AC CONDUCTANCE AND NON-SYMMETRIZED NOISE AT FINITE FREQUENCY IN QUANTUM WIRE AND CARBON NANOTUBE

Adeline CRÉPIEUX [1], Cristina BENA [2,3] and Inès SAFI [2]

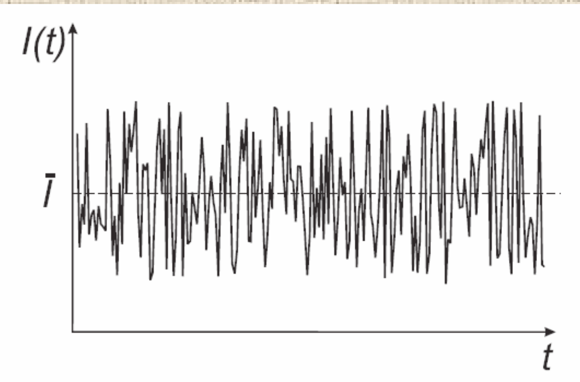
1 – Centre de Physique Théorique, CNRS Marseille, France

2 – Laboratoire de Physique des Solides, CNRS Orsay, France

3 – Institut de Physique Théorique, CEA Saclay, France

*Physical Review B **78**, 205422 (2008)*

CURRENT FLUCTUATIONS



ORIGINS OF NOISE

- High temperature : Johnson-Nyquist noise
- High voltage : Shot noise
- High frequency : Quantum noise
- We neglect the 1/f noise

ZERO FREQUENCY AND ZERO TEMPERATURE: SHOT NOISE

$\Rightarrow$ Schottky relation $S^+(\omega=0)=e^*|\langle \delta j\rangle|$ where $S^+(\omega)=FT\left\{\frac{1}{2}\left[\langle \delta j(0)\delta j(t)\rangle+\langle \delta j(t)\delta j(0)\rangle\right]\right\}$

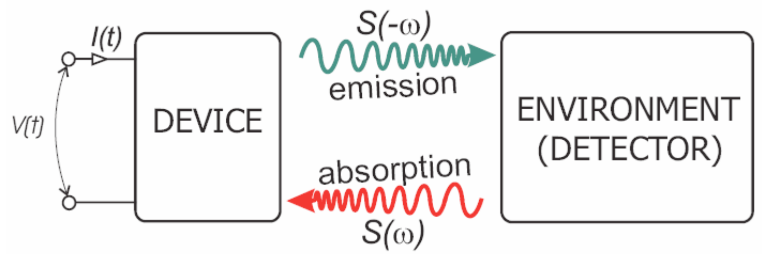
symmetrized noise

HIGH FREQUENCY NOISE MEASUREMENTS

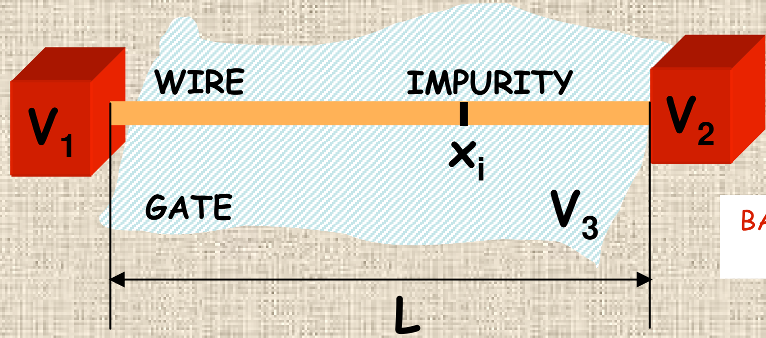
- High-frequency measurement in a diffusive wire $1-20\,\mathrm{GHz}$ ***SCHOELKOPF et al., PRL (1997)***
- On-chip detection using SIS junction $\rightarrow 100\ \mathrm{GHz}$ ***DEBLOCK et al., Science (2003)***
- Direct measurement in a QPC $4-8\,\mathrm{GHz}$ ***ZAKKA-BAJJANI et al., PRL (2007)***

What is measured is $S(\omega)=\int dt e^{i\omega t}\langle \delta j(0)\delta j(t)\rangle$

non-symmetrized noise



THE SYSTEM



MODEL

$$H = H_0 + H_B + H_V$$

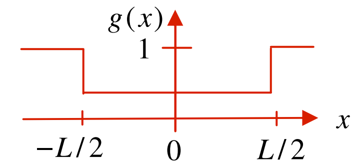
$$H_0 = \frac{\hbar v_F}{2}\int_{-\infty}^{\infty} dx\left[\Pi^2 + \frac{1}{g^2(x)}(\partial_x \Phi)^2\right]$$

COULOMB INTERACTIONS PARAMETER

$$H_B = \lambda \cos\left[\sqrt{4\pi}\Phi(x_i,t) + 2k_F x_i\right]$$

BACKSCATTERING AMPLITUDE

$$H_V = -\int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} eV(x)\partial_x \Phi(x,t)$$



WE CALCULATE PERTURBATIVELY

- The non-symmetrized noise: $S_{nm}(\omega) = \int dt e^{i\omega t}\langle \delta j_m(0)\delta j_n(t)\rangle$ where $n,m = 1,2,3$
- The AC conductance: $G_{nm}(t-t') = \left.\frac{\delta I_n(t)}{\delta V_m(t')}\right|_{V_m=0}$ where $\begin{cases} \delta I_n(t) = \langle \delta j_n(t)\rangle \\ V_n(t) = v_n \cos(\omega t) \end{cases}$

RESULT

$$S_{nm}(\omega) = S^+_{nm}(\omega) - \hbar\omega \mathrm{Re}[G_{nm}(\omega)]$$

with $S^+_{nm}(\omega) = \frac{1}{2}\int dt e^{i\omega t}\left(\langle \delta j_m(0)\delta j_n(t) + \delta j_m(t)\delta j_n(0)\rangle\right)$ ***symmetrized noise***

GENERALIZED KUBO-TYPE FORMULA

$$S^-_{nm}(\omega) = S_{nm}(\omega) - S_{nm}(-\omega) = -2\hbar\omega \mathrm{Re}[G_{nm}(\omega)]$$

SAFI AND SUKHORUKOV (unpublished)

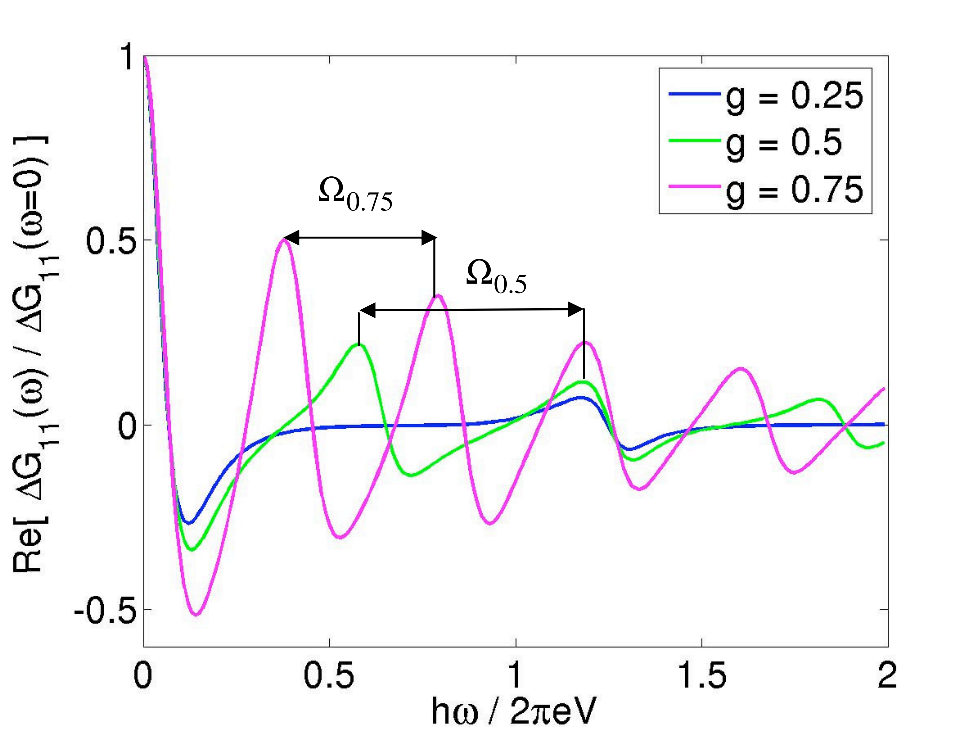
AC CONDUCTANCE

WEAK-BACKSCATTERING LIMIT OF THE EXCESS AC CONDUCTANCE

$\Delta G_{11}(\omega) = G_{11}(\omega) - G_{11}(\omega)\big|_{V=0}$ where $V = V_2 - V_1$ ***source-drain voltage***

FOR g=1: $\Delta G_{11}(\omega) = 0$ ***because of the linearity of the I-V caracteristic***

FOR g≠1: ***oscillations with frequency with a pseudo-period related to the wire frequency*** $\omega_L = v_F / gL$



$x_i = 0$

$T = 0$

$\lambda / eV = 0.01$

$g\hbar\omega_L / eV = 0.05$

The pseudo-period depends on L and g:

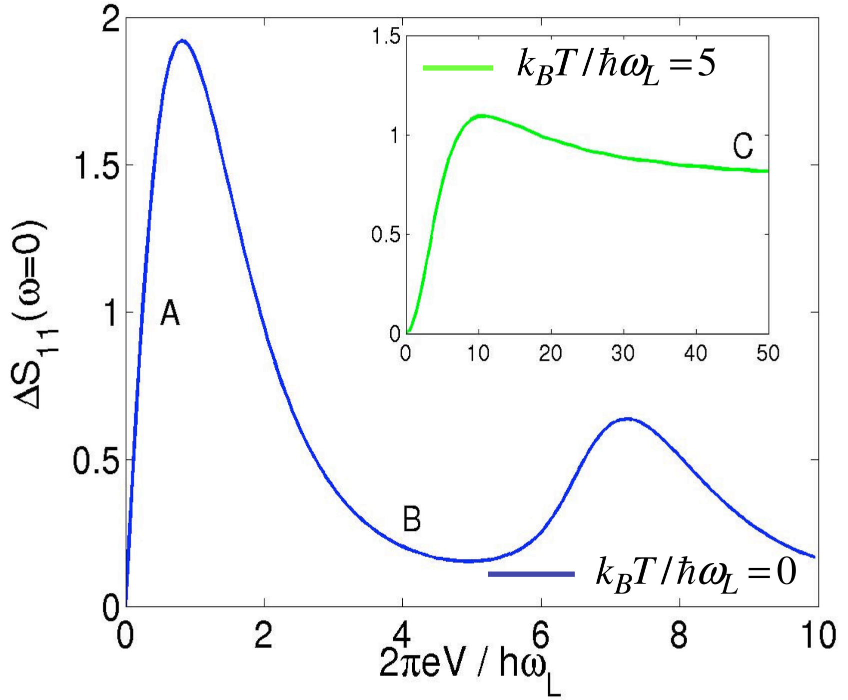
$$\Omega_g = \frac{hv_F}{gLeV}$$

ZERO-FREQUENCY NOISE

IN THE WEAK-BACKSCATTERING LIMIT

$$S_{nm}(\omega=0)=eI_B\coth\left(\frac{eV}{2k_BT}\right)+2k_BT\left[\frac{e^2}{h}-2\frac{\partial I_B}{\partial V}\right]$$

where I_B is the backscattering current



- **REGION A:** short-wire limit $eV<\hbar\omega_L$

$\Rightarrow$ **linear variation** with voltage

$\Rightarrow$ qualitative agreement with experiments on carbon nanotubes

WU et al., PRL 99, 156803 (2007)
HERRMANN et al., PRL 99, 156804 (2007)

- **REGION B:** long-wire limit $eV>\hbar\omega_L$

$\Rightarrow$ **oscillations** whose envelope has a power-law dependence

- **REGION C:** high temperature limit

$$k_BT>\hbar\omega_L$$

$\Rightarrow$ behaves like the noise of an infinite length interacting wire: **power-law variation**

FINITE-FREQUENCY NON-SYMMETRIZED NOISE

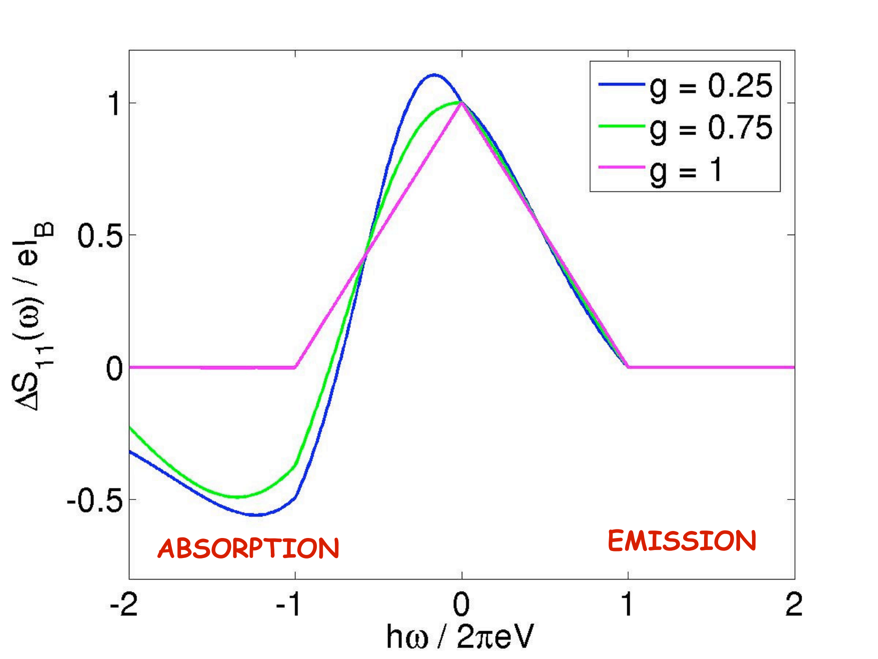
WE CALCULATE $\Delta S_{nm}(\omega) = S_{nm}(\omega) - S_{nm}(\omega)|_{V=0}$

FOR g=1: the non-symmetrized excess noise is **symmetric**

$S_{nm}(\omega) = S^{+}_{nm}(\omega) - \hbar\omega \mathrm{Re}[G_{nm}(\omega)] \Rightarrow \Delta S_{nm}(\omega) = \Delta S^{+}_{nm}(\omega)$ because $\Delta G_{11}(\omega) = 0$ when $g = 1$

FOR g≠1: the non-symmetrized excess noise becomes **asymmetric**

SHORT-WIRE LIMIT $g\hbar\omega_L / eV = 1$



$x_i = 0$
$T = 0$
$\lambda / eV = 0.01$

FINITE-FREQUENCY NON-SYMMETRIZED NOISE

LONG-WIRE LIMIT

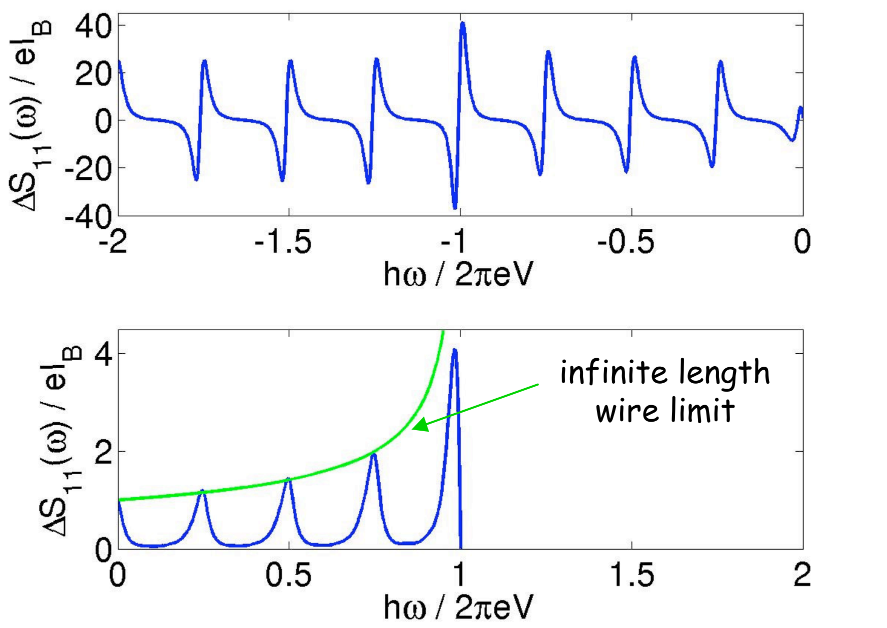
$g = 0.25$

$x_i = 0$

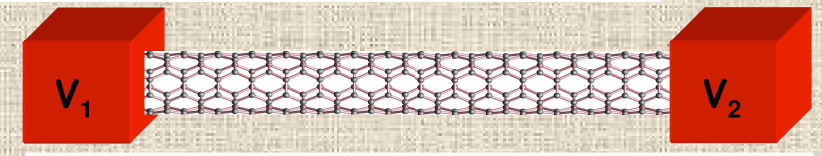
$T = 0$

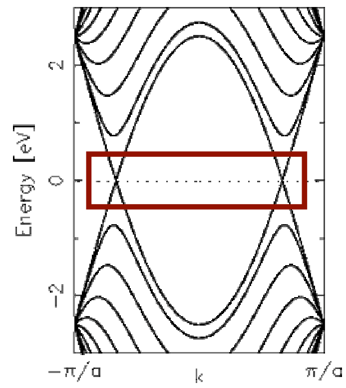
$\lambda / eV = 0.01$

$g\hbar\omega_L / eV = 0.01$



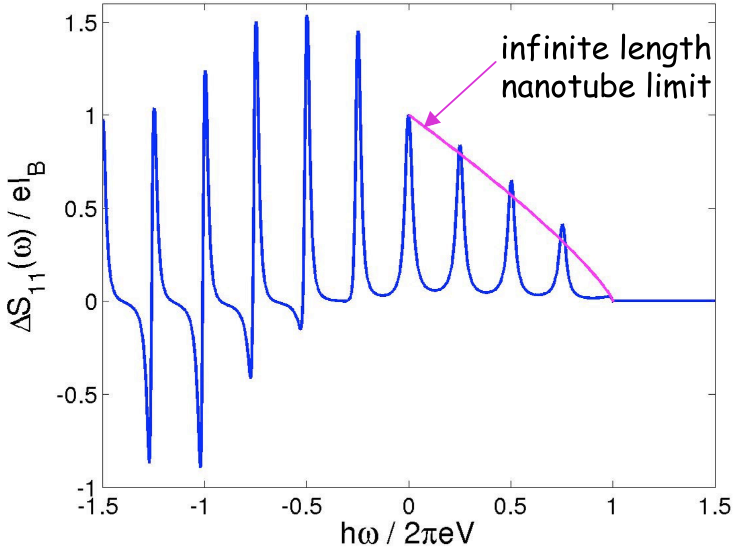
CARBON NANOTUBE





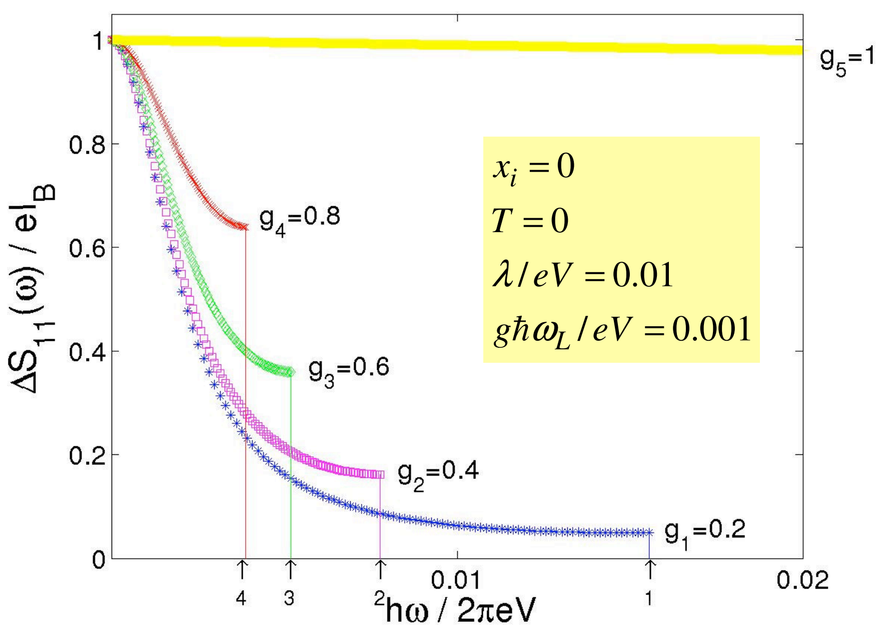
four channels of conduction

$\begin{cases} \text{charge sector with } g < 1 \\ \text{3 others sectors with } g = 1 \end{cases}$

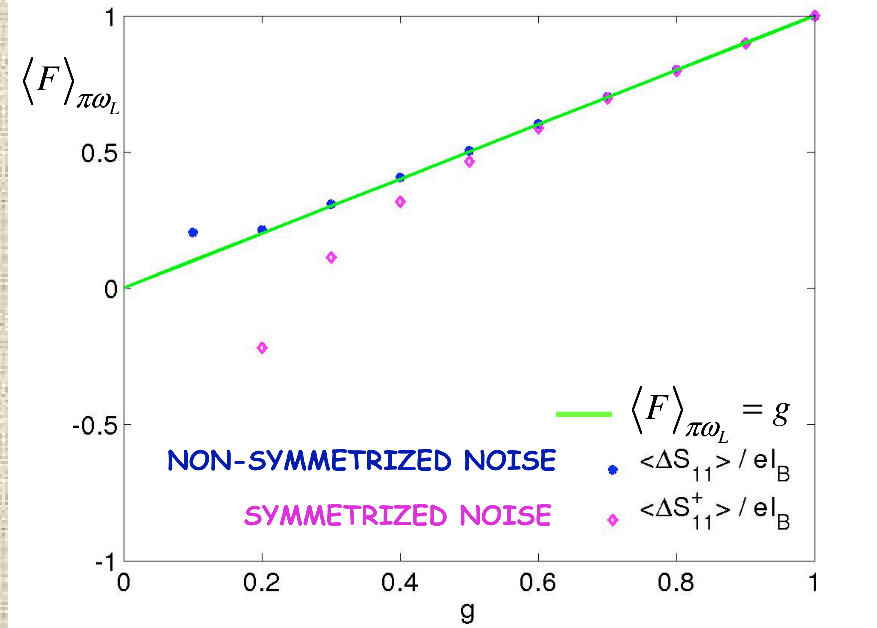


AVERAGE NON-SYMMETRIZED NOISE

EMISSION EXCESS NOISE



AVERAGED FANO FACTOR



WE CALCULATE THE AVERAGE OVER THE FIRST HALF PERIOD

$$\langle F \rangle_{\pi\omega_L} = \frac{\langle \Delta S_{11}(\omega) \rangle_{\pi\omega_L}}{eI_B} \quad \text{where} \quad \langle \Delta S_{11}(\omega) \rangle_{\pi\omega_L} = \frac{1}{\pi\omega_L}\int_0^{\pi\omega_L} \Delta S_{11}(\omega) d\omega$$

$\Rightarrow$ we obtain $\boxed{\langle F \rangle_{\pi\omega_L} \approx g}$

$\Rightarrow$ it should be possible to extract the value of the interaction parameter g

CONCLUSION

- **Simple relation** between the AC conductance the non-symmetrized noise

$$S_{nm}(\omega) = S^{+}_{nm}(\omega) - \hbar\omega \mathrm{Re}[G_{nm}(\omega)]$$

- In the presence of Coulomb interactions, the non-symmetrized noise is asymmetric:

Emission noise ($\omega > 0$) ≠ Absorption noise ($\omega < 0$)

- At low-temperature and for a long wire or a long nanotube, we obtain **oscillations** with a period related to L and g

- The **average non-symmetrized excess noise** over the first half period gives the value of g

$$\frac{\langle \Delta S_{11}(\omega) \rangle_{\pi\omega_L}}{eI_B} \approx g$$

ACKNOWLEDGMENTS TO: C. GLATTLI, H. BOUCHIAT, R. DEBLOCK
T. KONTOS, B. REULET, E. SUKHORUKOV