Universal Theory of Nonlinear Luttinger Liquids

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in collaboration with

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Earlier work:

Bosons: PRL **100** 206805 (2008)

Outline

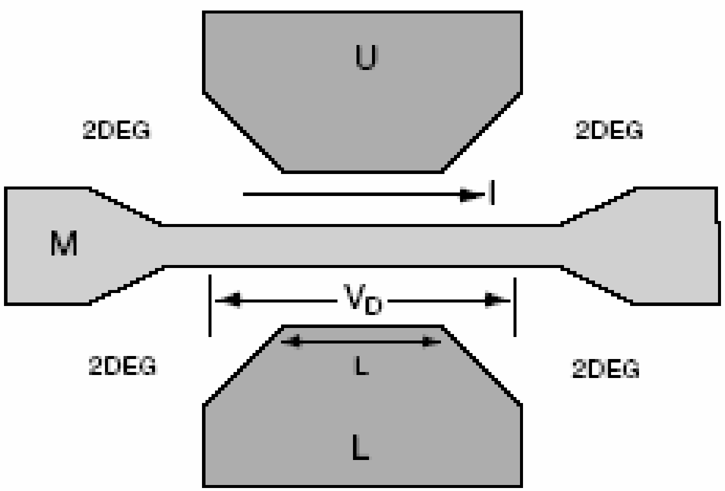
Universal Theory of Nonlinear Luttinger Liquids

curvature + interaction in 1D?

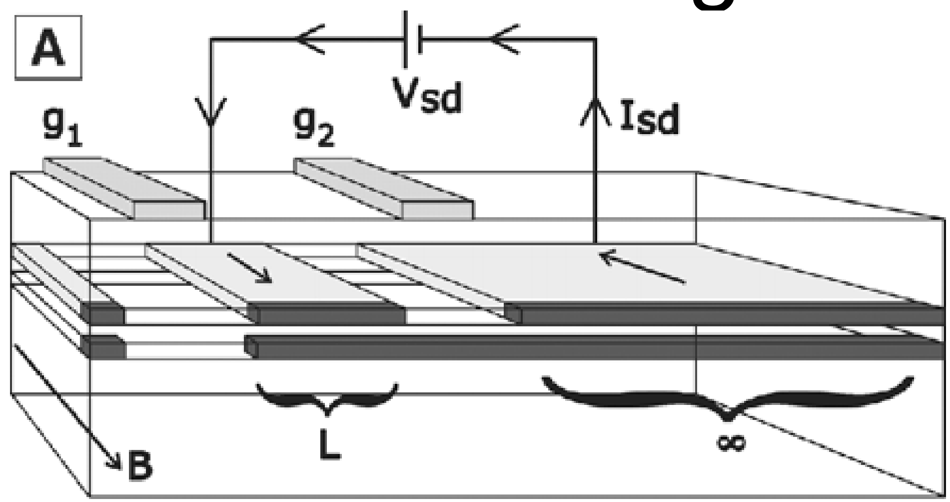
- Linear Luttinger liquid theory (bosonization)
- Universal Theory of Nonlinear Luttinger Liquids: **refermionize!**
- Universality beyond low energy limit
- Conclusions

Motivation

- Transport in nanowires: Coulomb drag, momentum-energy resolved tunneling



(a)



- Ultracold atomic gases
- Neutron scattering on spin chains
- Numerical methods for simulation of dynamics in 1D

Luttinger liquid theory

aka ~~bosonization~~ (Haldane,1981)

Refermionize !

Based on Tomonaga-Luttinger model:

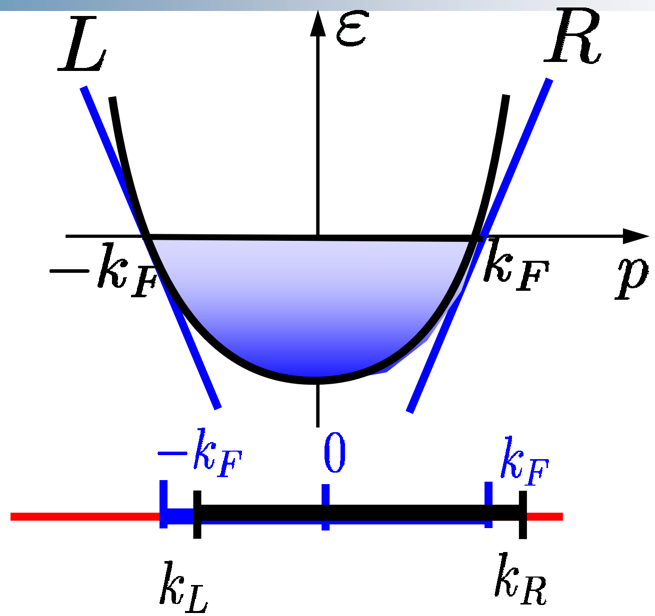
1D interacting fermions with ~~strictly linear~~ single particle spectrum

excitations of the system of interacting fermions ⟷ linear bosonic sound waves

Bosonization–~~linear~~ quantum hydrodynamics

nonlinear, but still universal

Bosonization for fermions without spin



local current $\partial_x \theta \propto \delta k_R + \delta k_L$

local density $-\partial_x \varphi \propto \delta k_R - \delta k_L$

Linear hydrodynamics:

$$H = \frac{v}{2\pi}\int dx \left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\varphi)^2 \right]$$

v sound velocity

$$K = \frac{v_F}{v}$$

Luttinger Liquid parameter

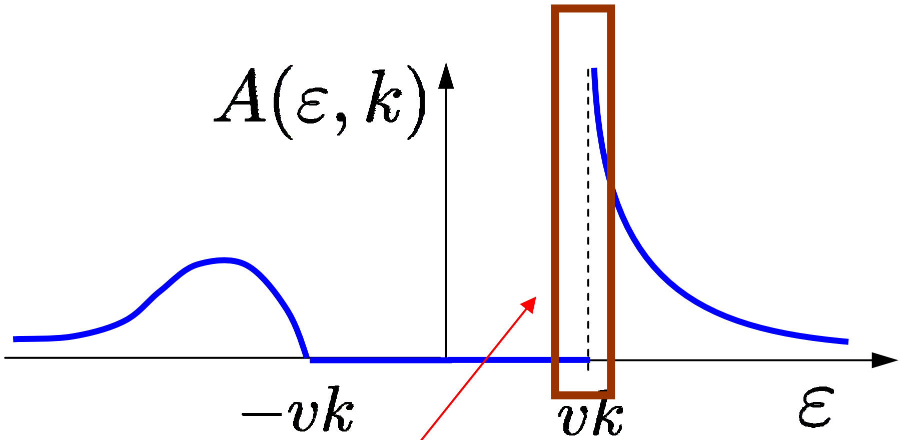
For Galilean systems depends on velocity, but controls correlation functions

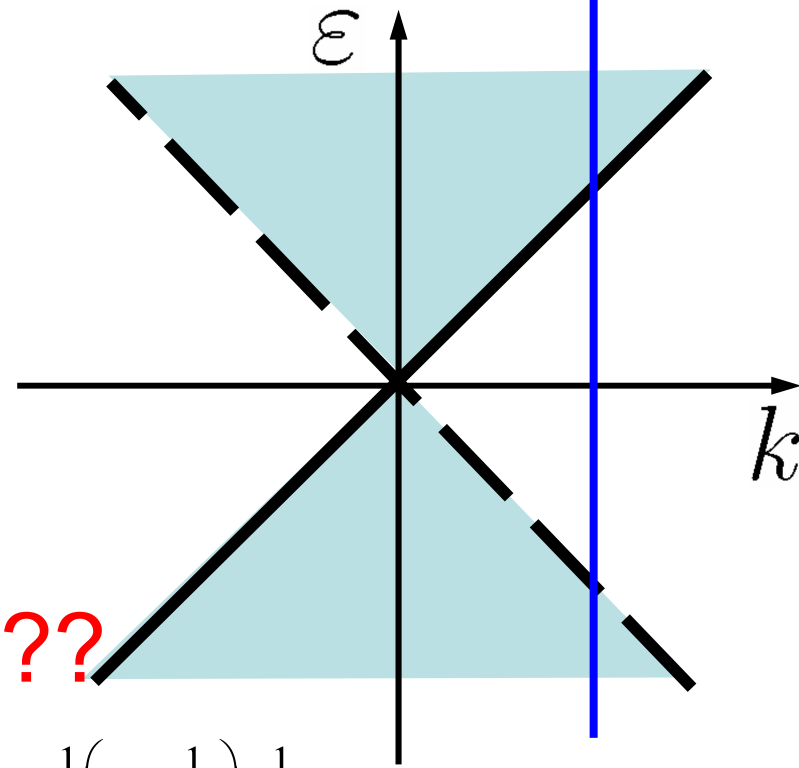
Spectral function in Tomonaga-Luttinger

Fermionic field: $\Psi_R^+ \propto e^{i\varphi - i\theta}$

Spectral function: probability to tunnel in a fermion

$$A(\varepsilon, k) = -\mathrm{Im}G^{ret}(\varepsilon, k)/\pi$$

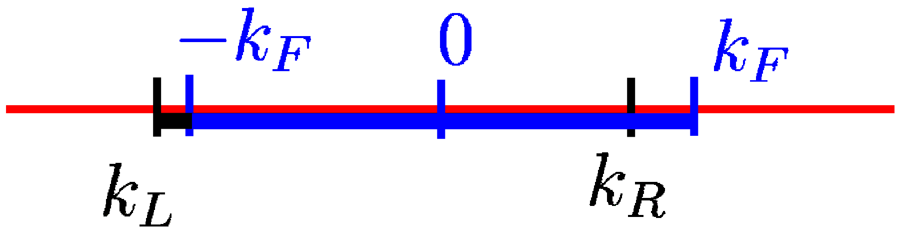




Is it true for real systems???

$$A(\varepsilon, k) \propto \frac{\theta\left[\varepsilon^2 - \mathrm{v}^2 k^2\right]}{\varepsilon - \mathrm{v}k}\left(\varepsilon^2 - \mathrm{v}^2 k^2\right)^{\frac{1}{4}\left(K + \frac{1}{K}\right) - \frac{1}{2}}$$

Spectrum nonlinearity in bosonization



$$\delta k_{R(L)} \propto \partial_x \theta \mp \partial_x \varphi$$

$$\xi_k = \pm \mathrm{v_F} k + \frac{k^2}{2m}$$

$$H_{kin}(x) = \int_{k_L(x)}^{k_R(x)} \xi_k \frac{dk}{2\pi} \propto \mathrm{v_F}\left[(\partial_x\theta)^2 + (\partial_x\varphi)^2\right]$$

$$\propto \frac{1}{m}\left(\partial_x\varphi \pm \partial_x\theta\right)^3$$

Nonlinear quantum field theory ☹

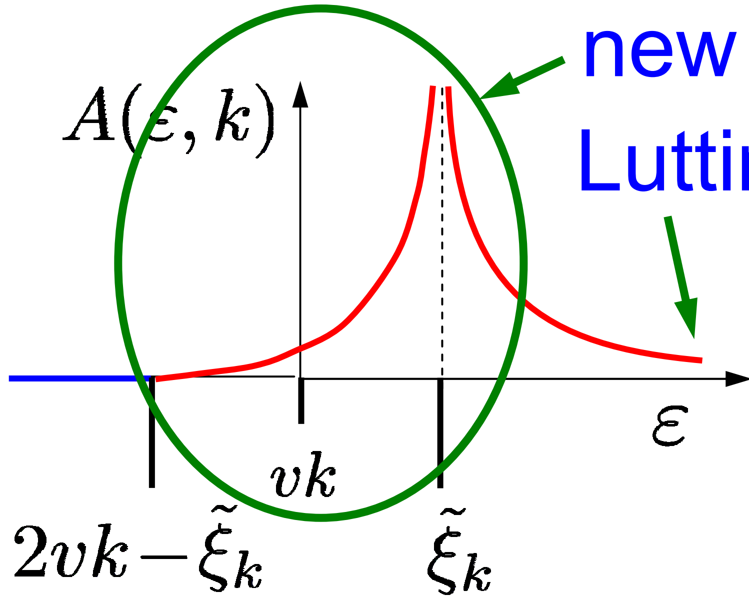
Corrections to $A(\varepsilon, k)$ in powers of $\frac{1}{m}$

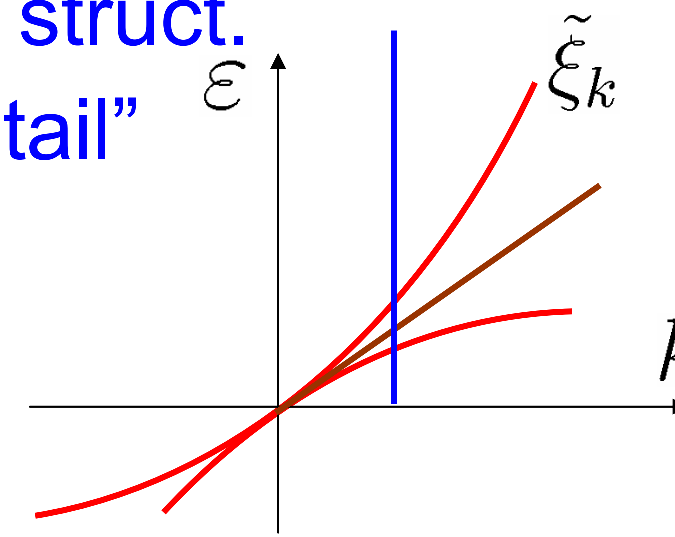
diverge near $\varepsilon - \mathrm{v}\, k \sim \dfrac{k^2}{2m}$ in all orders

Effect of Nonlinear Dispersion Relation

$$|\varepsilon - \mathrm{v}k| << \mathrm{v}k, \;\; k/k_{\mathrm{F}} \to 0$$

Finite mass - new energy scale $\delta\omega = \frac{k^2}{2m_*}$





Universal crossover function $A(\varepsilon, k) \propto A\left(\frac{\varepsilon - vk}{k^2/2m_*}\right)$

New Exponents

$$A(\varepsilon, k) \propto |\varepsilon - \tilde{\xi}_k|^{\gamma_{\rm true}}$$

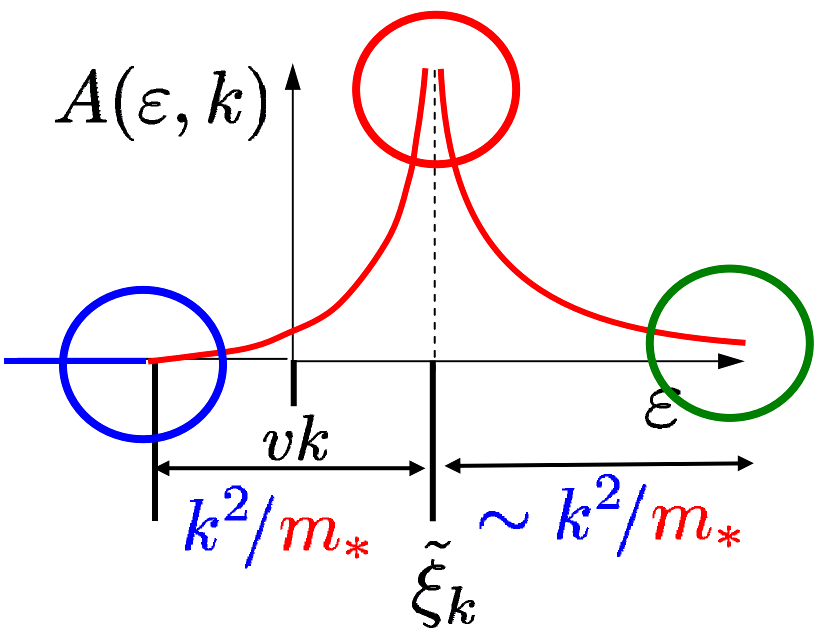
$$\gamma_{\rm true} = \frac{1}{2}\left[\left(1 - \frac{1}{\sqrt{K}}\right)^2 + \left(1 - \sqrt{K}\right)^2\right] - 1$$

$$\frac{1}{m_*} = \frac{\mathrm{v}}{\sqrt{K}}\frac{\partial \mathrm{v}}{\partial \mu} + \frac{\mathrm{v}^2}{2K\sqrt{K}}\frac{\partial K}{\partial \mu}$$

R.G. Pereira et al, 06

$$A(\varepsilon, k) \propto \left(\varepsilon - \mathrm{v}k + \frac{k^2}{2m}\right)^{\gamma}$$

$$\gamma = \left(\frac{1}{2\sqrt{K}} - \frac{\sqrt{K}}{2}\right)^2 + \left(1 + \frac{1}{2\sqrt{K}} + \frac{\sqrt{K}}{2}\right)^2 - 1$$



$$A(\varepsilon, k) \propto (\varepsilon - vk)^{\gamma_L}$$

$$\gamma_L = \frac{1}{4}\left(\frac{1}{K} + K\right) - \frac{3}{2}$$

Diagonalization of Tomonaga-Luttinger

$$H = \frac{\mathrm{v}}{2\pi}\int dx\left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\varphi)^2\right]$$

θ and φ canonically conjugate bosonic fields

Diagonalization: canonical Bogoliubov rotation

$$\tilde{\varphi} = \varphi/\sqrt{K} \qquad \tilde{\theta} = \theta\sqrt{K}$$

$\tilde{\theta}$ and $\tilde{\varphi}$ are still canonically conjugate

$$H = \frac{\mathrm{v}}{2\pi}\int dx\left[(\partial_x\tilde{\theta})^2 + (\partial_x\tilde{\varphi})^2\right]$$

⇓ Refermionize

Free fermionic quasiparticle Hamiltonian

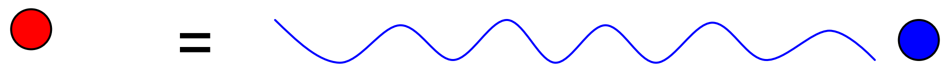
Refermionization of Tomonaga-Luttinger

$$\tilde{H}_1 = iv\int dx\left(:\tilde{\Psi}^\dagger_{\mathrm{L}}(x)\nabla\tilde{\Psi}_{\mathrm{L}}(x): - :\tilde{\Psi}^\dagger_{\mathrm{R}}(x)\nabla\tilde{\Psi}_{\mathrm{R}}(x):\right)$$

A.V.Rozhkov, 2005

Fermion via “new” fermionic quasiparticle:

$$\Psi^\dagger_{\mathrm{R}}(x) = \tilde{F}^\dagger_{\mathrm{R}}(x)\tilde{\Psi}^\dagger_{\mathrm{R}}(x)$$



fermion string + quasiparticle

Non-local “string” operator (like Jordan-Wigner):

$$\tilde{F}^\dagger_R(x) = \exp\left[-i\int^x dy\left(\frac{\delta_+}{2\pi}\tilde{\rho}_R(y) + \frac{\delta_-}{2\pi}\tilde{\rho}_L(y)\right)\right]$$

Universal phase shifts

$$\frac{\delta_+}{2\pi} = 1 - \frac{1}{2\sqrt{K}} - \frac{\sqrt{K}}{2} \qquad \frac{\delta_-}{2\pi} = \frac{1}{2\sqrt{K}} - \frac{\sqrt{K}}{2}$$

Universal Hamiltonian

$$\tilde{H}_1 = iv \int dx \left(:\tilde{\Psi}^\dagger_{\mathrm{L}}(x) \nabla \tilde{\Psi}_{\mathrm{L}}(x): - :\tilde{\Psi}^\dagger_{\mathrm{R}}(x) \nabla \tilde{\Psi}_{\mathrm{R}}(x): \right)$$

$$\tilde{H}_2 = \frac{1}{2m_*} \int dx \left(:(\nabla \tilde{\Psi}^\dagger_{\mathrm{L}})(\nabla \tilde{\Psi}_{\mathrm{L}}): + :(\nabla \tilde{\Psi}^\dagger_{\mathrm{R}})(\nabla \tilde{\Psi}_{\mathrm{R}}): \right)$$

Quasiparticles remain free!*

New scaling limit:

$$\frac{\varepsilon - \mathrm{v}\,k}{k^2 / 2m_*} \to \mathrm{const} \qquad \frac{k}{k_F} \to 0$$

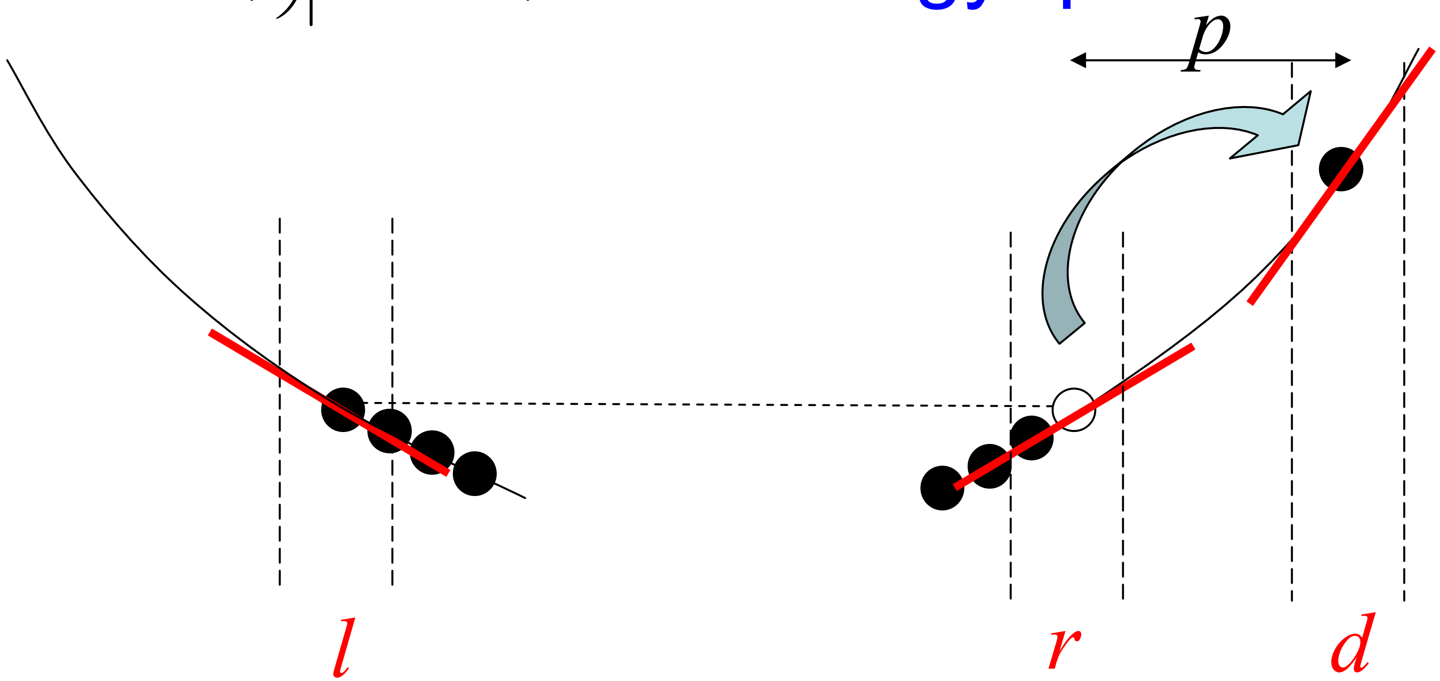
Universal crossover function $A(\varepsilon, k) \propto A\left(\frac{\varepsilon - vk}{k^2/2m_*} \right)$

* for original interactions decaying faster than ~1/x^2

Understanding new singularities

$$\left|\varepsilon - \left(\mathrm{v}\,p + \frac{p^2}{2m_*}\right)\right| << \frac{p^2}{2m_*}$$

One "high energy" particle + "low energy" particle-hole pairs



3 subband model: mobile impurity

Luttinger Liquid

Anderson's orthogonality catastrophe

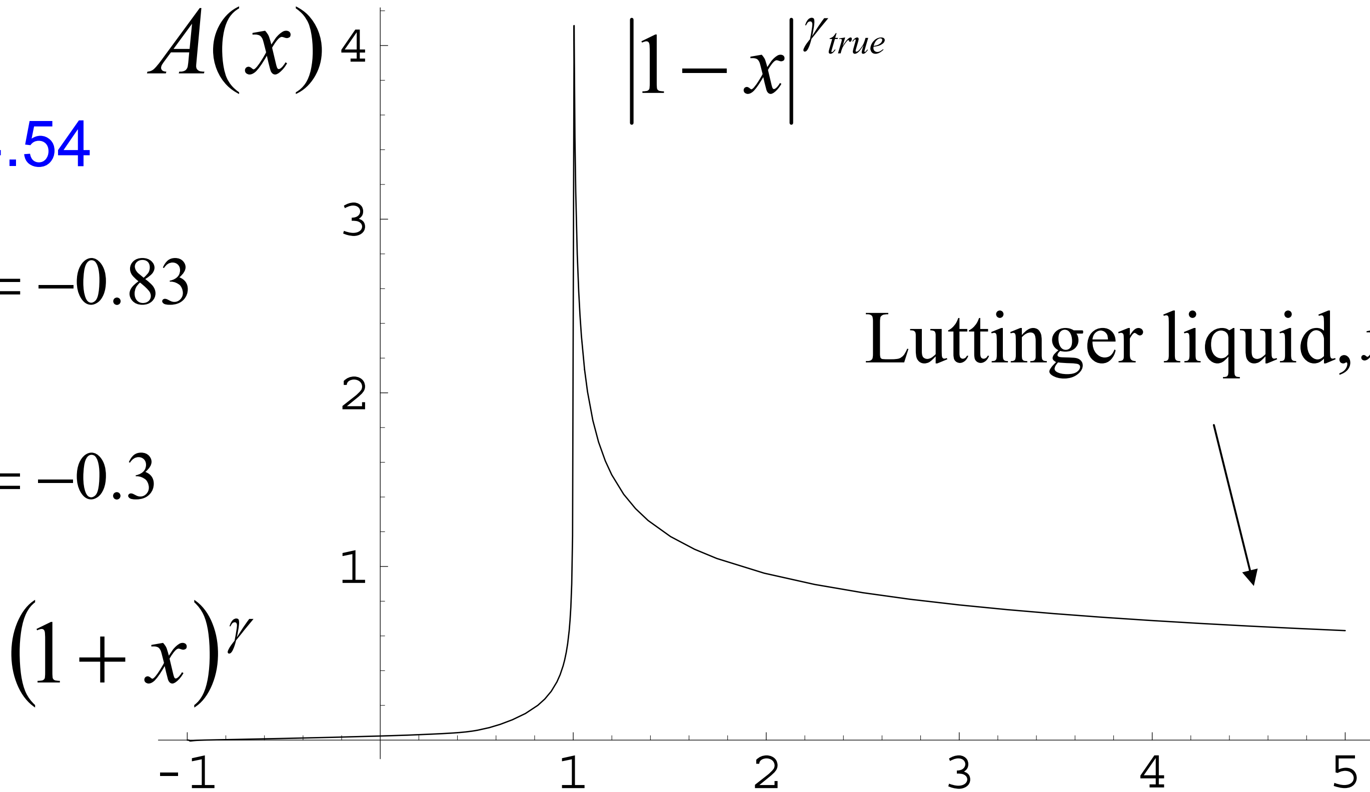
$$\gamma_{true} = \left(\frac{\delta_-}{2\pi}\right)^2 + \left(\frac{\delta_+}{2\pi}\right)^2 - 1$$

Universal crossover

K=4.54

$$\frac{\delta_-}{2\pi} = -0.83$$

$$\frac{\delta_+}{2\pi} = -0.3$$



$$x = \frac{\varepsilon - \mathrm{v}\,k}{k^2/2m_*}$$

Noninteracting quasiparticles, but complicated correlator ⟷ Infinite dimensional determinant

(Scalapino et al 1981, Hirsh 1985)
(Klich-Abanin-Levitov, 2002-2005)

Evaluate numerically

Other 1D Systems

Spectral function of 1D bosons (Lieb-Liniger)

$$A(\varepsilon, k) \propto \left|\varepsilon - \tilde{\xi}_k\right|^{\gamma_{\mathrm{true}}}$$

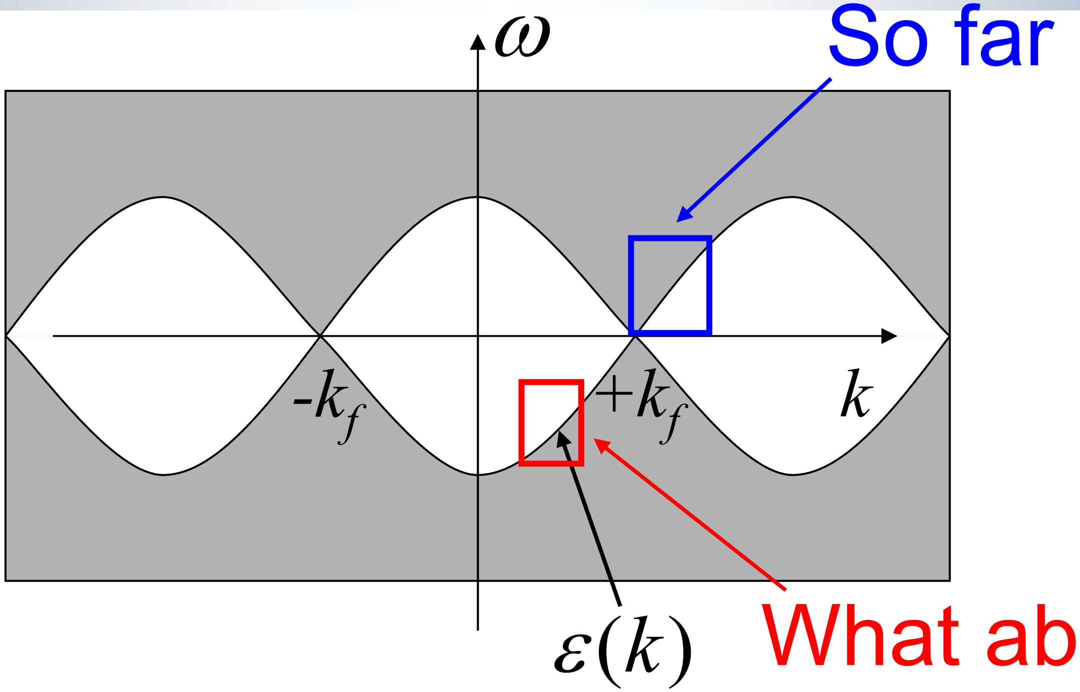
$$\gamma_{\mathrm{true}} = -\frac{1}{\sqrt{K}} + \frac{1}{2K}$$

S=1/2 XXZ antiferromagnet in a field along z-axis

$$S^{-+}(k,\omega) = \sum_j e^{-ikj} \int dt\, e^{i\omega t} \left\langle S_j^-(t) S_0^+(0) \right\rangle$$

$$S^{-+}(k,\omega) \propto \mathrm{const} + \left| \frac{1}{\omega - \left(v|k-\pi| \pm \frac{(k-\pi)^2}{2m_*} \right)} \right|^{\pm\frac{1}{\sqrt{K}} - \frac{1}{2K}}$$

Beyond low energy limit (unpublished)



Momentum dependent phase shifts

For Galilean systems related to

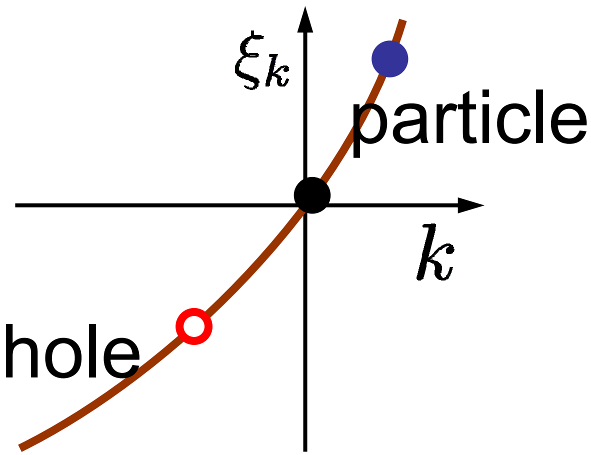
$$\delta_-(k), \delta_+(k) \longleftrightarrow \frac{\partial \varepsilon(k)}{\partial k}, \frac{\partial \varepsilon(k)}{\partial n}$$

Position of the edge defines the singularities!

Conclusions

1D fermions:

curvature + interaction



$$A(\varepsilon, k) \propto A\left(\frac{\varepsilon - vk}{k^2/2m_*}\right)$$

New exponents