

Interaction-Induced Beats of Friedel Oscillations in Quantum Wires

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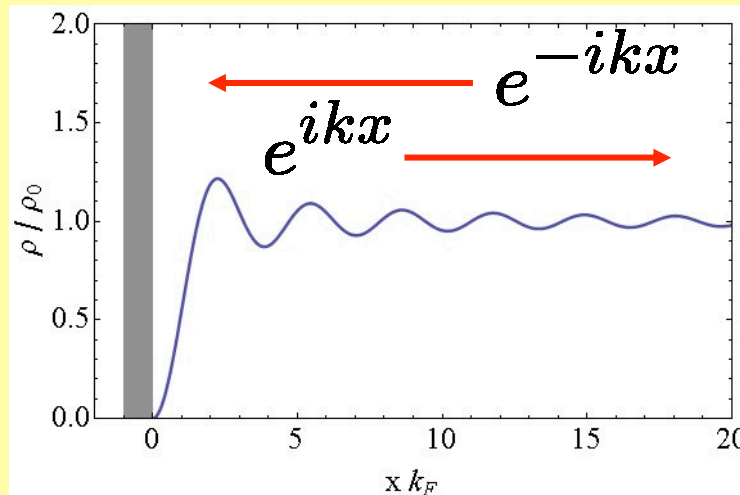
In collaboration with A.Komnik, Universität Heidelberg

PRL 100, 146602 (2008)

“Classical” Friedel oscillations

- Friedel oscillations (1D): Electron density modulation in vicinity of a hard-wall potential.

full reflection
at $x=0$



coherent superposition
of incoming & outgoing
electron wavefunctions

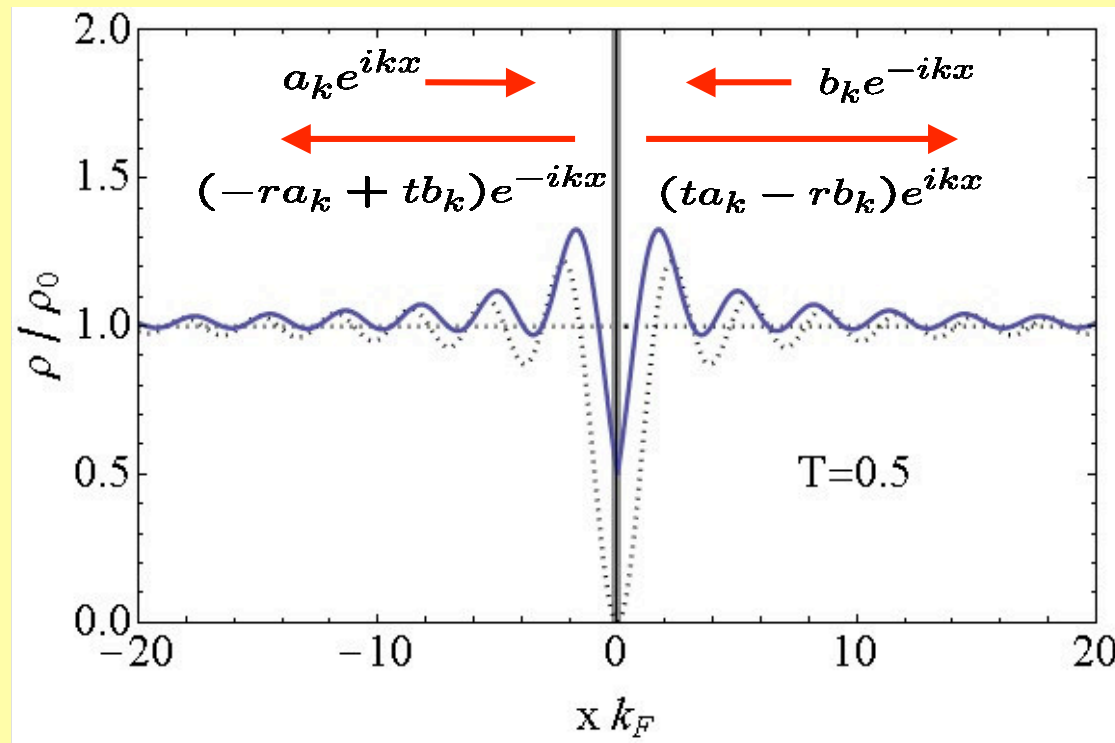
$$\frac{\delta\rho(x)}{\rho_0} \sim \frac{\sin(2k_F x)}{2k_F x}$$

(at zero temperature)

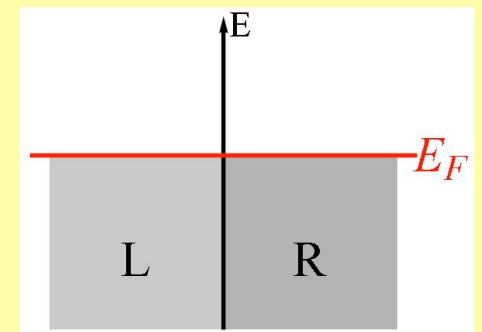
$$\text{oscillation period } \lambda = \pi/k_F$$

Friedel oscillations at finite barrier

- Finite barrier, transmission $T = \sqrt{t^*t}$

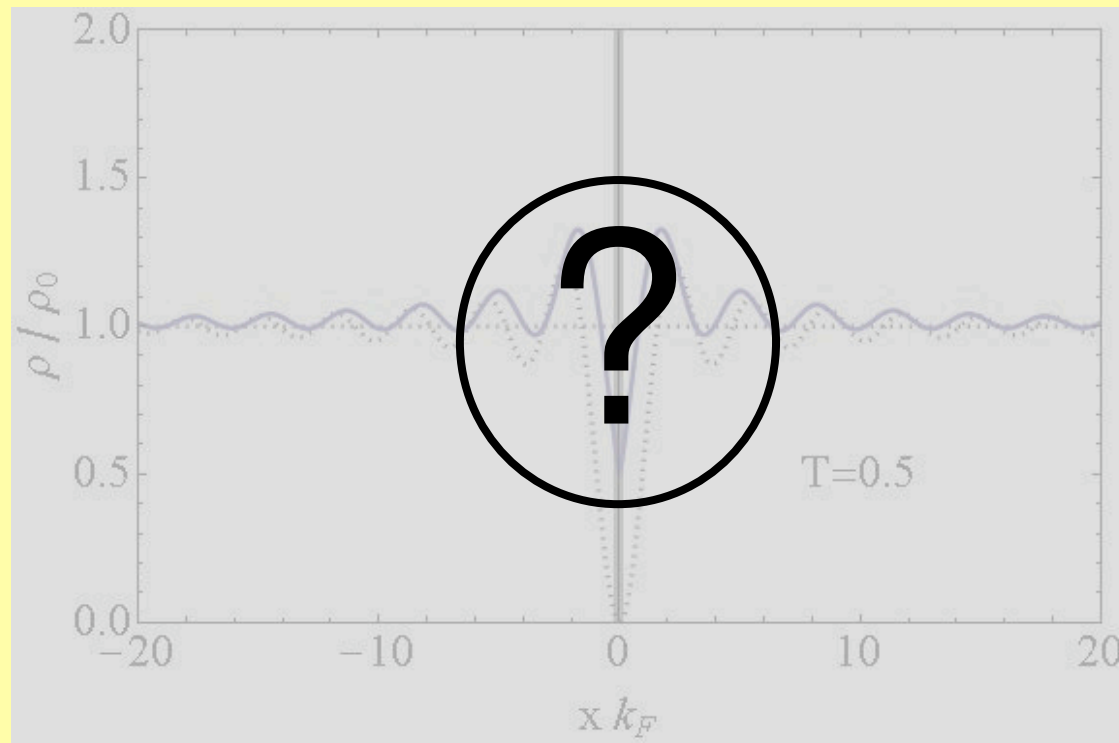


oscillation period $\lambda = \pi/k_F$

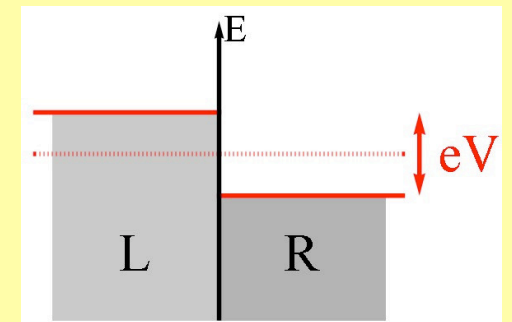


Friedel oscillations at finite barrier

- Finite barrier, transmission $T = \sqrt{t^*t}$

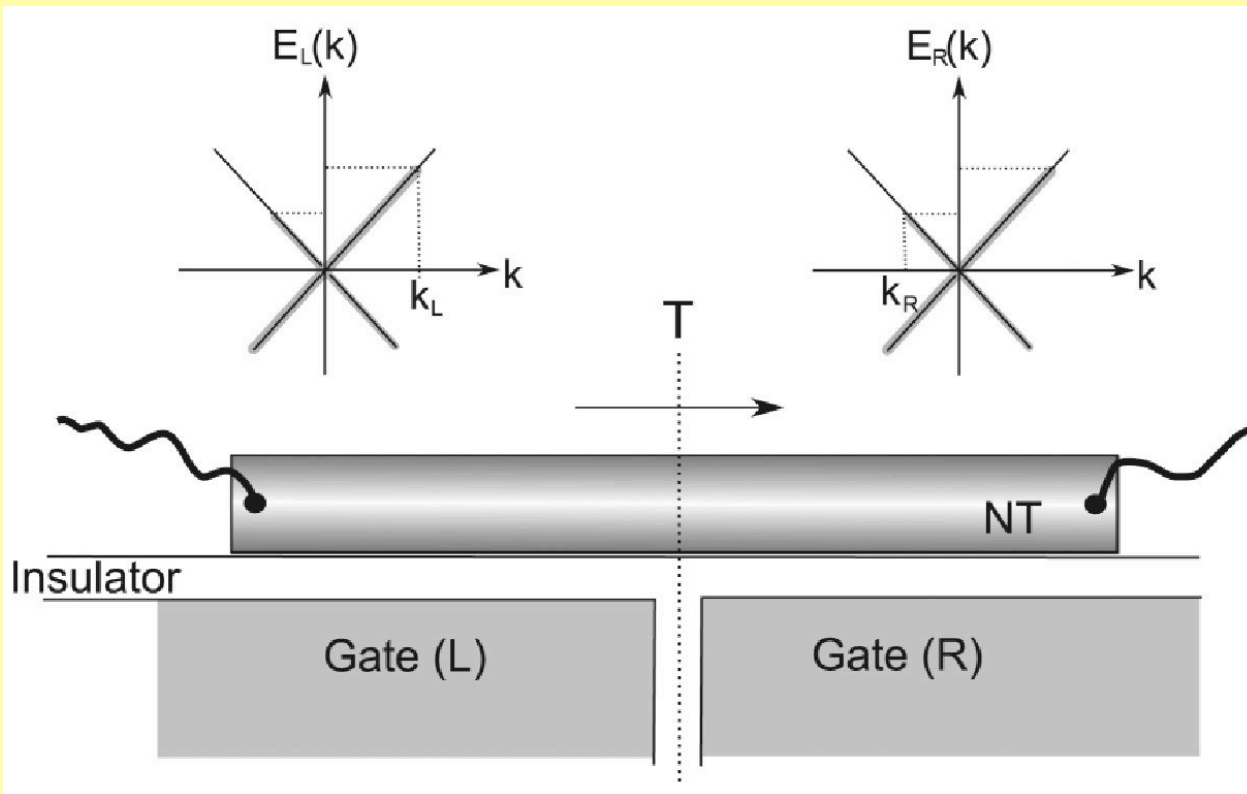


oscillation period $\lambda = ?$



Setup

let's be more specific:



linear dispersion:

$$E(k) = v_F |k|$$

finite bias voltage
+
appropriate gating

$$E_{F,L/R} = v_F k_{L/R} = v_F k_F \pm V/2$$

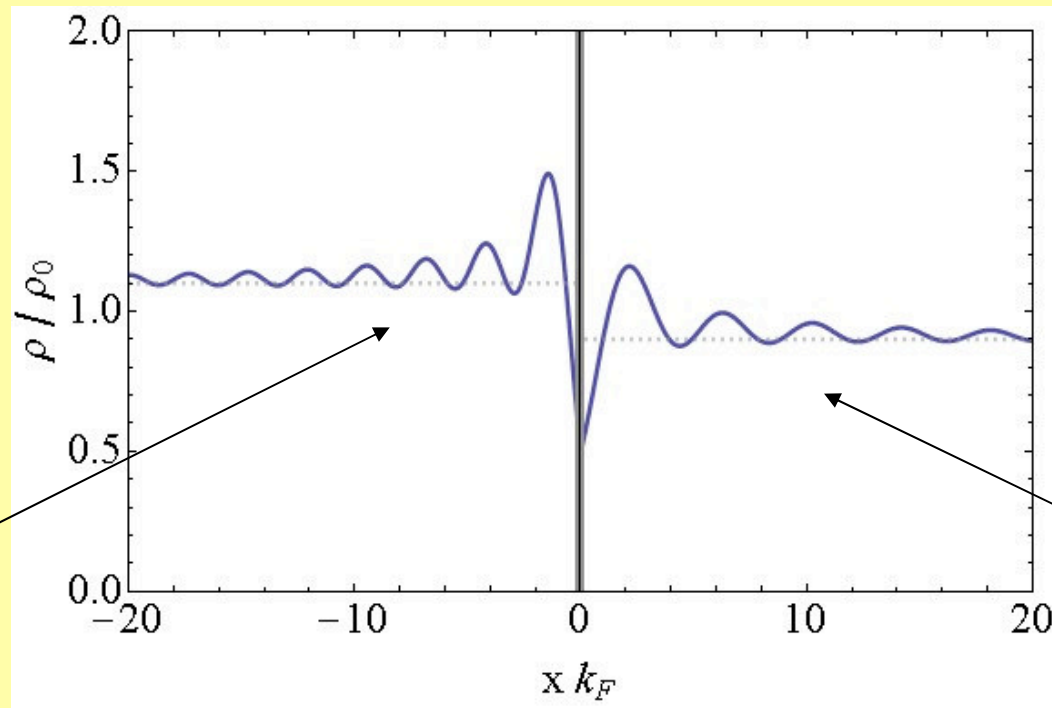
Friedel oscillations in non-equilibrium

$$\rho_{L/R} = \underbrace{\rho^{(0)}}_{(i)} \pm \underbrace{\frac{(1-T)V}{2v_F\pi}}_{(ii)} - \underbrace{\frac{\sqrt{1-T}}{2\pi} \frac{\sin(2|x|k_{L/R} - \varphi_r) + \sin(\varphi_r)}{|x|}}_{(iii)}$$

- i) The full constant average density
- ii) A constant shift proportional to $\pm V/2$ on the L/R side (\rightarrow Landauer dipole)
- iii) A space dependent oscillating part

Friedel oscillations in non-equilibrium

$$\rho_{L/R} = \rho^{(0)} \pm \frac{(1-T)V}{2v_F\pi} \frac{\sqrt{1-T} \sin(2|x|k_{L/R} - \varphi_r) + \sin(\varphi_r)}{|x|}$$



$$\lambda_L = \frac{\pi}{k_L}$$

$$\lambda_R = \frac{\pi}{k_R}$$

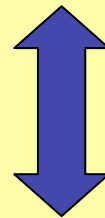
ρ_L only depends on k_L and ρ_R only depends on k_R ...

... in the absence of interactions!

e-e interactions

How to include e-e interactions?

1. perturbatively: weak interaction expansion,
treat system exactly in tunnelling amplitude



“dual” calculations

2. non-perturbatively: Tomonaga-Luttinger liquid (TLL)
weak tunnelling between two
half-infinite interacting systems

weak interaction expansion

spectrum of density oscillations:

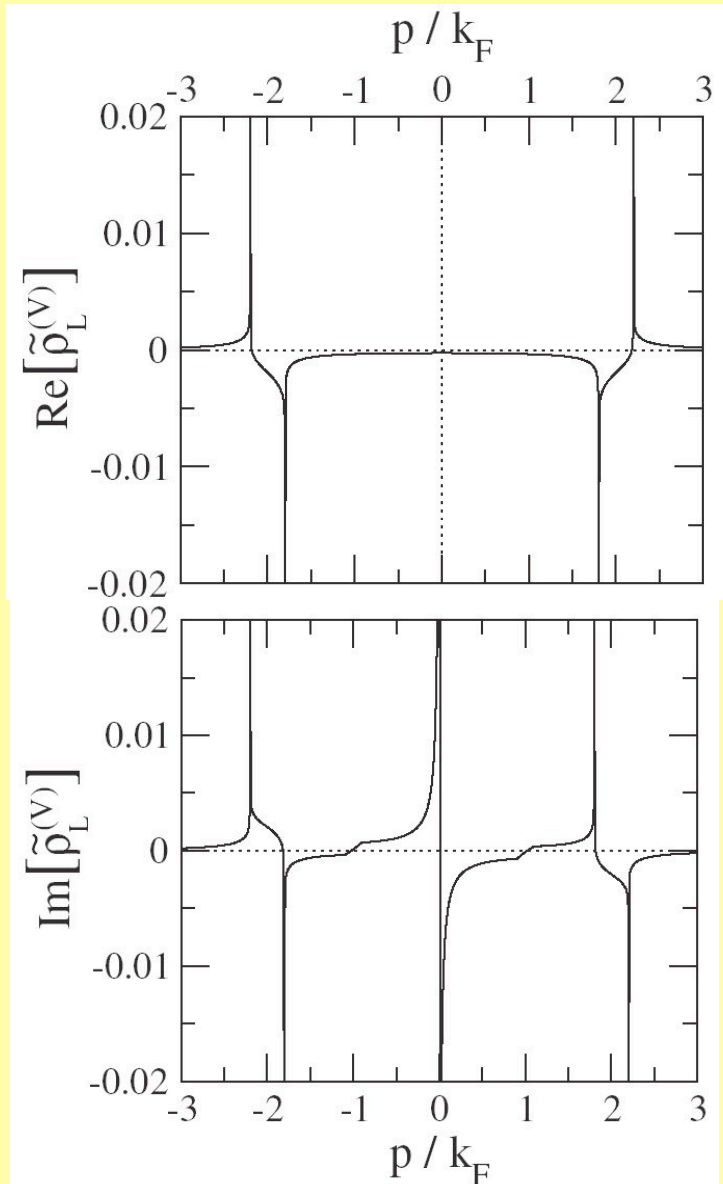
$$\begin{aligned}\tilde{\rho}_L(p) &= \int dx e^{ipx} \delta\rho_L(x) \\ &= \tilde{\rho}_L^{(LD)} + \tilde{\rho}_L^{(osc)}\end{aligned}$$

$\tilde{\rho}_L^{(LD)}$ comes from interaction with the **Landauer dipole**

$\tilde{\rho}_L^{(osc)}$ comes from interaction with oscillating part of the density

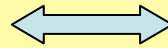
log-singularities at $p = \pm 2k_{L/R}$

→ beating pattern of density oscillation in real space

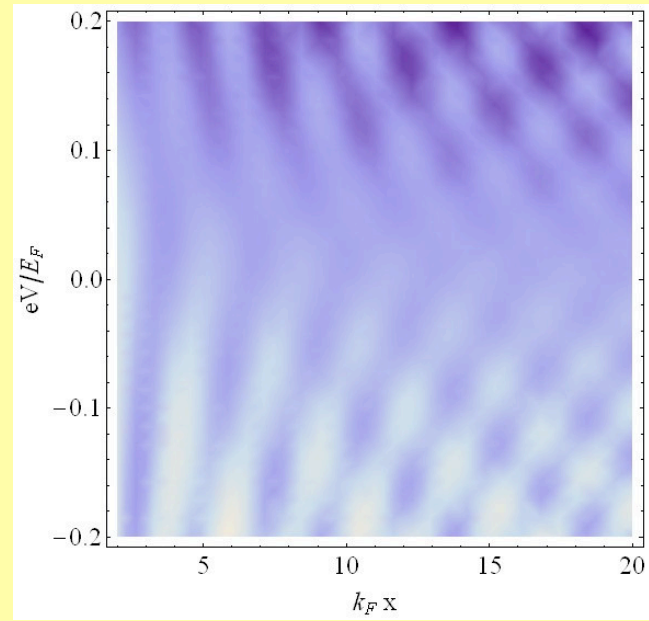
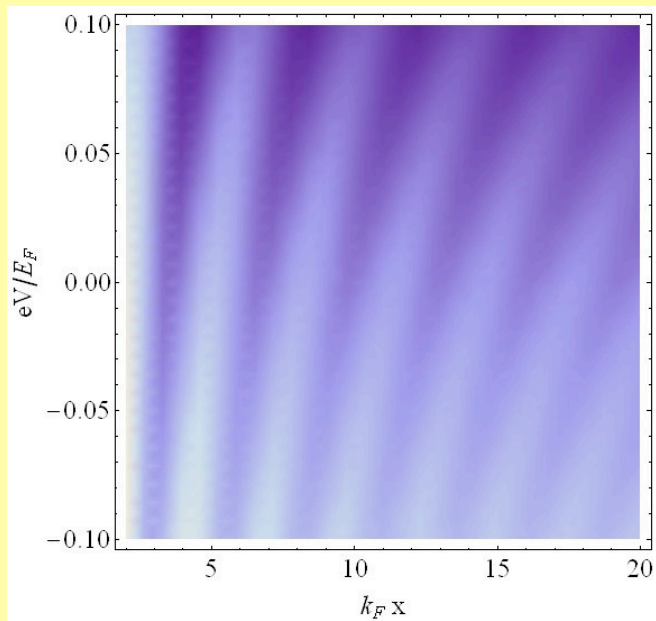
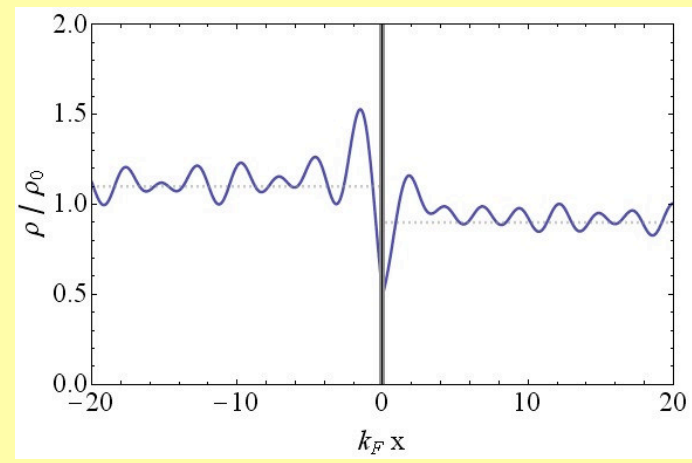
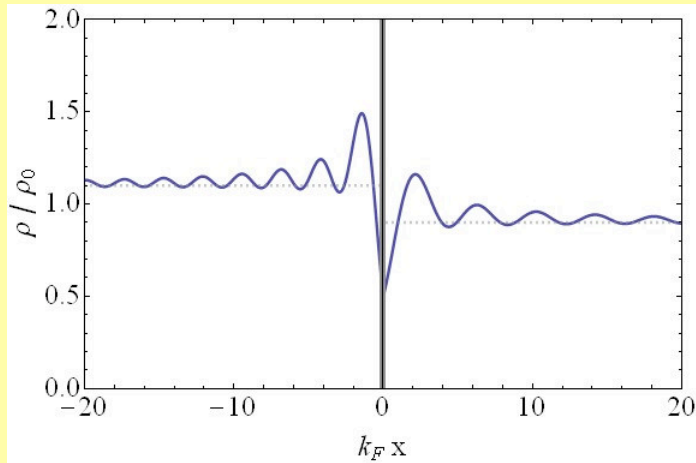


beating pattern of density oscillations

no e-e interactions

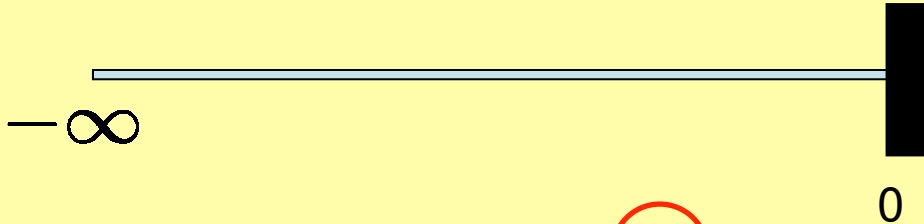


with e-e interactions



FO in a Tomonaga-Luttinger liquid

half-infinite system (one open boundary)

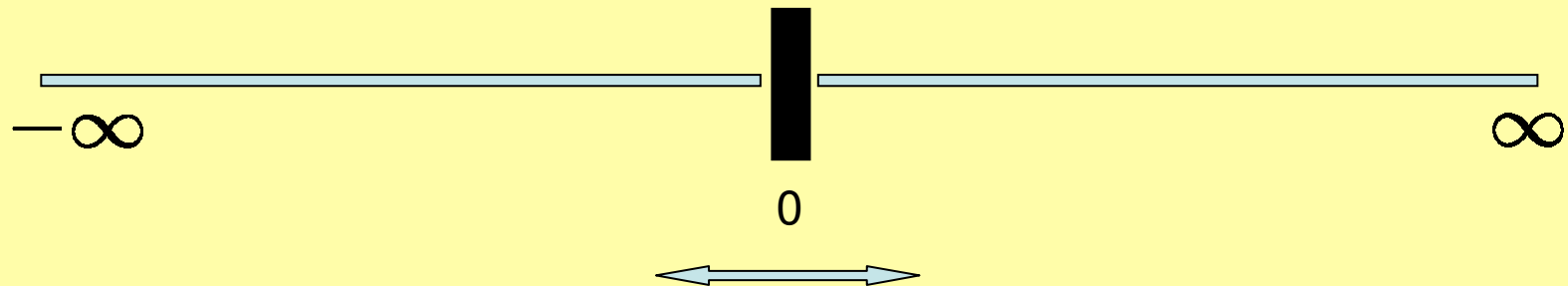


Egger, Grabert PRL 95
Fabrizio, Gogolin PRB 95

$$\rho(x) = \rho_0 - \frac{\sin(2k_F x)}{\pi a_0} \left| \frac{a_0}{2x} \right|^g$$

Interaction parameter

$$g = \frac{1}{\sqrt{1 + U/(\pi v_F)}}$$



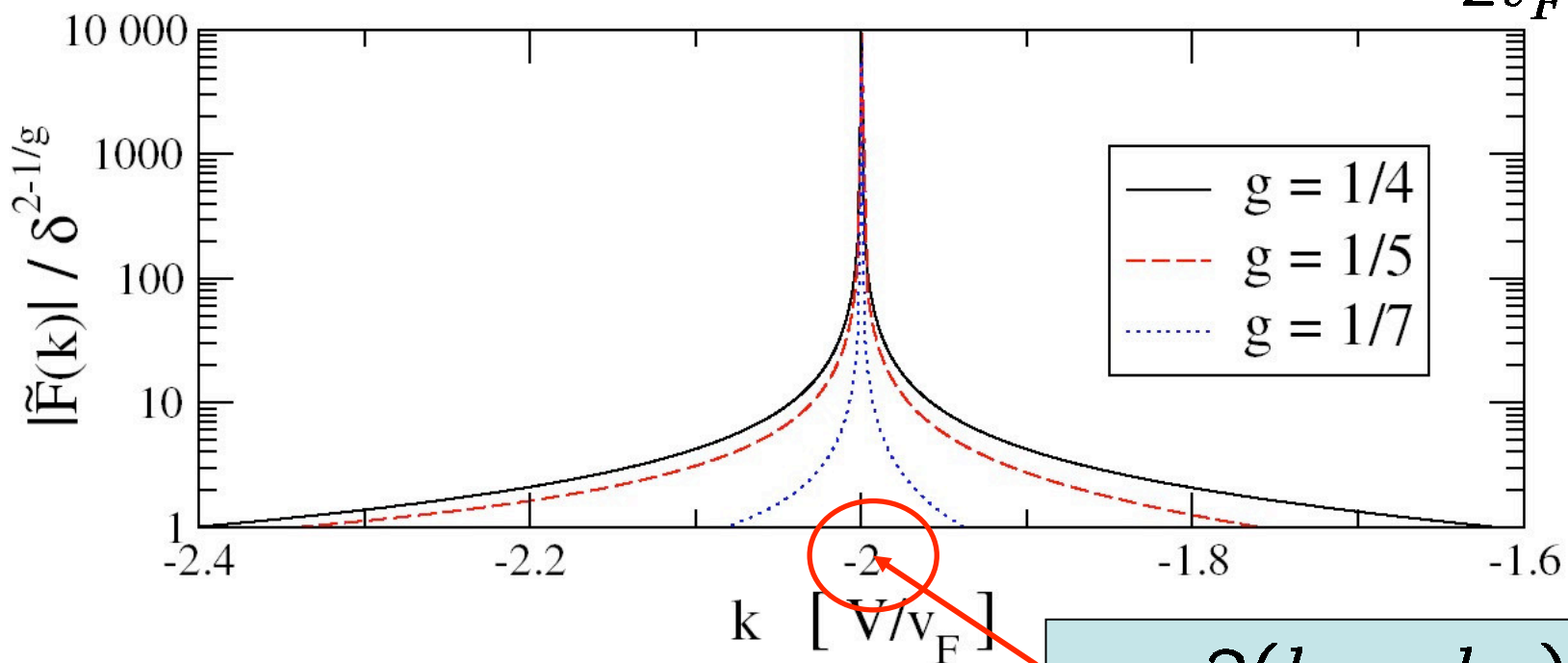
assume L and R weakly coupled $H = H_0 + H_T$ Kane, Fisher PRB 92

non-eq. FO in TLL model

$$\delta\rho_L(x) \propto \gamma^2 |x|^{-g} \sum_{\pm} (\pm) e^{\pm i2k_L x} \text{sign}(x) F(x)$$

Fourier transform of $F(x)$:

$$k_L = k_F + \frac{V}{2v_F}$$



$$= -2(k_L - k_R)$$

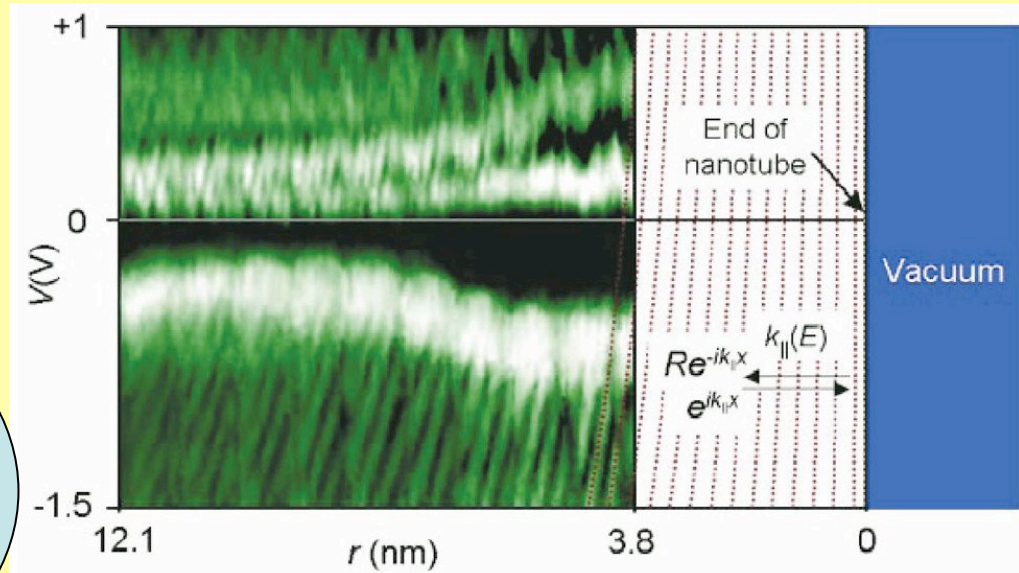
Experiments are possible...

Real Space Imaging of 1D Standing Waves:

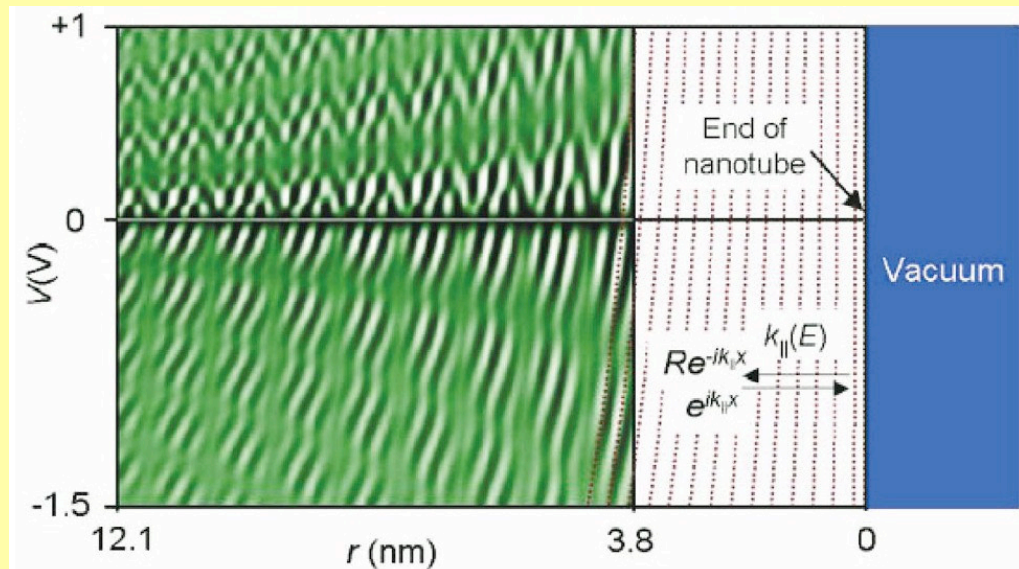
- boundary = open end of NT
- beating pattern due to underlying lattice

Jinhwan Lee, S. Eggert,
H.Kim, S.-J. Kahng,
H Shinohara, and Y Kuk

PRL 93, 166403 (2004)



Experiment



Theory

Summary

We have discussed the density profile of FO in a 1D wire with different Fermi wave vectors on either side of an imbedded impurity.

- without e-e interactions: oscillations with local value of π/k_F
- interacting system: additional peak in the spectral function
 - beating pattern in density oscillations in real space.
Interaction effect!

We have shown this for two different situations

1. using exact scattering states, treating interactions perturbatively
2. considering a weak link between two TLL

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