Studying "deconfined quantum criticality" using classical dimers

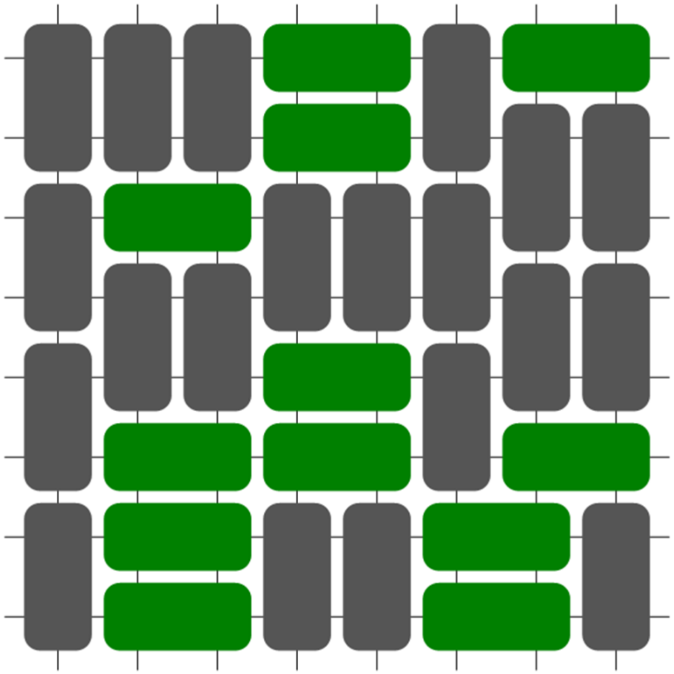
Stephen Powell

University of Nottingham

Outline

- Dimer models
  - Ordering/confinement transitions
  - Effective gauge theory
  - Critical theory for ordering transition
- Monomers at the confinement transition
  - Theory: RG flows
  - Numerical results
- Deconfined criticality
  - Higher-charge monopoles

Dimer models



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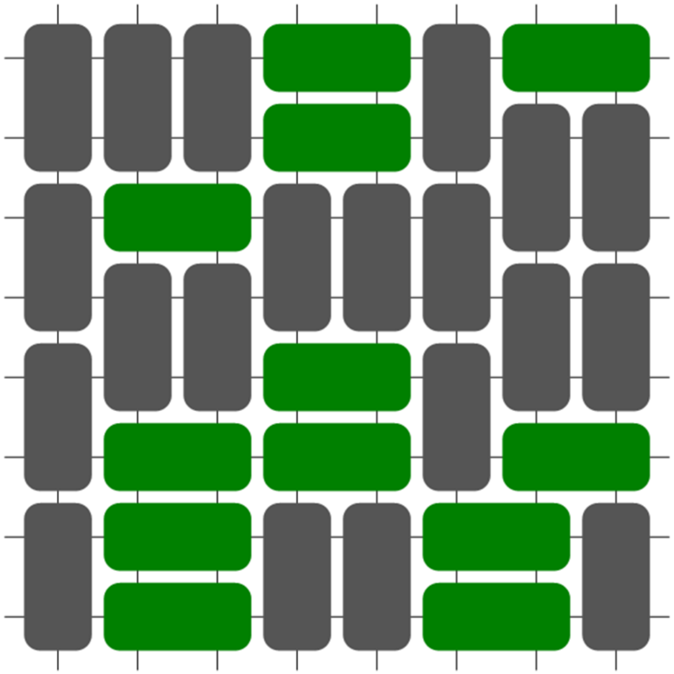
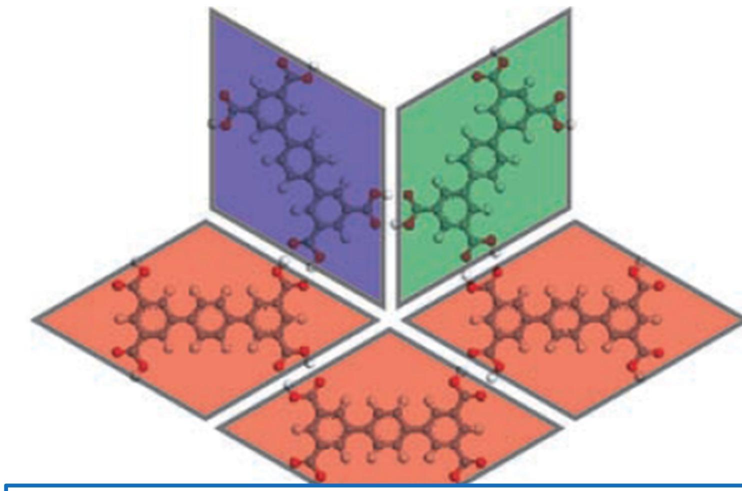
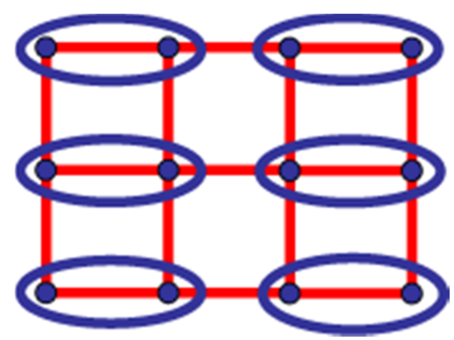




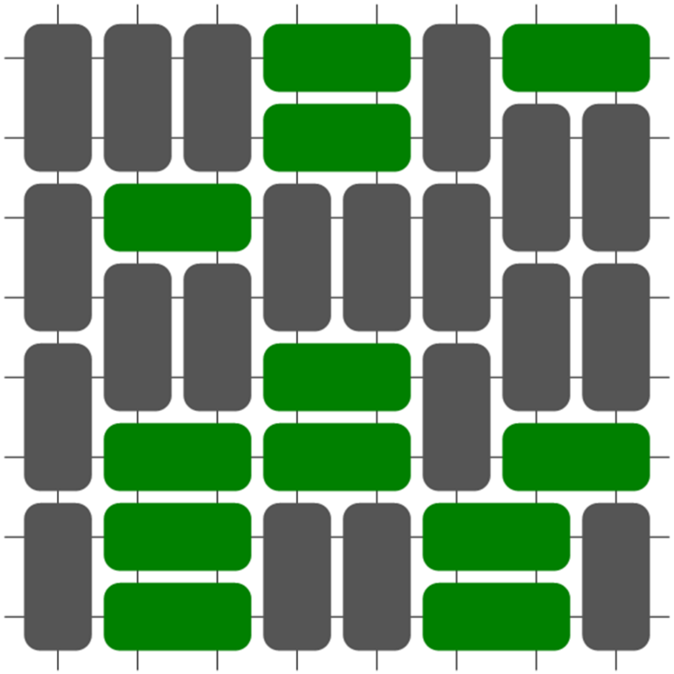
Blunt et al., Science **322**, 1077 (2008)



 $= \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$

Pauling, Anderson, ...

Dimer models



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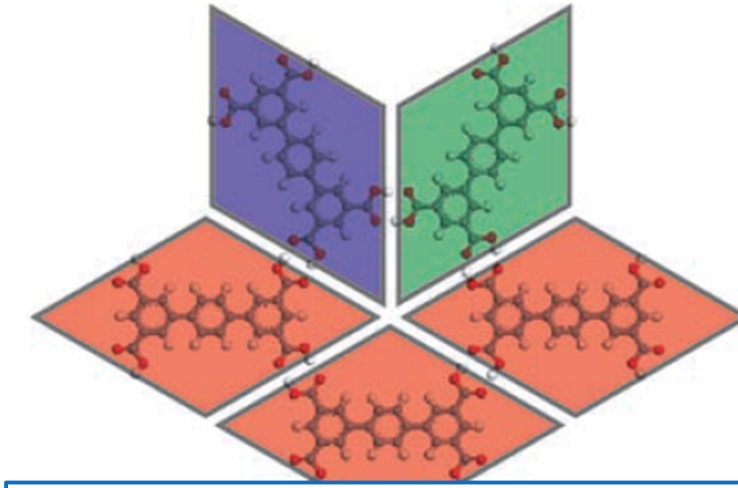
Kasteleyn, Physica **27**, 1209 (1961)



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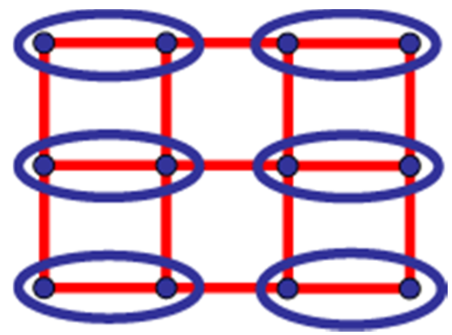

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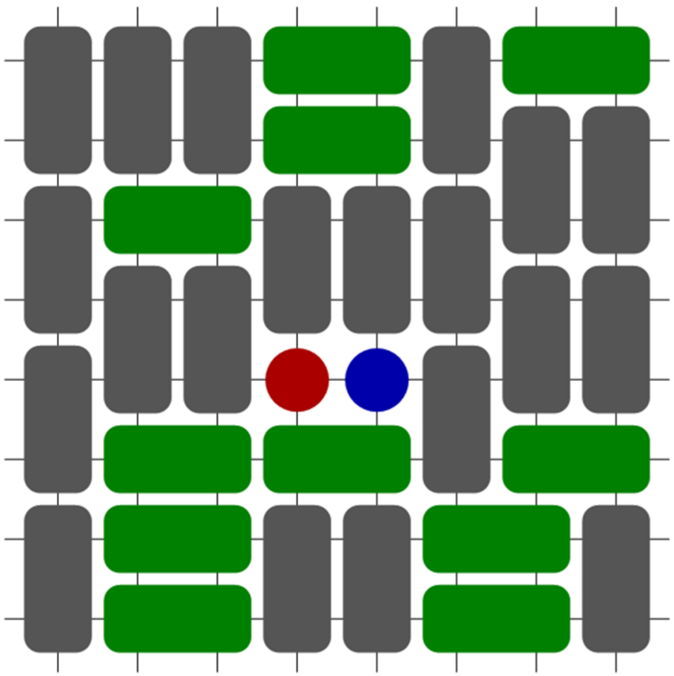

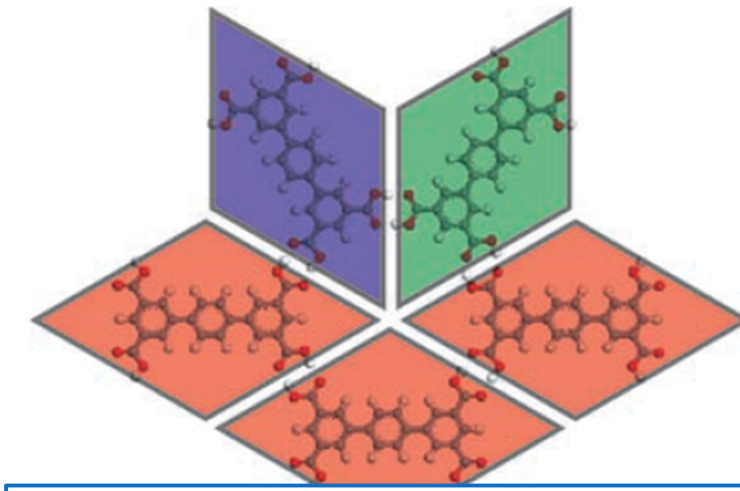

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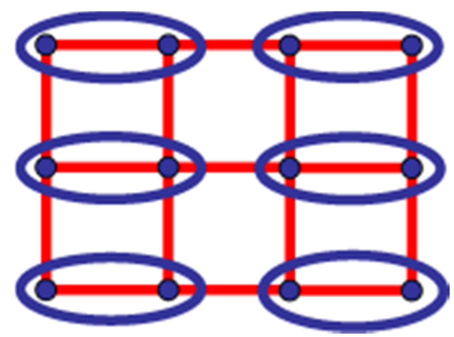

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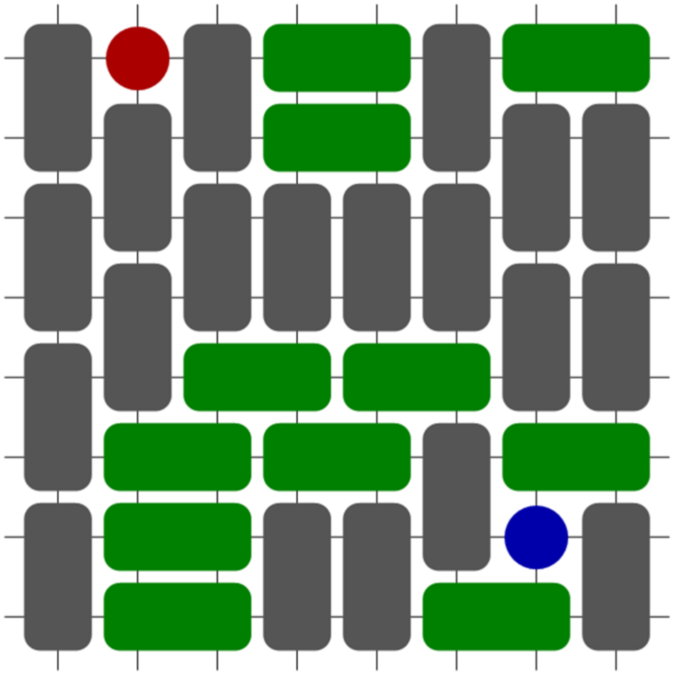

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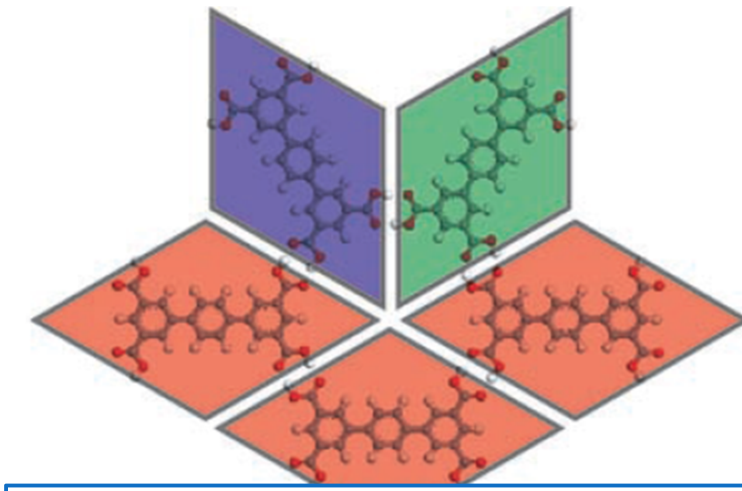

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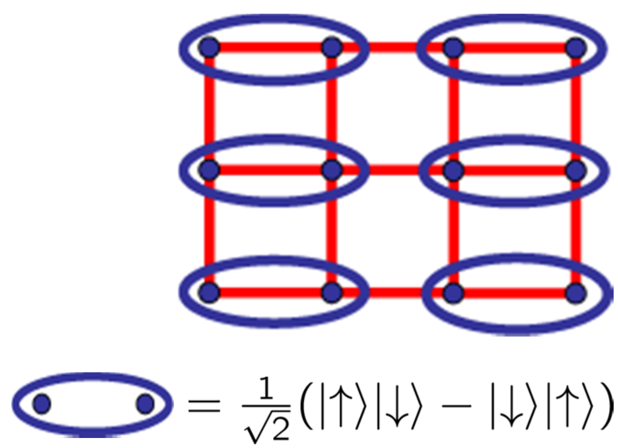

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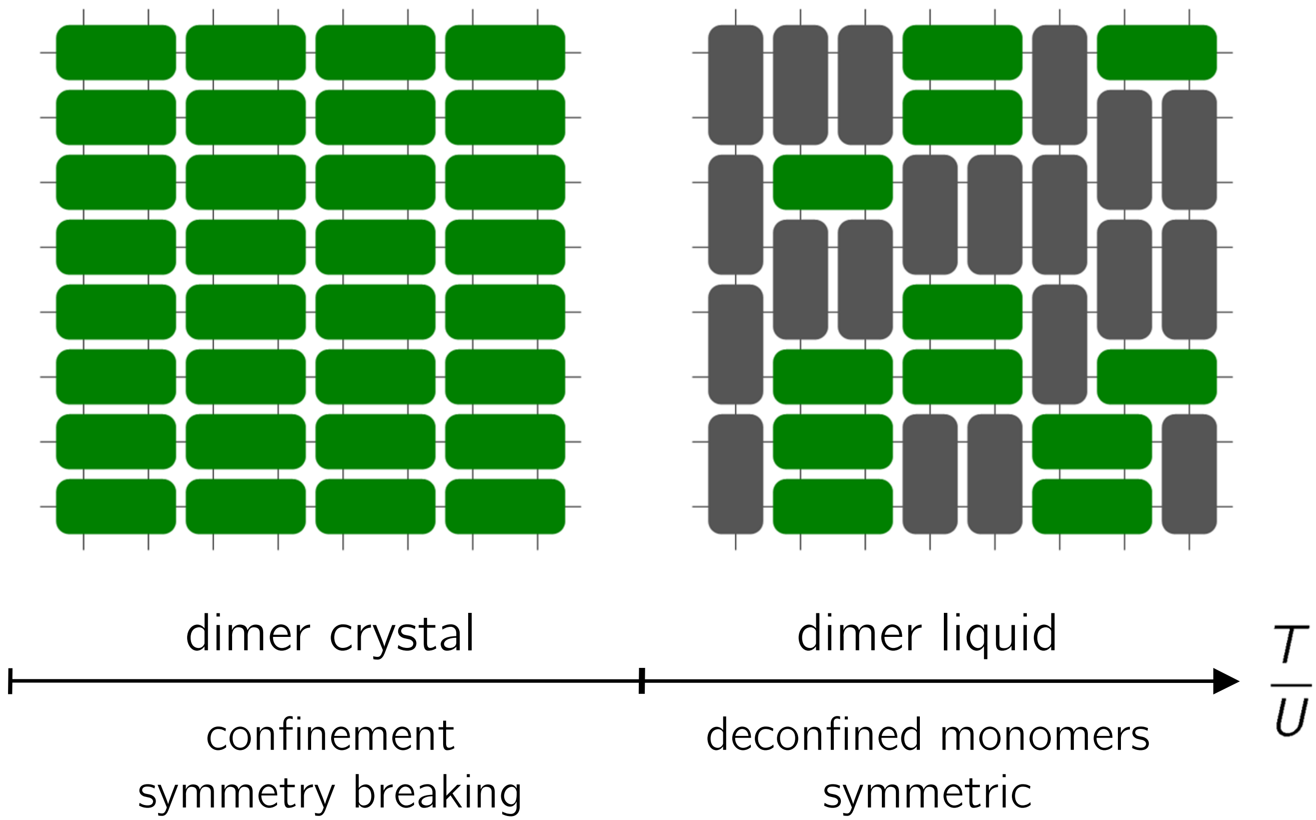
Dimer ordering transition

Interaction between dimers: $E = -UN_{\parallel}$



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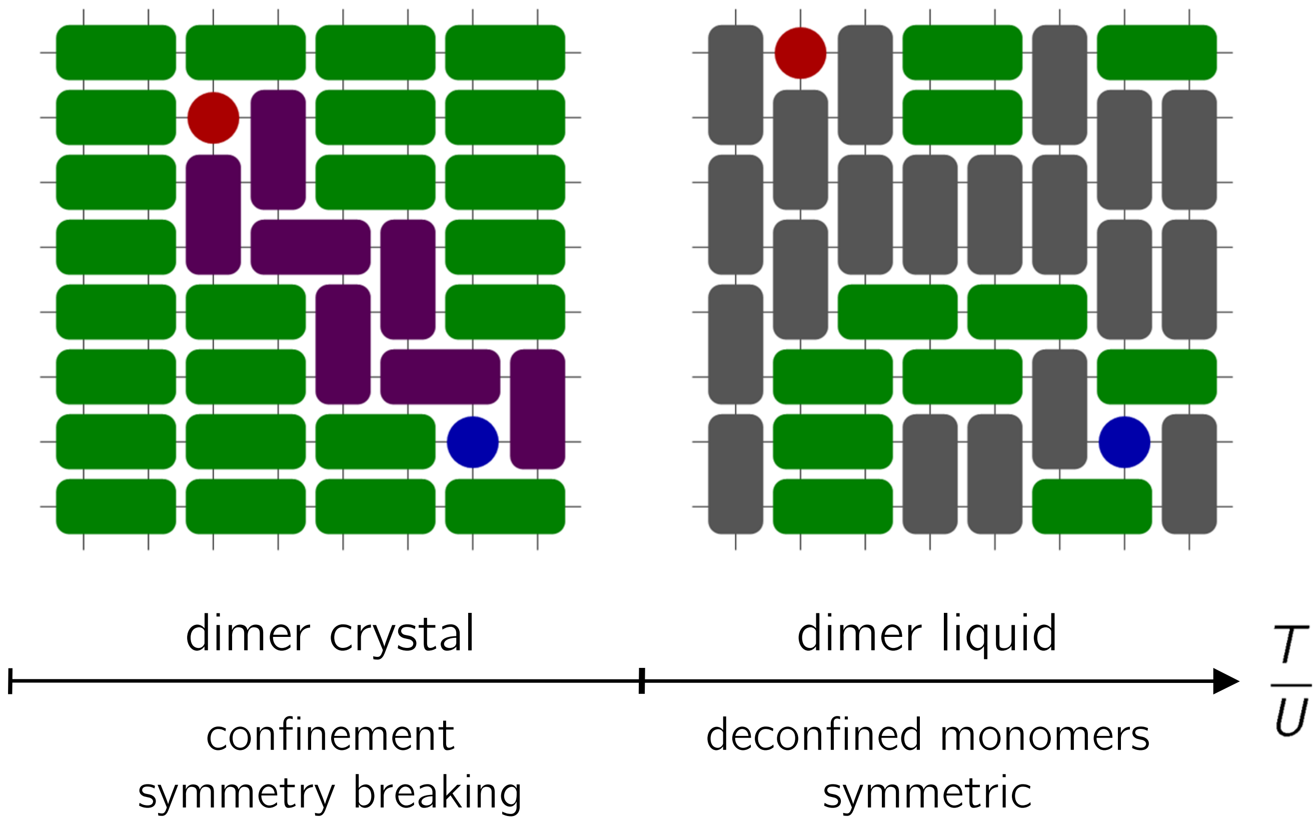

Cubic dimer model

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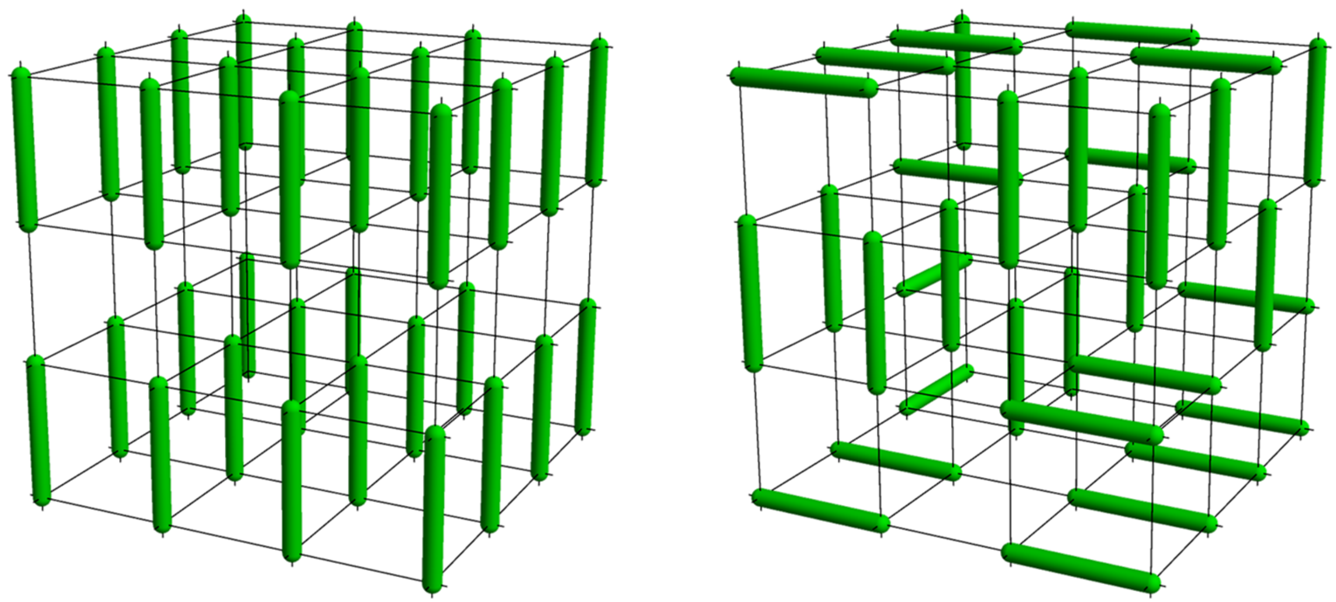

dimer crystal | dimer liquid → $\frac{T}{U}$

confinement
symmetry breaking

deconfined monomers
symmetric

Alet et al., PRL **97**, 030403 ('06)

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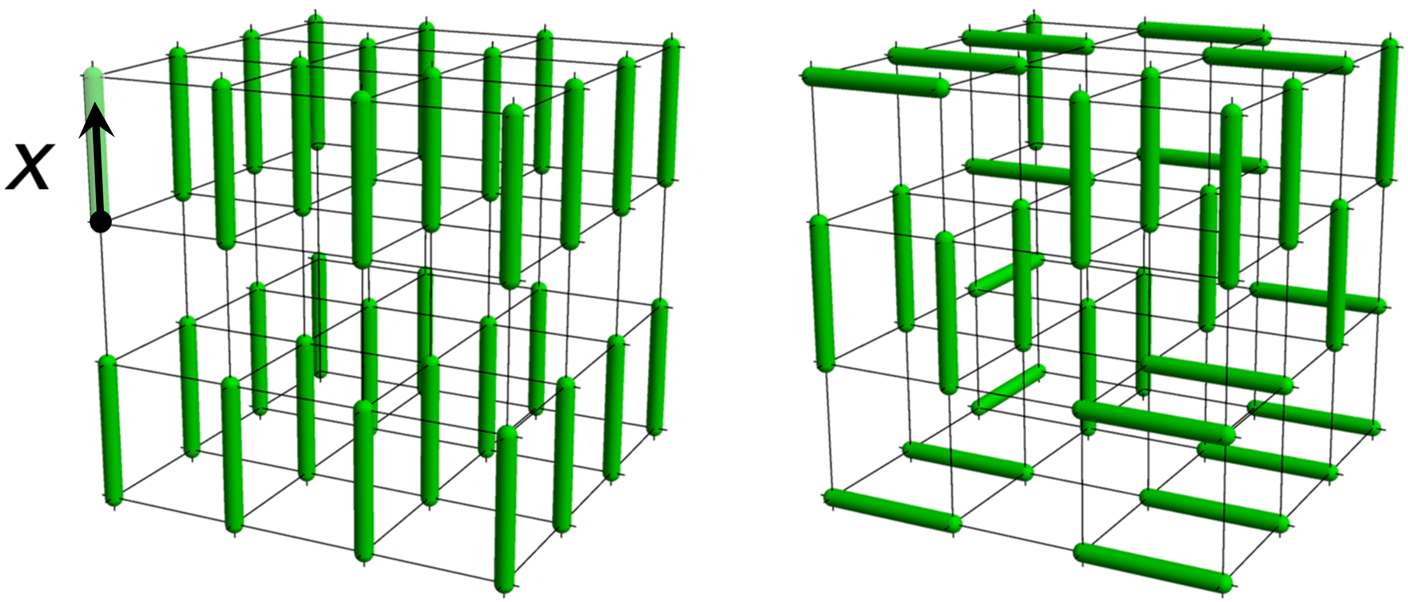

dimer crystal	dimer liquid	$\frac{T}{U}$
confinement symmetry breaking $\langle \boldsymbol{X} \rangle \neq \mathbf{0}$	deconfined monomers symmetric $\langle \boldsymbol{X} \rangle = \mathbf{0}$	

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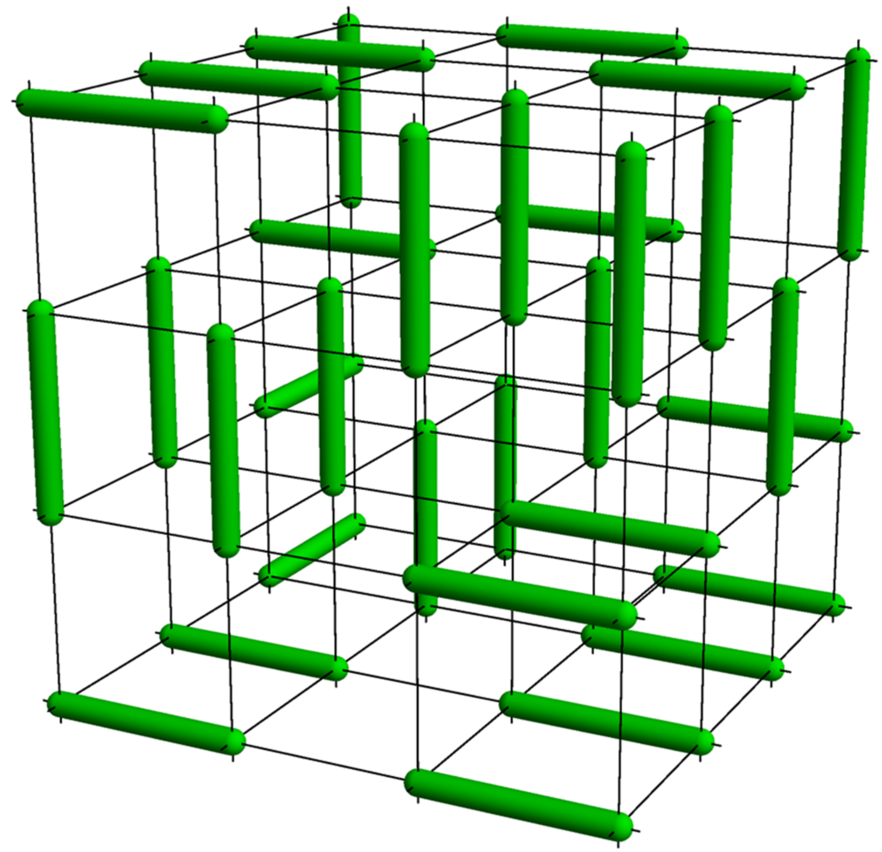
Effective gauge theory

Constraint: one dimer touches each site



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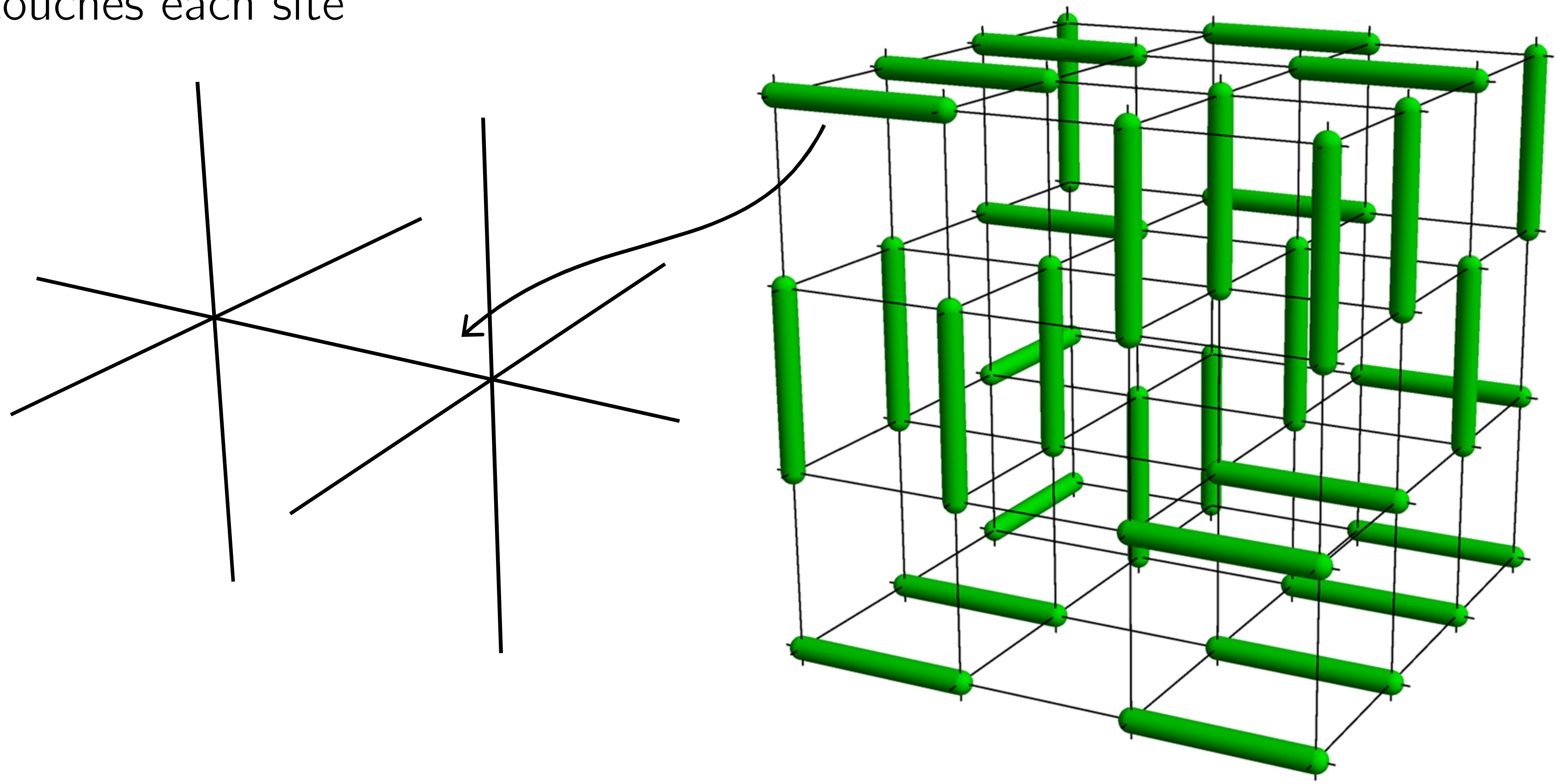

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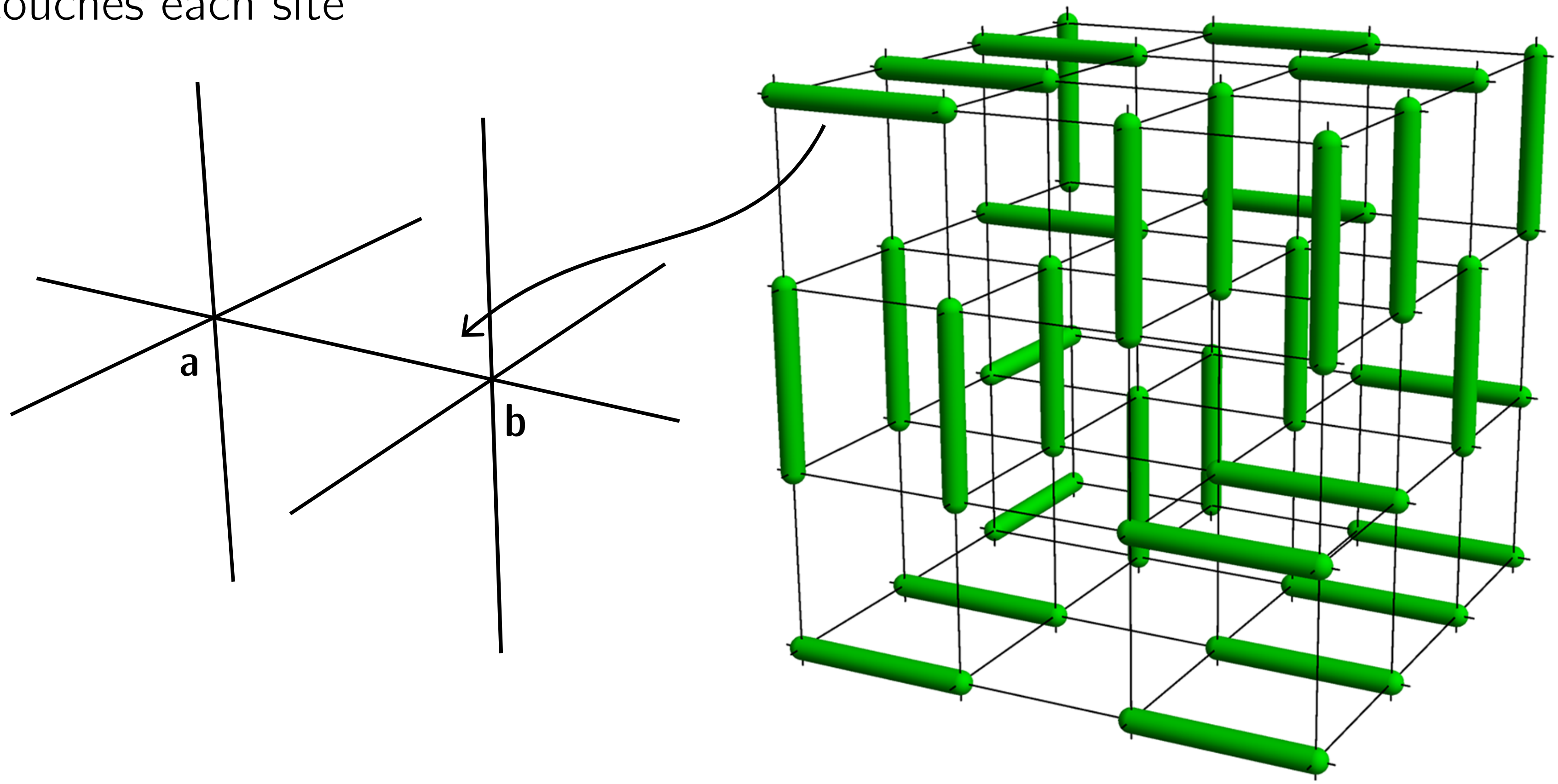

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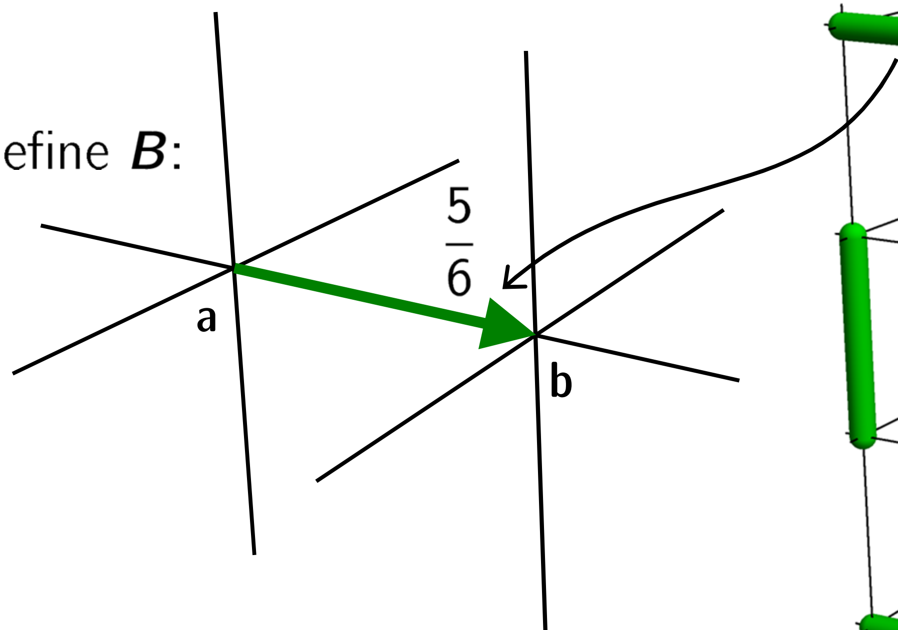

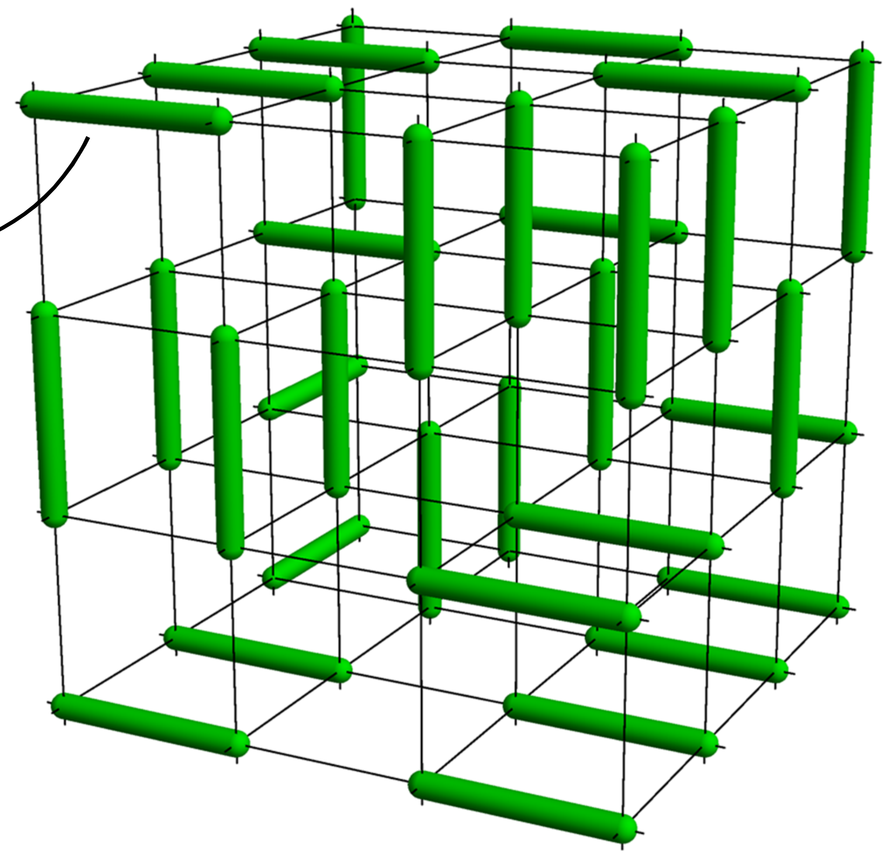

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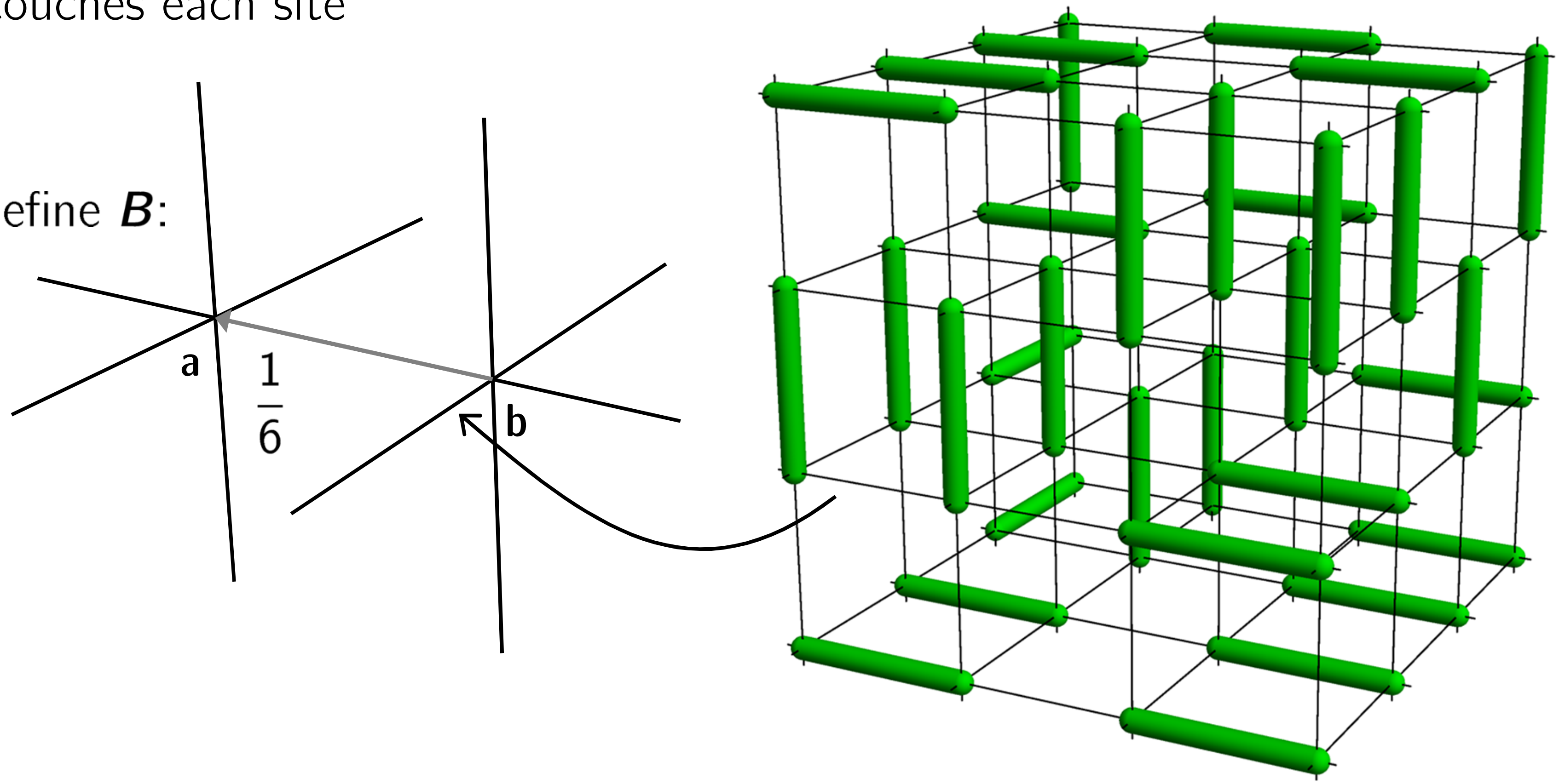

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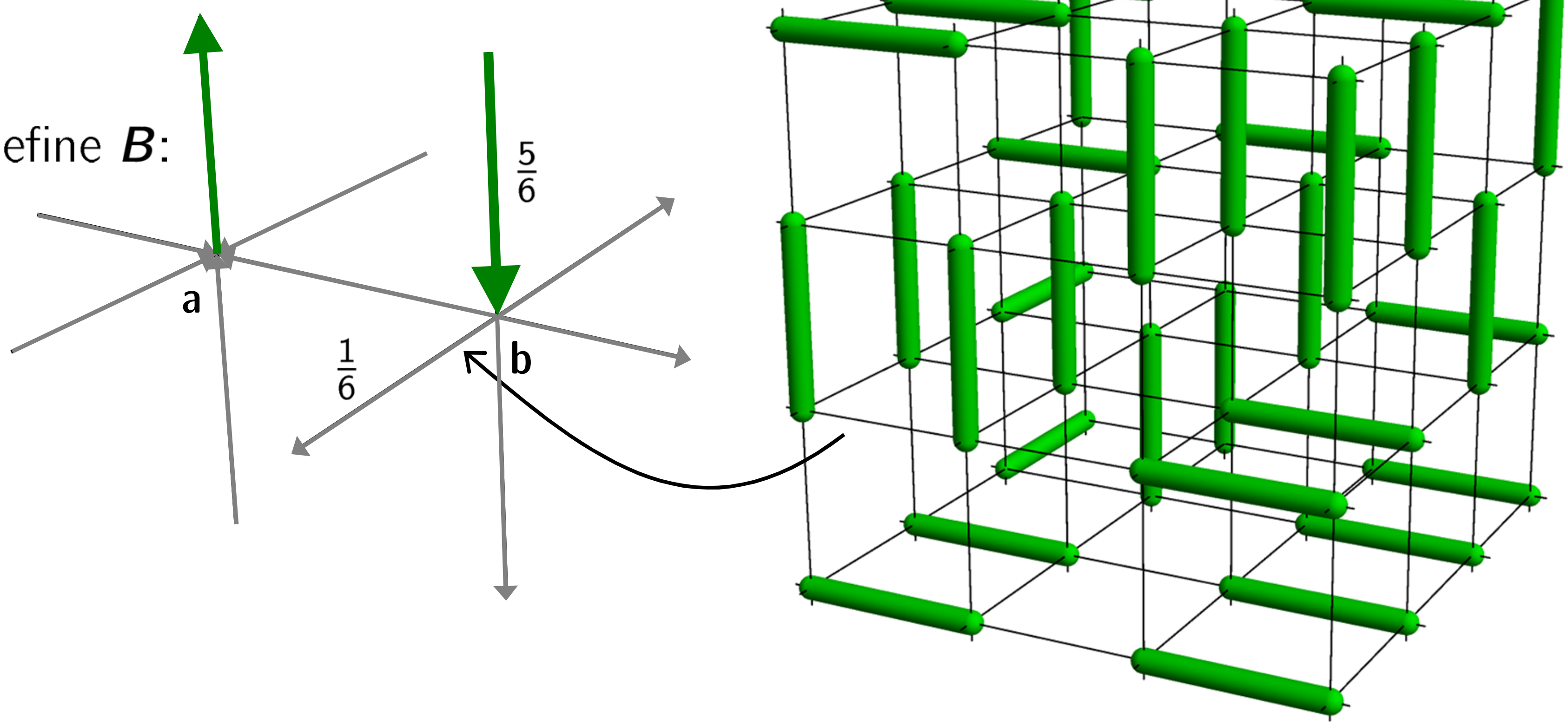

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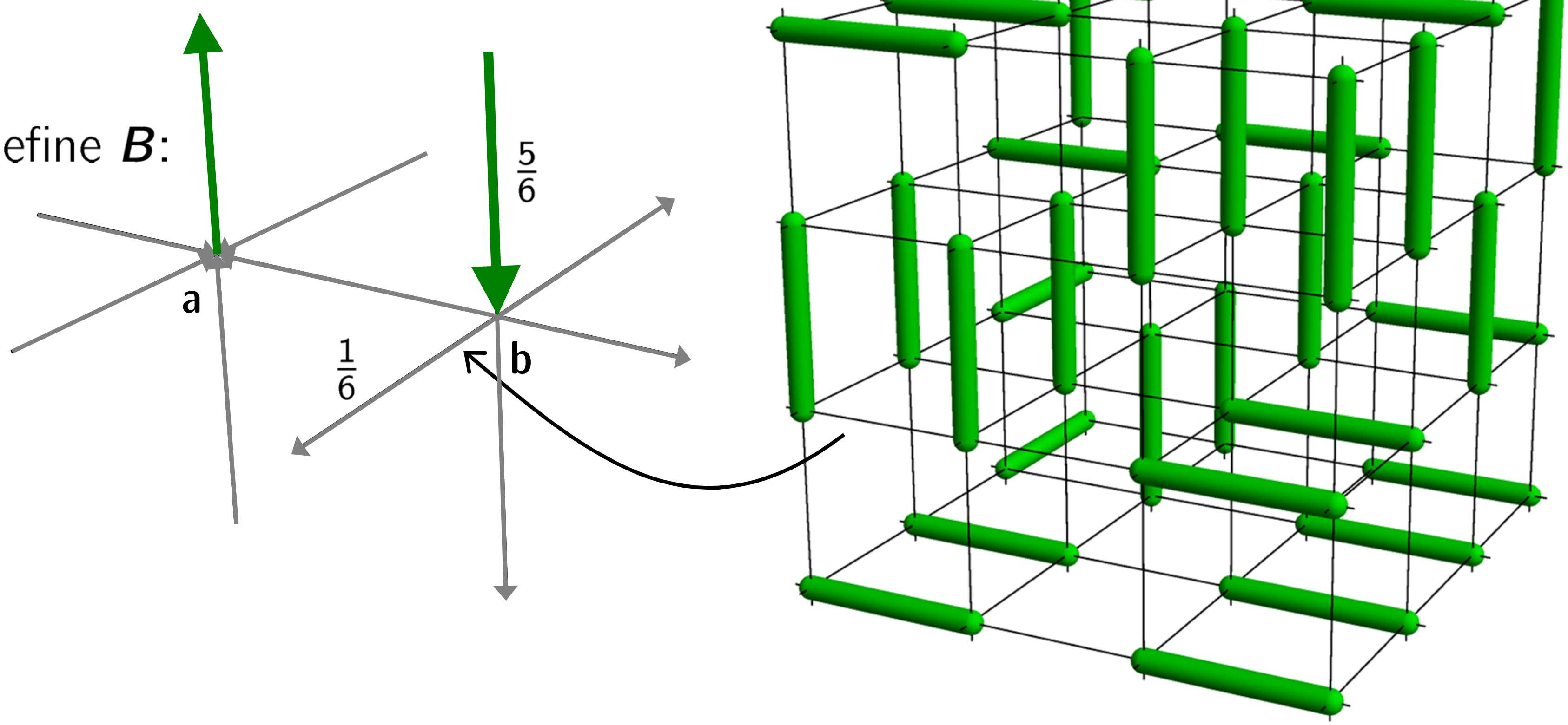

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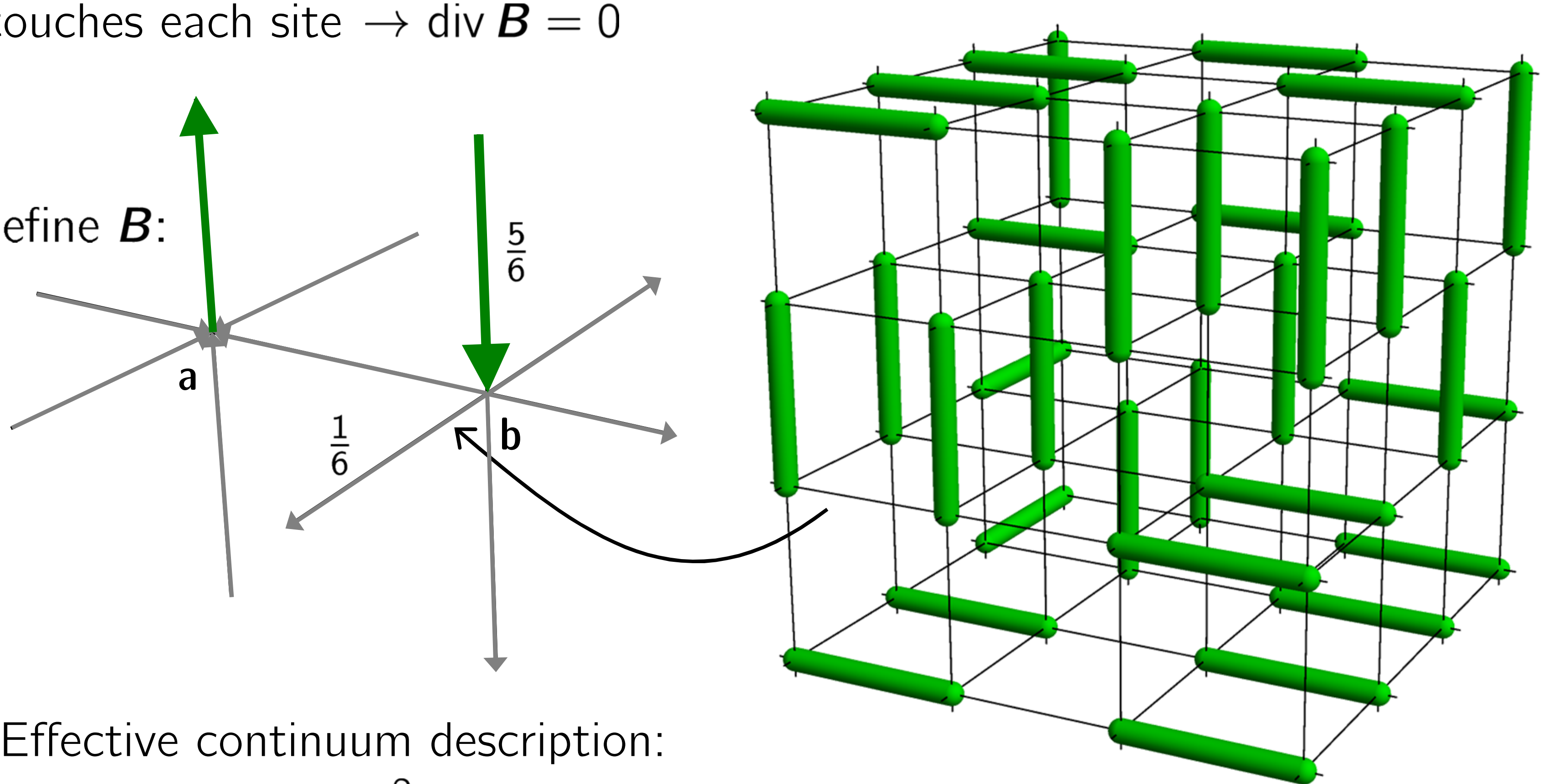

Effective continuum description:

$$\mathcal{F}(\boldsymbol{B}) = K|\boldsymbol{B}|^2 + \cdots$$
$$= K|\boldsymbol{\nabla} \times \boldsymbol{A}|^2 + \cdots$$

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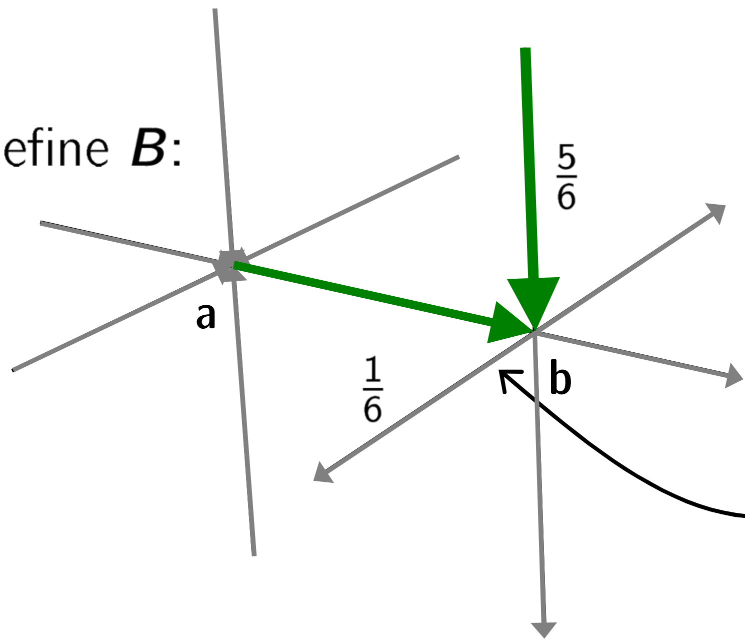

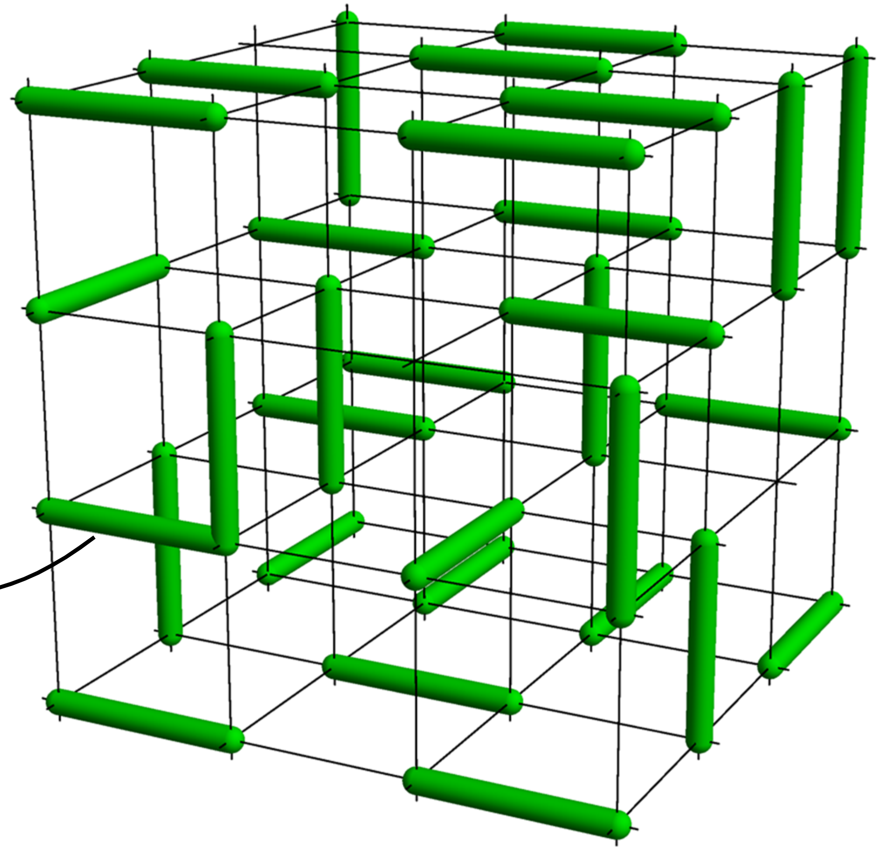

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Effective gauge theory

Defects in constraint: "monopoles"

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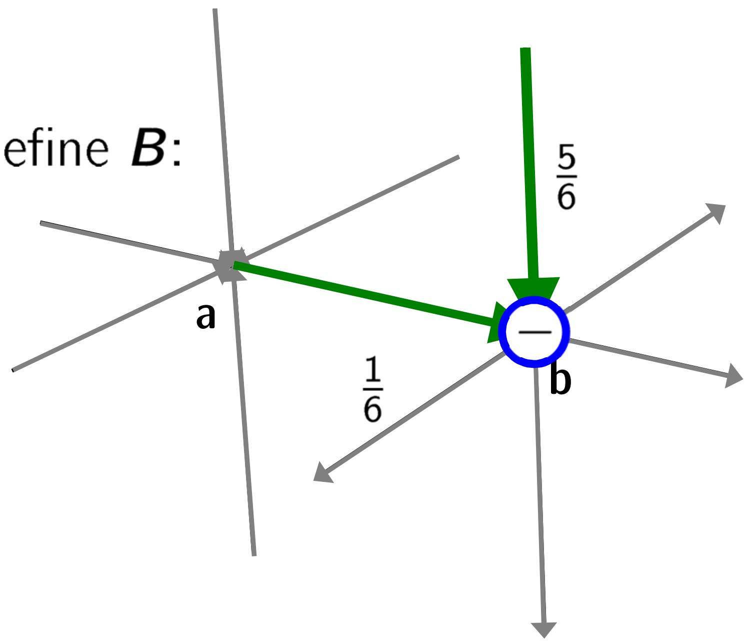

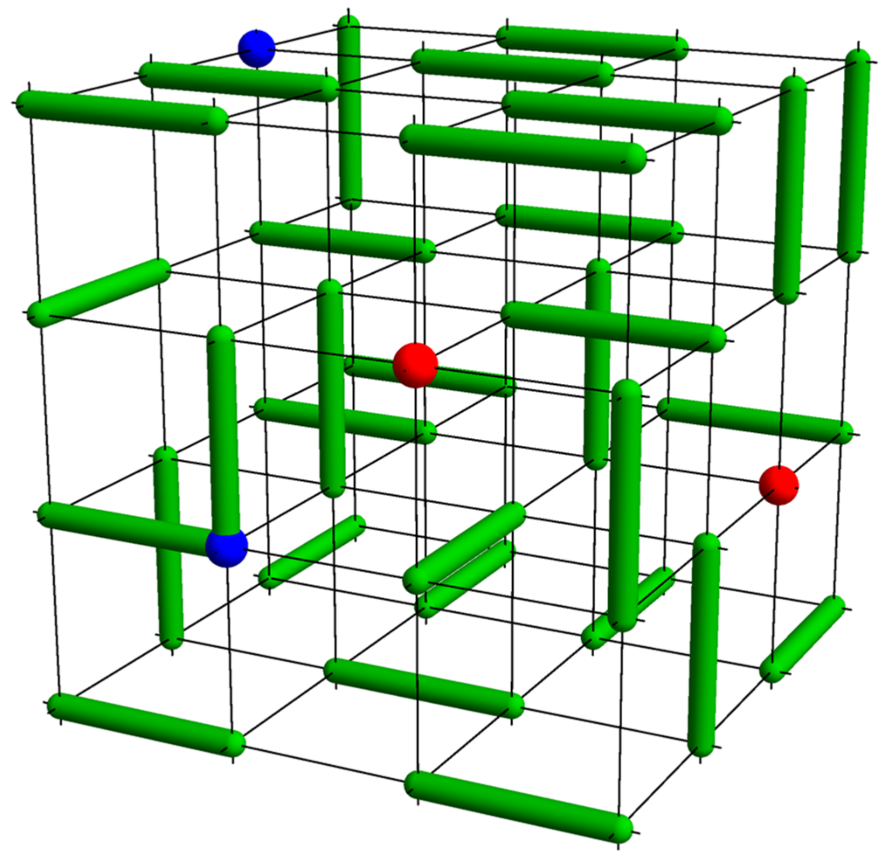

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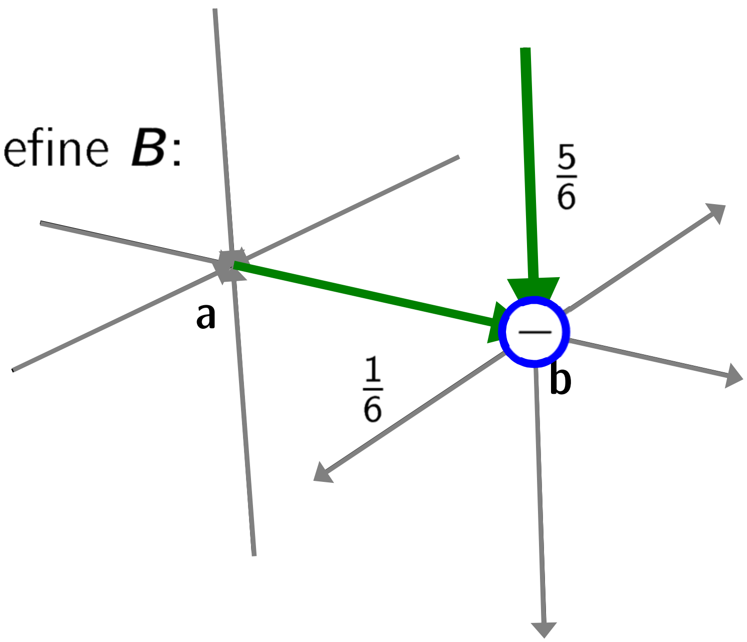

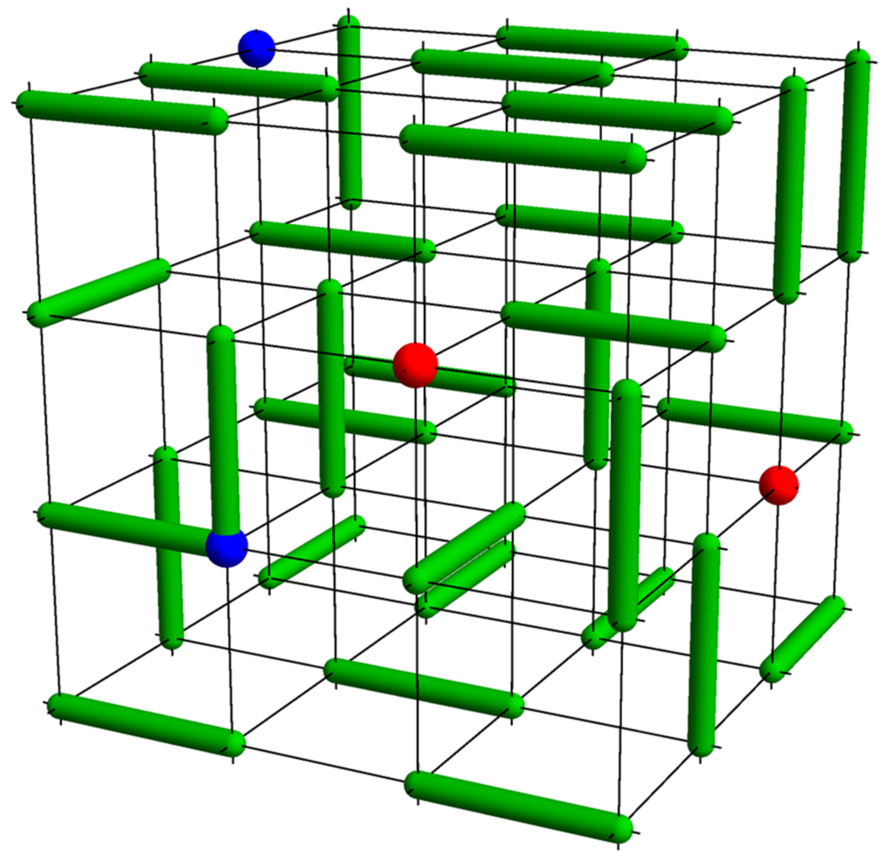

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Effective interaction: $U_{\text{m}}(\boldsymbol{R}) \sim \dfrac{K}{|\boldsymbol{R}|}$

Huse et al., PRL **91**, 167004 (2003)
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Dimer ordering transition

Order parameter: $\boldsymbol{X}$

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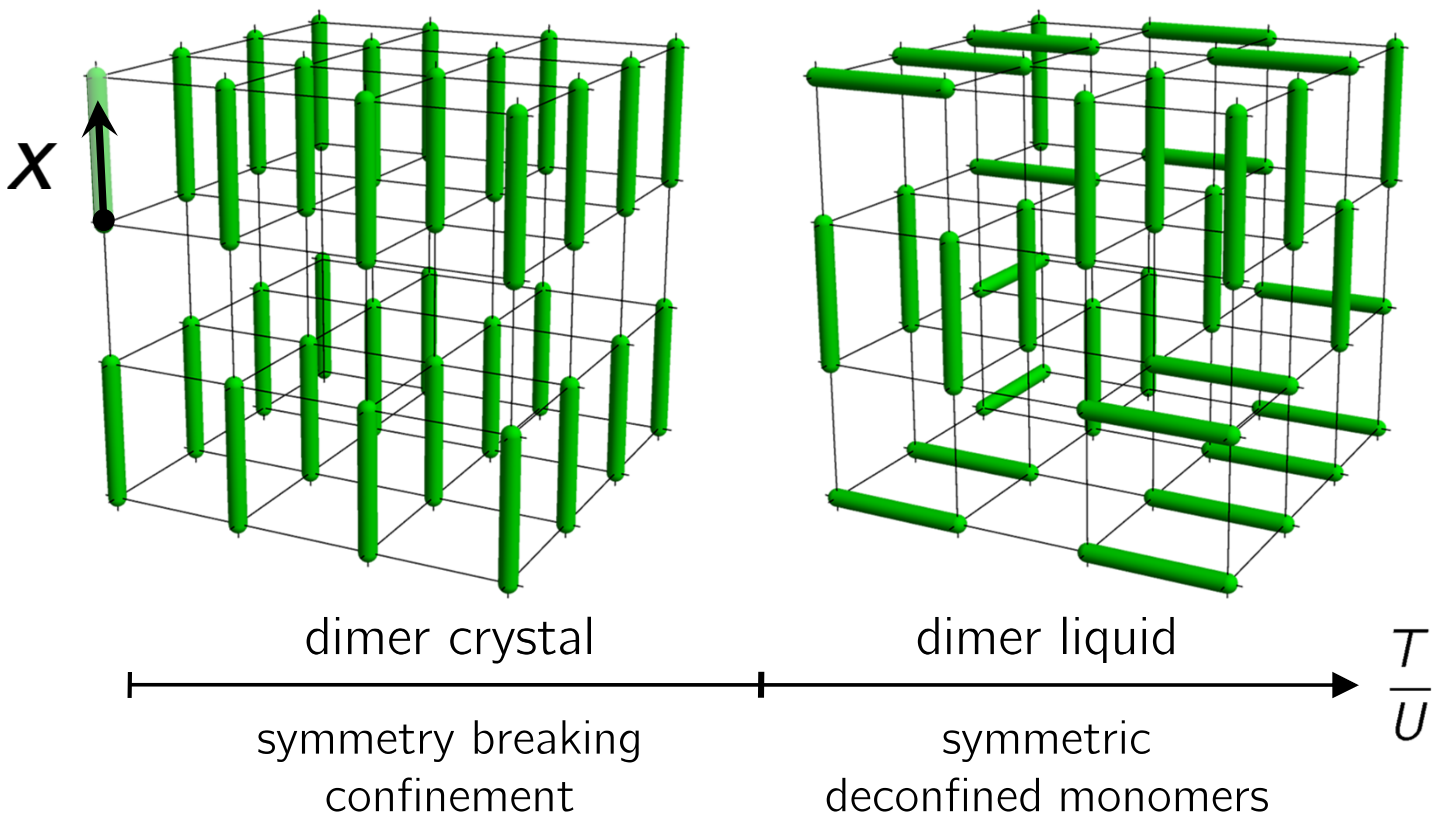


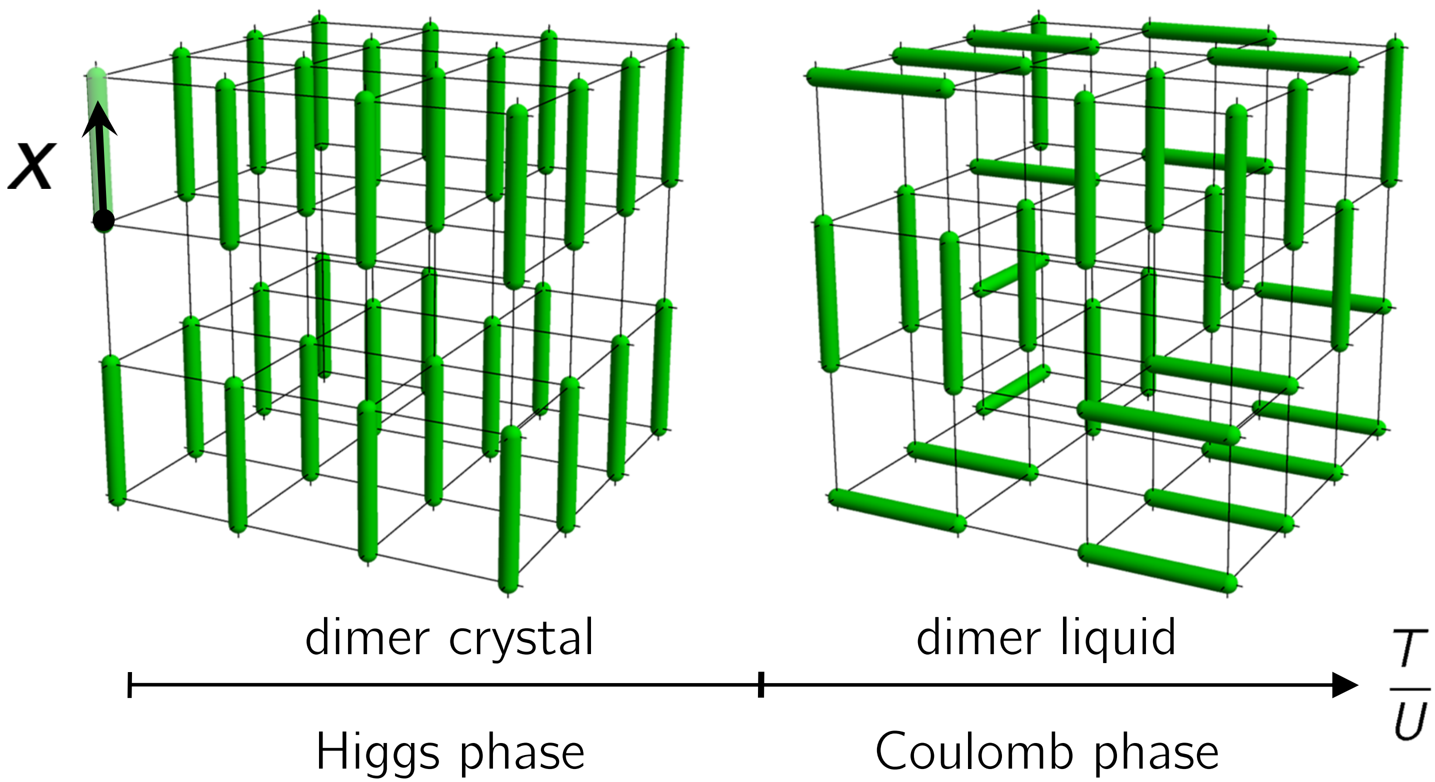
SP & Chalker, PRL (2008); Charrier et al., PRL (2008); Chen et al., PRB (2009)

Dimer ordering transition

Critical (Higgs) theory:

$$\mathcal{F} = K|\nabla \times \boldsymbol{A}|^2 + |(\nabla - \mathrm{i}\boldsymbol{A})\boldsymbol{\varphi}|^2 + r\boldsymbol{\varphi}^\dagger\boldsymbol{\varphi} + u(\boldsymbol{\varphi}^\dagger\boldsymbol{\varphi})^2$$

Order parameter: $X_\mu = \boldsymbol{\varphi}^\dagger \boldsymbol{\sigma}^\mu \boldsymbol{\varphi}$ ← SU(2) "matter" field



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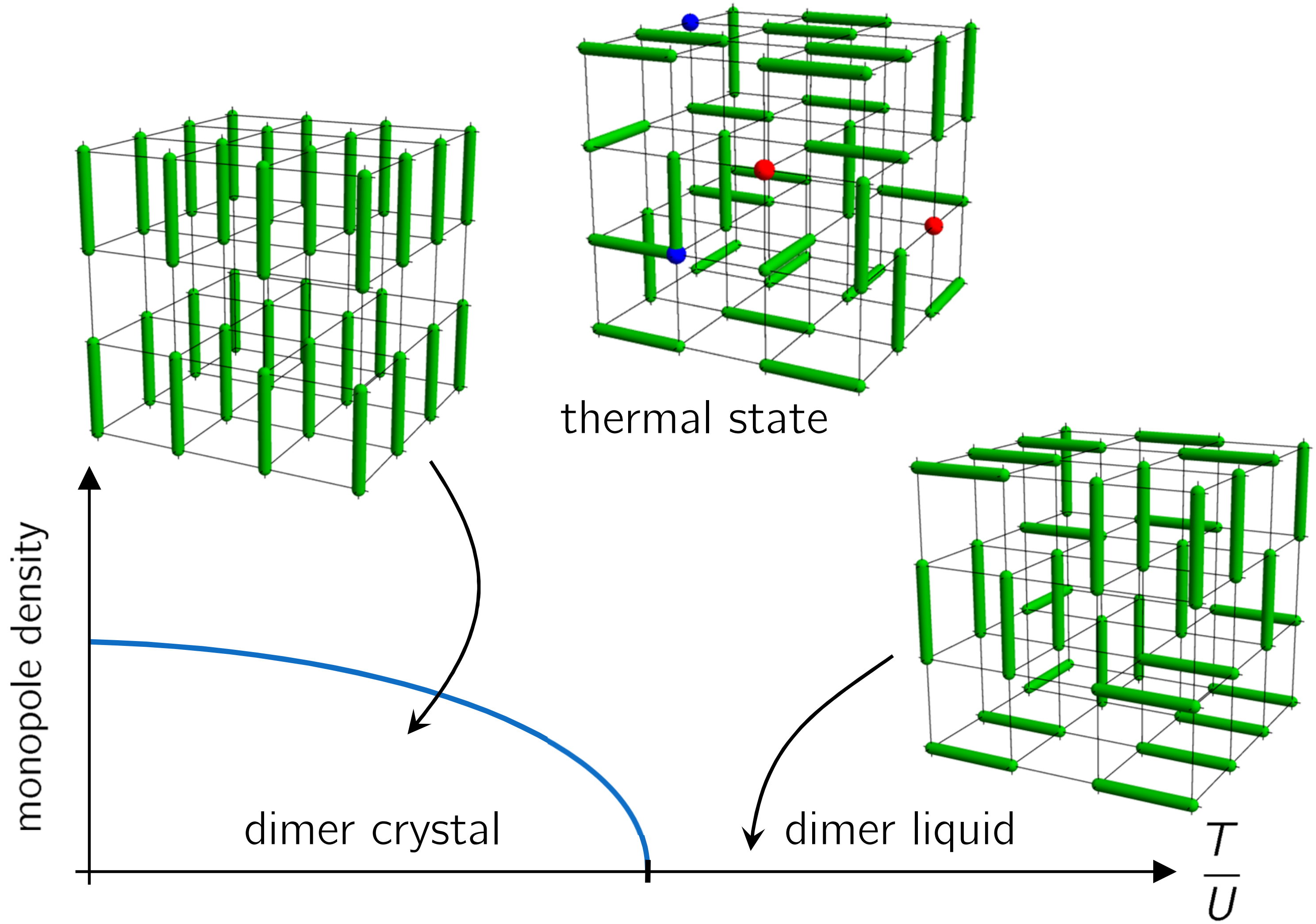
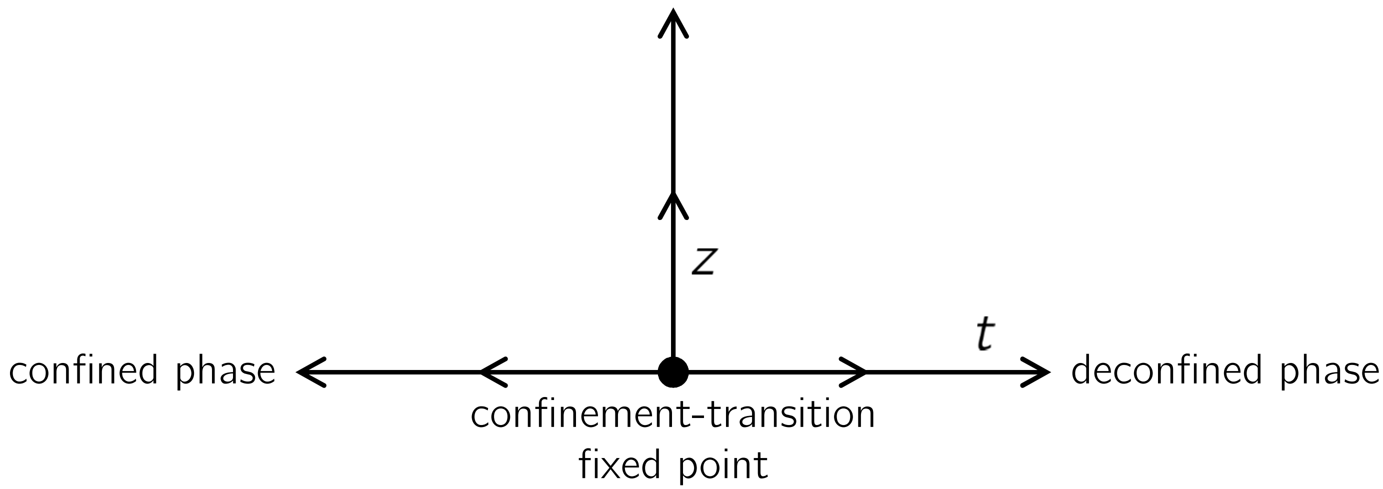
Dimer model with monomers



G. J. Sreejith & SP, PRB **89**, 014404 (2014)

Dimer model: RG flows

Relax $\Delta \to \infty$ limit: $$E = \Delta \sum_i (\mathrm{div}_i\, B)^2 - U N_{\parallel}$$



reduced temperature $t = (T - T_C)/T_C$
monopole fugacity $z = \mathrm{e}^{-\Delta/T}$

SP, PRL **109**, 065701 (2012); PRB **87**, 064414 (2013)

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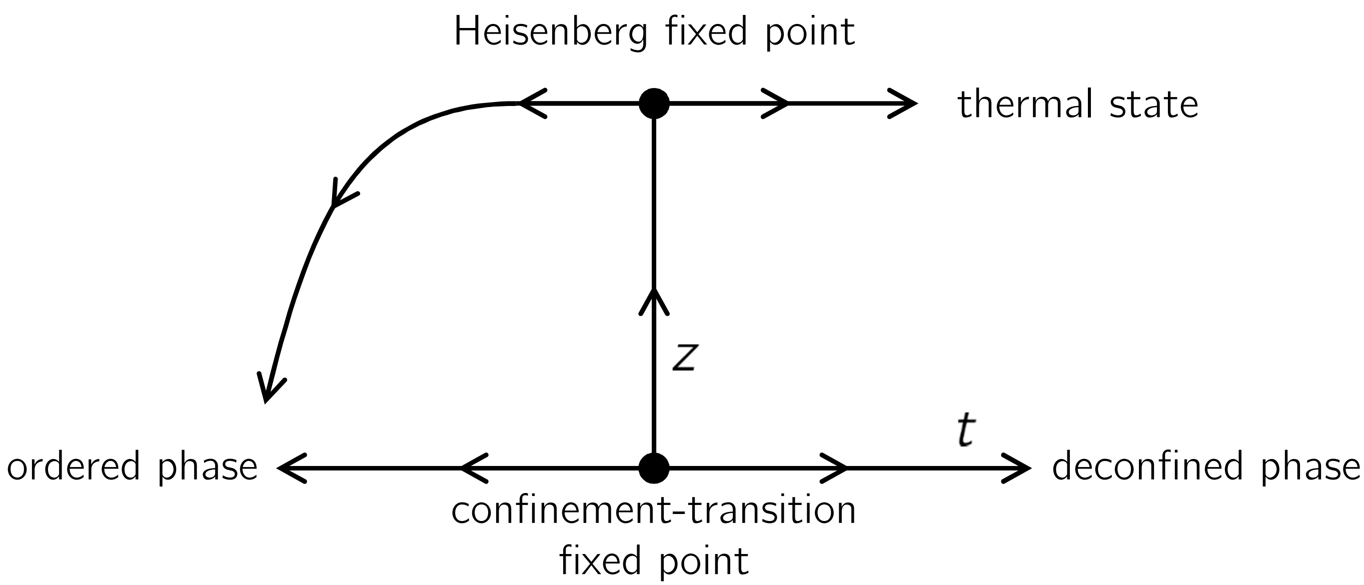

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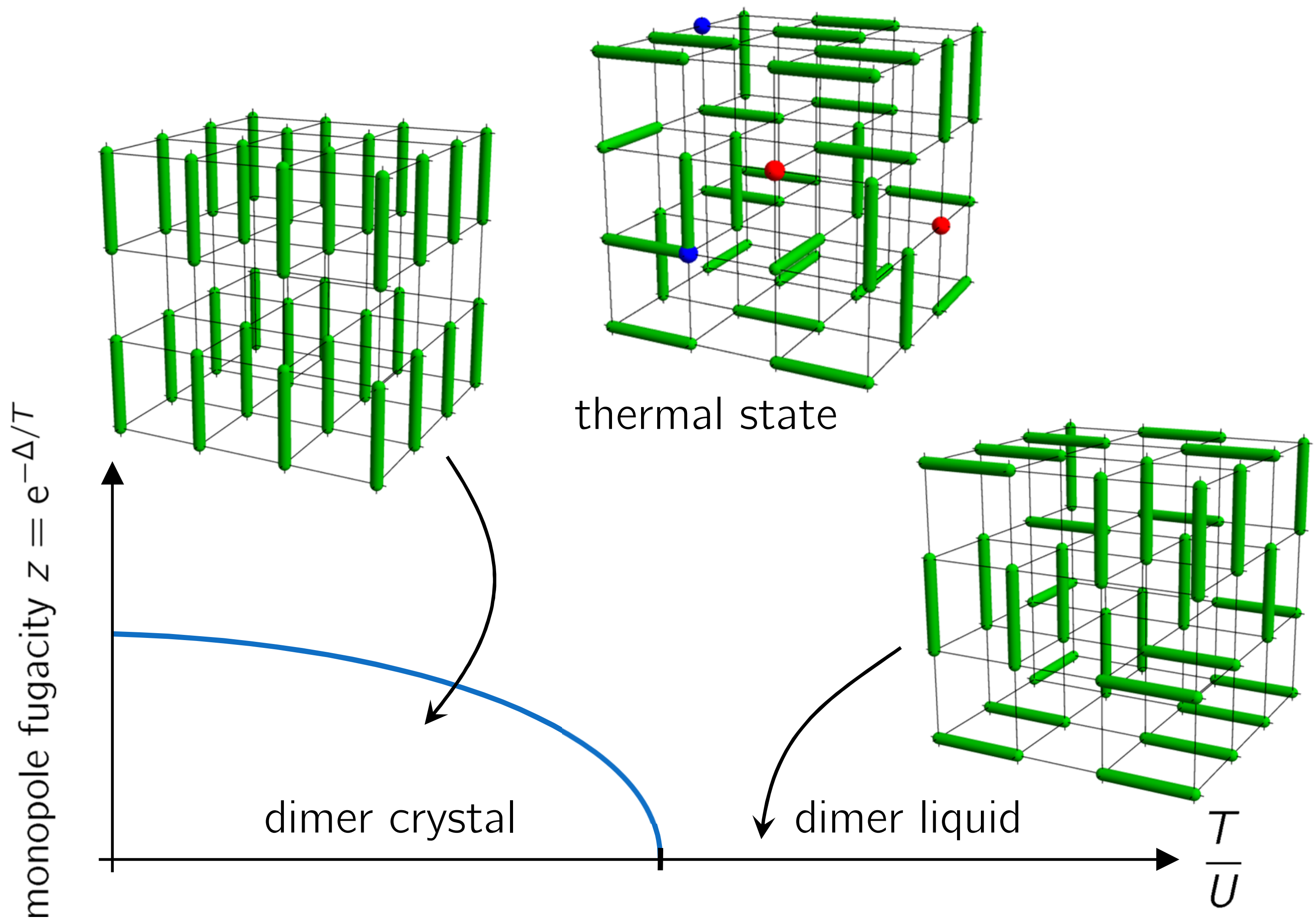

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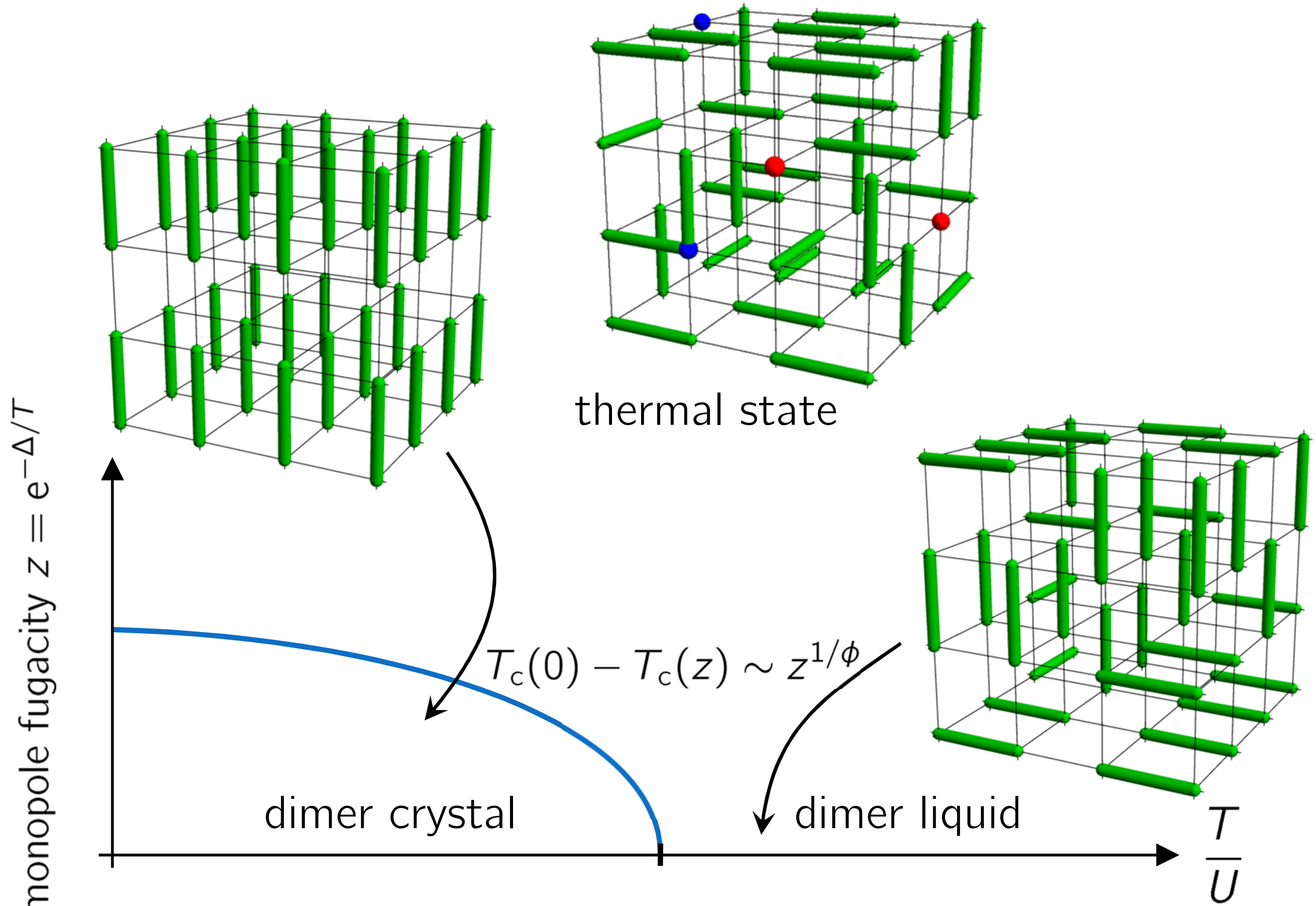
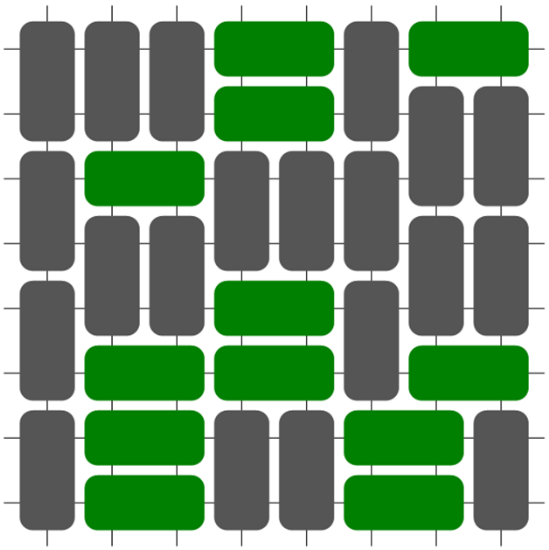


Crossover exponent $\phi = y_z/y_t > 0$

G. J. Sreejith & SP, PRB **89**, 014404 (2014)

Dimer model: Monte Carlo

Directed-loop Monte Carlo algorithm



Sandvik & Moessner, PRB **73**, 144504 (2006)

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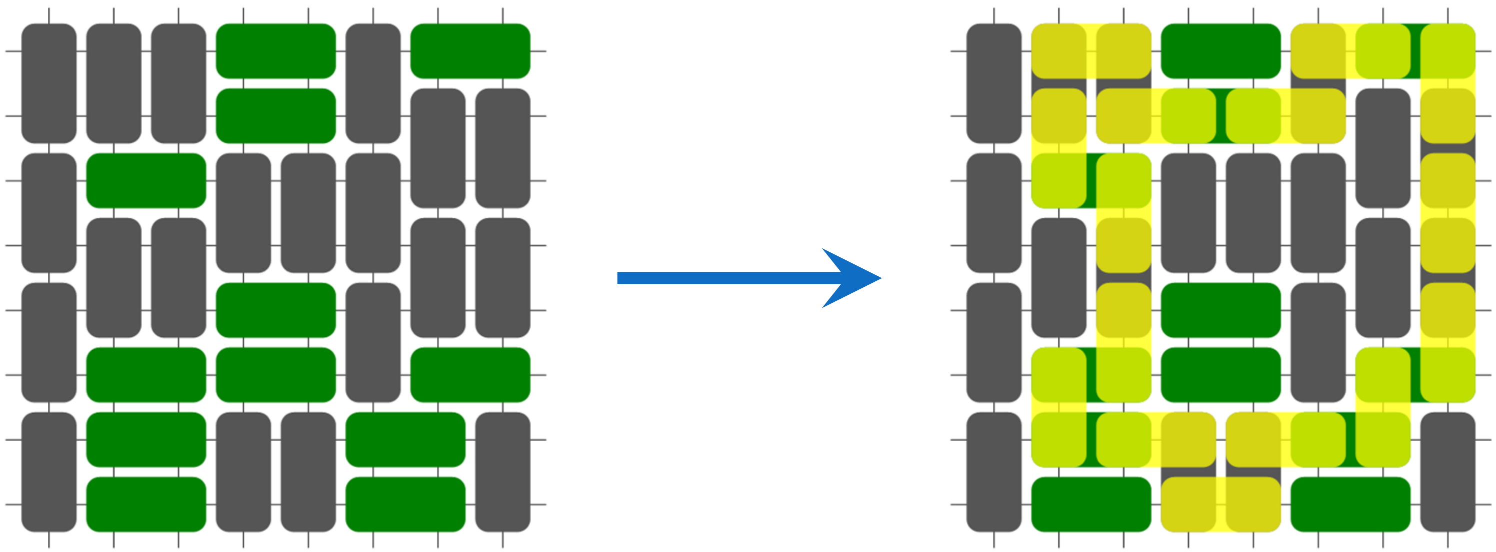

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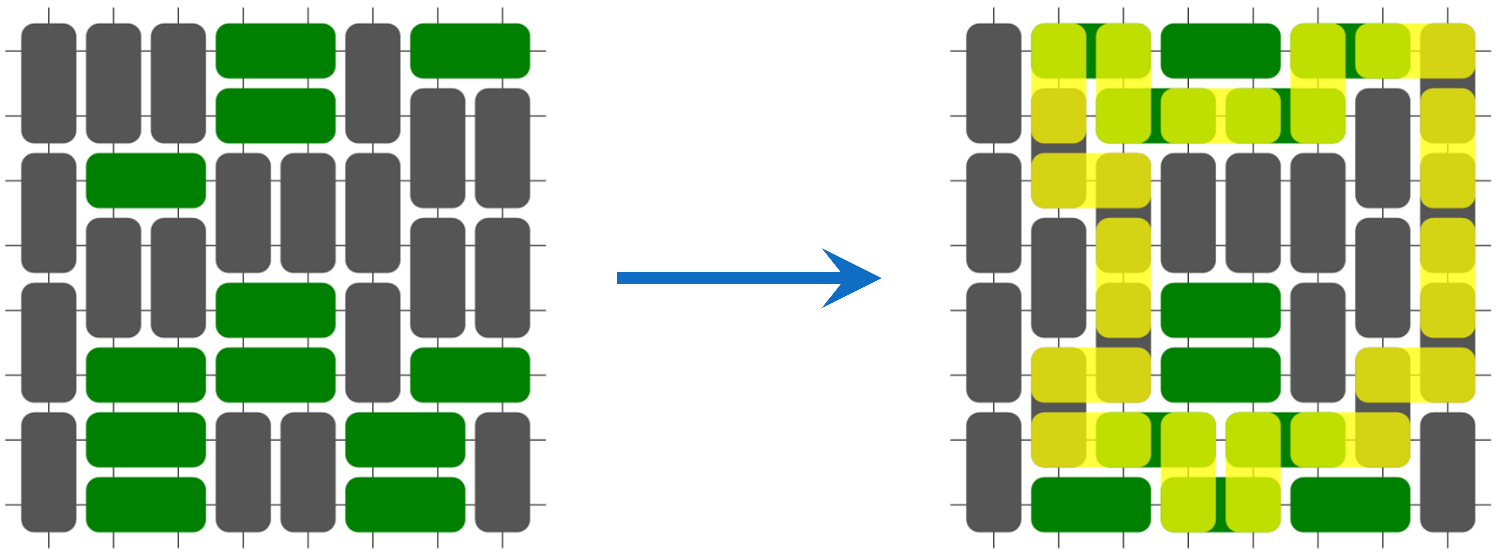

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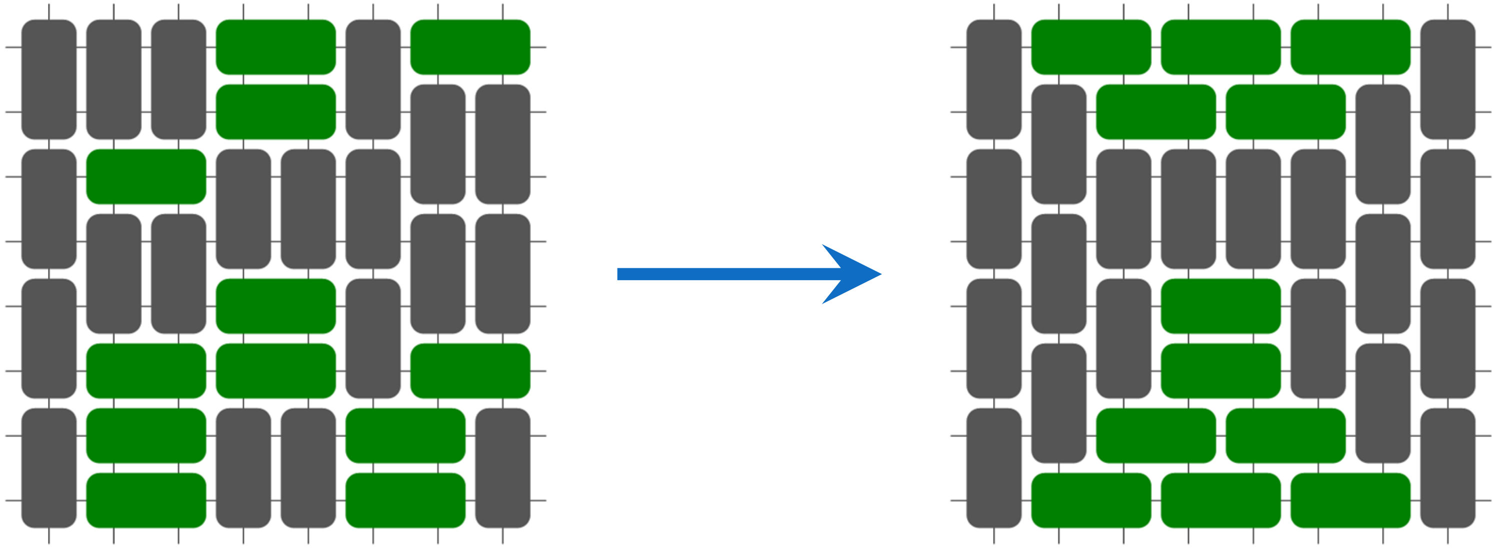

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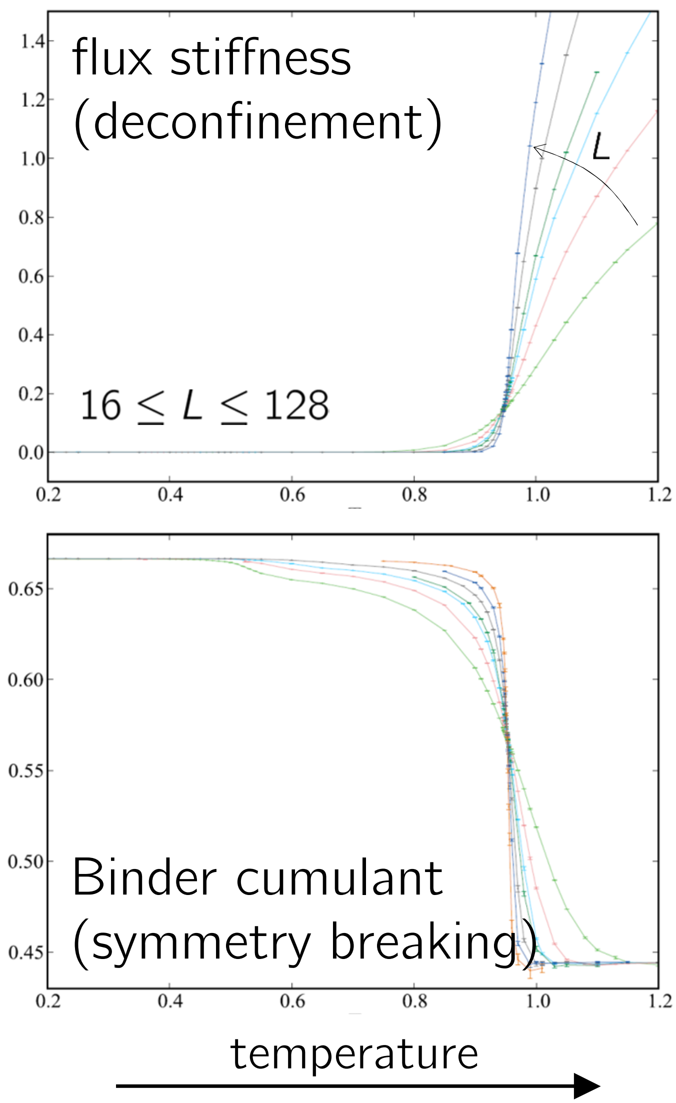
Dimer model: Numerical results

$z = 0 \qquad \Delta/T = \infty$



Sreejith & SP, PRB (2014)

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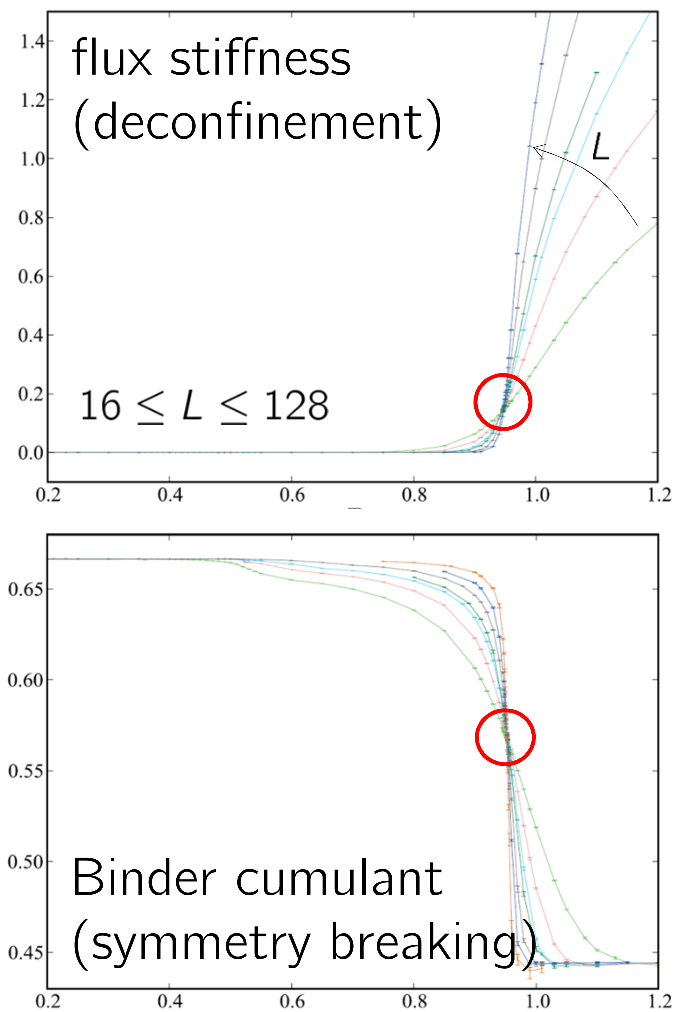

temperature

Sreejith & SP, PRB (2014)

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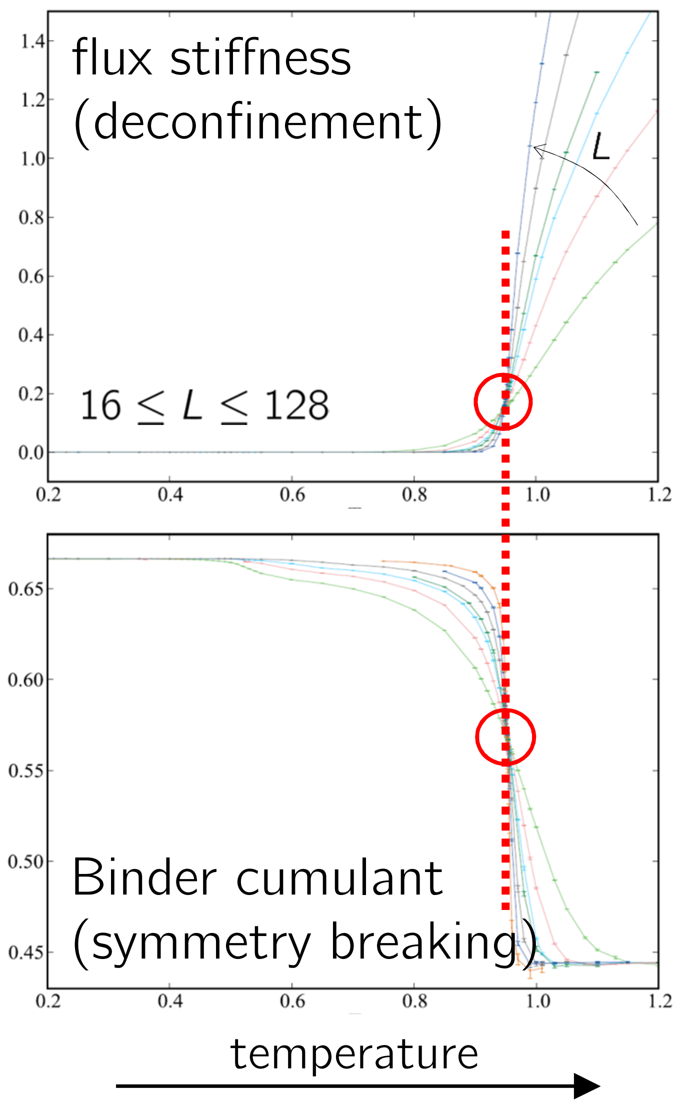

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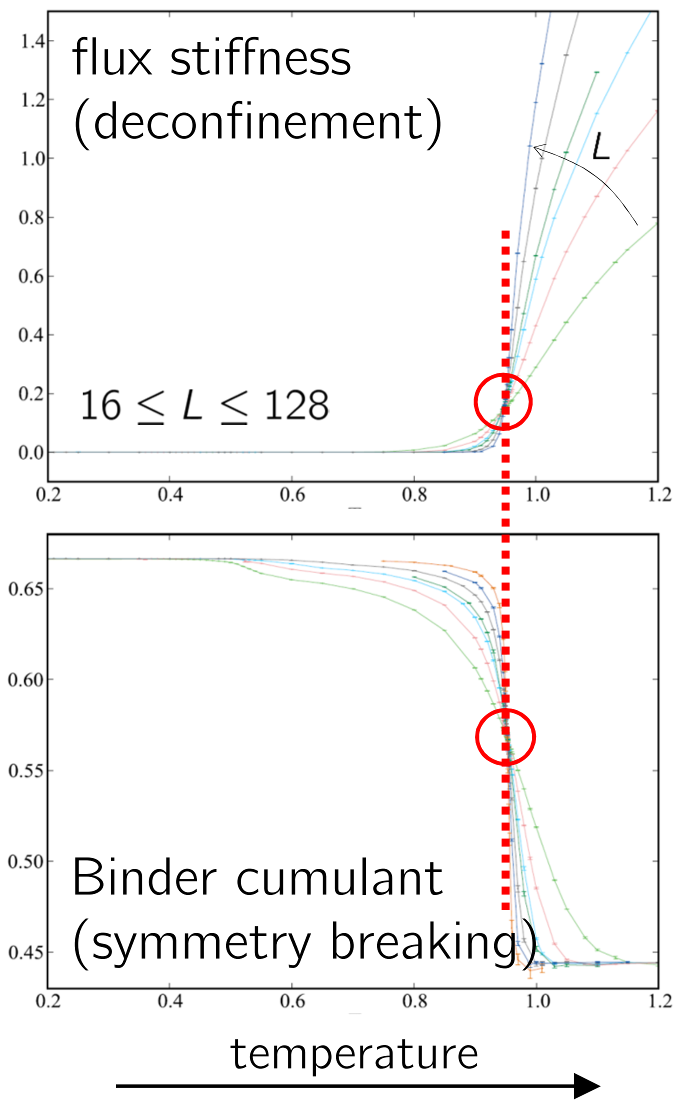
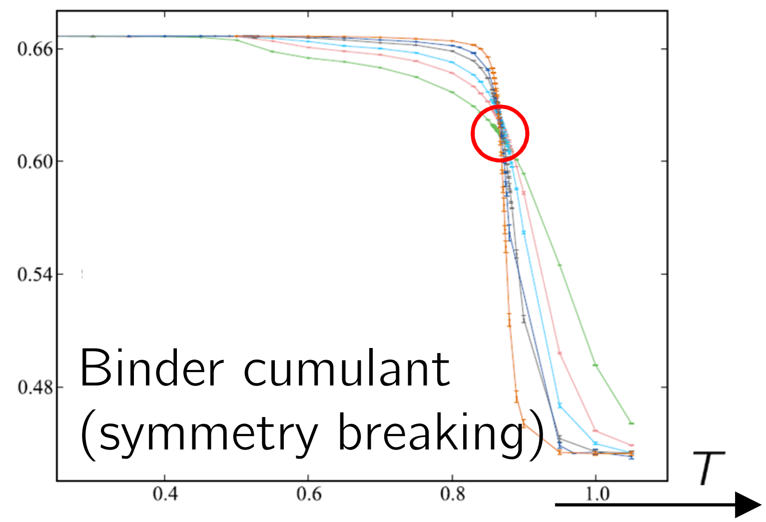


$z = 10^{-3} \qquad \Delta/T \simeq 7$



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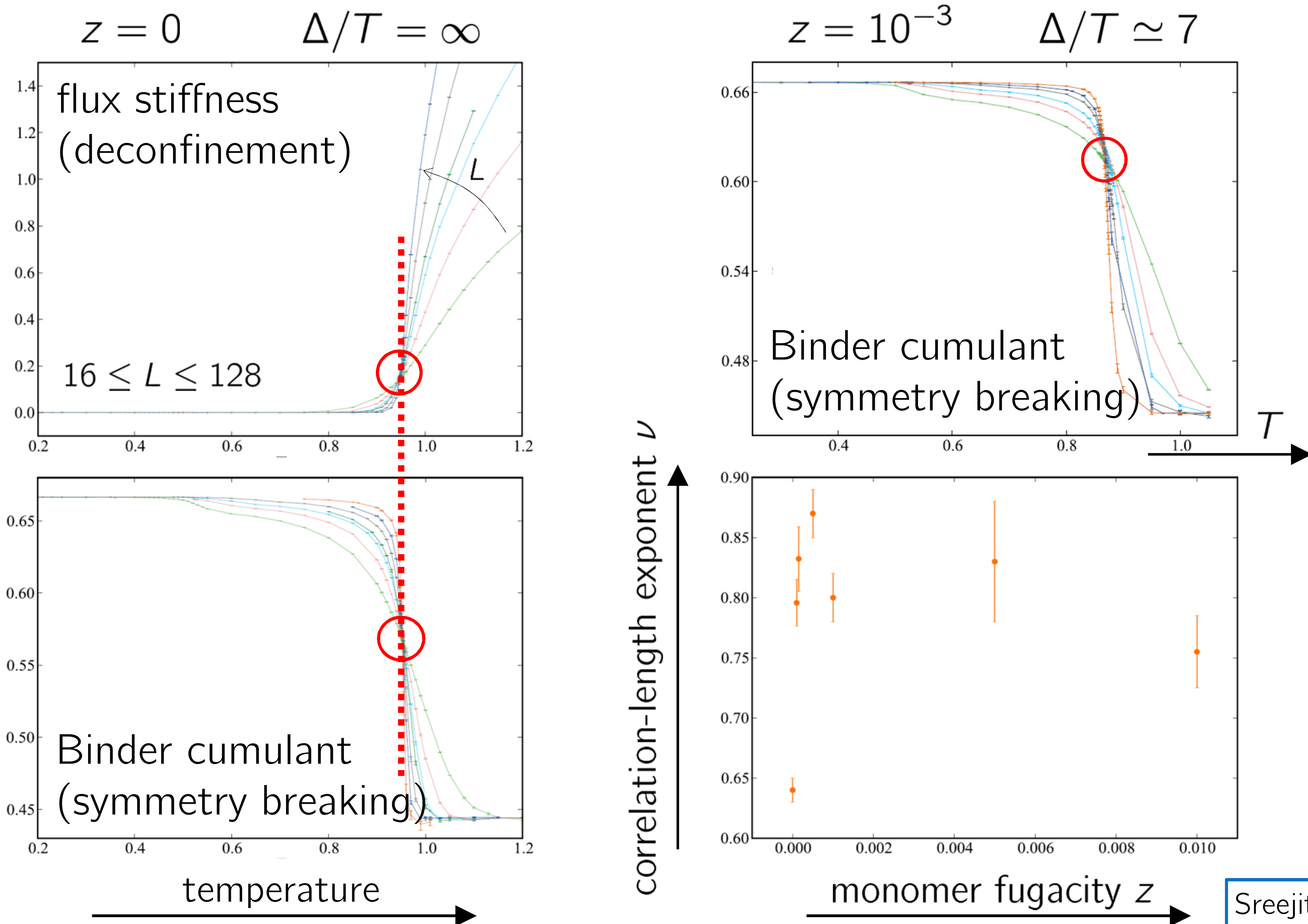

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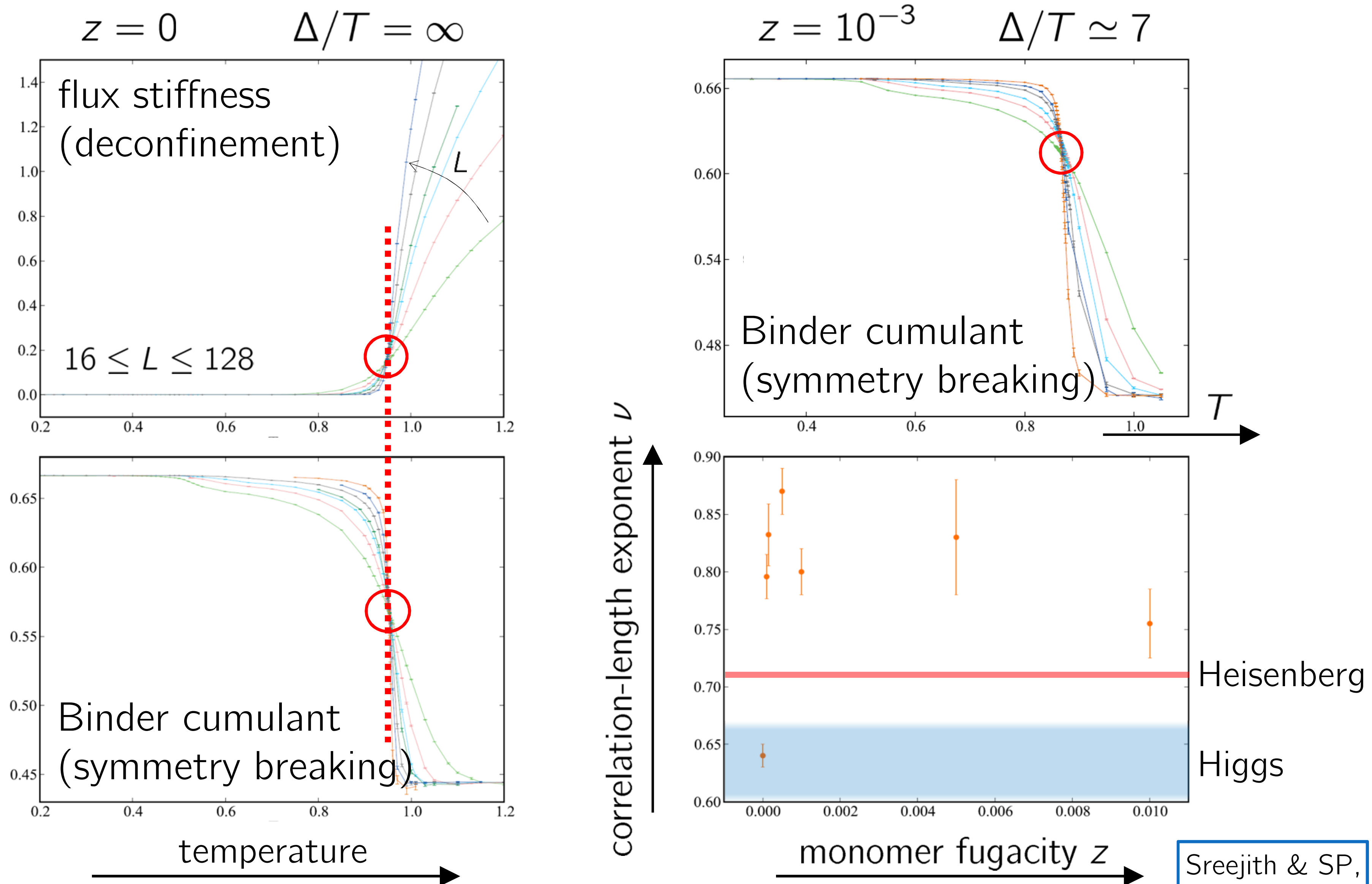
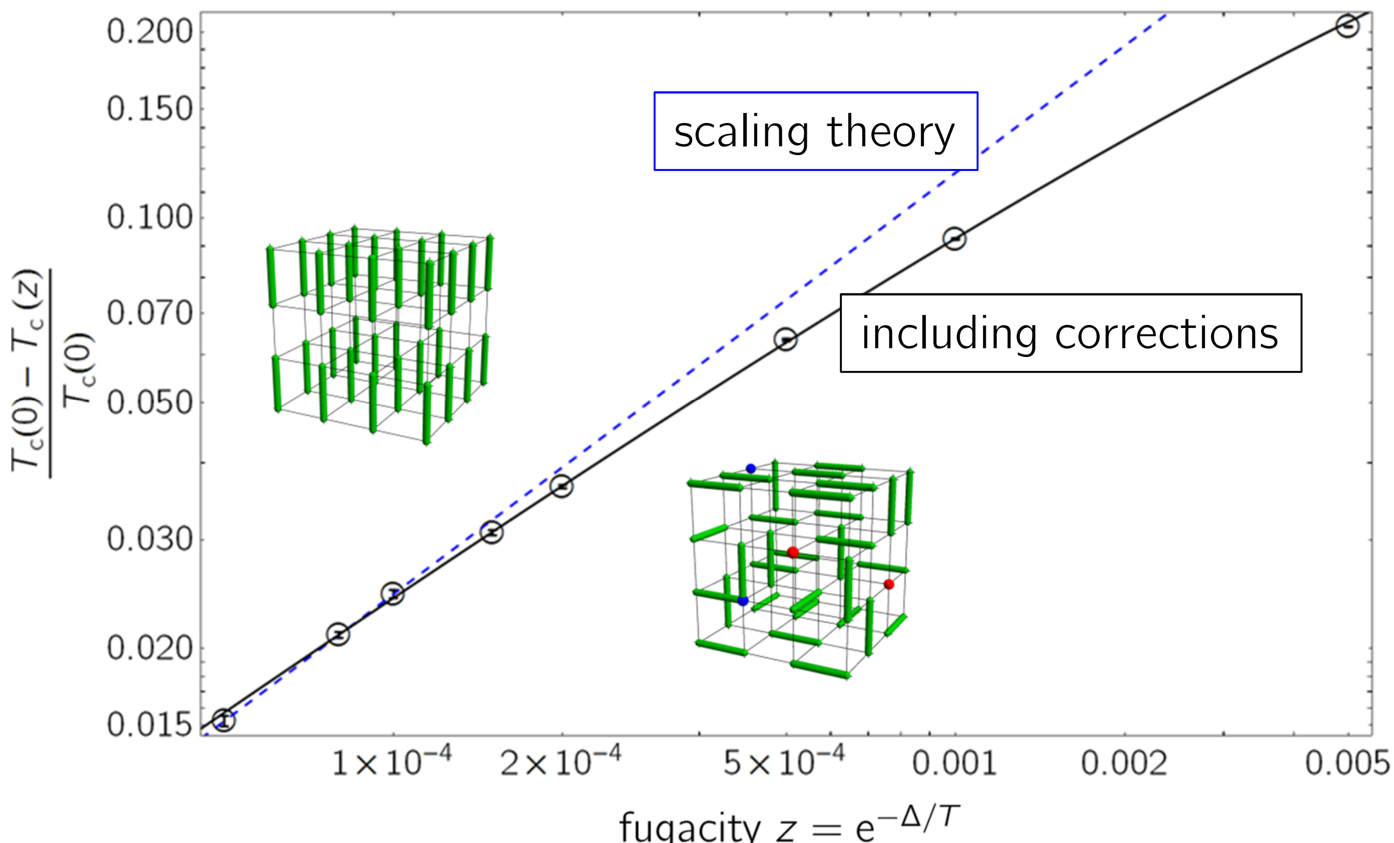


Sreejith & SP, PRB (2014)

Dimer model: Numerical results

Phase boundary at $z > 0$: $T_c(0) - T_c(z) \sim z^{1/\phi} +$ (large) corrections

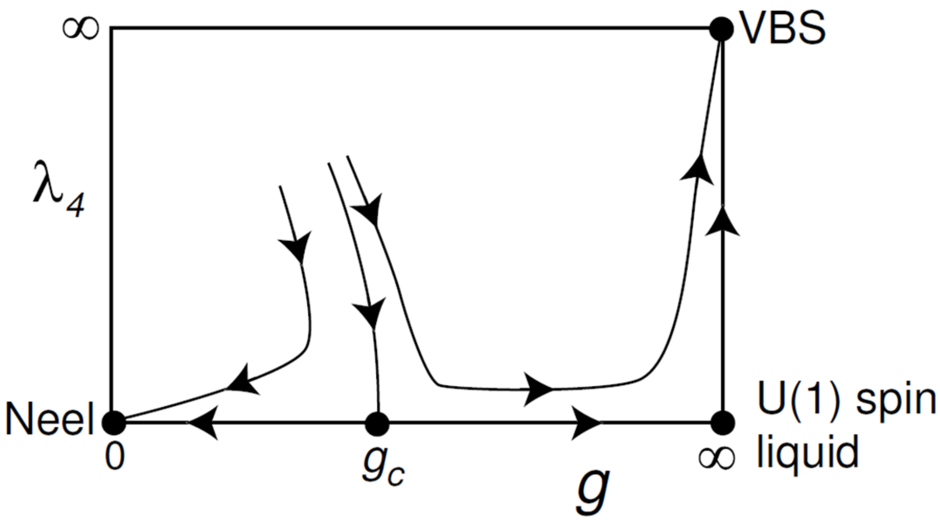
$\phi = 1.46 \pm 0.08$



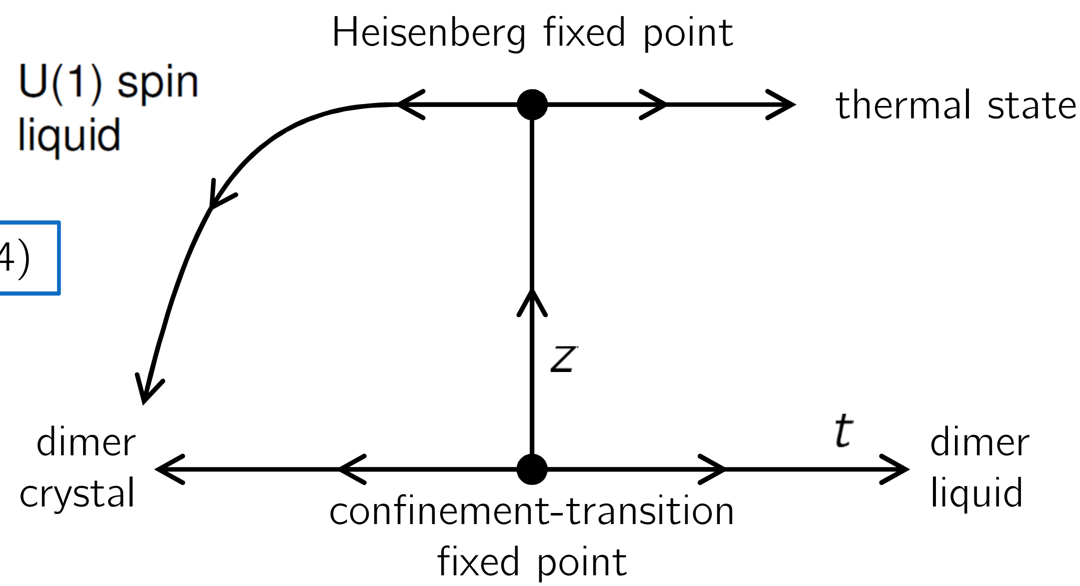
system sizes up to $L^3 = 128^3 \simeq 2 \times 10^6$
monomer fugacity $5 \times 10^{-5} \leq z \leq 10^{-2}$

Sreejith & SP, PRB (2014)

Deconfined criticality



Senthil et al., PRB **70**, 144407 (2004)



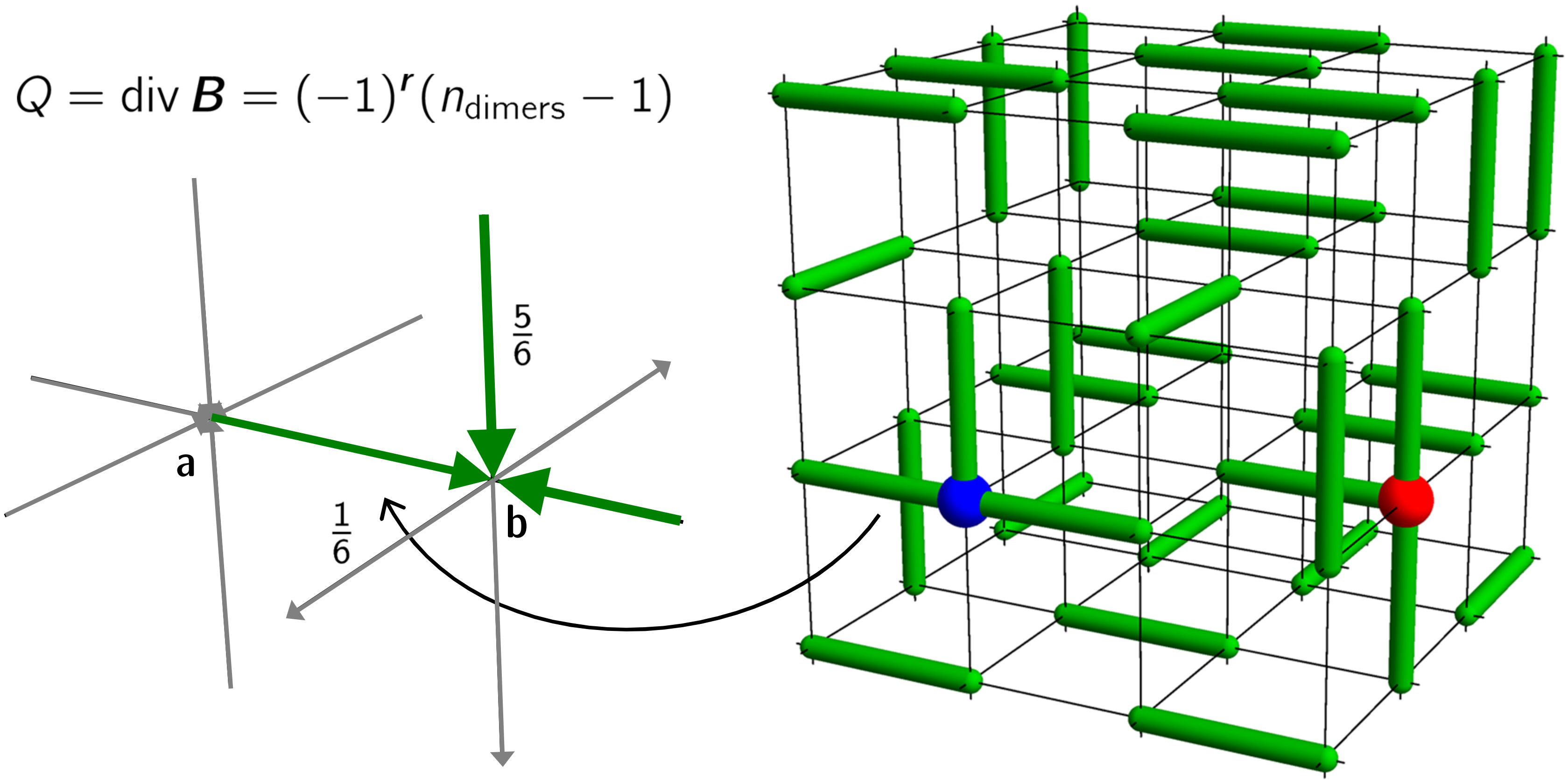
reduced temperature $t = (T - T_C)/T_C$
monopole fugacity $z = e^{-\Delta/T}$

Comparison of exponents

	Model	ν	η_{VBS}	y_z
Sreejith & SP, PRB (2014)	CDM ($v_4 = +1$)	0.64(1) ($\mathcal{B}$)		2.421(8) (G_{m})
		0.61(1) (K)		2.28(13) (T_{c})
Charrier & Alet, PRB (2010)	CDM ($v_4 = +1$)	0.60(4) ($\mathcal{B}$)		
		0.61(4) (K)		
Lou, Sandvik, & Kawashima, PRB (2009)	JQ (square)	0.67(1)	0.20(2)	2.40(1)
		0.69(2)	0.20(2)	2.40(1)
Pujari, Damle, & Alet, PRL (2013)	JQ (honeycomb)	0.54(5)	0.28(8)	2.36(4)
Harada et al., PRB (2013)	JQ (square and honeycomb)	0.59	0.35	2.33

Higher-charge monopoles

$$Q = \operatorname{div} \boldsymbol{B} = (-1)^r (n_{\text{dimers}} - 1)$$



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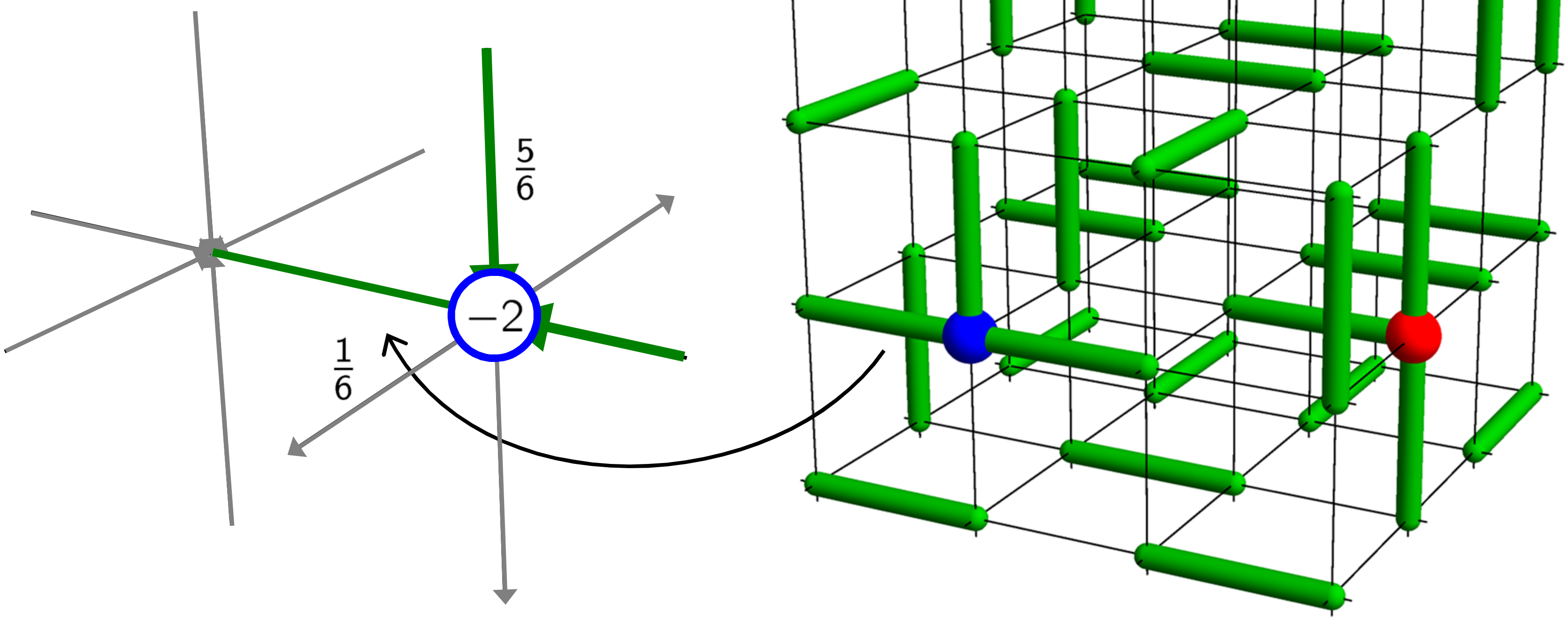

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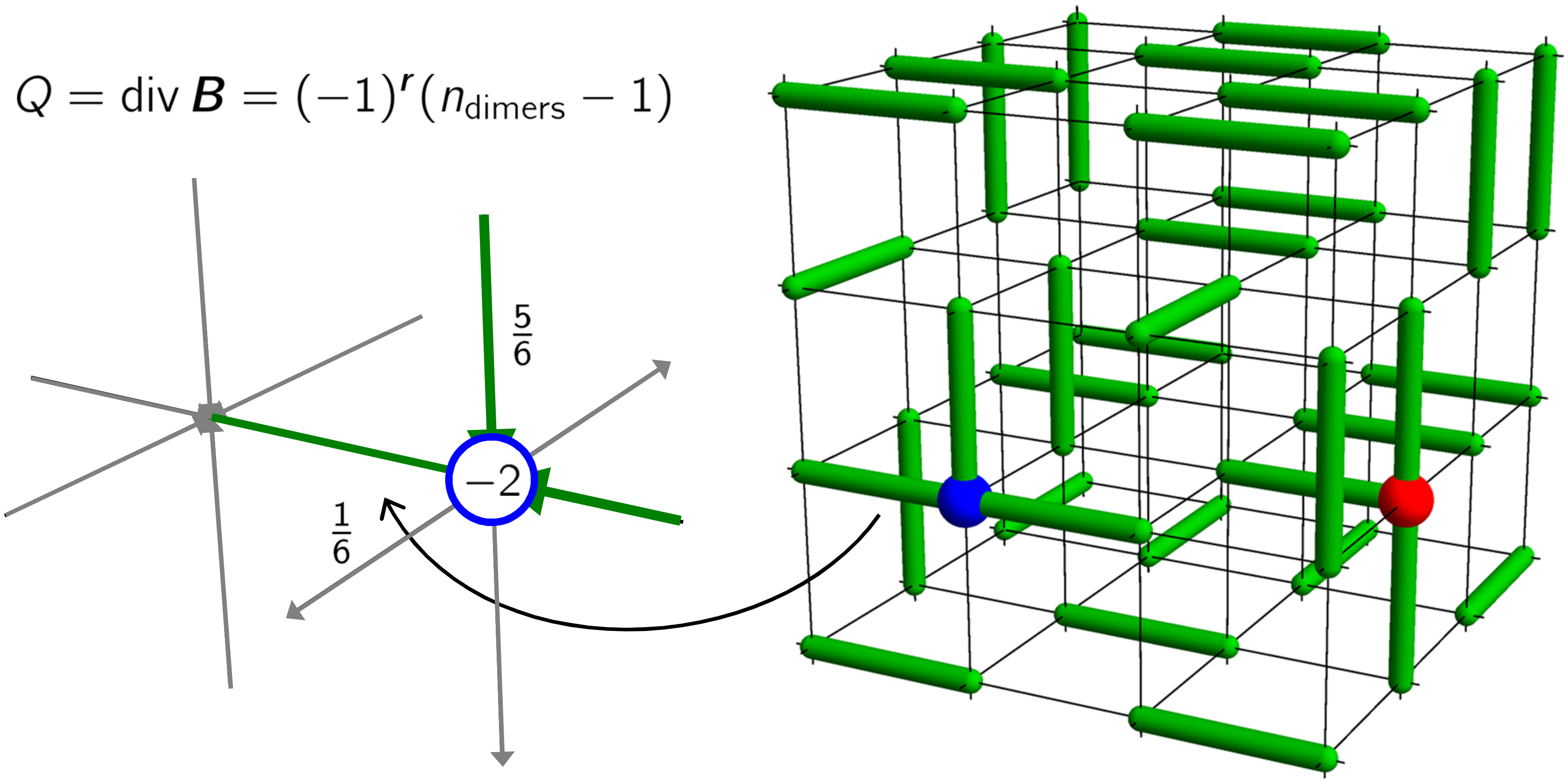

$$G_q(\boldsymbol{r}_+, \boldsymbol{r}_-) = \sum_{\text{config'ns}} \mathrm{e}^{-E/T} \prod_{\boldsymbol{r}} \delta_{Q_r, 2\delta_{rr_+} - 2\delta_{rr_-}}$$

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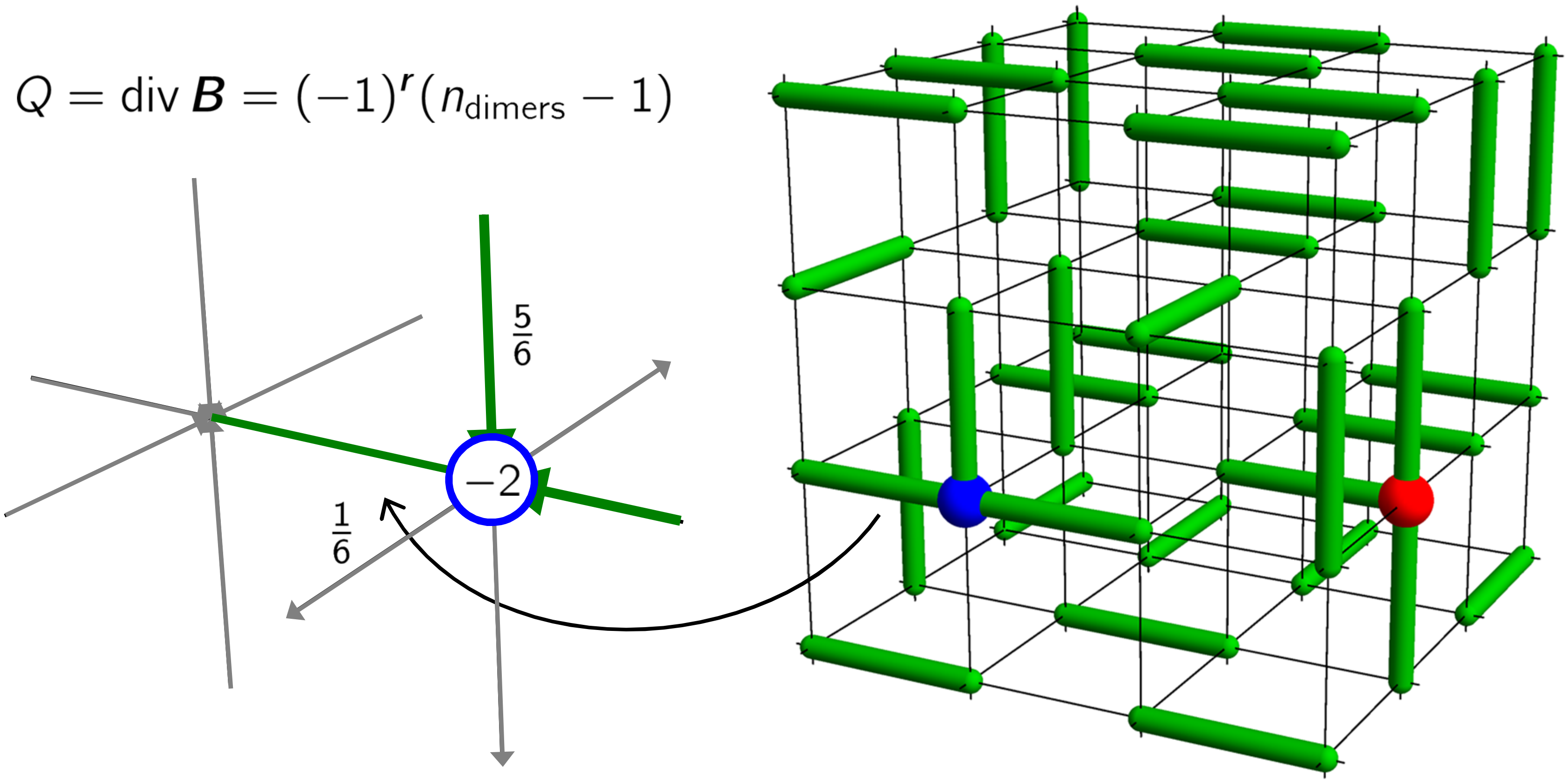

$$G_q(\boldsymbol{r}_+, \boldsymbol{r}_-) = \sum_{\text{config'ns}} \mathrm{e}^{-E/T} \prod_{\boldsymbol{r}} \delta_{Q_{\boldsymbol{r}}, 2\delta_{\boldsymbol{r}\boldsymbol{r}_+} - 2\delta_{\boldsymbol{r}\boldsymbol{r}_-}} \sim \frac{1}{|\boldsymbol{r}_+ - \boldsymbol{r}_-|^{2(d-y_q)}}$$

Summary

Using classical dimers...

- Noninteracting dimers on cubic lattice have liquid phase with deconfined monomers
- Continuous transition from dimer liquid to dimer crystal in presence of aligning interactions
- Critical theory for this transition describes it through condensation of SU(2) matter fields, charged under a U(1) gauge theory

G. J. Sreejith & SP, PRB **89**, 014404 (2014)

Support: SNIC (Sweden), Nottingham HPC

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...to study deconfined criticality

- This critical theory is proposed to described "deconfined critical point" in 2D quantum spin models
- Critical exponents show reasonable agreement between the two transitions, with significant corrections to scaling
- Monopoles, which are complicated topological objects for spin models, are simple point defects in the dimer model

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