

**Electron coherence
in a finite-length Luttinger liquid
&
in an electronic Mach-Zehnder interferometer**

Heung-Sun Sim

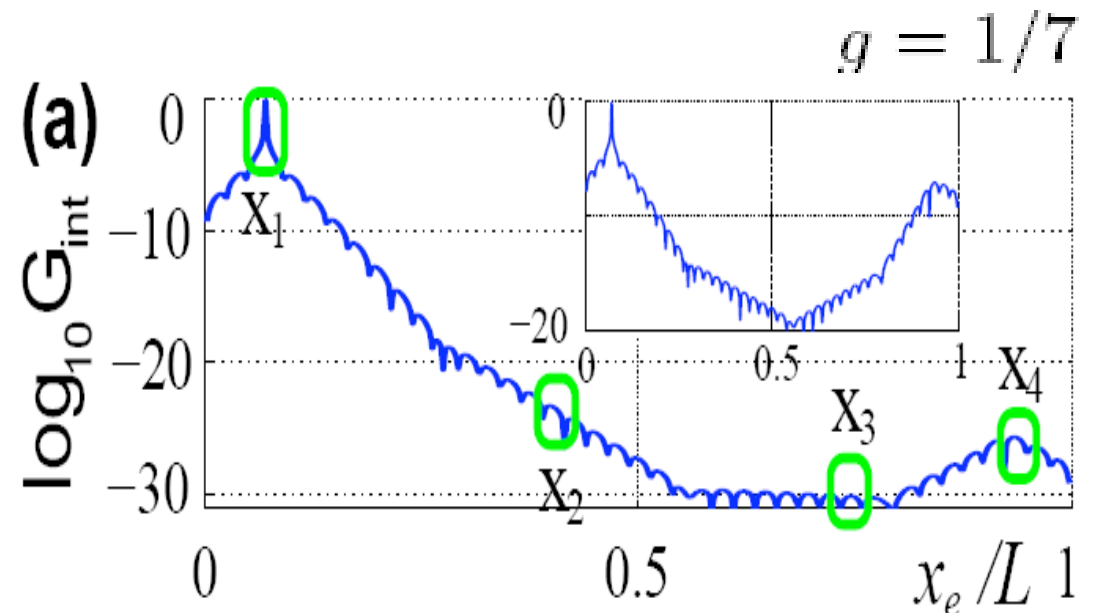
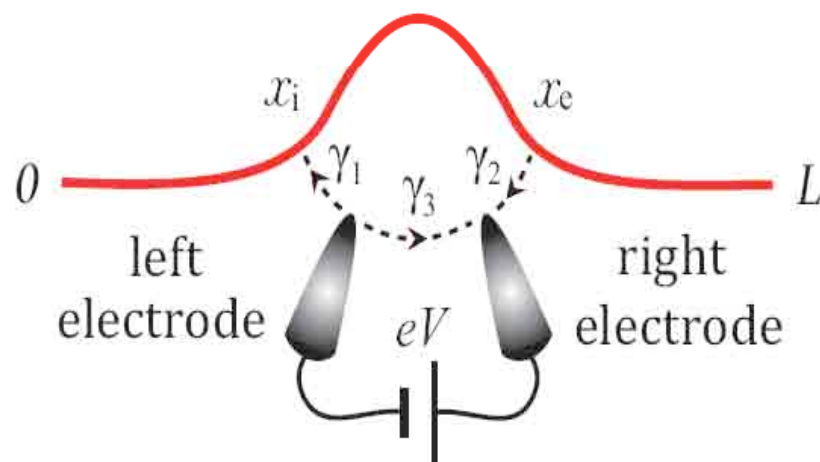
**Physics, KAIST, Korea
hssim@kaist.ac.kr**

Outline I

- Finite-length 1D quantum wire (Luttinger liquid):

electron fractionalization/recombination \rightarrow coherence?

- Electron **fractionalization**:
plasmons modes and zero modes
- Separation and recombination of the fractions:
revival of coherence, **multiple** coherence lengths

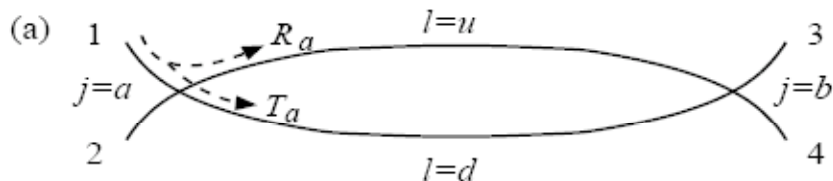


Outline II

- Electronic Mach-Zehnder interferometer in nonequilibrium

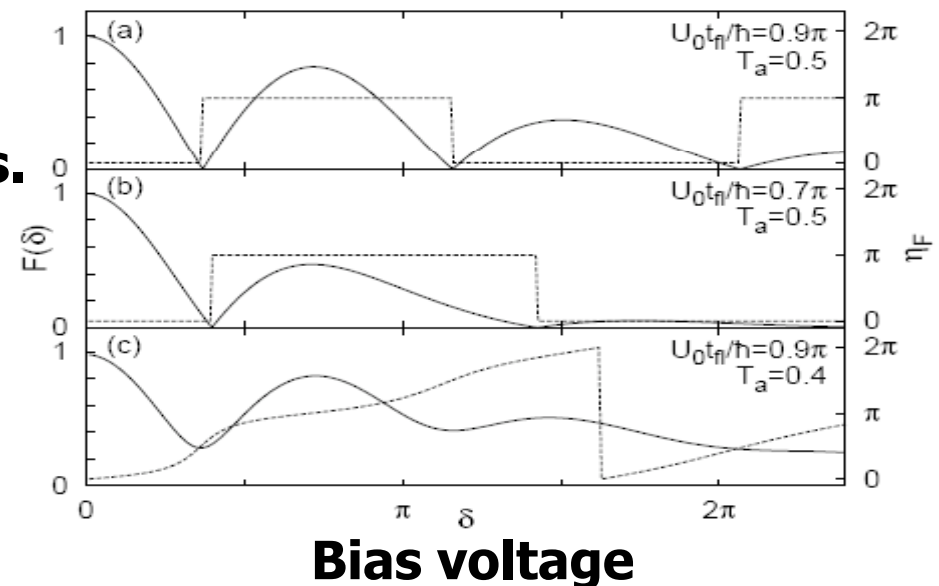
nonequilibrium charge density ensemble \rightarrow coherence?

- Shot noise at the beam splitter:
nonequilibrium ensemble of electron density
- Nonequilibrium ensemble + Interaction:
lobe pattern, π phase jump (consistent with experiments)



$$D \cos(\Phi_B + \eta_D) \equiv \langle \Re[e^{i\Phi_B} \sum_m P_m \langle e^{i\hat{\delta}(t_0)} \rangle_m] \rangle_{t_0}$$

Vis.



I. Revival of electron coherence in a finite-length 1D quantum wire

Conventional behavior of spatial coherence:

$$e^{-t/\tau_\phi}$$

exponential decay with single coherence length

(1D, disorder free, e-e interaction)

$$e^{-|x-y|/l_\phi}$$

Exp.: Hansen et al., PRB (01).

Rouilleau et al., PRL (08).

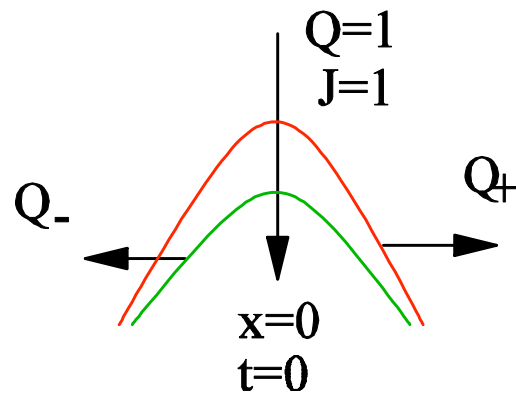
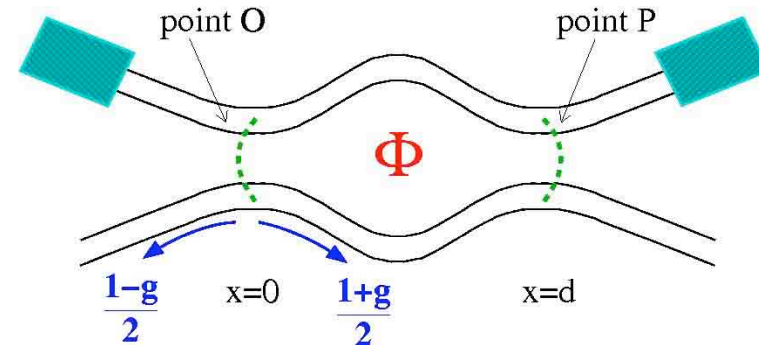
Th.: Seelig and Buttiker, PRB (01).

Karyn Le Hur, PRL (05).

Dephasing in an infinite Luttinger liquid

Dephasing results from charge fractionalization!

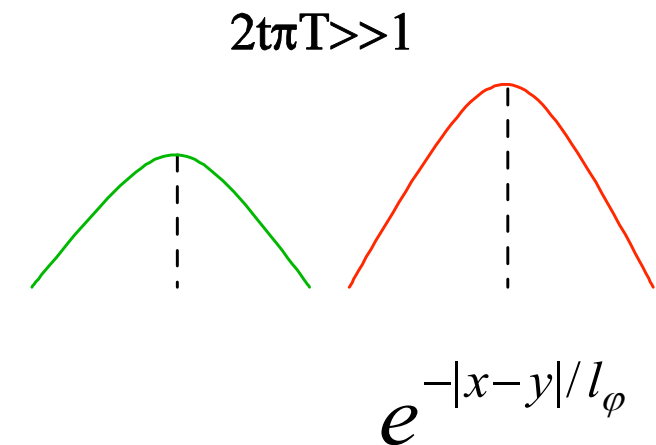
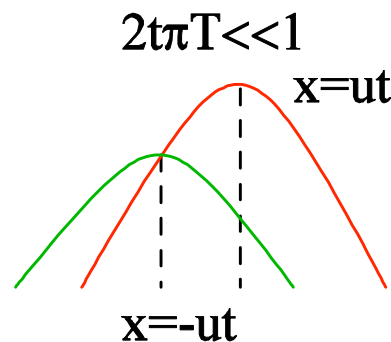
Karyn Le Hur, PRL (05)



$$Q_{\pm} = (1 \pm g) / 2,$$

$$Q_+ + Q_- = 1$$

(spinless case)



$$e^{-|x-y|/l_{\phi}}$$

$$l_{\phi} \propto 1/T$$

An electron loses its character (coherence) exponentially with time.

Reference:
(charge fractionalization in LL)

Th.: K.-V. Pham et al., PRB (00)

Exp.: H. Steinberg et al., Nature Phys. (08)

What happens in a finite-length Luttinger liquid?

Phase coherence length?

Boundary bounce (finite-size) effect?

$$k_B T \gtrsim \epsilon$$

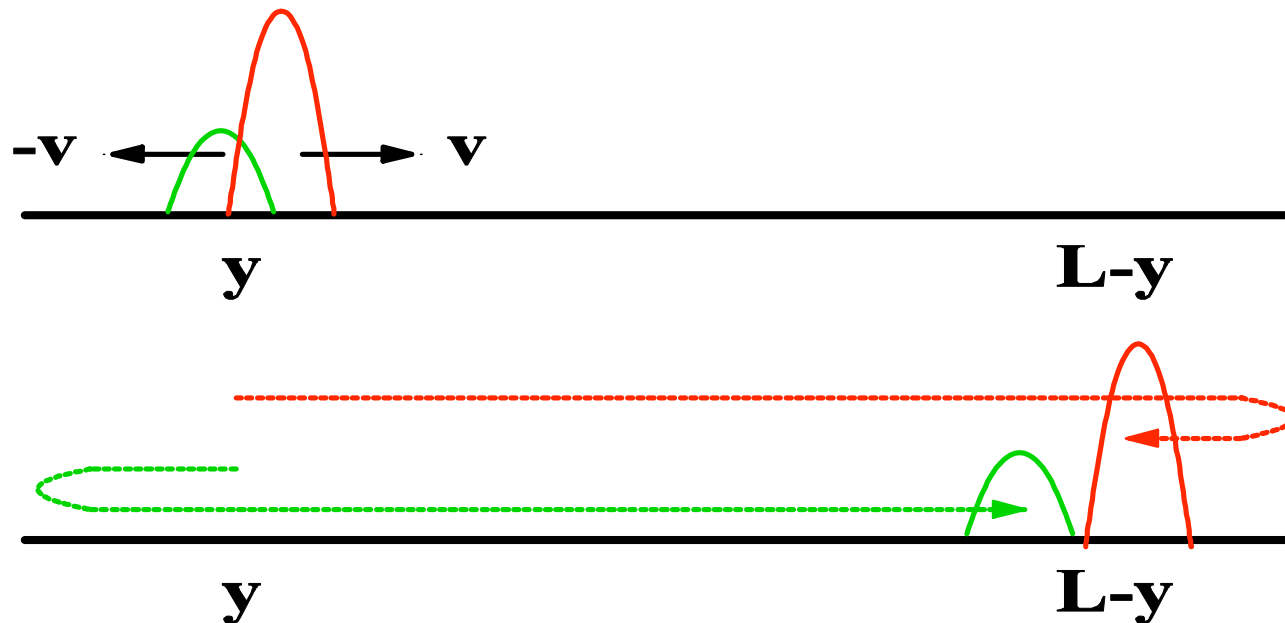
Spinless case, first

Speculations

Those absent in an infinite Luttinger liquid:

(1) Boundary reflection effects:

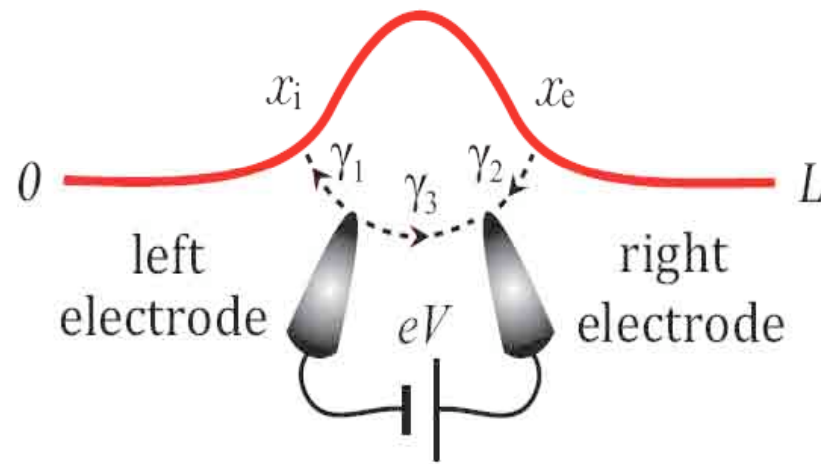
→ Revival of coherence at a position far away from the input point by more than the coherence length?



(2) Zero mode (the number of excess charges)

→ Important in a finite wire

Interferometry

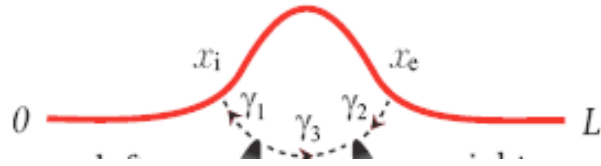


Interference current (leading-order term) via elastic cotunneling

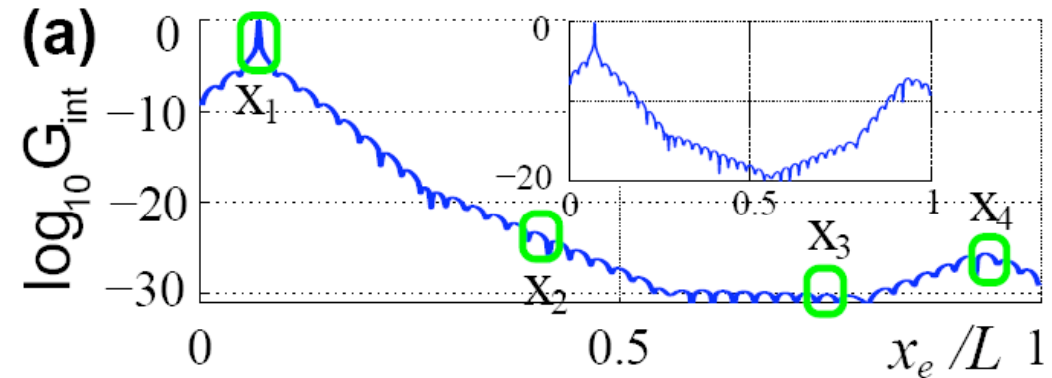
$$I_{\text{int}}(x_i, x_e) = I_0 \mathcal{P} \int d\omega d\omega' A(x_i, x_e; \omega) \frac{f_L(\omega') - f_R(\omega')}{\omega' - \omega}$$

(cf.) J. Koenig and Y. Gefen, PRB (02)

Result (spinless case): coherence revival, multiple coherence lengths



$$G_{\text{int}} \sim e^{-|x_e - x_i|/\ell_\phi(n)}$$



$$x_i = 0.07L, \quad k_B T = 5\epsilon, \quad g = 1/7.$$

For $k_B T \gtrsim \epsilon$ and $x_e \in [x_i, L - x_i]$

$$\ell_\phi^{-1}(n) = \ell_{\phi,T}^{-1} + \ell_{\phi,\text{spless}}^{-1}(n),$$

$$\ell_{\phi,\text{spless}}^{-1}(n) = \ell_{\phi,T}^{-1} \left[\frac{g^{-1} + g - 2}{2} - 2n(1 - g) \right]$$

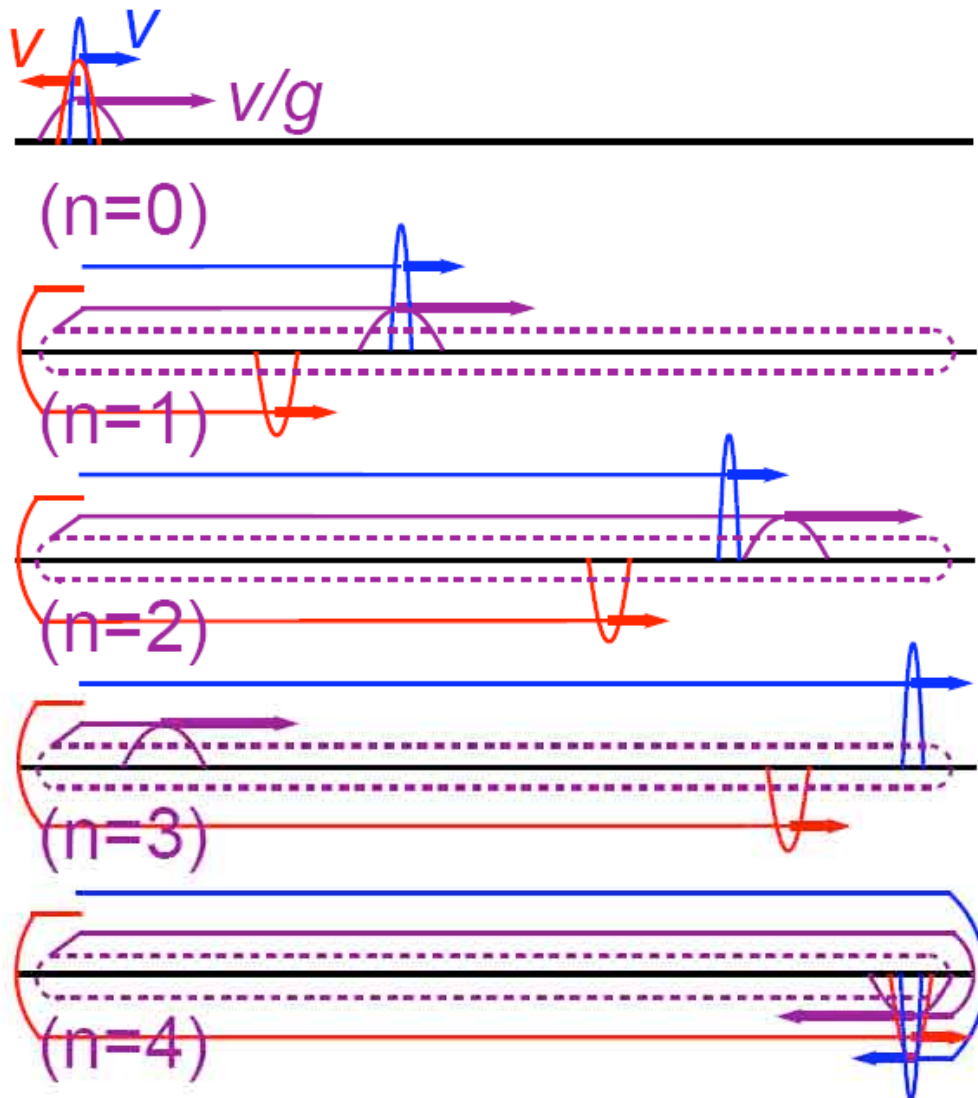
Interaction effect

$$\ell_{\phi,T} = \frac{\hbar v}{\pi k_B T}.$$

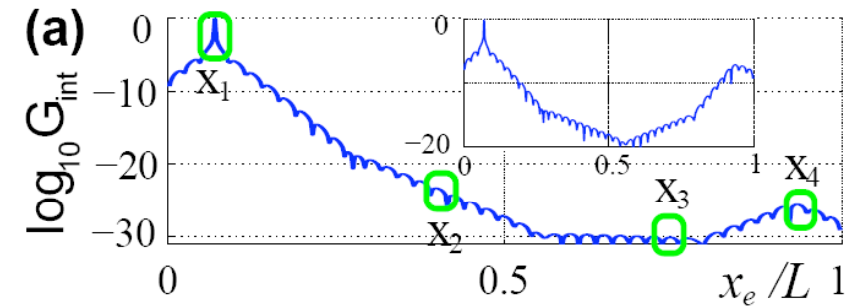
Thermal

$$\ell_\phi(n) \propto T^{-1}$$

Interpretation (spinless case): Fractionalization, separation, recombination



n : # of the round trip with length $2L$



Fractionalization
(for the tunneling into
a right-moving SP state):

Two bosonic plasmon modes

- **Blue** mode: main fraction
- **Red** mode

One fermionic zero mode

- **Purple** mode, v/g

Boundary bouncing!

Bosonization: Hamiltonian, Field operator

M. Fabrizio and A.O. Gogolin, PRB (95)

$$H = \epsilon \sum_{q>0} n_q b_q^\dagger b_q + \hbar \pi v N^2 / (2gL)$$

$$\begin{aligned} \Psi(x) &= \psi_+(x) + \psi_-(x) & \Psi(0) = \Psi(L) = 0 \\ \psi_\pm(x) &= (\pm i / \sqrt{2L}) \sum_{k>0} e^{\pm i k x} c_k \end{aligned}$$

fractionalization

zero

**plasmon
(main)**

plasmon

$$\psi_+(x, t) = \frac{e^{i(k_F + \frac{\pi}{2L})x}}{\sqrt{2\pi a}} e^{i\phi_0(x, t)} e^{i[c_+ \varphi(x - vt) + c_- \varphi(-x - vt)]}$$

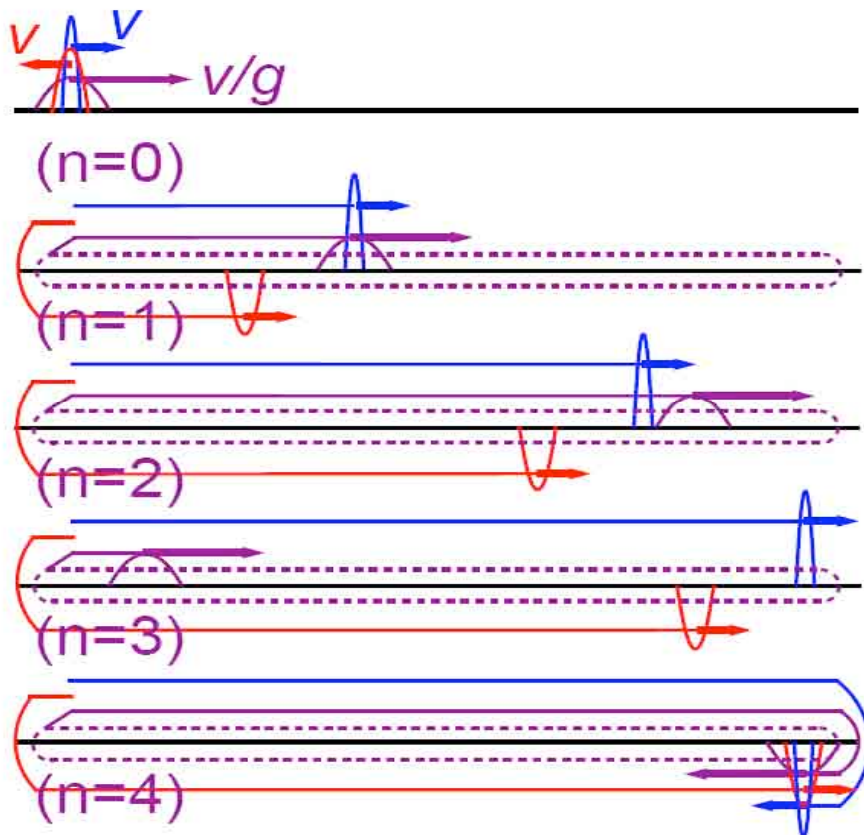
$$\phi_0(x, t) = \pi(x - g^{-1}vt)N/L - \chi \quad [\chi, N] = i$$

$$c_\pm = (g^{-1/2} \pm g^{1/2})/2$$

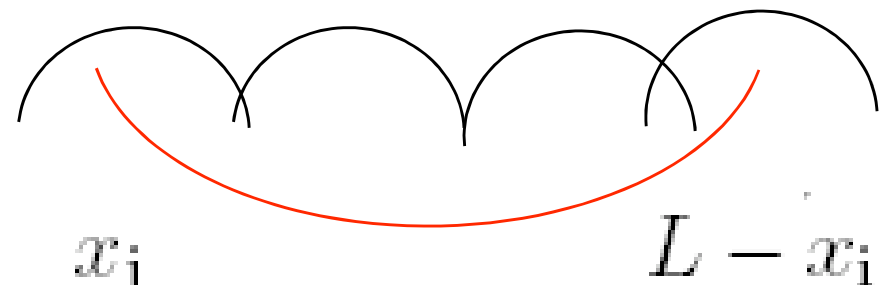
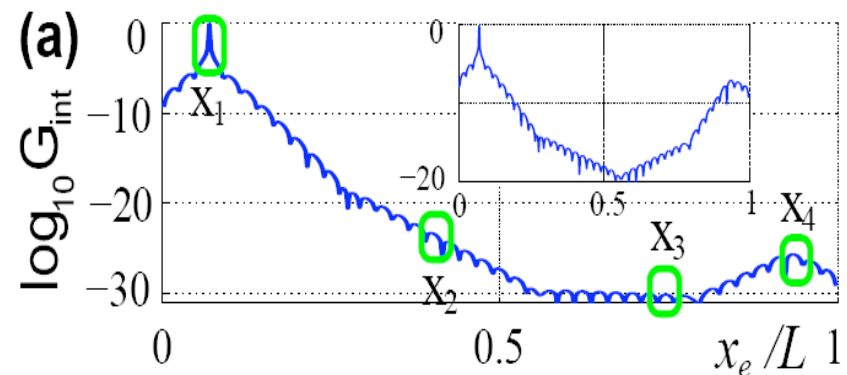
$$\varphi(z) = \sum_{q>0} \sqrt{\frac{\pi}{qL}} e^{iqz - aq/2} b_q + \text{h.c.}$$

Interpretation (spinless case):

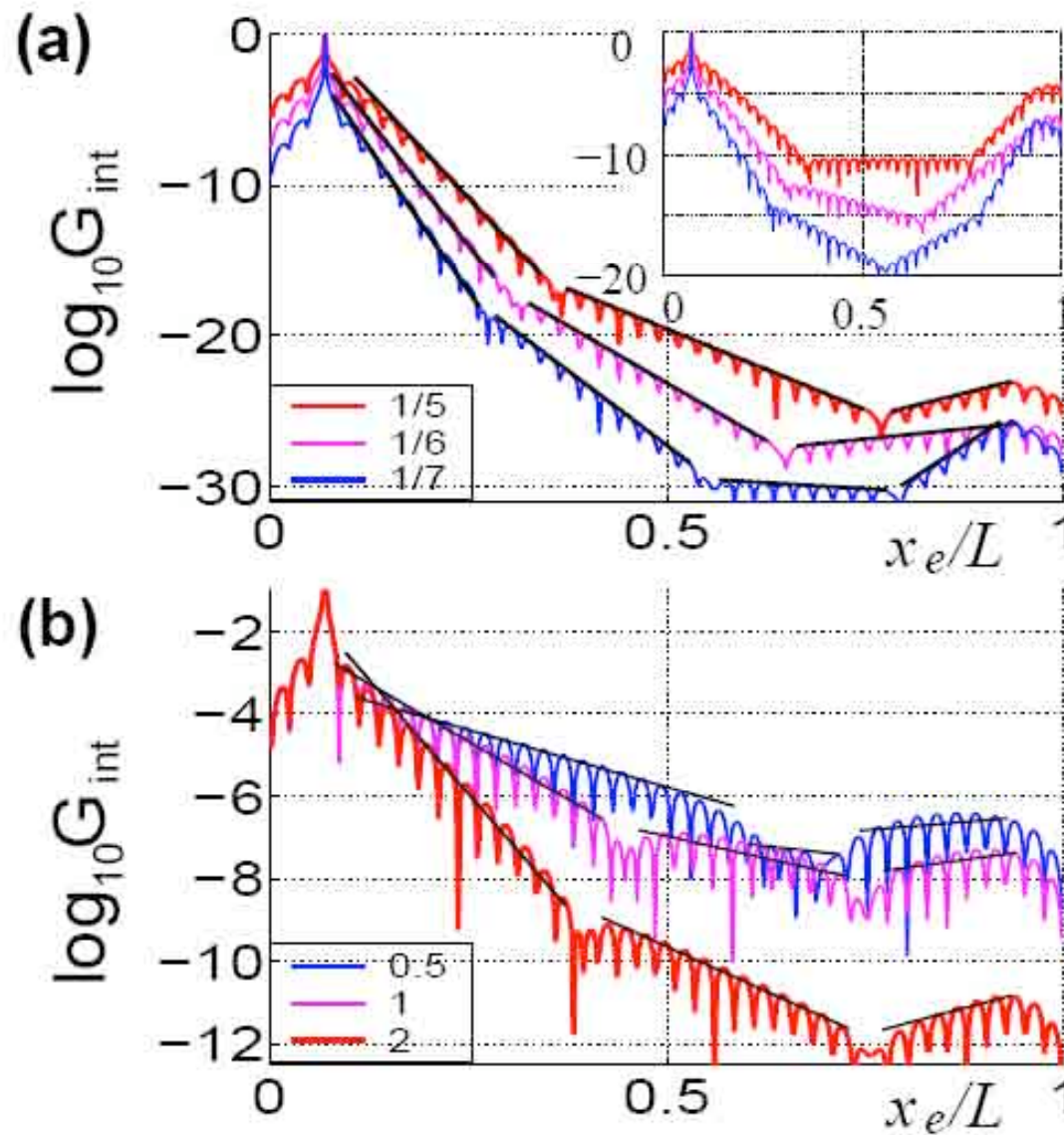
- The amplitude of the interference is contributed by the events **when the blue mode (the main fraction) arrives at x_e .**
- At those times the amplitude is determined by **the overlap of the blue mode with the others.**



n : # of the round trip with length $2L$



Result: dependence on temperature and g



$$k_B T = 5\epsilon$$

$$\ell_{\phi}^{-1}(n) = \ell_{\phi,T}^{-1} + \ell_{\phi,\text{spless}}^{-1}(n),$$

$$\ell_{\phi,\text{spless}}^{-1}(n) = \ell_{\phi,T}^{-1} \left[\frac{g^{-1} + g - 2}{2} - 2n(1 - g) \right]$$

$$\ell_{\phi,T} = \frac{\hbar v}{\pi k_B T}.$$

$$n: 0, 1, \dots, n_{\text{max}}$$

$$g = 1/5$$

Spinful case

Spin-charge separation:

the spin modes move slower than the charge mode by the factor g

$$\ell_{\phi}^{-1}(n) = \ell_{\phi,T}^{-1} + \ell_{\phi,\text{ch}}^{-1}(n) + \ell_{\phi,\text{sp}}^{-1},$$

$$\ell_{\phi,\text{ch}}^{-1}(n) = \ell_{\phi,T}^{-1} \left[\frac{g^{-1} + g - 2}{4} - 2n(1 - g) \right],$$

$$\ell_{\phi,\text{sp}}^{-1} = \ell_{\phi,T}^{-1} \frac{g^{-1} - 1}{2}.$$

Dephasing due to the charge part

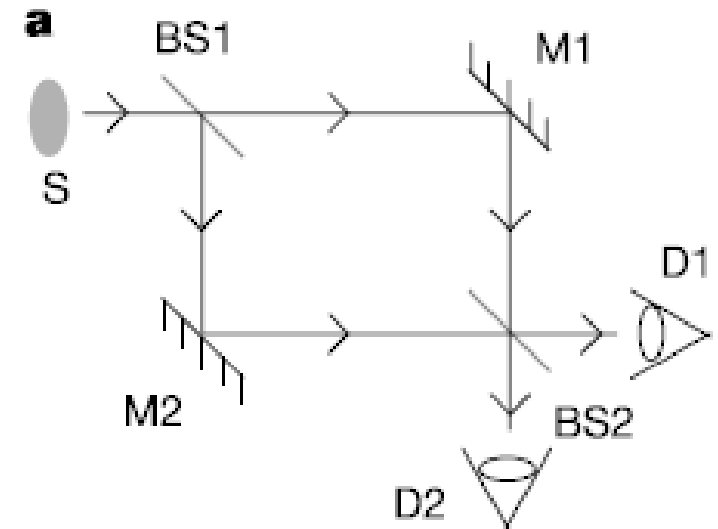
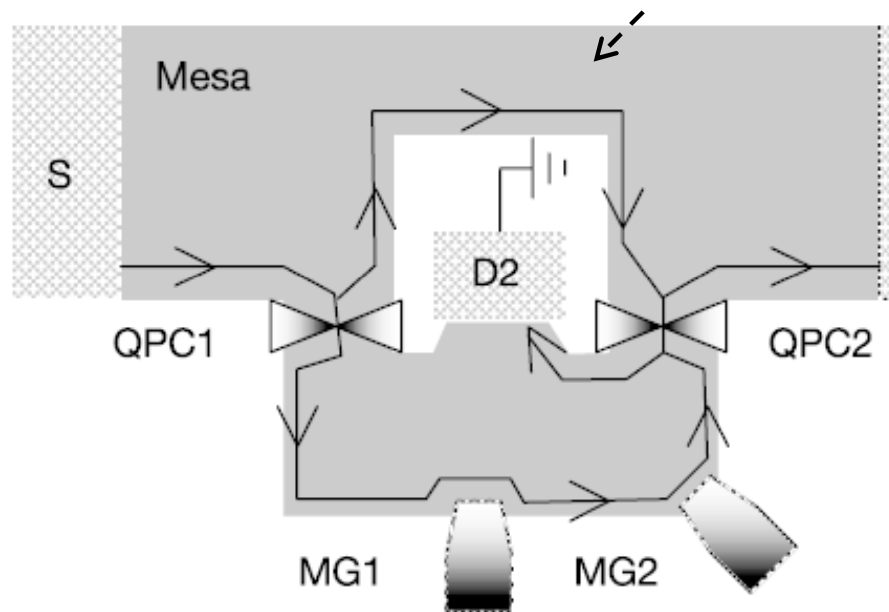
Due to spin-charge separation

II. Electronic Mach-Zehnder interferometry (E-MZI)

The first E-MZI:

Y. Ji, M. Heiblum et al., Nature 422, 415 (2003).

Integer quantum Hall edge channel

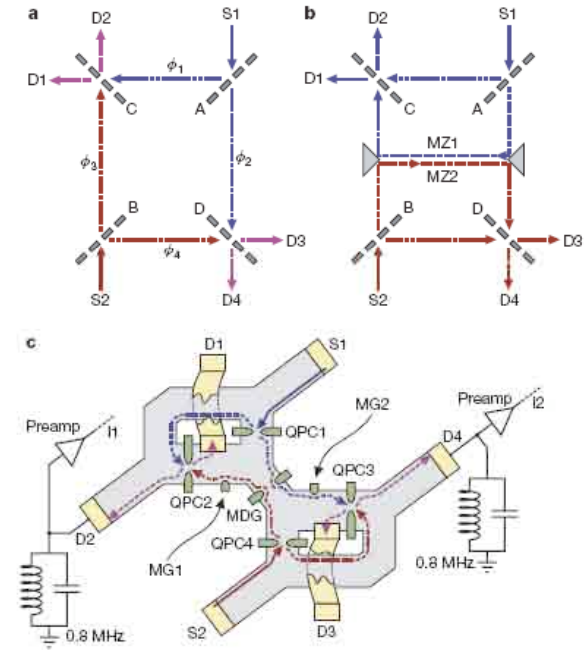


Generalization: Multi-electron interferometers

Two-electron (Bell-state) interferometer

Exp: Neder et al., Nature (2007).

Th.: Samuelsson, Sukhorukov, Buttiker, PRL (2004)

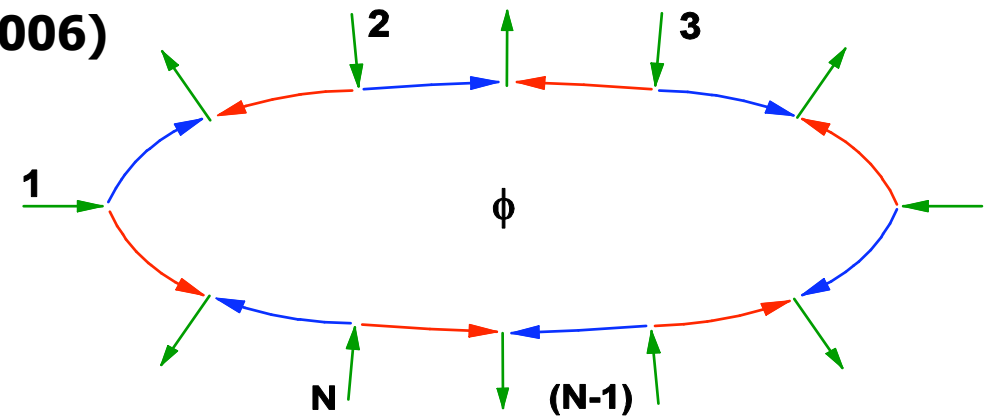


N-electron (GHZ-state) Aharonov-Bohm interferometer

Th.: H.-S. Sim and E. V. Sukhorukov, PRL (2006)

$$\langle \delta I_{D1} \delta I_{D2} \dots \delta I_{DN} \rangle \propto \cos \left(2\pi \frac{\Phi}{\phi_0} + \sum \phi_i \right)$$

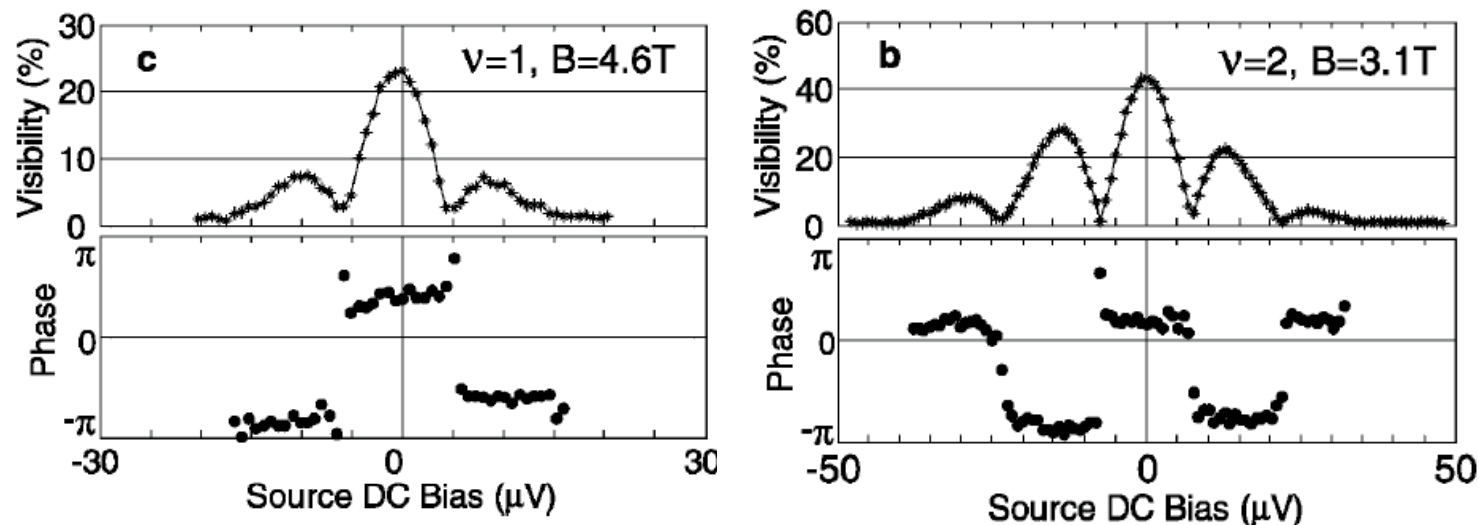
$$|\psi_N\rangle = p e^{i\varphi_p} |\uparrow \dots \uparrow\rangle + q e^{i\varphi_q} |\downarrow \dots \downarrow\rangle$$



Puzzle in E-MZI: Lobe patterns (Filling factor = 1)

Bias-dependent lobe patterns in E-MZI:

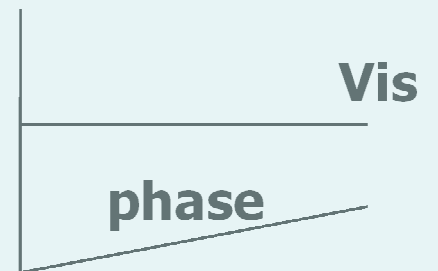
- I. Neder, M. Heiblum et al., PRL vol. 96, 016804 (2006).
- P. Roulleau, P. Roche et al., PRB vol. 76, 161309(R) (2007).



**QPC transmission
Probability = 0.5**

- The visibility shows bias-dependent lobe patterns.
- Phase jump of π at the minima of the visibility.

Noninteracting case:



Origin of the Lobes?

Maybe **e-e interactions**.

Proposed theories:

- **Intra-edge interaction, tunneling limit, bosonization: no lobe**

Chalker, Gefen, and Veillette, PRB 76, 085320 (2007).

- **Inter-edge interaction, tunneling limit, resonance, bosonization: lobe structure**

Sukhorukov and Cheianov, PRL 99, 156801 (2007).

Levkivskyi and Sukhorukov, PRB (2008) arXiv:0801.2338.

- **Intra-edge interaction, shot noise, beyond tunneling, filling factor one: lobe structure (phenomenological)**

weak interaction regime

Youn, Lee, and HS Sim, PRL 100, 196807 (2008).

rather strong interaction regime

Neder and Ginosar, PRL 100, 196806 (2008).

Basic question: how many nonequilibrium electrons in E-MZI?

$$N = \frac{t_{\text{fl}}}{\tau_V} = \frac{L|e|V}{2\pi\hbar v_F}$$

Injection time $\tau_V = 2\pi\hbar/(|e|V)$

Flight time $t_{\text{fl}} \equiv L/v_F$

Experimental situation:

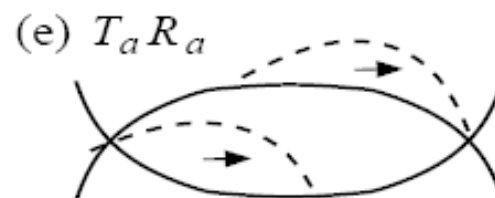
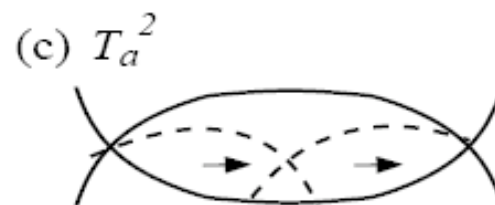
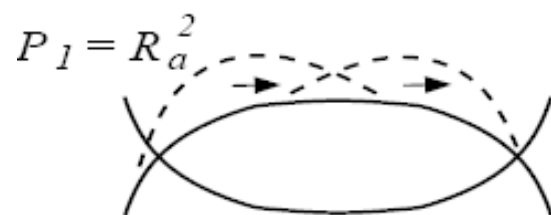
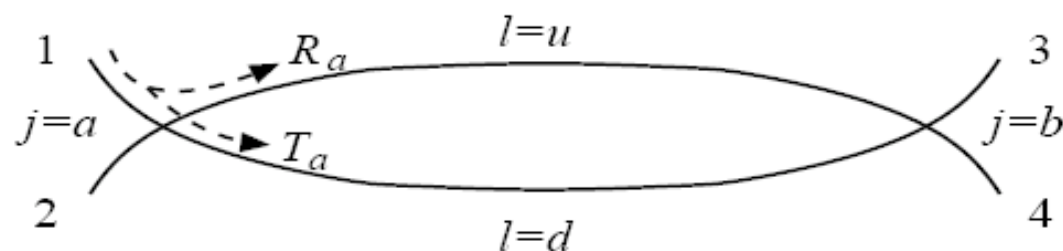
$N = 0.4 - 1$ at $eV = 8 \mu\text{V}$

$(L = 10 \mu\text{m}, v_F = (2-5) * 10^4 \text{ m/s})$

→ More than one nonequilibrium electron can occupy the E-MZI each time.

Key intuition: Nonequilibrium density ensemble

Shot noise at the $j=a$ beam splitter generates nonequilibrium ensemble!

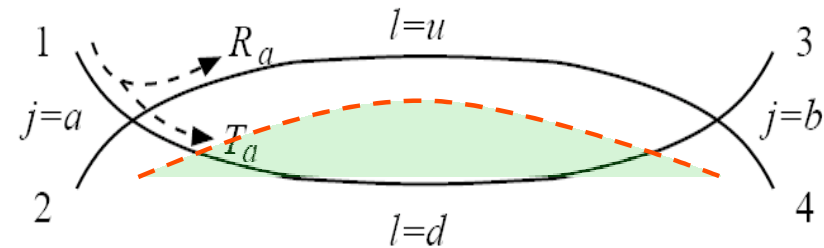
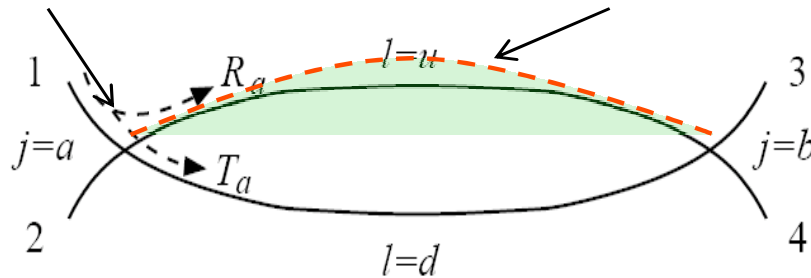


Effects of nonequilibrium density ensemble: A crude two-electron model

- Shot noise generates two possible nonequilibrium configurations:

Interfering electron

Nonequilibrium electron density



Probability of the configuration:

$$P_1 = R_a$$

$$P_2 = T_a$$

phase shift due to the interaction:

$$e^{i\delta}$$

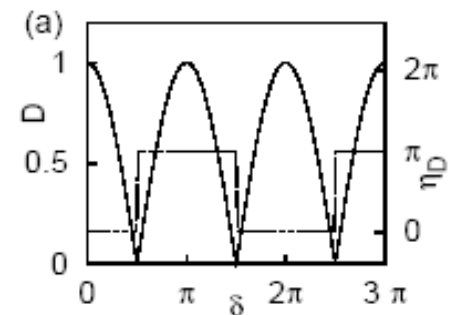
$$e^{-i\delta}$$

Transmission probability to the i=3 drain: $T_a = 0.5$

$$\mathcal{T} \equiv \mathcal{T}(E_0) = \mathcal{T}_0 + \mathcal{T}_1 D \cos(\Phi_B + \eta_D)$$

$$D \cos(\Phi_B + \eta_D) = \text{Re}[e^{i\Phi_B} (P_1 e^{i\delta} + P_2 e^{-i\delta})] = \cos \delta \cos \Phi_B$$

$$\mathcal{T}_0 = T_a T_b + R_a R_b, \quad \mathcal{T}_1 = 2\sqrt{T_a T_b R_a R_b},$$



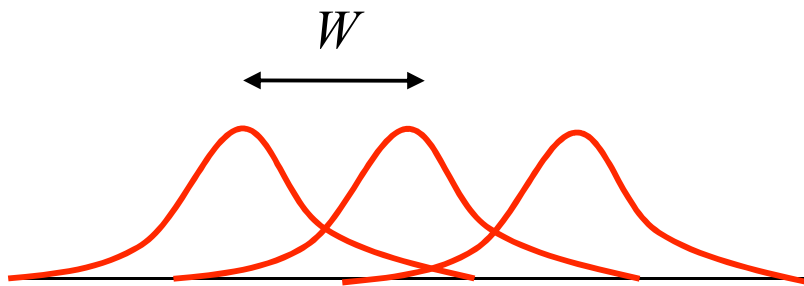
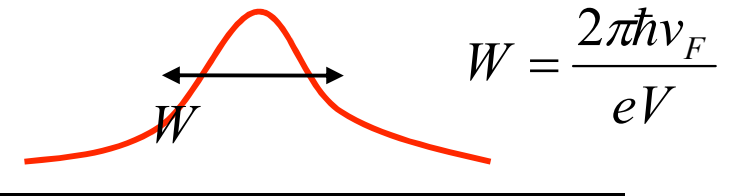
Beyond the two-electron model: a moving train of packets

Equivalent two pictures for nonequilibrium noninteracting electrons

$$|\Psi(t)\rangle_{\text{nint}} = \hat{U}(t) \prod_{E=0}^{|e|V} c^\dagger(E) |0\rangle = \hat{U}(t) \prod_{n=-\infty}^{\infty} d^\dagger(nW) |0\rangle$$

Packets

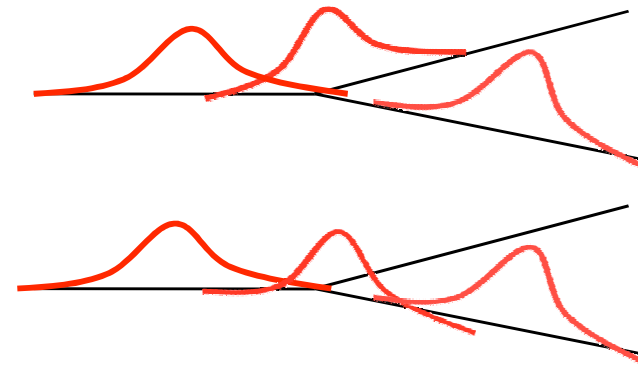
$$d_x^+ = \int_0^{eV} \frac{dE}{\sqrt{2\pi\hbar v_F}} e^{iEx/\hbar v_F} c^+(E)$$



A moving train of packets

Martin and Randauer

**Nonequilibrium ensemble of
packet configurations
at Y-junction:**



Calculation details :

interfering electron + the other nonequilibrium electrons

$$|\Psi\rangle = |\psi(E_0)\rangle \otimes |\Phi_{E_0}\rangle$$

Interfering electron
Plane wave basis

$$|\psi(E_0)\rangle = r|u(E_0)\rangle + t|d(E_0)\rangle$$

$$|r|^2 = R_a \quad |t|^2 = T_a$$

Nonequilibrium electrons
Packet basis

$$|\Phi_{E_0}\rangle = \sum_m c_m |\Phi_{E_0,m}\rangle$$

$$P_m = |c_m|^2$$

Sum over the elements of
The nonequilibrium density ensemble

Interaction between the interfering electron and the packets
→ configuration-dependent phase shift!

$$\hat{U}_{\text{ph}}(t_0) = \sum_{l=u,d} e^{-i \frac{U_0}{\hbar} \int_{t_0}^{t_0+t_{\text{fl}}} dt \hat{N}_l(t)} |l(E_0)\rangle \langle l(E_0)|$$

$$\hat{N}_l(t) = \int_{x \in l} \hat{n}(x, t) dx$$

time ensemble index $t_0 \in [0, \tau_V]$

U_0 is the interaction strength

Approximation: we ignored other interactions, e.g., between the packets.

Results: nonequilibrium dephasing

Transmission probability to the $i=3$ drain:

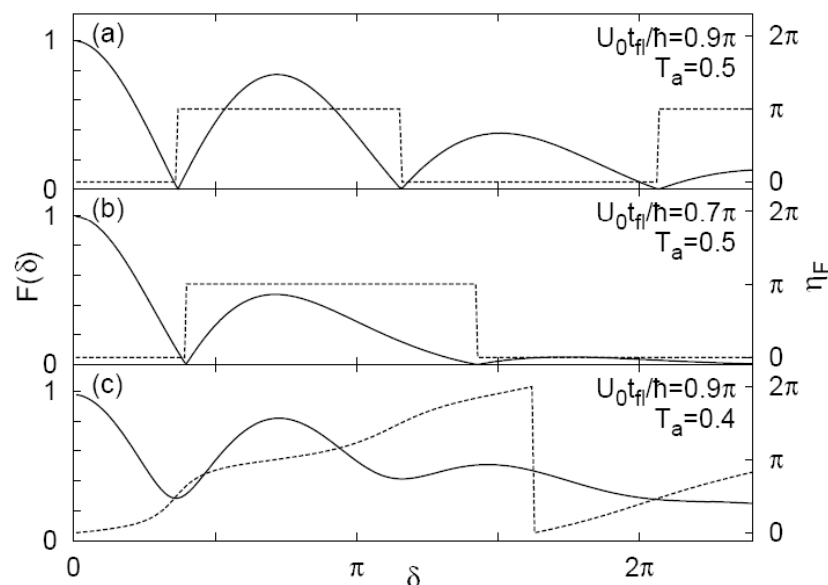
$$\mathcal{T} \equiv \mathcal{T}(E_0) = \mathcal{T}_0 + \mathcal{T}_1 D \cos(\Phi_B + \eta_D)$$

$$D \cos(\Phi_B + \eta_D) \equiv \langle \Re[e^{i\Phi_B} \sum_m P_m \langle e^{i\hat{\delta}(t_0)} \rangle_m] \rangle_{t_0}$$

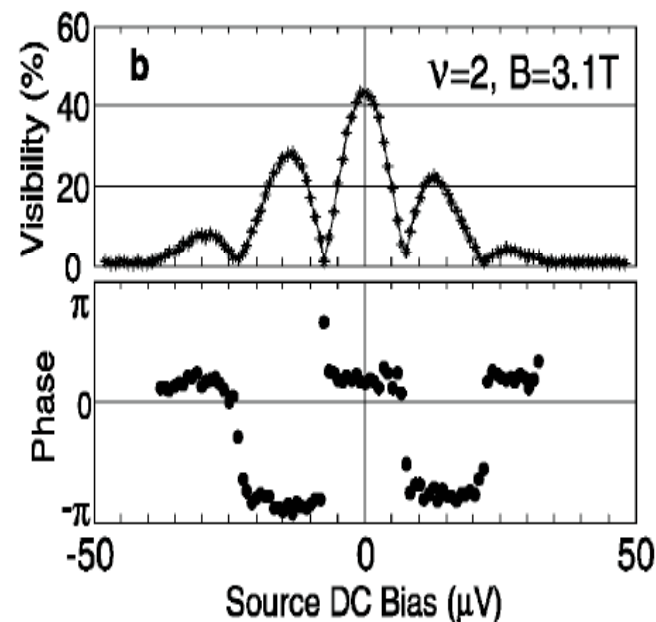
$\langle \cdots \rangle_{t_0}$ means the ensemble average over $t_0 \in [0, \tau_V]$

$$I = (e^2/h)V\mathcal{T}(V)$$

$$dI/dV = (e^2/h)\mathcal{T}_0[1 + F(V) \cos(\Phi_B + \eta_F(V))]$$



$$\delta \equiv N \frac{U_0 t_H}{\hbar} = \frac{|e|V t_H}{2\pi\hbar} \frac{U_0 t_H}{\hbar}$$

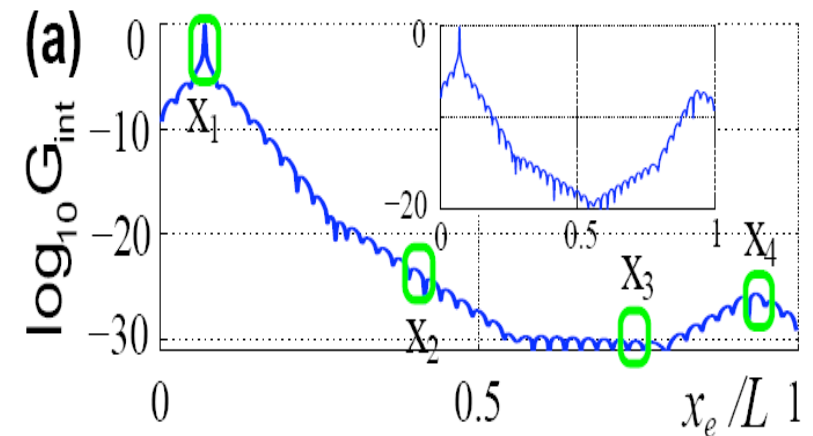


Summary: e-e interaction modifies interference signals in a nontrivial way!

- Finite-length 1D quantum wire (Luttinger liquid)

J. U. Kim, W.-R. Lee, H.-W. Lee, and H.-S. Sim, preprint

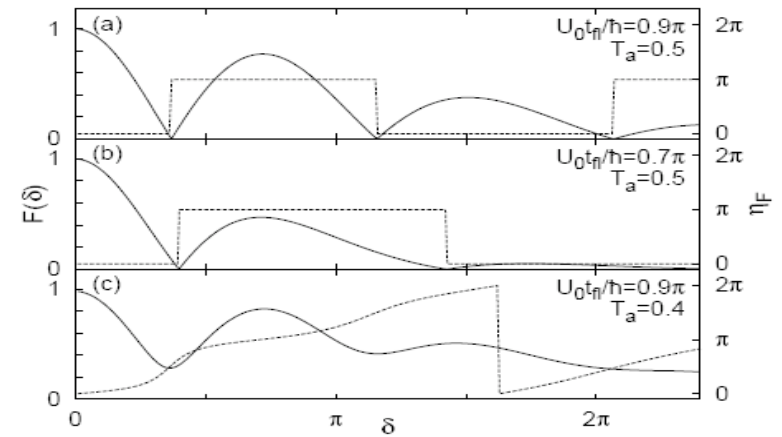
- Electron **fractionalization** and recombination of the fractions
 \rightarrow **revival** of coherence,
multiple coherence lengths



- Electronic Mach-Zehnder interferometer in nonequilibrium

S.-C. Youn, H.-W. Lee, and H.-S. Sim, PRL 100, 196807 (2008)

- **nonequilibrium charge density ensemble** of electron density
 \rightarrow bias-dependent **lobe pattern**,
 phase jumps of π



Acknowledgement

- **Quantum electron transport theory group in KAIST**

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Seok-Chan Youn **(Ph.D student)**

- **Postech**

Hyun-Woo Lee

J. U. Kim, W.-R. Lee, H.-W. Lee, and H.-S. Sim, preprint

S.-C. Youn, H.-W. Lee, and H.-S. Sim, PRL 100, 196807 (2008)

Thank you for your attention!