Quantum Wires Coupled to Dissipative Enviroments

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In collaboration with:

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Phys. Rev. Lett. 97, 076401 (2006)

Workshop on Correlations and Coherence in Quantum Matter,
Évora (Portugal), November 10-14

How to pin a Wigner Crystal Using a Metallic Gate

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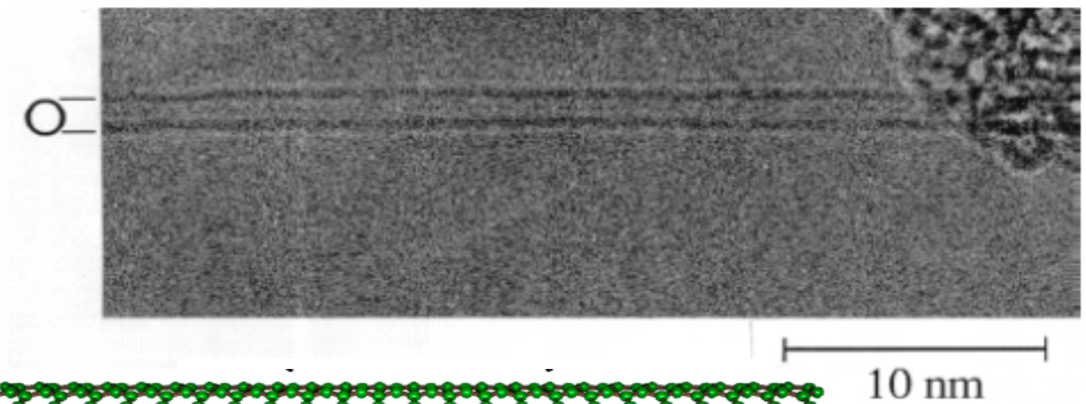
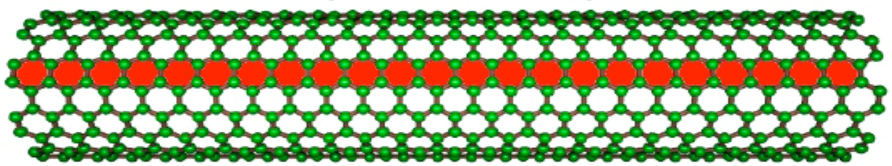
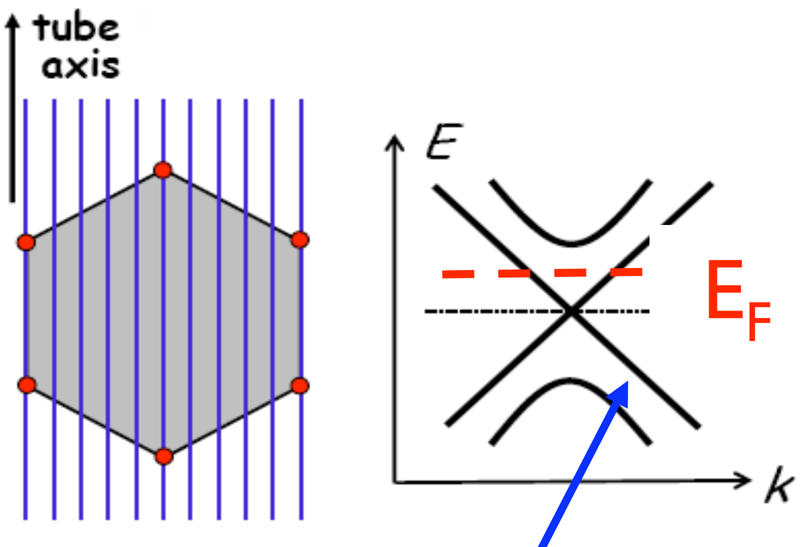
Give me a 1D wire!
Single Wall Carbon Nanotubes (SWCNT's)

Armchair CNT: Perhaps the cleanest 1D metal...





飯島さん (Iijima)

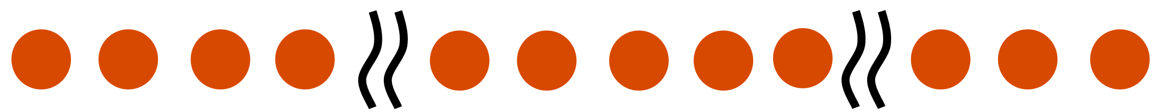


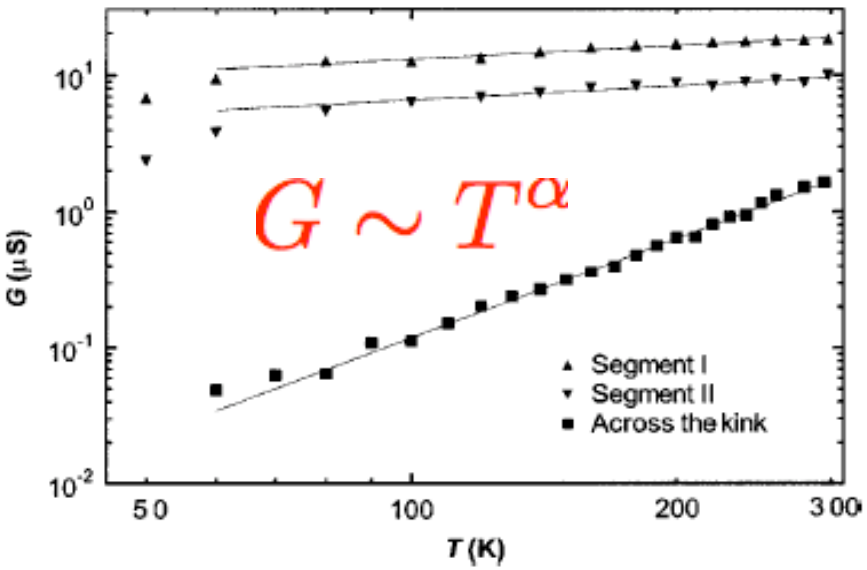
Slightly doped

朝永 (Tomonaga)-Luttinger liquid (TLL) behavior

SWCNT's: A fluctuating Wigner Crystal

Coulomb potential in 1D nanostructures is poorly screened





[Yao *et al* Nature 402 (1999)]

SWCNT's almost a Wigner Crystal

$$\langle \rho(x)\rho(0)\rangle\Big|_{8k_F} \propto \frac{\cos(8k_F x)}{x^{8g}}$$

$$\lambda_0 = \frac{2\pi}{2n_{\mathrm{ch}}\,k_F} = \frac{2\pi}{8k_F},$$

$$n_{\mathrm{ch}} = 2\times 2$$

Small Tomonaga-Luttinger parameter

(e.g. B Gao *et al* PRL 92 (2004), *g* = *0.2*)

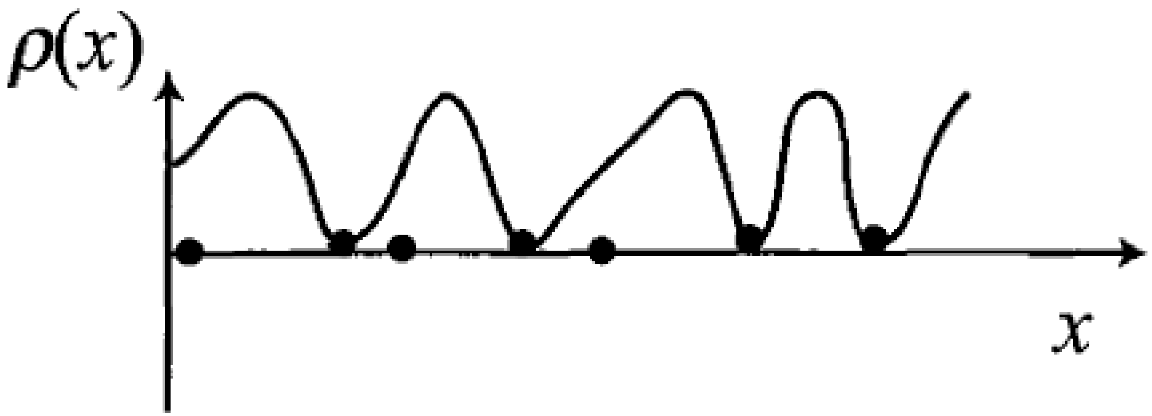
$$g = \left[1 + \frac{8e^2}{\pi\hbar v_F}\ln\left(\frac{R_s}{R}\right)\right]^{-\frac{1}{2}}$$

Yet, a fluctuating crystal...

$$\langle \rho(x)\rangle = \rho_0 = \mathrm{const.}$$

Can we pin this fluctuating Wigner Crystal ?

Of course, disorder will do it, but ...

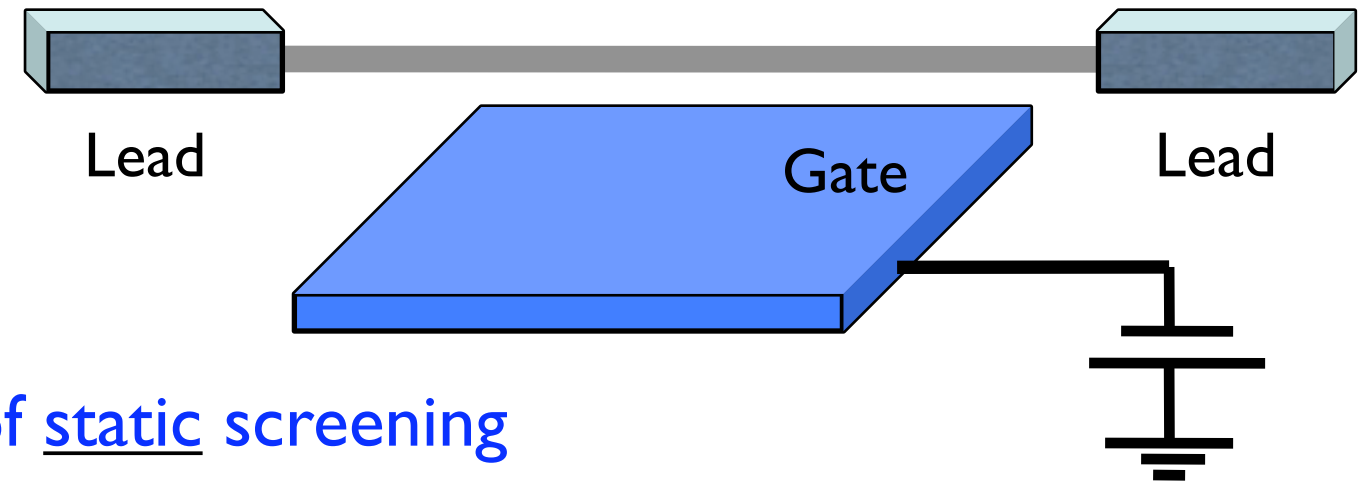


The electron density wave will 'adapt' to the disorder and we will end up seeing the disorder itself (the localized electron waves) but not the Wigner crystal.
[Boundaries or isolated impurities will lead to (exponentially, at finite T) decaying Friedel oscillations]

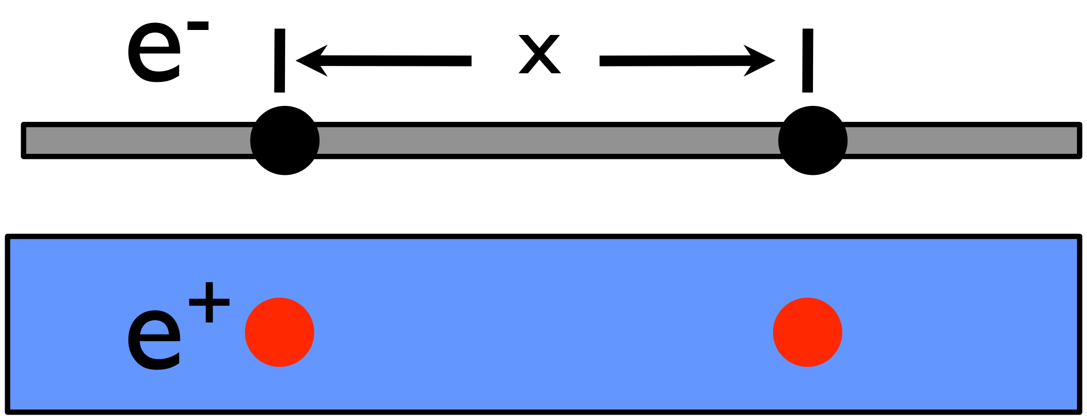
We want a true ground state with true long range order

How about gates?

Tuners of the chemical potential in mesoscopic systems



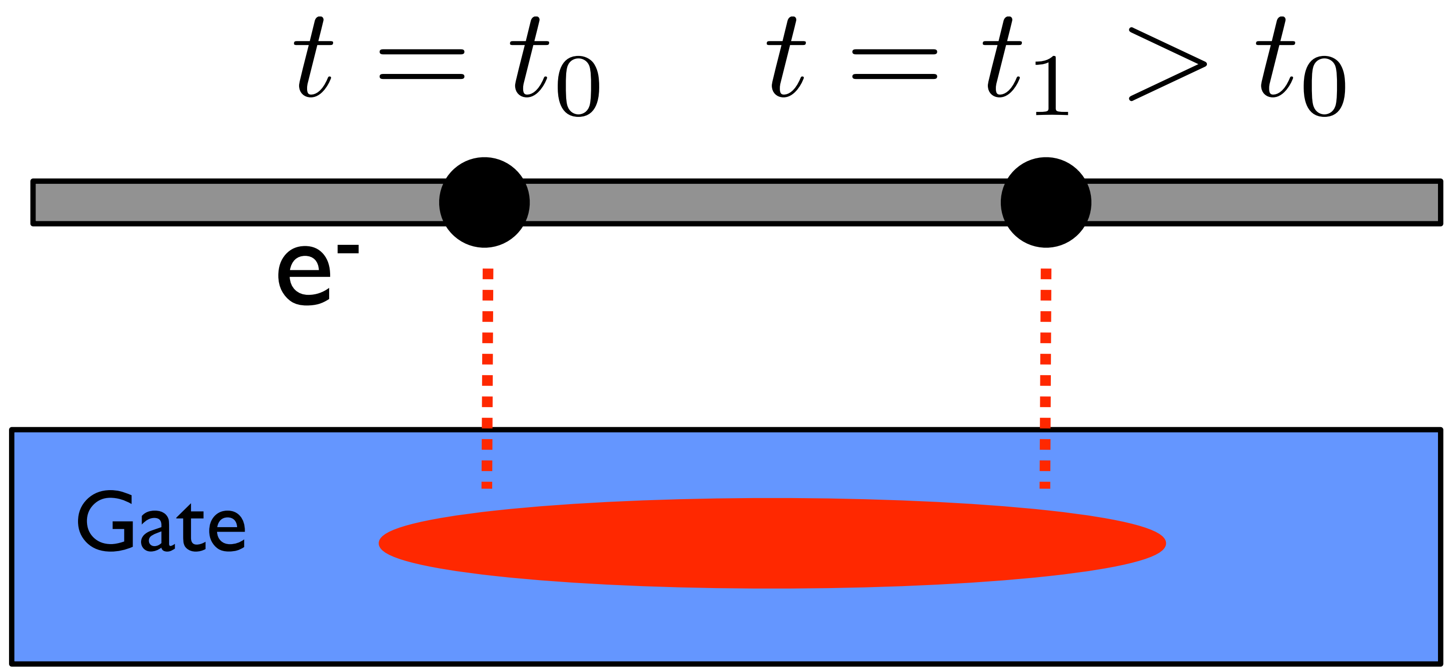
Sources of static screening



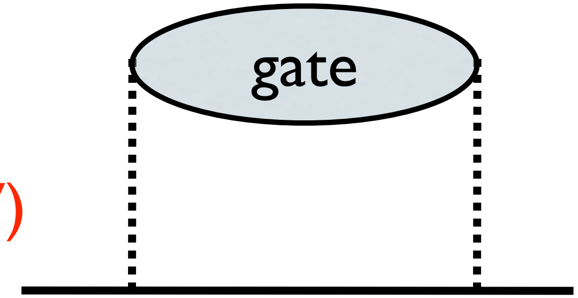
$$V_{\text{Coulomb}}(x) \sim \frac{1}{|x|} \Rightarrow V_{\text{dipole}}(x) \sim \frac{1}{|x|^3}$$

But can a gate pin the 1D Wigner Crystal?

However, the screening due to gates is dynamic



Model of the gate



Integrate out gate's degrees of freedom

F Guinea PRL 53 (1984); F Sols & F Guinea, PRB 36 (1987)

$$S_{\rm D} = \frac{1}{2\hbar}\int d\tau d\mathbf{r}\, d\tau' d\mathbf{r}'\, \rho(\mathbf{r},\tau) V_{\rm scr}(\mathbf{r},\mathbf{r}',\tau-\tau')\rho(\mathbf{r}',\tau')$$

$$V_{\rm scr}(\mathbf{r},\mathbf{r}',\tau) = -\frac{1}{\hbar}\langle V_{\rm C}(\mathbf{r},\tau)V_{\rm C}(\mathbf{r}',0)\rangle; \quad V_{\rm C}(\mathbf{r},\tau) = \int d\mathbf{r}'\, \frac{\rho_{\rm gate}(\mathbf{r}',\tau)}{4\pi\epsilon|\mathbf{r}-\mathbf{r}'|}$$

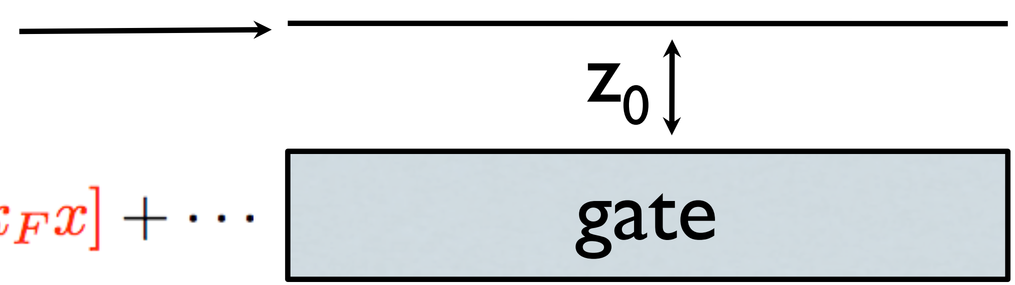
Static screening part: image potential

$$V_{\rm scr}(x,z_0,\tau) \simeq W_{\rm scr}(x,z_0)\delta(\tau) + \boxed{\frac{S(x,z_0)}{|\tau|^2}} \qquad |\tau| \gg \tau_c \sim \omega_p^{-1}$$

Ohmic dissipation: e.g. particle-hole excitations in the gate

$$\rho(\mathbf{r}) = \rho(x)\delta(y)\delta(z-z_0)$$

$$\rho(x) = \frac{1}{\pi}\partial_x\phi(x) + \mathcal{A}\cos\left[2\phi(x)+2k_F x\right] + \cdots$$



Model of 1D wire

Use bosonization $\rho(x) = \frac{1}{\pi}\partial_x\phi(x) + \mathcal{A}\cos\left[2\phi(x) + 2k_F x\right] + \cdots$

$$\begin{aligned}
S[\phi] &= S_0[\phi] + S_D[\phi], \\
S_0[\phi] &= \frac{1}{2\pi g}\int dx d\tau \left[\frac{1}{v}\left(\partial_\tau\phi\right)^2 + v\left(\partial_x\phi\right)^2\right], \quad v_F = vg, \\
S_D[\phi] &= S_{FD}[\phi] + S_{BD}[\phi], \\
S_{FD}[\phi] &= \int \frac{dq d\omega}{(2\pi)^2} f(q,\omega)\left|\phi(q,\omega)\right|^2
\end{aligned}$$

Forward (FD) $q \approx 0,$

Backward (BD) $q \approx Q = 2p^2 k_F$

$$f(q,\omega) \sim \begin{cases} \ln(1/q)q^2|\omega| \Rightarrow \text{Irrelevant (3D gate)} \\ q|\omega| \Rightarrow \text{Marginal (2D gate)}, \end{cases}$$

$$S_{BD}[\phi] = -\frac{\eta}{\pi}\int dx d\tau d\tau' \frac{\cos 2p\left[\phi(x,\tau) - \phi(x,\tau')\right]}{|\tau - \tau'|^{s+1}}$$

Ohmic s=1

Low-energy effective field theory

$$S[\phi] = S_0[\phi] + S_D[\phi],$$

$$S_0[\phi] = \frac{1}{2\pi g}\int dx d\tau \left[\frac{1}{v_{\rm pl}}\left(\partial_\tau\phi\right)^2 + v_{\rm pl}\left(\partial_x\phi\right)^2\right],$$

$$S_D[\phi] = -\frac{\alpha}{\pi v_{\rm pl}\tau_c}\int dx d\tau d\tau' \frac{\cos 2p\left[\phi(x,\tau)-\phi(x,\tau')\right]}{|\tau-\tau'|^{1+s}}$$

$$n_{\rm ch} = p^2$$

$p = 1$ spin polarized e^-

$p = \sqrt{2}$ spinful e^- ($g < 1/3$)

$p = 2$ nanotubes ($g < 1/5$)

Weak-coupling RG

$$\frac{dv}{d\ell} = -4p^2 g v \alpha, \qquad \frac{dg}{d\ell} = -4p^2 g^2 \alpha,$$

$$\alpha = (v\tau_c)\,\tau_c^{1-s}\eta$$

$$\frac{d\alpha}{d\ell} = \left(2 - s - 2p^2 g\right),$$

$$\ell = \log(\omega_c/T)$$

** QCP at $g^* = (2\text{-}s)/2p^2$ and $\alpha^* = 0$ with $z = 1$!!

Critical phase (Tomonaga-Luttinger liquid) for $g > g^*$ and $\alpha \rightarrow 0^+$

$$\mathrm{Re}\,\sigma(\omega > 0) \sim \alpha\,\mathcal{D}\left(\frac{\omega}{\omega_c}\right)^{\mu-4} \qquad \mu = 2p^2 g + s + 1$$

Strong coupling (SC) phase (?) for $g < g^*$ and $\alpha \rightarrow 0^+$

Correlation length near QCP $\qquad \dfrac{\xi_1}{a} \approx e^{-\pi/(2-s)p\sqrt{\alpha-\alpha_c}}$

SCHA or how to get insights into the SC phase

Replace the non-linear term

$$S_{BD}[\phi] = -\frac{\eta}{\pi}\int dx d\tau d\tau' \frac{\cos 2p\left[\phi(x,\tau) - \phi(x,\tau')\right]}{|\tau-\tau'|^2}$$

by a quadratic action

$$S_{\text{SCHA}}[\phi] = -\frac{1}{2}\int dx d\tau d\tau' \, \Sigma(\tau-\tau')\left[\phi(x,\tau)-\phi(x,\tau')\right]^2 \qquad \Sigma(\tau \gg \tau_c) \simeq \frac{\tilde{\eta}}{\pi|\tau|^2}$$

Optimize free energy: $\tilde{\eta} \sim a\left(\eta a\right)^{\frac{1}{1-2p^2 g}}$

Drude-like conductivity $\sigma(\omega) = \dfrac{iD}{\omega + i/\tau_d}, \quad \tau_d^{-1} \sim v_F \tilde{\eta}$

True CDW order

$$\langle \rho(x)\rho(0)\rangle_{Q=2pk_F} \approx a^{-1}\langle e^{2ip\phi(x)}e^{-2ip\phi(0)}\rangle \to \text{const.}$$

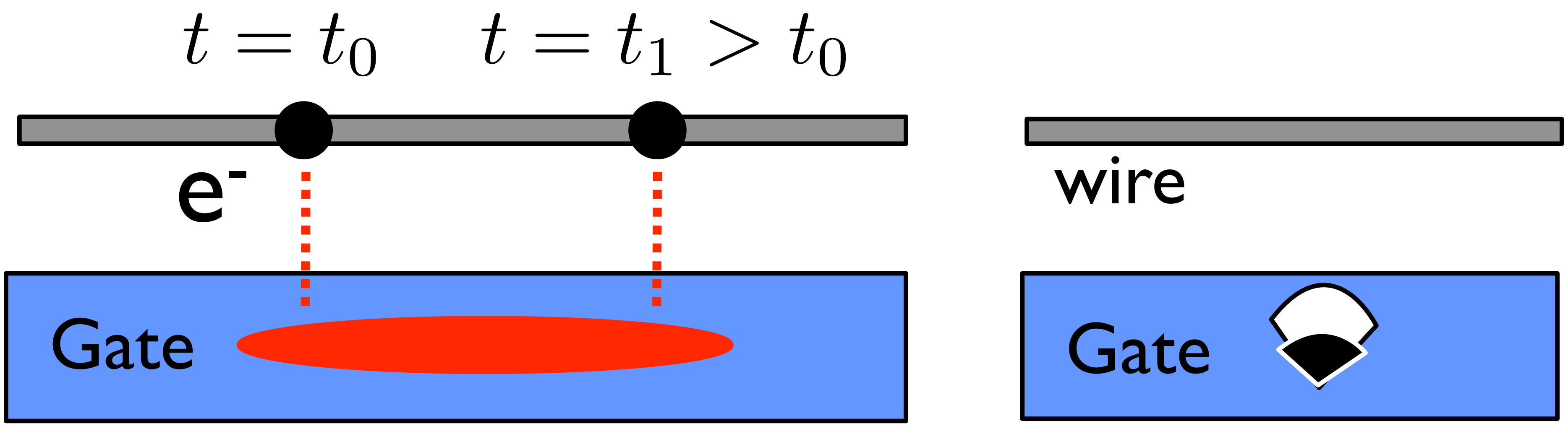
$$as\ |x| \to +\infty$$

"Seizing" of the vacuum

J Kogut and L Susskind PRD 11 (1975)

AH Castro-Neto, CC Chamon & C Nayak PRL 79 (1997)

Understanding why it will get pinned...



• The gate acts like a measurement apparatus, which performs a continuous measurement of the wire density.

• The quantum wire is very susceptible of measurements at the Fourier component $q = 2\, n_{ch}\, k_F$

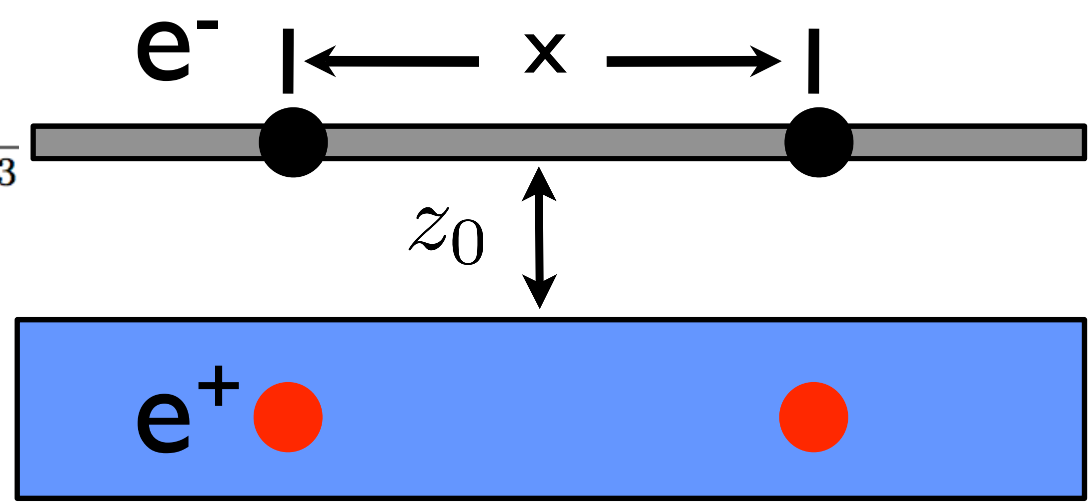
• Invoking the Quantum Zeno Effect that then

$$\langle \rho(q = 2 n_{\mathrm{ch}} k_F) \rangle = \mathrm{const.}$$

The two competing effects of a metallic gate...

Static screening

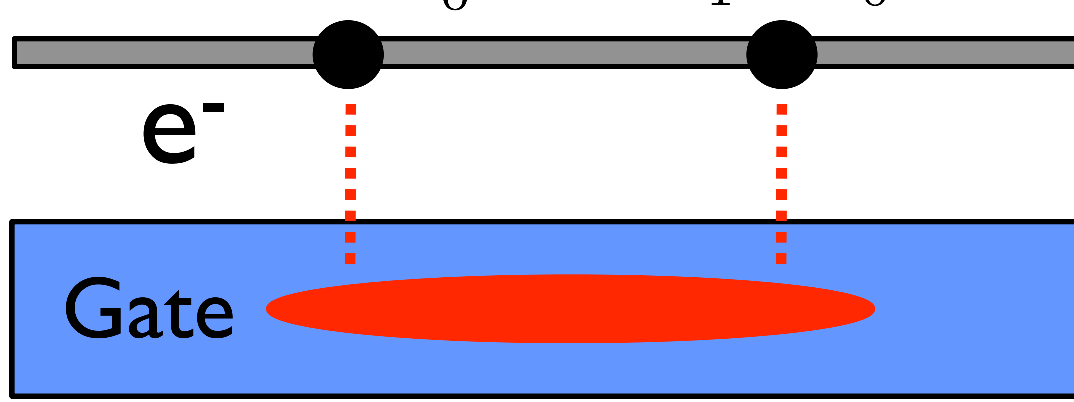
$$V_{\text{Coulomb}}(x) \sim \frac{1}{|x|} \Rightarrow V_{\text{dipole}}(x) \sim \frac{1}{|x|^3}$$



$$g = \left[1 + \frac{8e^2}{\pi\hbar v_F} \ln\left(\frac{R_s}{R}\right)\right]^{-\frac{1}{2}}$$

$$R_s \sim z_0$$

Dynamic screening = dissipation



Estimates (for a nanotube):

$$\alpha \approx g \left(\frac{k_F^{\text{Gate}}}{k_F}\right)^2 \frac{e^{-16 k_F z_0}}{\sigma_{\text{Gate}}}$$

Hint: Use a highly resistive 2D metal, or better, a granular system as gate (no metallic screening at q = 0)

Is this the whole story?

Large N: yet another QCP…

Introduce unit vector: $\mathbf{n}(x,\tau) = (\cos 2p\phi(x,\tau), \sin 2p\phi(x,\tau))$

O(2) dissipative NLσM $G_0^{-1}(q,\omega) = \eta|\omega| + \kappa_p[(\omega/v)^2 + q^2]$

$$S[\mathbf{n},\lambda] = \frac{1}{2}\int \frac{dq d\omega}{(2\pi)^2} G_0^{-1}(q,\omega)|\mathbf{n}(q,\omega)|^2 + \frac{i}{2}\int dx d\tau \lambda(x,\tau)\left[\mathbf{n}^2(x,\tau) - 1\right],$$

S Sachdev, A V Chubukov & A Sokol PRB 51 (1995), S Pankov et al PRB 69 (2004)

O(2)→ O(N) + Large N limit: saddle point for $\lambda = -i\kappa_p \xi^{-2}$

$$N\int \frac{dq d\omega}{(2\pi)^2} \frac{1}{\eta|\omega| + \kappa_p[(\omega/v)^2 + q^2 + \xi^{-2}]} = 1$$

** Another QCP with z = 2 (z = 1.97(3) MC)

$\xi^{-1} \sim (\eta_c - \eta)^\nu$ $\nu = 1$ $(\nu \simeq 0.689(6)$ (MC))

MC calculations (dissipative XY model)

P Werner, M Troyer & S Sachdev J Phys Soc Jpn Supl. 74 (2005)

Large N: yet another phase…

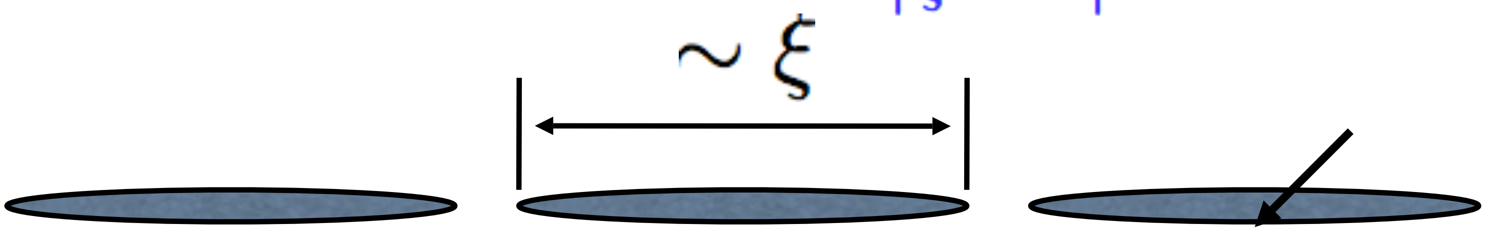
$$\Phi(x,\tau) = a\langle \rho(x,\tau)\rho(0)\rangle_{Q=2pk_F}$$

$$\Phi(x,\tau) \simeq \langle e^{2ip\phi(x,\tau)} e^{-2ip\phi(0)}\rangle = \langle \mathbf{n}(x,\tau)\cdot\mathbf{n}(0,0)\rangle$$

Disordered phase $\eta < \eta_c \Rightarrow \xi^{-1} \neq 0$ $\xi^{-1} \sim (\eta_c - \eta)^{\nu}$

$$\Phi(0,\tau) \sim \frac{1}{\tau^2}, \quad \Phi(x,0) \sim \frac{e^{-|x|/\xi}}{|\xi^{-1}x|}$$

"puddles" = single dissipative rotor



Ordered phase

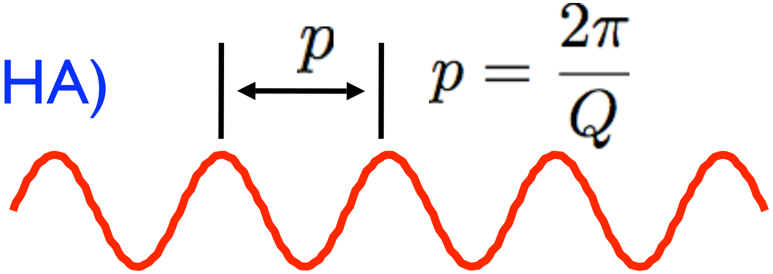
(same structure as SCHA)

$$\Phi(x,\tau) = N_0^2[1 + C_0(x,\tau)]$$

$$C_0(x,0) \sim \frac{1}{|x|},$$

$$p = \frac{2\pi}{Q}$$

$$C_0(0,\tau) \sim \frac{1}{\sqrt{|\tau|}}$$



Expansion about the uncoupled rotor limit

$$S[\varphi] = \sum_{m=1}^{M} \left[\int_0^{\hbar\beta} d\tau \, \frac{1}{2C} \left(\frac{d\varphi_m(\tau)}{d\tau} \right)^2 - \frac{\alpha}{\pi} \int_0^{\hbar\beta} d\tau d\tau' \, K(\tau - \tau') \cos\left[\varphi_m(\tau) - \varphi_m(\tau')\right] \right]$$
$$-K \int d\tau \cos\left[\varphi_m(\tau) - \varphi_{m+1}(\tau)\right].$$

Ohmic dissipation

Small

$$K(\tau) = (\pi/\hbar\beta)^2 \left|\sin(\pi\tau/\hbar\beta)\right|^{-2}$$

Uncoupled rotor (exact): $\langle e^{i\varphi_m(\tau)} e^{-i\varphi_m(0)} \rangle = \left(\frac{\pi\tau_c}{\hbar\beta} \right)^2 \frac{\mathcal{A}}{|\sin(\pi\tau/\hbar\beta)|^2}$

[H. Spohn and W. Zwerger J. Stat. Phys. 94 (1999)]

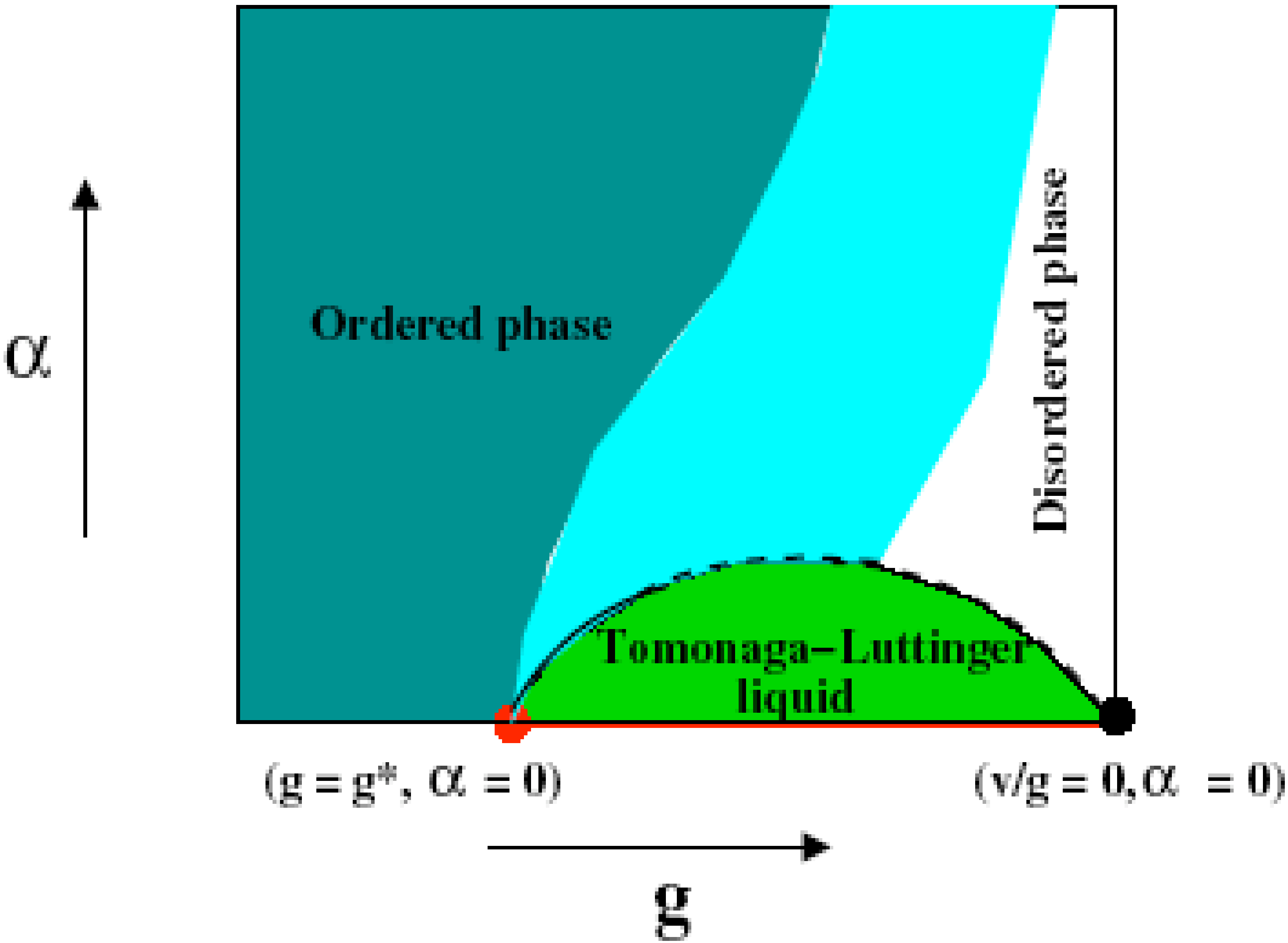
Perturbation theory in K is non-singular, e.g. free energy and

$$\langle e^{i\varphi_N(0)} e^{-i\varphi_1(0)} \rangle = \left(\frac{e^{-x_N/\xi_c}}{x_N/a_0} \right), \qquad \xi_c^{-1} = \ln(\mathcal{A}K\tau_c)/a_0$$

Care with $\alpha = 0$ limit: avoiding KT requires including a Berry phase

$$S_{\text{Berry}} = iV_0 \sum_m \int_0^{\hbar\beta} d\tau \, \dot{\varphi}_m(\tau)$$

Schematic phase diagram



Schematic phase diagram SWCNT + Gate