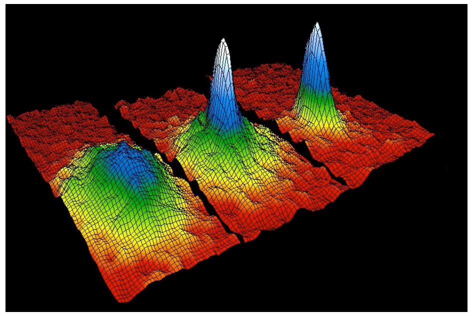
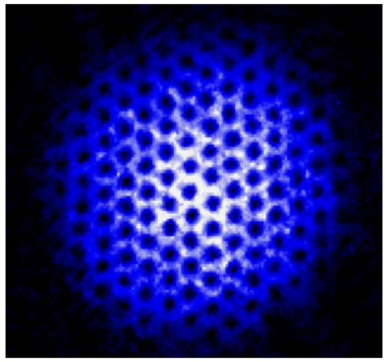
Rotational properties of two-species Bose gases

in the lowest Landau level

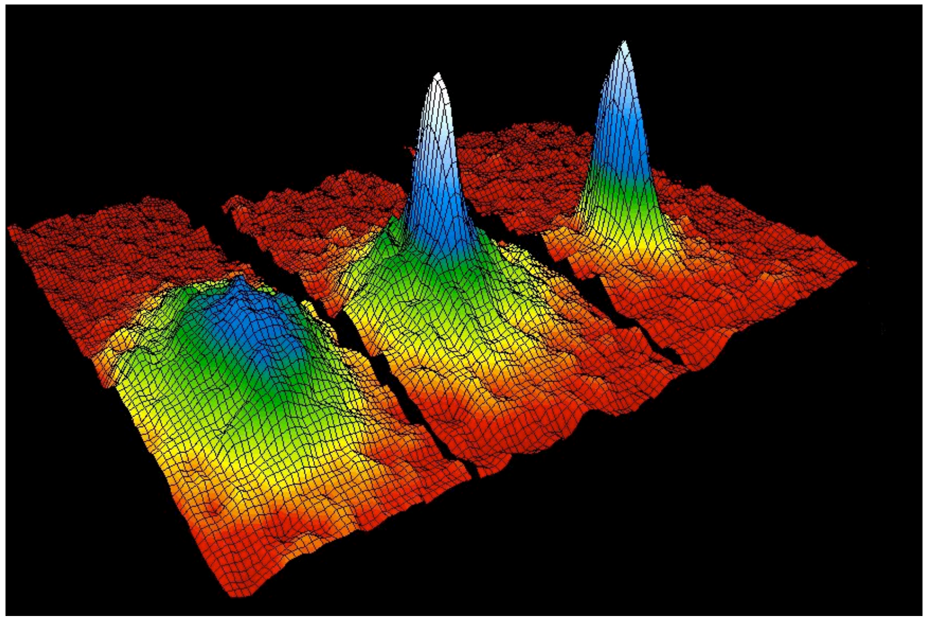


Susanne Viefers, University of Oslo



- Introduction to rotating Bose condensates
- Rotating bosons in the lowest Landau level, quantum Hall connection
- Two-species Bose gases
- Composite fermion approach: Results, puzzles
- Outlook

Atomic Bose condensates

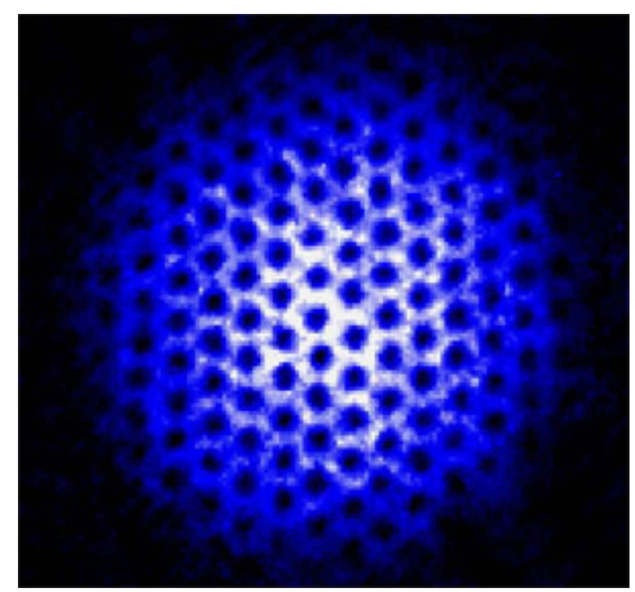


Alkali atoms in magnetic traps

Experimental traps well approximated by harmonic oscillator potential

$$T \leq \mu K, \quad N \approx 10^3 - 10^6$$

Rotating BEC (stirring): Angular momentum carried by *vortices* (vortex lattice). Several hundred vortices observed in experiment



[JILA]

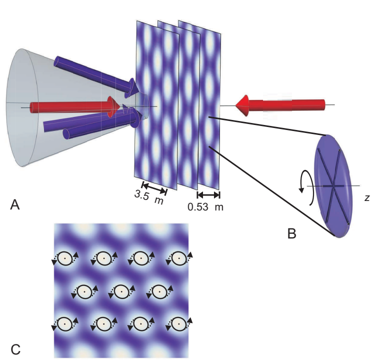
Rotating Bose condensates

- 1995: First atomic Bose condensate
- 1999: First vortex in rotating BEC (JILA, Paris)
- 2004: Abrikosov lattice in lowest Landau level (200 vortices).

Increasing rotation: Cloud flattens out (pancake shape), density decreases $\longrightarrow$ weaker interaction $\longrightarrow$ lowest Landau level.

Eventually: Vortex lattice predicted to melt, so system enters quantum Hall regime. [Other schemes for artificial B-fields more promising in practice]

- Recent reports of *small* systems (N<10) reaching FQH regime in novel type of optical lattice with local rotation of each site. [Gemelke et al, arXiv:1007:2677]

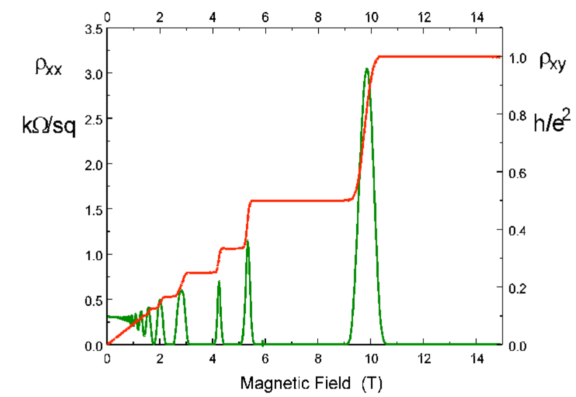


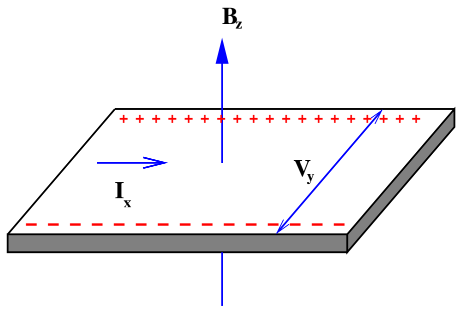
Lowest Landau level wave functions

- Historical motivation: Quantum Hall effect
- 2-dimensional electron gas in a strong perpendicular magnetic field at low T
- Electrons residing (mainly) in the lowest Landau level (LLL).

Single particle basis states:

$$|l\rangle = N_l z^l e^{-|z|^2/4}, \quad z = x + iy$$





N-particle (trial) wave functions constructed as *antisymmetric combinations* of these, i.e. homogeneous polynomials. Total angular momentum = degree of polynomial.

Construction of explicit trial wave functions by various schemes (in particular Laughlin, composite fermions) has proven very successful in exploring Quantum Hall physics. Not exact, but do capture essential properties (topological order).

$$\boxed{\psi_N(z_1, \ldots z_N) = \prod_{i<j} (z_i - z_j)^m e^{-\sum |z_i|^2/4}}$$

Bose condensate in the lowest Landau level

Mathematical equivalence in 2D between rotation (harmonic oscillator) and perpendicular magnetic field:

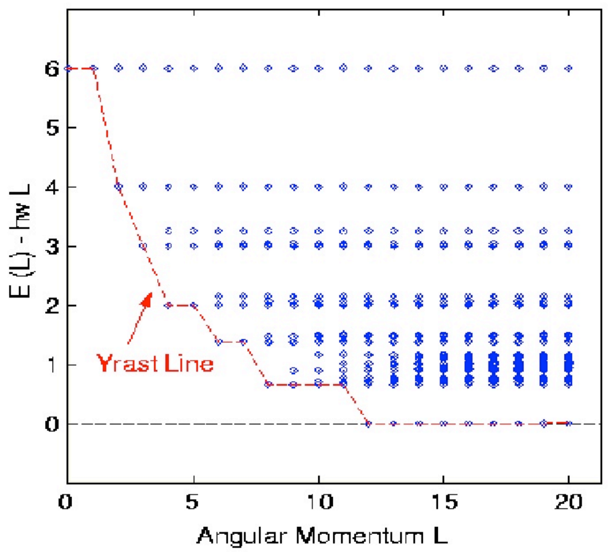
$$\boxed{\begin{aligned}\mathcal{H} &= \frac{1}{2m}\left(p_x^2+p_y^2\right)+\frac{1}{2}m\omega^2(x^2+y^2)\\ &= \frac{1}{2m}\left(\mathbf{p}-\mathbf{A}\right)^2+\omega L_z\end{aligned}}$$

$$L_z = xp_y - yp_x$$

$$\mathbf{A} = m\omega\,(-y,x) \;\rightarrow\; \mathbf{B}\equiv\nabla\times\mathbf{A} = 2m\omega\hat{z}$$

Eigenstates: Landau levels in the effective 'magnetic' field **B**. Dilute gas: LLL.

In the absence of interaction: Lowest N-body state *with given L* is highly degenerate. The interaction lifts this degeneracy and selects the lowest ("yrast") state. (Yrast = "most dizzy")



N-particle states: *symmetric* homogeneous polynomials. Total angular momentum = degree of polynomial.

Composite fermion scheme modified and shown to work successfully for the entire yrast line, *including low angular momenta* [Wilkin et al; SV, Hansson Reimann; Korslund & SV; SV & Taillefumier]

Two-component systems

- Much recent interest in (rotational) properties of *two-species* Bose systems.
- E.g. mixture of two types of atoms, two isotopes of the same atom, two hyperfine states of the same atom (all experimentally realized)
- Several parameters can in principle be varied -- inter- vs intraspecies interaction, particle numbers, masses..
- Rich physics. E.g. miscible to immiscible phase transition (non-rotating) as interspecies interaction gets large; defects such as coreless vortex lattices (square, triangular).
- Convenient language: "Pseudospin" 1/2 (at least for homogeneous interaction), label the species "up" and "down".

Two-component systems: Slow rotation

[M.L. Meyer, G.J. Sreejith, SV, PRA 2014]

- Study 2-species Bose gas in LLL with homogeneous contact interaction
- Recent work [Papenbrock et al 2012] identified a class of analytically exact many-body eigenstates for low angular momenta, $L \leq M \quad (M \geq N)$
- Beyond Papenbrock? Study low L regime in terms of composite fermions, exploiting (pseudo)spin analogy for homogeneous interaction. Not *a priori* expected to work...

$$\psi_{CF} = P_{LLL} \left[\Phi_z \Phi_w J_{N,M} \right]$$

Slater determinants for up/down species. Determine total angular momentum, pseudospin...

$$J_{N,M} = \prod_{i<j}^{N} (z_i - z_j) \prod_{k<l}^{M} (w_k - w_l) \prod_{i,l}^{N,M} (z_i - w_l)$$

- Choice of Slater determinants in general not unique, ie several CF candidates -- may or may not be linearly independent....

Full CF diagonalization

- Diagonalization of interaction operator in space spanned by *all* (compact "highest weight") CF candidate states

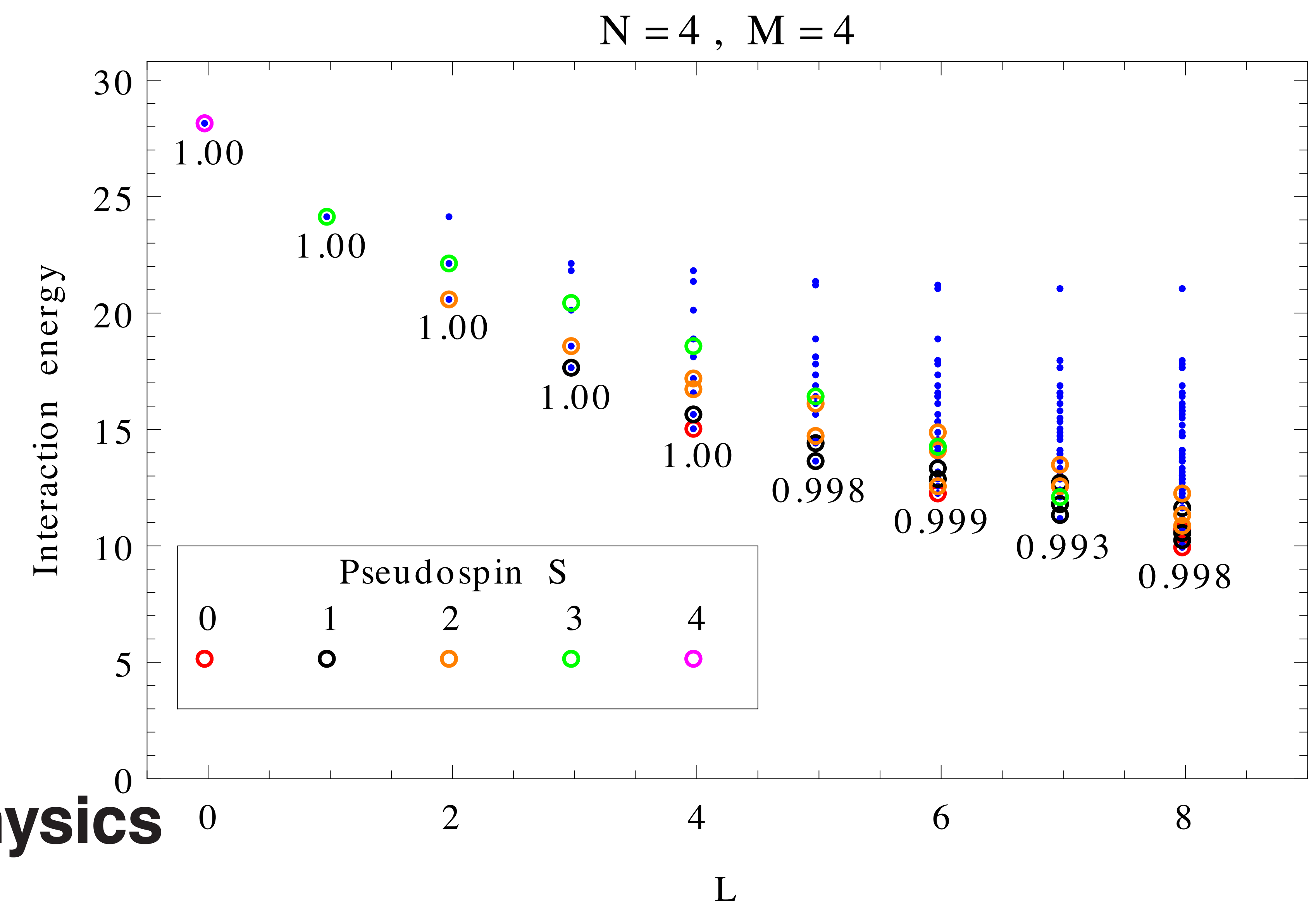
- $L \leq M$: All states (including Papenbrock) are reproduced *exactly*.

- $L > M$: Not exact eigenstates but very good approximations

- However: Very computationally demanding, in particular to identify linearly independent CF candidates. No major gain compared to full numerical diagonalization.

Simple states

- Have identified a special class (subset) of CF candidates whose contribution "stands for most of the good overlaps"
- Technically, characterized by no more than one CF per lambda level in the CF Slater determinants.
- Major computational simplification, much smaller CF state space, while still very good accuracy.



“Missing states” puzzle

[M.L. Meyer, O. Liabøtrø]

- As mentioned, the number of apparent CF candidates is generally too large, i.e. many of them turn out not linearly independent, or even identical, after projection.
- Similar problems were recently discussed in the context of higher bands for electronic FQH states, without succeeding to reveal the underlying mathematical structures. [Balram, Wojs, Jain PRB 2013]
- Ongoing: Systematic study and classification of linear dependence relations for simple states.

Outlook

- More general understanding of redundancies in CF formalism?
- Go beyond homogeneous interaction, study vortex structures, anyonic quasiparticles in QH regime
- Future experiments in this regime (Penn State)??