Strong magnetic field collective modes in doped graphene and in a standard 2D electron gas

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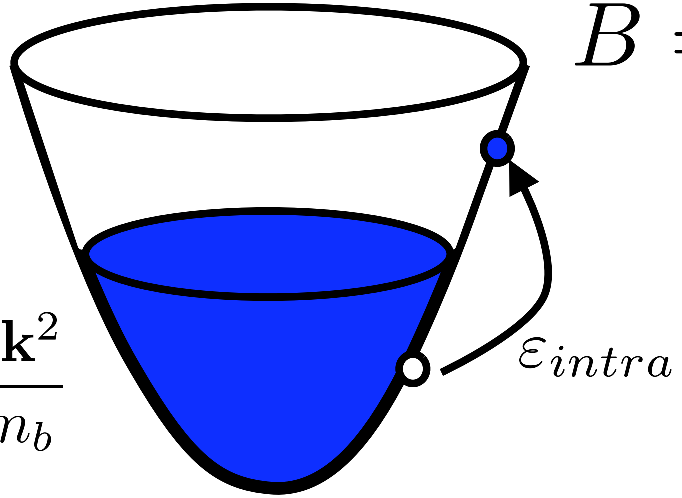
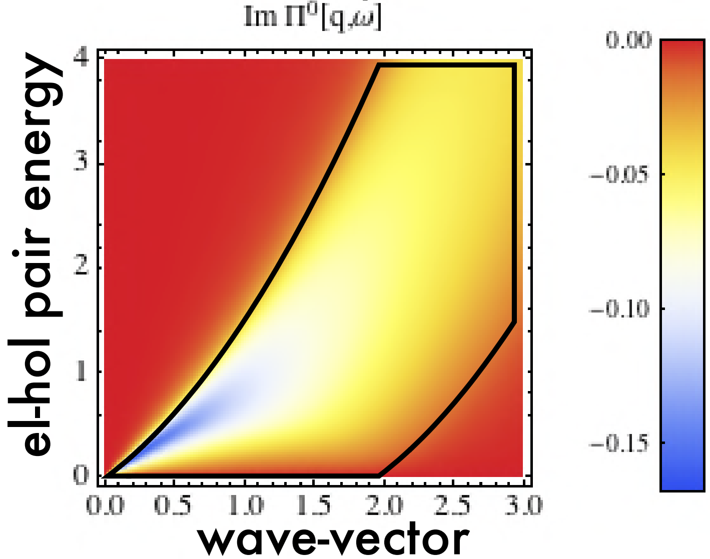
Particle-hole excitations in a standard 2DEG

$$\varepsilon_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m_b}$$



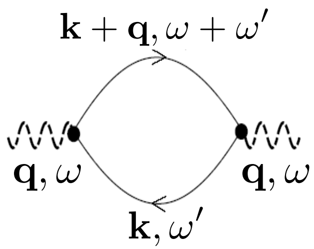
$$B = 0$$

Particle-hole excitation spectrum



The PHES is defined as the (ω, q) region of non-zero spectral weight

$$S(\mathbf{q},\omega) = -\frac{1}{\pi}\mathrm{Im}\Pi(\mathbf{q},\omega)$$



Particle-hole polarization:

$$i\Pi^0(\mathbf{q},\omega) = \int \frac{d\omega' d\mathbf{k}}{(2\pi)^3}\left[G^0(\mathbf{k},\omega')G^0(\mathbf{k}+\mathbf{q},\omega'+\omega)\right]$$

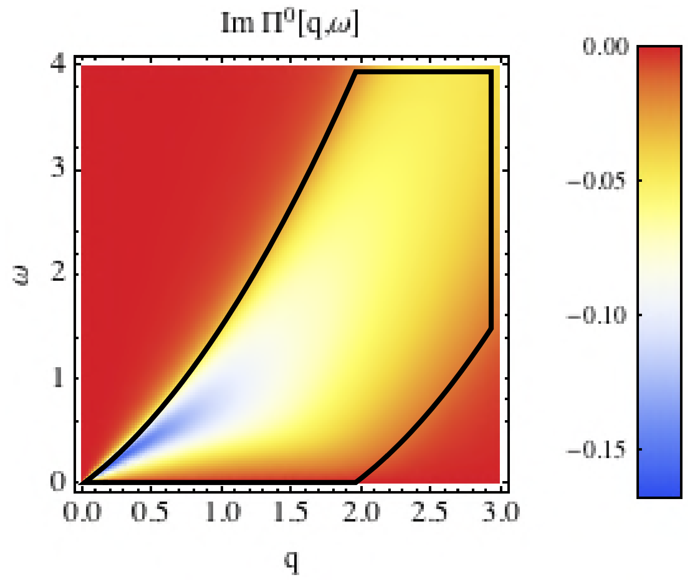
Interacting 2DEG: RPA theory

In RPA approximation the electron-electron interaction is treated self-consistently

$$\Pi^{RPA}(\mathbf{q},\omega) = \frac{\Pi^0(\mathbf{q},\omega)}{1 - v(\mathbf{q})\Pi^0(\mathbf{q},\omega)}$$

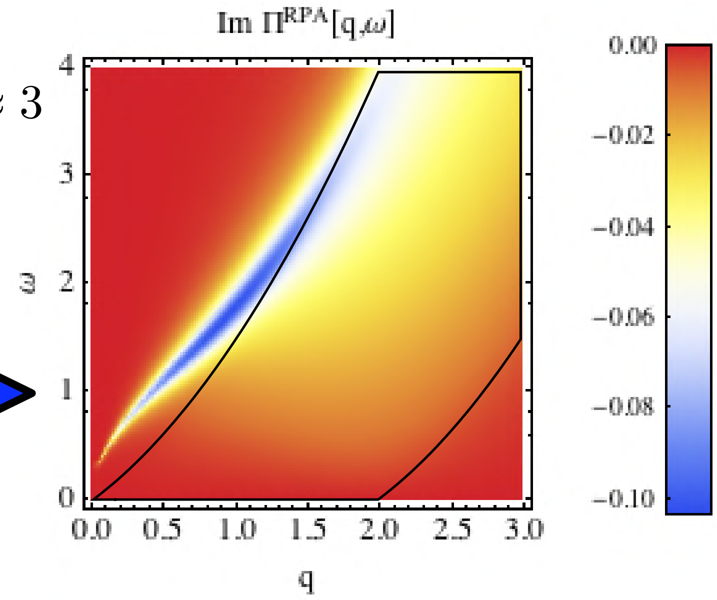
$$B = 0$$

2D Fourier transform of unscreened Coulomb potential $v(\mathbf{q}) = \frac{2\pi e^2}{\epsilon_b |\mathbf{q}|}$



$$r_s \equiv \frac{2m_b e^2}{\hbar^2 \varepsilon_b k_F} \approx 3$$

el-el interactions



The plasmon mode emerges from the continuum when (long-range) interactions between electrons are included

PHES of a standard 2DEG in a magnetic field

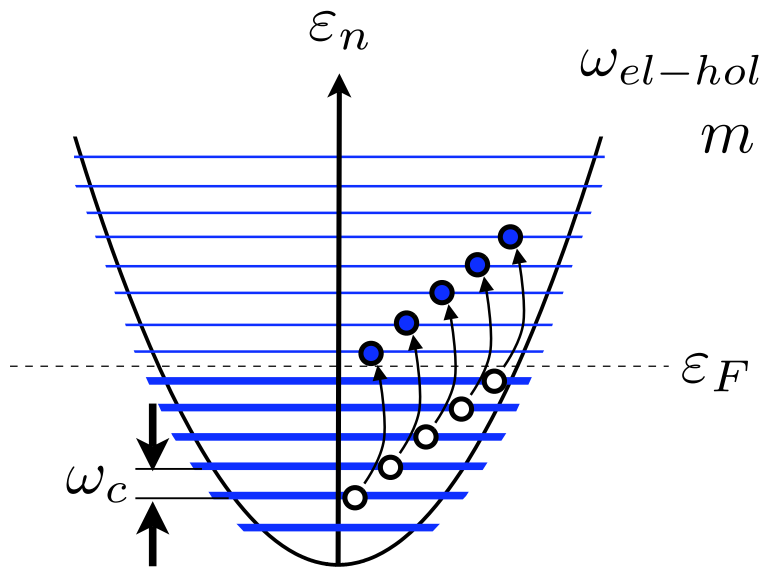
Landau levels (LLs)

$$\varepsilon_n = \hbar \frac{eB}{m_b}\left(n + \frac{1}{2}\right)$$

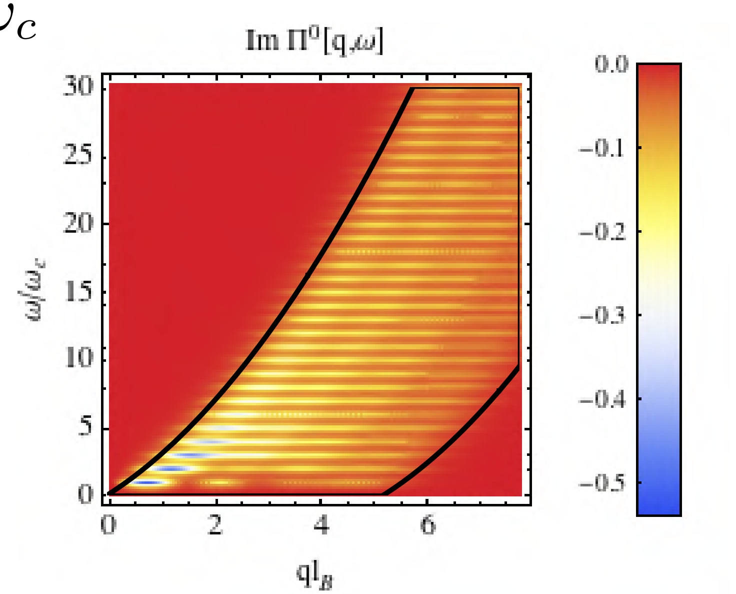
$$B \neq 0$$

"Density independent" cyclotron frequency

$$\omega_c = \frac{eB}{m_b}$$



$$\omega_{el-hol} \sim m\omega_c$$
$$m \in \mathbb{N}$$

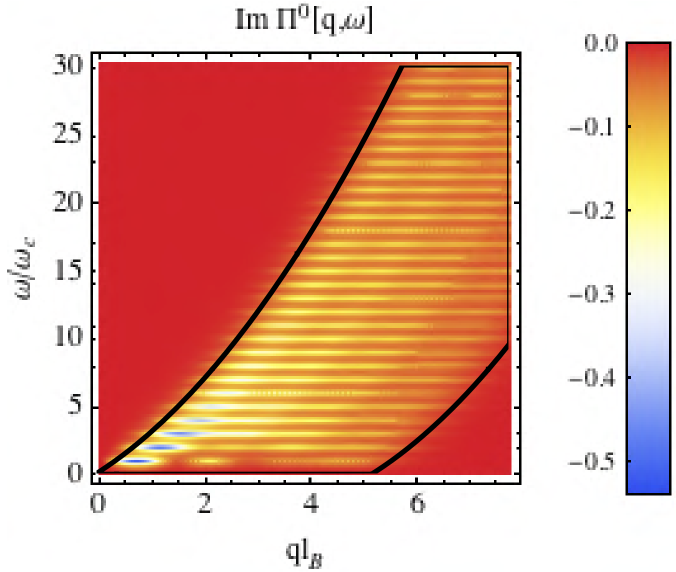


LL quantization
+
Equidistant LL separation

(horizontal) magneto-excitons

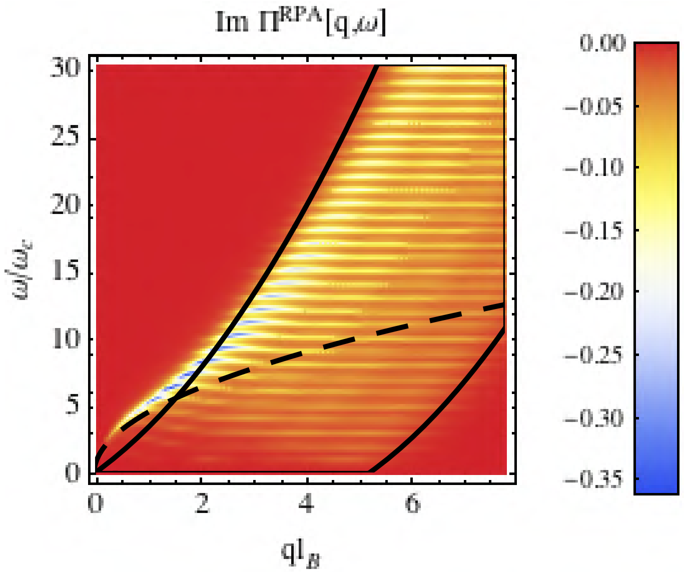
C. Kallin & B. I. Halperin. Phys. Rev. B 30, 5655 (1984)

Interacting 2DEG in a magnetic field: RPA theory



$$r_s \equiv \frac{2m_b e^2}{\hbar^2 \varepsilon_b k_F} \approx 3$$

el-el interactions



- Interactions lead to a transfer of spectral weight from the long-wave length region of the PHES to the plasmon mode, modified by the magnetic field (upper hybrid mode).
- The magneto-excitons acquire a dispersion due to the inclusion of Coulomb interaction.

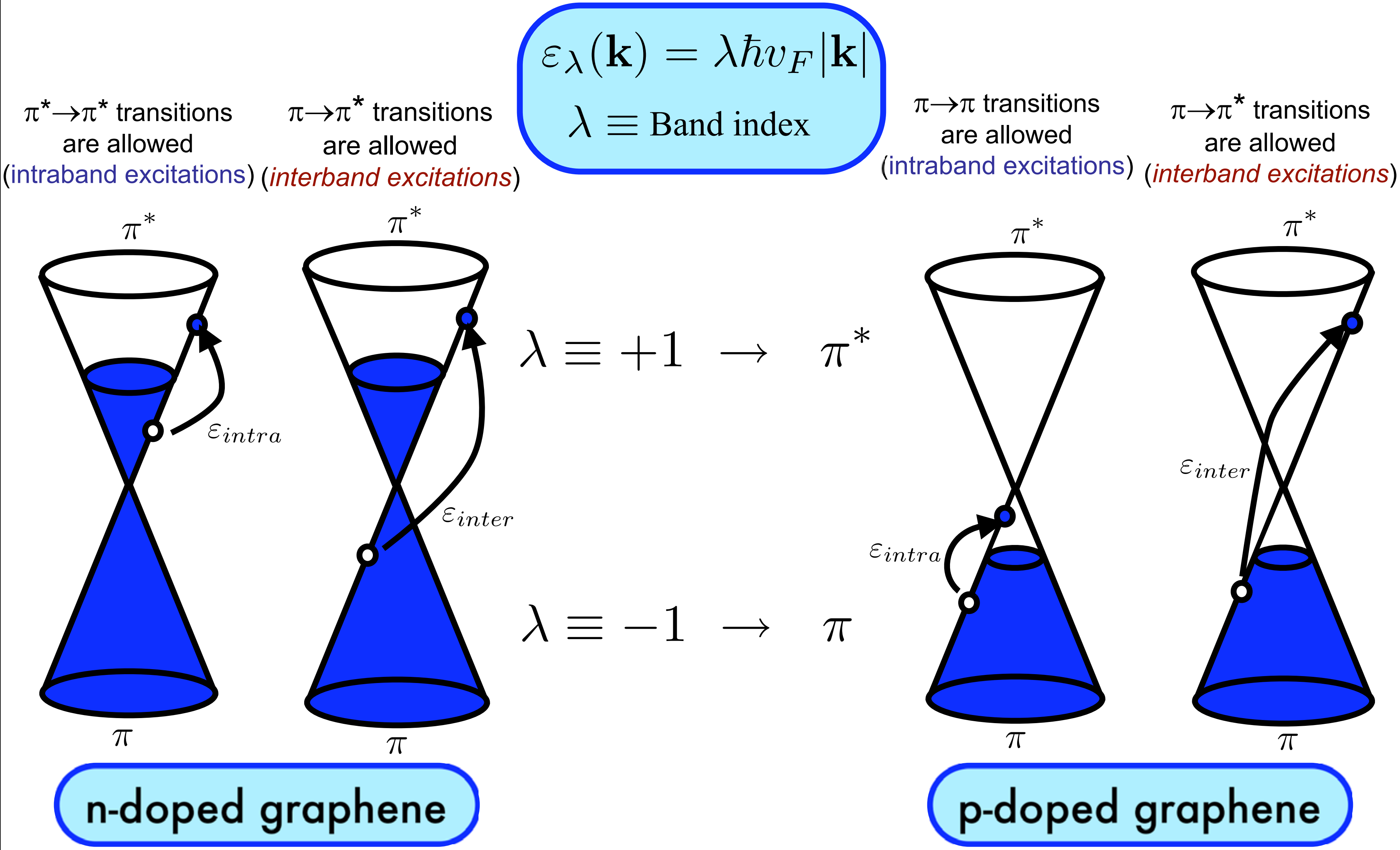
UHM fitting

$$\omega_{UH} = \sqrt{\omega_c^2 + \omega_{p,cl}^2}$$

$$\omega_c^2 = \left(\frac{eB}{m_b}\right)^2$$

$$\omega_{p,cl}^2 = \frac{2\pi e^2 n_0}{\varepsilon_b m_b} q$$

Particle-hole excitations in doped graphene



PHES in graphene: B=0

Particle-hole polarization function for graphene

$$\Pi^0(\mathbf{q},\omega) = -\frac{g_s g_v}{(2\pi)^2}\sum_{\lambda,\lambda'=\pm}\int d^2k \frac{n_F[\varepsilon_\lambda(\mathbf{k})] - n_F[\varepsilon_{\lambda'}(\mathbf{k}+\mathbf{q})]}{\omega + \varepsilon_\lambda(\mathbf{k}) - \varepsilon_{\lambda'}(\mathbf{k}+\mathbf{q})) + i\delta} F_{\lambda\lambda'}(\mathbf{k},\mathbf{q})$$

Chirality factor (wave-function overlap)

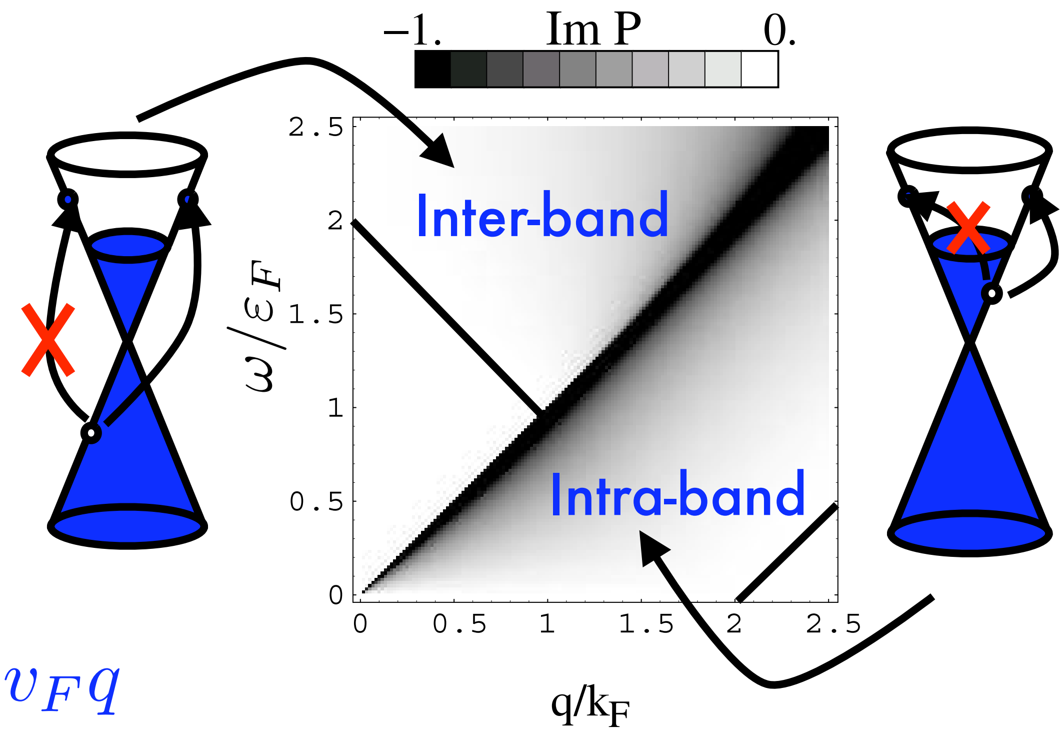
$$F_{\lambda,\lambda'}(\mathbf{k},\mathbf{q}) = \frac{1+\lambda\lambda'\cos\theta}{2}$$

⇓

Absence of backscattering

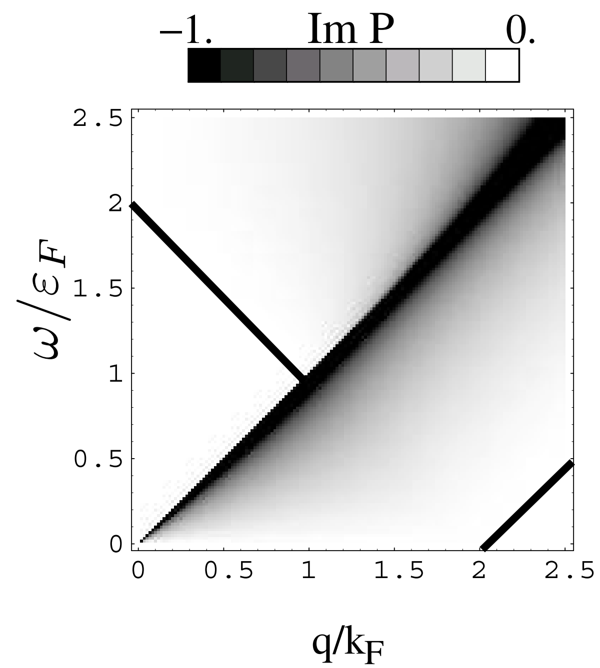
⇓

Concentration around $\omega = v_F q$

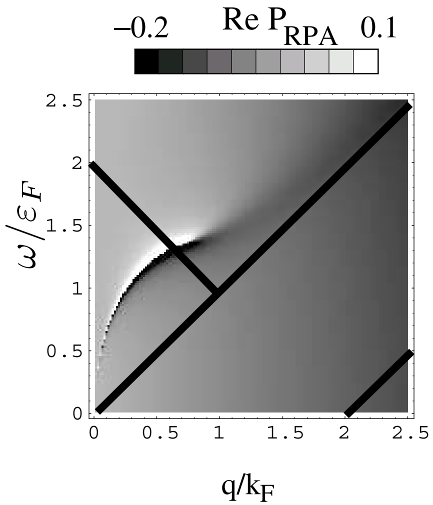


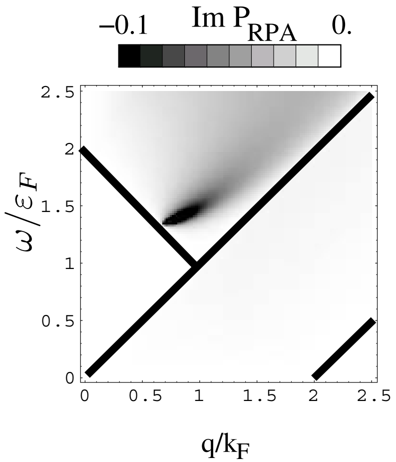
B. Wunsch, T. Stauber, F. Sols & F. Guinea. New Journal of Physics 8, 318 (2006)

Interacting graphene at B = 0: RPA theory



el-el interactions





- Interactions lead to a transfer of spectral weight of the PHES to the plasmon mode
- Landau damping when the plasmon mode enters the inter-band region of the PHES

B. Wunsch, T. Stauber, F. Sols & F. Guinea. New Journal of Physics 8, 318 (2006)

Particle-hole polarization in graphene: B≠0

Decomposition in inter- and intra-band contributions

$$\Pi_n^{\lambda}(\mathbf{q},\omega) = \sum_{\lambda'}\sum_{n'=0}^{n-1}\Pi_{nn'}^{\lambda\lambda'}(\mathbf{q},\omega) + \sum_{\lambda'}\sum_{n'=n+1}^{N_c}\Pi_{nn'}^{\lambda\lambda'}(\mathbf{q},\omega) + \Pi_{nn}^{\lambda-\lambda}(\mathbf{q},\omega)$$

$$\Pi^0(\mathbf{q},\omega) = \sum_{n=1}^{N_F}\Pi_n^{\lambda_F}(\mathbf{q},\omega) + \Pi^{vac}(\mathbf{q},\omega)$$

Vacuum (inter-band) contribution

$$\Pi^{vac}(\mathbf{q},\omega) \equiv -\sum_{n=1}^{N_c}\Pi_n^{\lambda=1}(\mathbf{q},\omega)$$

Filling factor

$$\nu = 4N_F + 2$$

Cutoff in the LL index

$$N_c \sim \frac{10^4}{B[T]}$$

$$\Pi_{nn'}^{\lambda\lambda'}(\mathbf{q},\omega) \equiv \frac{\overline{\mathcal{F}}_{nn'}^{\lambda\lambda'}(\mathbf{q})}{\lambda\xi_n - \lambda'\xi_{n'} + \omega + i\delta\mathrm{sgn}(\omega)}$$

Form factor of the polarization function at B ≠ 0

$$\overline{\mathcal{F}}_{nn'}^{\lambda\lambda'}(\mathbf{q}) = e^{-l_B^2q^2/2}\left(\frac{l_B^2q^2}{2}\right)^{n_>-n_<}\left\{\lambda 1_n^* 1_{n'}^*\sqrt{\frac{(n_<-1)!}{(n_>-1)!}}\left[L_{n_<-1}^{n_>-n_<}\left(\frac{l_B^2q^2}{2}\right)\right] + \lambda' 2_n^* 2_{n'}^*\sqrt{\frac{n_<!}{n_>!}}\left[L_{n_<}^{n_>-n_<}\left(\frac{l_B^2q^2}{2}\right)\right]\right\}^2$$

PHES of graphene in a magnetic field

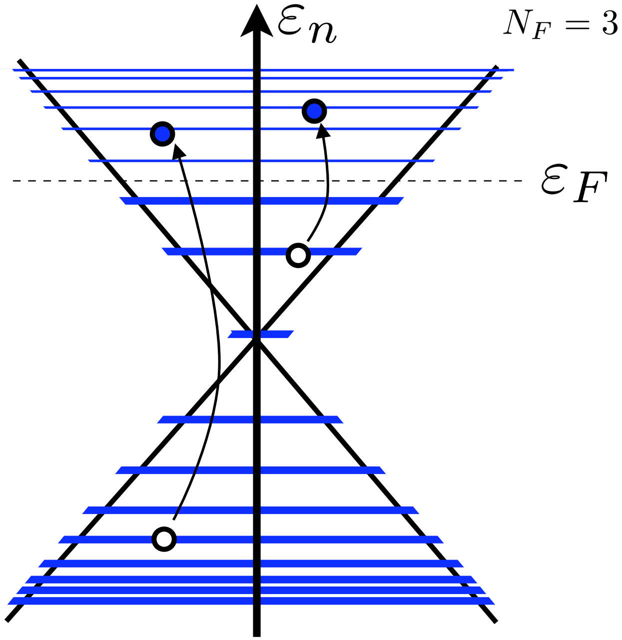
Landau levels (LLs)

$$\varepsilon_{\lambda,n} = \lambda\hbar\frac{v_F}{l_B}\sqrt{2n}$$

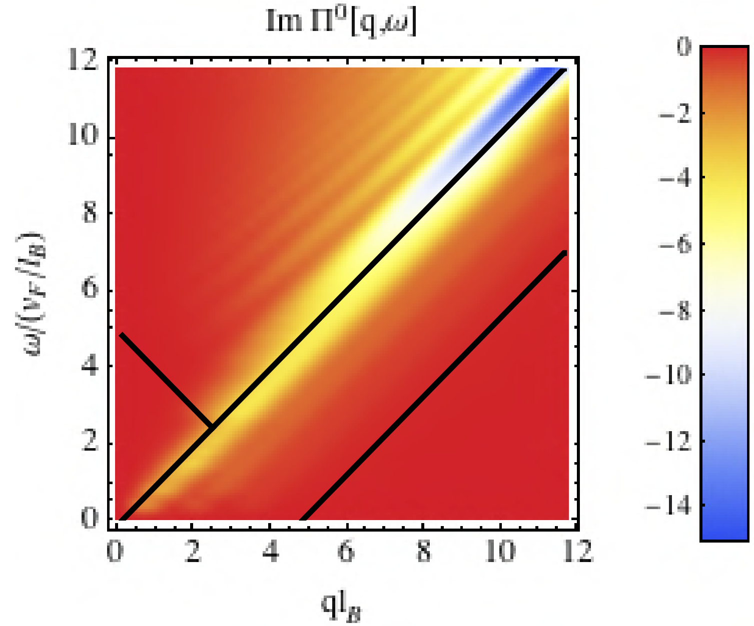
$$B \neq 0$$

"Density dependent" cyclotron frequency

$$\omega_c(\varepsilon_F) = \frac{eB}{\varepsilon_F/v_F^2}$$



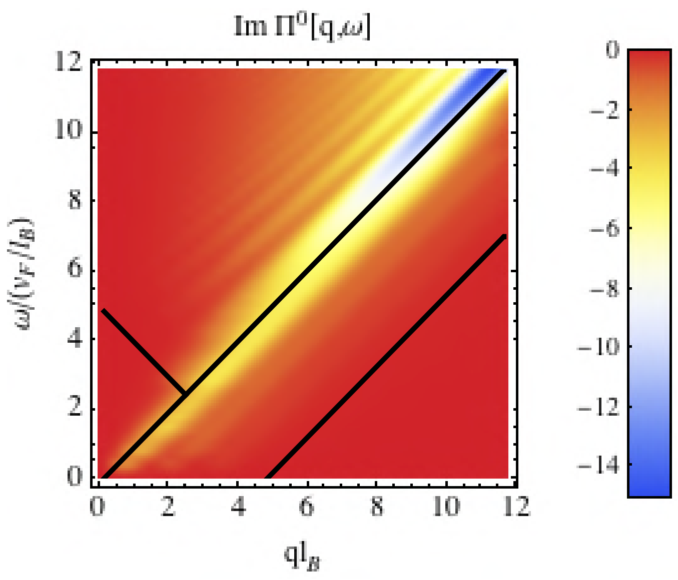
$N_F = 3 \Rightarrow 2k_F = 2\sqrt{6} \simeq 4.9 l_B$



LL quantization
+
Non-equidistant LL separation
+
Chirality factor

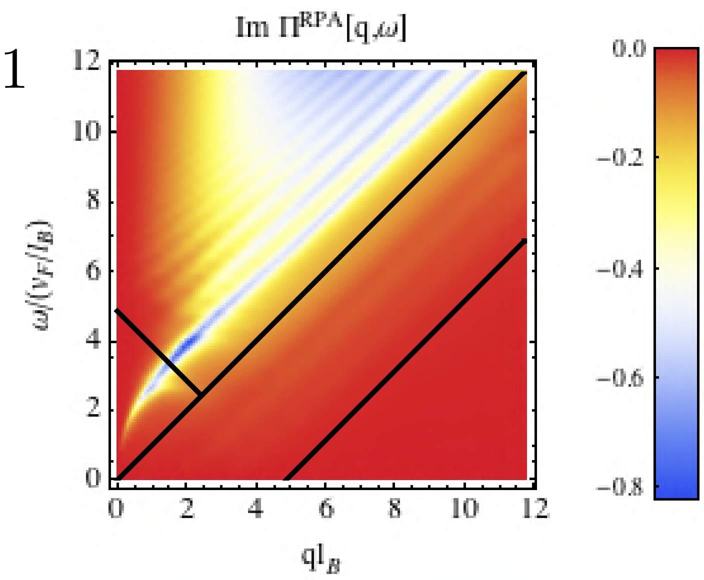
- blurred horizontal magneto-excitons
- dispersive modes: (linear) magneto-plasmons

Interacting graphene in a magnetic field: RPA theory



$$r_s \equiv \frac{e^2}{\epsilon_b \hbar v_F} \approx 1$$

el-el interactions



- Interactions lead to a transfer of spectral weight from the PHES to the upper-hybrid mode
- Relatively weak renormalization of the intra-band region of the spectrum.
- The linear magneto-plasmons are more pronounced due to the inclusion of Coulomb interaction.

Hydrodynamical theory of linear response

- Euler and continuity equations:

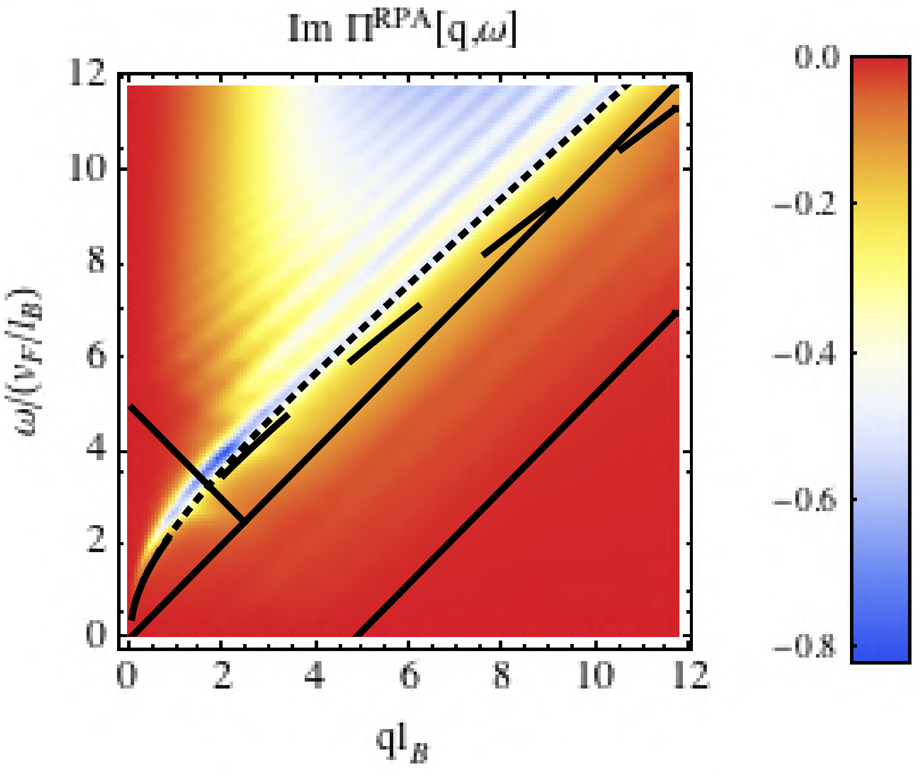
$$\frac{d}{dt}\left(\frac{\varepsilon_F}{v_F^2}\mathbf{J}(\mathbf{r},t)\right) = e\boldsymbol{\nabla}P(\mathbf{r},t) + en(\mathbf{r},t)\boldsymbol{\nabla}\int d\mathbf{r}'\frac{e^2}{|\mathbf{r}-\mathbf{r}'|}[n(\mathbf{r}',t)-n_0] - e\mathbf{J}(\mathbf{r},t)\times\mathbf{B},$$

$$\frac{\partial n(\mathbf{r},t)}{\partial t} = \frac{1}{e}\boldsymbol{\nabla}\cdot[\mathbf{J}(\mathbf{r},t)]$$

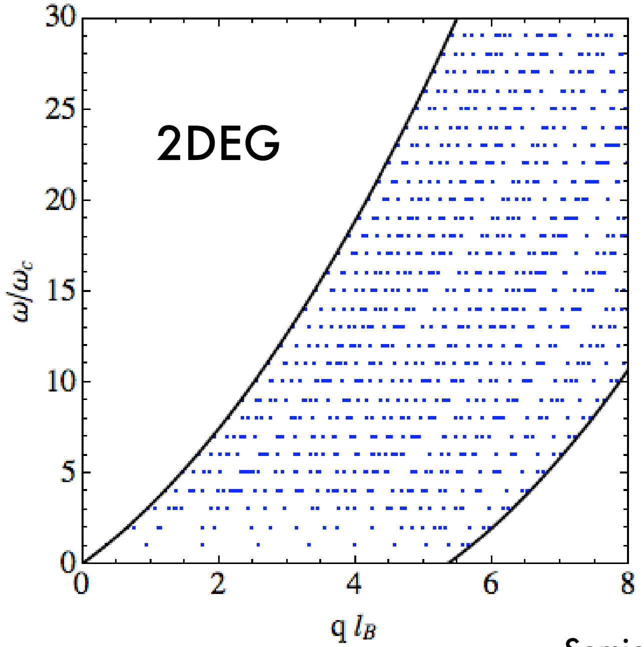
- Approximate dispersion relation:

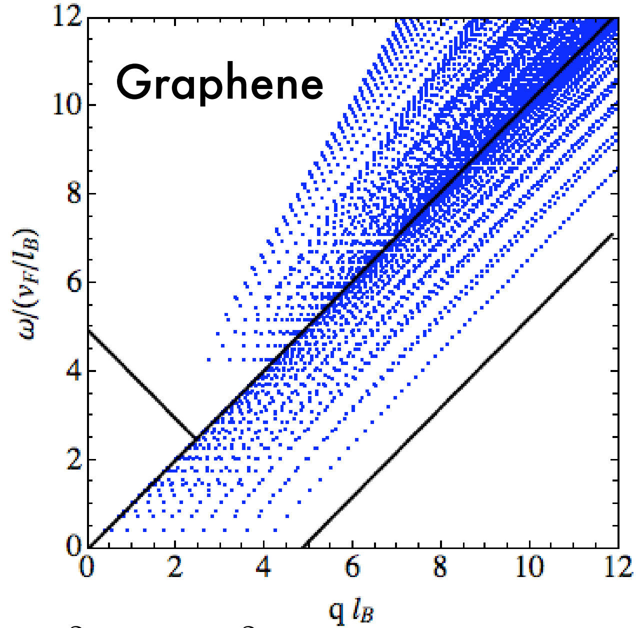
$$\omega(q) \simeq \sqrt{\# v_F^2 q^2 + \frac{2\pi e^2 n_0 v_F^2}{\epsilon_b \varepsilon_F} q + \omega_c(\varepsilon_F)^2}$$

$$\# = \begin{cases} 1/2 & \text{— — — Hydrodynamic} \\ 3/4 & \text{.............. RPA} \end{cases}$$



Semiclassical approximation for the PHES





Semiclassical quantization of the cyclotron orbit

$$p^2 = (2n+1)/l_B^2$$

$$\begin{cases} \omega = (n'-n)\omega_c \\ q = l_B^{-1}\sqrt{2\left[n'+n+1-\sqrt{(2n'+1)(2n+1)}\cos\theta\right]} \end{cases}$$

$$p^2 = 2n/l_B^2$$

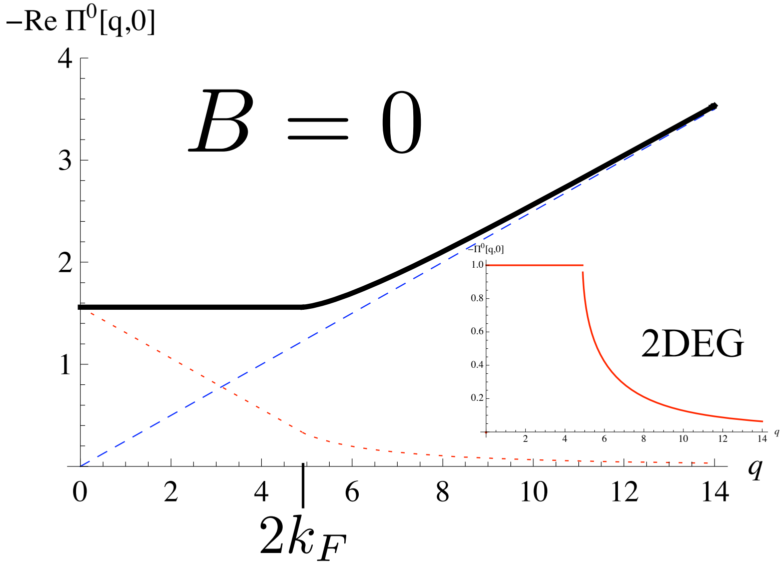
$$\begin{cases} \omega = v_F l_B^{-1}[\sqrt{2n'}-\lambda\sqrt{2n}] \\ q = l_B^{-1}\sqrt{2[n'+n-2\sqrt{n'n}\cos\theta]} \end{cases}$$

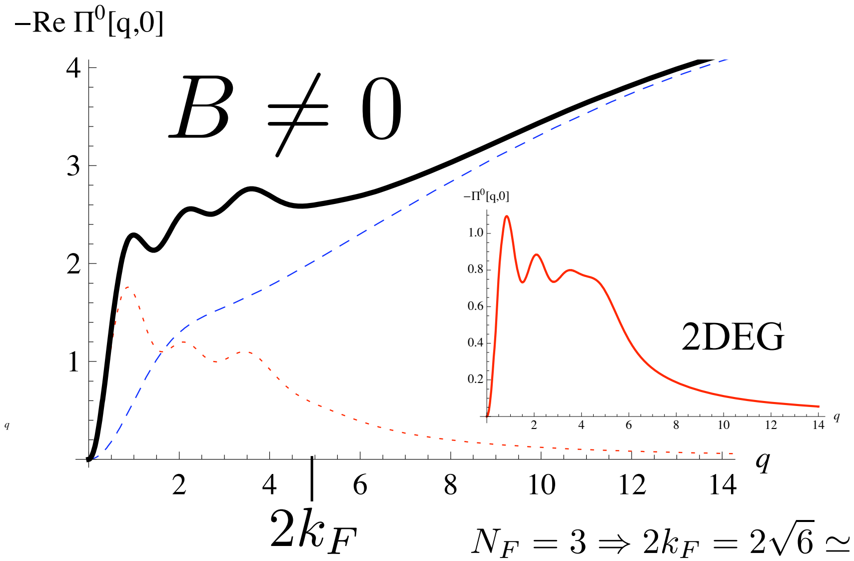
Chirality factor: $\mathrm{Prob}(\theta) = \dfrac{1\pm\cos\theta}{2}$

- Semiclassical results reproduce the characteristics features of the non-interacting PHES calculated in the quantum regime (Π^0 not Π^{RPA})
- Intuitive explanation of the discrete nature of the PHES in a magnetic field.

Static screening in graphene: B=0 vs. B≠0

$$\omega = 0$$





$N_F = 3 \Rightarrow 2k_F = 2\sqrt{6} \simeq 4.9 l_B$

$$q < 2k_F \Longrightarrow -\Pi^0(q,0) = const \propto \rho(\varepsilon_F)$$

$$q > 2k_F \Longrightarrow -\Pi^0_{graph}(q,0) \propto q$$

$$q > 2k_F \Longrightarrow -\Pi^0_{2DEG}(q,0) \propto q^{-2}$$

- - - - - - Inter-band contribution
.... Intra-band contribution
——— Total static contribution

Relevance of the valence band in the physics of graphene

E. H. Hwang & S. D. Sarma. Phys. Rev. B 75, 205418 (2007)

Conclusions

• We have studied the particle-hole excitation spectrum of graphene in a magnetic field and compare it to the standard 2DEG.

• Landau level quantization yields *linear magneto-plasmon* modes in contrast to the 2DEG, where the equidistant LL structure leads to pronounced horizontal magneto-exciton modes.

• Inelastic (Raman) light scattering could be used to reveal the existence and measure the dispersion relation of the linear magneto-plasmons, as well as the upper hybrid mode, in graphene.

	2DEG	Doped graphene
$B = 0, r_s = 0$	Continuum PHES	Intra- & inter-band continuum PHES
$B = 0, r_s \neq 0$	Continuum PHES + Plasmon	Intra- & inter-band continuum PHES + Plasmon
$B \neq 0, r_s = 0$	Discretized PHES (magneto-excitons)	Discretized PHES (linear magneto-plasmons)
$B \neq 0, r_s \neq 0$	Discretized PHES (ME) + upper hybrid mode	Discretized PHES (LMP) + upper hybrid mode

RR, J-N. Fuchs and M.O. Goerbig, arXiv:0809.2667