

# In-plane FFLO instability in multilayered S/F systems

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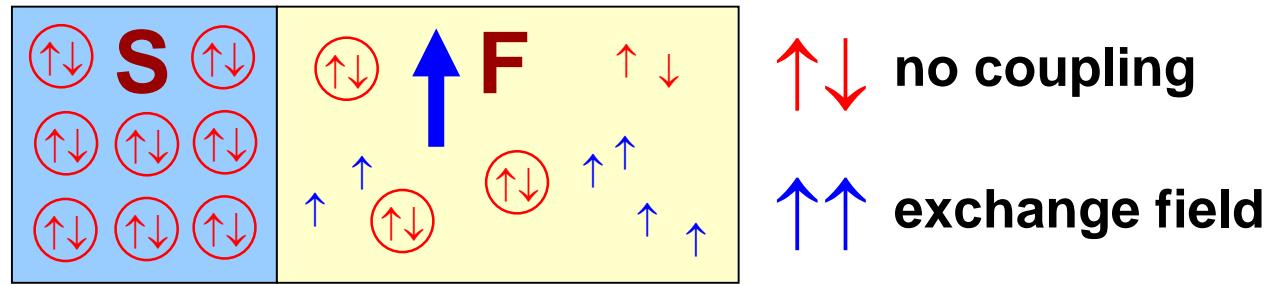
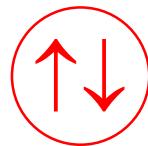


# Outline

1. Proximity effect in S/F systems
2. Can Meissner response become paramagnetic?
3. In-plane FFLO states in S/F systems
4. How experimentally observe the FFLO states?

# Proximity effect in S/F systems

Exchange field (energy) in the ferromagnet:  $\hat{H} = \dots + \vec{h} \cdot \hat{\sigma}$



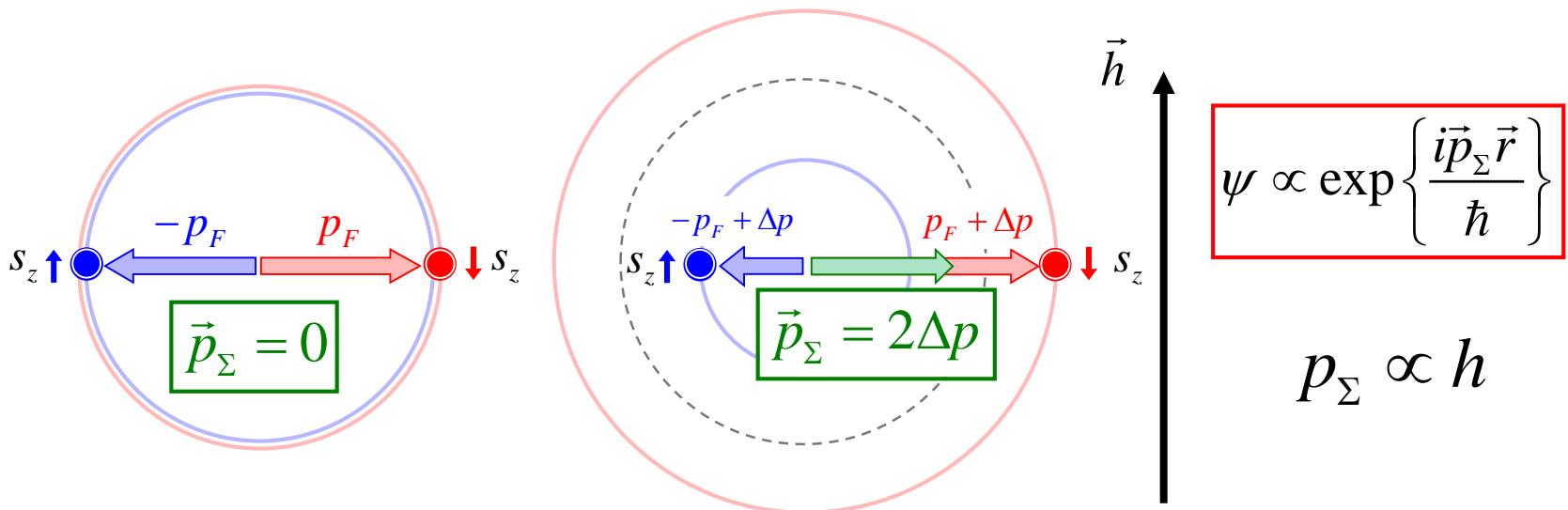
**Singlet:**  $\uparrow\downarrow - \downarrow\uparrow$   
 $S = 0$

**Triplet:**  $\left\{ \begin{array}{ll} \uparrow\downarrow + \downarrow\uparrow & S_z = 0 \\ \uparrow\uparrow & \\ \downarrow\downarrow & S_z = \pm 1 \end{array} \right.$

# Proximity effect in S/F systems

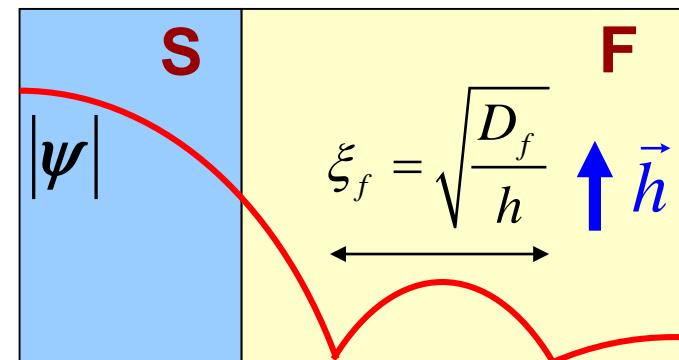
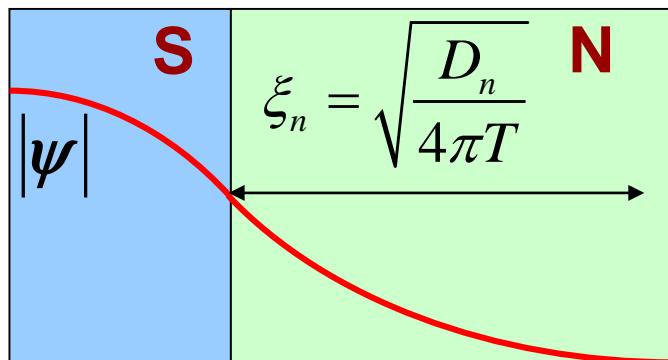
Exchange field (energy) in the ferromagnet:

$$\hat{H} = \dots + \vec{h} \cdot \hat{\vec{\sigma}}$$



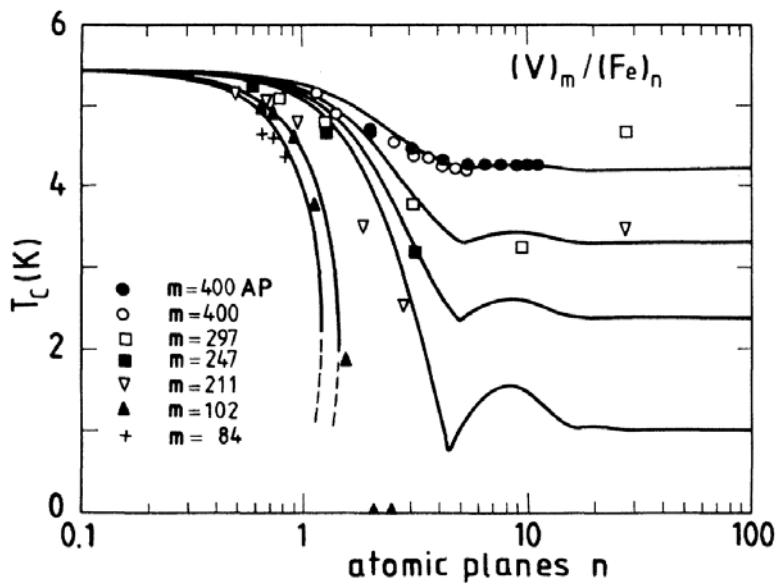
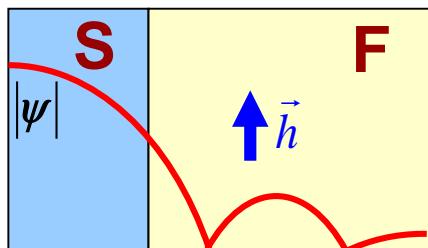
P. Fulde and R. A. Ferrell, Phys. Rev. (1964).

A. I. Larkin and Yu. N. Ovchinnikov, JETP (1964).



Transversal FFLO states

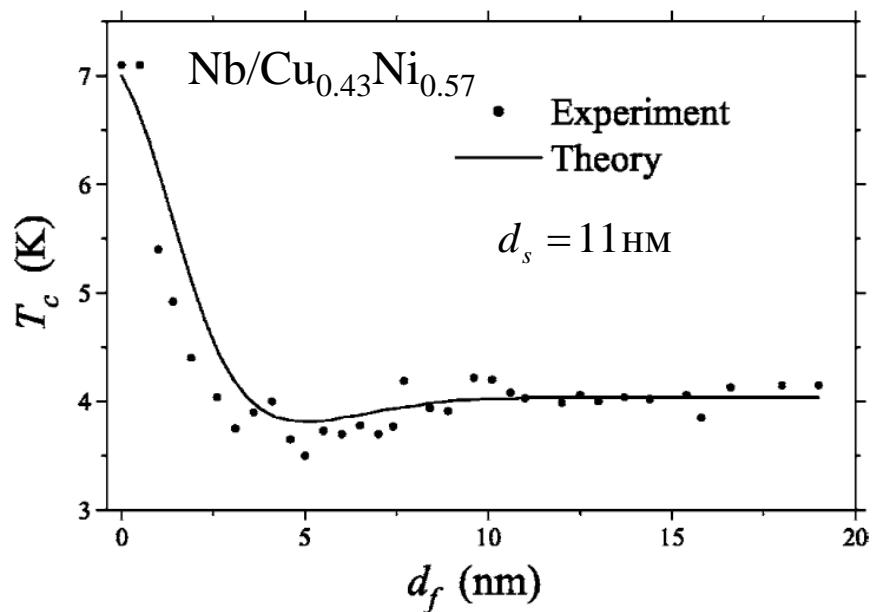
# Dependence of the critical temperature on the thickness of the F layer



A.I. Buzdin and M.Yu. Kupriyanov,  
JETP Lett. (1990)

Z.Radović, A.I. Buzdin, J.R. Clem,  
Phys. Rev. B (1991)

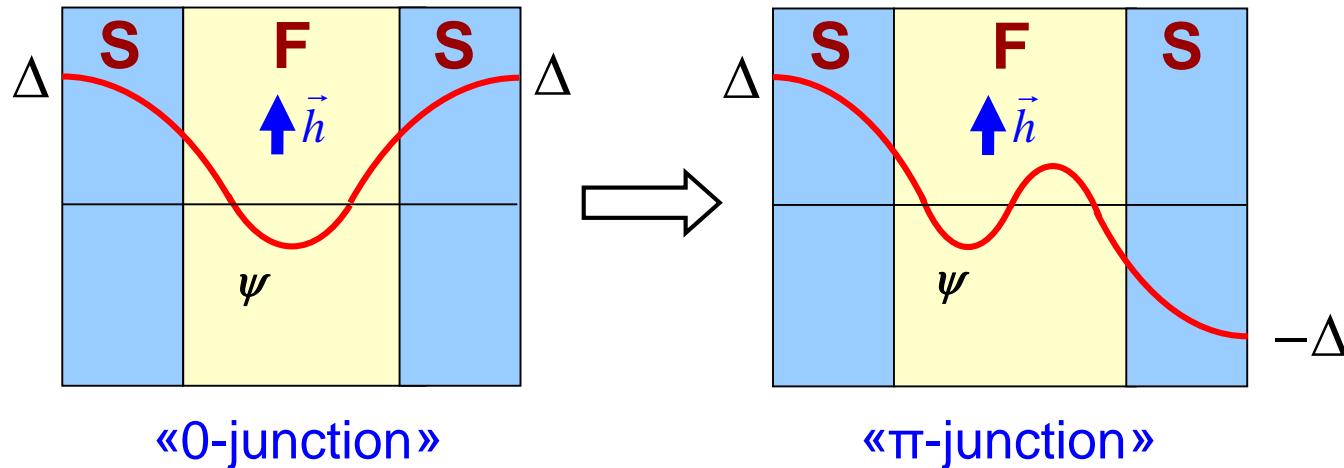
H.K. Wong *et al.*, Journ. Low Temp. Phys. (1986)



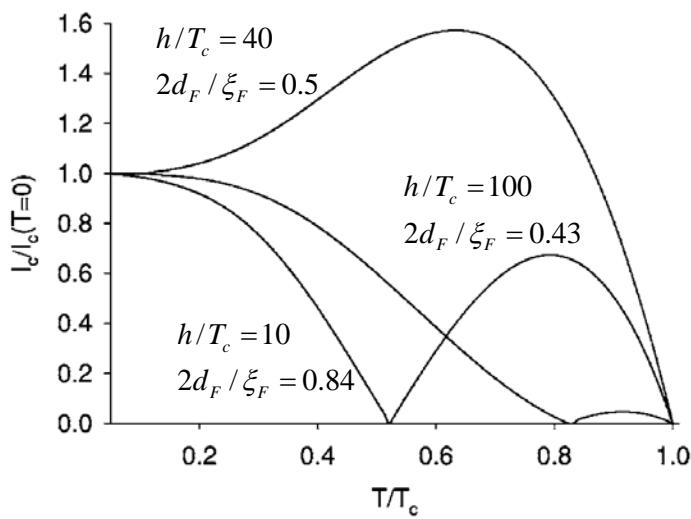
V.V. Ryazanov, V.A. Oboznov, A.S. Prokof'ev  
and S.V. Dubonos, JETP Lett. (2003)

Ya.V. Fominov, N.M. Chtchelkatchev,  
A.A. Golubov, Phys. Rev. B (2002)

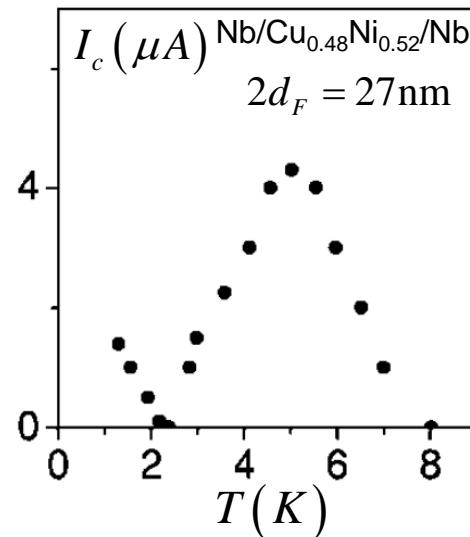
# Formation of $\pi$ -junctions



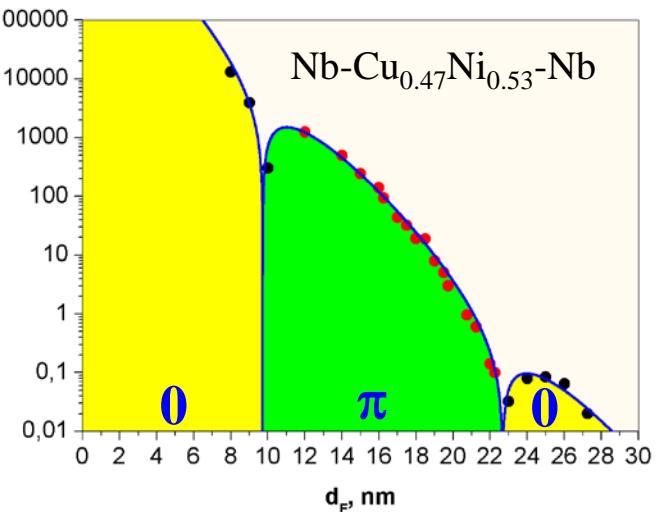
A.I. Buzdin and M.Yu. Kupriyanov, JETP Lett. (1990)



A.I. Buzdin, Rev. Mod. Phys. (2005)



V.V. Ryazanov *et al.*,  
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V.A. Oboznov, V.V. Bol'ginov,  
A.K. Feofanov, V.V. Ryazanov,  
A. I. Buzdin, Phys. Rev. Lett. (2006)

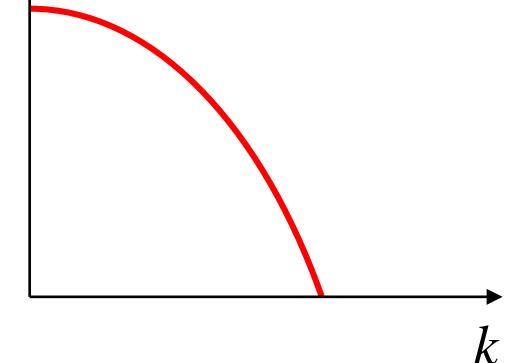
# Phenomenological description of FFLO states

$$\psi \propto \exp(i \vec{k} \cdot \vec{r})$$

$$m > 0$$

$$F[\psi] = \int dV \left\{ \alpha(T - T_{c0}) |\psi|^2 + \frac{\hbar^2}{4m} |\nabla \psi|^2 + \dots \right\}$$

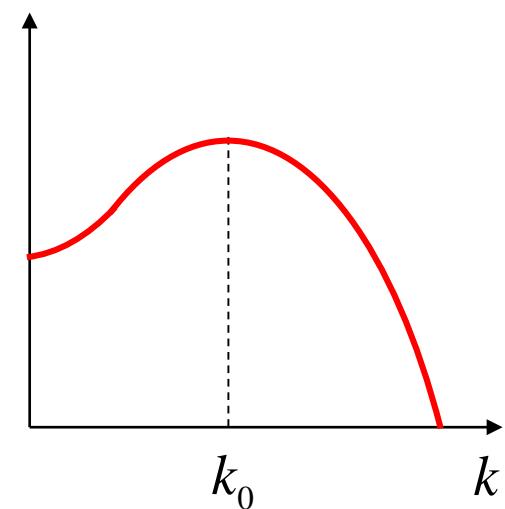
$$T_c(k)$$



$$m < 0$$

$$F[\psi] = \int dV \left\{ \alpha(T - T_{c0}) |\psi|^2 - \gamma |\nabla \psi|^2 + \eta |\nabla^2 \psi|^2 + \dots \right\}$$

$$T_c(k)$$

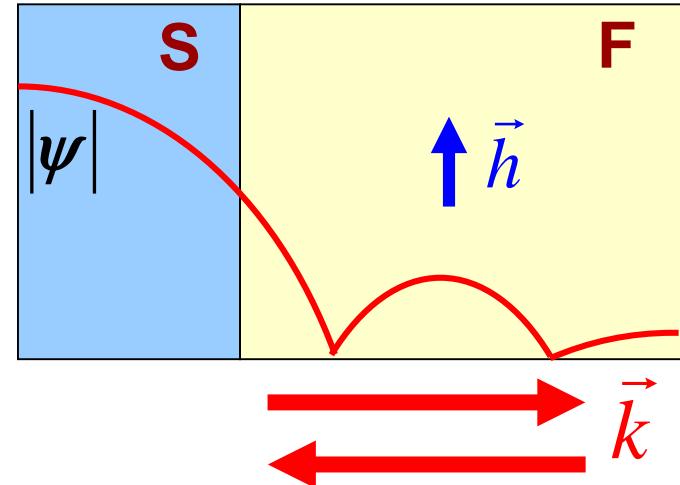


$$\psi_0(\vec{r}) = \sum_{|\vec{k}|=k_0} C_{\vec{k}} \exp(i \vec{k} \cdot \vec{r})$$

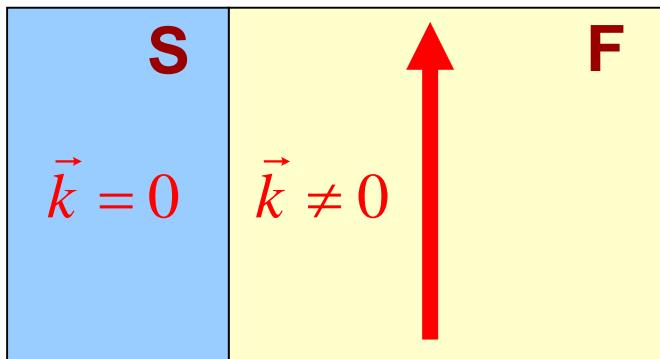
# Transversal FFLO states

$$\psi_0(\vec{r}) = \sum_{|\vec{k}|=k_0} C_{\vec{k}} \exp(i\vec{k}\vec{r})$$

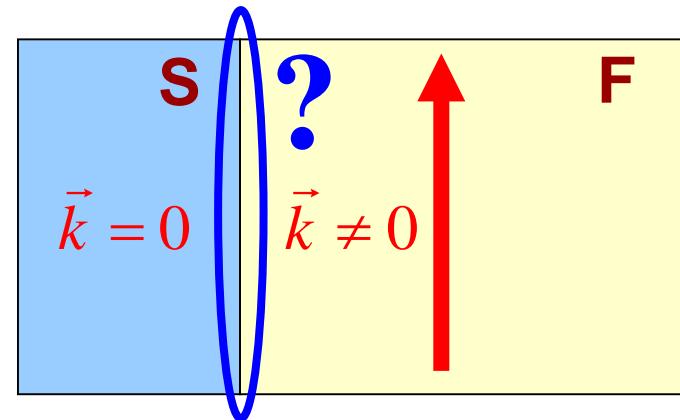
+ in-plane homogeneity



Can in-plane FFLO states exist?

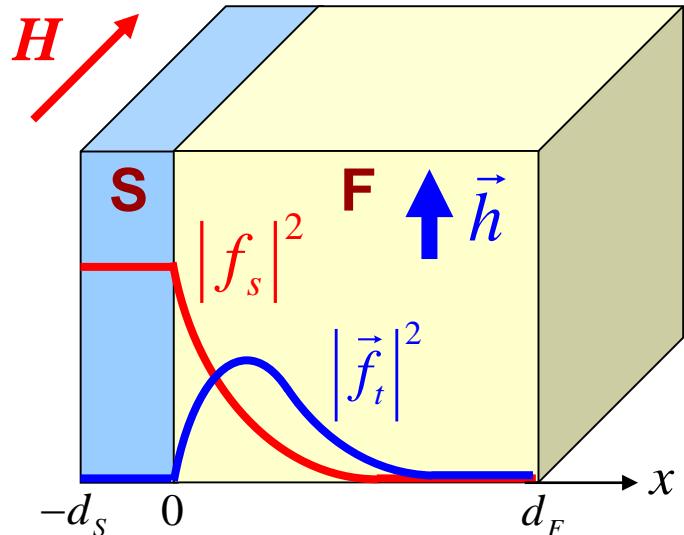


Yu. A. Izumov, Yu. N. Proshin and  
M. G. Khusainov, UFN **172**, 113 (2002)

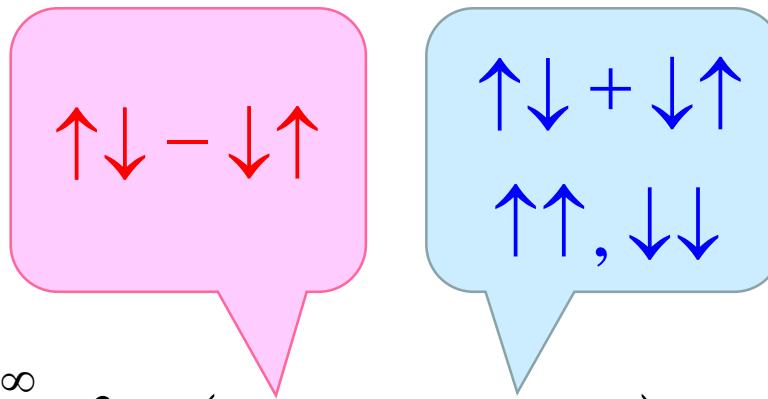


Ya.V. Fominov, M.Yu. Kupriyanov and  
M.V. Feigelman, UFN **173**, 113 (2003)

# Paramagnetic Meissner effect in dirty S/F bilayers



$$\vec{j} = -\frac{1}{4\pi} \lambda^{-2} \vec{A}$$

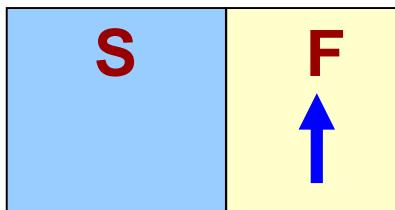


$$\lambda^{-2} = \frac{16\pi^2 T_c}{d_0} \sum_{n=0}^{\infty} \int \sigma \left( |f_s|^2 - |\vec{f}_t|^2 \right) dx$$

A blue oval contains the inequality  $|\vec{f}_t|^2 > |f_s|^2$ . An arrow points to another blue oval containing the expression  $\lambda^{-2}(x) < 0$ , indicating that the two are equivalent under certain conditions.

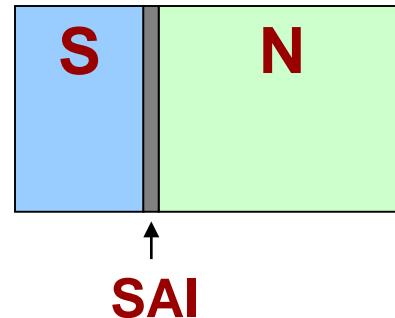
Local paramagnetic response

# Global paramagnetic Meissner effect

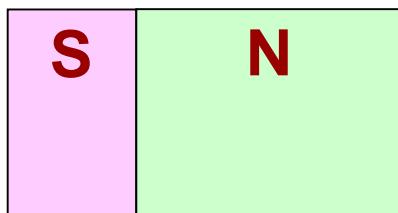


$$\lambda_S^{-2} > 0 \quad \lambda_F^{-2} < 0$$

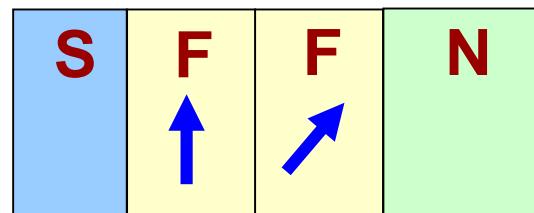
F.S. Bergeret, A.F. Volkov, K.B. Efetov,  
Phys. Rev. B **64**, 134506 (2001)



T. Yokoyama, Y. Tanaka, N. Nagaosa,  
Phys. Rev. Lett. **106**, 246601 (2011)



Y. Asano et al., A. A. Golubov,  
Y. V. Fominov, Y. Tanaka,  
Phys. Rev. Lett. **107**, 087001 (2011)



M. Alidoust, K. Halterman, J. Linder,  
Phys. Rev. B **89**, 054508 (2014)

# Paramagnetic Meissner effect and FFLO instability

$$\vec{j} = -\frac{1}{4\pi} \lambda^{-2} \vec{A} = -\frac{\delta F_A}{\delta \vec{A}} \quad \longrightarrow \quad F_A = \frac{1}{8\pi} \int \lambda^{-2} \vec{A}^2 dV$$

$$F_A = \frac{1}{8\pi} \int \lambda^{-2} \left( \vec{A} - \frac{\Phi_0}{2\pi} \nabla \varphi \right)^2 dV$$

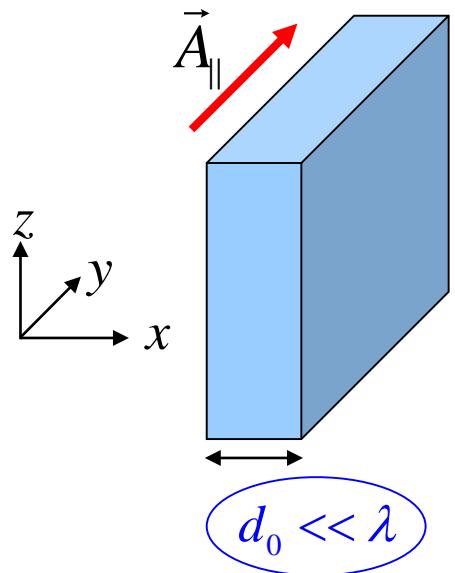
↓  
 $\vec{A} = 0$

$$F_A = \frac{\Phi_0^2}{32\pi^3} \int \lambda^{-2} (\nabla \varphi)^2 dV$$

$$\lambda^{-2} = \frac{4\pi e^2 n_s}{m}$$

1. Uniform superconducting state can become unstable
2. Meissner response can not be paramagnetic

# In-plane FFLO states in film-film S/F structures

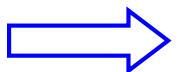


$$\hat{f} = \hat{f}(x) \exp\left\{ik\vec{r}_{\parallel}\right\}$$

$$F_A = \left( \vec{A}_{\parallel} - \frac{\Phi_0}{2\pi} \vec{k} \right)^2 \frac{S}{8\pi} \int \lambda^{-2} dx$$

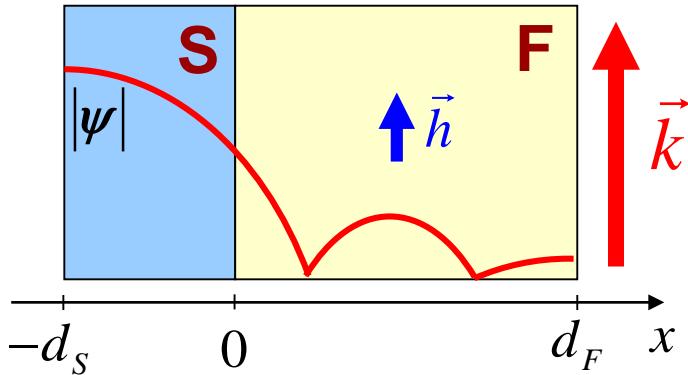
The criterion of instability: vanishing of the Meissner response

$$\lambda^{-2} = \frac{1}{d_0} \int \frac{e^2 n_s}{2m} dx < 0$$



**In-plane FFLO state**

# Inhomogeneous FFLO states in thin S/F bilayers



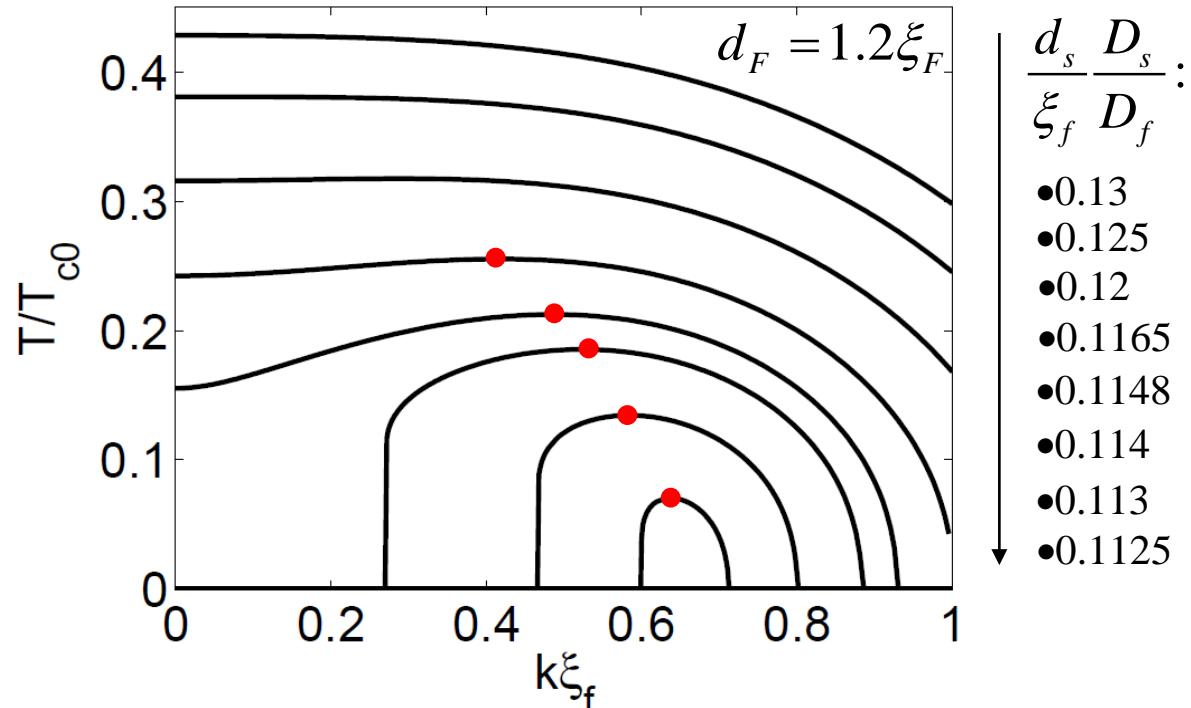
$$\left\{ \begin{array}{l} \frac{D}{2} \frac{\partial^2 \hat{f}}{\partial x^2} - \left( \omega_n + \frac{D}{2} \mathbf{k}^2 \right) \hat{f} - \frac{i}{2} \left( \vec{h} \hat{\sigma} \hat{f} + \hat{f} \vec{h} \hat{\sigma} \right) + \hat{\Delta} = 0 \\ \Delta \ln \frac{T_c(\mathbf{k})}{T_{c0}} + \sum_{n=-\infty}^{\infty} \left( \frac{\Delta}{n + \frac{1}{2}} - 2\pi T_c(\mathbf{k}) f_{12}^S \right) = 0 \end{array} \right.$$

$$\Delta = \Delta(x) \exp \left\{ i \vec{k} \vec{r}_{\parallel} \right\}$$

$$\hat{f}(\mathbf{k}) = \left( f_s + \vec{f}_t \hat{\sigma} \right) i \hat{\sigma}_y$$

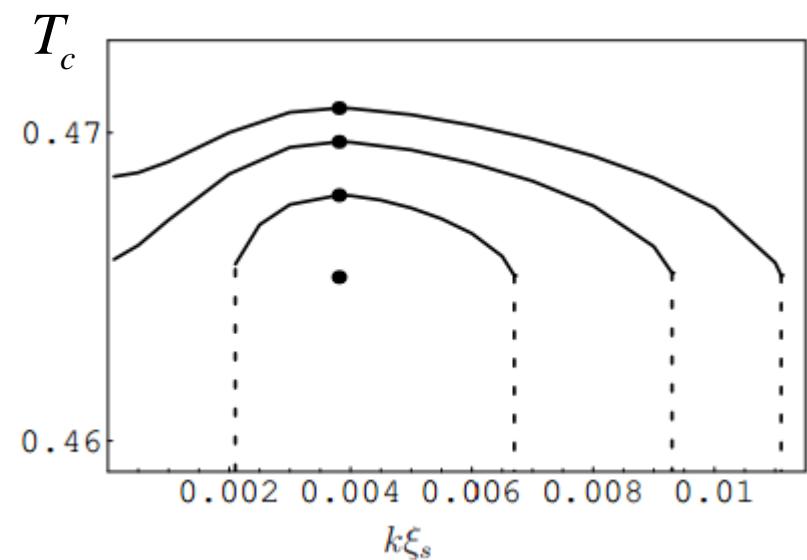
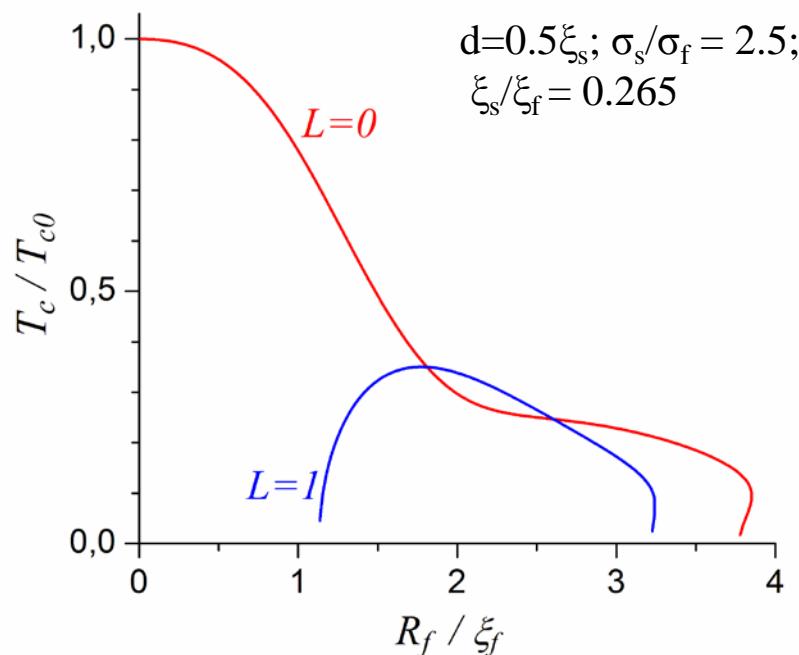
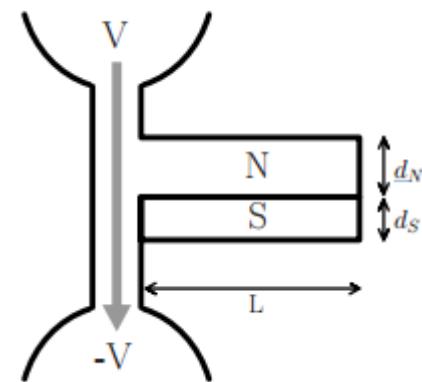
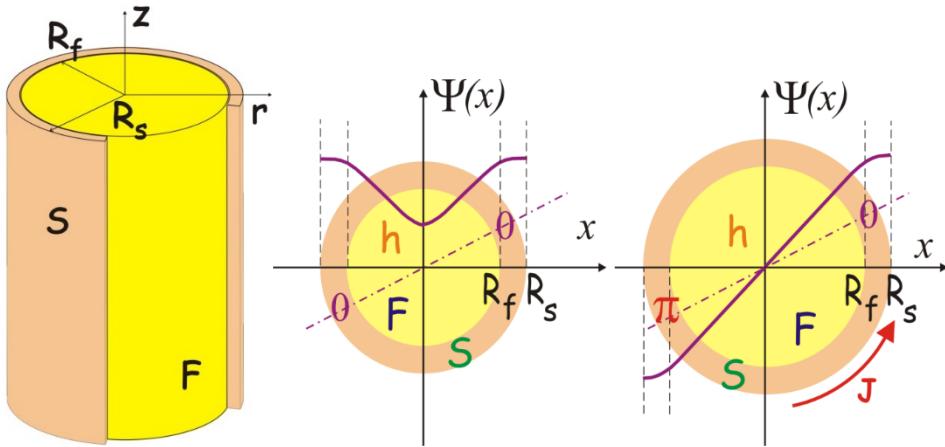
$$T_c = \max \{ T_c(\mathbf{k}) \}$$

$$T_c \ll T_{c0}$$



$$\frac{D_f}{D_s} \gg \frac{h}{T_{c0}} \quad d_F \sim \xi_f \quad \frac{h}{T_{c0}} \xi_f \leq d_S \leq \frac{D_f}{D_s} \xi_f$$

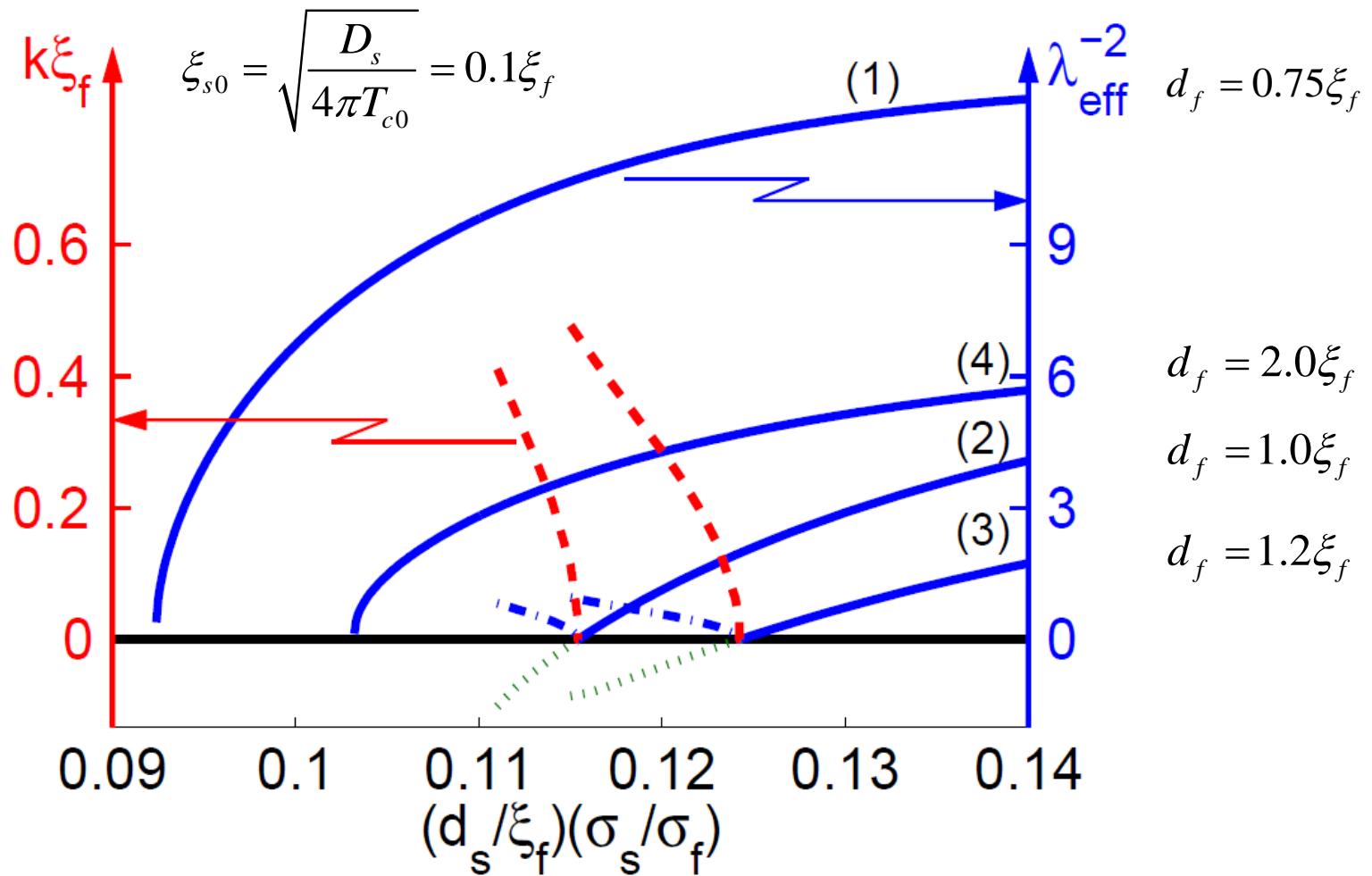
# Other systems with in-plane FFLO instability



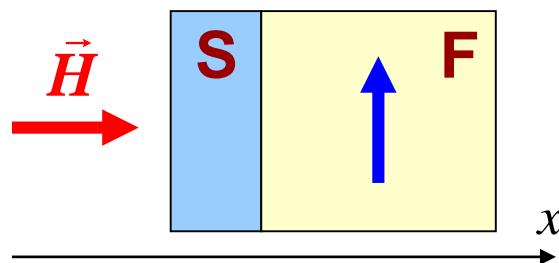
A.V. Samokhvalov, A.S. Mel'nikov and  
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I.V. Bobkova and A.M. Bobkov, Phys.  
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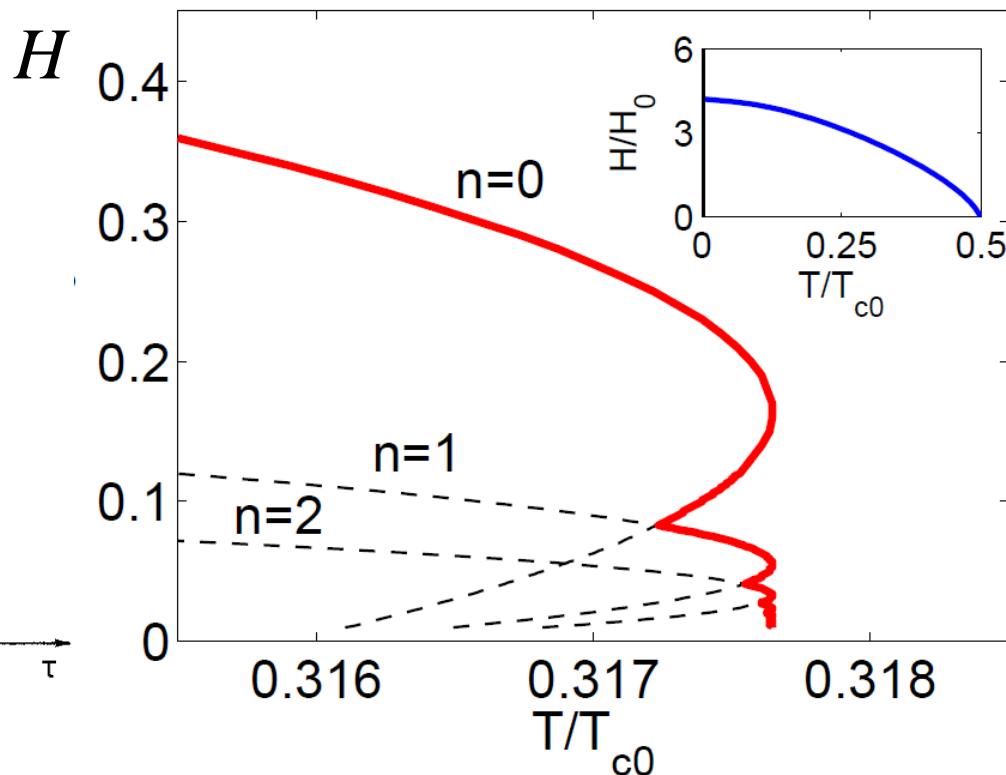
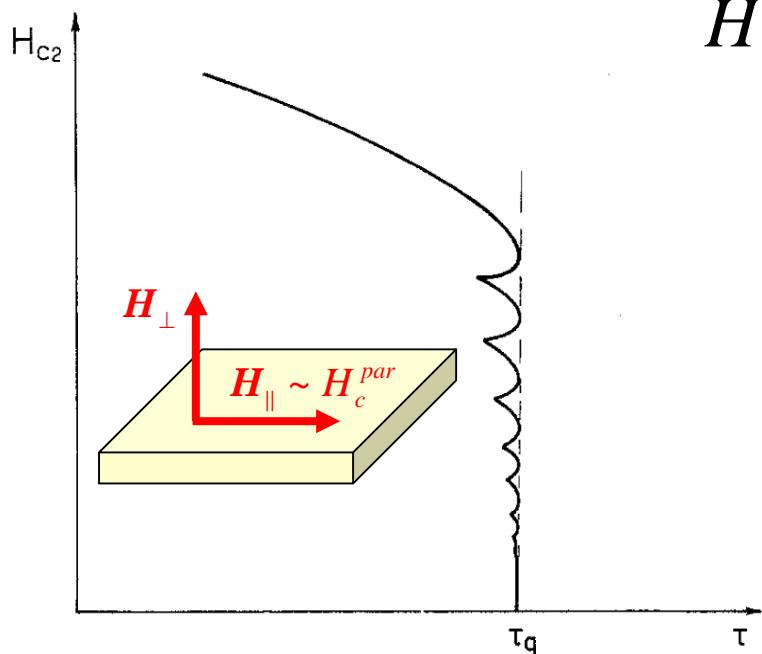
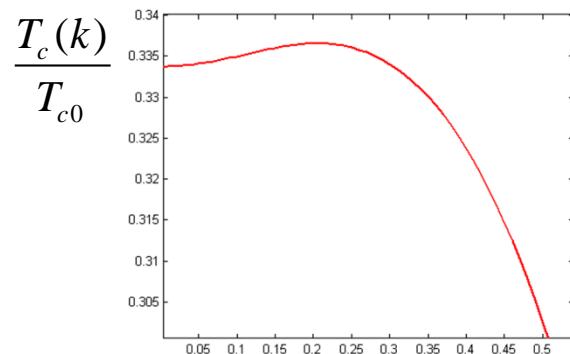
# Inhomogeneous FFLO states in S/F bilayers



# Phase diagram of S/F bilayers in the in-plane FFLO state

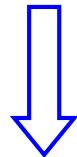


$$k^2 \rightarrow 4eH\left(n + \frac{1}{2}\right)$$

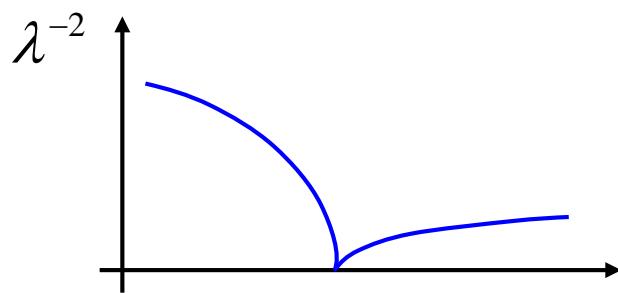


# Summary

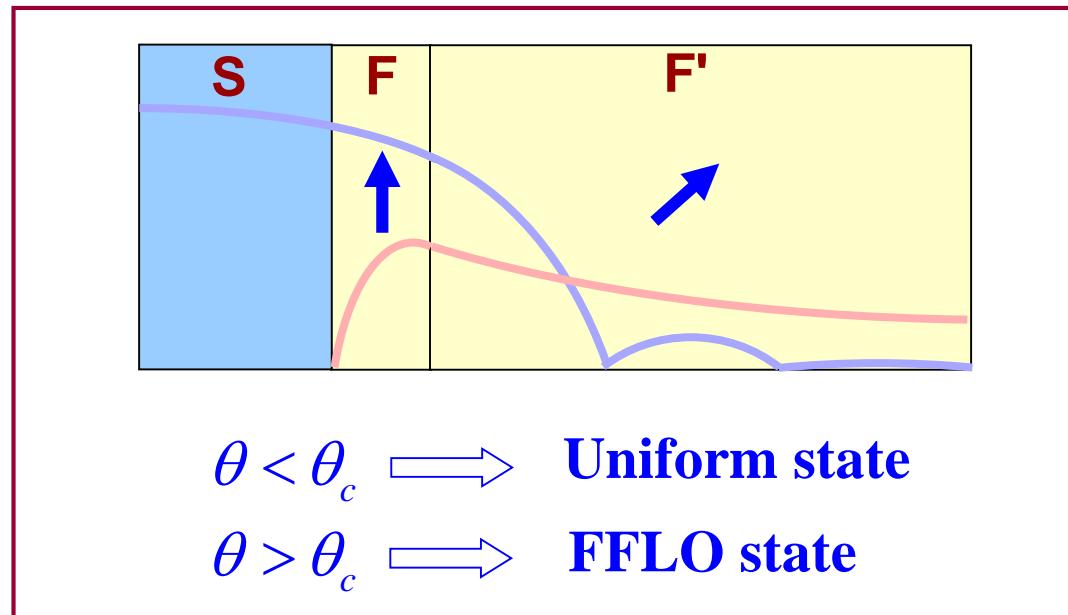
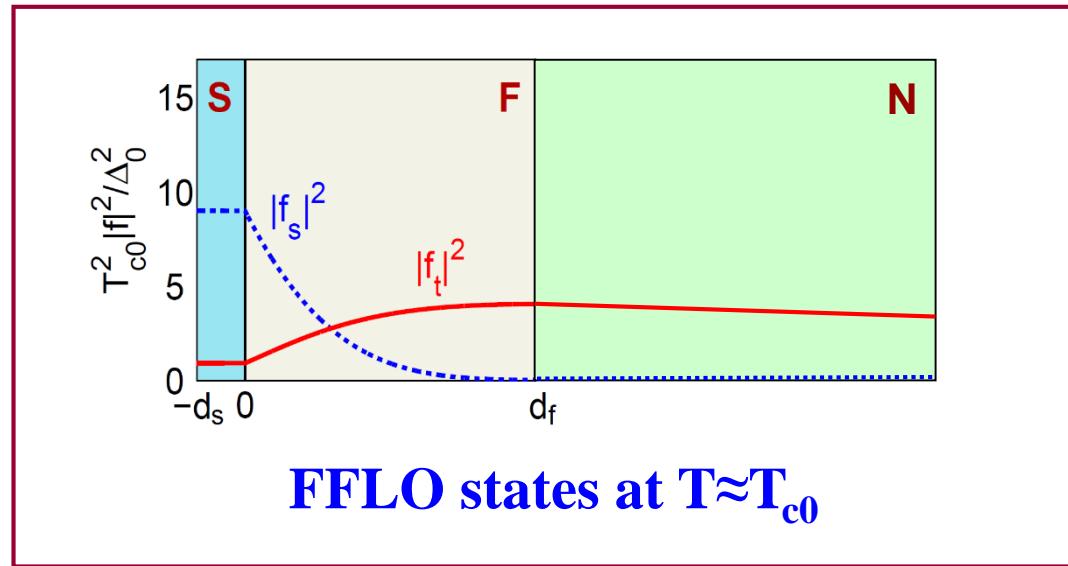
Paramagnetic  
Meissner effect



In-plane FFLO states



# In-plane FFLO states in S/F/N and S/F/F' systems



**Thank you for attention!**