Density fluctuations in a very elongated Bose gas : from ideal gaz to quasi-condensate

Isabelle Bouchoule, Jérôme Estève, Jean-Baptiste Trebbia, Thorsten Schumm, Alain Aspect, Chris Westbrook, Karen Kheruntsyan and Gora Shlyapnikov

Institut d'Optique, Palaiseau.

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Ultracold gases and quantum correlated systems physics

Cold atoms experiments :

- Large variety of confining potentials
- Interaction parameter may be changed
- No coupling to a noisy environment
- Fermions and/or Bosons
- ...

Correlated systems in cold atomic gases :

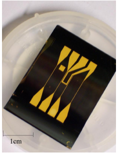
- MOTT transition with fermions and Bosons
- BEC-BCS cross over in fermion gases
- Fermionic gases at unitarity (infinite interactions)
- Fermionic gases with nonequal Fermi levels
- ...

Physics in systems of reduced dimension

- Very different physics from 3D systems
- Enhanced effect of interactions
- 1D case : exact solutions exist

Contribution of cold atom field

- Realisation of reduced dimensional systems by strong confinement in 1 or 2D
- Well controlable systems
- Chip experiment well suited to study 1D physics



Outline

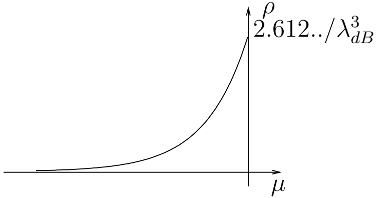
1. Theoretical results on weakly interacting 1D gases
   - Homogeneous gases
   - harmonically trapped gas
2. Experimental study of the cross-over towards quasi-bec
   - Observation of bunching effect
   - Inhibition of bunching in the quasi-bec regime
3. Experimental proof of the failure of Hartree-Fock to explain the transition towards a quasi-bec
4. Other results
5. Conclusion

Outline

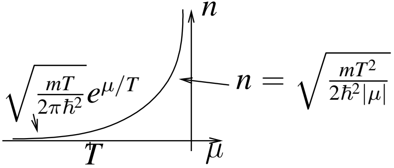
Ideal 1D Bose gases

No BEC phenomena in 1D systems.

3D gases : excited states population saturates



1D gases : No saturation



Physics is very different in 1D systems.
Enhanced fluctuations
Physics governed by interactions

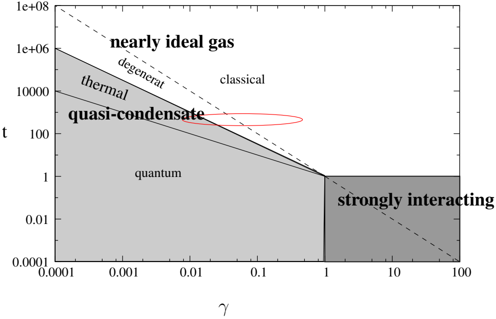
Interacting 1D Bose gas

Coupling constant : g

$$H = -\frac{\hbar^2}{2m}\int dz \psi^+ \frac{\partial^2}{\partial_z^2}\psi + \frac{g}{2}\int dz \psi^+\psi^+\psi\psi,$$

Exact solution :Lieb-Liniger Thermodynamic : Yang-Yang (60')

Parameters : $t = T\hbar^2/mg^2$, $\gamma = mg/\hbar^2 n$

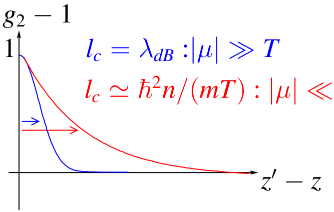


Nearly ideal gas regime : bunching phenomena

For each quantum state, Boltzmann distribution

$\rightarrow$ particle number fluctuations : $\langle n^2 \rangle - \langle n \rangle^2 = \underbrace{\langle n \rangle}_{\text{shot noise}} + \underbrace{\langle n \rangle^2}_{\text{bunching}}$

$$\langle n(z)\, n(z') \rangle = \langle n \rangle^2 g_2(z'-z)$$



Bunching effect $\rightarrow$ density fluctuations.
correlation length increases with density

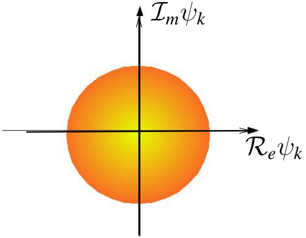
Bunching : correlation between particles. Quantum statistic

Highly degenerate gas : classical field limit

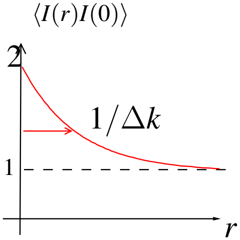
Thermal classical field ψ : $H = \sum_k \epsilon_k |\psi_k|^2$

Mode k :
gaussian distribution

$\langle \psi_k^+ \psi_k^+ \psi_k \psi_k \rangle = 2 \langle \psi_k^+ \psi_k \rangle^2$



Interferences between all modes :

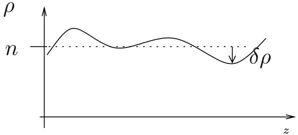


$I(r) = |\psi(r)|^2$
$\psi(r) = \sum_k \psi_k e^{ikr}$

bunching phenomena : speckle

Crossover towards quasi-condensate

⋆ Repulsive Interactions → Density fluctuations require energy



$$\int dz \delta\rho = 0 \Rightarrow$$
$$H_{int} = \frac{g}{2} \int dz \rho^2 = \frac{g}{2} \int dz \delta\rho^2 > 0$$

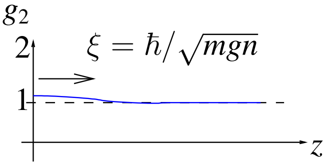
Reduction of density fluctuations at low temperature/high density

Cross-over temperature :

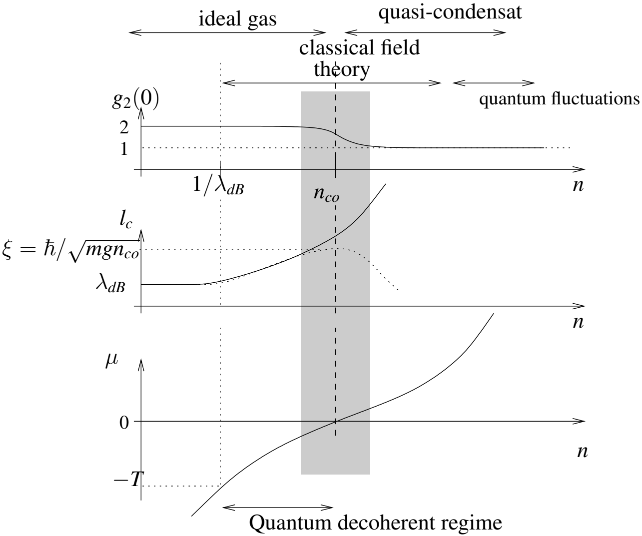
$$\frac{1}{N} H_{int} \propto gn \simeq |\mu| \quad \Rightarrow \quad \boxed{T_{c.o.} \simeq \frac{\hbar^2 n^2}{2m} \sqrt{\gamma}}$$

⋆ For $T \ll T_{c.o.}$: quasi-bec regime

- bunching effect killed



1D weakly interacting homogeneous Bose gas

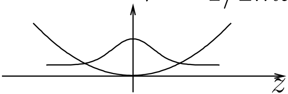


$$n_{co} = \frac{1}{\lambda_{dB}} t^{1/6}$$

$$t = \hbar^2 T/(mg^2)$$

Cross-over towards quasi-bec in a one-dimensional gas trapped in a harmonic potential

Local density approach :
$\mu(z) = \mu_0 - m\omega^2 z^2/2$



Cross-over towards quasi-bec when peak density n_0 reaches $n_{c.o.}$.
approaching crossover from ideal Bose gas side, we find

$$N_{c.o.} \simeq \frac{k_B T}{\hbar\omega} \ln\left(t^{1/3}\right) \qquad T_{c.o.} \simeq \frac{N\hbar\omega}{\ln\left(\left(\frac{N\hbar^3\omega}{mg^2}\right)^{1/3}\right)}$$

- Validity of LDA : $l_c \ll \frac{1}{n}\frac{dn}{dz}$

At cross-over : $l_c \simeq \frac{\hbar}{\sqrt{mgn_{co}}}$

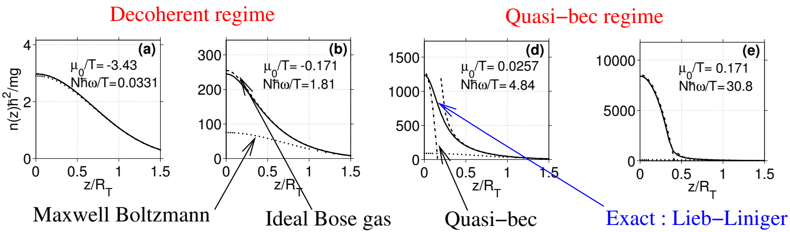
$\Rightarrow$ condition for LDA $\qquad \hbar\omega \ll (mg^2T^2/\hbar^2)^{1/3}$

If not satisfied : finite size condensation phenomena

Comparison with exact results

Lieb-Liniger and Yang-Yang thermodynamic : exact solution that describes the crossover

- Density profiles
  $t = k_B T \hbar^2/(mg^2) = 10^5, N_{c.o.}\hbar\omega/T = \frac{1}{3}\ln(t) = 3.6$



- Atom number at cross over in good agreement with approximate formula
- Decoherent quantum regime can be clearly identified

Realization in a 3D world

- 1D dynamic
  - Transverse confinement : $\omega_\perp$
  - Gas energy scales $\ll \hbar\omega_\perp$. At cross-over : $T \ll \hbar\omega_\perp$
  - transverse wave function of atoms : Gaussian ground state $\psi_\perp$
- Effective 1D coupling constant
  - Low energy scattering ($\hbar/\sqrt{mE} \ll R_e$) : scattering length a
  - Case $a \ll l_\perp = \sqrt{\hbar/m\omega_\perp}$: 3D collision physics

    $\Rightarrow$ $\boxed{g = 2\hbar\omega_\perp a}$
- LDA condition : $\omega \ll \omega_\perp (T/\hbar\omega_\perp)^{2/3}(a/l_\perp)^{2/3}$

  Easily satisfied
- t parameter achievable

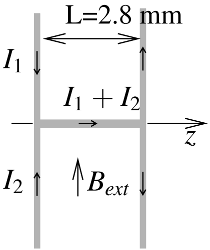
  1D condition $\Rightarrow t \ll (l_\perp/a)^2$

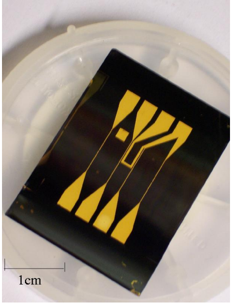
  $\Rightarrow$ difficult to obtain very high values of t experimentally.

Outline

Realisation of very anisotropic traps on an atom chip

Use of a H shape trap





Transverse confinement produced by central wire ($I_1 + I_2 = 3$A) and B_{ext} ($B_{ext} = 30$G) :

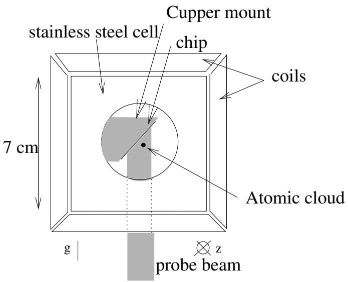
$$\omega_\perp = 2\pi \times 2800Hz$$

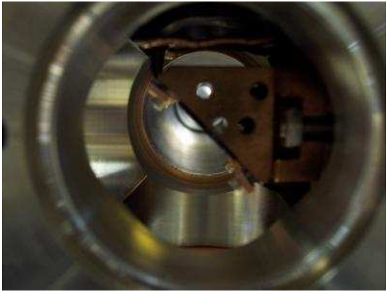
Longitudinal confinement :

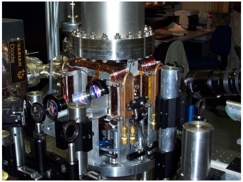
$$\omega_z \propto (I_1 - I_2)^2 : 6Hz \rightarrow 20Hz$$

Lower value limited by potential roughness

Experimental apparatus



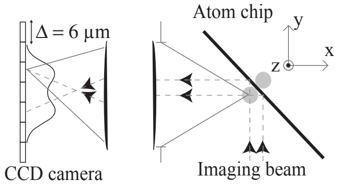




- ^{87}Rb atoms loaded from a dispenser source.
- Surface MOT transfered into the magnetic trap (3×10^6 atoms)
- Radio-frequency evaporative cooling : $T \simeq 1.5\hbar\omega_\perp$ for a few thousand atoms

Absorption images

- Imaging geometry



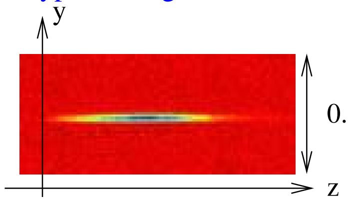
2 pictures taken :

* With atoms and trap still on
* Without atoms (delay of 200 ms)

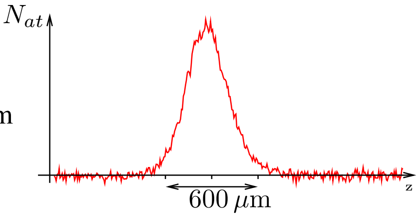
Atom per pixel : $n_{at} = \frac{\Delta^2}{\sigma} \ln(\frac{I_2}{I_1})$.

Error if variation of density on a scale smaller than pixel size and high optical density

- Typical image



- Transverse integration : $N_{at} = \sum_y n_{at}$

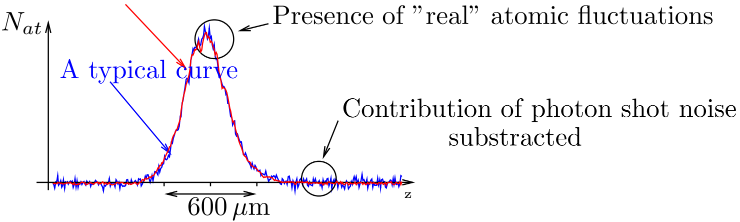


Noise measurement

Inspired by *Föling et al. Nature* **434**, *481*, M. Greiner et al., PRL **94**, 110401

Statistical analysis over about 300 images taken in the same condition.

Reference curve : average over 20 images (running average) and normalised to the same N_{tot}

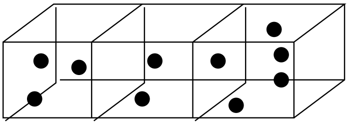


Running average : remove small long term drift
Normalisation to N_{tot} : remove shot to shot total atom number fluctuations

Atom-number fluctuations in an ideal Bose gas

Pixel size $\Delta \gg l_c$
Thermodynamic in each pixel.

- For each eigenstate : $\langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle + \langle n \rangle^2$
- G occupied eigenstates



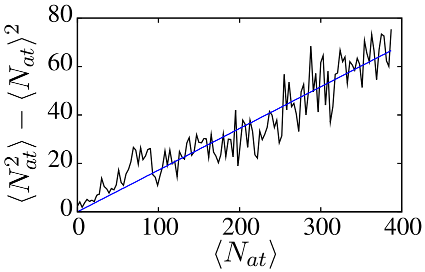
$$\langle N_t^2 \rangle - \langle N_t \rangle^2 = \langle N_t \rangle + \sum_i \langle n_i \rangle^2$$

For equally occupied states ,

$$\langle N_t^2 \rangle - \langle N_t \rangle^2 = \langle N_t \rangle + \frac{1}{G} \langle N_t \rangle^2$$

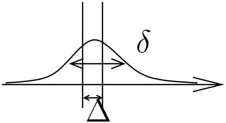
- To observe bunching : G should not be too high
- Ratio bunching/shot noise : $\langle N_t \rangle / G = psd$

Observation of atomic shot noise



High temperature cloud

Slope smaller than 1 : due to finite optical resolution δ



Contribution of each atom on the absorption in a given pixel reduced by a factor $\propto \frac{\delta}{\Delta}$

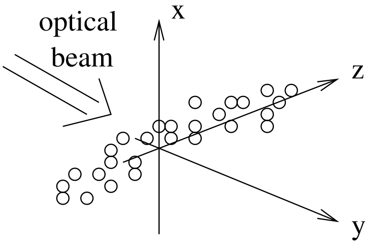
$\Rightarrow$ slope of shot noise reduced

For Gaussian resolution of rms width δ, reduction by $\kappa = \frac{\Delta}{2\sqrt{\pi}\delta}$.

Measured reduction $\kappa = 0.17 \rightarrow \delta = 10\mu$m.

Measured resolution : 8μm

Bunching effect in atom number fluctuations in the optical images



Integration along y and x :

$G_\perp \simeq \left(\frac{k_B T}{\hbar\omega_\perp}\right)^2$

Longitudinally :

Phase space volume : $\Omega = \Delta\sqrt{mk_BT} = \sqrt{2\pi}\hbar\ \Delta/\lambda_{dB}$

$\Rightarrow G_z \simeq \Omega/\hbar = \frac{\Delta}{\lambda_{dB}}$

Bunching term : reduced by $1/G_\perp Gz$

$$\rightarrow \langle N^2\rangle - \langle N\rangle^2 = \langle N\rangle + \frac{\lambda_{dB}}{\sqrt{2}\Delta}\left(\frac{\hbar\omega_\perp}{4k_BT}\right)^2\langle N\rangle^2$$

More precise calculation of the bunching effect

Fluctuations computed using the ideal Bose gas exact calculations

- For $|\mu| \gg k_B T$ (highly non degenerate),

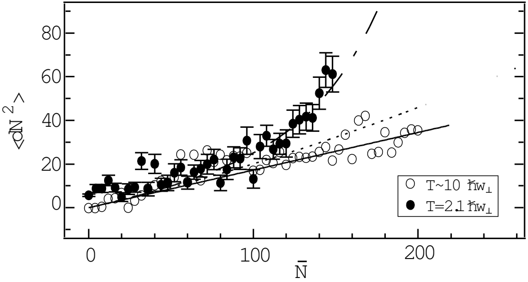
$$\langle N^2 \rangle - \langle N \rangle^2 = \langle N \rangle + \langle N \rangle^2 \frac{\lambda_{dB}}{\sqrt{2}\Delta} \tanh^2(\hbar\omega_\perp / 2k_B T)$$

  Effective number of populated states smaller when gas more degenerate.

- For $|\mu| \ll k_B T$ (highly degenerate),

$$\langle N^2 \rangle - \langle N \rangle^2 \simeq \langle N \rangle + \langle N \rangle^3 \frac{\hbar^2}{m k_B T \Delta^2}$$

Observation of atomic bunching and evidence for quantum decoherent regime



Temperature deduced from longitudinal profile.

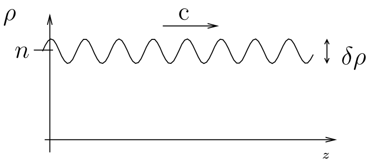
Dotted : non-degenerate gas approximation
Dashed-dotted : Exact formula

Expected atom number fluctuations in a one dimensional system in quasibec regime

Pixel size Δ much larger than healing length $\xi = \hbar/\sqrt{mgn}$
$\Rightarrow$ Relevant excitations are phonons

Energy of a phonon of wave vector k :

$$H_k = L\left(\frac{g}{2}\delta\rho_k^2 + n\frac{\hbar^2 k^2}{2m}\theta_k^2\right)$$



Thermodynamic equilibrium :

$$\frac{k_B T}{2} = L\frac{g}{2}\delta\rho_k^2 \qquad (k_B T \gg gn)$$

Atom number fluctuations :
$\mathrm{Var}N = \int_\Delta \int_\Delta \langle \delta\rho(z)\delta\rho(z')\rangle$

$$\Rightarrow \boxed{\mathrm{Var}N = \Delta \frac{k_B T}{g}}$$

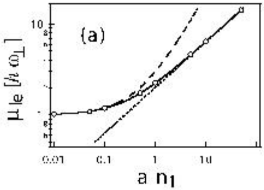
Expected atom number fluctuations in a nearly one dimensional system in quasibec regime

- $n \ll 1/a$: Purely one dimensional case recovered. ($g = 2\hbar\omega_\perp a$)
- n of the order or larger than $1/a$: Transverse breathing associated with a longitudinal phonon has to be taken into account.

Thermodynamic argument :

$$\mathrm{Var}(N) = k_B T \left.\frac{\partial N}{\partial \mu}\right)_T$$

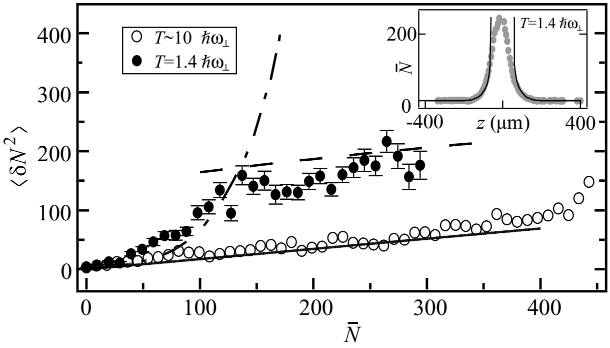
Analytic expression :
$\mu \simeq \hbar\omega_\perp \sqrt{1 + 4na}$



$$\Rightarrow \mathrm{Var}(N) = k_B T \Delta \frac{\sqrt{1 + 4na}}{2\hbar\omega_\perp a}$$

In good agreement with a 3D Bogoliubov calculation of $\mathrm{Var}(N)$.

Experimental results in the quasibec regime

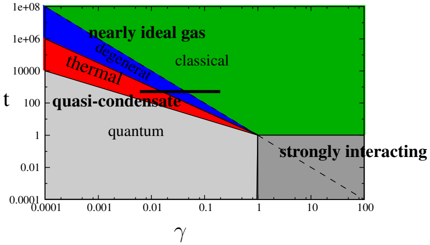


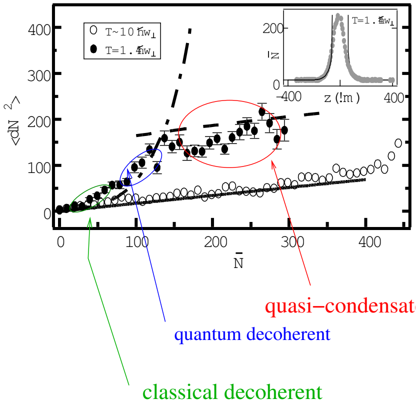
- Temperature fitted from the wings of the profile
- Good agreement with theory for low temperature

Estève et al. PRL **96**, *130403*

Conclusion on density fluctuations measurement

Most features of weakly interacting 1D Bose gases observed





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Success of Hartree-Fock theory in 3D Bose gases

3D ideal Bose gases : BEC for $\rho_c = 2.612.../\lambda_{dB}^3$
For weak interactions ($\rho a^3 \ll 1$), Mean-field theories accurate.

•For $\rho < \rho_c$: Hartree-Fock theory

variational method : non interacting Bosons that experienced V_{eff}.

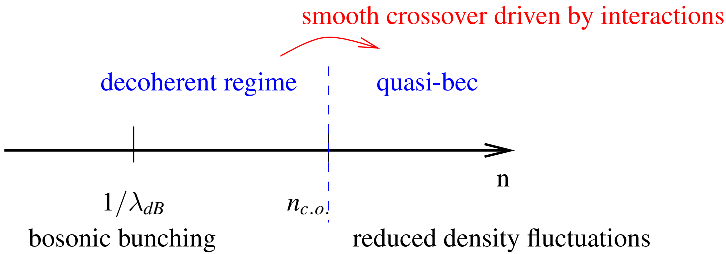
$$\Rightarrow \boxed{V_{eff}(r) = 2gn(r)}, \qquad g = 4\pi\hbar^2 a/m$$

- ρ_c unchanged
- shift of chemical potential by $2g\rho$.
- for a gas trapped in harmonic potential, small shift of N_c (*Gerbier et al., Phys. Rev. Lett. 92, 030405 (2004)*)

• Beyond mean-field

- Validity of Mean-Field (Landau-Ginzburg criteria) : $|T - T_c|/T_c > a\rho^{1/3}$.
- Beyond mean-field effects :
  - small shift of T_c,
  - change of critical exponent (*Donner et al., Sience 315 :1556 (2007)*)

Expected failure of Hartree-Fock theory to describe cross over towards quasi-bec in a 1D gas



- Hartree-Fock theory

Gas described by an ideal Bose gas

$\Rightarrow$ atomic correlations introduced by interactions neglected

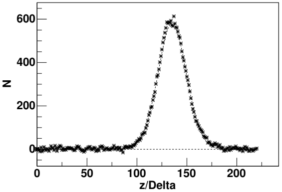
$\Rightarrow$ failure in 1D

Density profile through the transition towards quasibec

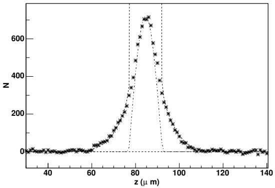
Very elongated trap : $2\pi * \omega_\perp = 2.75\text{kHz}, 2\pi * \omega_z = 15\text{Hz}.$

In situ density profile by absorption imaging

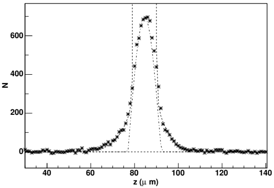
$k_BT = 5.7\hbar\omega_\perp$ $\mu = -3.6\hbar\omega_\perp$	$k_BT = 2.75\hbar\omega_\perp$ $\mu = 1.6\hbar\omega_\perp$	$k_BT = 2.3\hbar\omega_\perp$ $\mu = 1.4\hbar\omega_\perp$



Good agreement with ideal Bose gas



Appearance of quasi-bec at the center



Good agreement with quasibec shape

Quasi-bec shape : $\mu_{loc} = \mu - m\omega^2 z^2/2 = \hbar\omega_\perp\sqrt{1 + 4na}$

Calculation of Hartree-Fock longitudinal profile

We assume :

- Longitudinal local density approximation $\mu_{\text{loc}} = \mu - 1/2m\omega_z^2 z^2$

Local linear density is that of gas of independent Bosons that experience

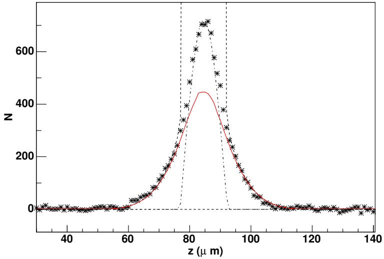
$$H_{HF} = \frac{\hat{p}_z^2}{2m} + H_{2D}^{harm} + 2g\rho(r)$$

ρ computed

- by iteration (μ small)
- by minimization over $\rho = e^{-r^2/2\sigma^2}(a + br^2 + cr^4 + dr^6)$

Assumptions verified *a posteriori*. (1D diagonalization of the effective longitudinal potential)

Failure of Hartree-Fock theory : a quasibec without condensation



Hartree-Fock calculation

*Population in the ground state $N_0/N_{tot} \simeq 3 \times 10^{-3} \ll 1$.

The appearance of quasi-bec is not explained by Hartree-Fock theory.

First failure of mean field theory in a weakly interacting regime.

J.-B. Trebbia et al. PRL **97**, *250403 (2006)*

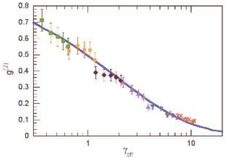
Outline

Other experimental results

- Phase fluctuations measurement in weakly interacting gases.
  $\langle \psi^{+}(z)\psi(0)\rangle = ne^{-mTz/2n\hbar^2}$

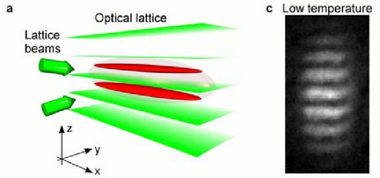
  Dettmer et al. PRL **87**, 160406 (2001), Richard et al. PRL **91**, 010405 (2003)
- Quantum phase fluctuations in weakly interacting 1D gas

  S. Hofferberth et eal., Nature Physics **4**, 489 (2008)
- Strongly interacting 1D gases : fermionization



Kinoshita et al. PRL **95**, 190406 (2005)

- 2D gases : Berinskii-Kosterlitz-Thouless transition



Hadzibabic et al., Nature 441 1118 (2006)

Outline

Conclusion

Prospects

- Study of 1D gases in the strong interaction regime.
  expected : $\omega_\perp = 40$ kHz, $n = 1$ at/μm
- Study of density fluctuations in 2D gases.
  Use of rf dressed potentials
- Study of correlation length of density fluctuations in 1D gases

Collaborators

Members of the chip experiment

- Chris Westbrook
- Jérôme Estève
- Thorsten Schumm
- Jean-Baptiste Trebbia
- Carlos Garrido-Alzar
- Julien Armijo

Theoreticians collaborators

- Karen Kheruntsyan
- Gora Shlyapnikov

Micro-fabrication

- LPN laboratory
- Dominique Mailly