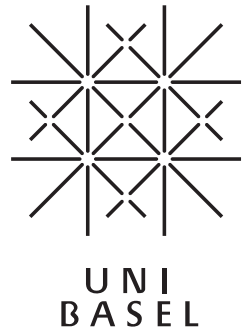


Chiral Electromagnetic Waves in Weyl Semimetals



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Outline:

Introduction to Weyl semimetals.

Model of Weyl semimetal: topological insulator multilayer.

Propagation of the EM field in Weyl semimetal.

EM field in the vicinity of the magnetic domain walls.

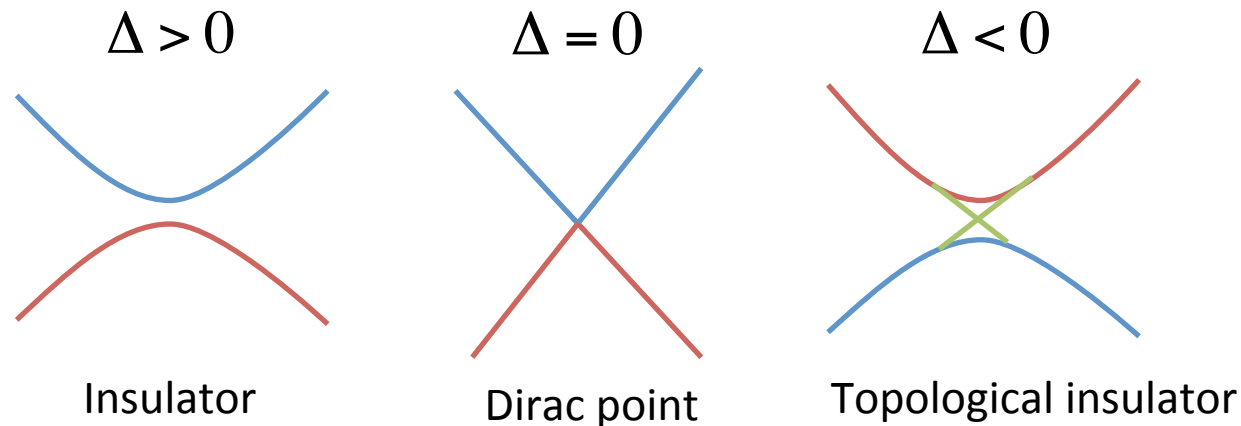
Minimal model of the topological insulator

3D crystal band structure

2 bands of opposite parity + 2 spin directions

$$H(k) = \begin{pmatrix} \Delta & iv_F k_- & 0 & iv_F k_z \\ -iv_F k_+ & -\Delta & -iv_F k_z & 0 \\ 0 & iv_F k_z & \Delta & -iv_F k_+ \\ -iv_F k_z & 0 & iv_F k_- & -\Delta \end{pmatrix} \quad \text{Massive 4 component Dirac Fermion} \quad E_{\pm} = \pm \sqrt{v_F^2 p^2 + \Delta^2}$$

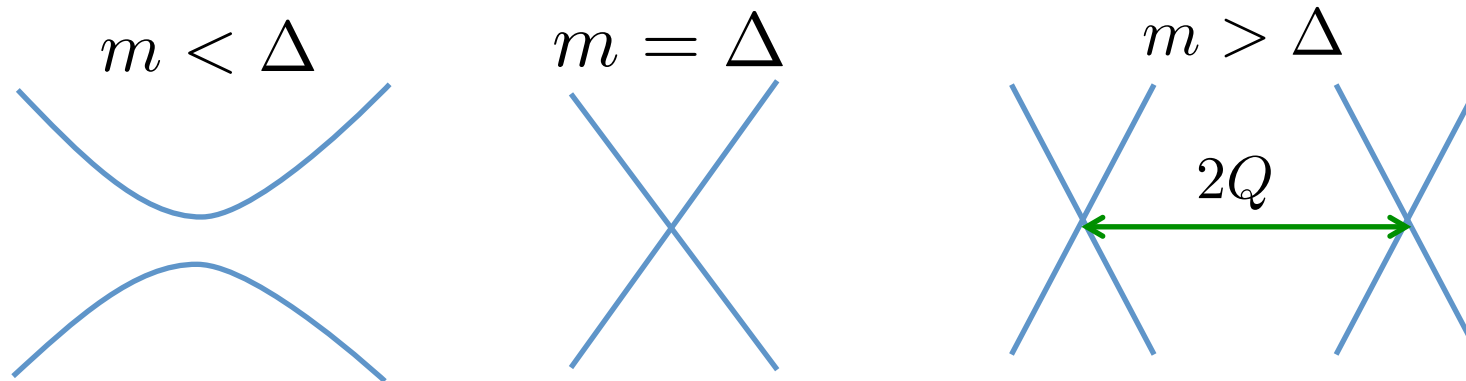
Sign change of the mass of the Dirac fermion:



1. Massless Dirac fermion occur at the critical point between an insulator and a topological insulator.
2. Conduction and valence bands touch at the isolated point in 3D BZ.
3. Doubly degenerate if TR and Inversion symmetries are preserved.
4. 4 component Dirac fermion is decomposed into a pair of 2 component Weyl fermions of opposite chirality.

$$H = \begin{pmatrix} v_F \mathbf{p} \cdot \boldsymbol{\sigma} & \Delta \\ \Delta & -v_F \mathbf{p} \cdot \boldsymbol{\sigma} \end{pmatrix} + m \sigma^z$$

Breaking TRS separates the degenerate single Dirac point into two Weyl points.



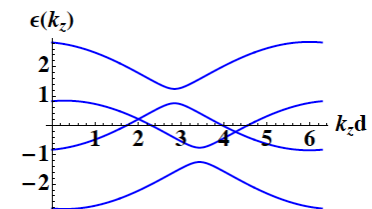
$$H_R = +v_F \sigma(\mathbf{p} + Q\hat{z})$$

$$H_L = -v_F \sigma(\mathbf{p} - Q\hat{z})$$

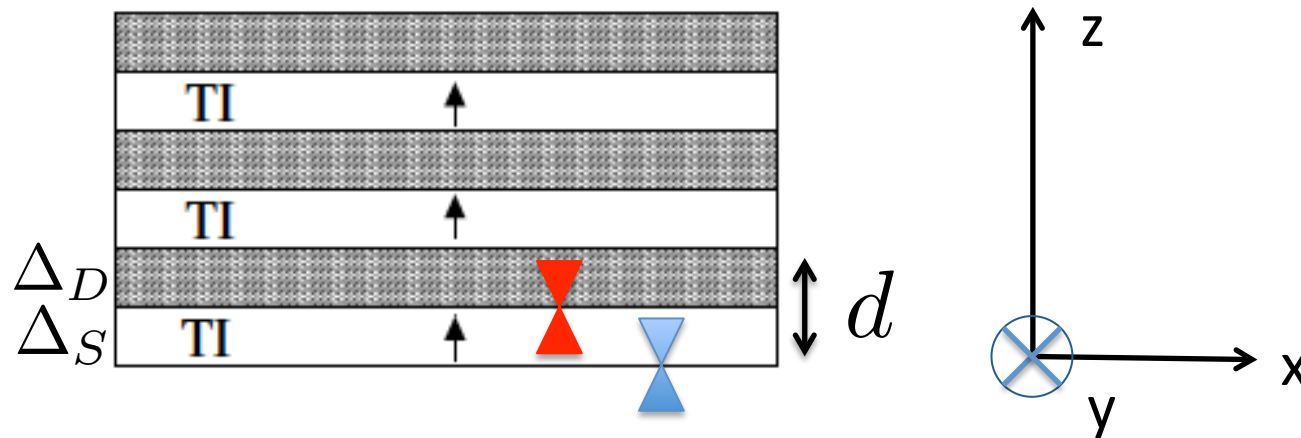
$$p_{z\pm} = \pm \frac{1}{v_F} \sqrt{m^2 - \Delta^2}$$

Wan, et al.; Yang, et al; Burkov and Balents. (2011)

Broken inversion symmetry. Halasz, Balents; A.Z., Burkov. (2012)



Burkov and Balents, PRL 2011

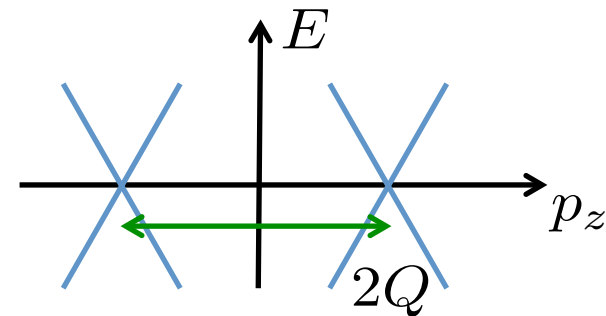


Broken TR symmetry due to magnetization: $\mathbf{M}(x, y, z) = M \hat{z}$

$$\mathcal{H}(\mathbf{p}) = v_F p_y \sigma^x - v_F p_x \sigma^y + (|m| - \Delta(p_z)) \text{sign}(M) \sigma^z - \mu$$

$|m|$ Is exchange energy due to magnetically ordered impurities

$$\Delta(p_z) = [\Delta_S^2 + \Delta_D^2 + 2\Delta_S \Delta_D \cos(p_z d)]^{1/2}$$



Numerics for Cd_3As_2 :

Wang et al. PRB 85, 195320 (2012)

Experimental signatures of Dirac spectrum in Cd_3As_2 :

ARPES: M. Neupane et al. Nature Com. 2014

Positive and linear magneto-resistance: Feng et al., 1405.6611, Liang et. al.1404.7794

Shubnikov-de Haas oscillations: He et al. 1404.2557

Propagation of the electromagnetic wave in the Weyl semimetal

$$\nabla \times \nabla \times \mathbf{E}(\omega, \mathbf{r}) = \frac{\omega^2}{c^2} \epsilon(\omega) \mathbf{E}(\omega, \mathbf{r}) \quad \epsilon(\omega) = \begin{pmatrix} \epsilon_x(\omega) & i\gamma(\omega) & 0 \\ -i\gamma(\omega) & \epsilon_x(\omega) & 0 \\ 0 & 0 & \epsilon_z(\omega) \end{pmatrix}$$

The gyrotropy parameter:

$$\gamma(\omega) = \gamma_0(\omega) \text{sign}(M) \quad \gamma_0(\omega) = \frac{2\alpha \tilde{v}_F Q}{\pi \omega}$$

Weyl semimetal with broken TR symmetry is an optically gyrotropic media with the gyrotropy parameter proportional to the sign of the magnetization. Faraday rotation is expected.

$$\epsilon_a(\omega) = 1 + c_a \frac{\alpha}{3\pi} \left[\ln \left| \frac{4\Lambda^2}{4\mu^2 - \omega^2} \right| - \frac{4\mu^2}{\omega^2} + i\pi \text{sign}(\omega) \Theta(|\omega| - 2|\mu|) \right]$$

$$c_x = 1, c_z = \tilde{v}_F^2 / v_F^2 \quad \alpha = \frac{e^2}{4\pi\epsilon_0 \tilde{v}_F}$$

Direction of the propagation is perpendicular to the gyration axis:

$$\mathbf{E}(x, y) = E \left(1, \frac{c^2 q_y^2 - \omega^2 \epsilon_x(\omega)}{c^2 q_x q_y + i\omega^2 \gamma(\omega)}, 0 \right) e^{ixq_x + iyq_y}$$

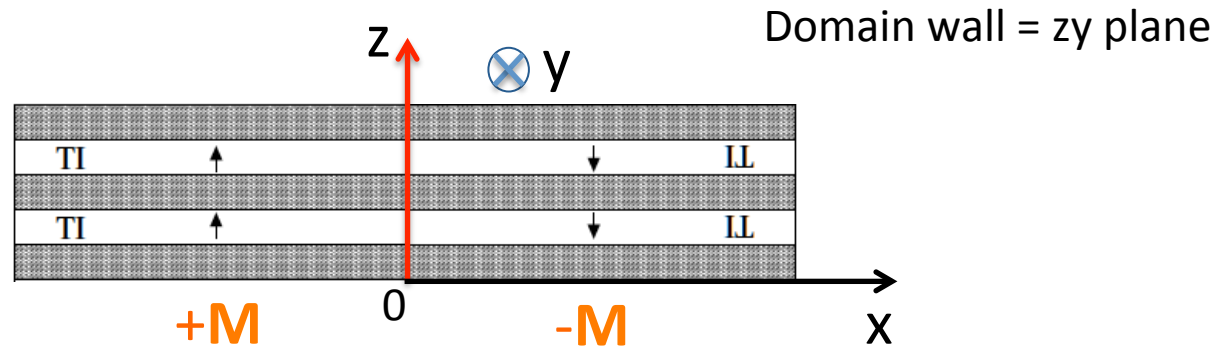
$$q_x^2 + q_y^2 = \frac{\omega^2}{c^2} \epsilon_x(\omega) \left[1 - \frac{\gamma^2(\omega)}{\epsilon_x^2(\omega)} \right]$$

Condition for the positive real values of the dielectric constant:

$$2|\mu| > |\omega| > \sqrt{\frac{\alpha}{3\pi} \frac{2|\mu|}{\left[1 + \frac{2\alpha}{3\pi} \ln |\Lambda/\mu|\right]^{1/2}}}$$

$$\text{Im}\epsilon_x(\omega) = 0, \quad \text{Re}\epsilon_x(\omega) > 0$$

Magnetic domain wall in the Weyl semimetal.



$$\mathbf{M}(x, y, z) = -|M|\text{sign}(x)\hat{z}$$

$$\gamma(\omega) \rightarrow \gamma(\omega, x) = -\gamma_0(\omega)\text{sign}(x)$$

$$\epsilon_a(\omega) \rightarrow \epsilon_a(\omega)$$

Difference from surface plasmons, Dyakonov waves... $\gamma_0(\omega) > 0, \omega > 0$
 Zhukov, Raikh: Optical isomers (2000) $\epsilon_x(\omega) > 0$

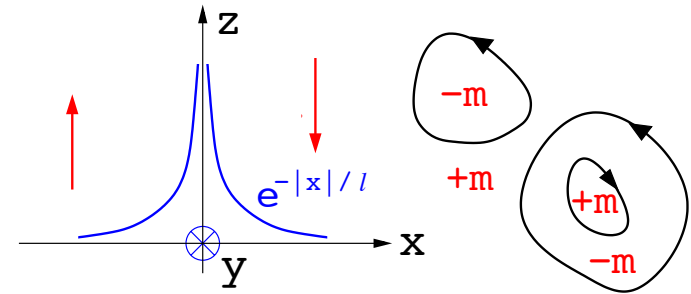
$$-\partial_x^2 E_y(x) - q \frac{2\gamma_0(\omega)}{\epsilon_x(\omega)} \delta(x) E_y(x) = \mathcal{E}(\omega) E_y(x)$$

Attractive for positive q

$$\mathcal{E}(\omega) = - \left[\frac{q\gamma_0(\omega)}{\epsilon_x(\omega)} \right]^2 \quad q = \omega \sqrt{\epsilon_x(\omega)} / c$$

$$E_y(x, y) = E_y(0) e^{iqy - |x|/\ell},$$

$$E_x(x, y) = iE_y(x, y) \frac{\epsilon_x^2(\omega) + \gamma_0^2(\omega)}{2\epsilon_x(\omega)\gamma_0(\omega)} \text{sign}(x)$$



Localization length of the
EM field at the domain wall

$$\ell = \frac{\pi}{2\alpha} \frac{c \sqrt{\epsilon_x(\omega)}}{\tilde{v}_F Q}$$

$$\ell \sim 1\mu m \quad \mu \sim 0.1eV \quad \text{Near infrared}$$

Main conclusions.

1. Weyl semimetal with broken TR symmetry is an optically gyrotropic media with the gyrotropy parameter proportional to the sign of the magnetization.
2. Chiral electromagnetic wave can propagate in a region of space where the gyration vector flips its direction.