Interaction-Induced Beats of Friedel Oscillations in Quantum Wires

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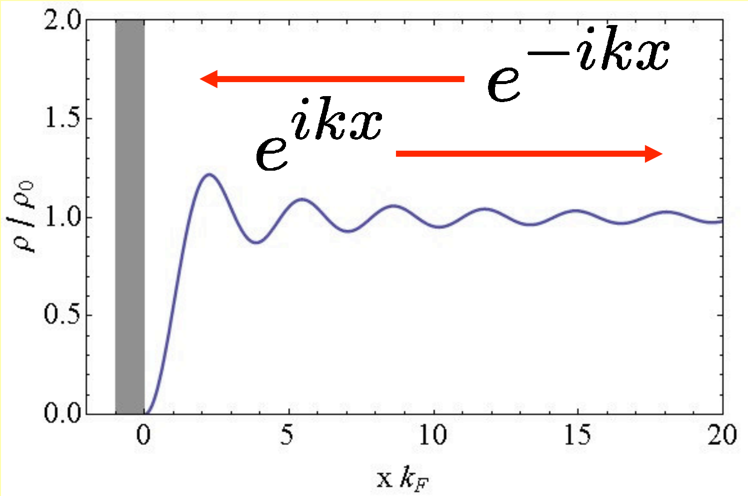
"Classical" Friedel oscillations

- Friedel oscillations (1D): Electron density modulation in vicinity of a hard-wall potential.

full reflection at $x=0$



coherent superposition of incoming & outgoing electron wavefunctions

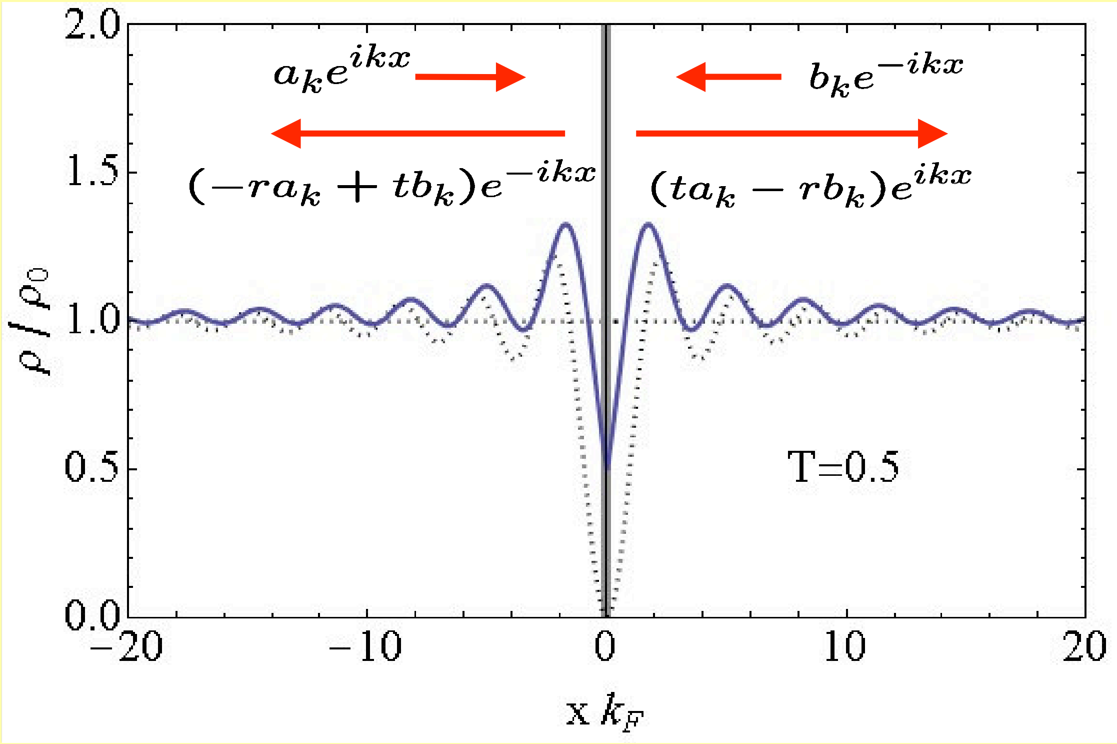
$$\frac{\delta\rho(x)}{\rho_0} \sim \frac{\sin(2k_F x)}{2k_F x}$$

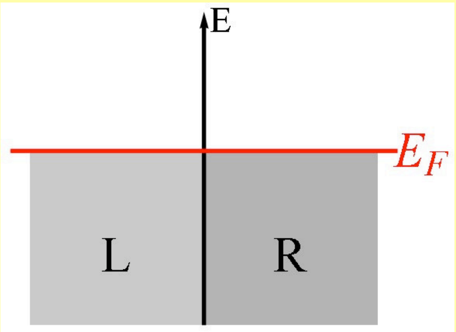
(at zero temperature)

oscillation period $\lambda = \pi / k_F$

Friedel oscillations at finite barrier

- Finite barrier, transmission $T = \sqrt{t^* t}$





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Friedel oscillations at finite barrier

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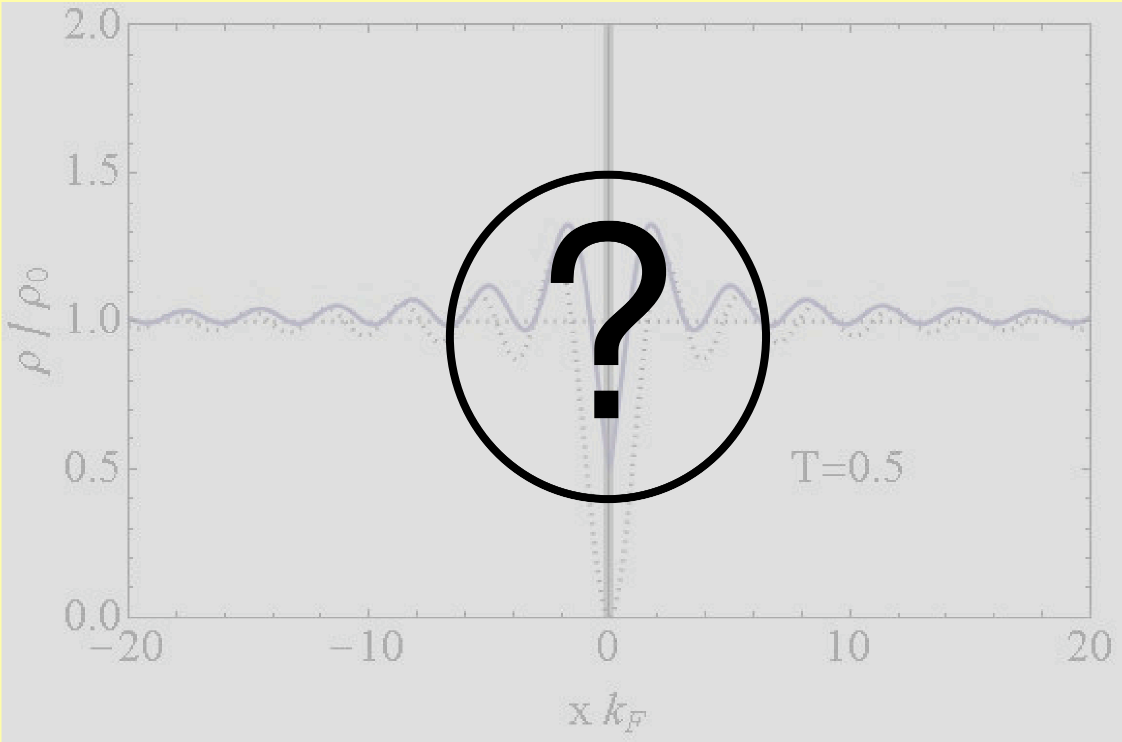
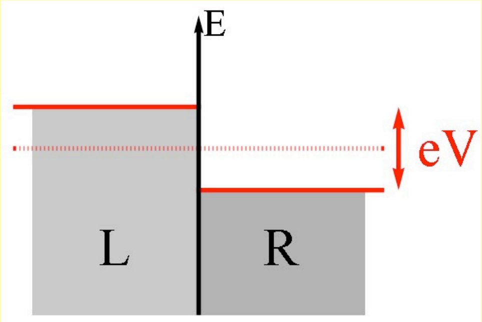
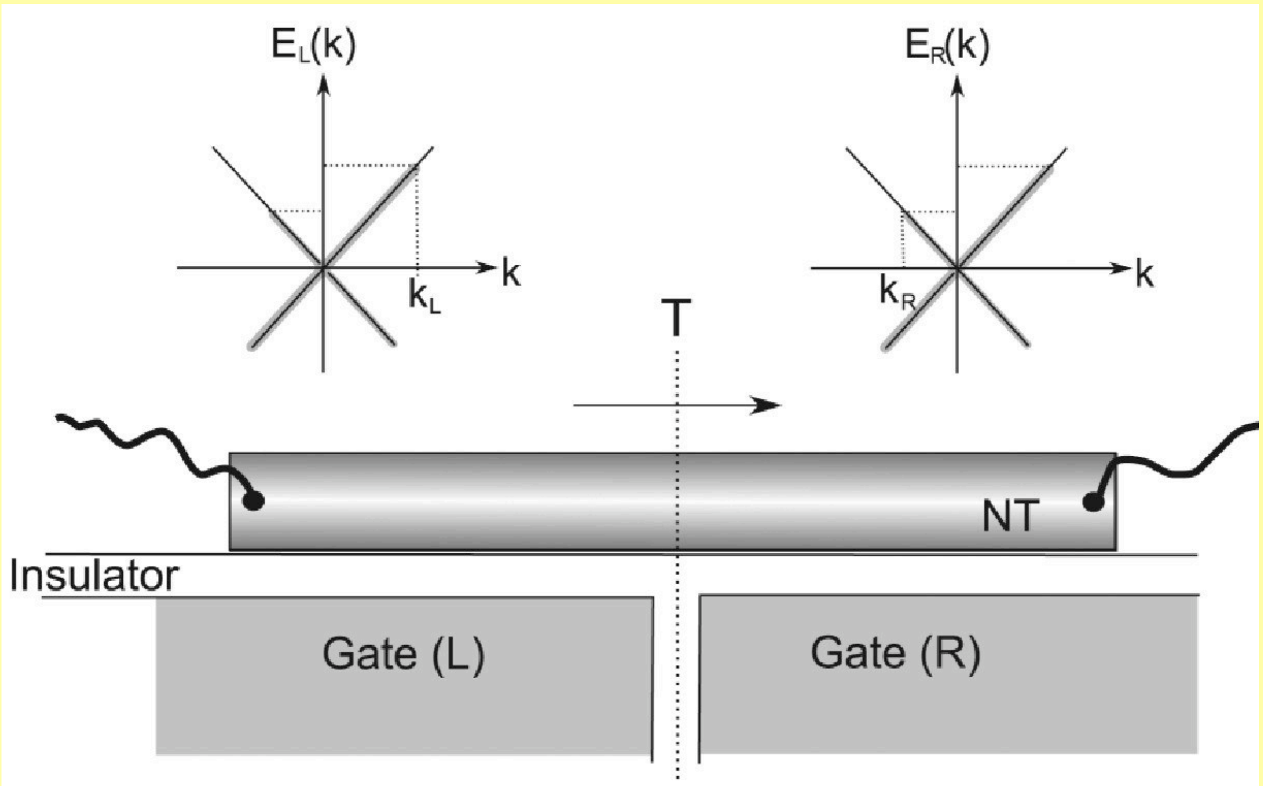




oscillation period $\lambda = ?$

Setup

let's be more specific:



linear dispersion:

$$E(k) = v_F|k|$$

finite bias voltage
+
appropriate gating

$$E_{F,L/R} = v_F k_{L/R} = v_F k_F \pm V/2$$

Friedel oscillations in non-equilibrium

$$\rho_{L/R} = \underbrace{\rho^{(0)}}_{\text{(i)}} \pm \underbrace{\frac{(1-T)V}{2v_F\pi}}_{\text{(ii)}} - \underbrace{\frac{\sqrt{1-T}}{2\pi}\frac{\sin(2|x|\,k_{L/R}-\varphi_r)+\sin(\varphi_r)}{|x|}}_{\text{(iii)}}$$

i) The full constant average density

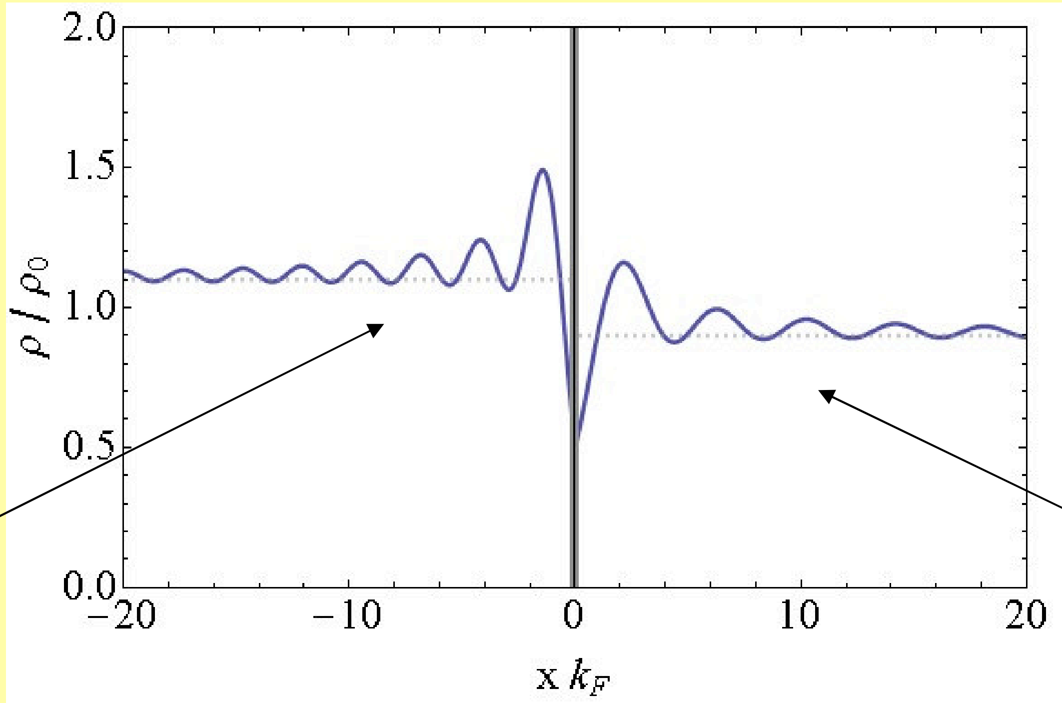
ii) A constant shift proportional to $\pm V/2$ on the L/R side (→ Landauer dipole)

iii) A space dependent oscillating part

Friedel oscillations in non-equilibrium

$$\rho_{L/R} = \rho^{(0)} \pm \frac{(1-T)V}{2v_F\pi} - \frac{\sqrt{1-T}}{2\pi}\,\frac{\sin(2|x|\,k_{L/R} - \varphi_r) + \sin(\varphi_r)}{|x|}$$

$$\lambda_L = \frac{\pi}{k_L}$$



$$\lambda_R = \frac{\pi}{k_R}$$

ρ_L only depends on k_L and ρ_R only depends on k_R …

… in the absence of interactions!

e-e interactions

How to include e-e interactions?

1. perturbatively: weak interaction expansion,
   treat system exactly in tunnelling amplitude

↕ "dual" calculations

2. non-perturbatively: Tomonaga-Luttinger liquid (TLL)
   weak tunnelling between two
   half-infinite interacting systems

weak interaction expansion

spectrum of density oscillations:

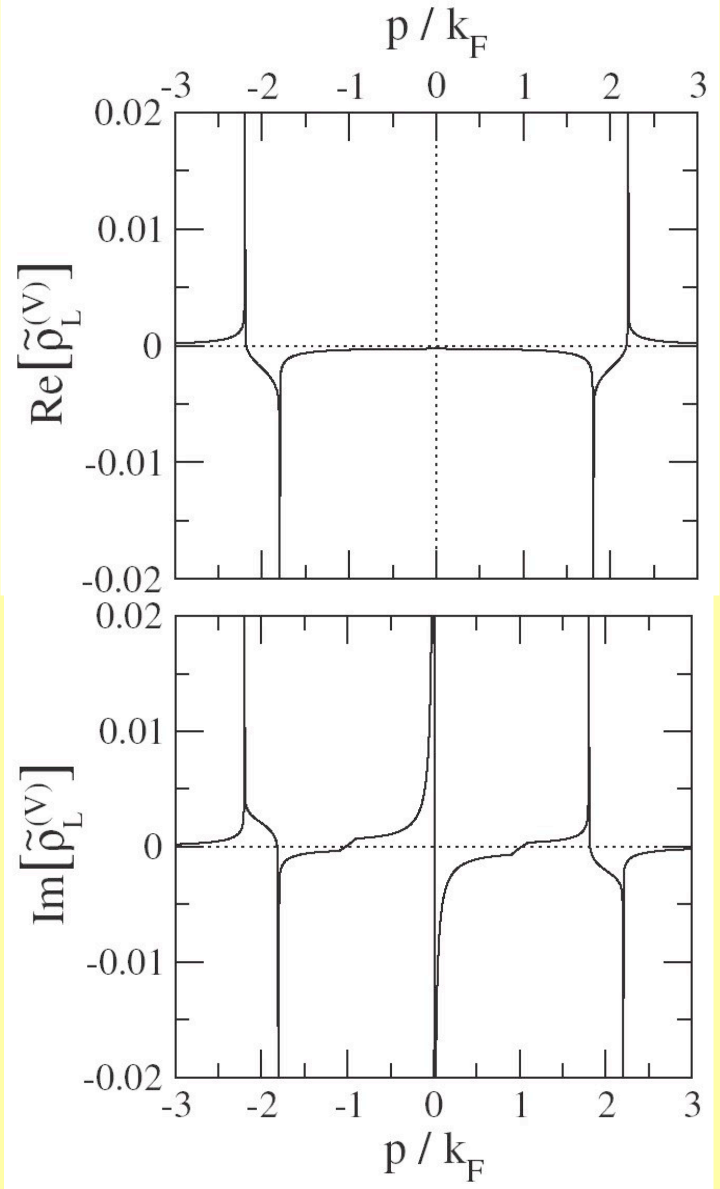
$$\tilde{\rho}_L(p) = \int dx\, e^{ipx}\, \delta\rho_L(x)$$
$$= \tilde{\rho}_L^{(LD)} + \tilde{\rho}_L^{(osc)}$$

$\tilde{\rho}_L^{(LD)}$ comes from interaction with the **Landauer dipole**

$\tilde{\rho}_L^{(osc)}$ comes from interaction with oscillating part of the density

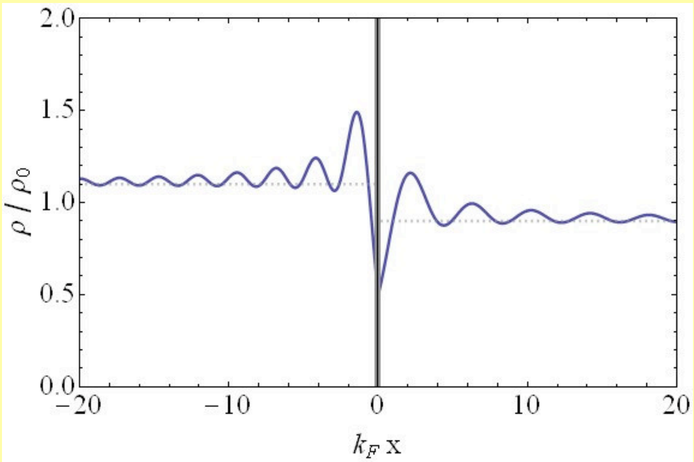
log-singularities at $p = \pm 2k_{L/R}$

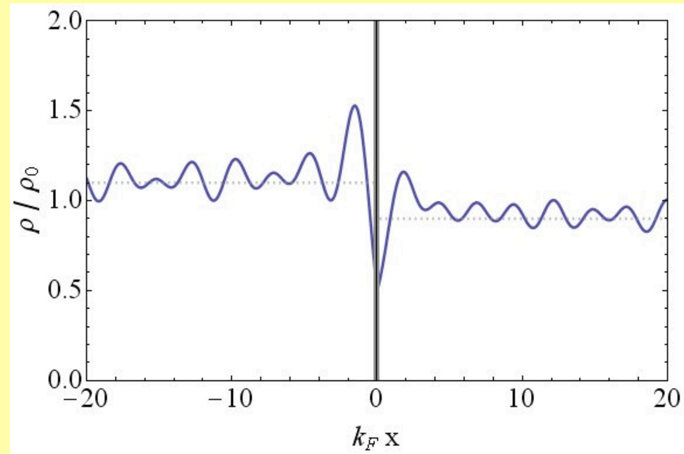
→ beating pattern of density oscillation in real space

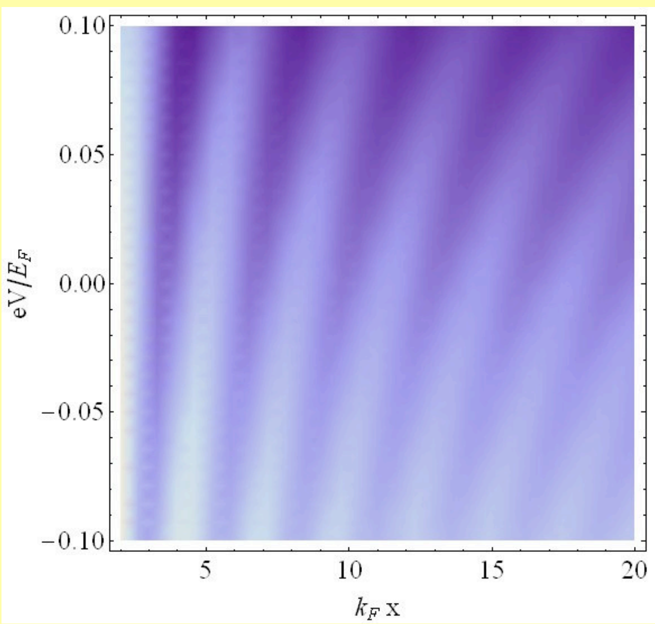


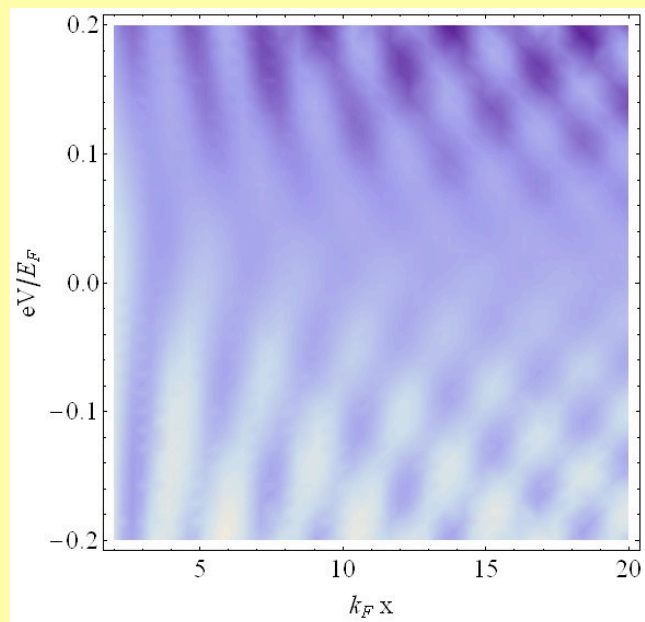
beating pattern of density oscillations

no e-e interactions ⟺ with e-e interactions



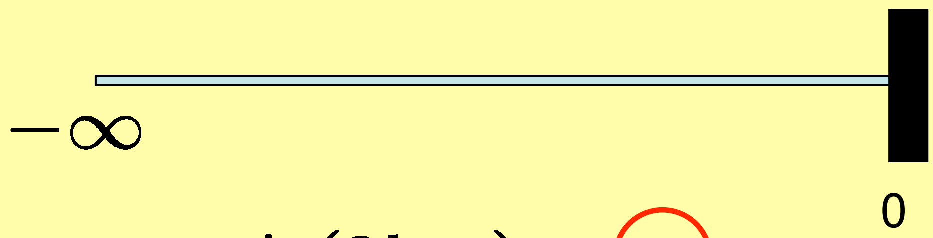






FO in a Tomonaga-Luttinger liquid

half-infinite system (one open boundary)

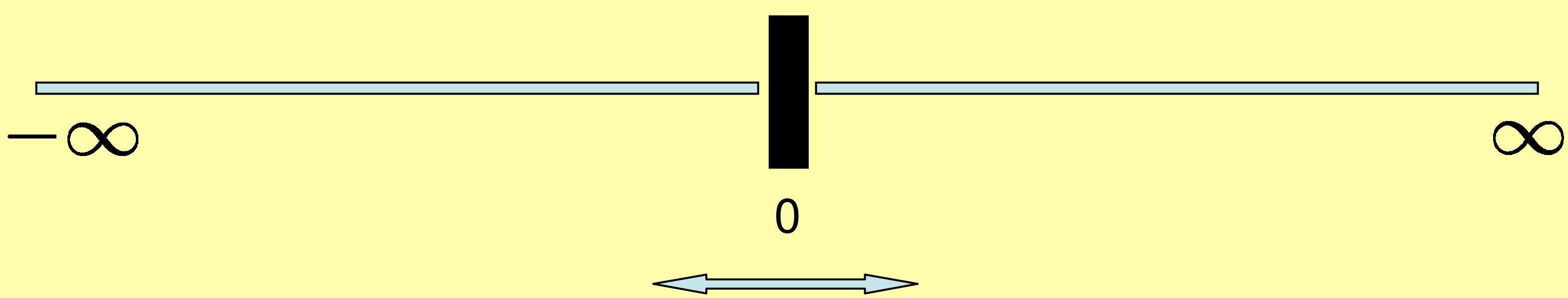


Egger, Grabert PRL 95
Fabrizio, Gogolin PRB 95

$$\rho(x) = \rho_0 - \frac{\sin(2k_F x)}{\pi a_0} \left| \frac{a_0}{2x} \right|^g$$

Interaction parameter

$$g = \frac{1}{\sqrt{1 + U/(\pi v_F)}}$$



assume L and R weakly coupled $H = H_0 + H_T$

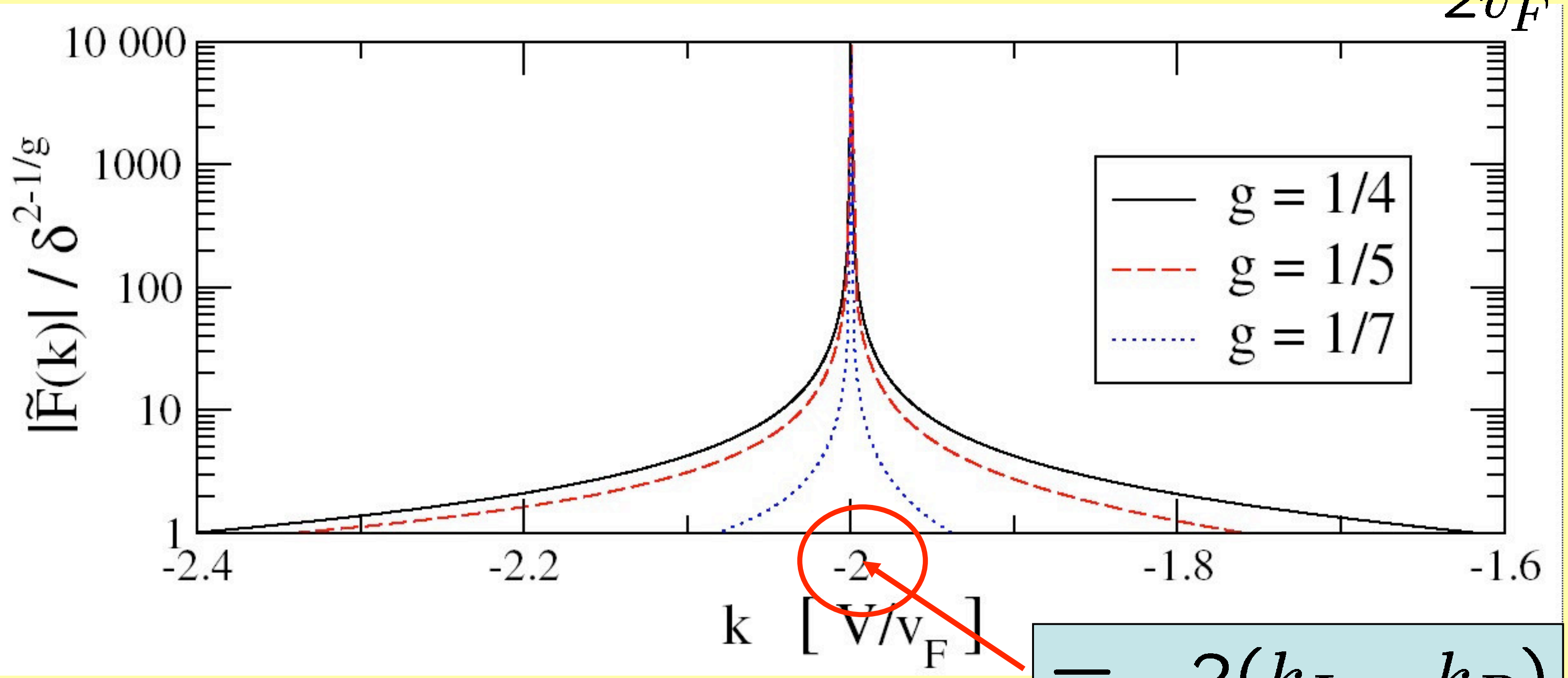
Kane, Fisher
PRB 92

non-eq. FO in TLL model

$$\delta\rho_L(x) \propto \gamma^2 |x|^{-g} \sum_{\pm} (\pm) e^{\pm i 2 k_L x} \mathrm{sign}(x)\, F(x)$$

$$k_L = k_F + \frac{V}{2v_F}$$

Fourier transform of *F(x)*:



$$= -2(k_L - k_R)$$

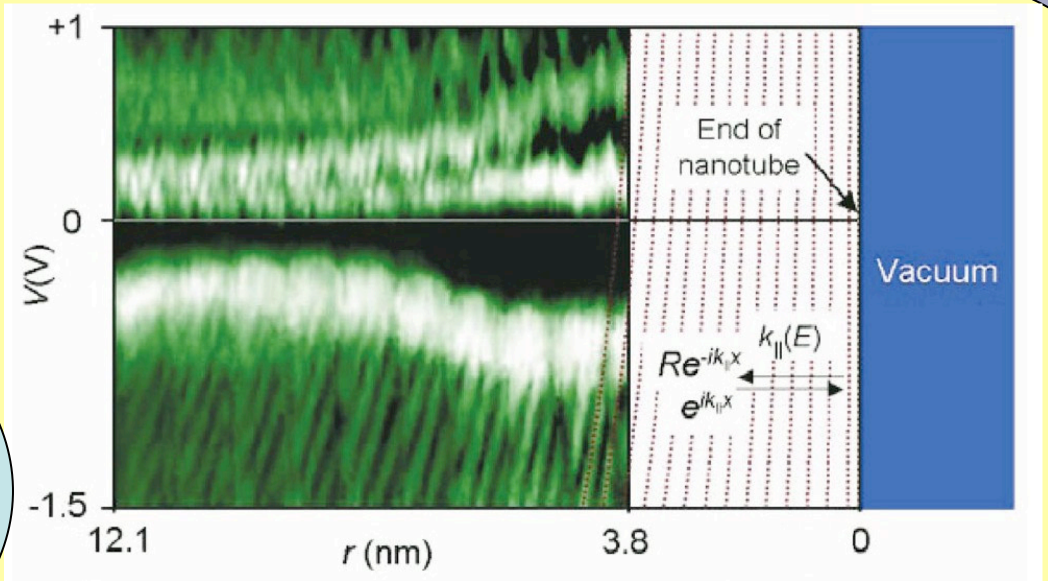
Experiments are possible…

Real Space Imaging of 1D Standing Waves:

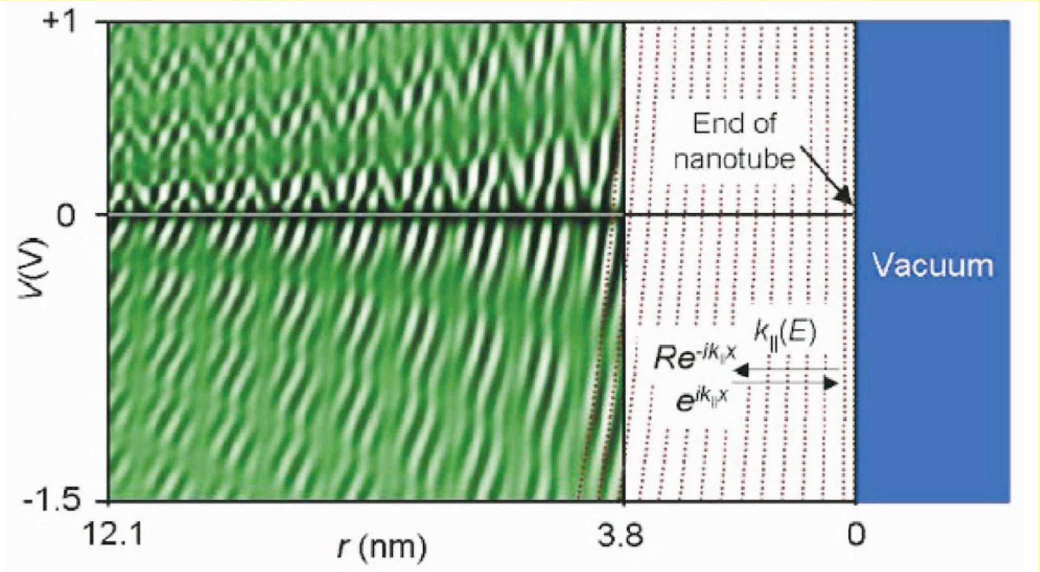
- boundary = open end of NT
- beating pattern due to underlying lattice

Jinhwan Lee, S. Eggert, H.Kim, S.-J. Kahng, H Shinohara, and Y Kuk

PRL 93, 166403 (2004)



Experiment



Theory

Summary

We have discussed the density profile of FO in a 1D wire with different Fermi wave vectors on either side of an imbedded impurity.

- without e-e interactions: oscillations with local value of π/k_F
- interacting system: additional peak in the spectral function

$\longrightarrow$ beating pattern in density oscillations in real space. Interaction effect!

We have shown this for two different situations

1. using exact scattering states, treating interactions perturbatively
2. considering a weak link between two TLL

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