TRANSPORT PHENOMENA FOR BOSONS IN Y-JUNCTION AND ITS RELATED SYSTEMS

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Phys. Rev. Lett. **100**, 140402 (2008)

OUTLINE

– SINGLE Y-JUNCTION

Transmission and reflection at junction

- Bosons filled in whole of Y-junction.
- We predict that the negative density reflection can be observed.
- This phenomenon is similar to that of the Andreev type.

– RING INTERFEROMETER WITH FLUX INSIDE THE RING

Transport between leads through the ring

- Bosons is filled in the whole of system.
- Effective magnetic flux exists inside the ring.
- Absence of Aharonov-Bohm effect.

Y-SHAPED CONFINEMENT

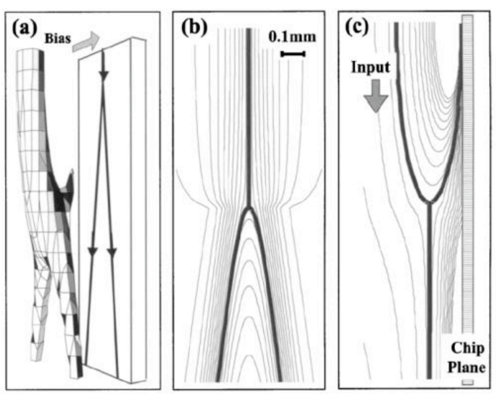
WHY Y-SHAPED CONFINEMENT IN COLD ATOM SYSTEMS?

- Control of the motion of BEC
- Role of beam splitter

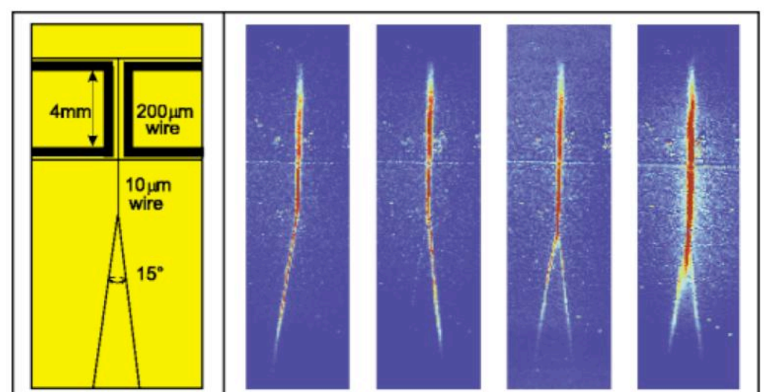
HOW TO MAKE Y-SHAED SYSTEMS

- Magnetic optical trap over micro-fabricated atom chips

D. Cassettari *et al.,* Phys. Rev. Lett. **85** 5483 (2000)
D. Cassettari *et al.,* App. Phys. B **70**, 721 (2000)
J. Fortágh and C. Zimmermann, Rev. Mod. Phys. **79**, 235 (2007)



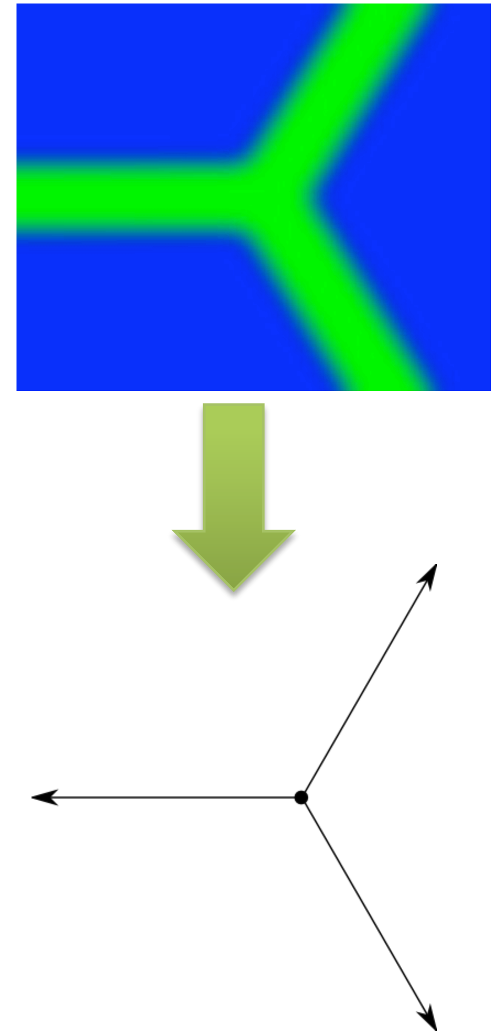
3D image and contour map for trap potential.



Circuit of atom chip, and image of guiding Y-shaped trapped BEC.

MODEL

$$\mathcal{H} = \mathcal{H}_{\mathrm{bulk}} + \mathcal{H}_{\mathrm{boundary}}$$



BULK PART

- Describe Y-shaped boson system as 1D Y-junction.
- Interaction between bosons is repulsive.
- Bosons are filled in the whole of the system.

Low-energy effective theory: Tomonaga-Luttinger liquid

$$\mathcal{H}_{\mathrm{bulk}} = \sum_{j=1,2,3} \frac{\hbar v}{2\pi} \int dx \left[K \left(\frac{\partial \theta_j}{\partial x} \right)^2 + \frac{1}{K} \left(\frac{\partial \varphi_j}{\partial x} \right)^2 \right]$$

K : Luttinger parameter. K>1 for repulsive bosons.

v : Velocity of low-energy excitation

BOUNDARY PART

- Non-trivial problem.
- Similar to the tunneling problem through a potential barrier

C. Nayak *et al.,* Phys. Rev. B **59**, 15694 (1999)

M. Oshikawa *et al.,* J. Stat. Mech. P02008 (2006)

Boundary nature → Boundary conditions + perturbation

What is stable, suitable boundary condition in low-energy limit?

BOUNDARY CONDITION

BOUNDARY CONDITION

Primary boundary condition: current conservation

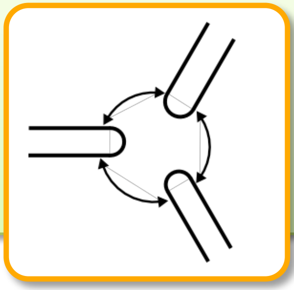
$$\partial_t \varphi_1(0,t) + \partial_t \varphi_2(0,t) + \partial_t \varphi_3(0,t) = 0$$

What boundary conditions are suitable in low-energy limit?

Disconnected limit: $\partial_t \varphi_{1,2,3}(0,t) = 0$

Perturbation: tunneling between branches.

Scaling dimension = 1/K <1 RELEVANT!

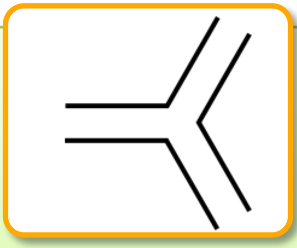


The tunneling amplitude is renormalized to infinity as the energy scale lowered.

Strongly coupled limit: $\partial_x \varphi_1(0,t) = \partial_x \varphi_2(0,t) = \partial \varphi_3(0,t)$

Perturbation: backscattering

Scaling dimension = 4K/3 >1 IRRELEVANT!



This fixed point to this boundary condition is the suitable terminal of the renormalization flow.

DYNAMICS & NEGATIVE REFLECTION

TIME EVOLUTION OF DENSITY CONFIGURATION

Time evolution of the expectation values of fields on each branch

$$\langle \rho_j(x,t)\rangle = \rho^0 + \rho_j^L(x+vt) + \rho_j^R(x-vt).$$

INITIAL CONDITION

On branch 1
$\rho_1^L(x,0) = \mathcal{D}_0(x) \quad \rho_1^R(x,0) = 0$

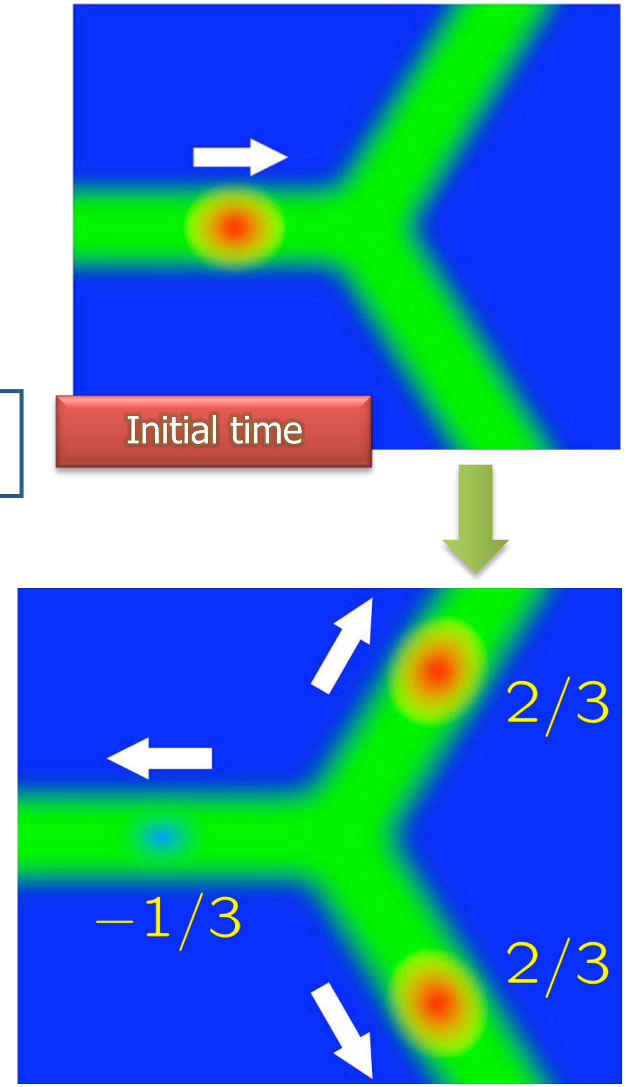
On branch 2 & 3
$\rho_{2,3}^L(x,0) = 0 \qquad \rho_{2,3}^R(x,0) = 0$

$$\bar{\rho}_1(x,t) = \bar{\rho}_0 + \mathcal{D}(vt+x) - \underline{\frac{1}{3}\mathcal{D}(vt-x)\Theta_S(vt-x)}$$

REFLECTION

$$\bar{\rho}_{2,3}(x,t) = \bar{\rho}_0 + \underline{\frac{2}{3}\mathcal{D}(vt-x)\Theta_S(vt-x)}$$

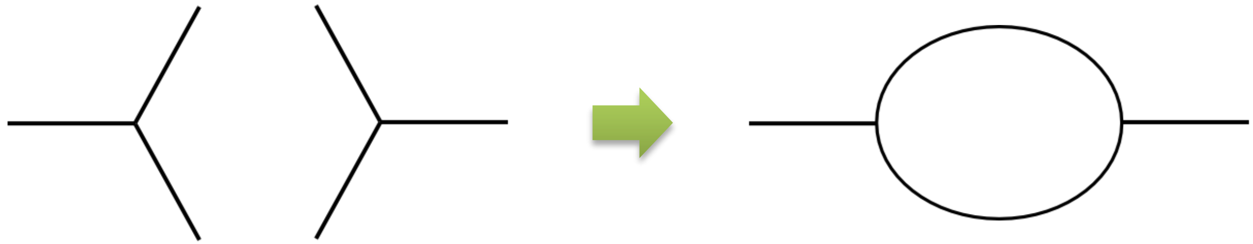
TRANSMISSION



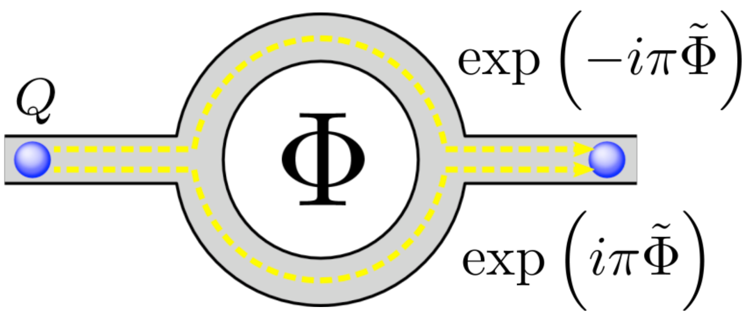
- Enhancement of total transmission: 4/3
- Negative density reflection: -1/3

RING-TYPE INTERFEROMETER

From Y-junction to ring-type interferometer geometry



SINGLE PARTICLE DISCUSSION



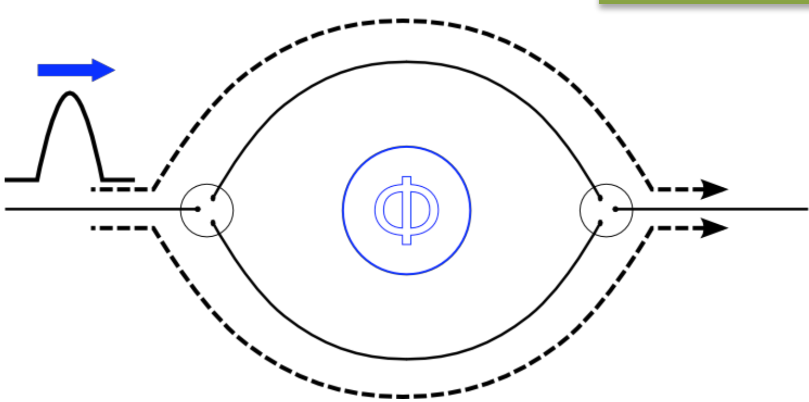
Transmission between leads

$$|g|^2 = |\psi_0 e^{i\pi\tilde{\Phi}} + \psi_0 e^{-i\pi\tilde{\Phi}}|^2 = 2|\psi_0|^2\left(1 + \cos 2\pi\tilde{\Phi}\right)$$

$$\tilde{\Phi} \equiv \Phi Q/h$$

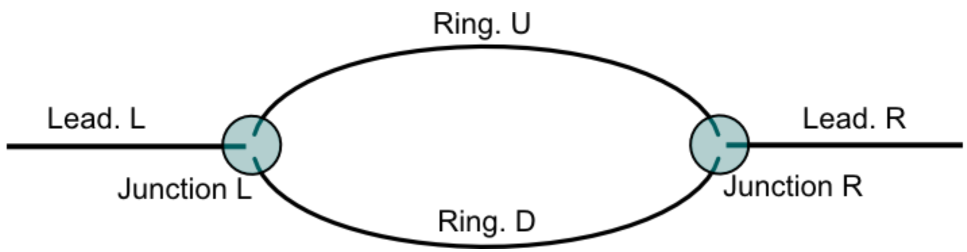
INTERFERENCE $\tilde{\Phi} = \mathbb{Z} + 1/2 \quad \Rightarrow \quad |g|^2 = 0$

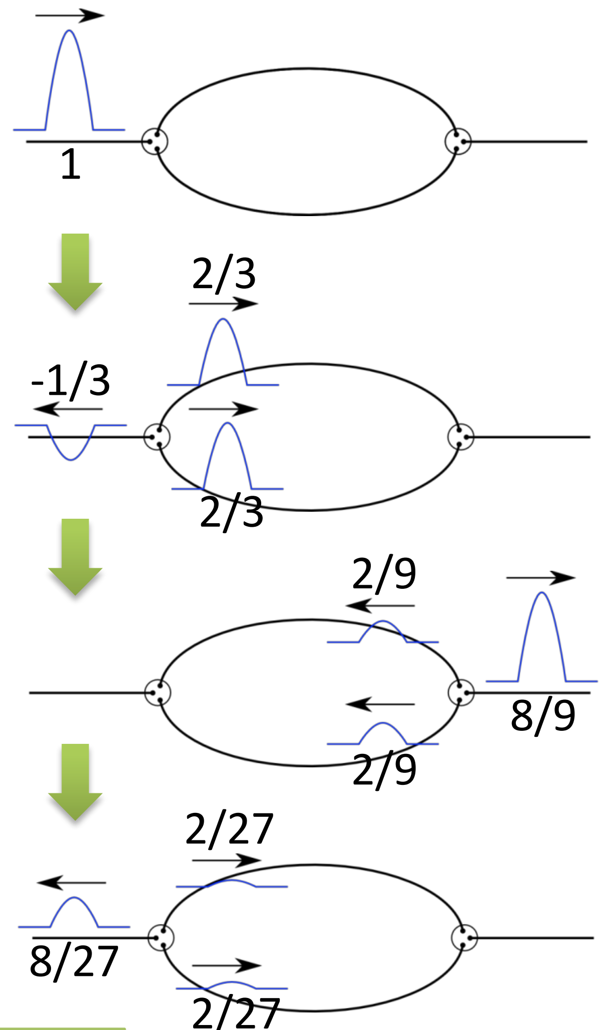
Aharonov-Bohm effect



For boson systems, is the same interference observed?

EFFECTIVE THEORY & RESULT





Gauge transformation & bosonization

<u>All branches</u> TL liquids

<u>Boundary condition</u>

Junction L

$$\partial_t \varphi_L = \partial_t \varphi_U + \partial_t \varphi_D \qquad \varphi_L = \varphi_U = \varphi_D$$

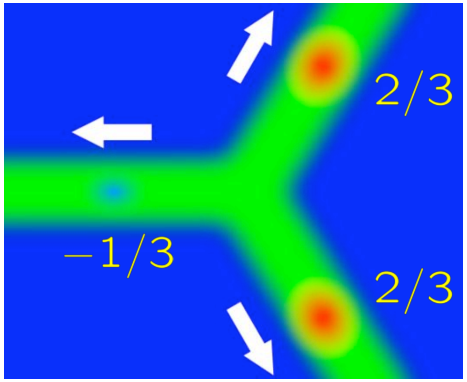
Junction R

$$\partial_t \varphi_U + \partial_t \varphi_D = \partial_t \varphi_R \qquad \varphi_U + \pi\tilde{\Phi} = \varphi_D - \pi\tilde{\Phi} = \varphi_R$$

Flux appears only in BCs. → absence in BCs for density.

NO INTERFERENCE → Absence of Aharonov-Bohm effect
MAGNETIC FLUX INSIDE RING → Just driving persistent current

SUMMARY



SINGLE-JUNCTION FOR BOSE LIQUID

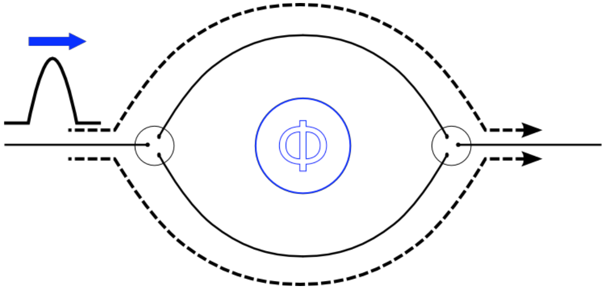
- Three TL liquid connected at junction.
- Connected boundary condition is stable in low-energy limit.

Negative reflection & Enhancement of total transmission

RING-TYPE INTERFEROMETER FOR BOSE LIQUID

- Dynamics of density packet in ring-type interferometer.
- Presence of effective magnetic flux inside ring.
- Strongly coupled boundary condition at junctions.

Absence of AB effect



THANK YOU FOR YOUR ATTENTION.