Supercurrent and singlet-doublet phase transitions of a quantum dot Josephson junction

Christoph Karrasch

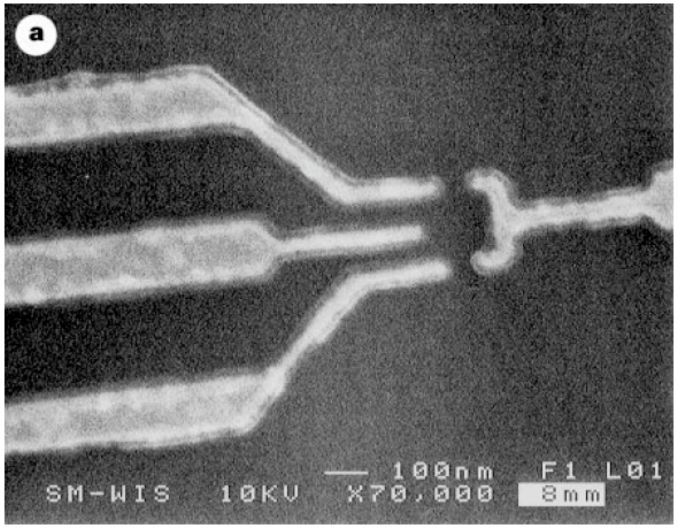
RWTH AACHEN

JARA Jülich Aachen Research Alliance

Evora, November 2008

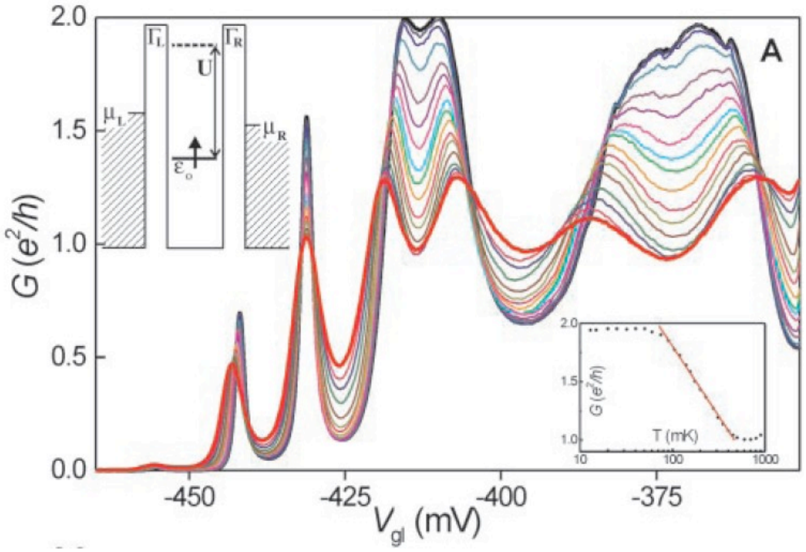
Quantum dots: Experimental situation

Quantum dots:



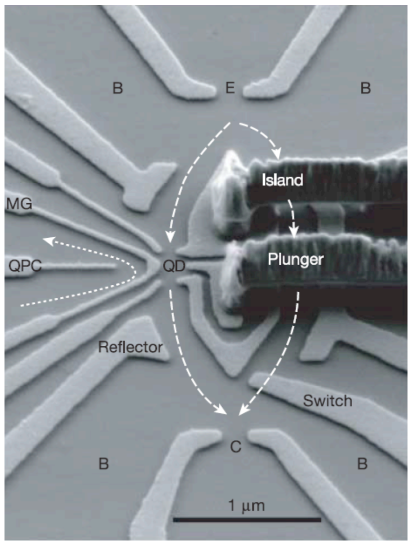
Goldhaber-Gordon et al., Nature 1998

Kondo effect:



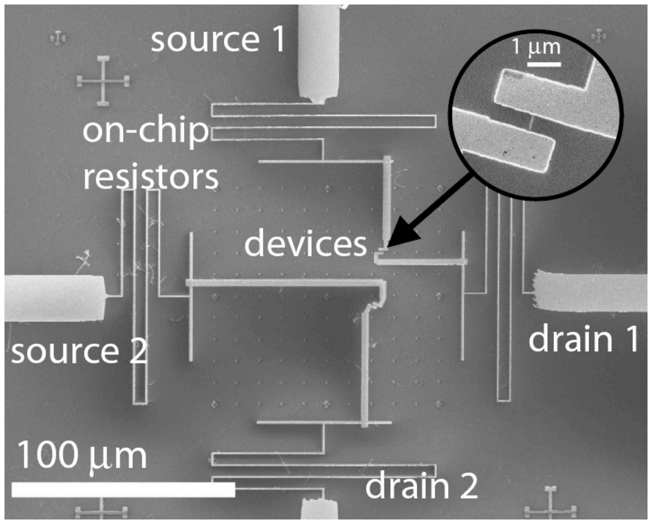
van der Wiel et al., Science 2000

Interferometric setups:



Avinun-Kalish et al., Nature 1995

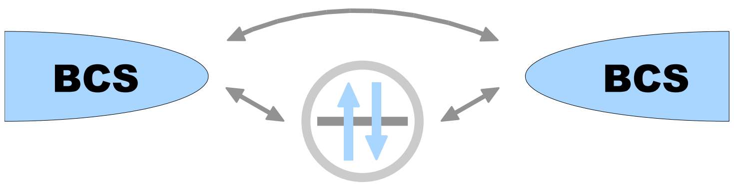
Superconducting electrodes:



Eichler et al., submitted to PRL

Quantum dots: Theoretical description

Great variety of **model parameters:**



ϵ	*impurity energy*
U	*Coulomb repulsion*
Δ	*superconducting energy gap*
ϕ	*superconducting phase*
Γ	*dot-lead hybridization*
Γ_L/Γ_R	*left-right asymmetry*
t_d	*direct coupling strength*

Possible approaches:

- Hartree-Fock

  cannot describe Kondo physics!

- numerical RG

  very accurate, but rather unflexible

- ...

There is a need for additional methods!

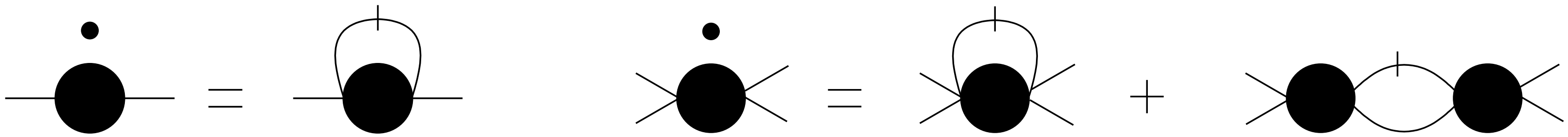
Functional renormalization group

General framework:

- consider m-particle vertex functions
- introduce cutoff Λ into the free Green function:

$$\mathcal{G}^0(i\omega) \rightarrow \Theta(|\omega| - \Lambda)\mathcal{G}^0(i\omega)$$

- differentiate w.r.t. Λ $\Rightarrow$ exact hierarchy of flow equations



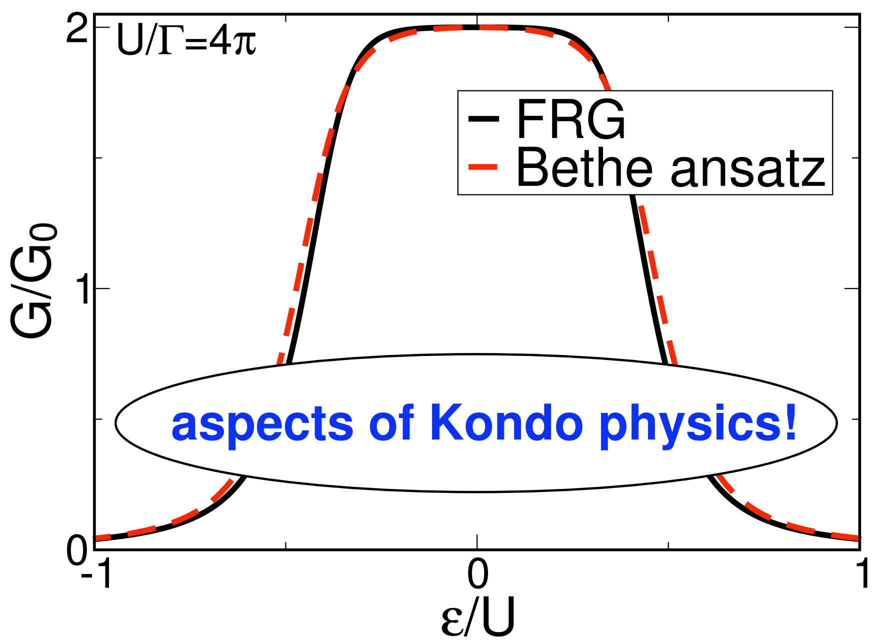
- in practice: truncate infinite hierarchy $\Rightarrow$ approximate method

Flow equations for SIAM:

$$\partial_\Lambda \epsilon_\Lambda = \frac{U_\Lambda \epsilon_\Lambda / \pi}{(\Lambda + \Gamma)^2 + \epsilon_\Lambda^2}$$

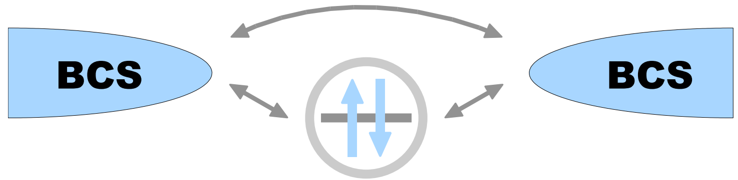
$$\partial_\Lambda U_\Lambda = \frac{2U_\Lambda^2 \epsilon_\Lambda^2 / \pi}{[(\Lambda + \Gamma)^2 + \epsilon_\Lambda^2]^2}$$

solve numerically!



The QD Josephson junction

Model Hamiltonian:



$$H^{\mathsf{dot}} = (\epsilon - U/2) \sum_{\sigma} d_{\sigma}^{\dagger} d_{\sigma} + U d_{\uparrow}^{\dagger} d_{\uparrow} d_{\downarrow}^{\dagger} d_{\downarrow}$$

quantum dot

$$H_{s=L,R}^{\mathsf{lead}} = \sum_{k\sigma} \epsilon_{sk} c_{sk\sigma}^{\dagger} c_{sk\sigma} - \Delta \sum_{k} \left[e^{i\phi_s} c_{sk\uparrow}^{\dagger} c_{s-k\downarrow}^{\dagger} + \mathsf{H.c.} \right]$$

BCS leads

$$H_{s=L,R}^{\mathsf{coup}} = -\, t_s \sum_{\sigma} c_{s\sigma}^{\dagger} d_{\sigma} + \mathsf{H.c.}$$

coupling QD-leads

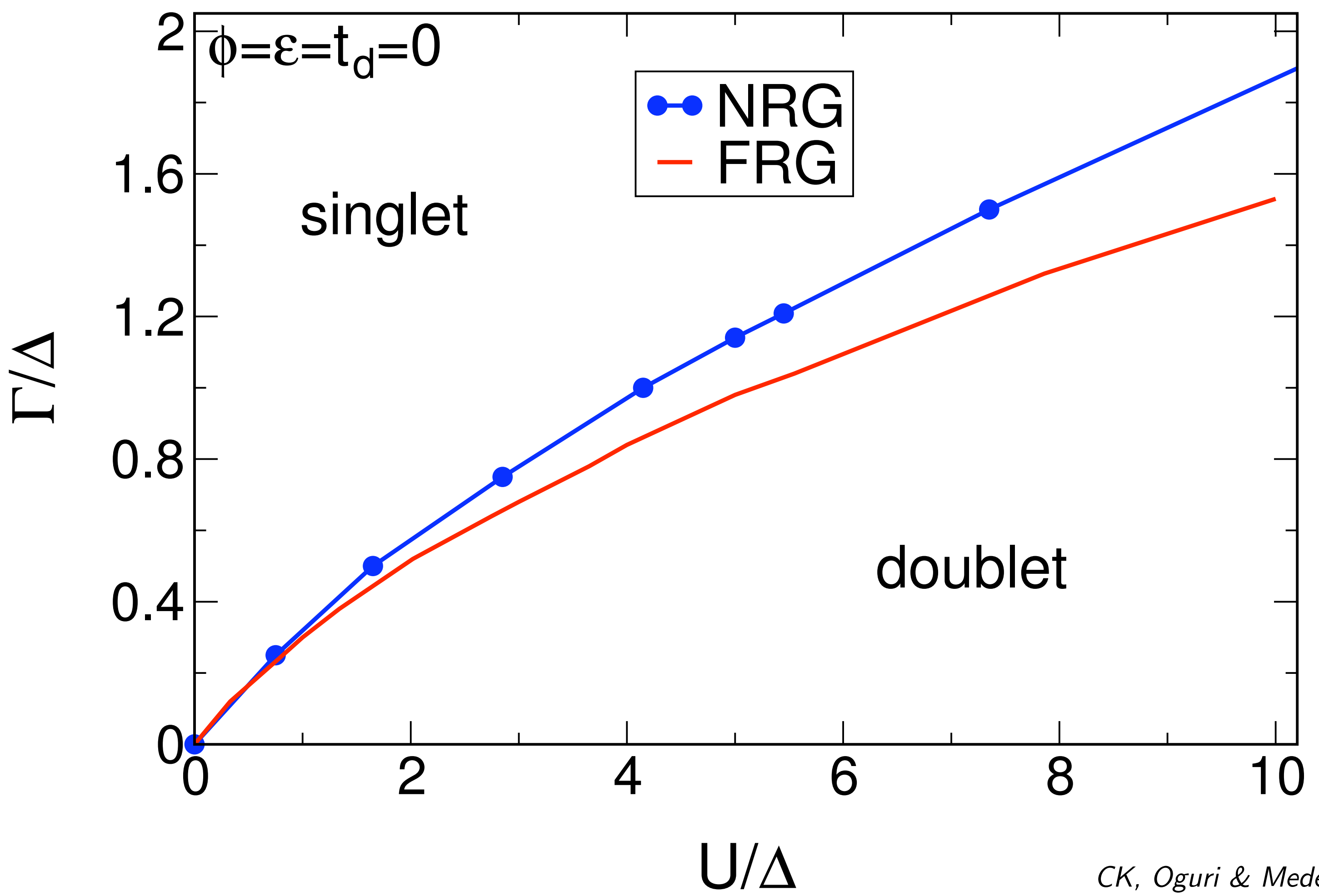
$$H^{\mathsf{direct}} = -\, t_d \sum_{\sigma} c_{L\sigma}^{\dagger} c_{R\sigma} + \mathsf{H.c.}$$

direct coupling

Next: obtain three FRG **flow equations** for the **self-energy Σ^{Λ}**, **anomalous self-energy $\Sigma_{\Delta}^{\Lambda}$** and **effective interaction U^{Λ}** straight-forwardly!

Phase diagrams

- $T_K \gg \Delta$ – **singlet phase**:
  Kondo effect active, Cooper pairs broken, singlet ground state
- $T_K \ll \Delta$ – **doublet phase**:
  Kondo screening disturbed, energy gap Δ, free spins

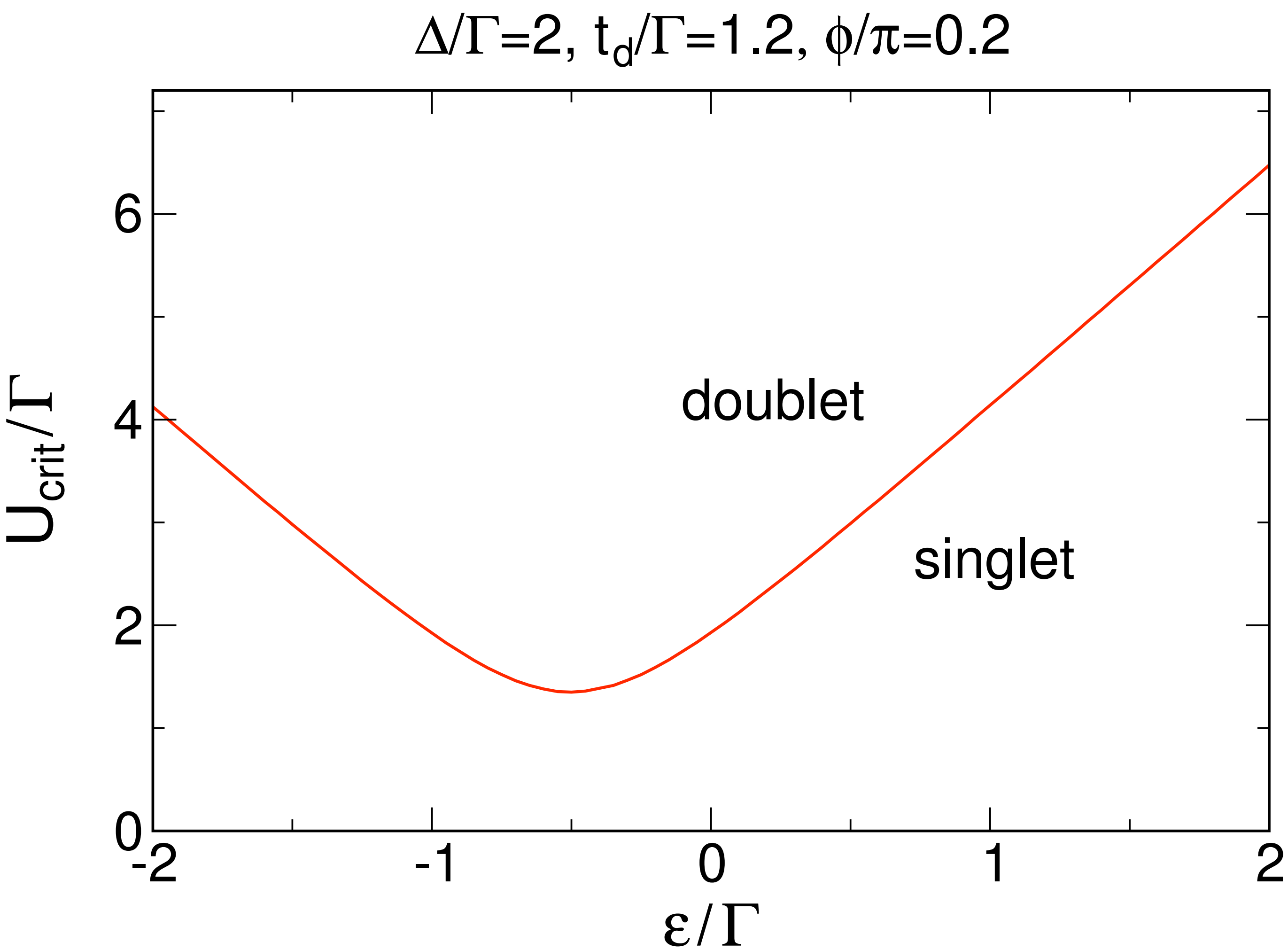


CK, Oguri & Meden, PRB 2008

Phase diagrams: Aharonov-Bohm situation

Finite coupling t_d between left and right leads

- treat atomic limit $\Delta = \infty$ exactly
- FRG calculations for arbitrary Δ

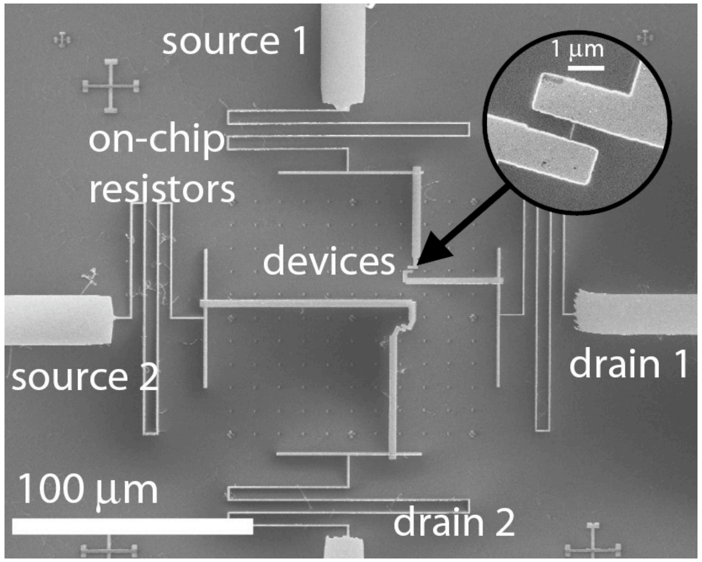


non-monotonic phase boundary

re-entrance behavior

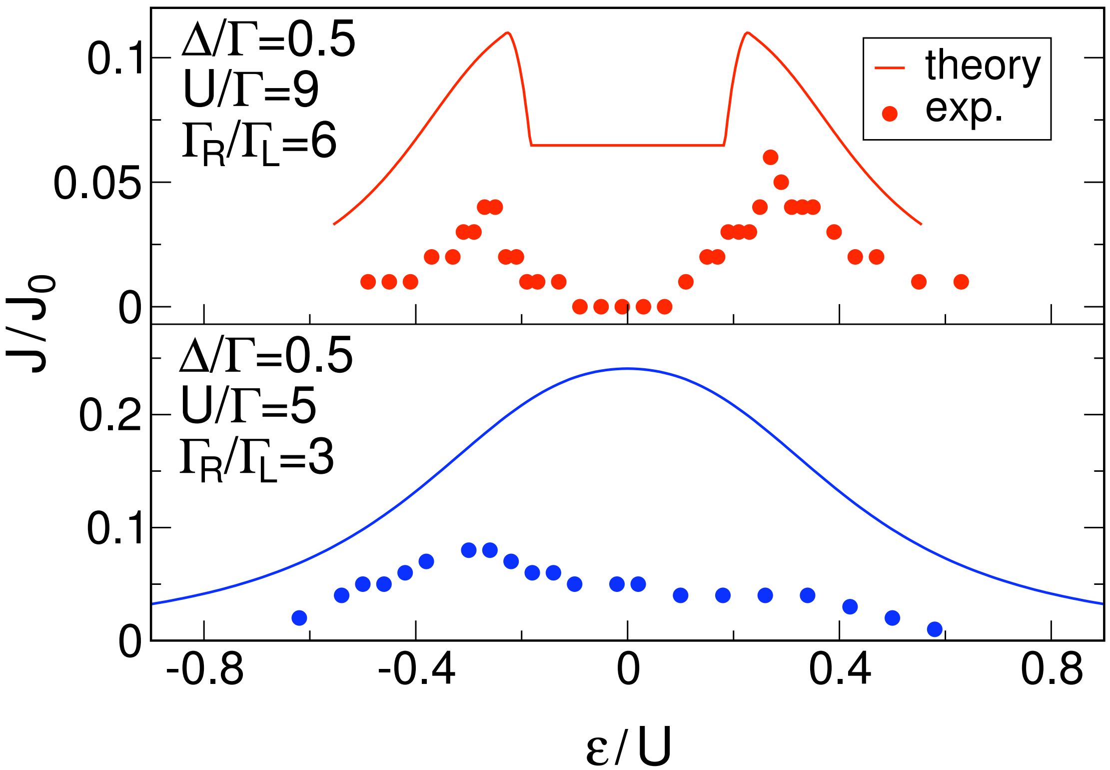
CK & V. Meden, arXiv:0810.3847

Josephson current



sandwich carbon nanotubes between superconducting electrodes

⟶ **measure Josephson current**



"doublet" situation

"singlet" situation

good agreement!

Eichler, Deblock, Weiss, CK, Meden, Schönenberger & Bouchiat, arXiv:0810.1671

Many thanks to. . .

Numerical RG calculations

Akira Oguri
Osaka City University

Quantum dot experiments

Alexander Eichler, Markus Weiss & Christian Schönenberger
University of Basel

Richard Deblock & Hélène Bouchiat
Université Paris-Sud

Volker Meden
RWTH Aachen

. . . and to you for your attention!