int \frac{\vec{k}\cdot\partial W_0/\partial\vec{v}}{(\omega - \vec{k}\cdot\vec{v})} d\vec{v} = 0$$

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Dispersion relation similar to that of electrostatic waves in a plasma

$$1 + \chi(\omega, \vec{k}) = 0$$

Mono-kinetic distribution

$$W_0(\vec{v}) = n_0 \delta(\vec{v} - \vec{v}_0)$$

$$1 - \frac{Q n_0}{M(\omega - \vec{k} \cdot \vec{v}_0)^2} = 0$$

For $v_0 = 0$, cold atom oscillations similar to plasma oscillations (compare with ω_{pe})

$$\omega = \omega_P \equiv \sqrt{\frac{Q n_0}{M}}$$

Effective atomic charge

$$q_{eff} = \sqrt{\varepsilon_0 Q}$$

Typical experimental value, $q_{eff} = 10^{-5}$ e

Atomic beam instability

Two distinct mono-kinetic distributions

$$W_0(\vec{v}) = n_0\delta(\vec{v}) + n_1\delta(\vec{v}-\vec{v}_1)$$
$$n_1 << n_0$$

Dispersion relation

$$1 - \frac{Qn_0}{M\omega^2} - \frac{Qn_1}{M(\omega - \vec{k}\cdot\vec{v}_1)^2} = 0$$

Cold atom oscillations can become unstable (SCARI)

$$\omega = \omega_r + i\gamma, \qquad \omega_r = \omega_{p0} \equiv \sqrt{\frac{Qn_0}{M}}$$

Maximum instability growth rate

$$\gamma = \frac{\sqrt{3}}{2}\omega_{p0}\left(\frac{n_1}{2n_0}\right)^{1/3}$$

SCARI:
Dynamical analogue to CARL
Similar to beam-plasma instability

Fluid description

Fluid equations for the cold gas

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla P}{Mn} + \frac{\vec{F}}{M} - \alpha \vec{v}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0, \qquad \nabla \cdot \vec{F} = Qn$$

Linearized equations

$P \propto n^{\gamma}$

Adiabaticity law

$$\left[\frac{\partial}{\partial t}\left(\alpha + \frac{\partial}{\partial t}\right) + \omega_P^2 n - u_s^2 \nabla^2\right]\tilde{n} = \left(\frac{u_s^2 \nabla \tilde{n}}{n_0} - \frac{\delta \tilde{F}}{M}\right) \cdot \nabla n_0$$

$$\nabla \cdot \delta \tilde{F} = Q\tilde{n}$$

Sound speed

$$u_s^2 = \frac{5}{3} \frac{P_0}{M n_0}$$

Plasma-acoustic hybrid mode

Perturbation analysis $\tilde{n}, \delta\vec{F} \propto \exp(i\vec{k}\cdot\vec{r} - i\omega t),$

Complex frequency $\omega = \omega_r + i\gamma$

Dispersion relation

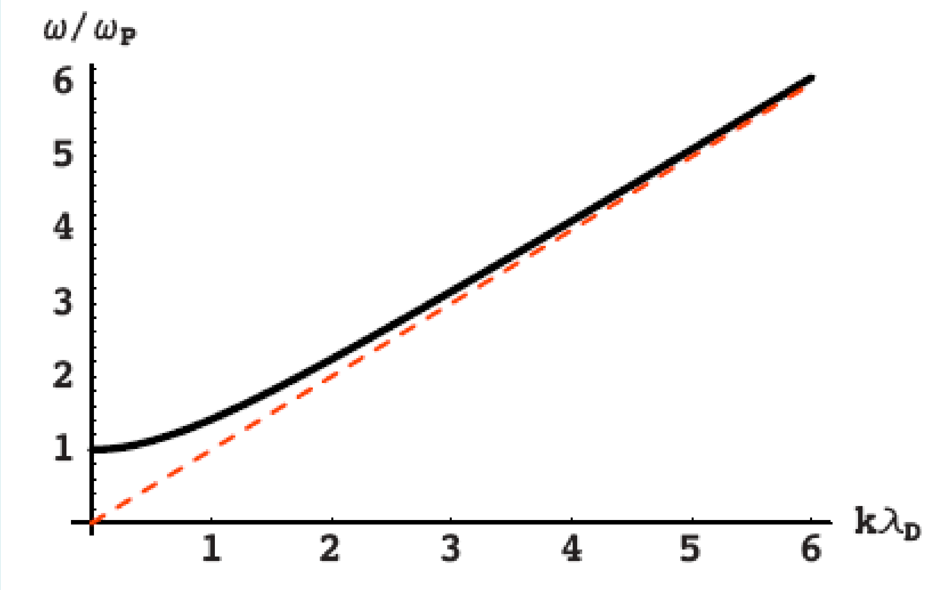
$$\omega_r^{\,2} = \omega_P^2 + k^2 u_s^2 + \frac{3}{4}\alpha$$

Damping rate

$$\gamma = \alpha / 2$$

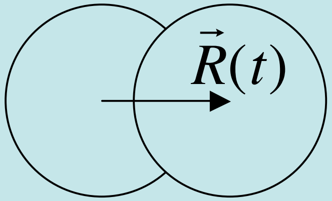
Limit of small viscosity

$$\omega^2 = \omega_P^2 + k^2 u_s^2$$



Centre of mass oscillations

Centre of mass position



$$\vec{R}(t) = \frac{1}{N}\int_V \vec{r}\,n(\vec{r})\,d\vec{r}$$

Electron-ion plasma

$$\frac{d^2\vec{R}}{dt^2} + \omega_M^2\vec{R} = \vec{f}(t)$$

Mie frequency

$$\omega_M = \frac{Q}{M}\frac{1}{R^3}\int_0^R n(r)r^2dr$$

Constant density profile, $n(r) = n_0$

$$\omega_M = \sqrt{\frac{Qn_0}{3M}}$$

Neutral gas confined in a MOT

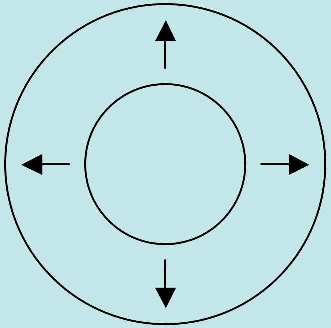
$$\frac{d^2\vec{R}}{dt^2} + \omega_D^2\vec{R} = \vec{f}(t)$$

Dipole frequency

$$\omega_D = \sqrt{K/M} \propto \sqrt{n_0}$$

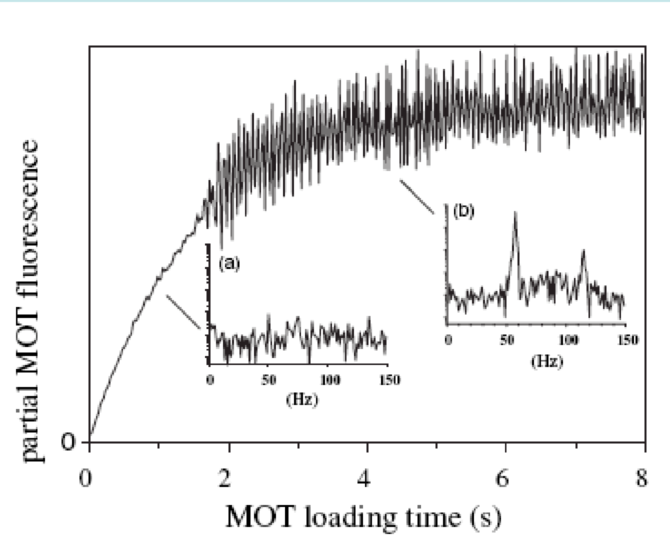
Volume oscillations

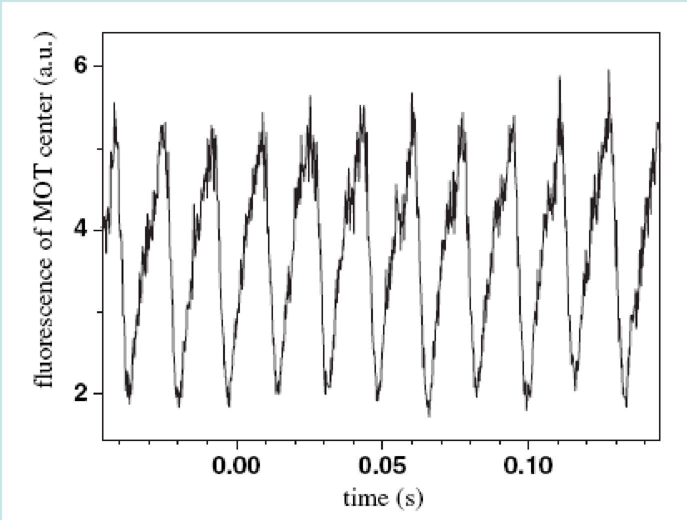
$$\frac{d^2\vec{R}}{dt^2} + \omega_P^2\vec{R} = \vec{f}(t)$$



$$\omega_P = 3\omega_M = \sqrt{\frac{Qn_0}{M}}$$

Observations: Centre of mass or volume oscillations?





Experiments by G. Labeyrie et al., PRL (2006)

Tonks-Dattner resonances

Internal oscillations in a Nonuniform cold gas

$$\nabla^2 \tilde{n} + k^2(\vec{r})\tilde{n} = \frac{\delta \vec{F}}{M u_S^2} \cdot \nabla n_0 + \frac{\nabla n_0}{n_0} \cdot \nabla \tilde{n},$$

$$k^2(\vec{r}) = [\omega^2 - \omega_P^2(\vec{r})] / u_S^2$$

a) Uniform slab

$$\frac{d^2 \tilde{n}}{dx^2} + \frac{1}{u_S^2}[\omega^2 - \omega_P^2(x)]\tilde{n} \approx 0$$

$$\omega_m^2 = \omega_P^2 \left[1 + \left(m + \frac{1}{2} \right)^2 \pi^2 \frac{\lambda_D^2}{L^2} \right]$$

m = 0, 1, 2, ...

b) Cylindrical geometry (plasma)

Parker, Nickel and Gould, PoF (1964)

c) Spherical geometry (neutral cold atom gas)

Mendonça et al., PRA (2008).

Internal structure of the hybrid modes in a sphere of ultra-cold neutral gas

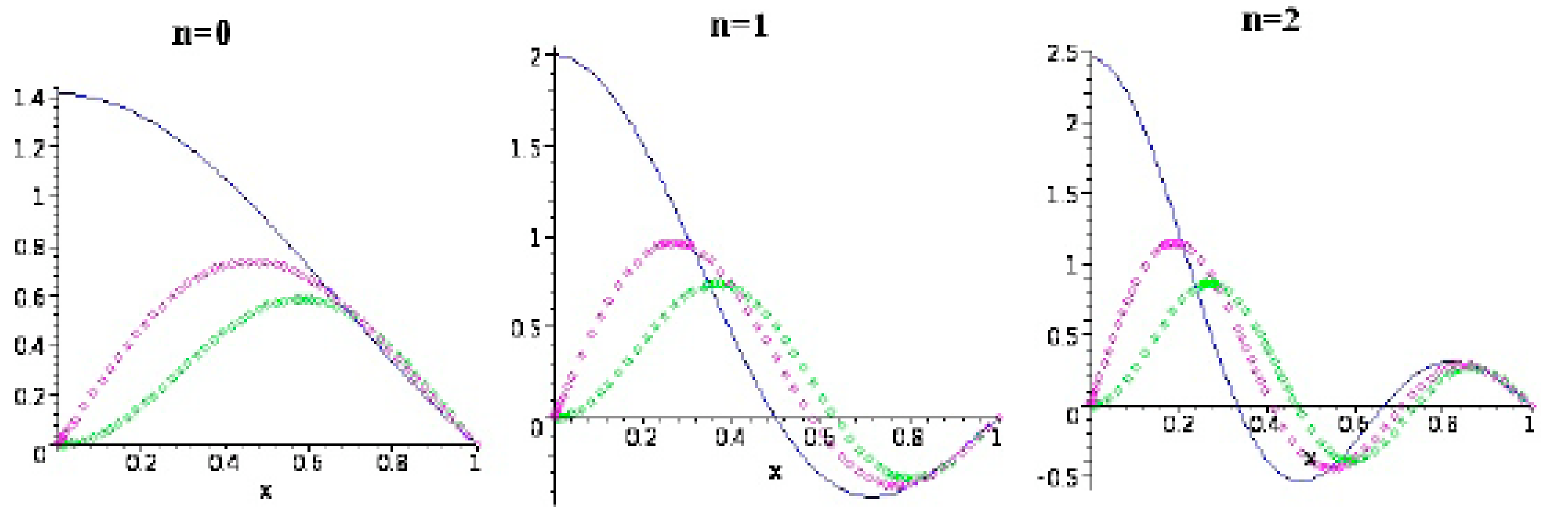


FIG. 2: *Profile of Tonks-Dattner modes, for* $n = 0, 1, 2$ *and* $l = 0, 1, 2$.

Nonlinear coupling between dipole and plasma (or TD) resonances

Density perturbations

$$\tilde{n}(\vec{r},t) = \tilde{A}(t)N(\vec{r})$$

Mathieu-type of equation

$$\frac{\partial^2 \tilde{A}}{\partial \tau^2} + \left[\nu + 2\varepsilon\cos(2\tau)\right]\tilde{A} + 2\varepsilon\sin(2\tau)\frac{\partial \tilde{A}}{\partial \tau} = 0$$

Variables and parameters

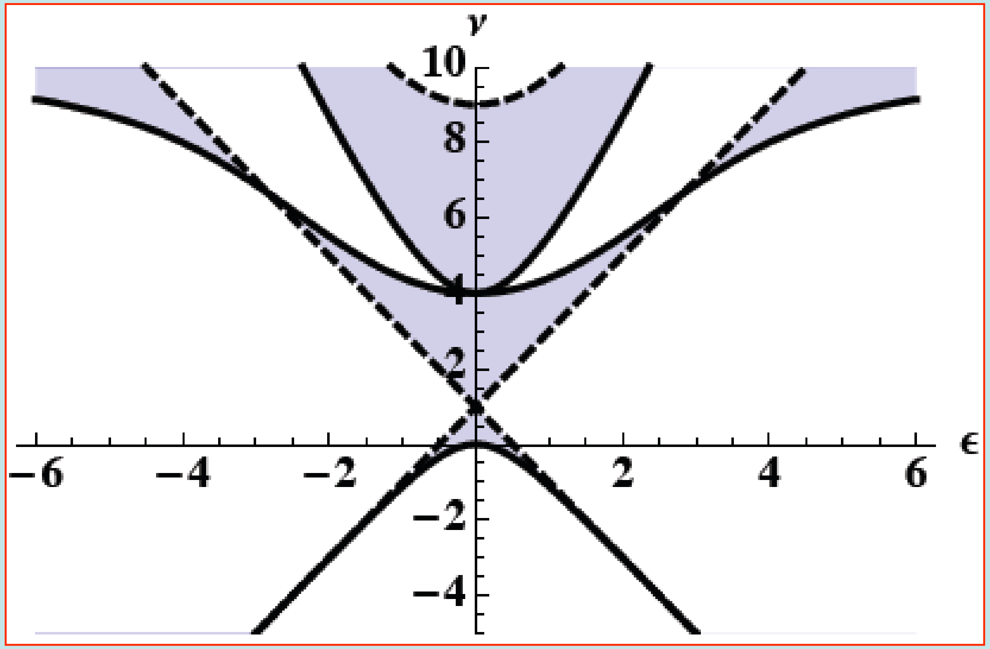
$$2\tau = \omega_D t + \varphi$$

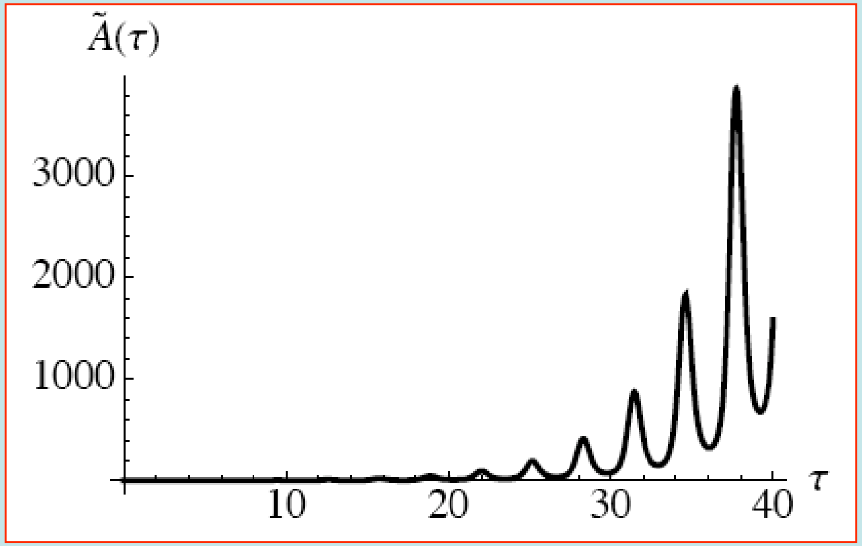
$$\nu = 4(\omega_P^2 + u_S^2 k^2)/\omega_D^2$$

$$\varepsilon = 2\vec{u}_0.\nabla \ln N(\vec{r})/\omega_D$$

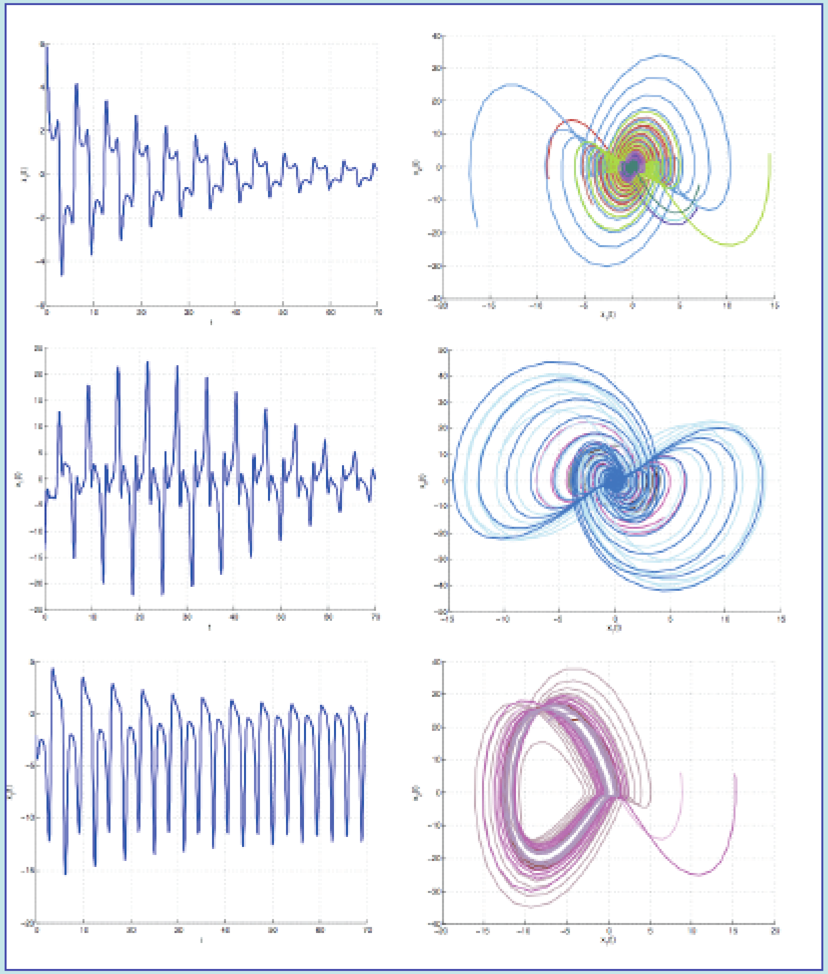
Terças, Mendonça and Kaiser (2008)

Stability range





Feedback stabilization



Coulomb-like explosions

Fluid equations

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla P}{Mn} + \frac{\vec{F}}{M} - \alpha \vec{v}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0, \qquad \nabla \cdot \vec{F} = Qn$$

High viscosity limit

$$\vec{v} \approx \frac{\vec{F}}{\alpha}$$

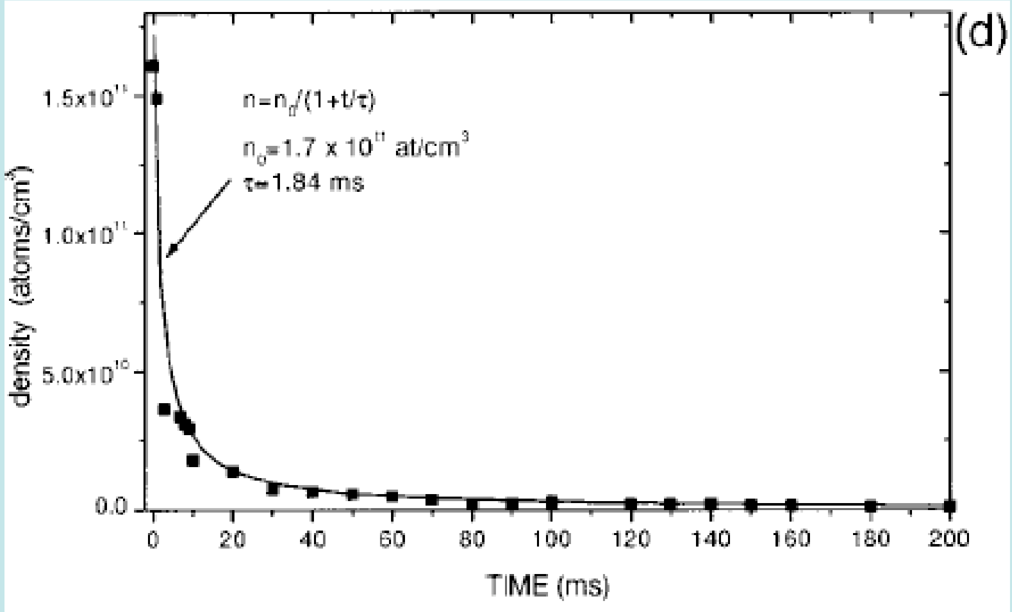
$$\frac{\partial n}{\partial t} = -\frac{1}{\alpha} \nabla \cdot \left[\vec{F}(\vec{r}) n(\vec{r}, t) \right]$$

Spherically expanding gas cloud

$$\frac{1}{n(t)} = \frac{1}{n_0} + \frac{Q}{\alpha} t$$

$$V/V_0 = 1 + t/\tau,$$

$$n/n_0 = (1 + t/\tau)^{-1}$$



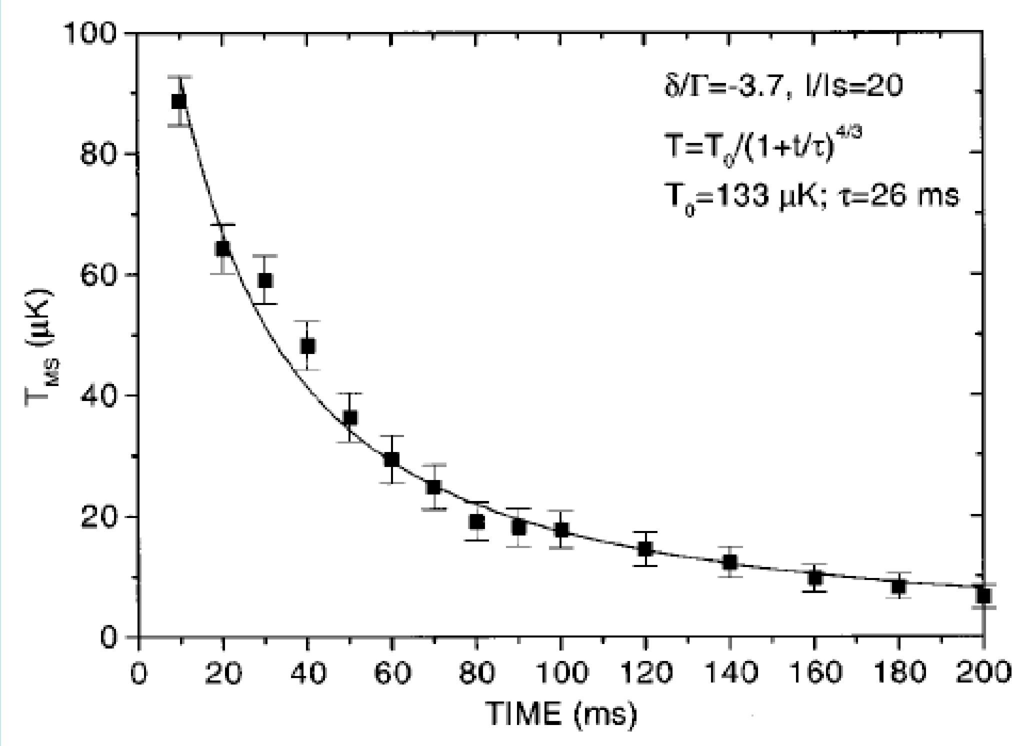
L.Pruvost et al, PRA(2000)

Atom cooling results from cloud expansion

Similar but slower than Coulomb explosion in plasmas

$$T_{MS} = \frac{T_0}{(1 + t/\tau)^{4/3}}$$

$$k_B T_{MS} = m \langle v^2 \rangle$$



Question: can collective effects lead to new cooling processes?

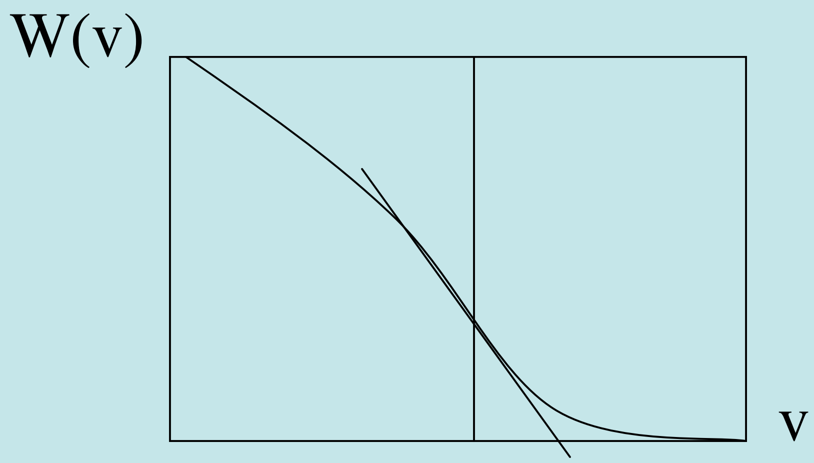
Atomic Landau damping

Back to kinetics

$$\varepsilon(\omega,\vec{k}) \equiv 1 + \chi(\omega,\vec{k}) = 0$$

$$\chi_r(\omega,\vec{k}) = -\frac{1}{\omega^2}(\omega_P^2 + k^2 u_s^2)$$

$$\chi_i(\omega,\vec{k}) = i\pi \frac{Q}{Mk^2}\left(\frac{\partial W}{\partial v}\right)_{\omega/k}$$



Non dissipative wave damping

$$\gamma = -\chi_i(\omega_r,k)/(\partial\chi_r/\partial\omega)_r$$

$$\gamma = \frac{\pi}{\omega}\frac{Q}{Mk^2}\left(\frac{\partial W}{\partial v}\right)_{\omega/k}$$

Diffusion in velocity space

Quasi-linear theory for a broad spectrum of fluctuations

$$I(t) = \int I(\vec{k},t)\, d\vec{k} \Big/ (2\pi)^3$$

$$\frac{d}{dt} I(\vec{k},t) = 2\gamma_k(t) I(\vec{k},t) + S(\vec{k},t)$$

Diffusion equation

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla + \frac{\partial}{\partial \vec{v}} \cdot \overline{D} \cdot \frac{\partial}{\partial \vec{v}} \right) W_0(\vec{v},t) = 0$$

$$\overline{D} \propto \int I(\vec{k},t) \frac{\vec{k}\vec{k}}{(\omega - \vec{k} \cdot \vec{v})} \frac{d\vec{k}}{(2\pi)^3}$$

Fluctuations: an additional obstacle to atom cooling

Waves and solitons in cylindrical geometry

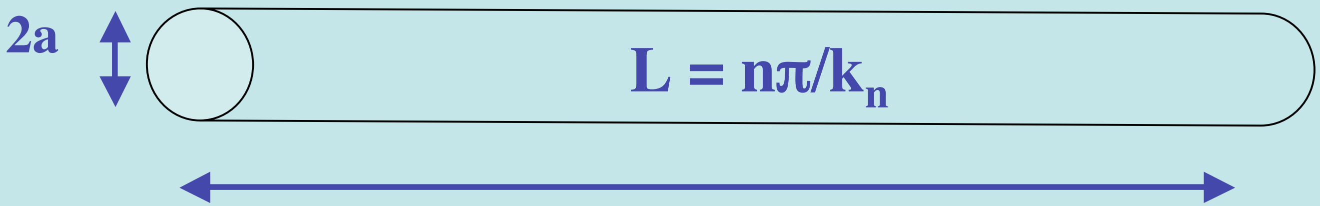
Linear waves (Trivelpiece-Gould)

$$\omega_{n,\ell,m}^2 = \omega_p^2 \frac{k_n^2}{k_n^2 + k_{\perp(\ell,m)}} + v_{th}^2 k_n^2$$

$$k_{\perp \ell,m} = \frac{\alpha_{\ell,m}}{a}$$

Nonlinear waves (KdV solitons)

$$\tilde{n}(\xi,\tau) = \frac{2u_0}{\beta} \sec h^2 \left[\frac{1}{2} \sqrt{\frac{2u_0}{1-H}} (\xi - u_0 \tau) \right]$$



Good configuration for experiments

H. Terças, J.T. Mendonça and P.K. Shukla, PoP (2008).

Rydberg Plasmas

a. Creation of ultracold plasmas by photoionization of laser cooled Xe atoms [T.C. Killian et al., PRL (1999)]

b. Spontaneous evolution of a Rydberg could Xe gas, into a plasma

Creation of very could plasmas (an apparent contradiction)

$T_i \sim 30\ \mu K$, $T_e < 100$ mK (instead of 1-10 eV)

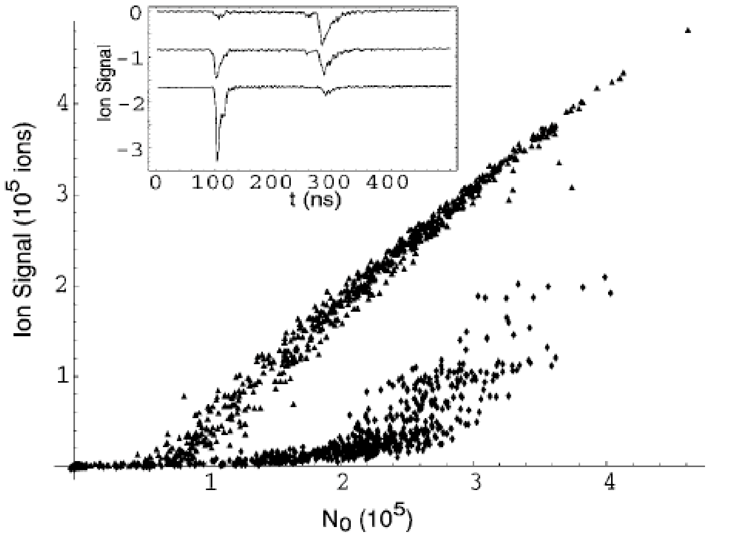


FIG. 1. Ion signals observed for different initial populations of the Rb 36*d* state. The two curves show the ion signals 2 μs (◆) and 12 μs (▲) delays after the dye laser excitation. The inset shows the time resolved signals obtained for $N_0 = 1.9 \times 10^5$ atoms at delays of 2 μs (upper trace), 5 μs (middle trace), and 12 μs (lower trace). In the upper trace there is no early ion signal and a large late atom signal while the reverse is true in the lower trace, indicating the formation of the plasma by 12 μs after laser excitation.

M.P. Robinson et al., PRL (2000)

Possible explanation

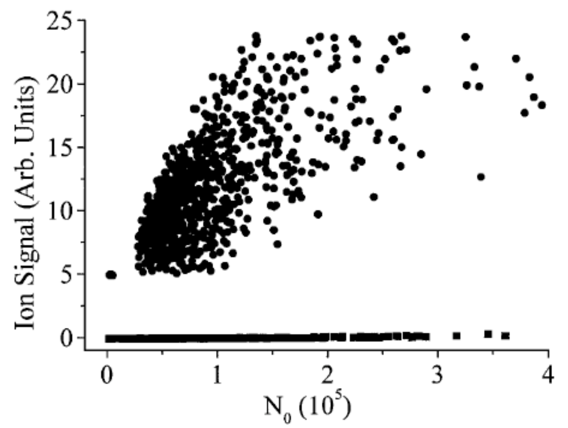


FIG. 4. Ion signals obtained with a delay of 3 μs after the excitation of the Cs 39d state with (●) and without (■) hot atoms. The signal hot atoms is offset by five units.

Existence of a small fraction of hot atoms (1% at room temperature)

Or maybe not [T. Pohl et al. PRA (2003)]

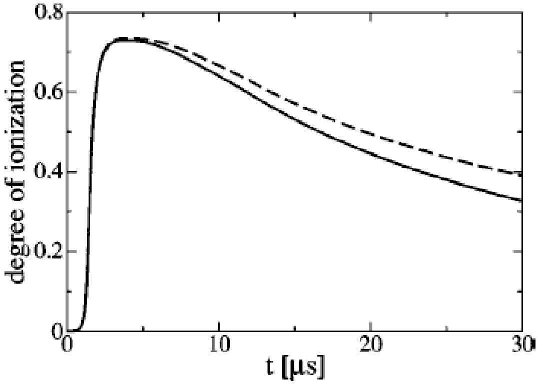


FIG. 1. Time evolution of the degree of ionization for the following initial conditions: atom density $\rho = 8\times10^9$ cm^{-3}, atom temperature $T_a = 140$ μK, plasma width $\sigma = 60$ μm, and initial principal quantum number of the Rydberg atoms, $n_0 = 70$. Results are shown with ionic correlations (solid) and without ionic correlations (dashed).

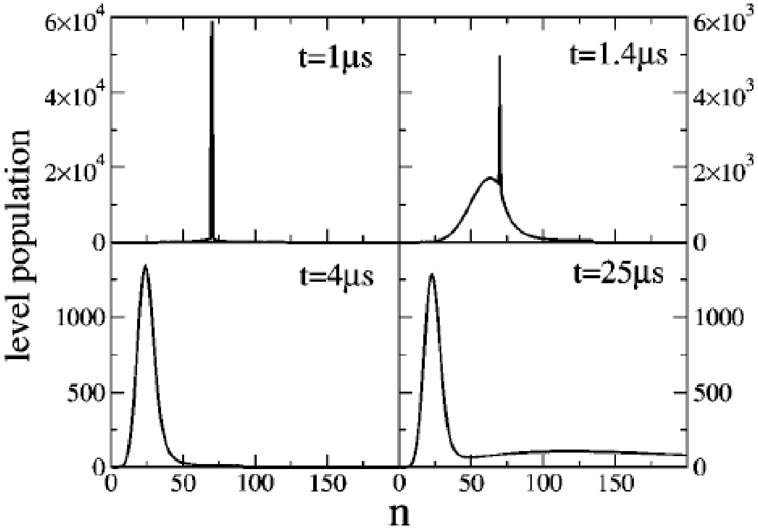


FIG. 3. Level distribution of Rydberg atoms after 1 μs, 1.4 μs, 4 μs, and 25 μs.

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Ambipolar diffusion model for plasma expansion

Expanding ion bubble

$$n_i(r,t) = \frac{N_i}{(4\pi D_a t)^{3/2}} \exp\left(-\frac{r^2}{4 D_a t}\right)$$

Diffusion coefficients

$$D_a = \frac{\mu_i D_i + \mu_e D_e}{\mu_i + \mu_e}, \qquad D_j = T_j / m_j \nu_j, \qquad \mu_j = q_j / m_j \nu_j \qquad j = e,i$$

Quasi-neutrality, $n_i \sim n_e$

Ambipolar electric field

$$\vec{E} = \frac{D_i \nabla n_i + D_e \nabla n_e}{\mu_i n_i + \mu_e n_e}$$

Loureiro, Mendonça, in progress (2007)

Conclusions

- The neutral cold atom gas can behave like a plasma;
- Collective effects are due to shadow-repulsive forces;
- Plasma-like oscillations and plasma-acoustic hybrid mode;
- SCARI dynamical instability (analogue to CARL);
- Dipole oscillations and Tonks-Dattner resonances;
- Trivelpiece-Gould solitons in a cylindrical atomic cloud;
- New areas of atomic physics can be explored.
- New experiments are being performed.