DYNAMICS AT A QUANTUM CRITICAL POINT: QMC + HOLOGRAPHY

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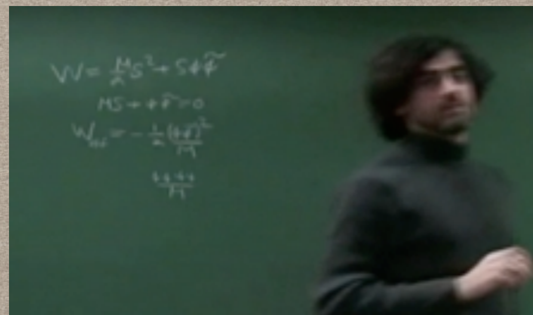
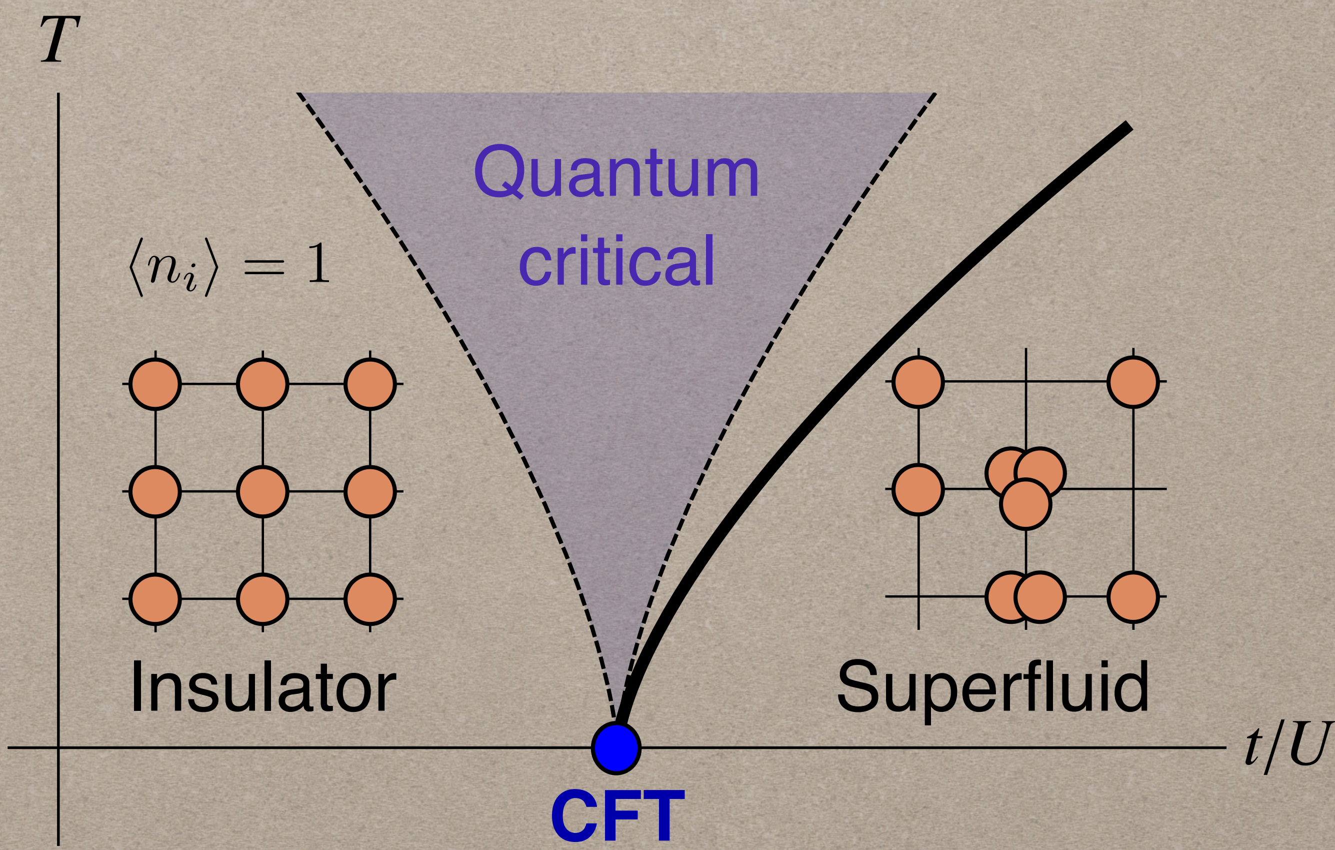
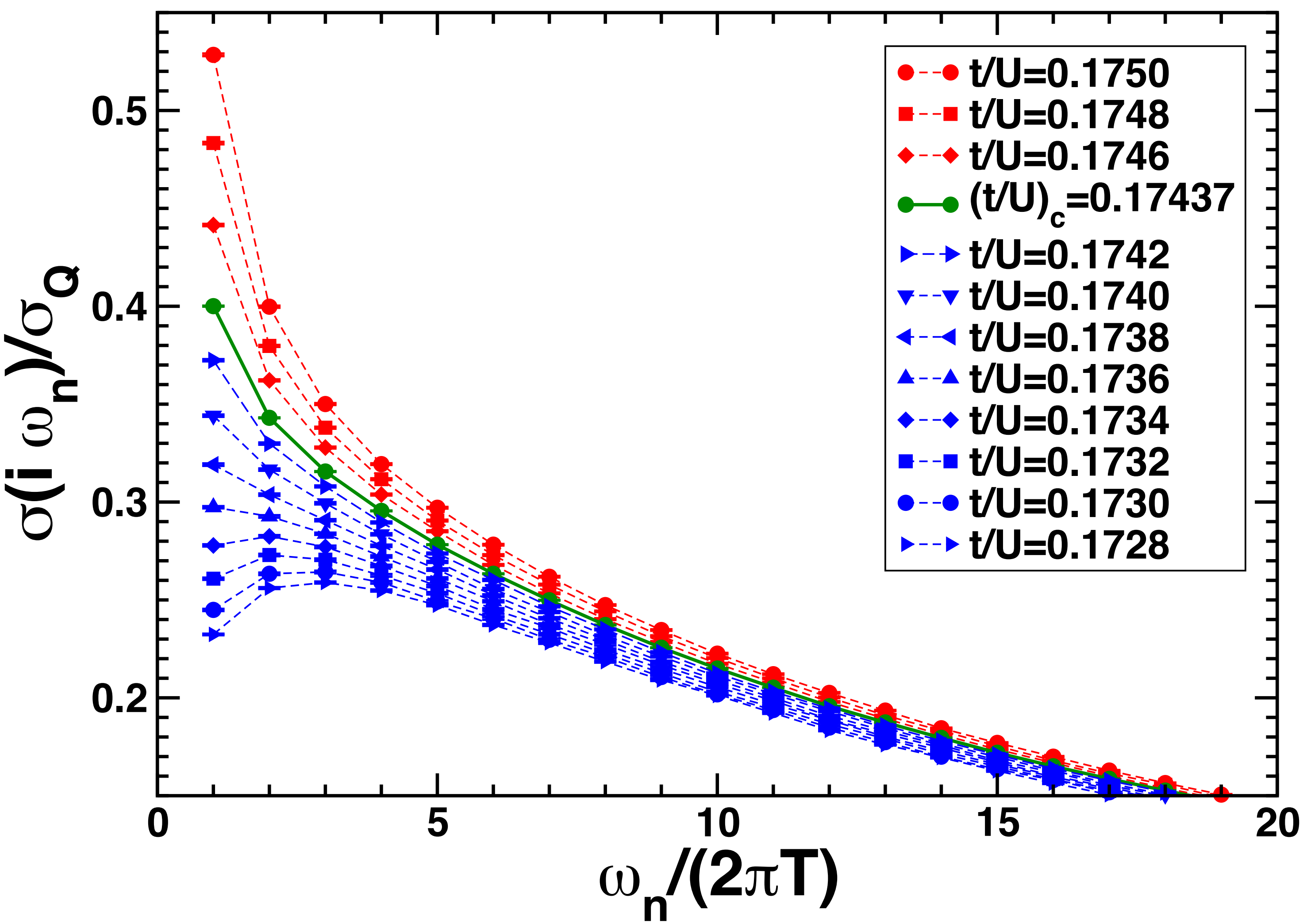


Bose-Hubbard Model in 2+1 D

$$\boxed{H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i}$$



- *Z=1 Emergent Lorentz Symmetry*
- *2+1D O(2) Universality quantum Rotors*



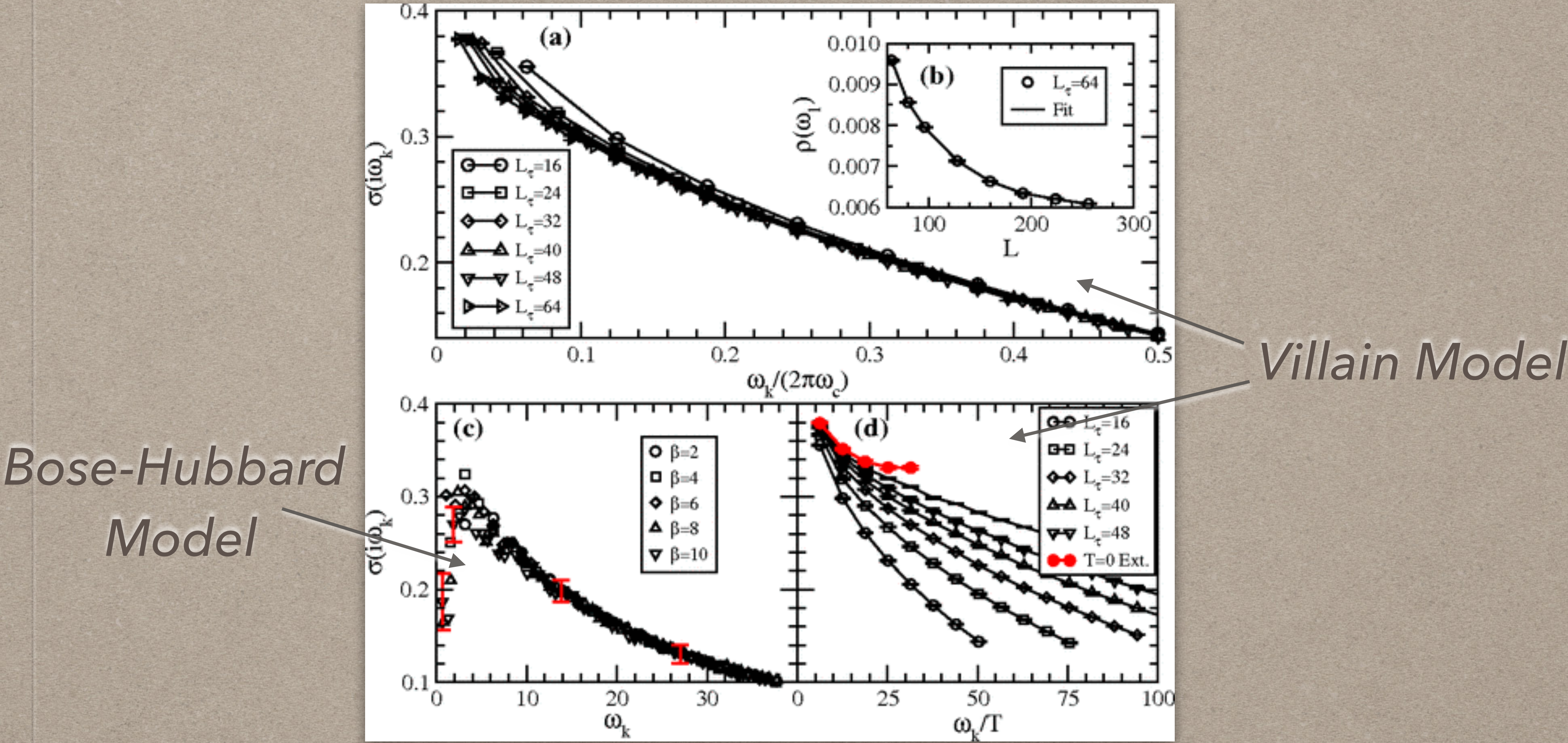
t/U=0.1750
t/U=0.1748
t/U=0.1746
(t/U)c=0.17437
t/U=0.1742
t/U=0.1740
t/U=0.1738
t/U=0.1736
t/U=0.1734
t/U=0.1732
t/U=0.1730
t/U=0.1728
σ(i ωn)/σQ
ωn/(2πT)
0.2
0.3
0.4
0.5
0
5
10
15
20

Conductivity: $T^{(d-2)/z}\sigma(\omega, T)$

Scaling:

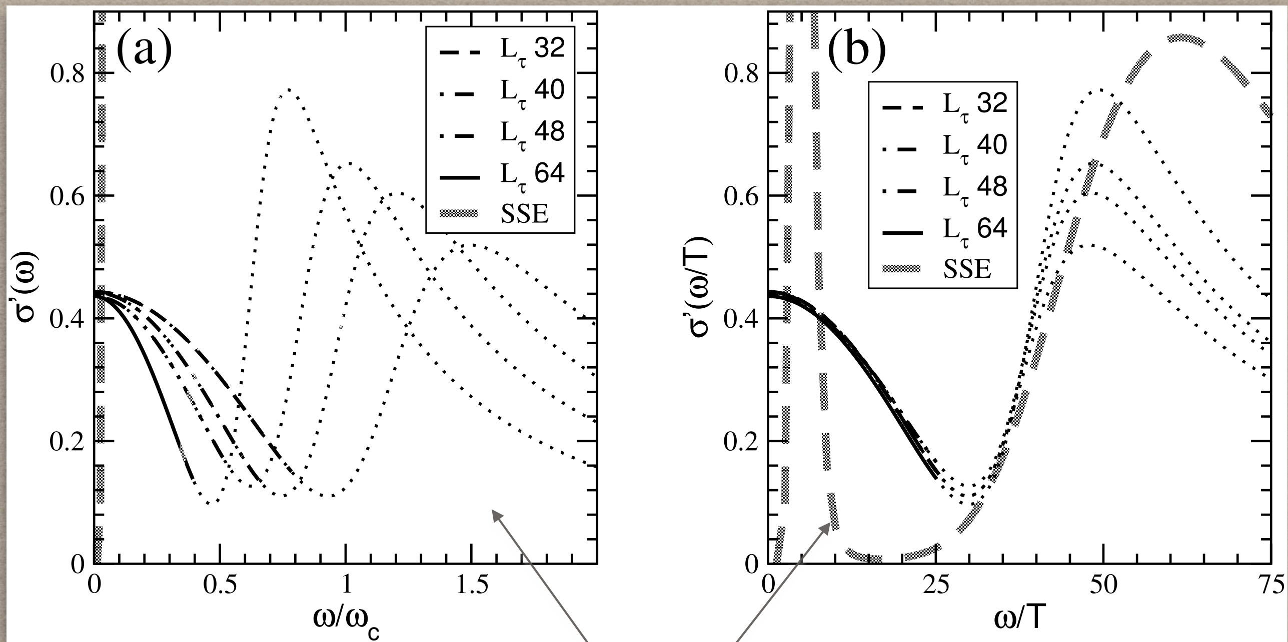
$$\boxed{\sigma(\omega/T, T \to 0) = (k_B T/\hbar c)^{(d-2)/z}\, \sigma_Q\, \Sigma(\hbar\omega/k_B T)}$$

$$\sigma_Q = (e^*)^2/h,$$



J. Smakov, ESS PRL 2005

Analytical Continuation



Max Ent

J. Smakov, ESS PRL 2005

Quantum Rotors

$$H_{\mathrm{qr}} = \frac{U}{2}\sum_{\mathbf{r}} \frac{1}{2}\left(\frac{1}{i}\frac{\partial}{\partial\theta_{\mathbf{r}}}\right)^2 - \mu\sum_{\mathbf{r}}\frac{1}{i}\frac{\partial}{\partial\theta_{\mathbf{r}}} - \sum_{\langle\mathbf{r},\mathbf{r}'\rangle} t\cos(\theta_{\mathbf{r}} - \theta_{\mathbf{r}'}) \ .$$

$$Z_{QR} \approx {\sum_{\{\mathbf{J}\}}}' \exp\left\{-\sum_{(\mathbf{r},\tau)}\left(\Delta\tau U\left(\frac{1}{2}\left[J^{\tau}_{(\mathbf{r},\tau)}\right]^2 - \frac{\mu}{U}J^{\tau}_{(\mathbf{r},\tau)}\right) - \ln\left(I_{J^x_{(\mathbf{r},\tau)}}(t\Delta\tau)\right) - \ln\left(I_{J^y_{(\mathbf{r},\tau)}}(t\Delta\tau)\right)\right)\right\} \ .$$

$$\Delta\tau \to 0$$

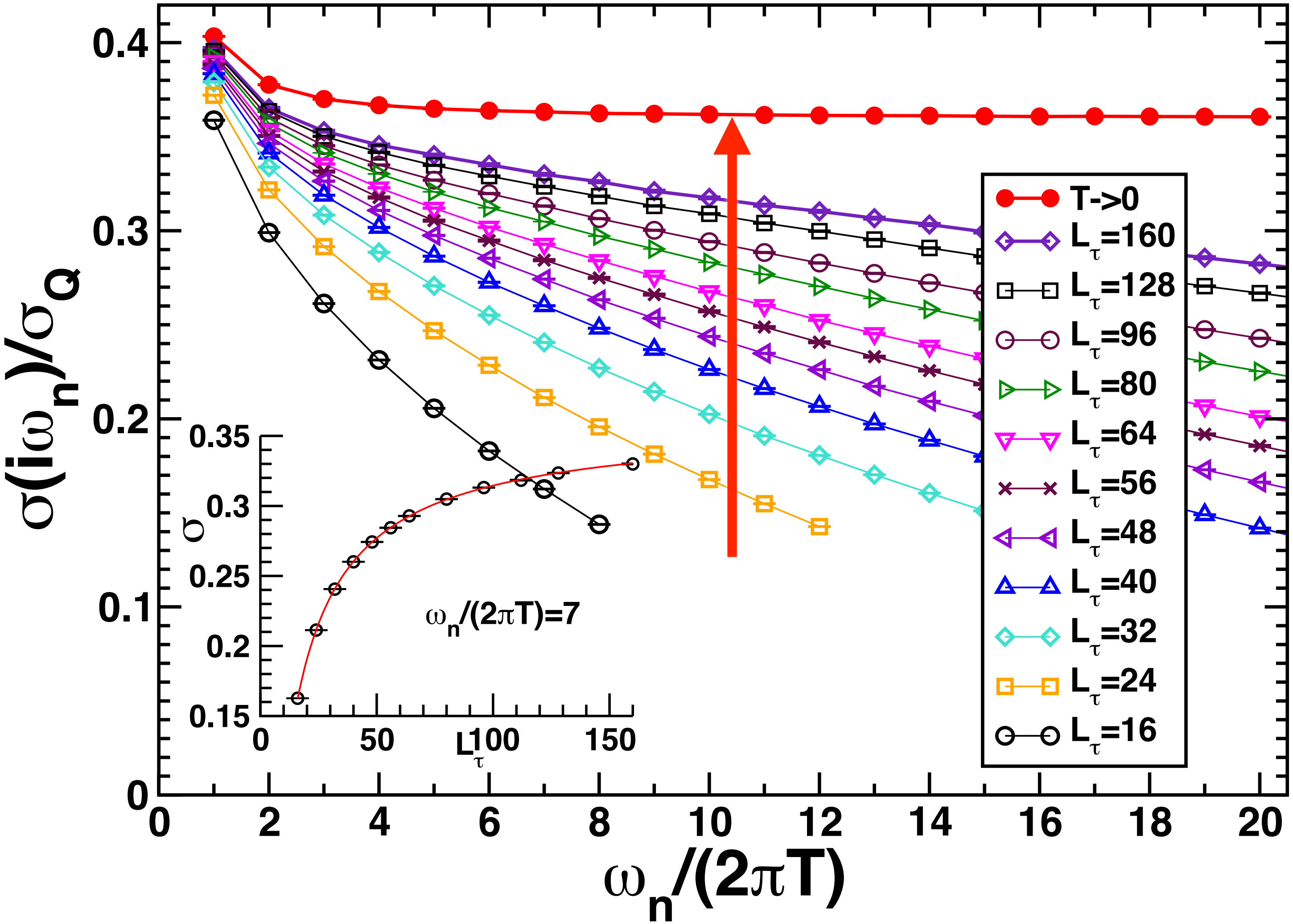
Villain Model

$$H_{\mathrm{V}} = \frac{U}{2}\sum_{\mathbf{r}} \frac{1}{2}\left(\frac{1}{i}\frac{\partial}{\partial\theta_{\mathbf{r}}}\right)^2 - \mu\sum_{\mathbf{r}}\frac{1}{i}\frac{\partial}{\partial\theta_{\mathbf{r}}} - \sum_{\langle\mathbf{r},\mathbf{r}'\rangle} t\sum_{m}(\theta_{\mathbf{r}} - \theta_{\mathbf{r}'} - 2\pi m)^2 \ .$$

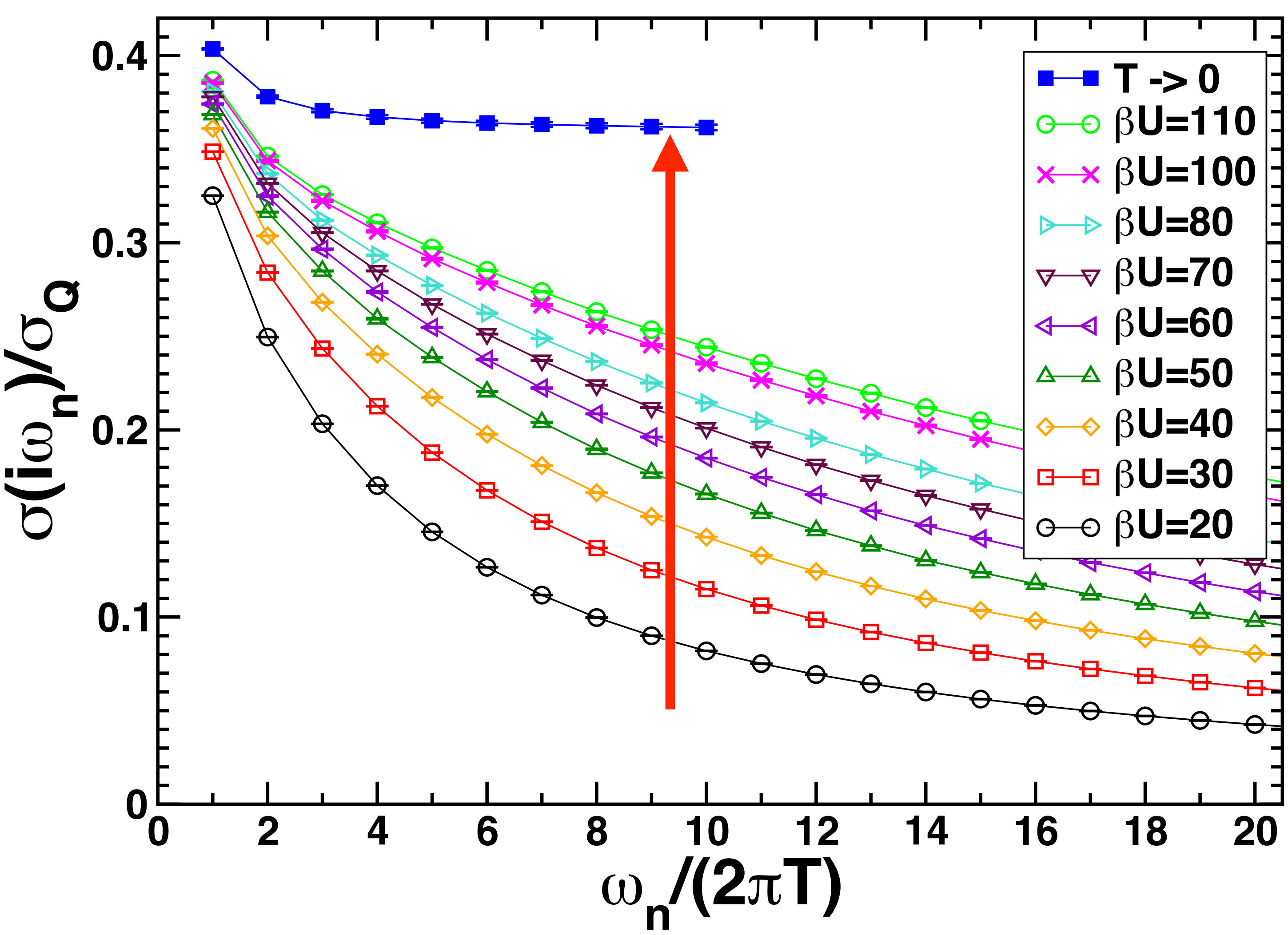
$$Z_V \approx {\sum_{\{\mathbf{J}\}}}' \exp\left\{-\frac{1}{K}\sum_{(\mathbf{r},\tau)}\left(\frac{1}{2}\left(\left[J^{x}_{(\mathbf{r},\tau)}\right]^2 + \left[J^{y}_{(\mathbf{r},\tau)}\right]^2 + \left[J^{\tau}_{(\mathbf{r},\tau)}\right]^2\right) - \frac{\mu}{U}J^{\tau}_{(\mathbf{r},\tau)}\right)\right\} \ .$$

$\Delta\tau = \frac{1}{\sqrt{Ut}}$ *Fixed*

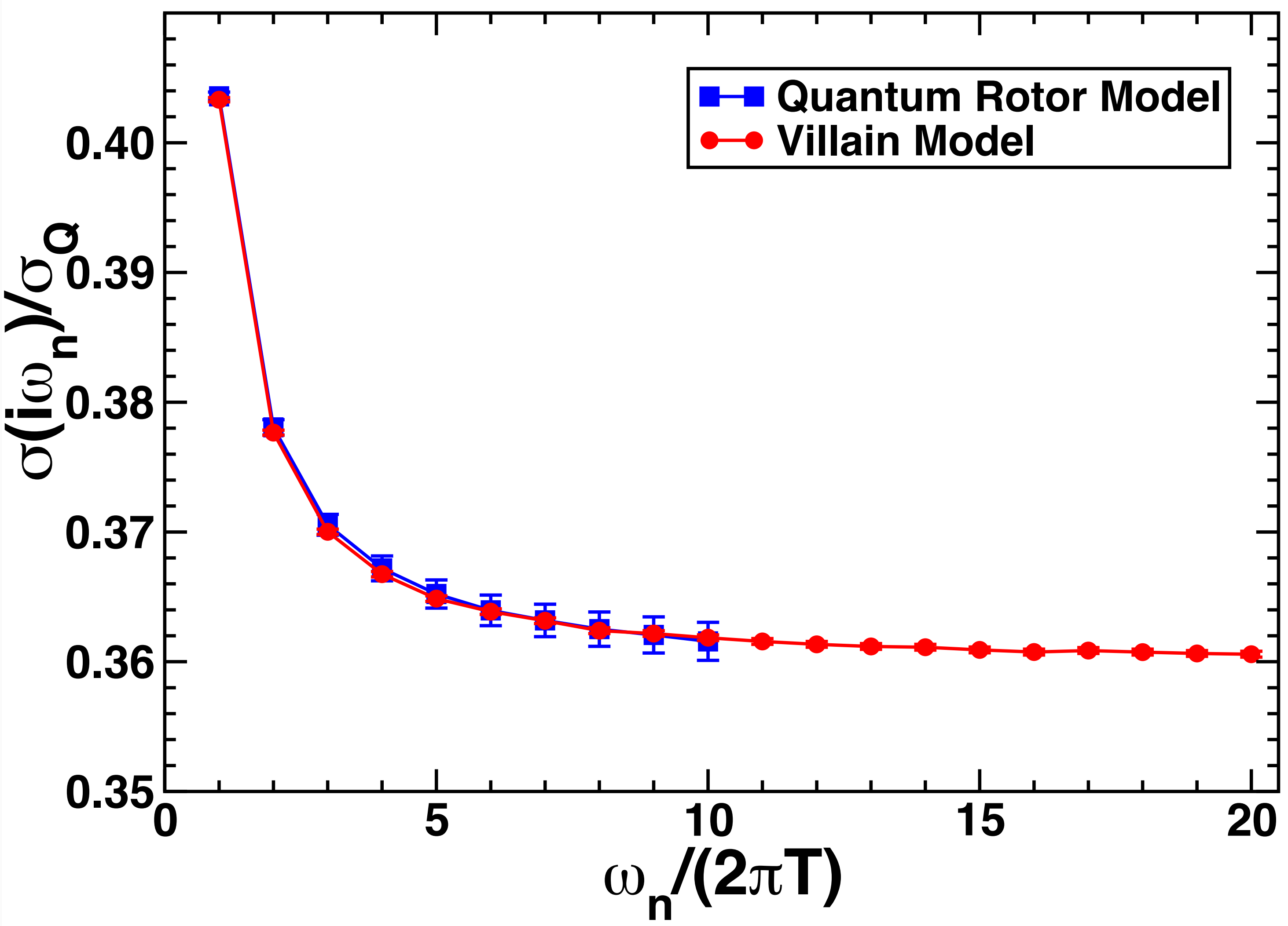
Villain Model



Quantum Rotors



$T \to 0$ *Extrapolated Results*



W. Witczak-Krempa, ESS, S. Sachdev Nat Phys 2014

OPE

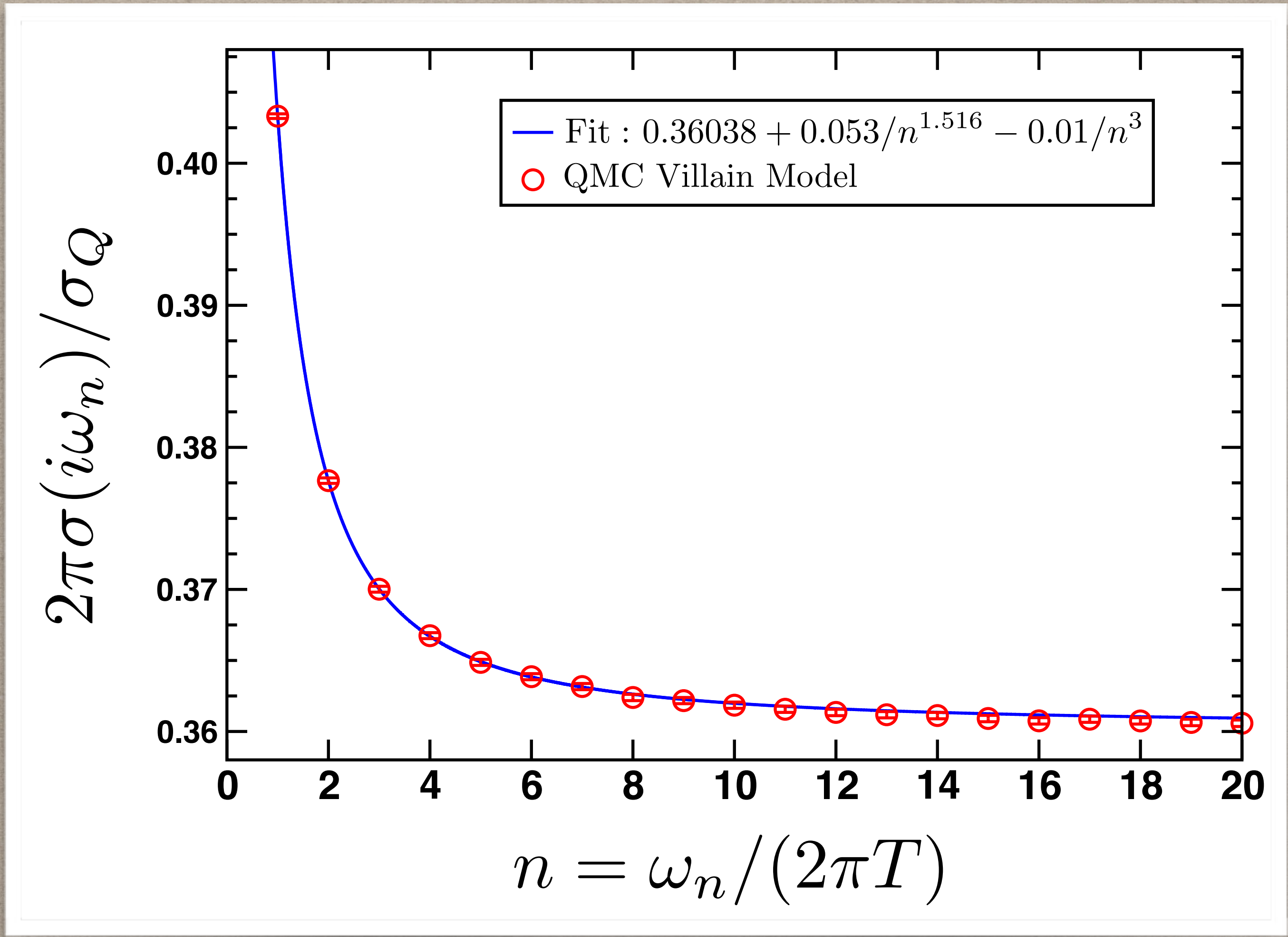
$$\lim_{|\omega_n|\gg p} J_x(\omega)J_x(-\omega+p) = -|\omega_n|\,\sigma_\infty\,\delta^{(3)}(p) - \frac{\mathcal{C}}{|\omega_n|^{\Delta-1}}\mathcal{O}(p)$$
$$+\frac{\mathcal{C}_T}{\omega_n^2}\Big[T_{xx}(p) - T_{yy}(p) - 12\gamma(T_{xx}(p)+T_{yy}(p))\Big] + \cdots$$

$$\Downarrow$$

$$\frac{\sigma(i\omega_n)}{\sigma_Q} = \sigma_\infty + b_1\left(\frac{T}{\omega_n}\right)^{\Delta} + b_2\left(\frac{T}{\omega_n}\right)^{3} + \cdots\,, \qquad \omega_n \gg T$$

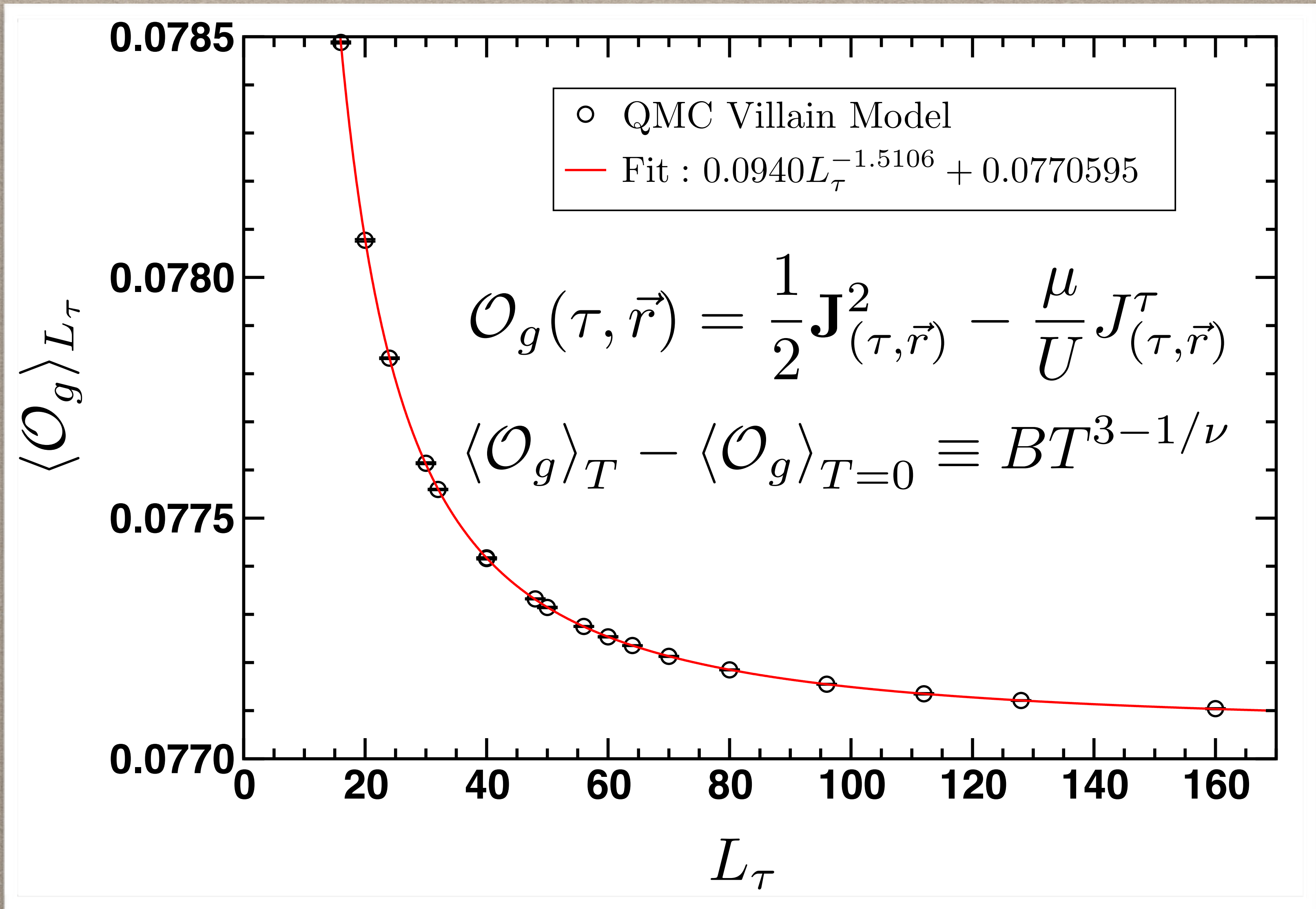
Thermal Operator, $\mathcal{O}_g$ *scaling dimension* $\Delta = 3 - 1/\nu$

E.Katz, S. Sachdev, ESS, W. Witczak-Krempa Arxiv (2014)



$$\sigma_\infty/\sigma_Q = 0.36038... \qquad \nu = 0.674$$

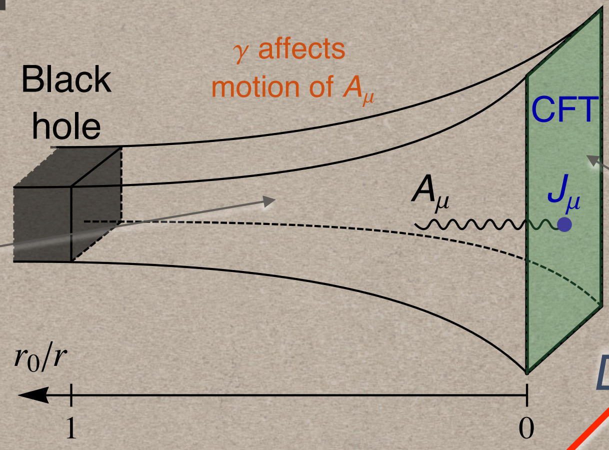
EXISTENCE OF THERMAL OPERATOR



$$\nu = 0.6714(10)$$

HOLOGRAPHY

$$r_0 = T(4\pi L^3/3)$$



AdS_4

CFT_3

Deviations from S-duality

$$S_{\rm bulk} = \int d^4x\sqrt{-g}\left[\frac{1}{\kappa^2}\left(R+\frac{6}{L^2}\right) - \frac{1}{4g_4^2}F_{ab}F^{ab} + \gamma\frac{L^2}{g_4^2}C_{abcd}F^{ab}F^{cd}\right] \qquad |\gamma| \le \frac{1}{12}$$

$8\pi G$

R, Riemann curv., L Radius of Curvature of AdS4, C Weyl curvature tensor

$$\sigma\left(\frac{\omega}{T}\to\infty\right) = 1/g_4^2$$

$$\sigma(w) = -\frac{i}{3w}\frac{\partial_u A_y}{A_y}\bigg|_{u=0}$$

2 Parameter fit in σ(∞) and γ

$$A_y'' + \left(\frac{f'}{f}+\frac{g'}{g}\right)A_y' + \frac{9w^2}{f^2}A_y = 0$$

$$f(u) = 1-u^3, \quad g(u) = 1+4\gamma u^3, \quad u = r_0/r$$

Modified Maxwell equation

R. Myers, S. Sachdev, A. Singh, PRD (2011)

W. Witczak-Krempa, S. Sachdev , PRB (2013)

REFINED ANALYSIS: INCLUDE SCALAR FIELD

$$S = \int d^4x\sqrt{-g_{\mathrm{Sch}}}\left[\frac{1}{\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4g_4^2}[1+\alpha\phi(u)]F_{ab}F^{ab}\right]$$

$8\pi G$

$$\sigma\left(\frac{\omega}{T}\to\infty\right) = 1/g_4^2$$

$$b_1 = \sigma_\infty\ \alpha\, a\, \frac{\Gamma(\Delta+1)}{2^\Delta}\left(\frac{4\pi}{3}\right)^\Delta$$

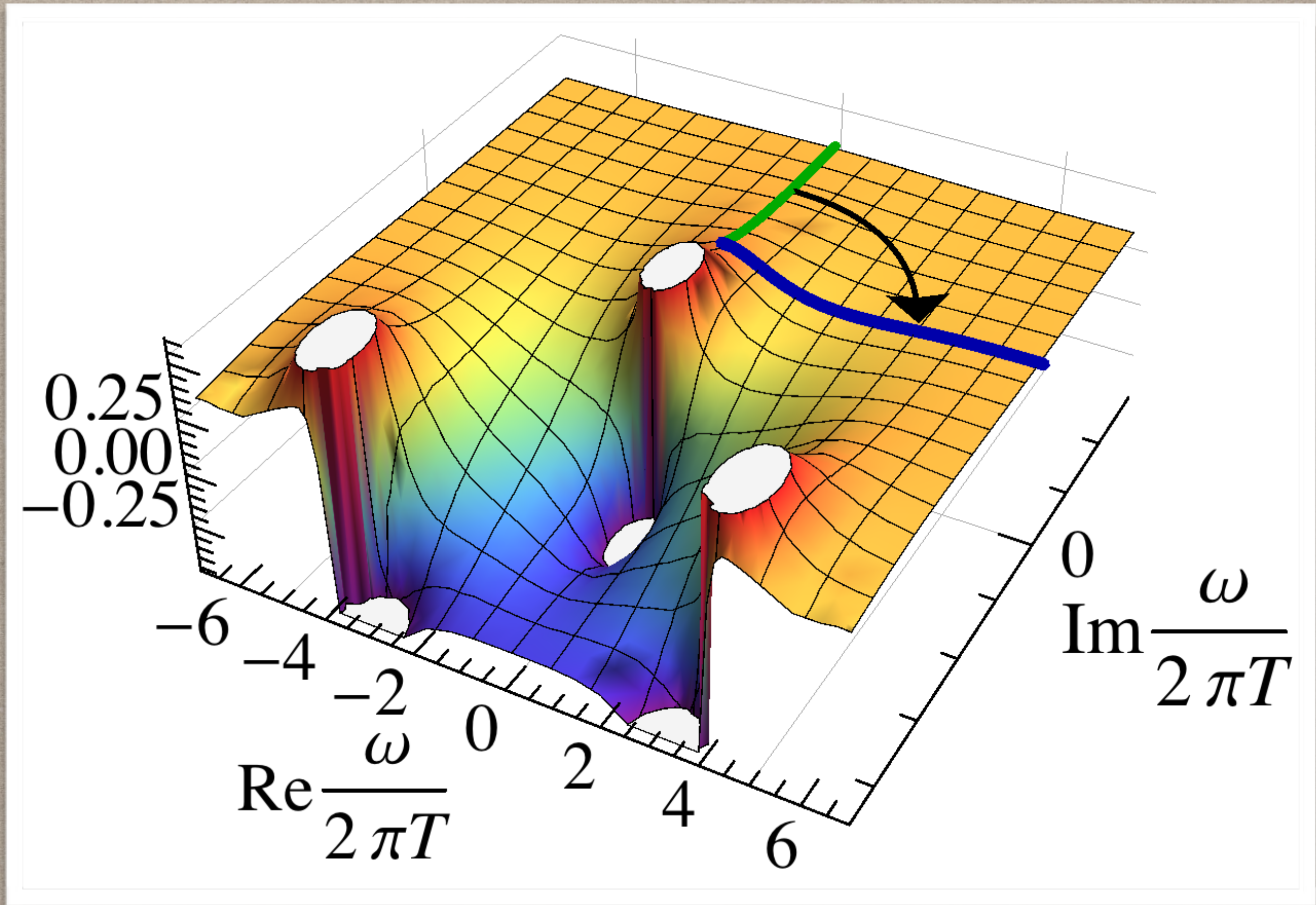
$$\Delta_g = 3 - 1/\nu$$

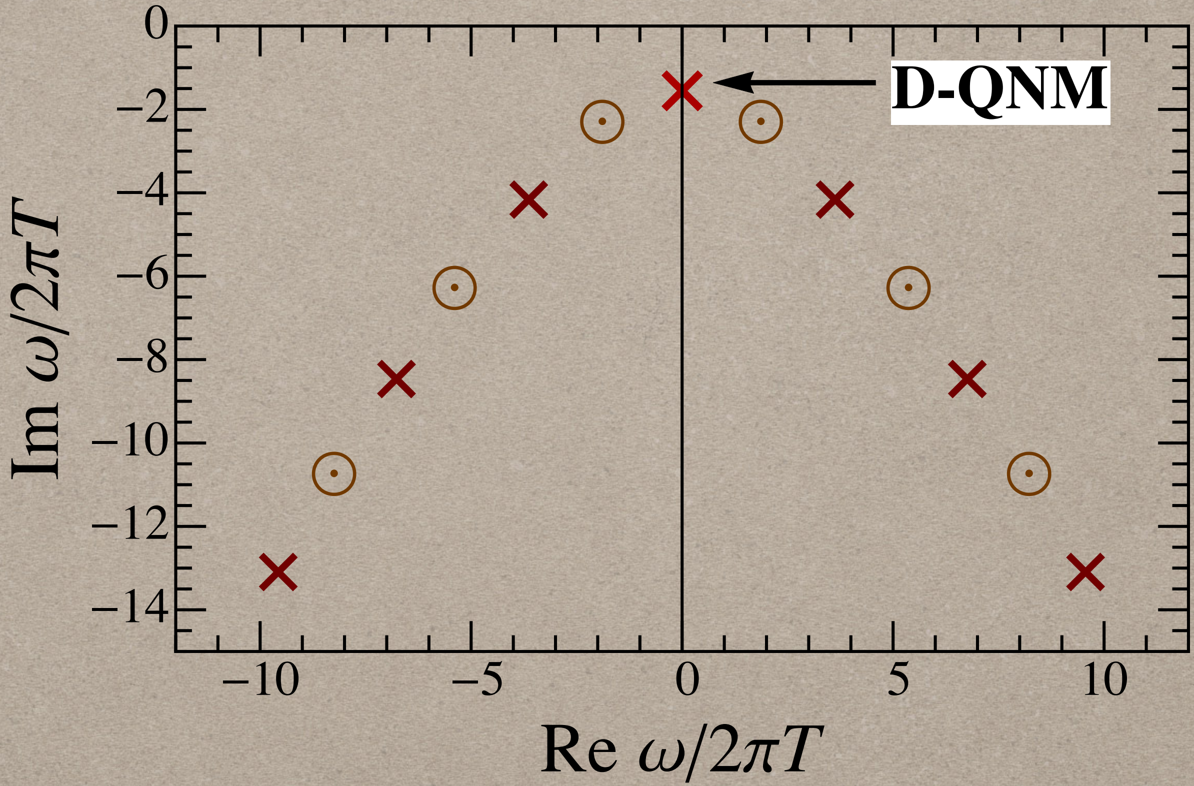
$$\frac{\sigma(i\omega_n)}{\sigma_Q} = \sigma_\infty + b_1\left(\frac{T}{\omega_n}\right)^\Delta + \cdots$$

$$((1+\alpha\phi)fA_y')' - w_n^2\frac{(1+\alpha\phi)}{f}A_y = 0 \quad ; \quad w_n \equiv \frac{3\omega_n}{4\pi T}$$

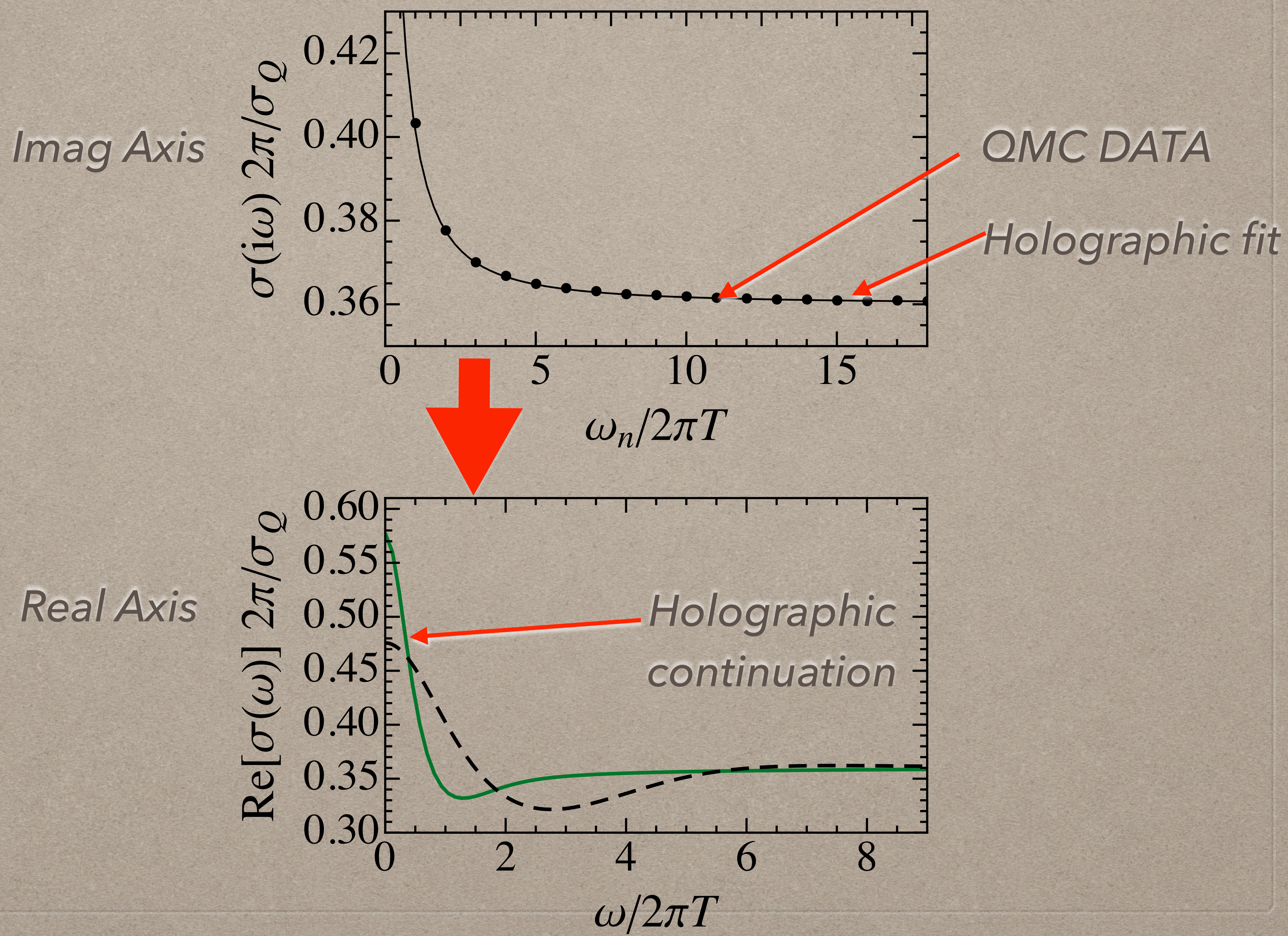
2 Parameter fit in σ(∞) and b_1

HOLOGRAPHIC CONDUCTIVITY IN THE COMPLEX PLANE





D-QNM
0
−2
−4
−6
−8
−10
−12
−14
Im ω/2πT
−10
−5
0
5
10
Re ω/2πT



Imag Axis
QMC DATA
Holographic fit
0.42
0.40
0.38
0.36
0
5
10
15
$\sigma(i\omega)\ 2\pi/\sigma_Q$
$\omega_n/2\pi T$
Real Axis
Holographic
continuation
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0
2
4
6
8
$\mathrm{Re}[\sigma(\omega)]\ 2\pi/\sigma_Q$
$\omega/2\pi T$

CONCLUSIONS

- *Particle like excitations*
- *Sum rules*

$$\boxed{\int_0^\infty d\omega\,[\mathrm{Re}\,\sigma(\omega/T) - \sigma(\infty)] = 0}$$

- *New QMC results for Universal $\sigma(i\omega/T)$ using 2 different models*
- *AdS/CFT continuation*