Electron-electron interactions and Dirac liquid behavior in graphene bilayers

arXiv:0805.0305

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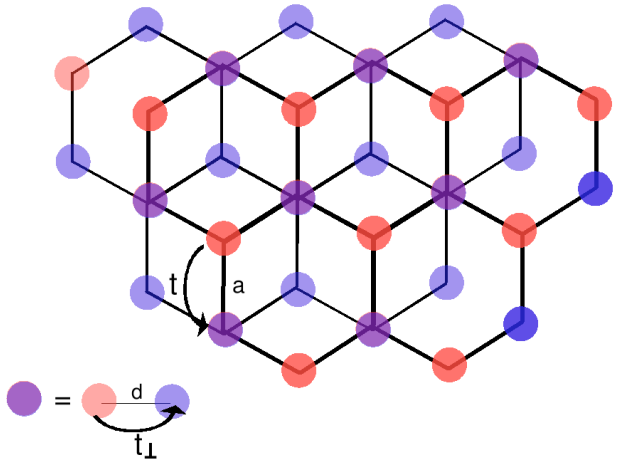
Boston University

Workshop on Correlations and Coherence in Quantum Matter
Évora 2008

Bilayer Graphene



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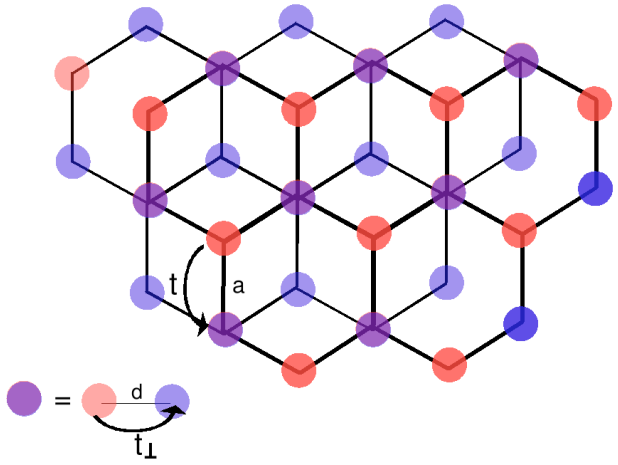

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$$H_0 = 4\sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \begin{pmatrix} 0 & v_F p e^{i\phi(\mathbf{p})} & -t_\perp & 0 \\ v_F p e^{-i\phi(\mathbf{p})} & 0 & 0 & 0 \\ -t_\perp & 0 & 0 & v_F p e^{-i\phi(\mathbf{p})} \\ 0 & 0 & v_F p e^{i\phi(\mathbf{p})} & 0 \end{pmatrix} \psi_{\mathbf{p}}$$

with $\psi^\dagger_{\mathbf{p}} = (c^\dagger_{\mathbf{p},A_1}, c^\dagger_{\mathbf{p},B_1}, c^\dagger_{\mathbf{p},B_2}, c^\dagger_{\mathbf{p},A_2})$

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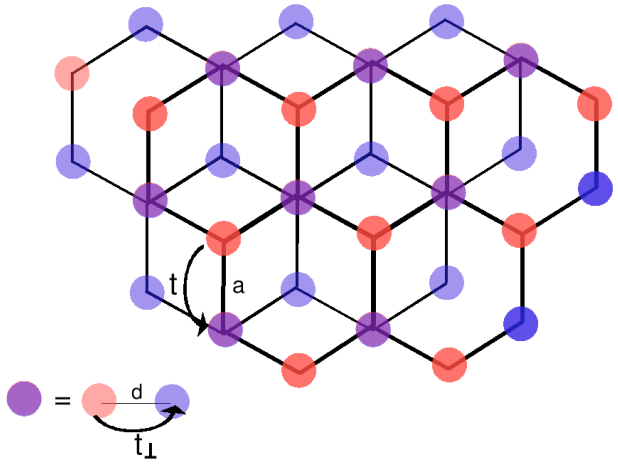

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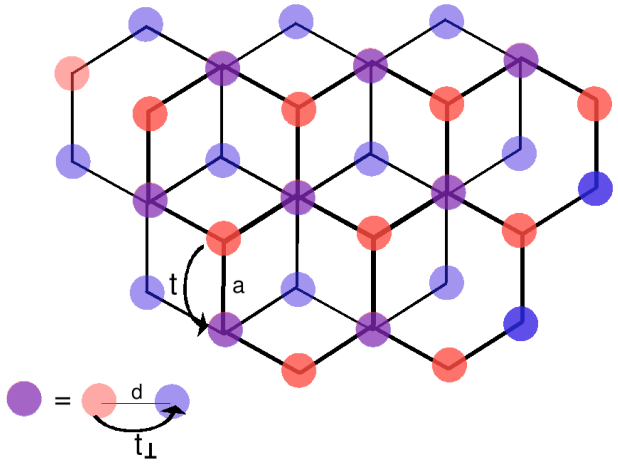

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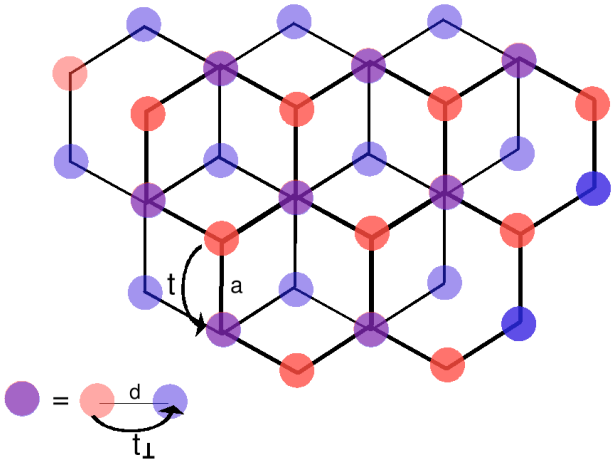
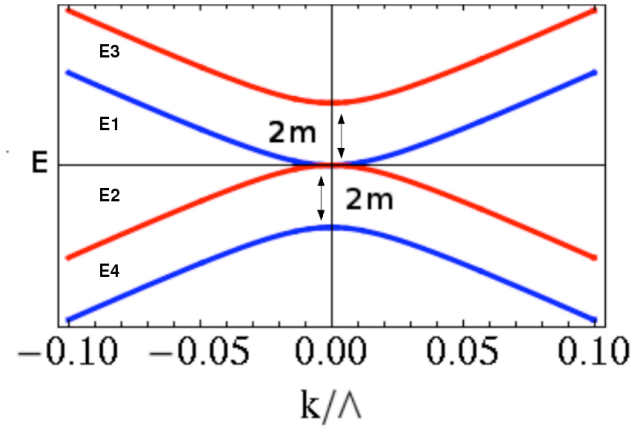


Non-interacting dispersion



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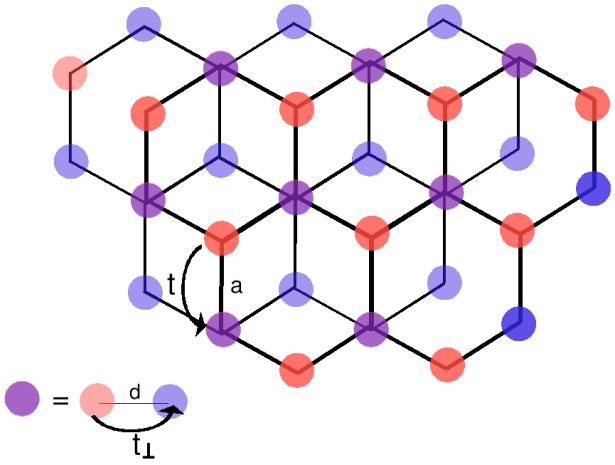

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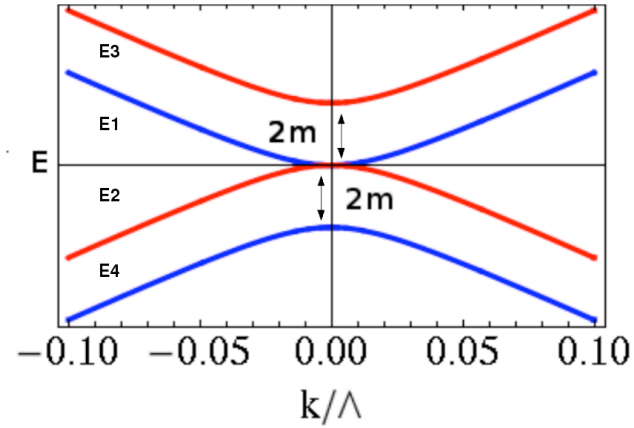

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Crossover energy scale: mv_F^2

non-relativistic $\leftrightarrow$ ultra-relativistic

Add Coulomb interactions:

$$V_{\pm}(\mathbf{k}) = \frac{2\pi e^2}{\epsilon} \frac{(1 \pm \exp\{-kd\})}{[2(k+\beta m)]}$$

β is the screening strength:

- $\beta = 0$ Hartree - Fock approximation
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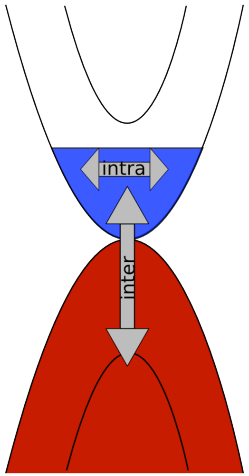

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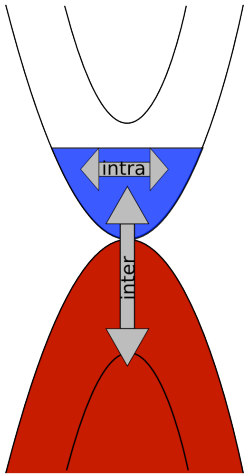

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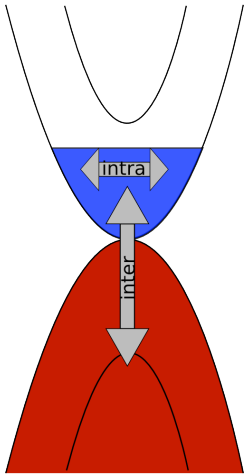

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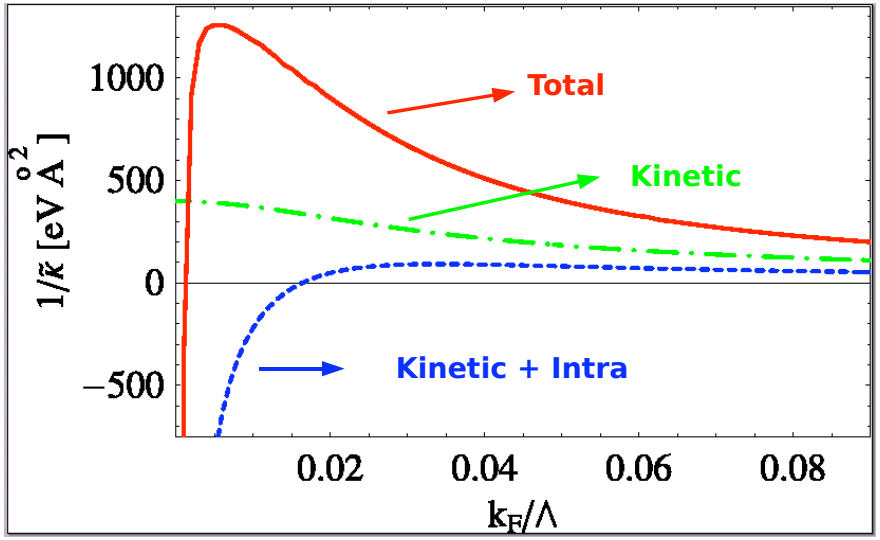

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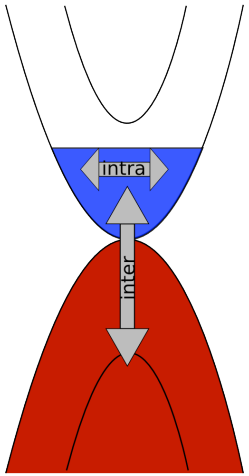

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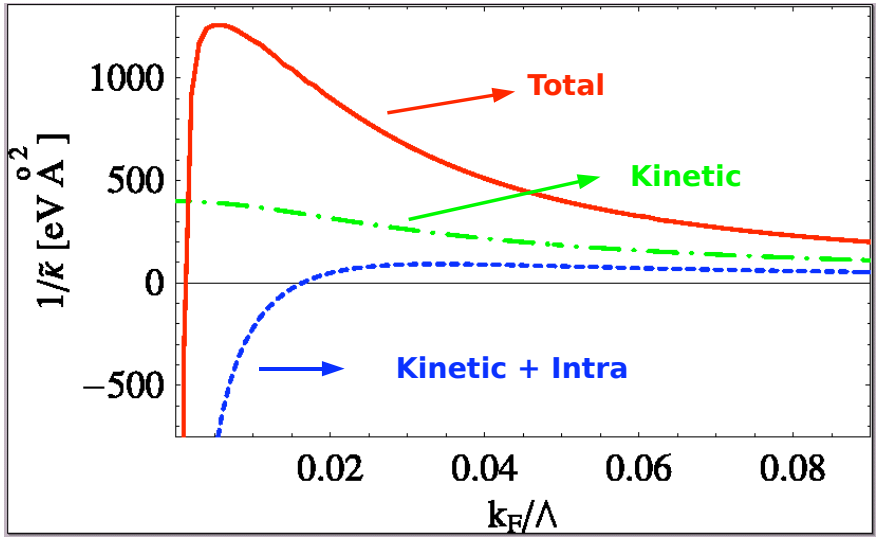

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- The Intra-band contribution is negative.
- The Inter-band contribution is positive.
- This competition is responsible for non - monotonic behavior.
- The inverse compressibility diverges negatively at very low densities.
- The “bump” signals a crossover from a 2DEG - like behavior to a graphene monolayer - like one.
- The effect is present only when the full, four bands model is considered.
- There exists a very small electron - hole asymmetry.

Energy of a quasiparticle in the ith band:

$$\epsilon_i(q) = \delta E/\delta n_i(\mathbf{q})|_{n_i=n_i^0}$$

- $E[\delta n_i]$ is the total energy $E = E_0 + E_{ex}$
- $\delta n_i(\mathbf{q}) = n_i(\mathbf{q}) - n_i^0(\mathbf{q})$
- $n_i^0(\mathbf{k})$ non-interacting occupation number

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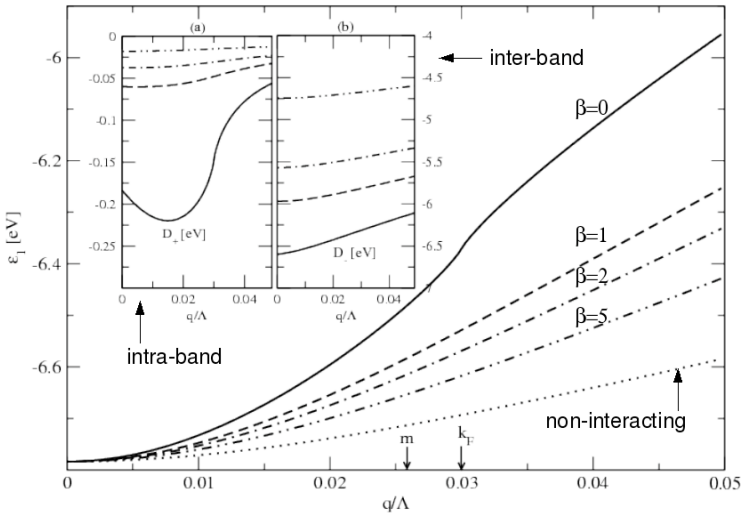
Renormalized dispersion



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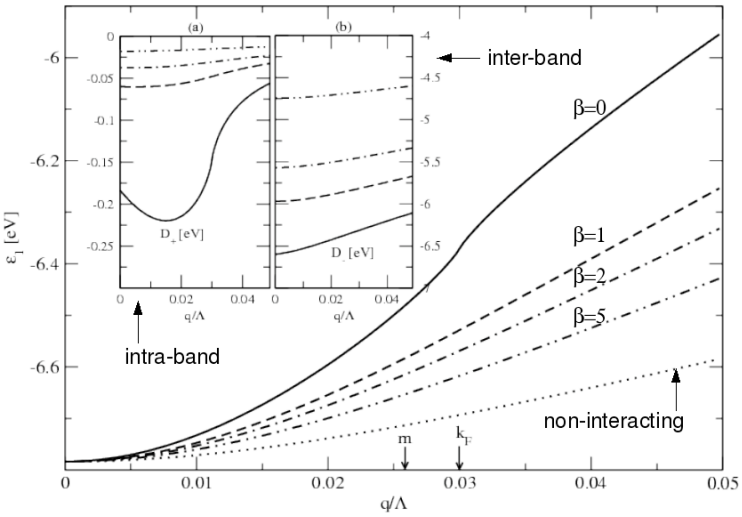
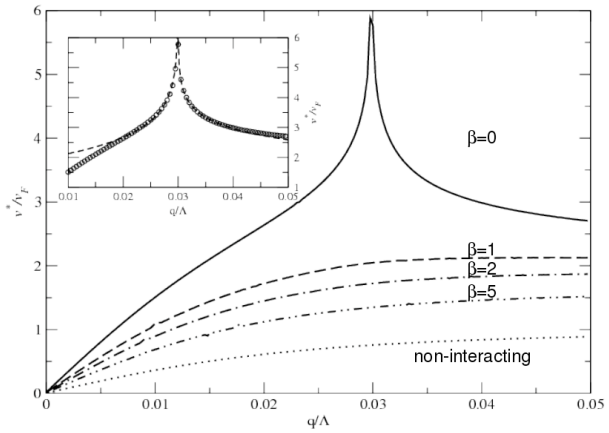


Screening is necessary:

diverging Fermi velocity as in 2DEG

Renormalized band velocity



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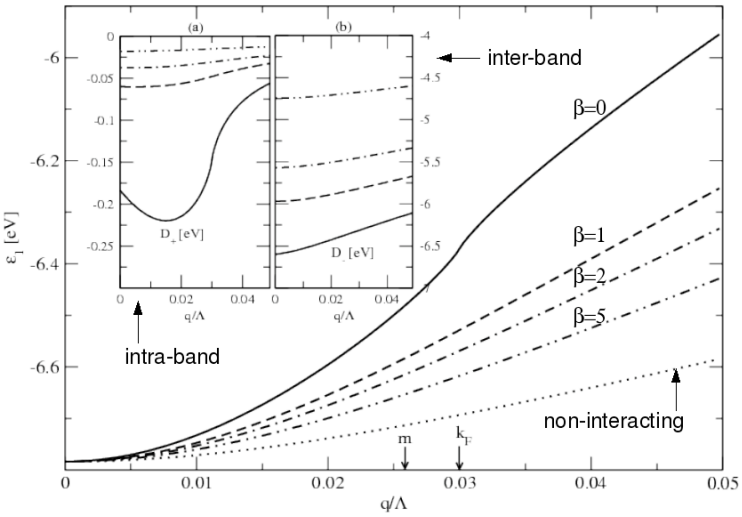

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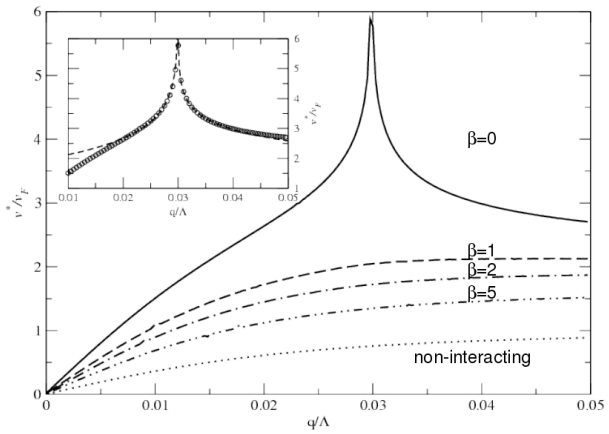

Relativistic dispersion recovered!

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Both mass and "light" velocity get renormalized

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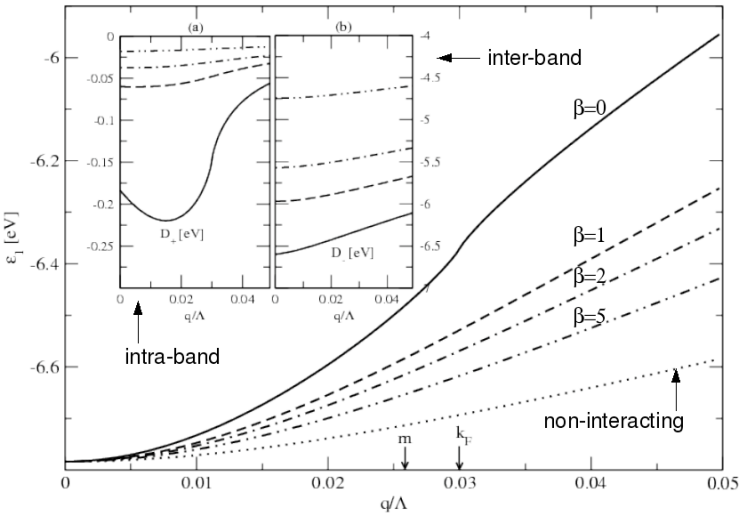

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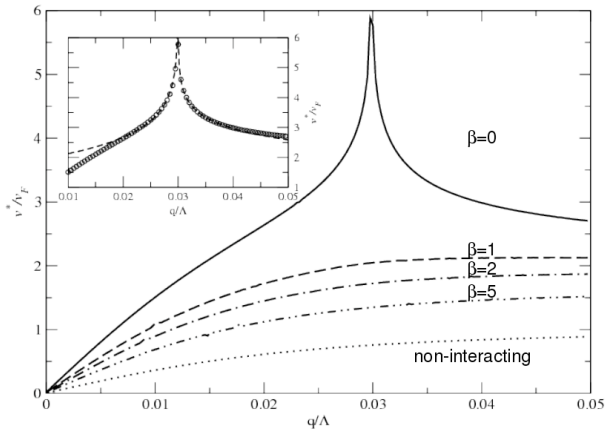

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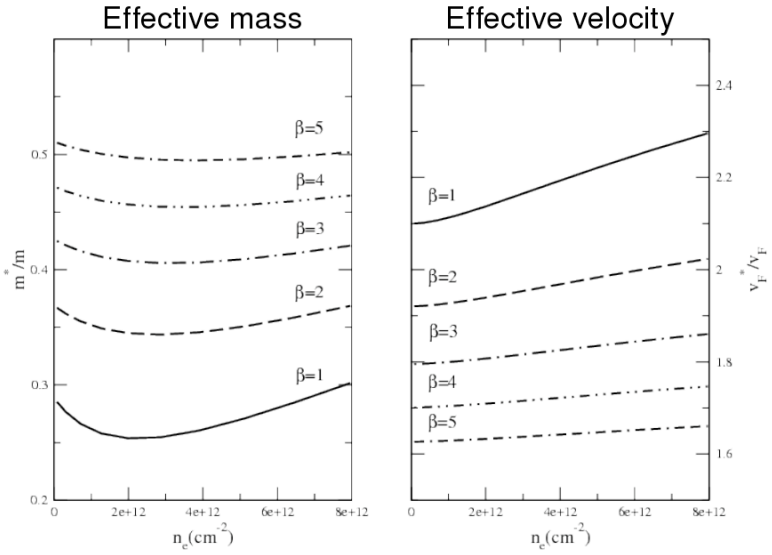
→due to inter-band interactions

e^- density dependence: m^* and v_F^*



Coulomb interactions:

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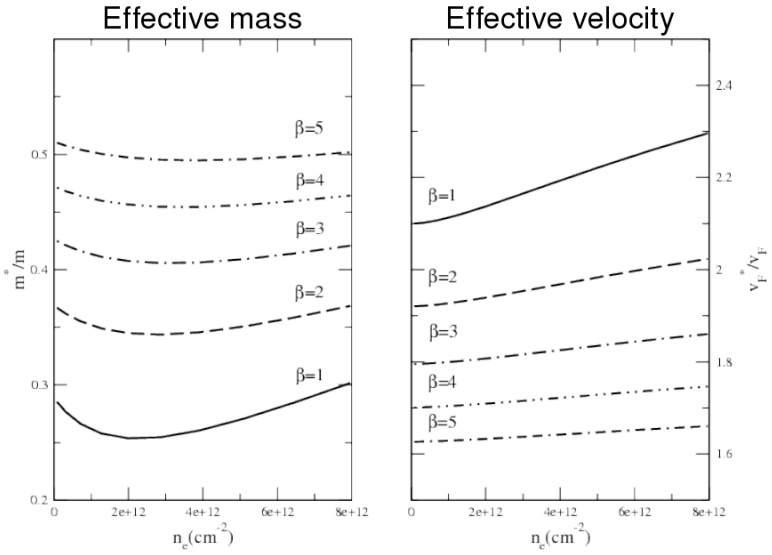
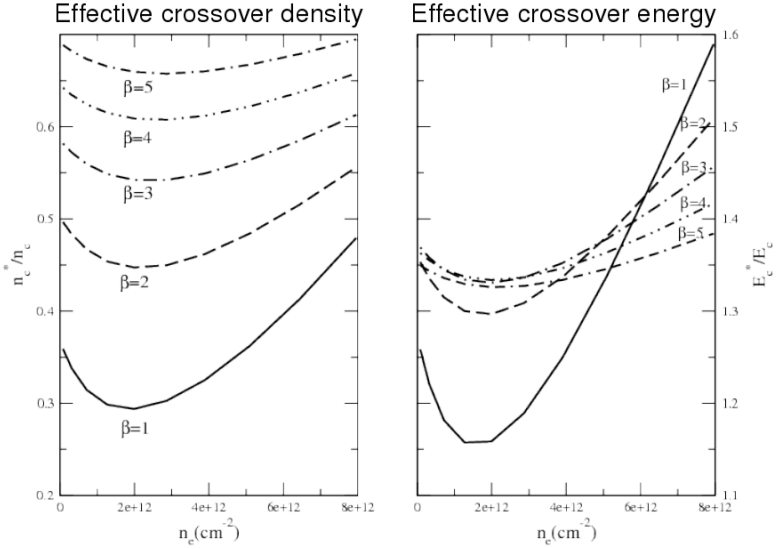
- increase the "light" velocity $v_F^* > v_F$

e^- density dependence: m^* and v_F^*



e^- density dependence: n_c^* and E_c^*



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Effect on the NR → UR crossover:

- increase the crossover energy $E_c = mv_F^2$: $E_c^* > E_c$
- decrease the crossover density $n_c = (mv_F)^2/\pi$: $n_c^* < n_c$

Non-interacting BLG in B-field: Landau Levels

$$\frac{E_n^{\pm}}{\omega_c} = \pm \left\{ n + \frac{1}{2} + 2r^2 - \frac{1}{2}\left[1 + 16r^4 + 16r^2\left(n + \frac{1}{2}\right)\right]^{1/2} \right\}^{1/2}$$

- n positive integer
- $\omega_c = v_F\sqrt{2eB/c}$ cyclotron frequency
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Interacting BLG in a B-field:

Quasiparticle dispersion with renormalized parameters m^*, v_F^*
$\Rightarrow$ Replace in LL

- $m \quad \longrightarrow \quad m^*$
- $v_F \quad \longrightarrow \quad v_F^*$

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Free parameters

- Bare "light" velocity $v_F = \frac{3ta}{2}$
- Bare "mass" $m = \frac{t_\perp}{2v_F^2}$
- Dimensionless coupling $\alpha = e^2/\hbar v_F \epsilon$
  $\rightarrow \quad \alpha \approx 0.5$ for SiO_2
- Screening strength: $\beta_{RPA} = 4\alpha\,(1 + E_F/m)$
  $\rightarrow \quad \beta \approx 1 - 5$ for typical densities

Interacting BLG in a B-field:

Quasiparticle dispersion with renormalized parameters m^*, v_F^*
$\Rightarrow$ Replace in LL

- $m \quad \rightarrow \quad m^*$
- $v_F \quad \rightarrow \quad v_F^*$

Non-interacting BLG in B-field: Landau Levels

$$\frac{E_n^{\pm}}{\omega_c} = \pm \left\{ n + \frac{1}{2} + 2r^2 - \frac{1}{2}\left[1 + 16r^4 + 16r^2\left(n + \frac{1}{2}\right)\right]^{1/2} \right\}^{1/2}$$

- n positive integer
- $\omega_c = v_F\sqrt{2eB/c}$ cyclotron frequency
- $r = mv_F^2/\omega_c$

Free parameters

- Bare "light" velocity $v_F = \frac{3ta}{2}$
- Bare "mass" $m = \frac{t_\perp}{2v_F^2}$
- Dimensionless coupling $\alpha = e^2/\hbar v_F \epsilon$
  $\longrightarrow \quad \alpha \approx 0.5$ for SiO_2
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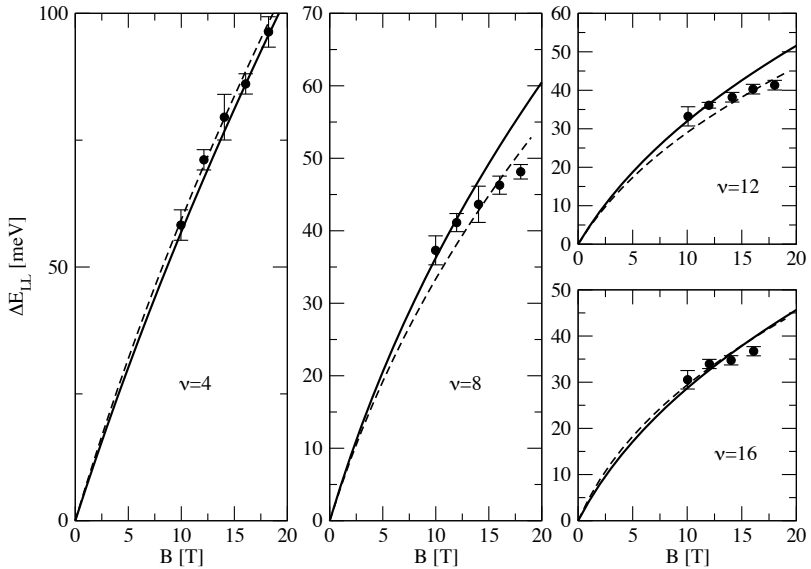
Cyclotron Resonance Data



Data from E. A. Henriksen *et al*, Phys. Rev. Lett. **100**, 087403.

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Interacting BLG in a B-field:

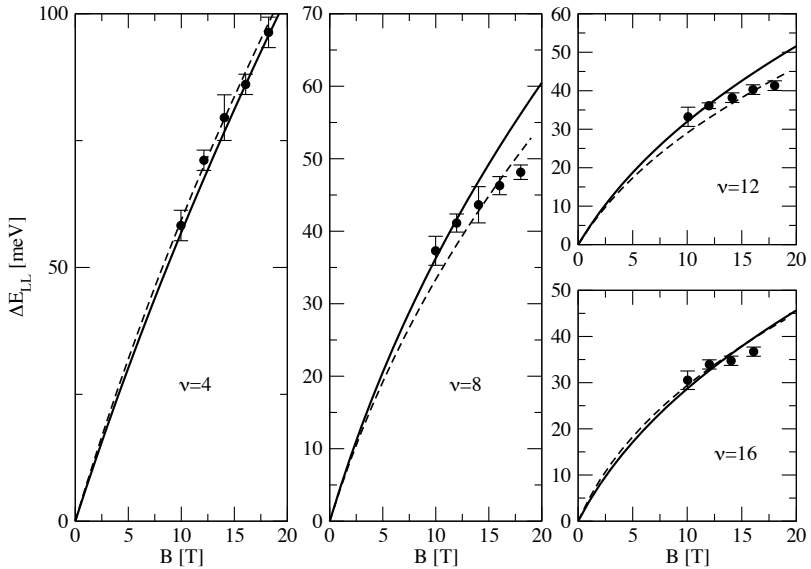
Quasiparticle dispersion with renormalized parameters m^*, v_F^*
$\Rightarrow$ Replace in LL

- $m \quad \longrightarrow \quad m^*$
- $v_F \quad \longrightarrow \quad v_F^*$

Results (solid line) $\longrightarrow$

- $\alpha = 0.5$
- $\beta = 4$
- $v_F = 0.76 \times 10^6$ m/s $\longrightarrow v_F^* = 1.2 - 1.3 \times 10^6$ m/s
- $t_\perp = 0.33$ eV $\longrightarrow t_\perp^* = 0.44 - 0.45$ eV

Cyclotron Resonance Data



Data from E. A. Henriksen *et al*, Phys. Rev. Lett. **100**, 087403.

- Within a Hartree-Fock-Thomas-Fermi approximation, after the inclusion of Coulomb interactions, the quasiparticle dispersion can still be described by a Lorentz invariant dispersion but with renormalized parameters.
- This is a consequence of inter-band interactions which are dominant.
- The interaction renormalizes up both in-plane (t) and out of plane ($t_\perp$) hopping parameters.
- Further corrections?