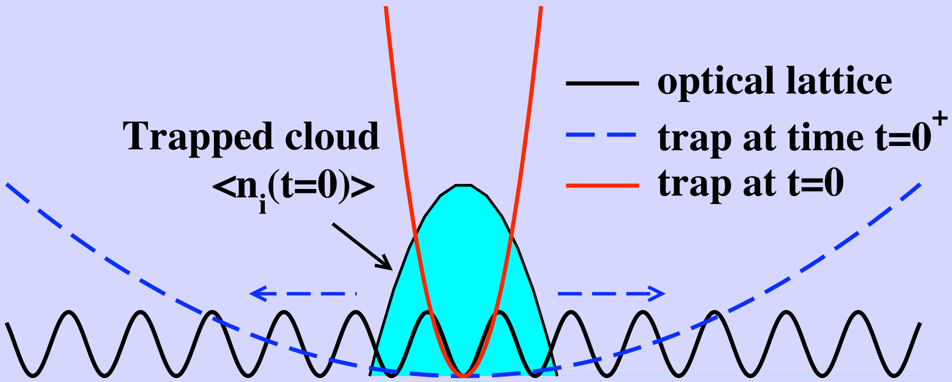
The expansion of strongly interacting fermions in a one-dimensional optical lattice

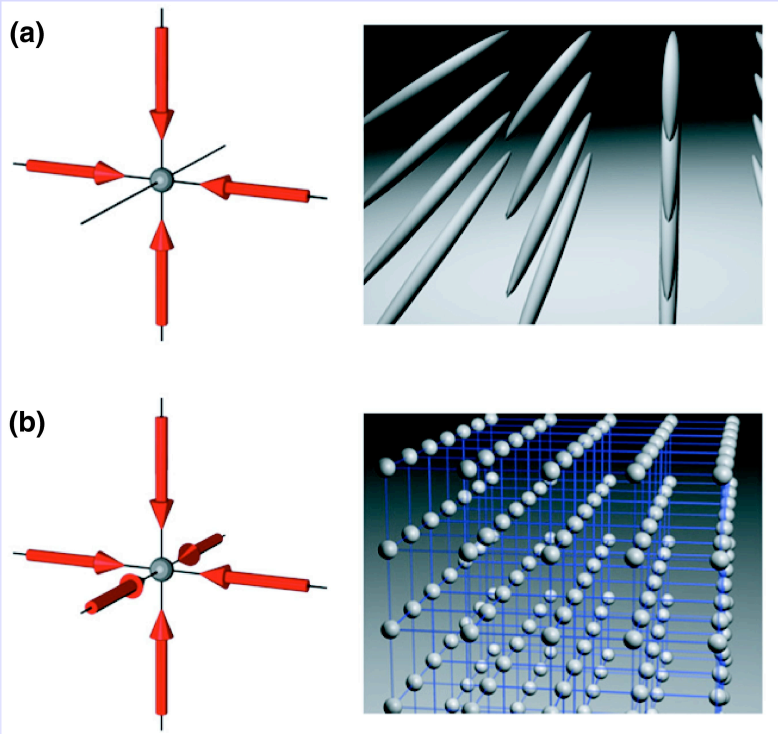


Fabian Heidrich-Meisner (RWTH Aachen University)

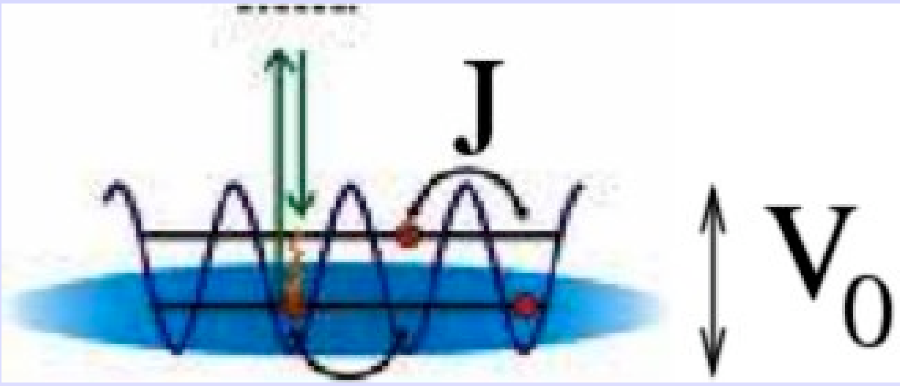
Marcos Rigol (UC Santa Cruz) – Alejandro Muramatsu (Stuttgart)

Adrian Feiguin (Microsoft Q, UCSB) – Elbio Dagotto (UTK & ORNL)

Optical lattices



Bloch, Dalibard, Zwerger, RMP (2008)



Bloch, Nature Physics 1, 23 (2005)

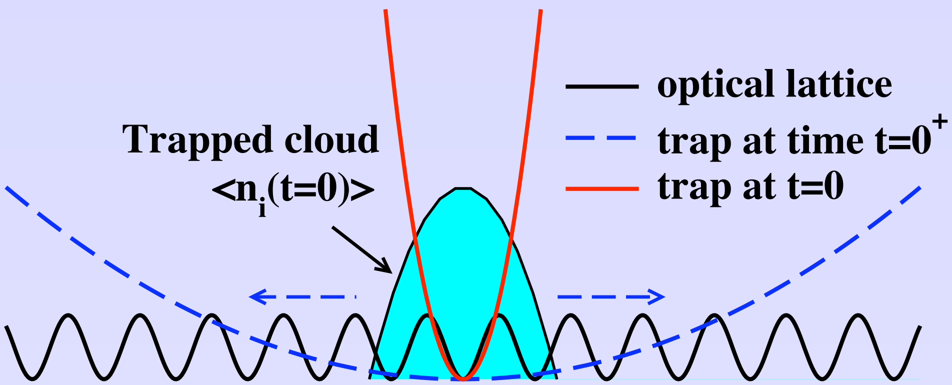
Interactions due to s-wave scattering
V_0: lattice depth
E_r: recoil energy

$J = J(V_0, E_r)$ **and** $U = U(V_0, E_r)$: $U \sim \int r^3 |w(r)|^4$, $w(r)$: **Wannier-function**

$U/J \propto \exp(2\sqrt{V_0/E_r})$: **tunability**

Motivation: Expansion

Experimental realization



Independent control of lattice and trap necessary

Realized in Clement et al. PRL (2005) & Fort et al. PRL (2005)

Kinoshita, Wenger, Weiss Nature (2006)

Lignier et al. PRL (2007) (Pisa)

Sequence: turn off narrow trap – expand into opt. lattice – perform time-of-flight $\Rightarrow$ measure MDF n_k

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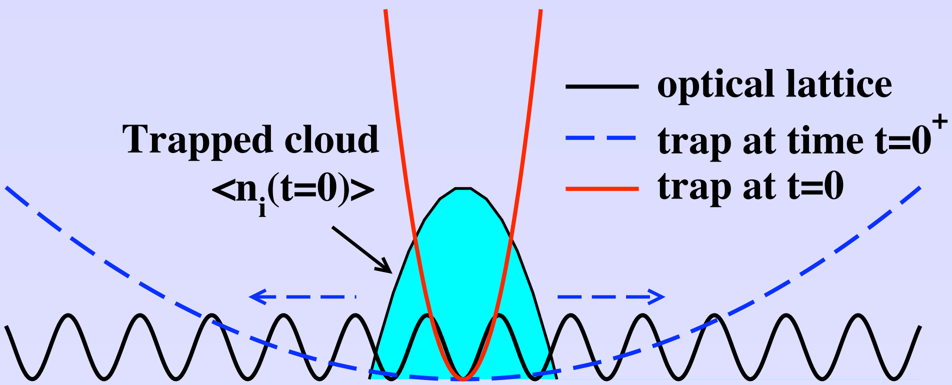

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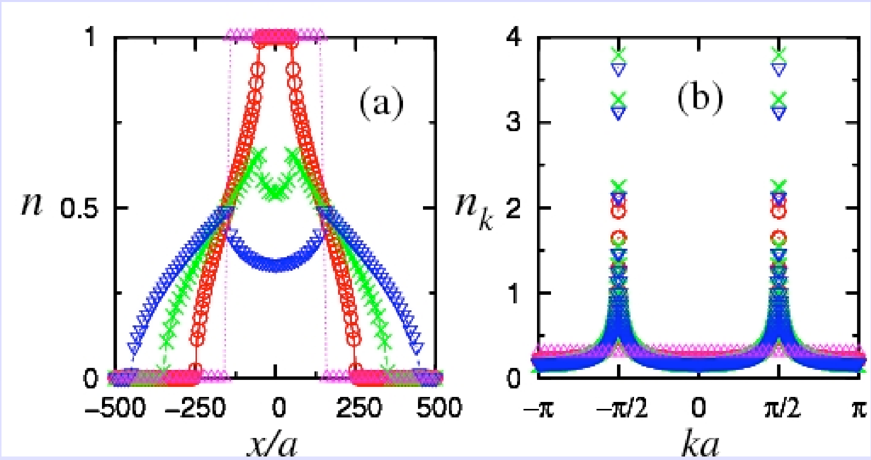
Hard-core bosons: Intriguing findings



Fock state $\longrightarrow$ Quasi-condensation

Rigol & Muramatsu: PRL (2004) & PRL (2005)

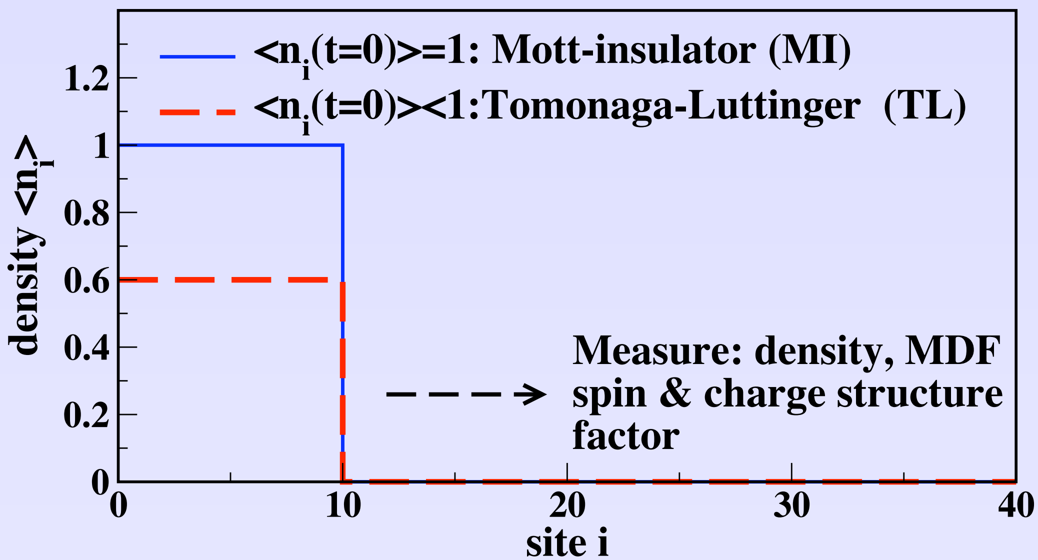
Main goals

- Characterize fermion expansion
- Time-dependent phenomena?
- Relation to equilibrium systems?

Set-up and numerical method

1D Hubbard model:

$$H_{\text{hubb}} = -J \sum_{i=1}^{N-1} (c^\dagger_{i+1,\sigma} c_{i,\sigma} + h.c.) + \sum_i [U n_{i,\uparrow}\; n_{i,\downarrow} + V n_i n_{i+1}]$$



Time $t \leq 0$**:** $H = H_{\text{hubb}} + H_{\text{conf}}$

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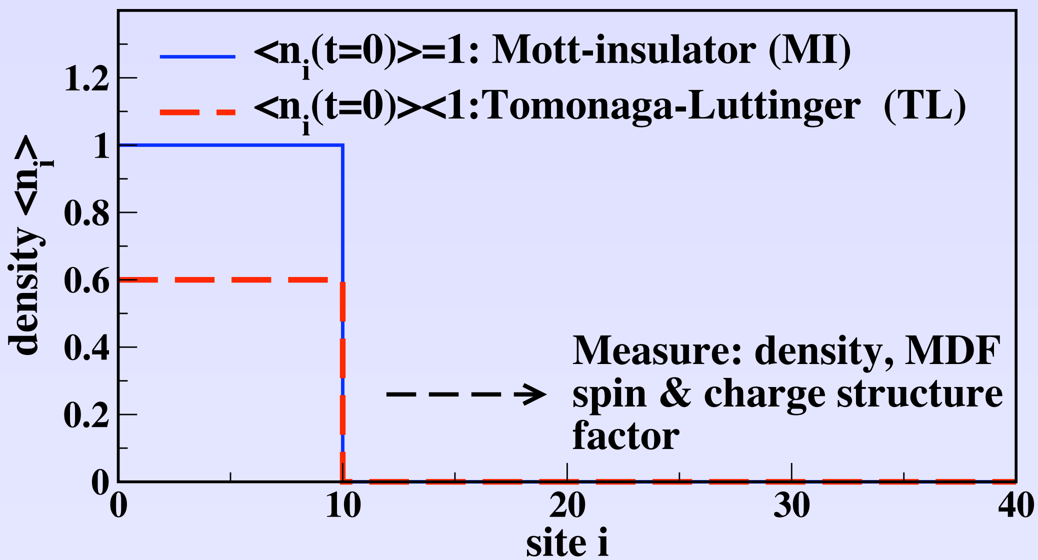
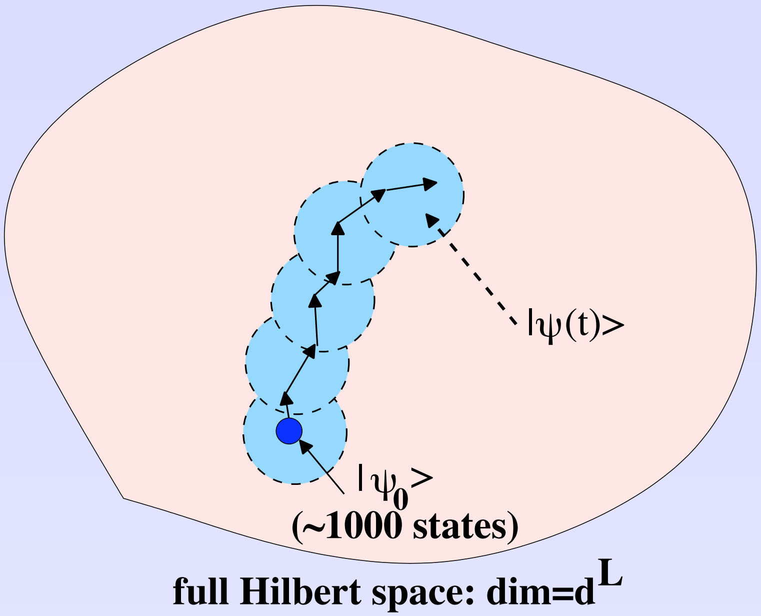


Time $t \le 0$: $H = H_{\rm hubb} + H_{\rm conf}$

Adaptive time-dependent DMRG



Method constructs truncated basis to accurately represent $|\psi(t)\rangle$

White & Feiguin PRL (2004)

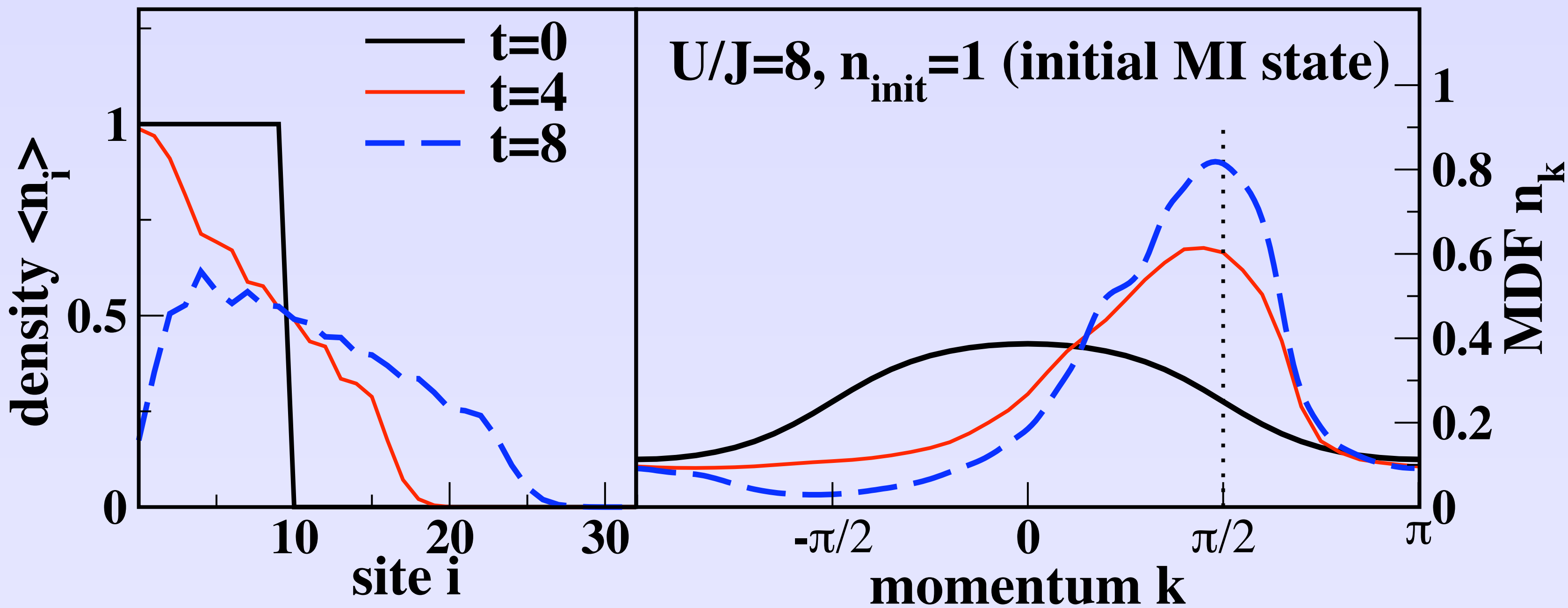
Daley et al. JStatMech: Theor. & Exp. (2004)

Review: Schollwöck RMP (2005)

Expansion from a MI state

Momentum distribution function (MDF)

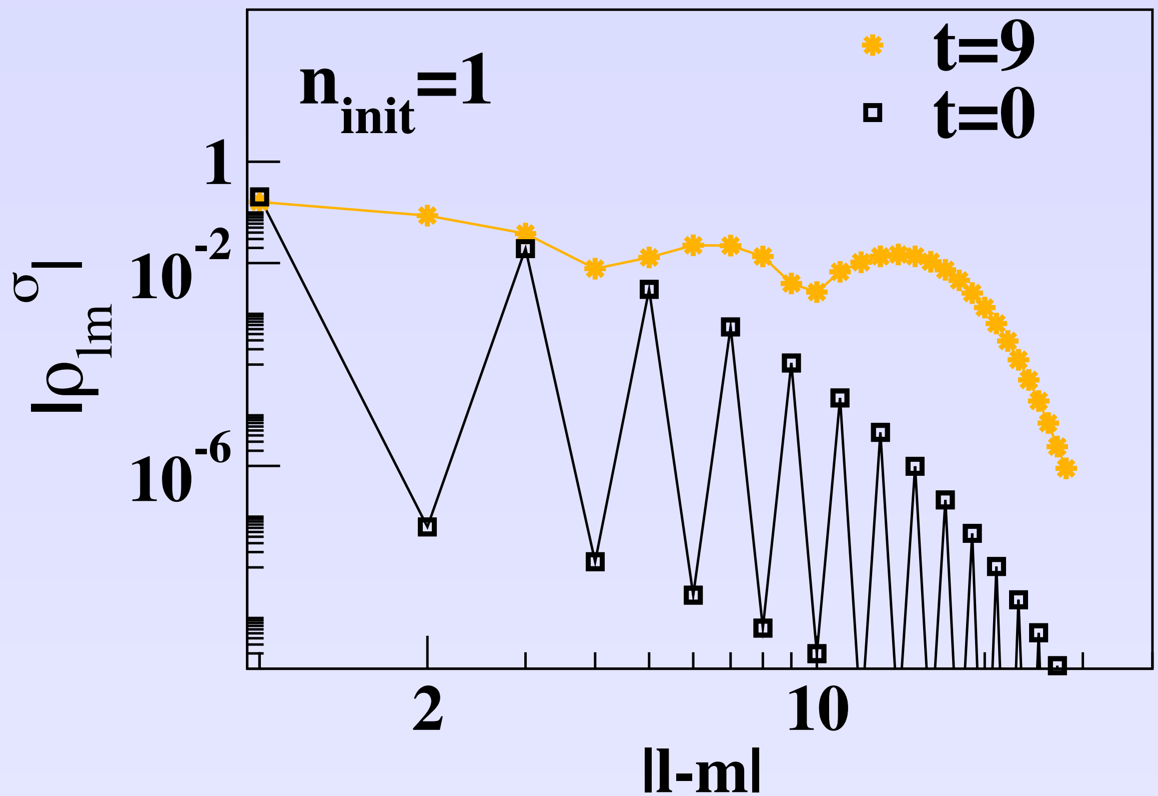
$$n_k = \frac{1}{N} \sum_{lm;\sigma} \mathbf{exp}[-i(l-m)k] \, \langle c^\dagger_{l,\sigma} c_{m,\sigma} \rangle$$



MDF peaks at $k_p \approx \pi/2$ since $E_{\mathrm{kin,av}} \approx 0$

Coherence properties

Spatial decay of OPDM: $\rho^{\sigma}_{lm} = \langle c^{\dagger}_{l,\sigma} c_{m,\sigma} \rangle$



Dynamical emergence of coherence, exponent as in ground state!

Similar to HCB case: Rigol & Muramatsu PRL (2004)

Ground-state reference systems

Construction of reference systems

Perform ground-state DMRG:

$$H_{\mathrm{ref}} = H_{\mathrm{hubb}}(J,U) + \sum_i \epsilon_i \, n_i$$

find set of ϵ_i such that at each given time t

$$\langle n_i \rangle_{\mathrm{ref},t} = \langle n_i(t) \rangle$$

$\Rightarrow$ **Minimize over ϵ_i**
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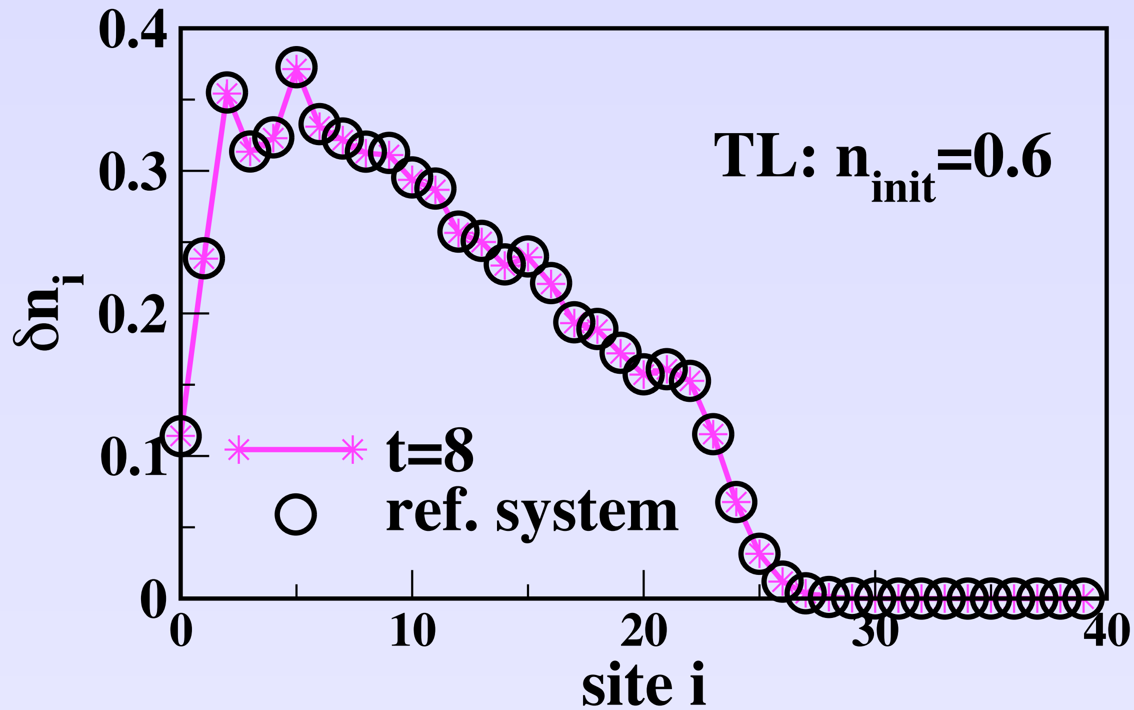
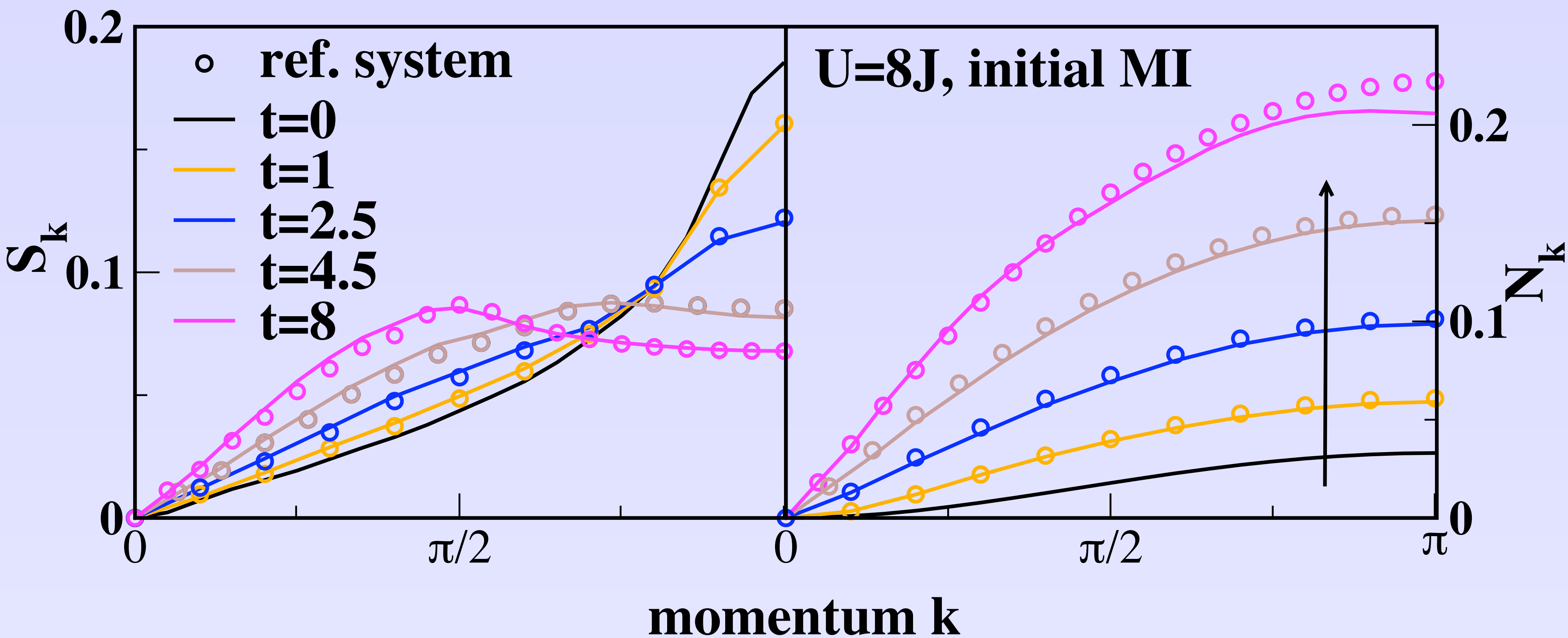
Reproduces charge and fluctuations

$$\delta n_i = \langle n_i^2 \rangle - \langle n_i \rangle^2$$



Spin and charge structure factors

S_k, N_k : **Fourier trafo of** $S_{ij} = \langle S_i^z S_j^z \rangle$ **&** $N_{ij} = \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle$

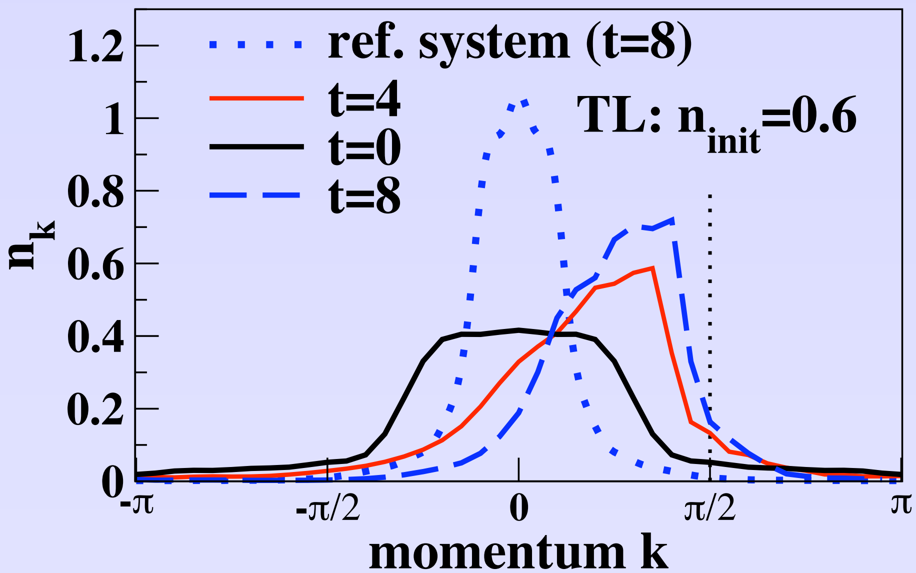


Excellent agreement between ref. systems and expanding cloud!

S_k, N_k: **functionals of density:** $N_k \approx \mathcal{F}_U\{n_i(t)\}$

MDF n_k and total energy E_0

Consider initial TL



Ref. system vs. cloud:
n_k very different

Total energies at $t = 8$:
$E_0 = -6.86J$ vs. $E_0^{\mathrm{ref}} = -10.74J$

Estimate kinetic energy from noninteracting dispersion:

$$T_{\mathrm{kin}} = \sum_k n_k \epsilon_k;\ \epsilon_k = -2J\cos(k)$$

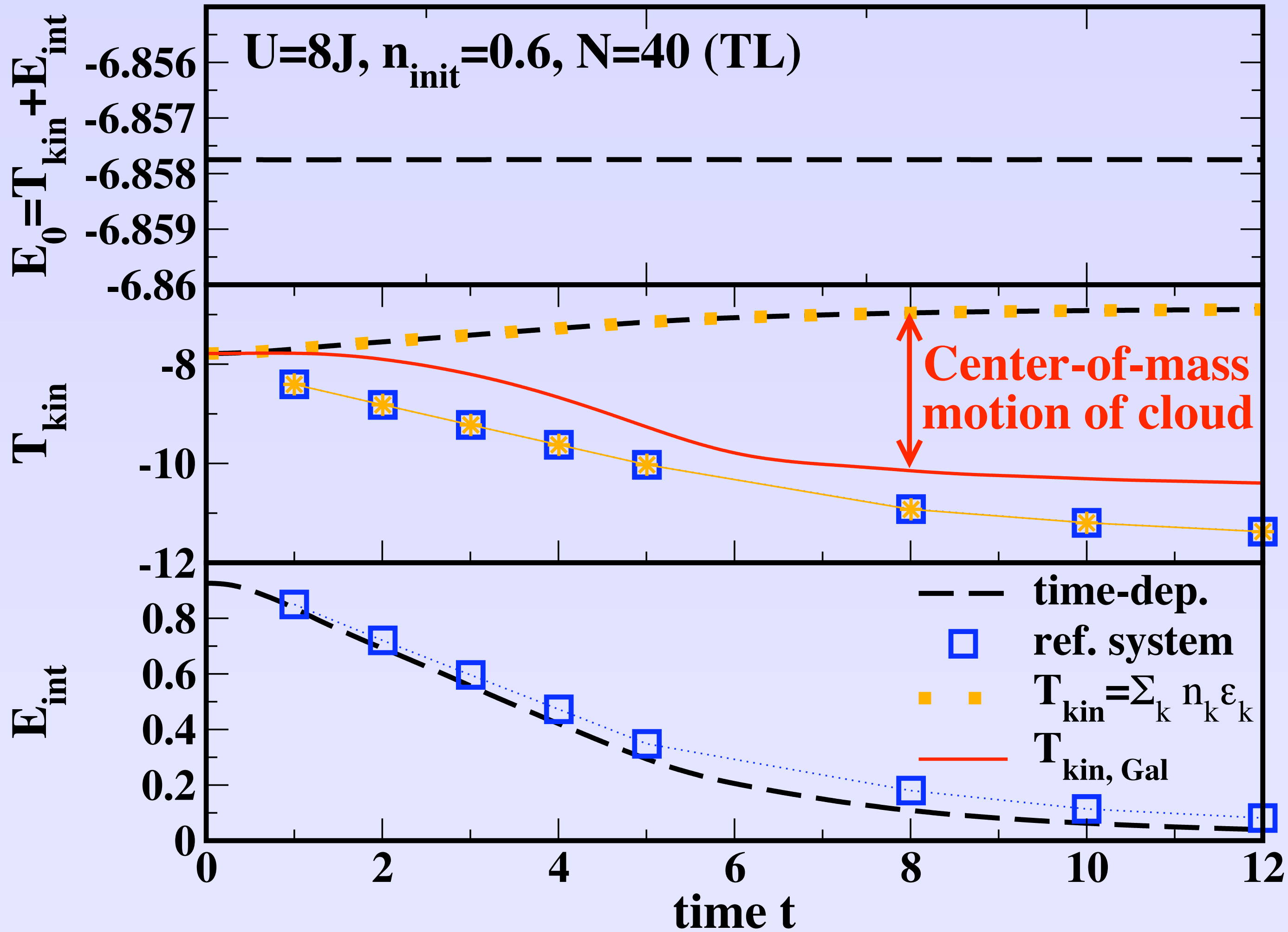
Compute cloud's kinetic energy in co-moving frame:

$$T_{\mathrm{kin}}^{\mathrm{Gal}} = \sum_k n_k \epsilon_{k-k_0}$$

Average momentum of cloud:

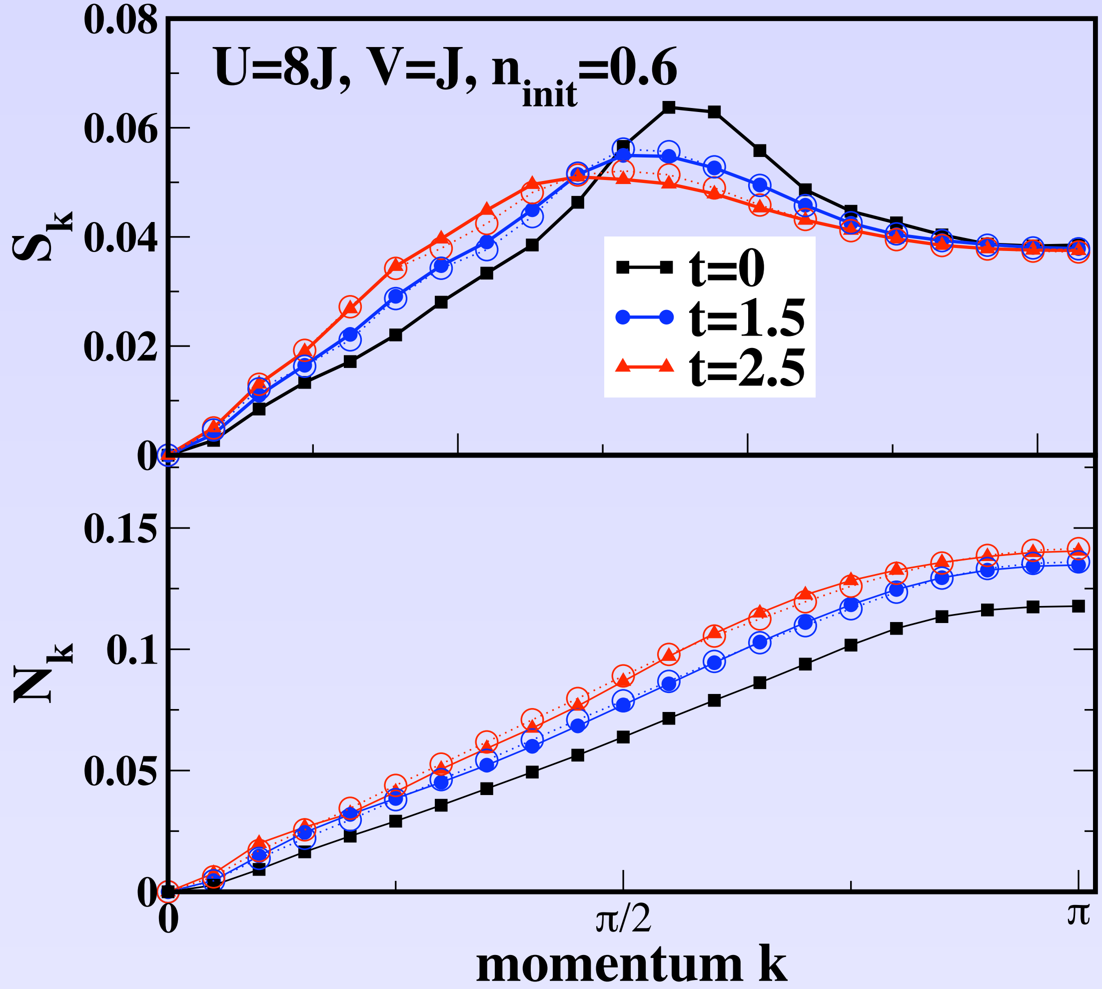
$$k_0 = \sum_k n_k k / \sum_k n_k$$

Kinetic and interaction energy



Energy difference due to finite momentum of cloud!

A nonintegrable system: the extended Hubbard model



$H = H(J,U) + V\sum_i n_i n_{i+1}$: Reference systems still work

Summary

1. Properties of the expanding cloud
   - MI: peak in the MDF at $k \approx \pi/2$
     Experimental identification of MI!
2. Ground-state reference systems
   - Excellent approximation to spin and density correlations
   - Moving cloud and reference system's
     kinetic energy similar up to a Galilean trafo
   - Integrable & Nonintegrable systems

HM, Rigol, Muramatsu, Feiguin, Dagotto: Phys. Rev. A 78, 013620 (2008)

Thank you for your attention!