Strongly interacting Fermi mixtures

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Outline

- Introduction
- Strongly interacting regime in Fermi gases
- Mixture of heavy and light fermions. BEC side
- BCS side
- How wide is the strongly interacting regime?

Two-component Fermi gases. Experiments ^{40}K ^{6}Li

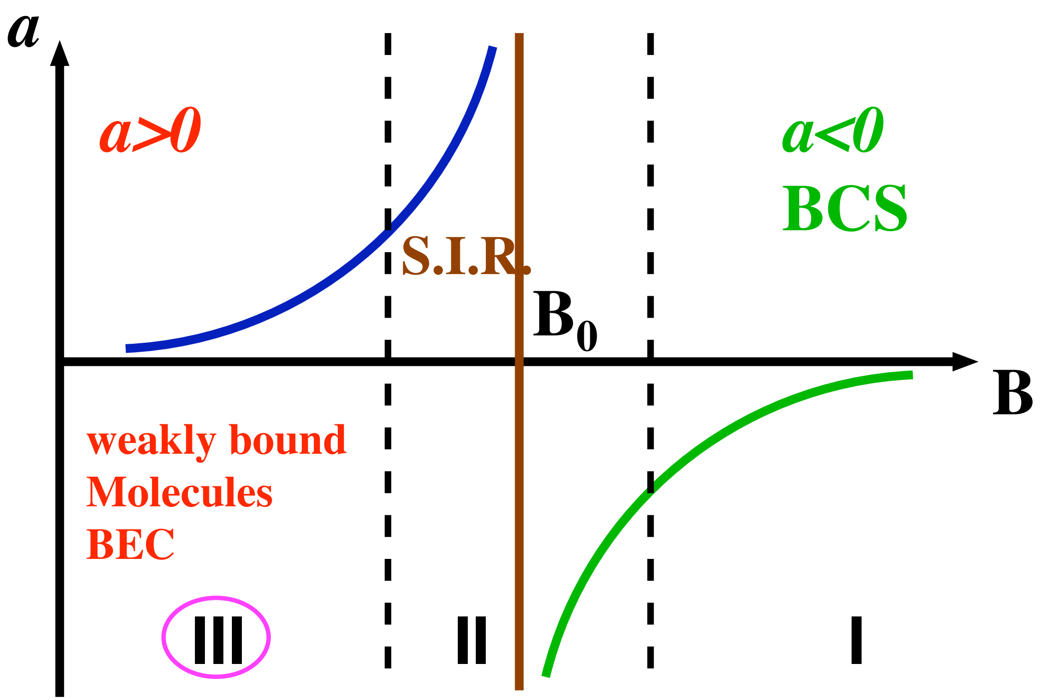
Dilute limit $nR_e^3 \ll 1$ Ultracold limit $\Lambda_T \gg R_e$

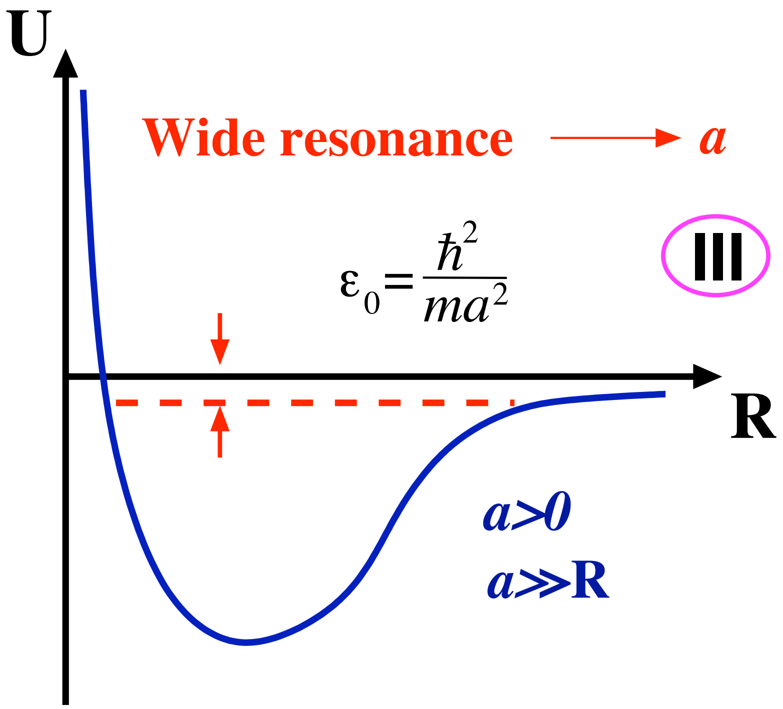
Quantum degeneracy $\rightarrow$ JILA 1998 ^{40}K

At present $n \sim 10^{13} - 10^{14}\mathrm{cm}^{-3}; \quad T \sim 1\mu\mathrm{K}$

Superfluid behavior through vortex formation $\rightarrow$ MIT

BEC of bosonic molecules $\rightarrow$ JILA, Innsbruck, ENS, MIT, Rice, Duke





Strongly interacting regime

$T = 0 \quad k_F|a| \gg 1$ → Only one distance scale $n^{-1/3}$

Only one energy scale $E_F \sim \hbar^2 n^{2/3}/m$

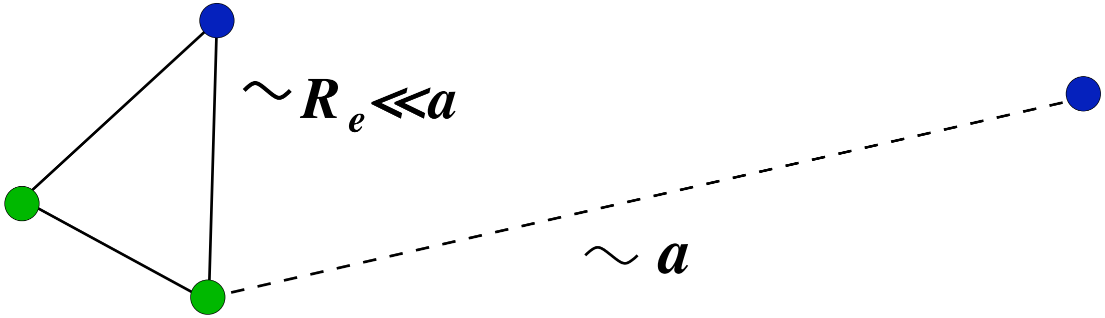
Universal thermodynamis (J. Ho)

Remarkable experiments with imbalanced gases (MIT and elsewhere)

Positive side of the resonance ($a > 0$). Gas of bosonic dimers

$na^3 \ll 1 \quad \Rightarrow$ weakly interacting Bose gas

Dimers → The highest rovibrational state ⇒ Remarkable collisional satability



$$\alpha_{rel} \sim (k_{eff} R_e)^{2?} \sim (R_e/a)^{2?} \Rightarrow C(\hbar R_e/m)(R_e/a)^s; \qquad s = 2.55$$

$\tau \sim (\alpha_{rel} n)^{-1} \sim$ seconds Petrov et al 2003)

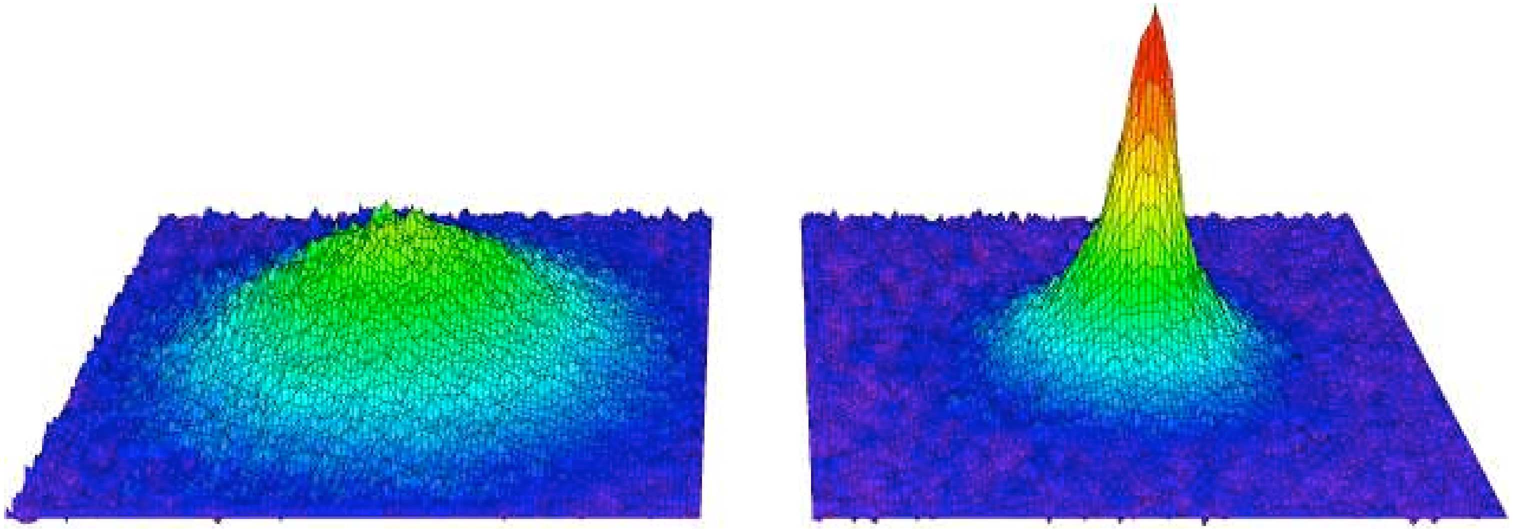
Bose-Einstein condensates of molecules

Suppressed relaxation **Fast elastic collisions** $a_{dd} = 0.6a$

Efficient evaporative cooling $\rightarrow$ BEC

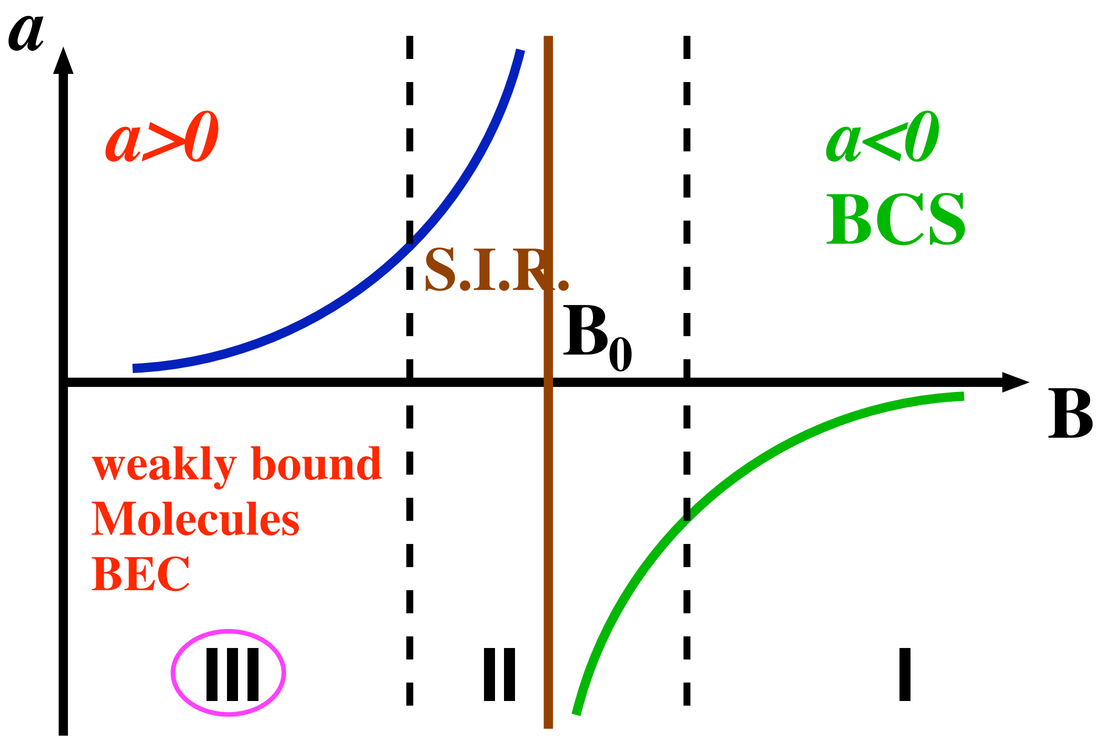
The largest diatomic molecules in the world,
with the size up to $\sim 3000\text{Å}$

BEC $\Rightarrow$ JILA, Innsbruck, MIT, ENS, Rice, Duke

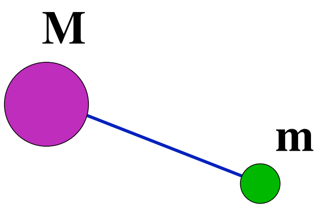


Mixtures of heavy and light fermions

Heavy and light fermions $^{6}\mathrm{Li}^{40}\mathrm{K}$ $^{6}\mathrm{Li}^{171}\mathrm{Yb}$



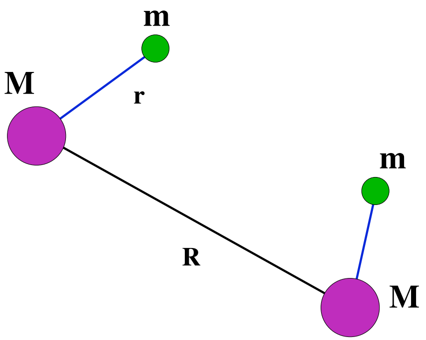
$a > 0 \Rightarrow$ weakly bound molecules BEC $a < 0 \Rightarrow$ BCS pairing



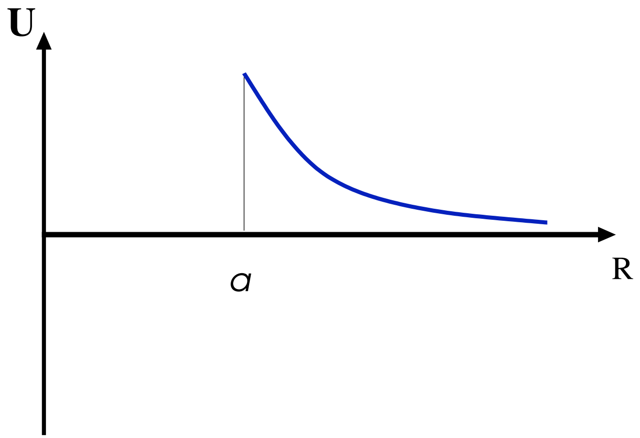
Long-range intermolecular repulsion

Molecules of heavy and light fermions Born-Oppenheimer picture

R ≫ a



$$U(R) = 2\left(\frac{\hbar^2}{maR}\right)\exp(-2R/a)$$

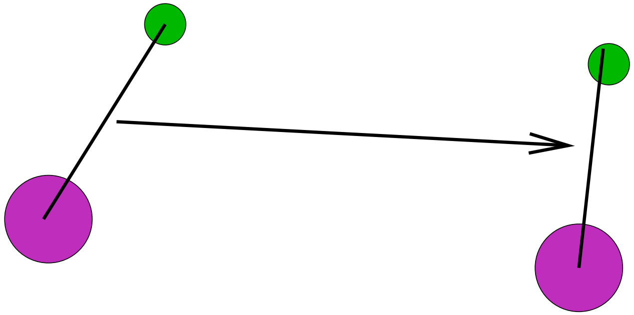


$$P \sim \exp\left(-0.9\sqrt{\frac{M}{m}}\right)$$

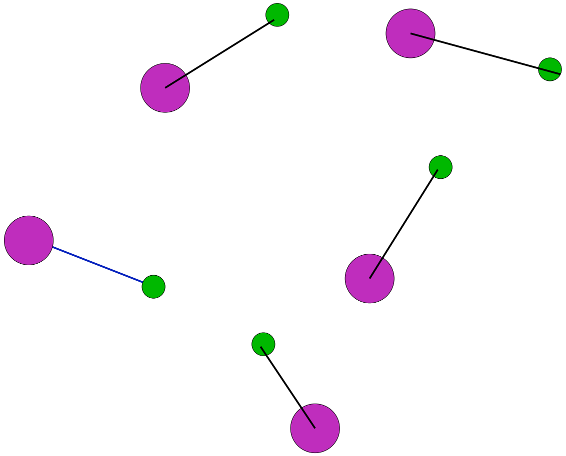
$M >>> m \rightarrow$ Collisional stability independent of a

Weakly interacting regime on the BEC side

Molecule-molecule scattering amplitude $a_{dd} \approx a \ln \sqrt{(M/m)}$



Condition of a weakly interacting regime $na_{dd}^3 \ll 1$



$k_F a \ll \dfrac{1}{\ln(M/m)}$ instead of $k_F a \ll 1$ for $M = m$

BCS regime for atomic Fermi gas at $a < 0$

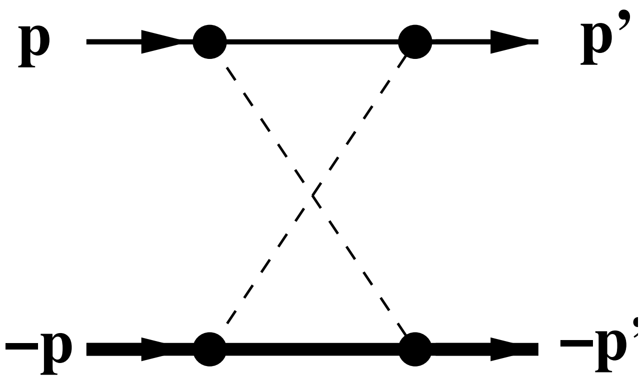
Superfluid pairing between heavy and light fermions

Transition temperature in the BCS approach $T_c \sim \sqrt{E_M E_m}\, \exp(-\pi/2k_F|a|)$

Effective interaction between heavy and light fermions in the medium

Gorkov-Melik-Barkhudarov second order contribution $\sim g^2\nu \sim g(k_F|a|)$

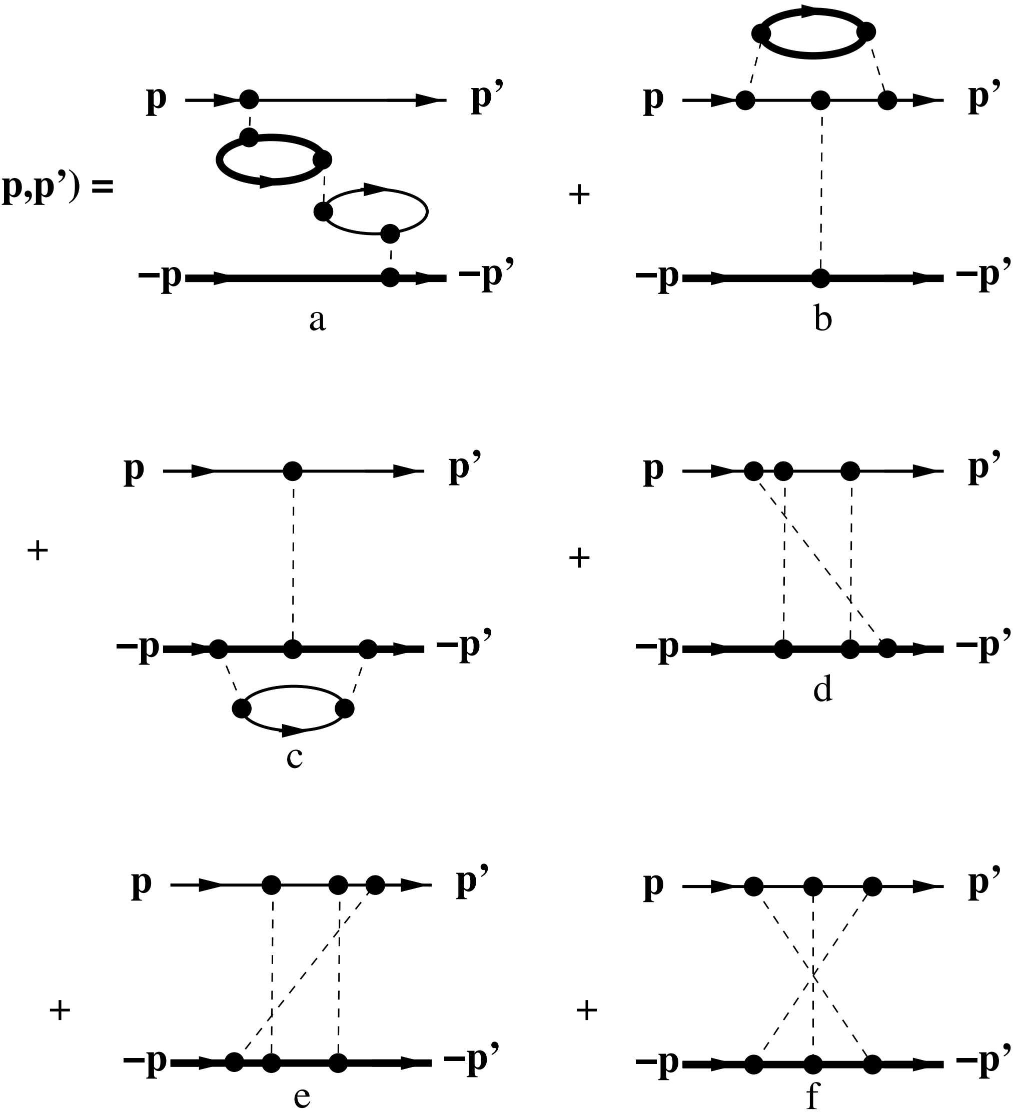
$\delta \mathbf{V}(\mathbf{p},\mathbf{p}') =$



$T_c = 0,825 E_M \exp(-\pi/2k_F|a|)$ M.Baranov, C.Lobo, G.S. (2008)

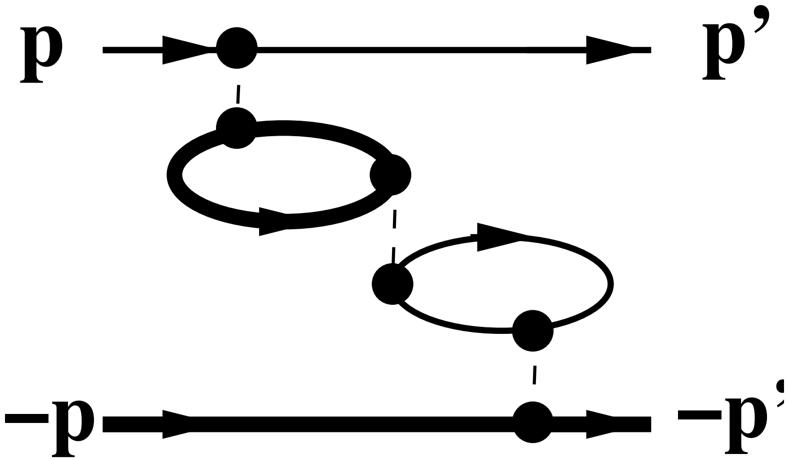
Third order processes

$\delta \mathbf{V}^{(3)}(\mathbf{p},\mathbf{p'}) =$



Small parameter of the theory

Third order processes. For example:



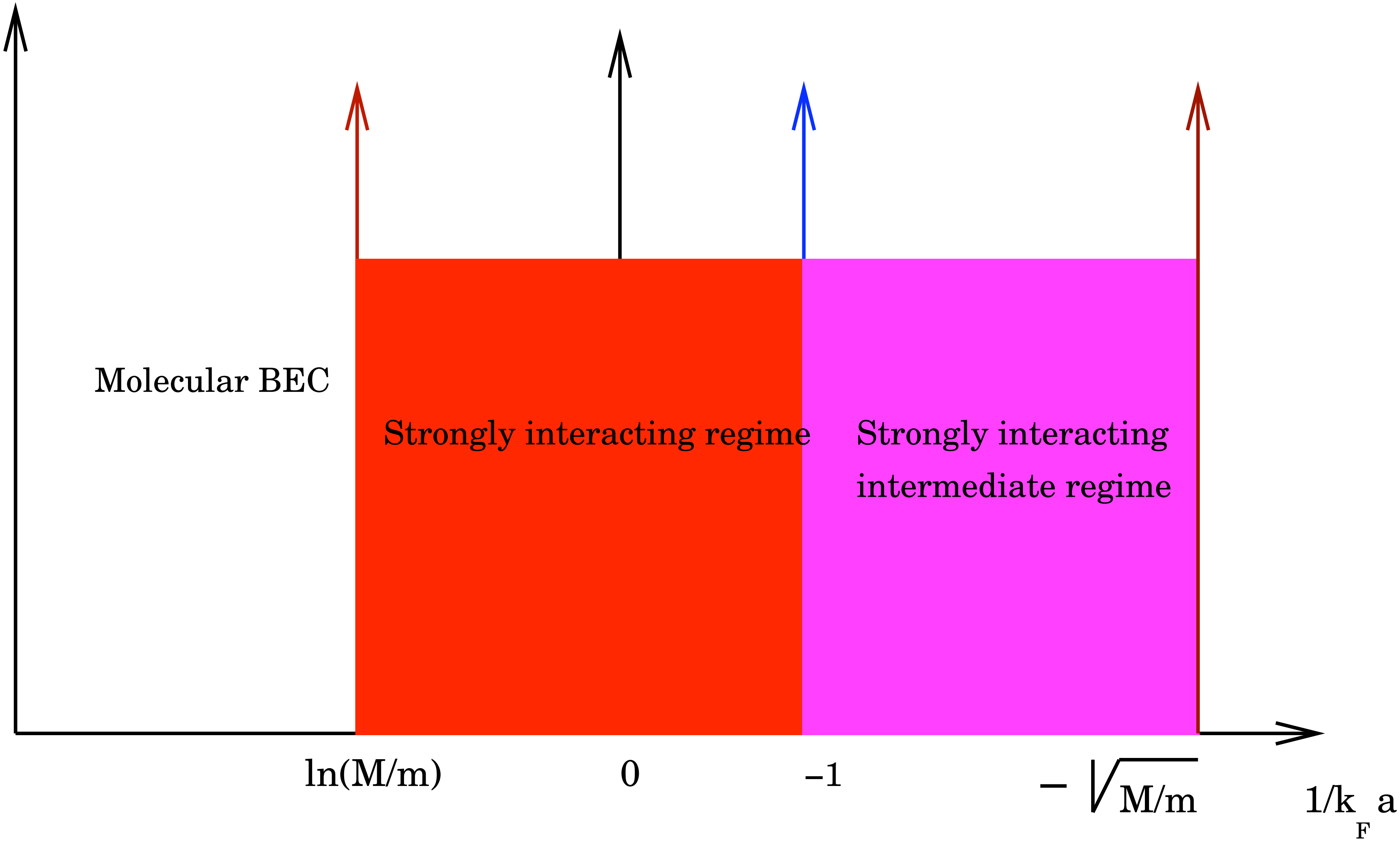
$$g_{eff} \sim g^3 \nu_M \nu_m; \quad \nu_M = Mk_F/(2\pi^2\hbar^2); \quad \nu_m = mk_F/(2\pi^2\hbar^2)$$

$$g_{eff}/g \sim (k_F a)^2 M/m$$

Small parameter of the theory

$$k_F|a|\sqrt{\frac{M}{m}} \ll 1$$

How wide is the strongly interacting regime ?



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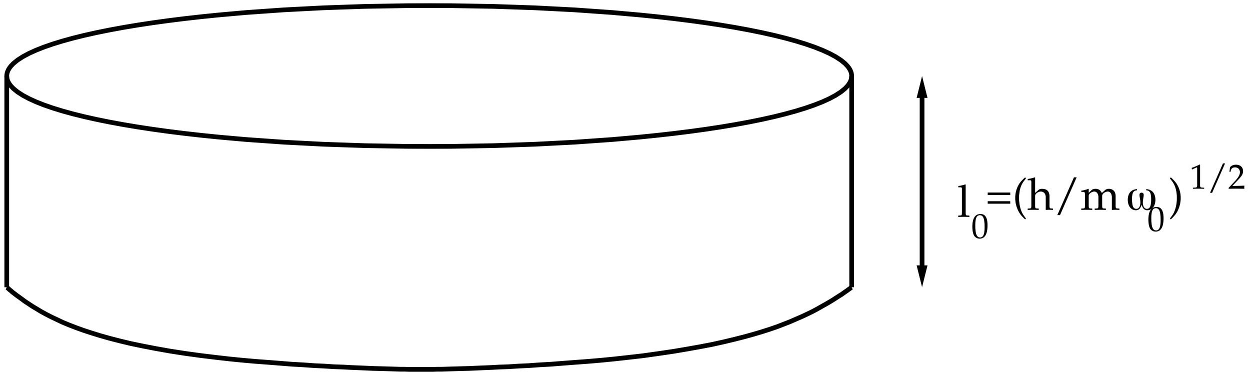
Mixture of heavy and light fermions in 2D



$$g=\frac{4\pi\hbar^2}{m}\frac{1}{\ln(\epsilon_0/E_F)}$$ Attractive interaction $\Rightarrow \epsilon_0 \ll E_F$

Small parameter of the theory $\Rightarrow \dfrac{1}{\ln(E_F/\epsilon_0)} \ll 1$

Quasi2D $m=M \Rightarrow \quad \epsilon_0 = B\left(\dfrac{\hbar\omega_0}{\pi}\right)\exp(-\sqrt{2\pi}l_0/|a|); \quad a<0$

$$\ln^{-1}(E_F/\epsilon_0)\rightarrow(\sqrt{2\pi}l_0/|a|-\ln(\hbar\omega_0/\pi E_F))^{-1}$$

No confinement-induced resonance $\Rightarrow$ small parameter $\dfrac{|a|}{l_0}\ll 1$

Superfluid transition in 2D

Kosterlitz-Thouless transition

BCS limit $\rightarrow$ transition temperature very close to T_c BCS (Miyake, 1983)

$$m = M \Rightarrow \quad T_c \sim E_F \exp\{-(1/2)\ln(E_F/\epsilon_0)\}$$

GM approach $\Rightarrow T_c = 0.3\sqrt{\epsilon_o E_F}$ (Baranov, Petrov, G.S.) (2003)

$M \gg m$ GM approach $T_C \approx \sqrt{\epsilon_0 E_F}$

Third order diagrams $g_{eff} \sim g^3 \nu_M \nu_m \sim \dfrac{g}{\ln^2(E_M/\epsilon_0)} \dfrac{M}{m}$

Small parameter of the theory $\sqrt{\dfrac{M}{m}} \ln^{-1}(E_m/\epsilon_0) \ll 1$

Quasi2D $\dfrac{|a|}{l_0}\sqrt{\dfrac{M}{m}} \ll 1$

One is able to see deviations from the BCS regime at $|a| \ll l_0$ for Li-K

Conclusions

- Strongly interacting regime becomes much wider in mixtures of heavy and light fermions
- Mixtures in quasi2D geometries are good candidates to see the effect