Studying correlated systems with PEPS

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- Some motivations: «trivial» vs «topological» spin liquids (on frustrated lattices)
- The Projected Entangled Pair State (PEPS) scheme
- «Holographic» framework & the bulk-edge correspondence

COLLABORATORS



Norbert Schuch, DP, J. Ignacio Cirac, and David Pérez-García,
Phys. Rev. B **86**, 115108 (2012)

DP, Norbert Schuch, David Pérez-García, and J. Ignacio Cirac,
Phys. Rev. B **86**, 014404 (2012)

Norbert Schuch, DP, J. Ignacio Cirac, and David Perez-Garcia
Phys. Rev. Lett. 111, 090501 (2013)

DP and Norbert Schuch, Phys. Rev. B 87, 140407 (2013)

DP, Norbert Schuch, J. Ignacio Cirac, Phys. Rev. B 88, 144414 (2013)

DP, Philippe Corboz, Norbert Schuch, J. Ignacio Cirac, PRB 2014

Exotic «spin liquids» beyond the «order parameter» paradigm

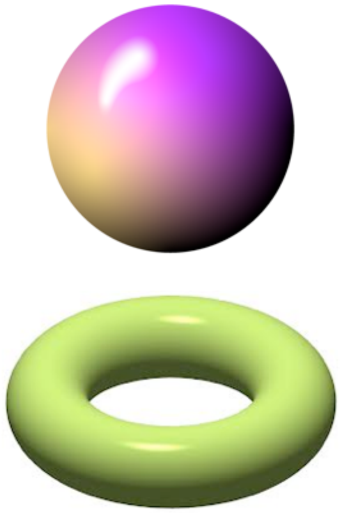
* no spontaneous broken symmetry

* no local order

* Topological order

- Do they exist in materials ?
  in simple models ?

- How to detect them ?



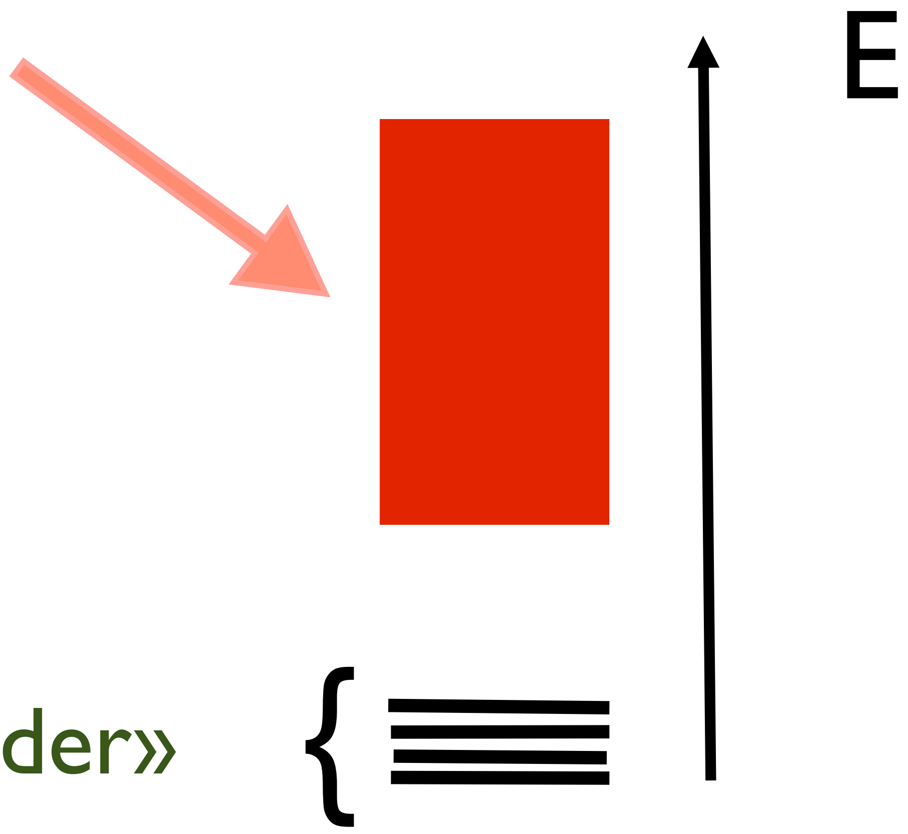
X. G.Wen

GS degeneracy (depends on **topology** of space)

Topological order can also be detected by entanglement measures !

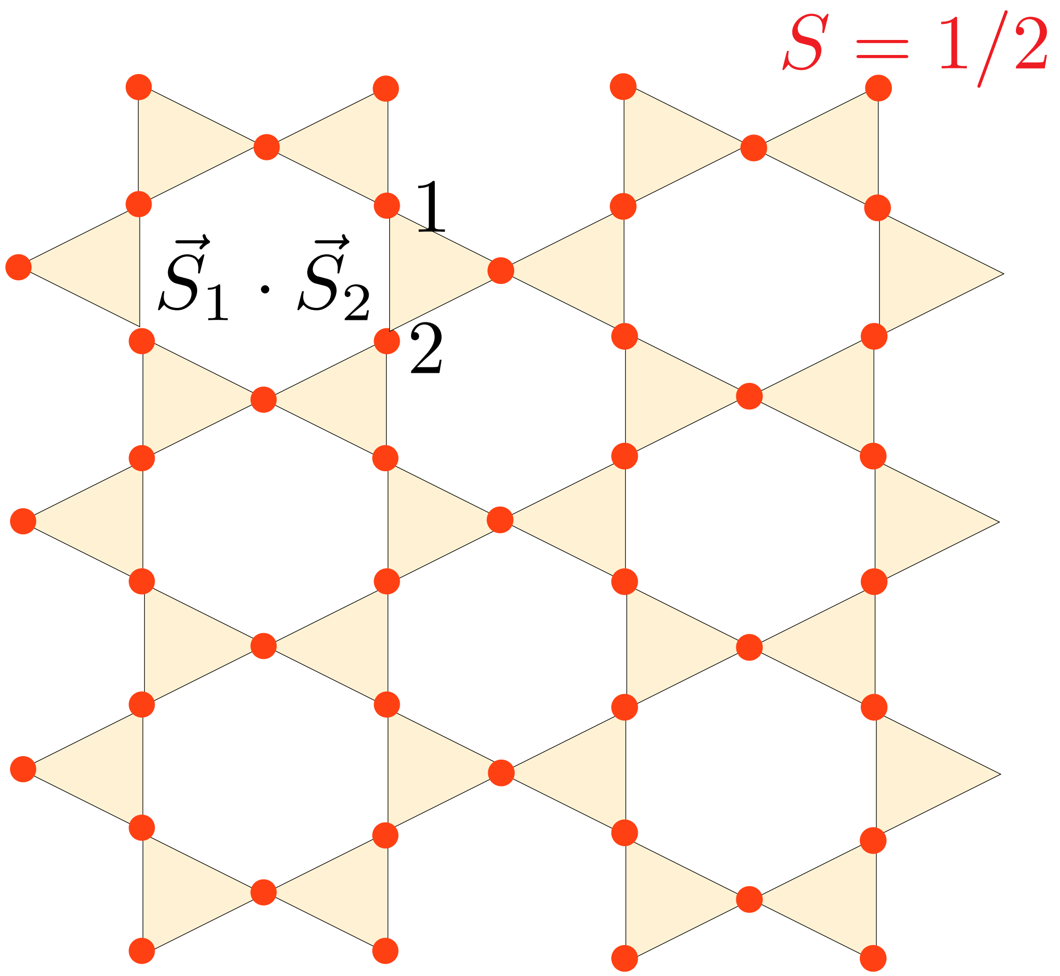
The many-body spectrum of a topological liquid

Excitations are fractional (created by pairs)



Degeneracy from «topological order»

Best candidate : spin-1/2 Heisenberg QAF on the Kagome lattice !



Herbertsmithite: P. Mendels (Orsay)
& Z. Hiroi (ISSP)

Numerical «evidence» (DMRG)
for a (gapped) spin liquid:

S. Yan, D.A. Huse & S. White, Science 2011
S. Depenbrock, I.P. McCulloch & U. Schollwock, PRL 2012

topological features ?

TWO TYPES OF SPIN LIQUIDS:

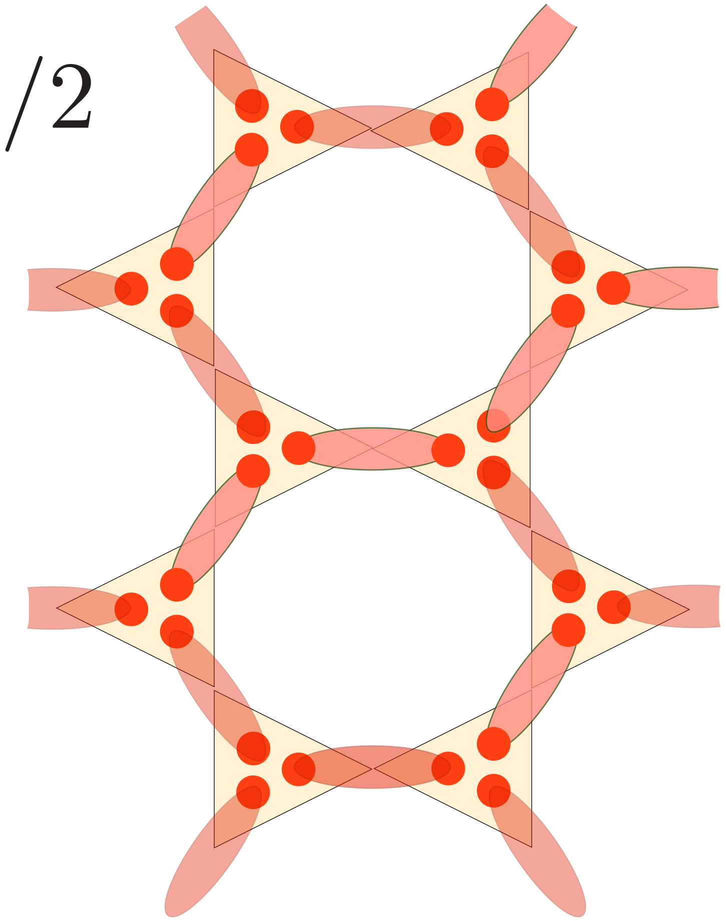
spin-S AKLT

$\mathrm{Bi_3Mn_4O_{12}(NO_3)}$ material

J. Lavoie et al., Nat. Phys. 6, 850 (2010)

M. Matsuda et al., Phys. Rev. Lett. 105, 187201 (2010)

$$S = z/2$$



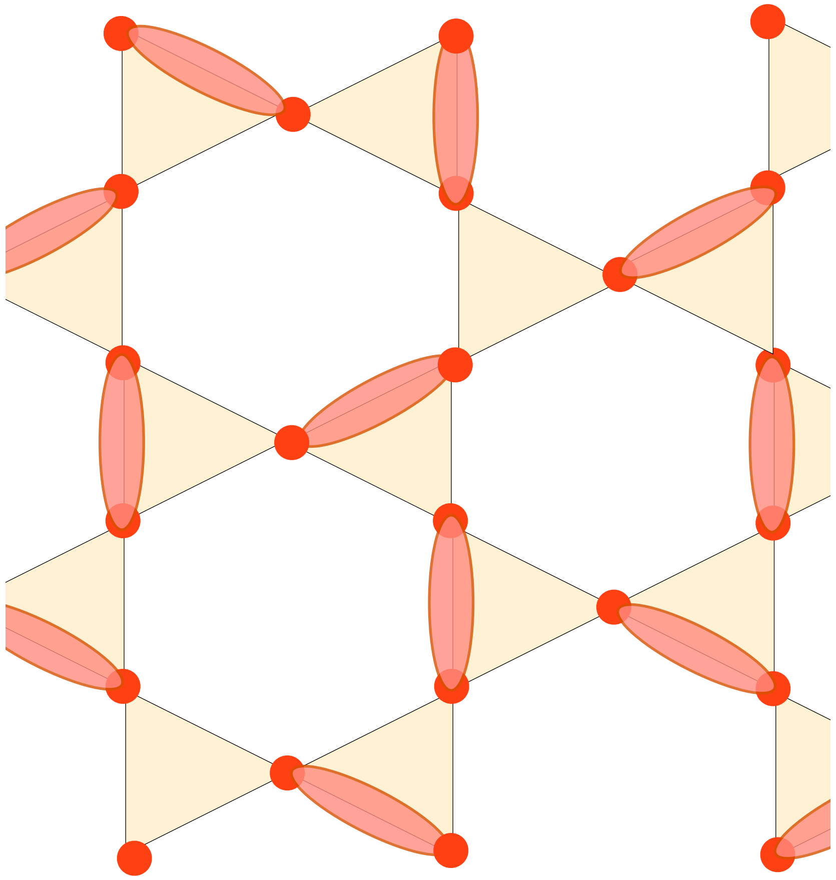
$$S = 0$$

«Trivial» liquid

spin-1/2 RVB

P. Fazekas and P.W. Anderson

Philosophical Magazine **30**, 423-440 (1974)

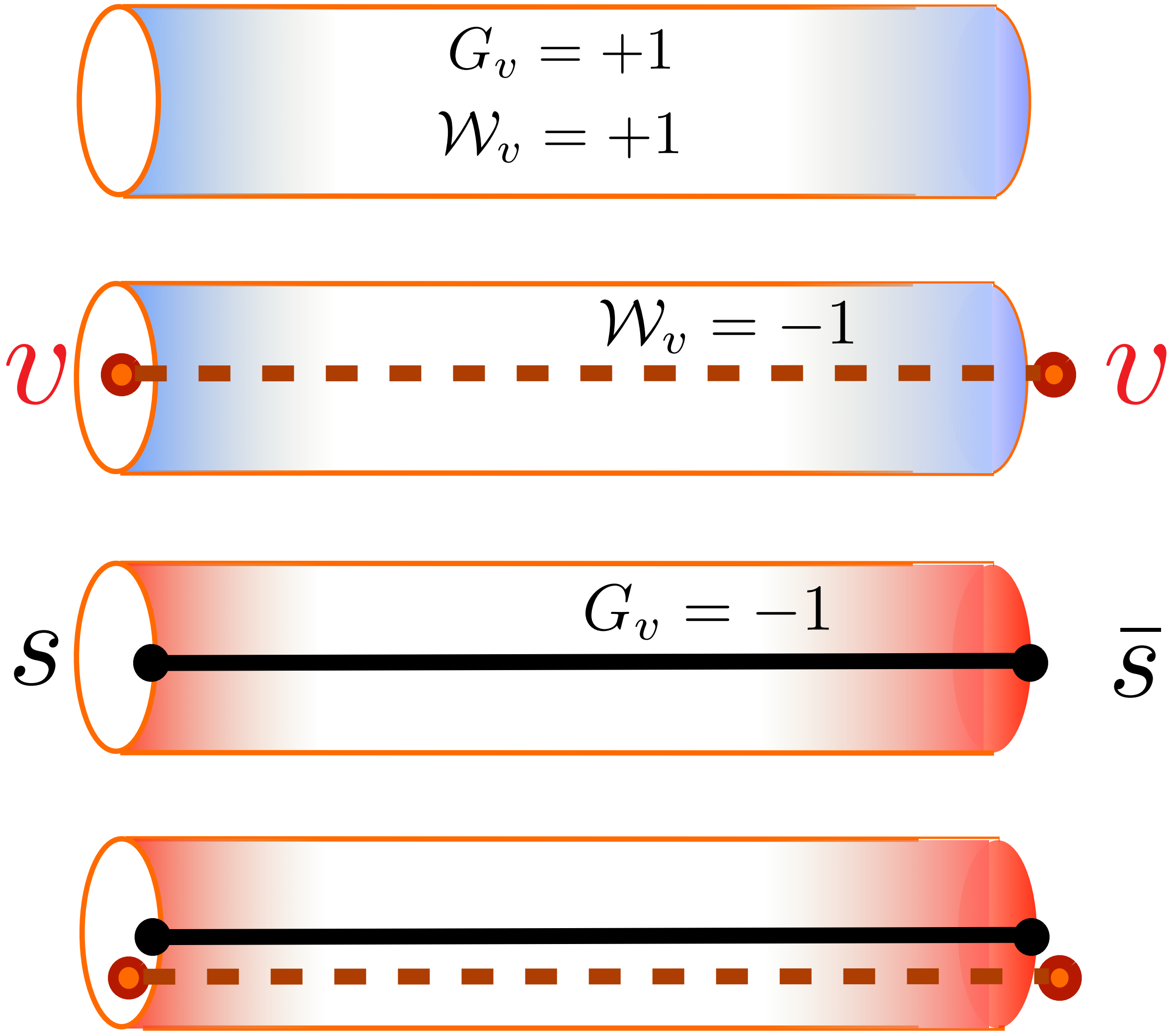


Equal-weight superposition
of NN singlet coverings

Topological liquid

Hasting-Oshikawa-LSM theorem

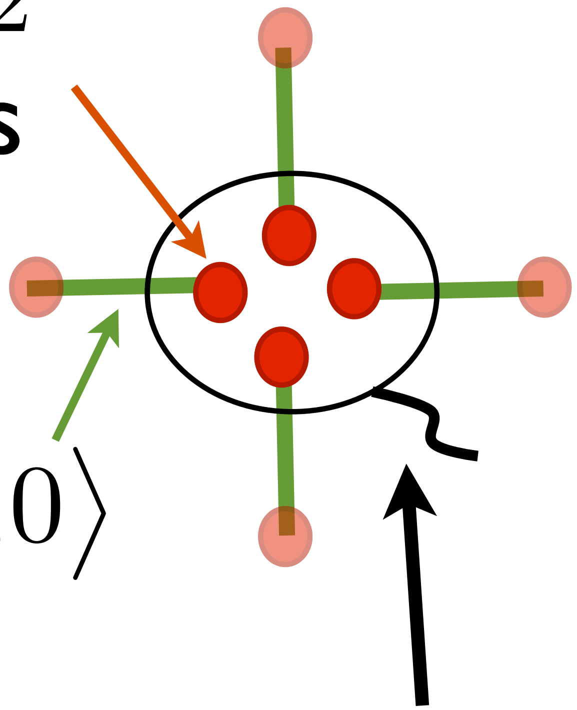
$\mathbb{Z}_2$ spin liquid :
topological GS inserting «spinons» and «visons»



same class as
Kitaev's Toric Code
(fixed point $\xi = 0$)

Projected Entangled Pair States (PEPS) construction

Ex.: the spin-2 AKLT

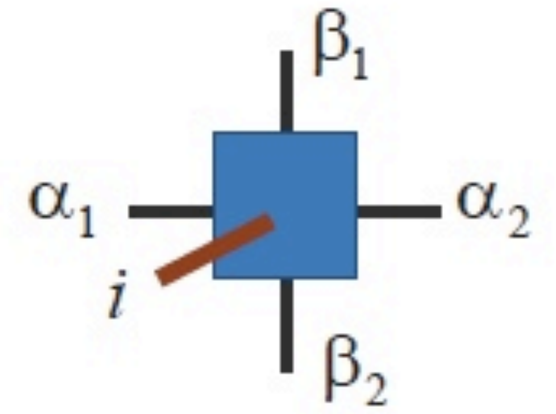


S=1/2

D=2 virtual states

$$|\mathcal{S}\rangle = |01\rangle - |10\rangle$$

Project onto physical subspace $d = 2S_{\text{phys}} + 1$:

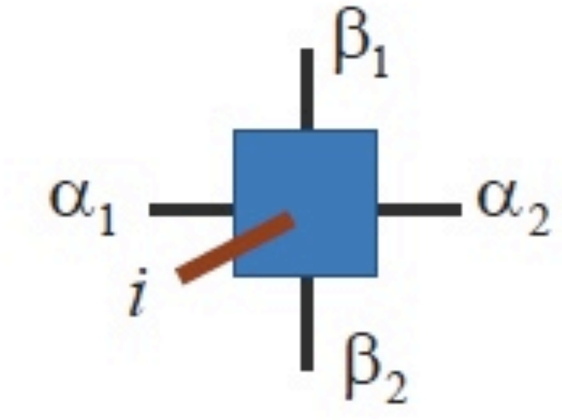


local tensor

The PEPS as a variational ansatz

$$|\Psi\rangle = \sum C_{i_1,i_2,\dots,i_N} |i_1, i_2, \dots, i_N\rangle$$

$$A^{i}_{\alpha_1,\alpha_2;\beta_1,\beta_2}$$



$$i = \{1, \cdots, d_{\text{phys}}\}$$

$$\alpha, \beta = \{1, \dots, D\}$$

dimension of auxilary (or virtual) space

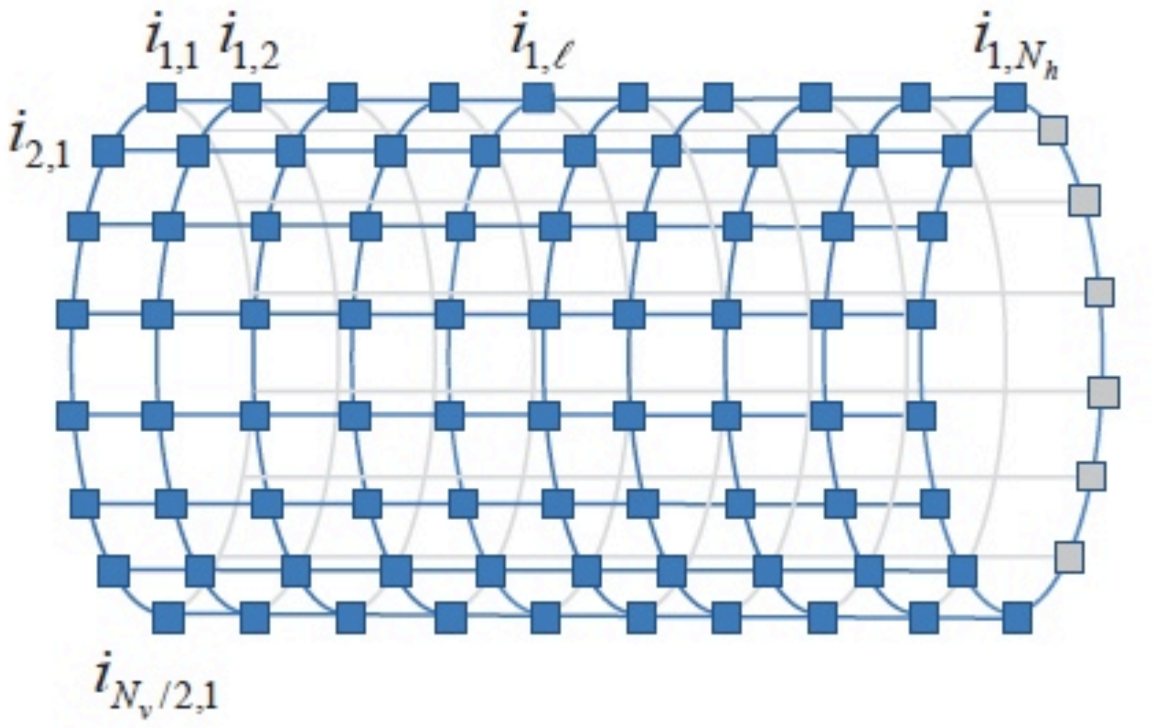
I. Cirac
F. Verstraete
G. Vidal

$$N = N_v N_h$$

Coefficients $C_{\{i_{1,1},\dots,i_{N_v,N_h}\}}$ of the wavefunction

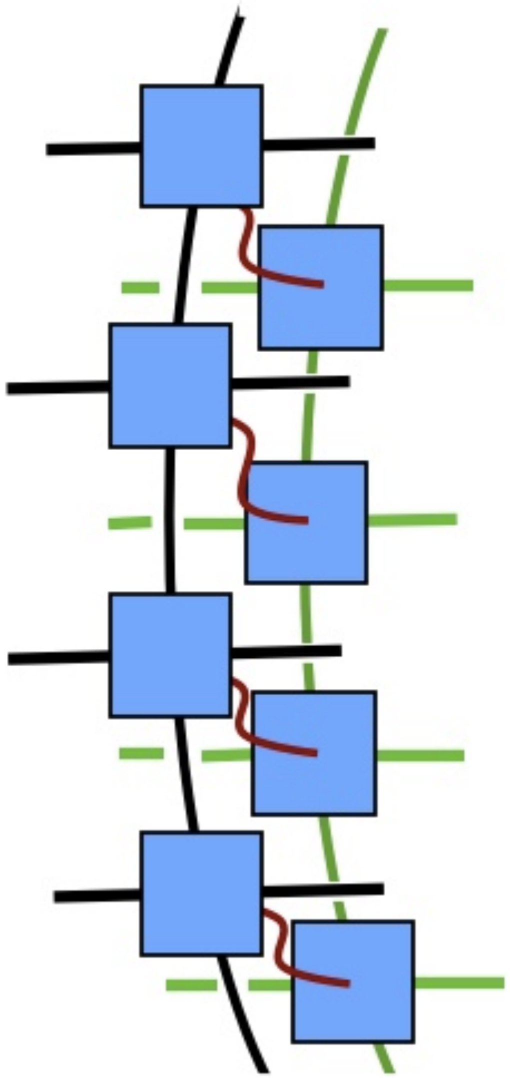
⇓

“contract” product of tensors



Build «double layer» tensor network by contracting physical variables

$\langle \Psi | \Psi \rangle$



«transfer matrix»

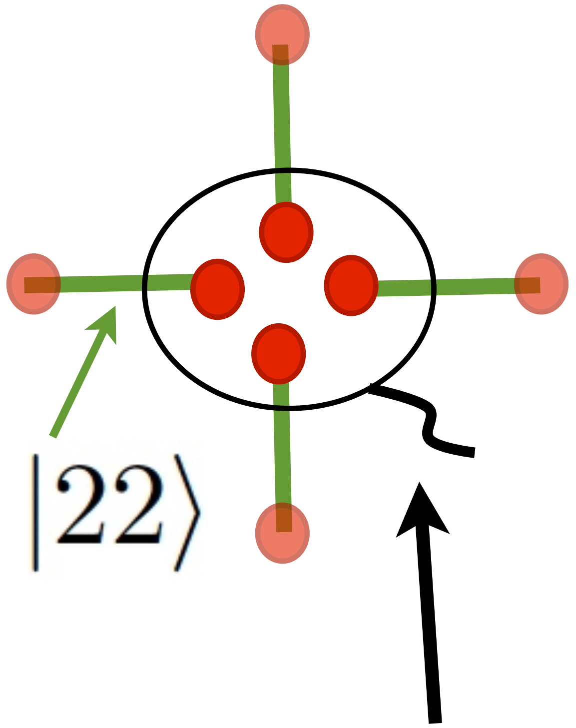
$D^{2N_v} \times D^{2N_v}$

Iterate product of TM's to build **infinite** cylinder

if D small enough **exact contractions** possible...

The spin-1/2 RVB can be written as a PEPS !

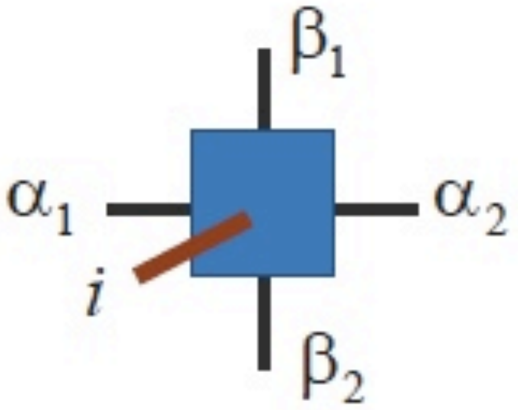
virtual states: $1/2 \oplus 0$
(D=3)



$$|\mathcal{S}\rangle = |01\rangle - |10\rangle + |22\rangle$$

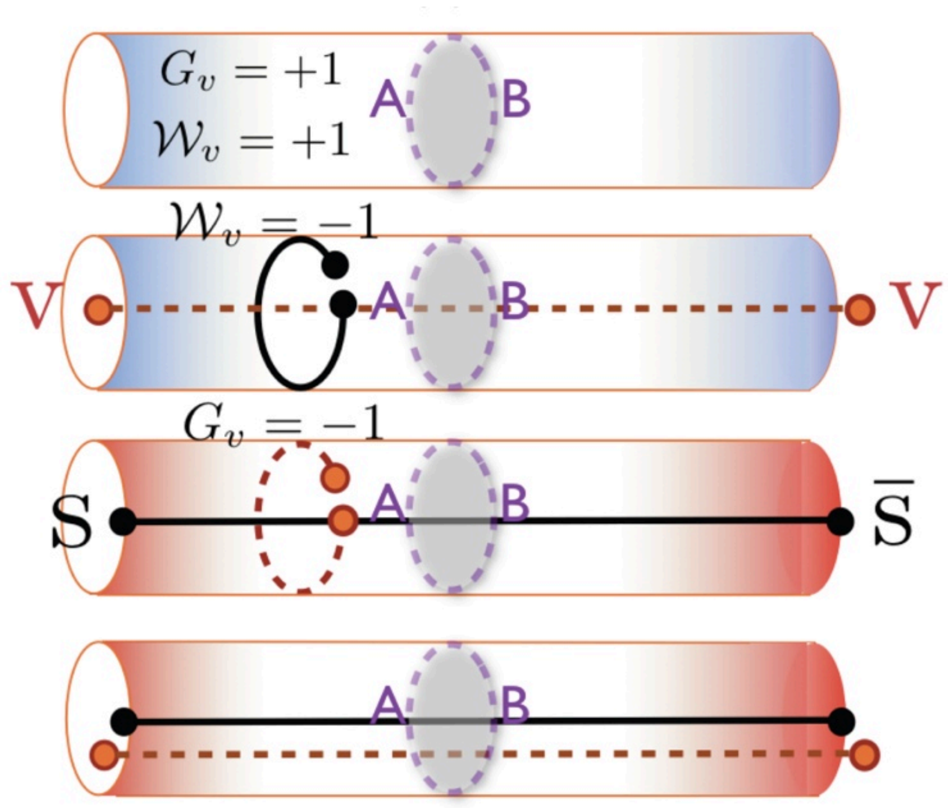
Project onto physical subspace S=1/2 (d=2)

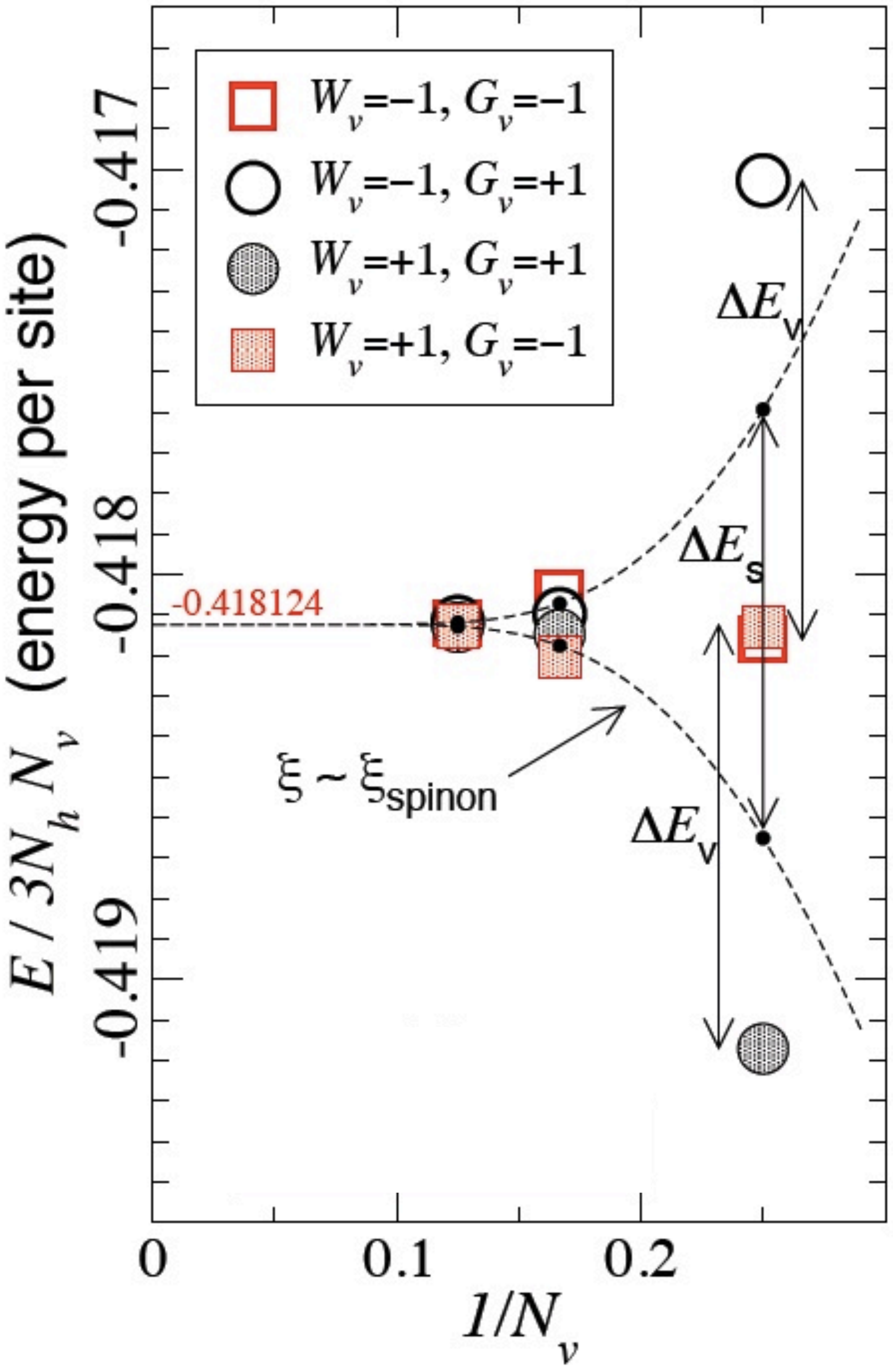
⇒



PEPS tensor

Finite size scaling of RVB energy



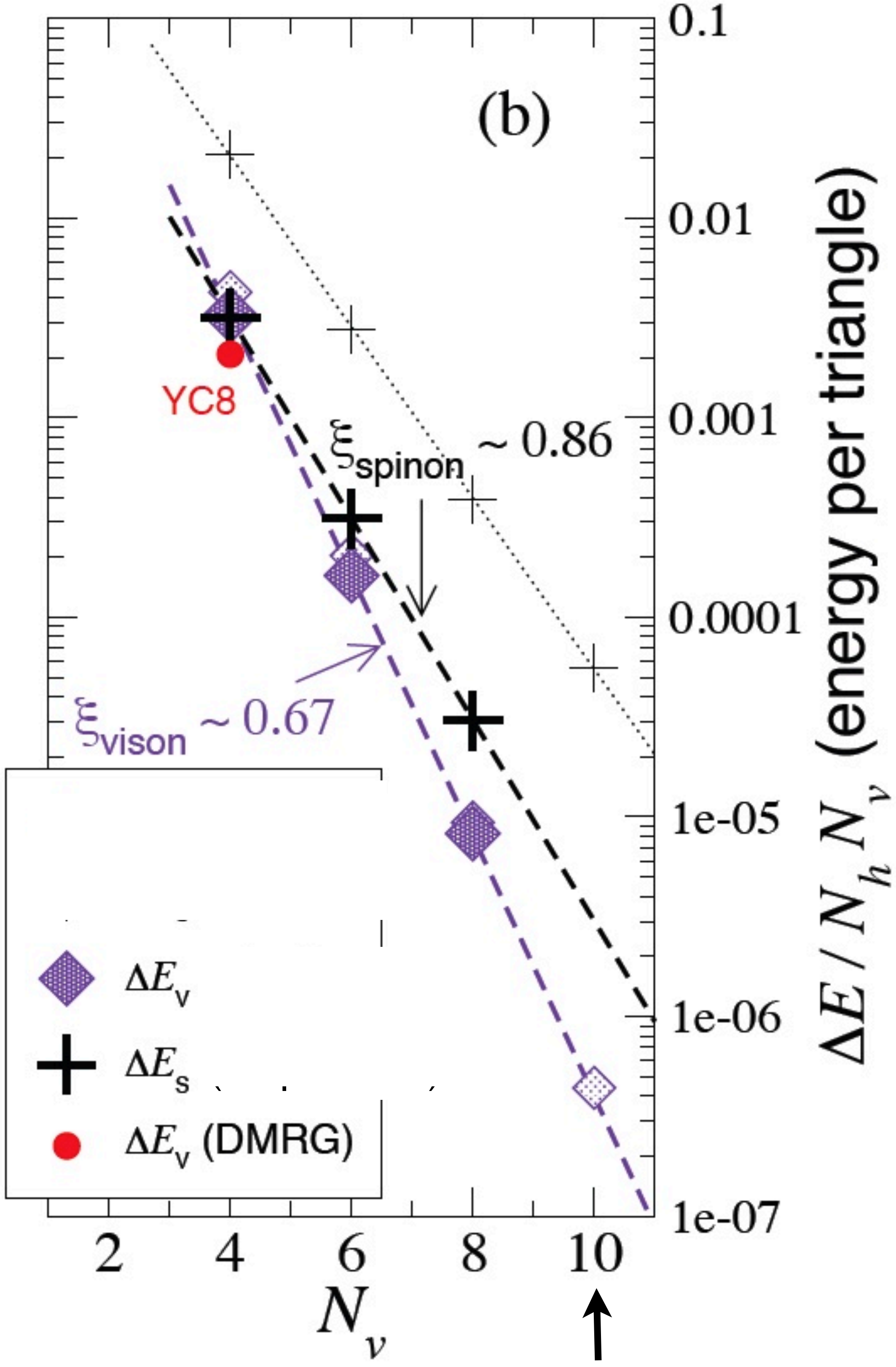


↑ ↑ ↑

YC16 YC12 YC8 cylinders

Gs energy splittings (semi-log scale)

$(N_h \to \infty)$



YC20

INFINITE CYLINDER

$$
\begin{aligned}
\Delta E_{\mathrm{s}} &= a N_h N_v \exp\left(-N_v/\xi_{\mathrm{spinon}}\right), \\
\Delta E_{\mathrm{v}} &= b N_h N_v \exp\left(-N_v/\xi_{\mathrm{vison}}\right),
\end{aligned}
$$

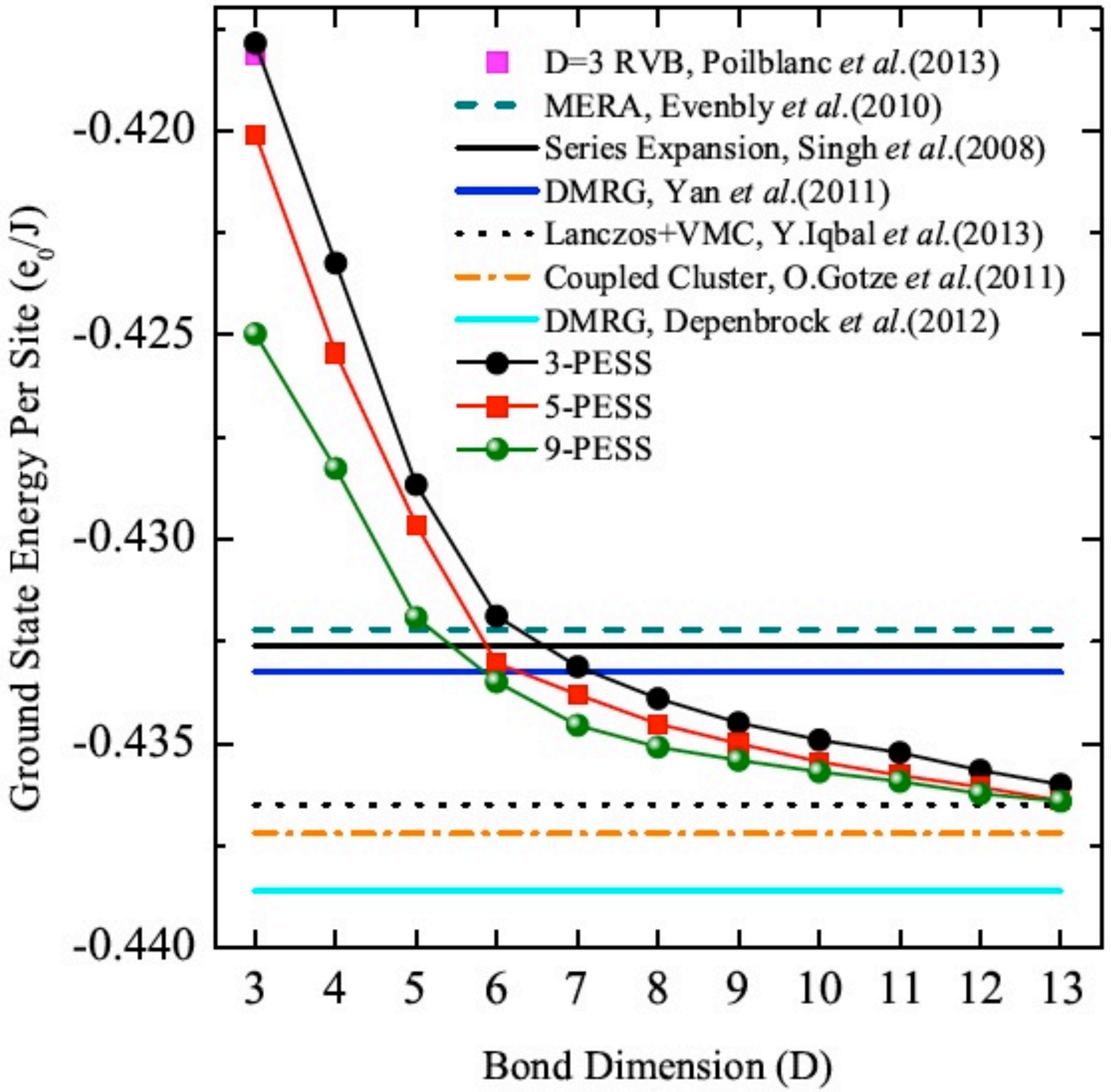
Very short coherence lengths

$\xi < 1$ unit cell

Improving the RVB / PEPS ...

«Projected Entangled Simplex»

Z. Y. Xie, J. Chen, J. F. Yu, X. Kong, B. Normand, and T. Xiang, arXiv:1307.5696



Simple update method
based on imaginary-time evolution

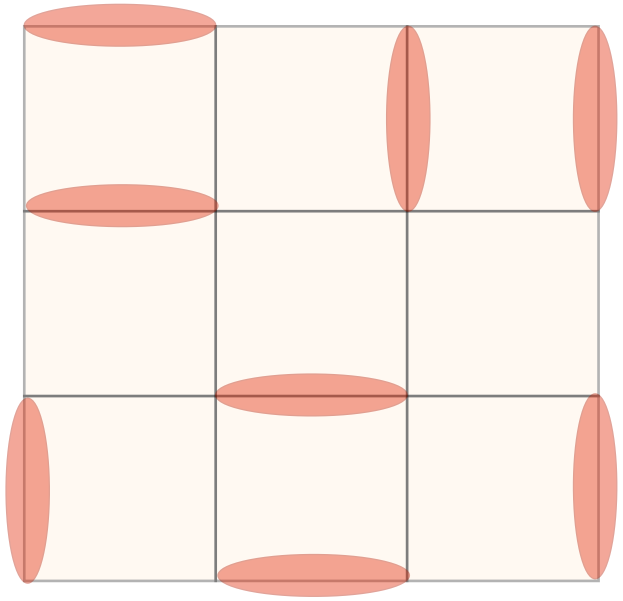
Fermionic **charge degrees of freedom** in the PEPS framework

The RVB spin liquid is the «parent» insulator of the high-Tc superconductor P.W. Anderson, T.M. Rice, etc...

Could a d-wave superconductor emerge from doping the RVB state
(on the square lattice) ?

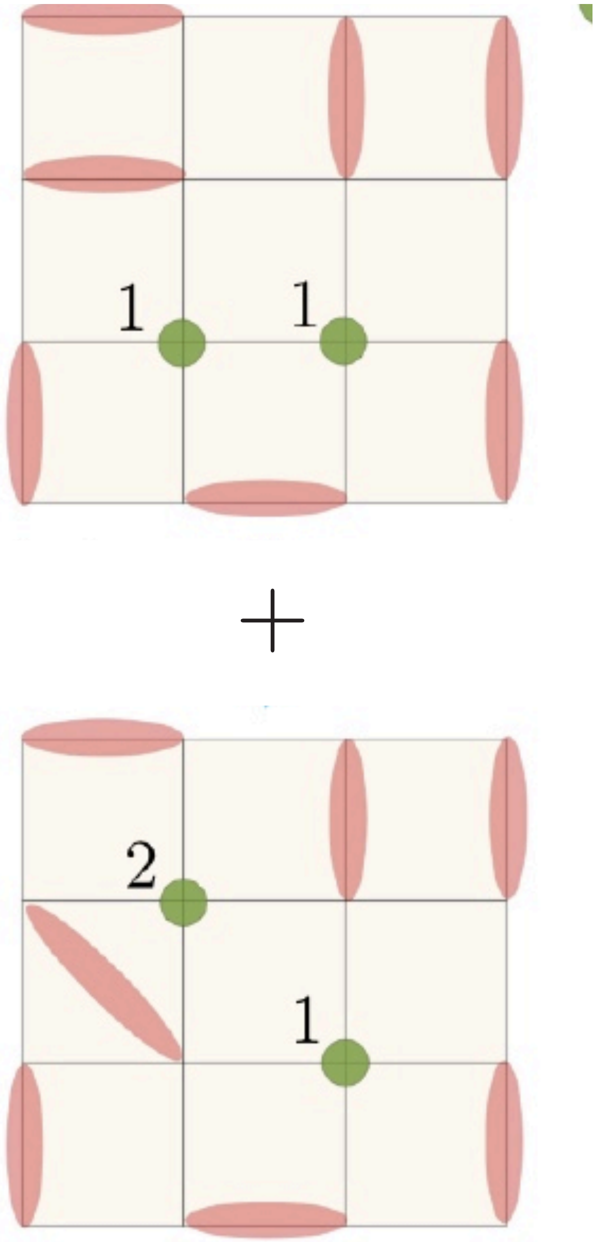
RVB spin liquid

$$\sum_{\mathrm{VB}}$$



doping

$$\sum_{\mathrm{VB+holes}}$$



$+ \cdots$

fermion representation !

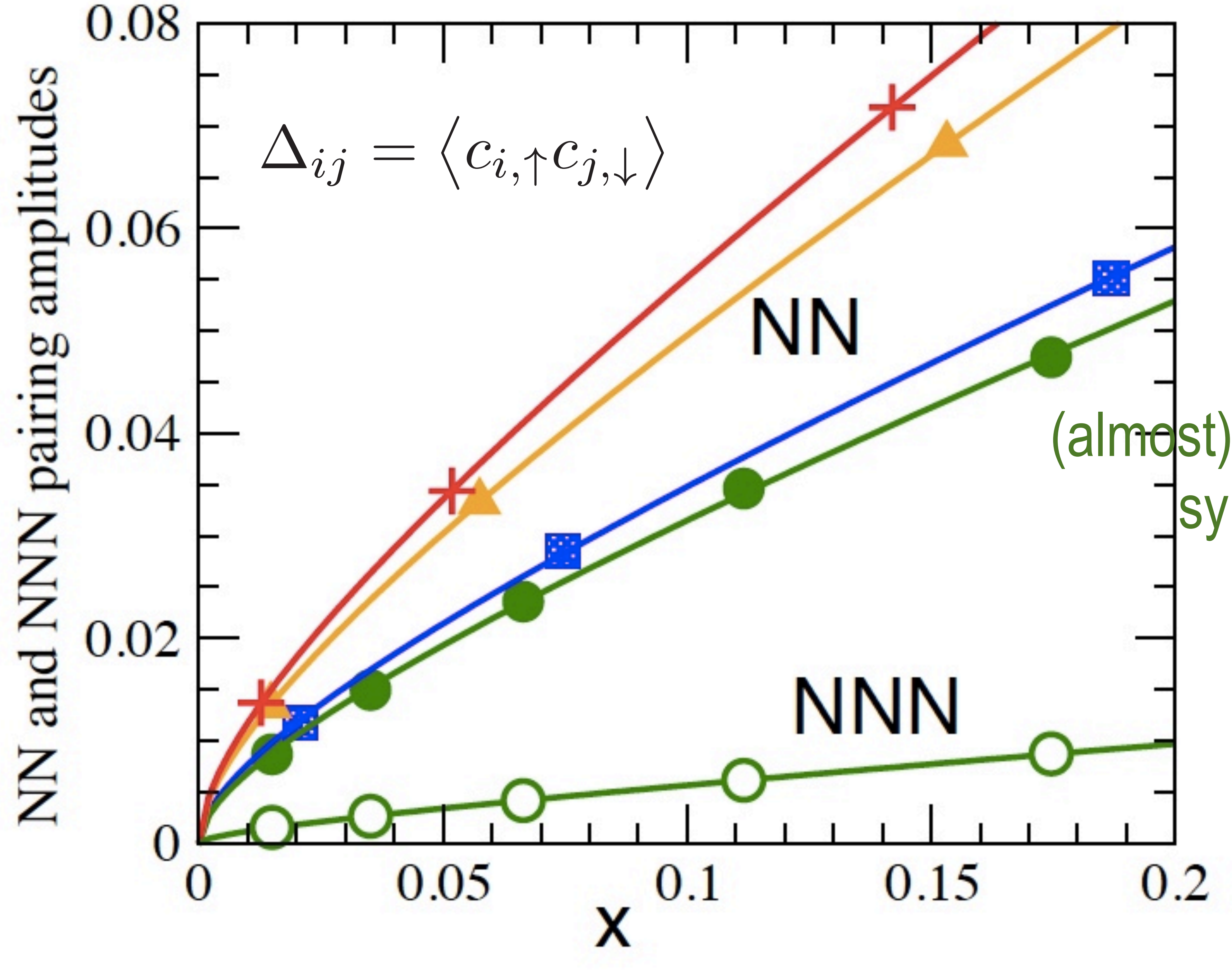
(Corboz, Evenbly, Verstraete, Vidal, PRA 2010)

Mott insulator

Superconductor ?

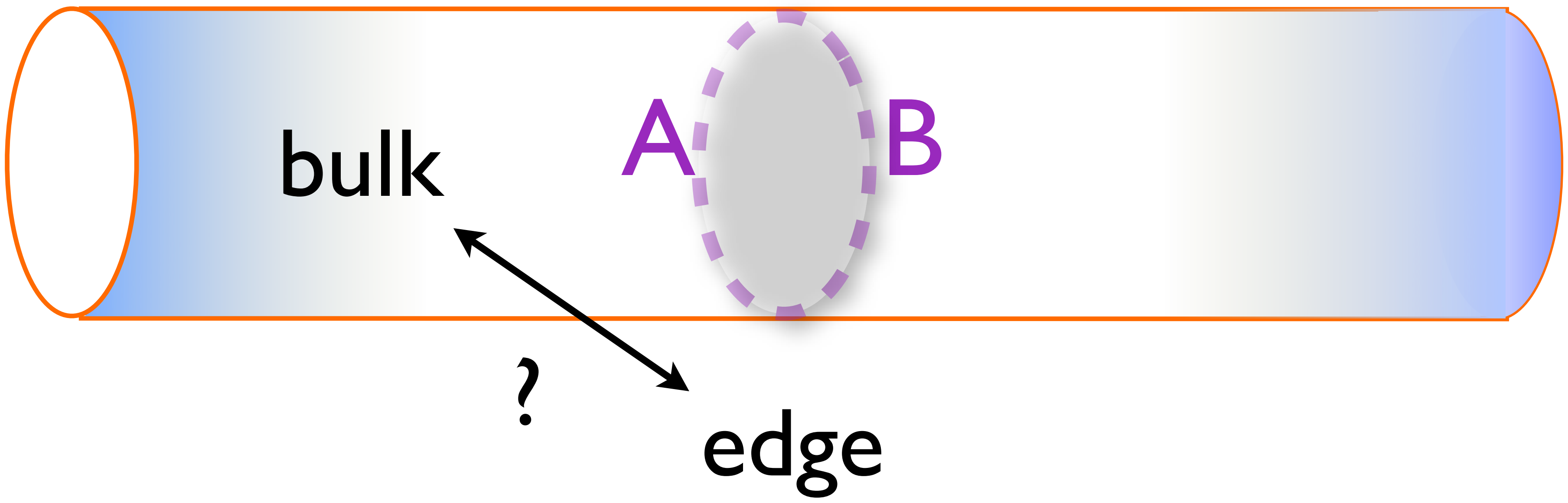
Superconducting pairing amplitude

d+is symmetry



(almost) pure d-wave symmetry

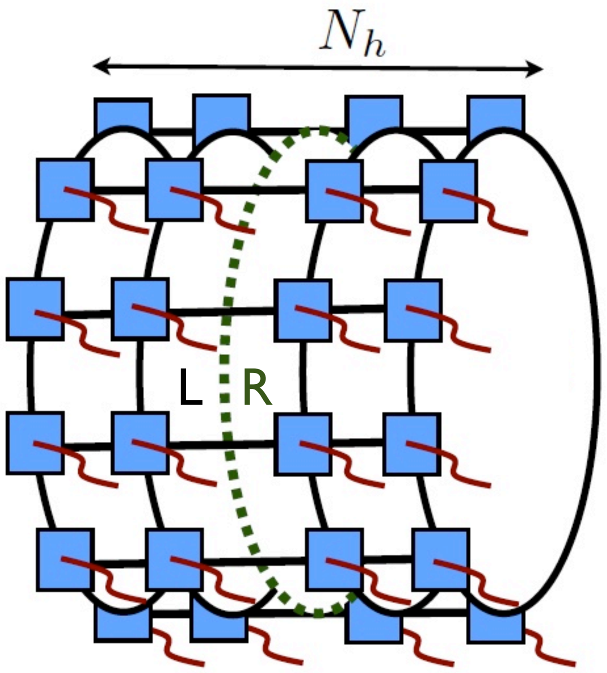
«Holographic» framework



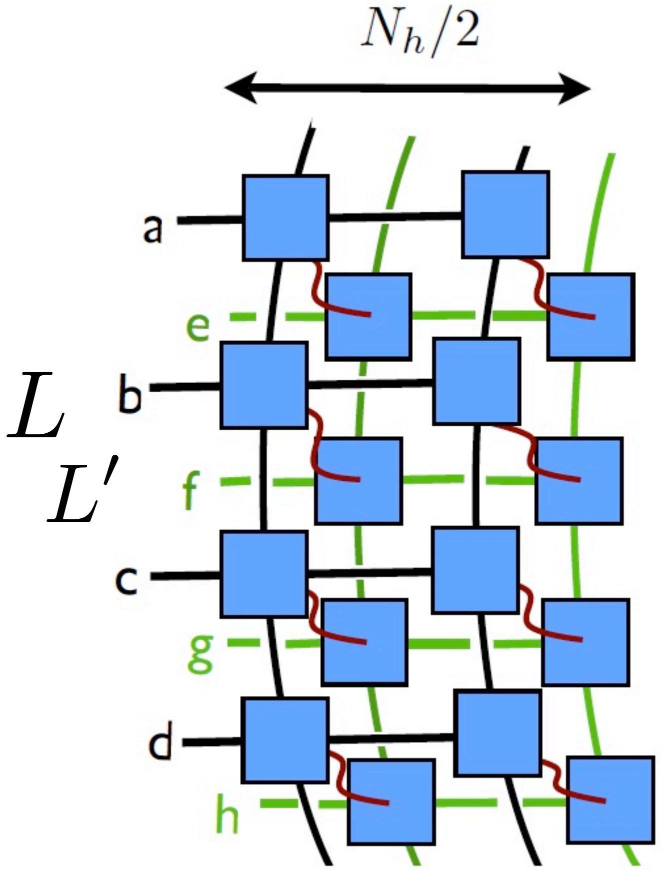
$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

Reduced density matrix

ρ_A



l A B



σ_b^2

lives” on the boundary

Basic formula: $\rho_A = U\sigma_b^2 U^\dagger$

isometry: maps 2D onto 1D

J. Ignacio Cirac, DP, Norbert Schuch, Frank Verstraete
Phys. Rev. B 83, 245134 (2011)

Entanglement entropy

$$S_{\mathrm{VN}} = -\mathrm{Tr}\{\rho_A \ln \rho_A\} = \text{-}\mathrm{Tr}\{\sigma_b \ln \sigma_b\}$$

(Von Neumann)

“area” law

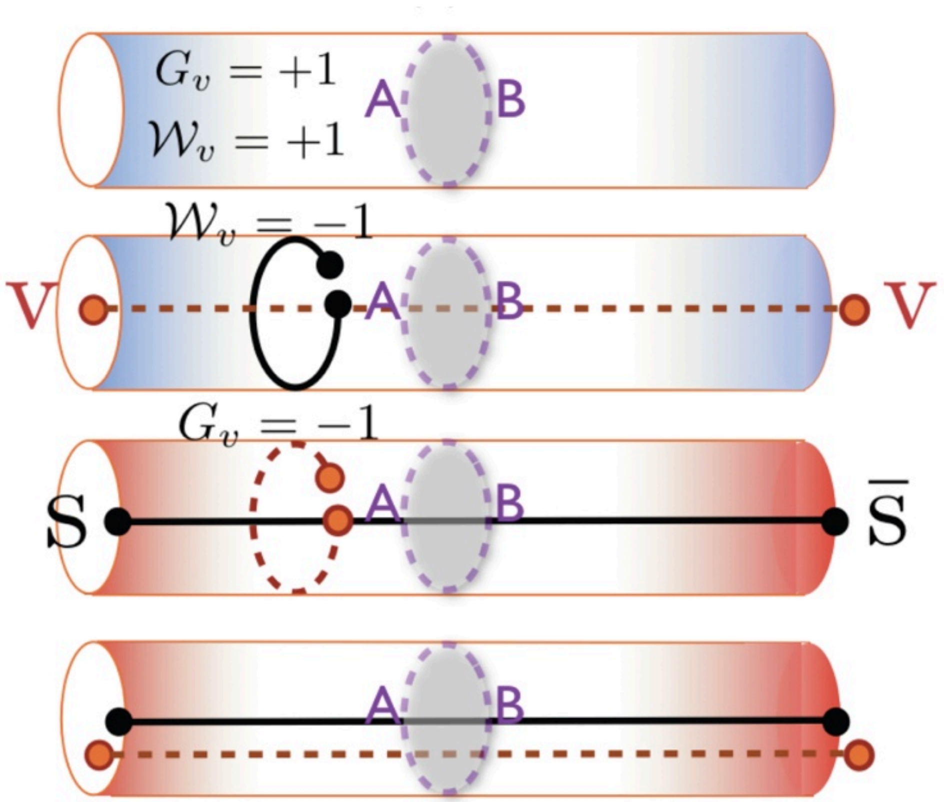
$$S_{\mathrm{VN}} \sim C N_v - \ln D$$

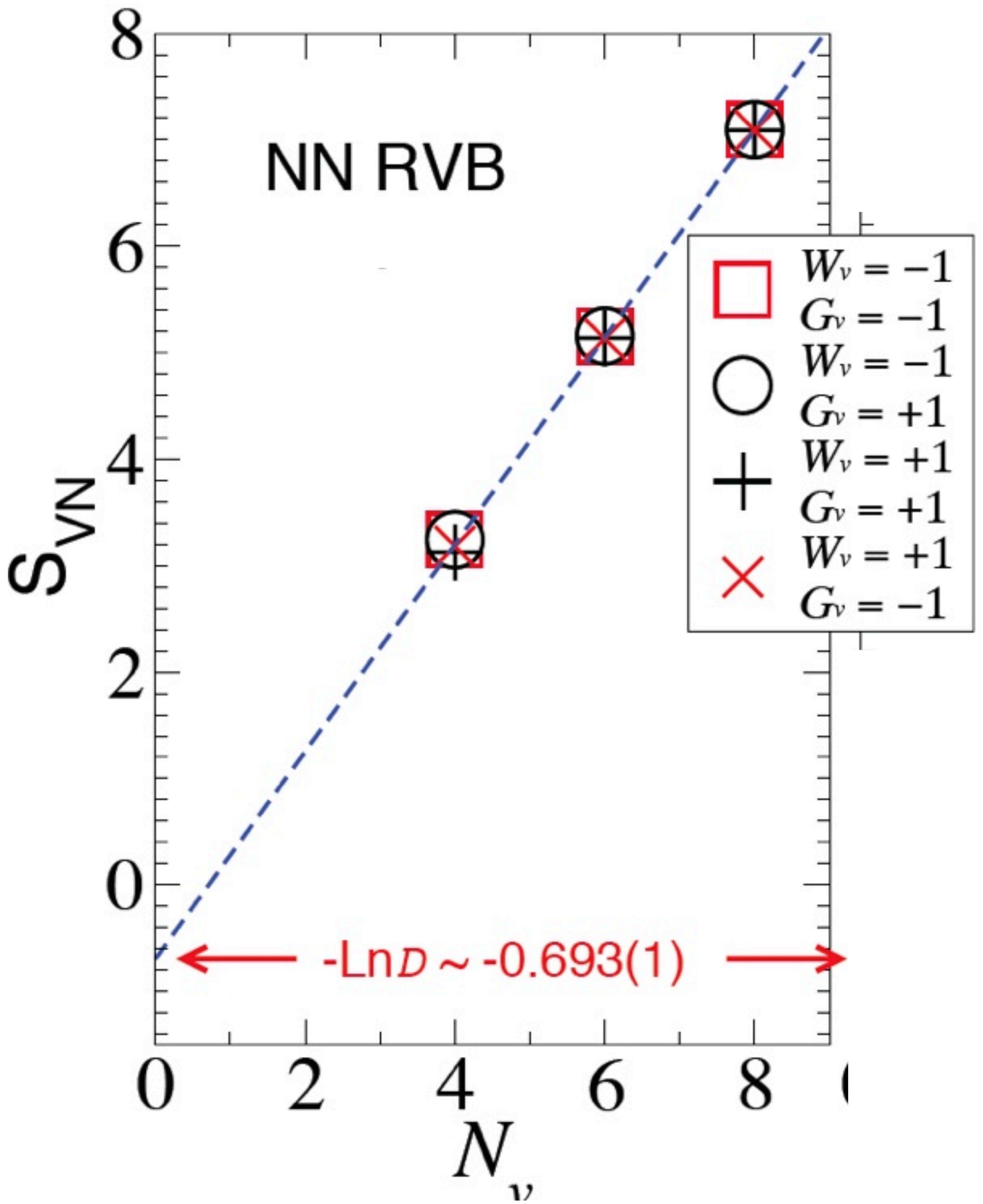
subleading correction to area law:
topological entropy

Kitaev & Preskill, 2006

Levin & Wen, 2006

Numerical results





$S_{\mathrm{TE}} \simeq -\ln 2$ → $\mathbb{Z}_2$ spin liquid

Entanglement Hamiltonian (acting on the edge)

$$\sigma_b^2 = \exp\left(-H_b\right)$$

The spectrum of Hb is in one-to-one
with the true edge spectrum !

Li & Haldane

local operator (on the edge):
$D \times D$ matrix $\Rightarrow$ basis of D^2 operators

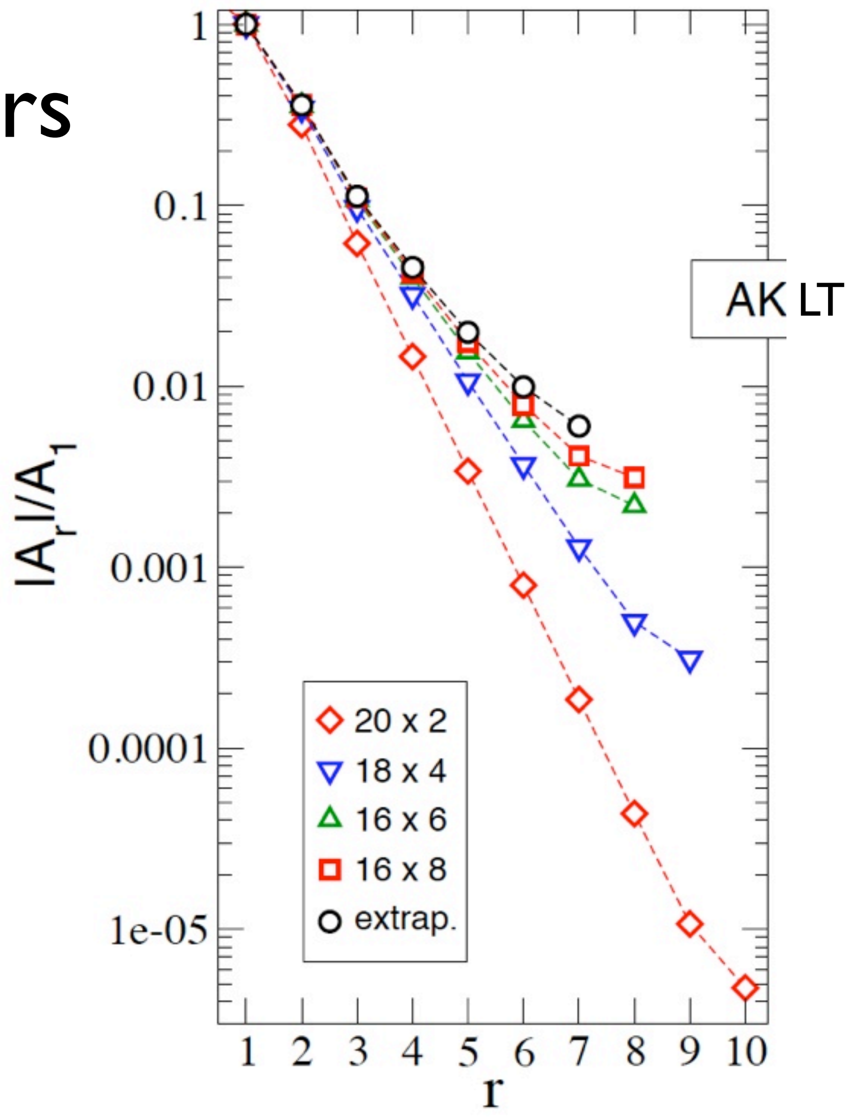
$$\begin{aligned} \mathcal{O}_{\rm edge} &= c_0 N_v + \sum_{\lambda,i} c_\lambda \hat{x}^i_\lambda + \sum_{\lambda,\mu,r,i} d_{\lambda\mu}(r)\, \hat{x}^i_\lambda \hat{x}^{i+r}_\mu \\ &+ \sum_{\lambda,\mu,\nu,r,r',i} e_{\lambda\mu\nu}(r,r')\, \hat{x}^i_\lambda \hat{x}^{i+r}_\mu \hat{x}^{i+r'}_\nu + \cdots , \end{aligned}$$

Is it local ?

Trivial gapped spin liquid

Short-range entanglement Hamiltonian

AKLT cylinders
(D=2)



$$H_b = A_0 N_v + \sum_{r,k} A_r \, \mathbf{S}_k \cdot \mathbf{S}_{k+r} + R\hat{X}$$

$$A(r) \sim \exp\left(-r/\xi_b\right)$$

ξ_b traks bulk correlation length

Topological gapped spin liquid

Entanglement Hamiltonian highly non-local

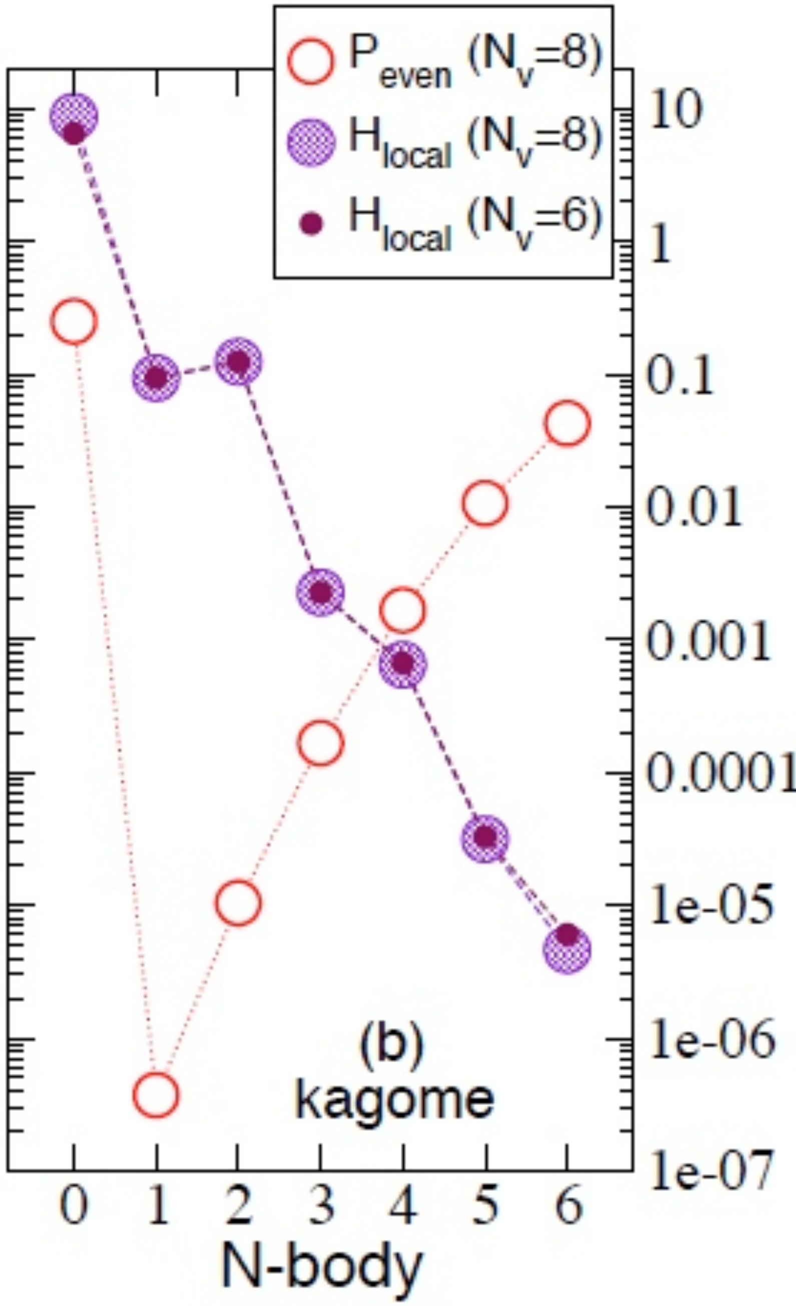
$$\tilde{H}_b = H_1 + \beta_\infty(\mathbf{1}^{\otimes N_v} - \mathcal{P}) \qquad \beta_\infty \to \infty$$

$$H_1 = H_{\mathrm{local}}\mathcal{P}$$

supported by the non-zero eigenvalue sector of the RDM

projector characterizing topological sectors

Exemple: Kagome RVB



CONCLUSION

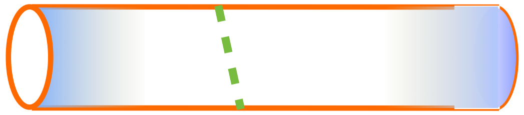
* Qualitative understanding of (simple) correlated phases (topo SL, SC, incompressible phases, supersolids, nematics,...)

* Systematic improvement can be made for physical Hamiltonians : iPEPS

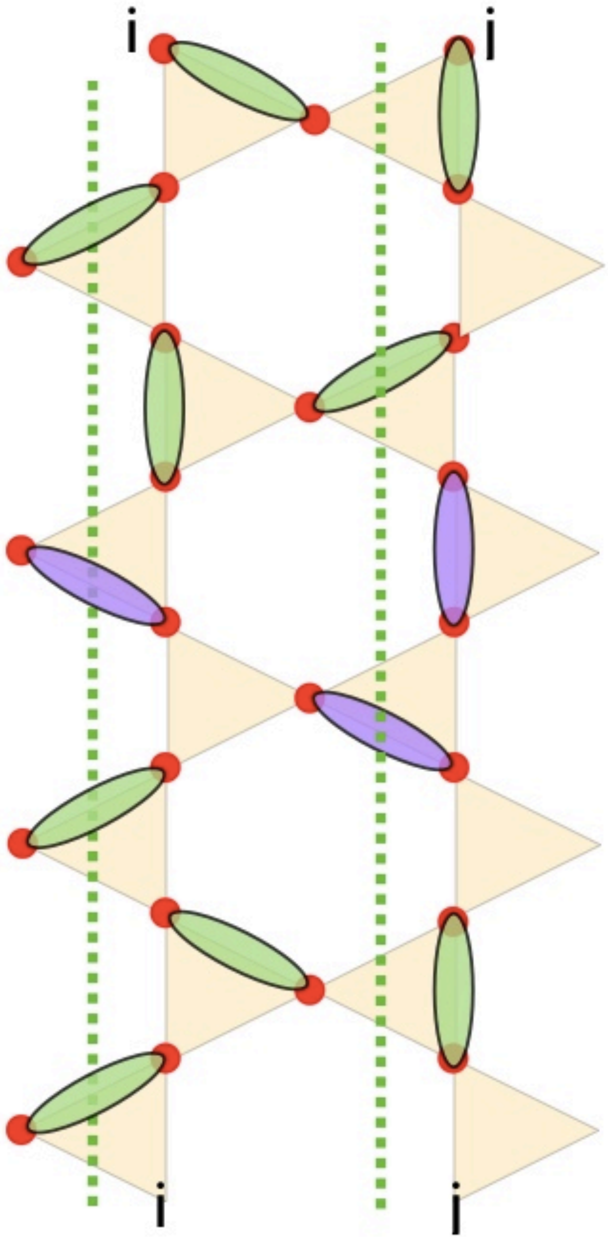
OUTLOOK

* Chiral RVB SL ?
* Attack microscopic models with optimization schemes

Fix the cylinder boundaries

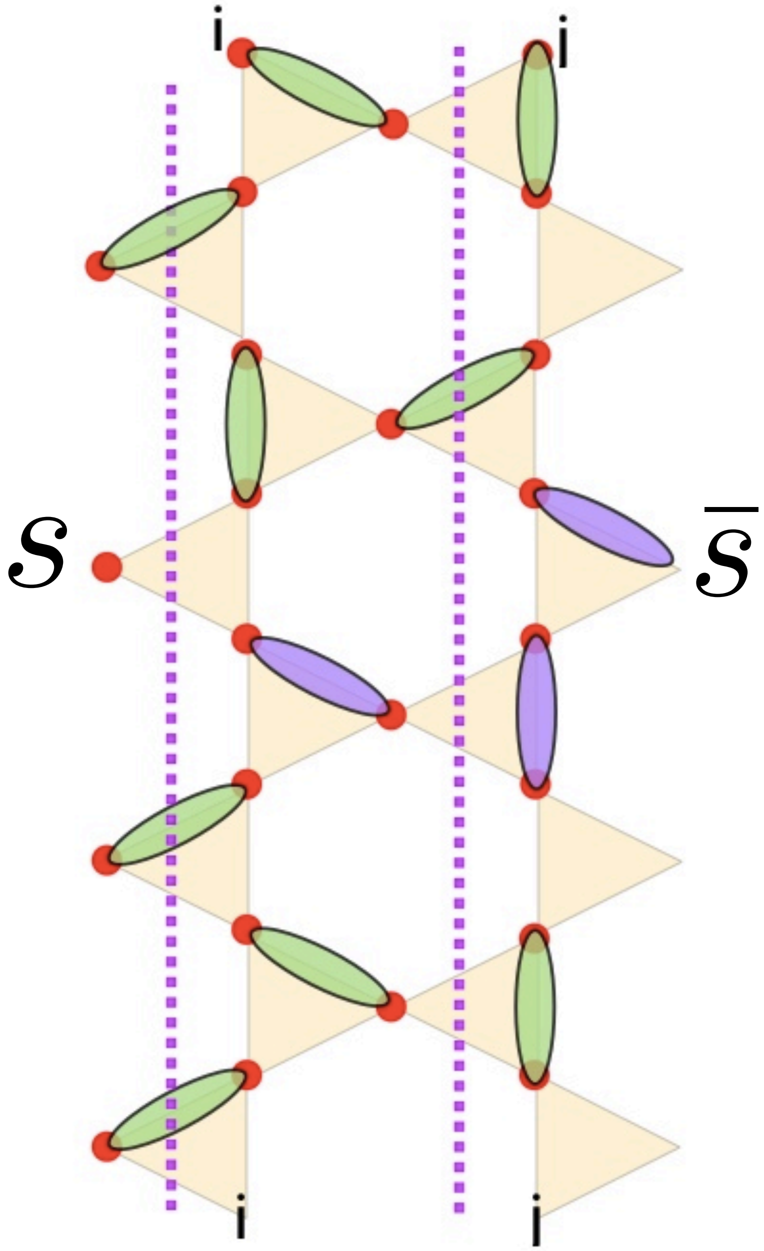


cylinder geometry



«even»

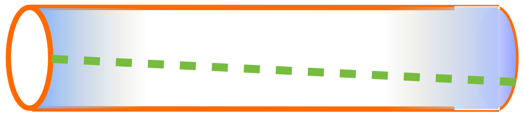
$$G_v = +1$$



«odd»

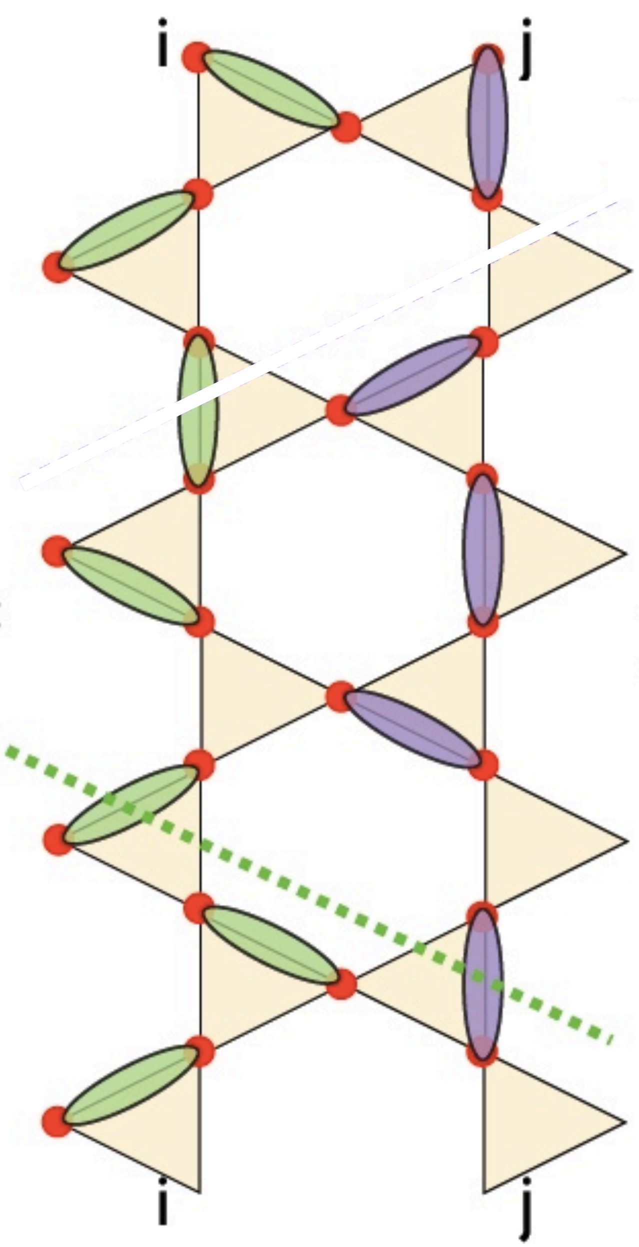
$$G_v = -1$$

Eigenstates of a «Wilson loop» operator

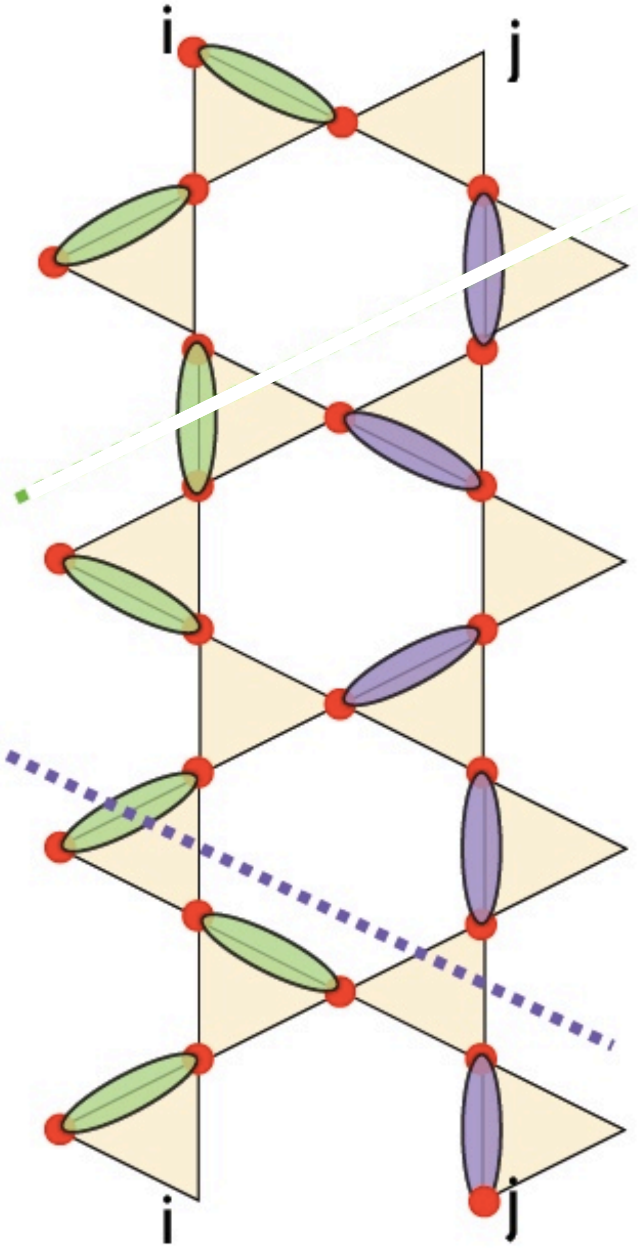


cylinder geometry

$$\Psi^{\pm}_{\mathrm{RVB}} =$$



$$\pm$$



$+$ no vison flux: $w_v = +1$

$-$ $\mathbb{Z}_2$ vison flux: $w_v = -1$

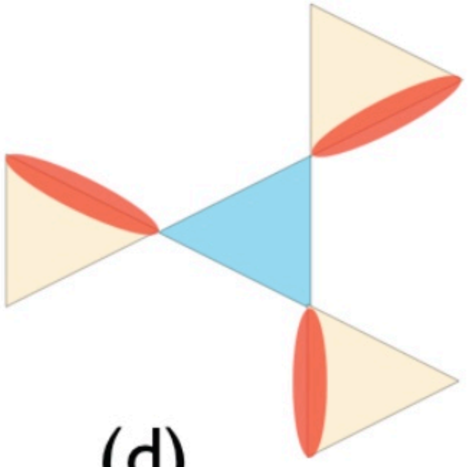
Improving the RVB / PEPS ...

Step 1: The «Simplex RVB»

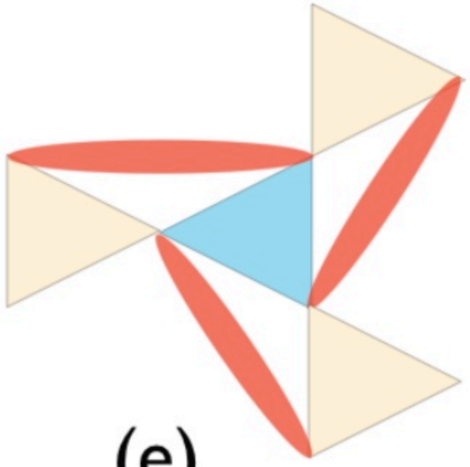
«defect triangles» cost energy !

dressing by introducing NNN singlets

(Zeng & Elser 90')

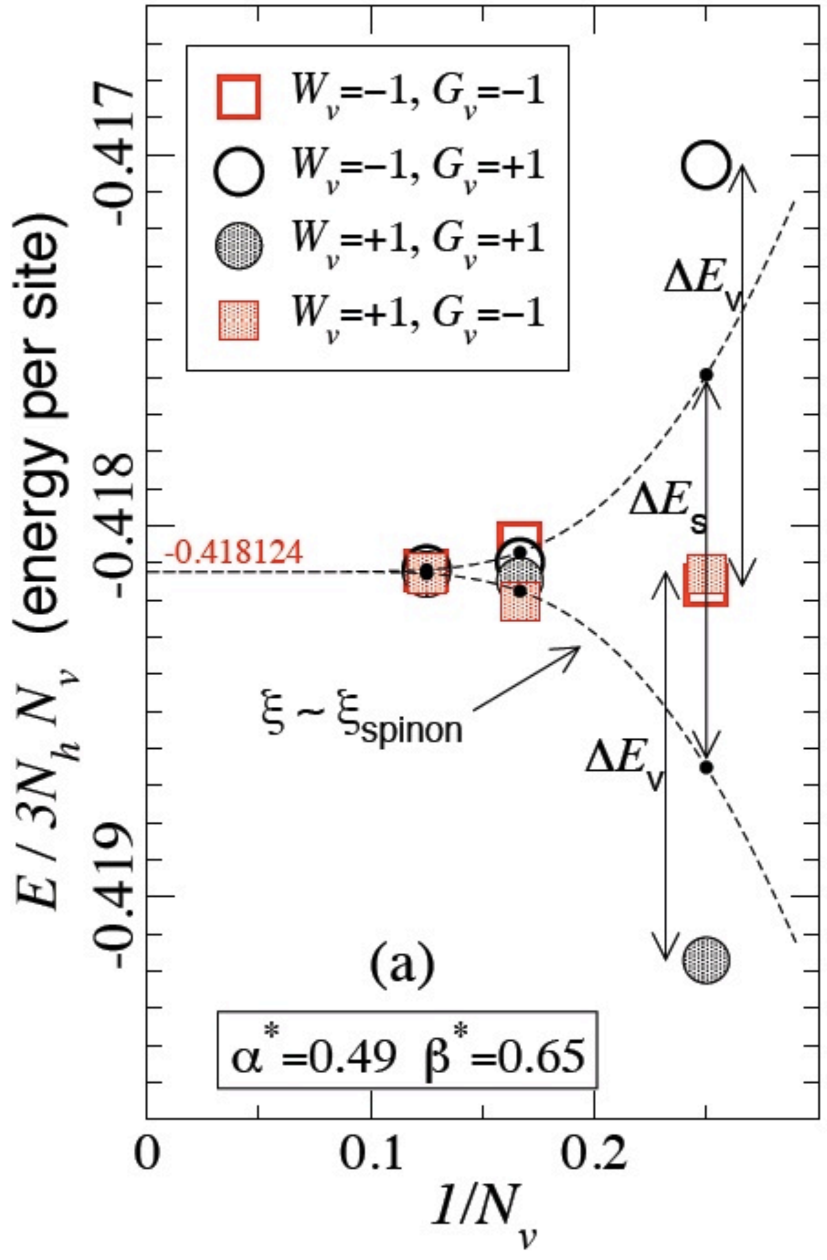


(d)

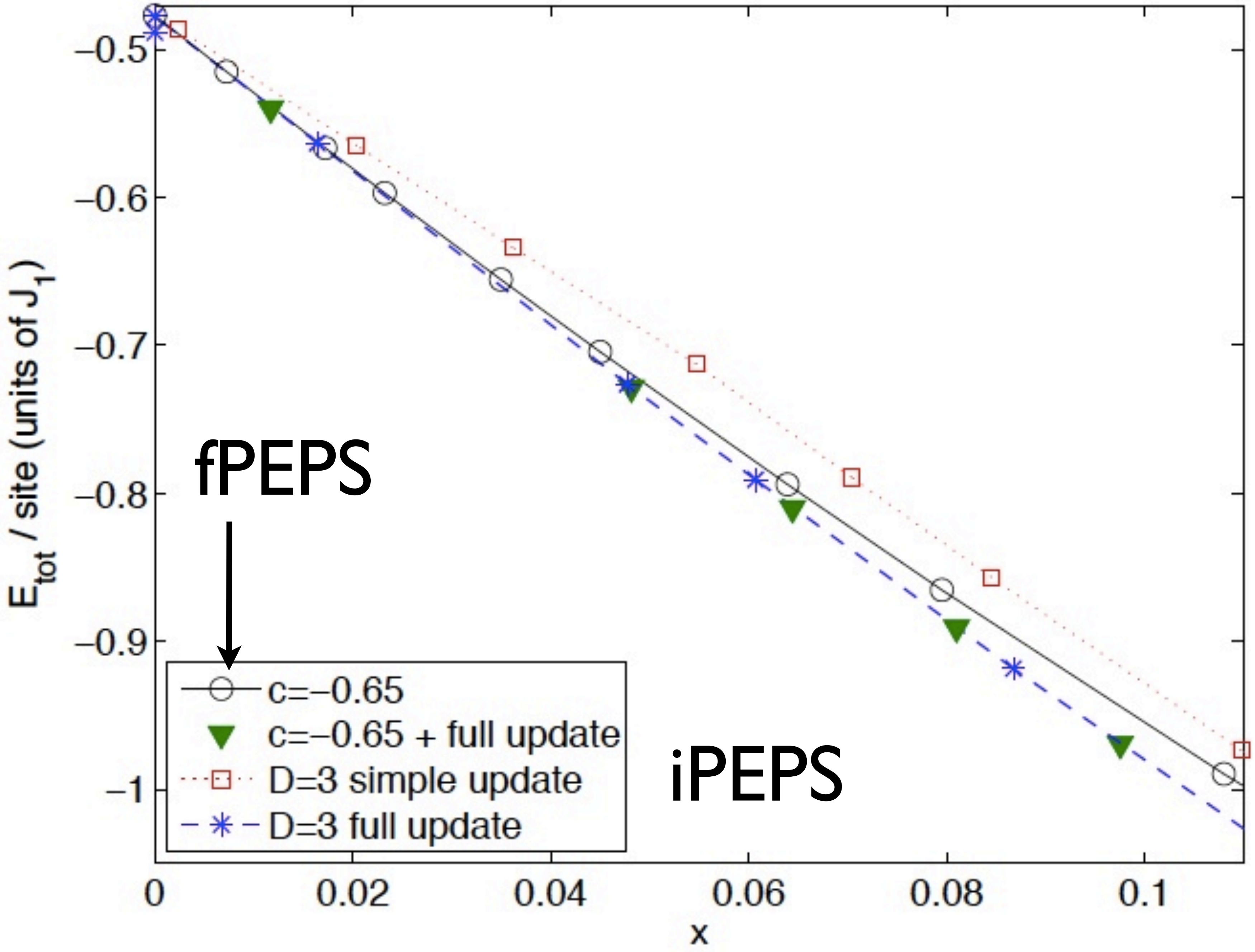


(e)

Finite size scaling of energy



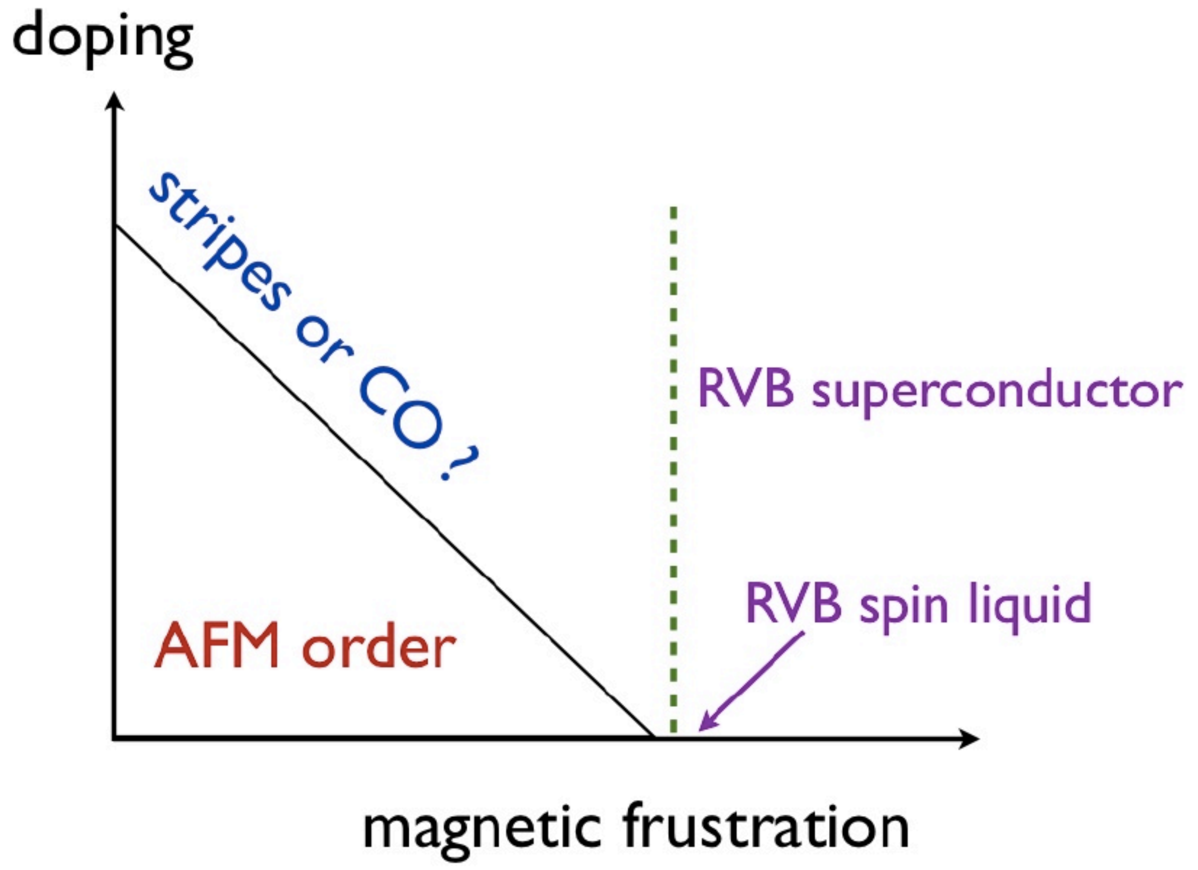
Variational energy : fPEPS vs iPEPS



Very good ansatz !

RVB superconductors from doping the RVB state (on the square lattice) ?

Idea #1: the RVB spin liquid is the «parent» insulator of the high-Tc superconductor P.W. Anderson, T.M. Rice, etc...



Idea #2: magnetic frustration is the key parameter to stabilize a (RVB) spin liquid on the 2D square lattice

Ling Wang, DP, Zheng-Cheng Gu, Xiao-Gang Wen, Frank Verstraete
PRL **111** 037202 (2013)

RVB superconductors for high-Tc superconductivity

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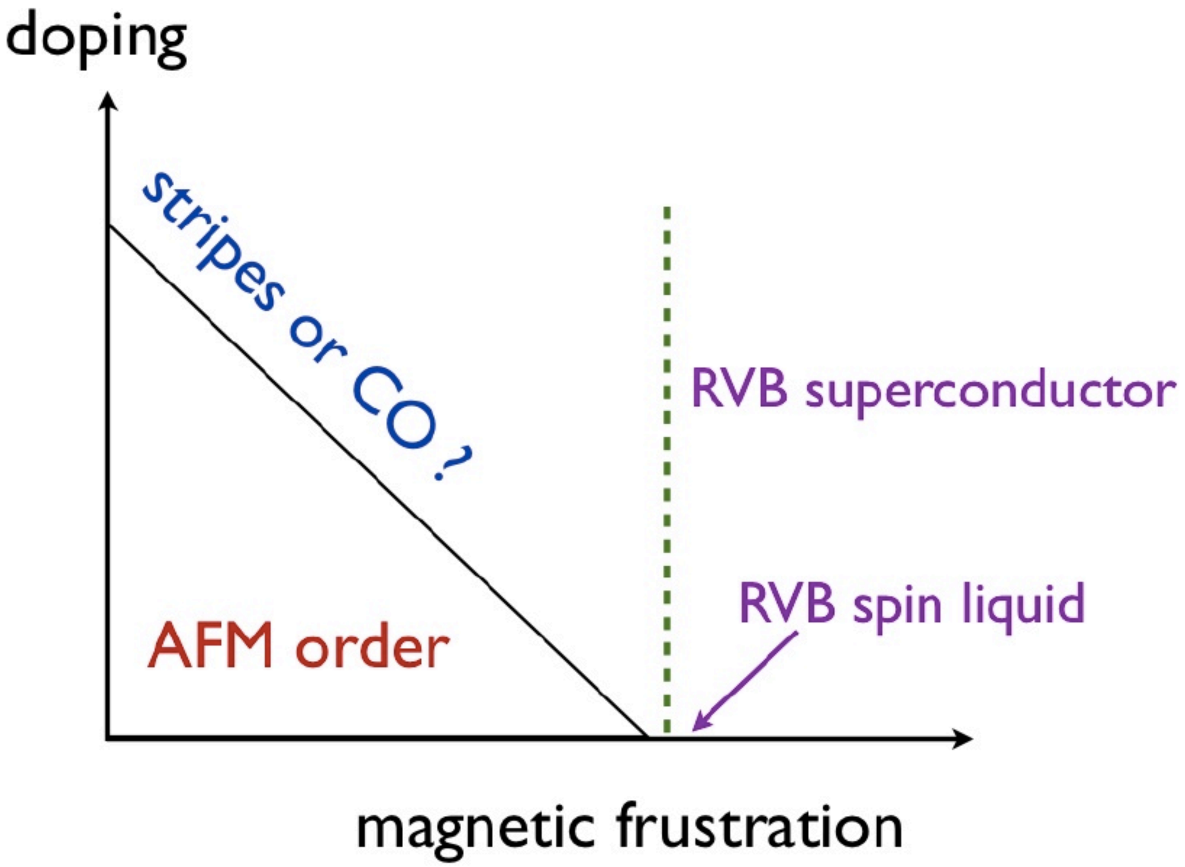

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