Engineering new non-Abelian systems

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Collaborators

- Erez Berg, Gil Refael, Ady Stern

  PRX **2**, 041002 (2012)
- Lukasz Fidkowski, Alexei Kitaev

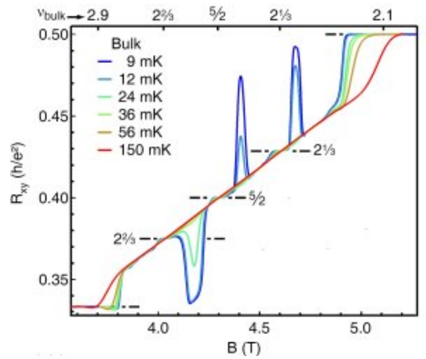
  (to be published)
- Roger Mong, Jason Alicea, David Clarke, Erez Berg, Ady Stern, Kirril Stengel, Paul Fendely, Chetan Nayak, Matthew Fisher , Yuval Oreg

  PRX **4**, 011036 (2014)
  arXiv: 1406.0846

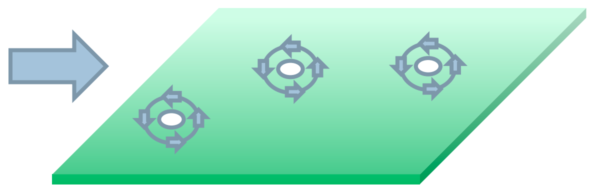
Outline

- Part 1: Fractionalized Majorana zero modes from hybrid superconductor / quantum Hall devices.
  - Majorana Fermions in1D top. superconductors
  - Parafermionic zero modes in Fractionalized 1D superconductors.
  - Twist defects
- Part 2: Fibonacci anyons from a 2D fractionalized superconductor.

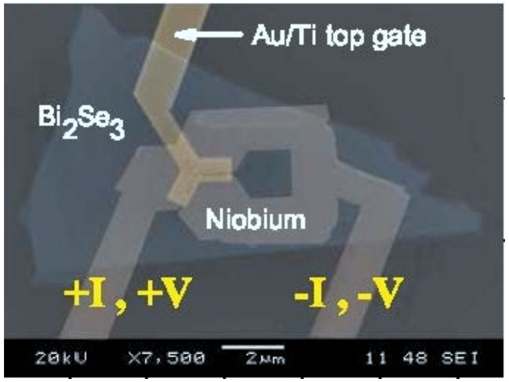
Proposed non-Abelian systems



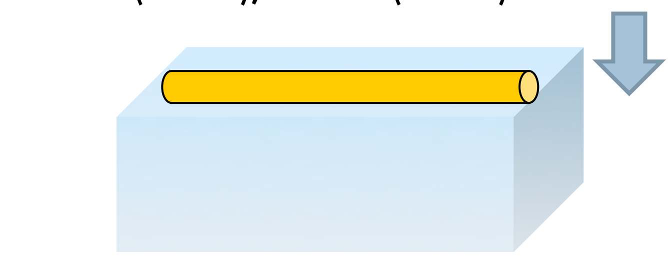
Fractional QH
Willet, Eisenstein, et al. (1987)
Moore & Read (1991)



2D p+ip
superconductors
Read, Green (2000)



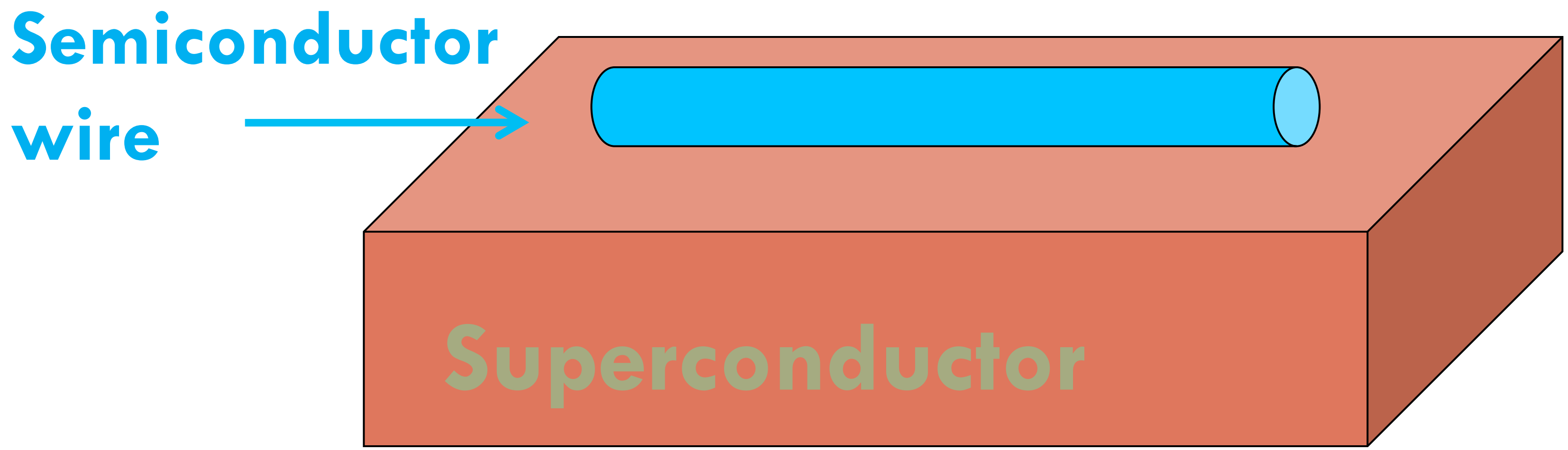
Superconductor - 3D Top. Insulator (Semiconductor) heterostructures
Fu & Kane (2008); Sau et al. (2010);
Lee (2009); Alicea (2010)...



1D Topological superconductors
Kitaev (2001); Fu and Kane (2009);
Lutchyn et al., Oreg et al. (2010);
Mourik et al. (2012)

“Ising” anyons

Topological 1D superconductor



- "Majorana Fermion" zero modes at the edges of the system.
- Two degenerate ground states, separated by an energy gap from the rest of the spectrum:

  Odd & Even number of electrons.
- Ground state degeneracy is "topological":

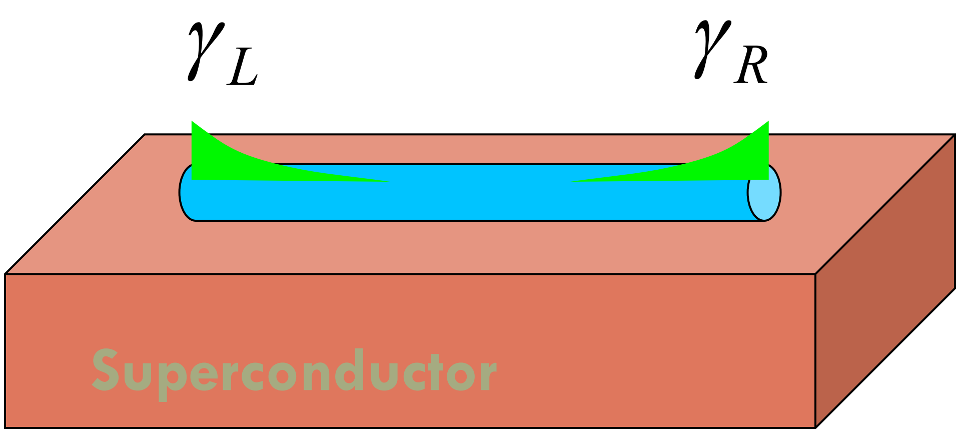
  no local measurement can distinguish between the two

  ground states!

Majorana Fermion zero modes

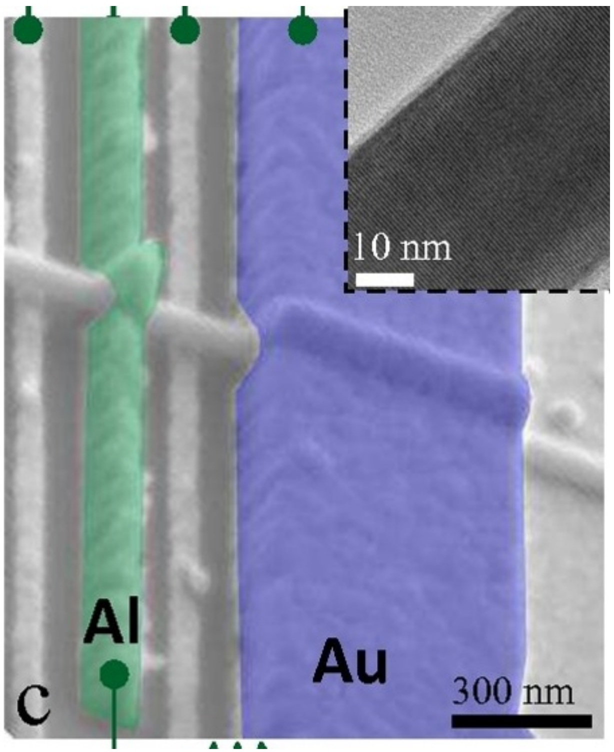
$$[\gamma_{L,} H] = 0 \qquad [\gamma_{R,} H] = 0 \qquad [(-1)^F, H] = 0$$

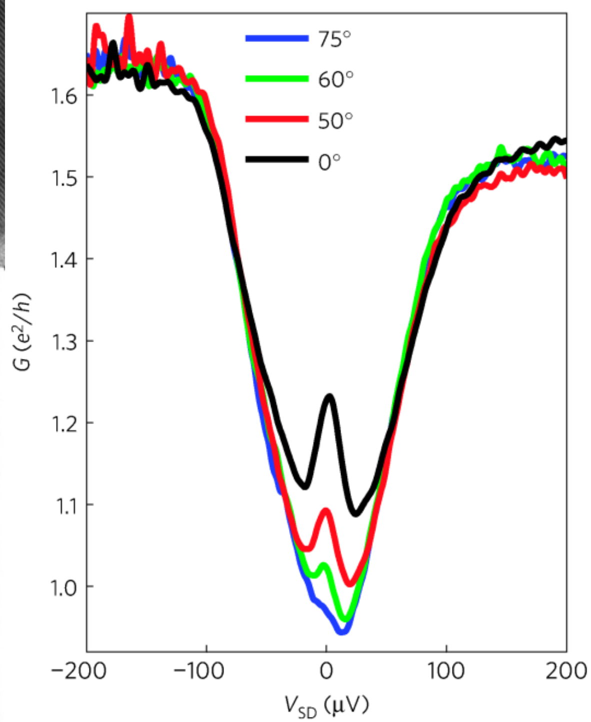
$$\gamma_i^\dagger = \gamma_i \qquad \{\gamma_i, \gamma_j\} = \delta_{ij} \qquad \{\gamma_i, (-1)^F\} = 0$$

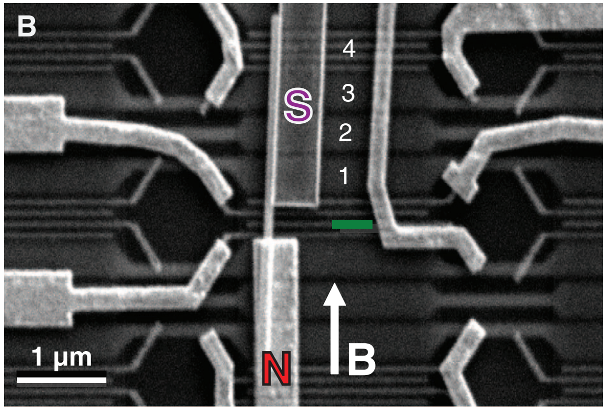


Topological 1D superconductor

Recent experimental realizations:





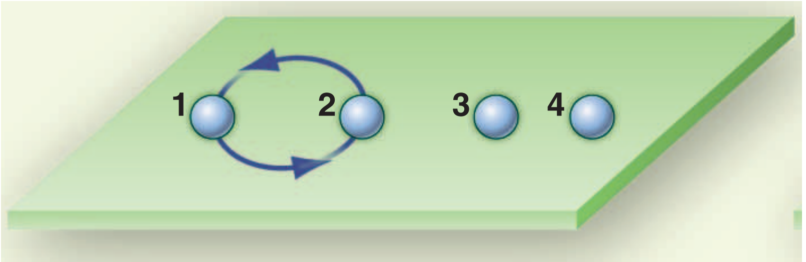


Das et al. (2013)

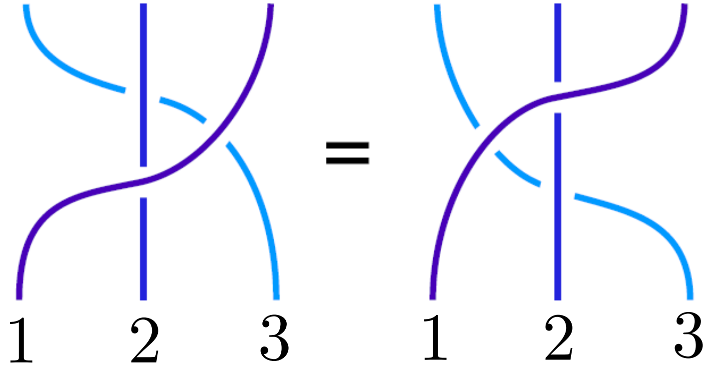
Mourik et al. (2012)

Non – Abelian statistics

$$|\psi_i\rangle \rightarrow \sum_j U_{ij}|\psi_j\rangle$$



Braid group:



Ising anyons (Majoranas):

$$e^{(\pi/4)\gamma_1\gamma_2} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

2D vortices: Ivanov, Read & Green,…

1D wire network: Alicea et.al (2010)

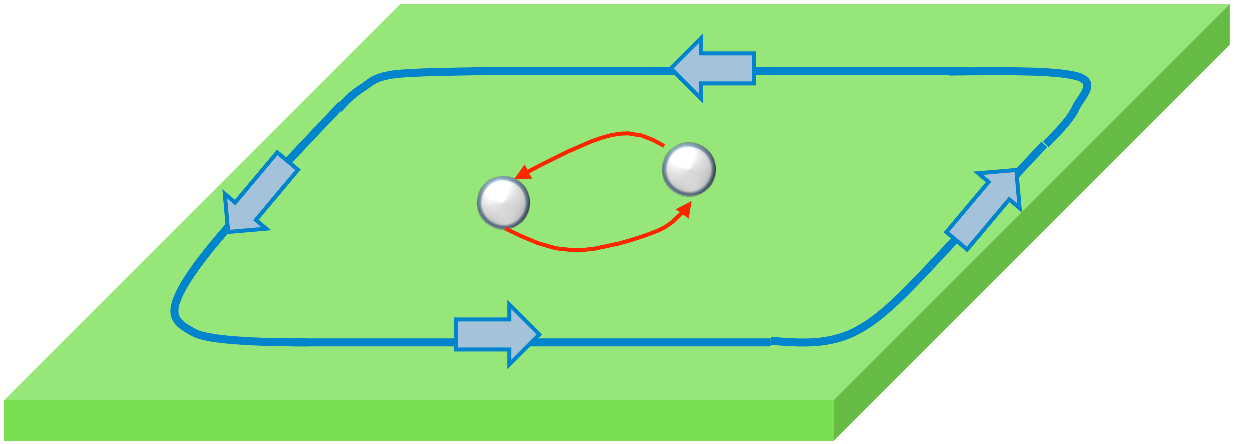
- Ising anyons braid matrices are **not universal** for quantum computation purposes.

Can we get something richer then Ising anyons in an experimentally accessible system?

- **In one dimension?**
- **No Go Theorem** (Fidkowski ; Turner, Pollman, Berg, 2010)
- Gapped, local Hamiltonians of **fermions (or bosons)** in **1D**, at best give Majorana fermions.

Beyond Majorana Fermions

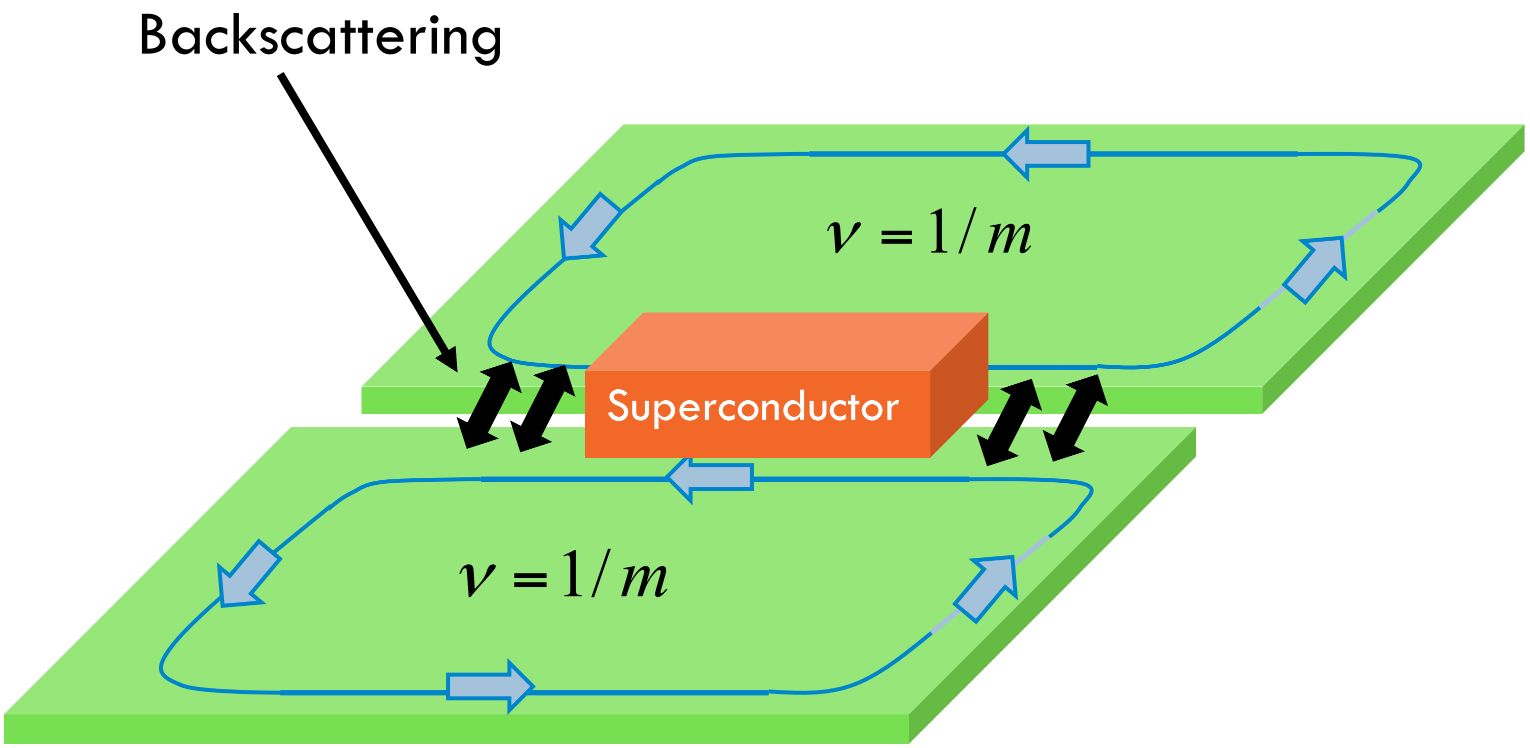
- Start with a topological phase which supports abelian anoyns
- For example, a Laughlin quantum Hall state: $\nu = 1/m$



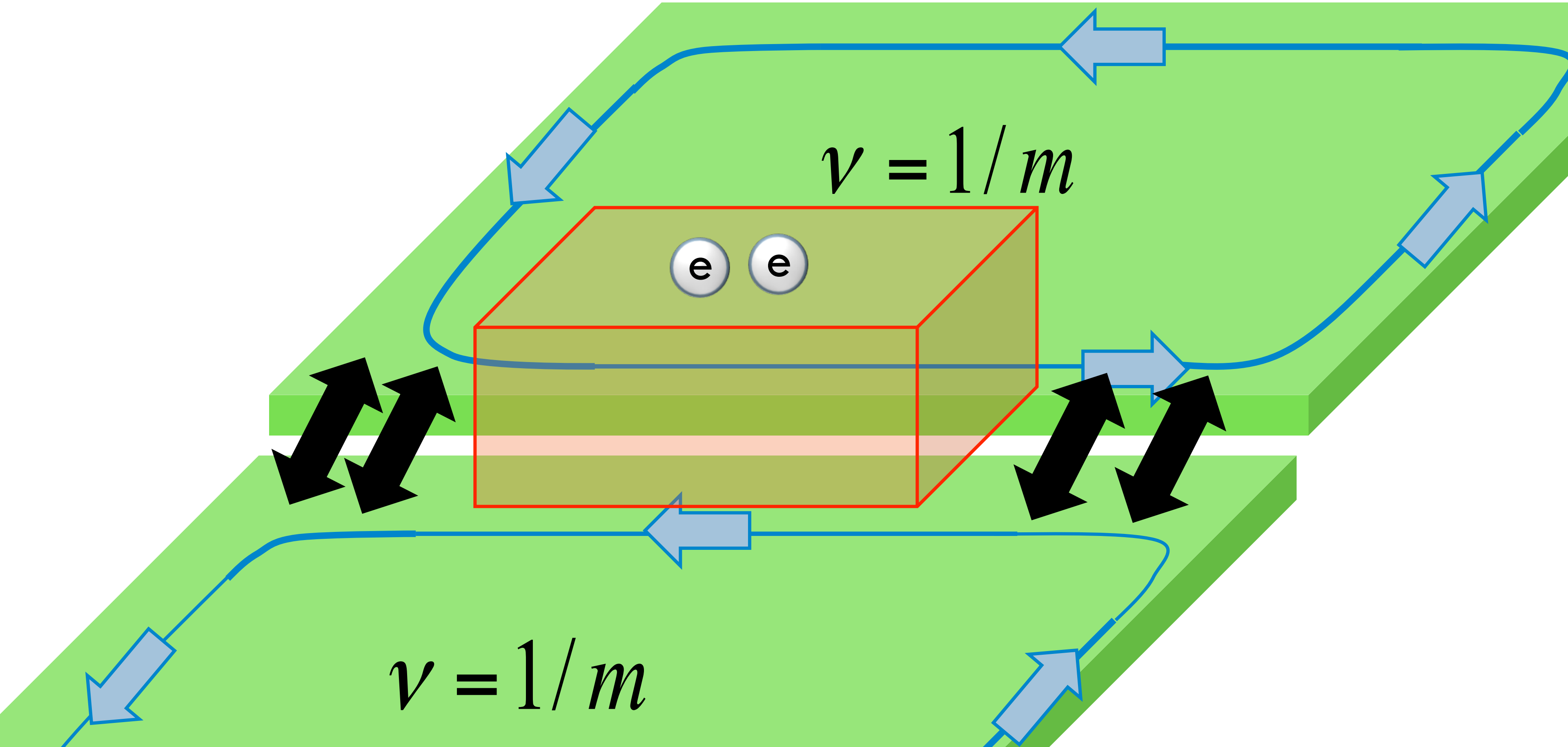
$$\psi \rightarrow \psi' = e^{i\pi/3}\psi$$

$$e^{*} = e/3$$

Fractionalized 1D superconductor

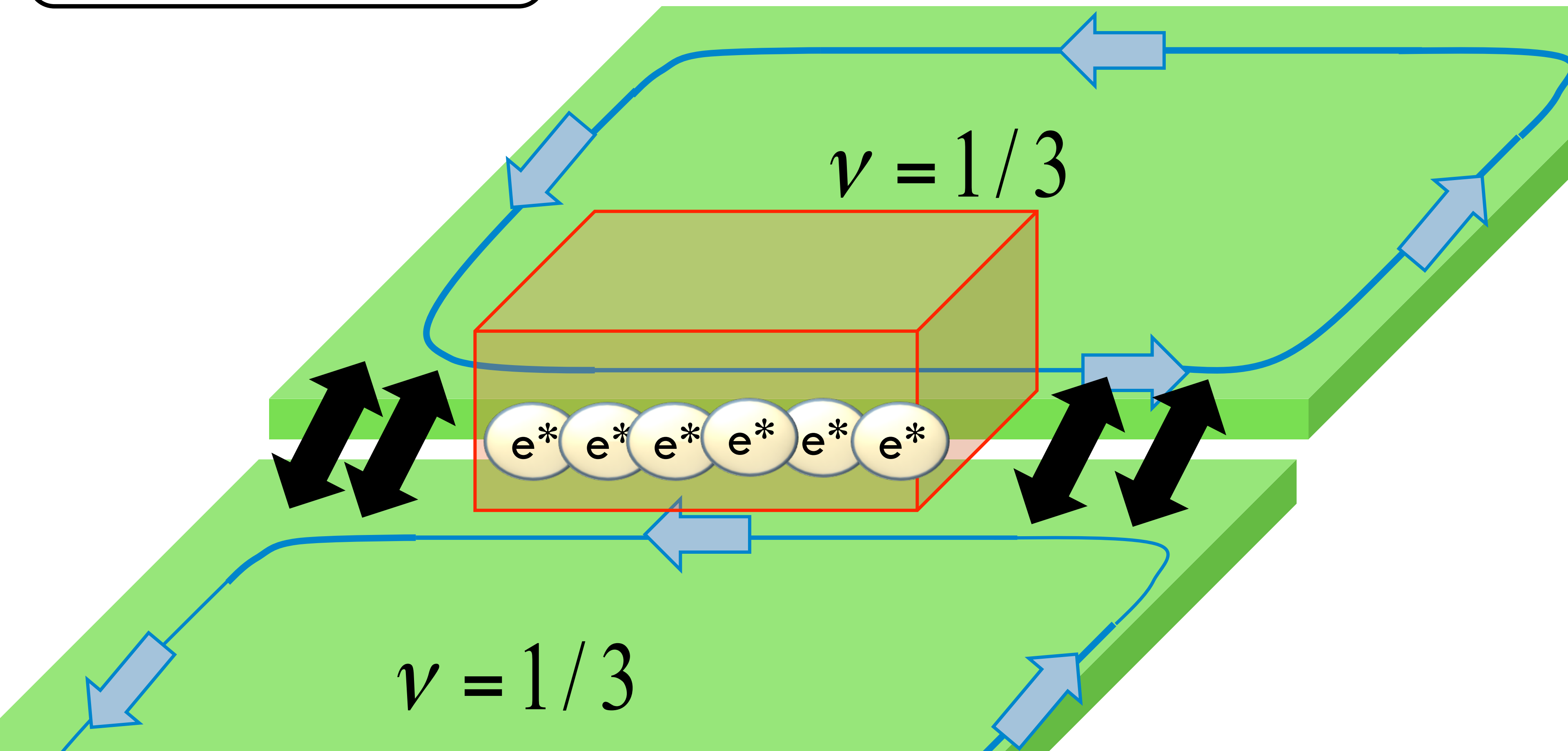


Fractionalized 1D superconductor



Fractionalized 1D superconductor

$$\boxed{\chi_{q.p} \rightarrow e^{i\pi/3}\chi_{q.p}}$$ ⇒ Z_6 Symmetry



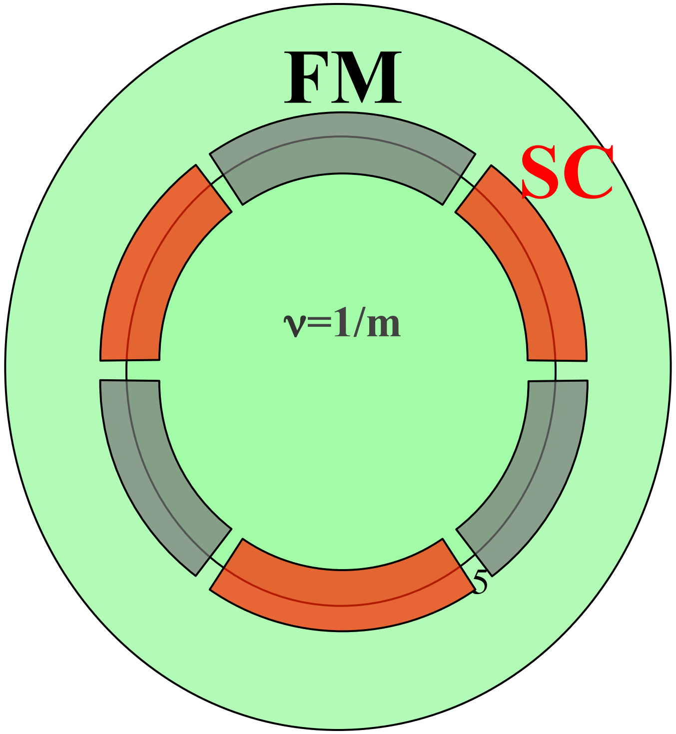
Effective Edge State Model

$$H = \frac{u}{2\pi\nu}\int dx\left[K(x)(\partial_x\phi)^2 + \frac{1}{K(x)}(\partial_x\theta)^2\right]$$
$$-\int dx\left[g_S(x)\cos(2m\phi) + g_F(x)\cos(2m\theta)\right]$$

$$\chi_R^{q.p} : e^{i(\phi+\theta)}$$
$$\chi_L^{q.p} : e^{i(\phi-\theta)}$$

$$\boxed{\psi_R\psi_L} \qquad \boxed{\psi_R^\dagger\psi_L}$$

$$\phi = \frac{\pi}{m}k \qquad \theta = \frac{\pi}{m}k$$



$$\boxed{[\phi(x), \theta(x')] = \frac{i\pi}{m}\Theta(x'-x)}$$

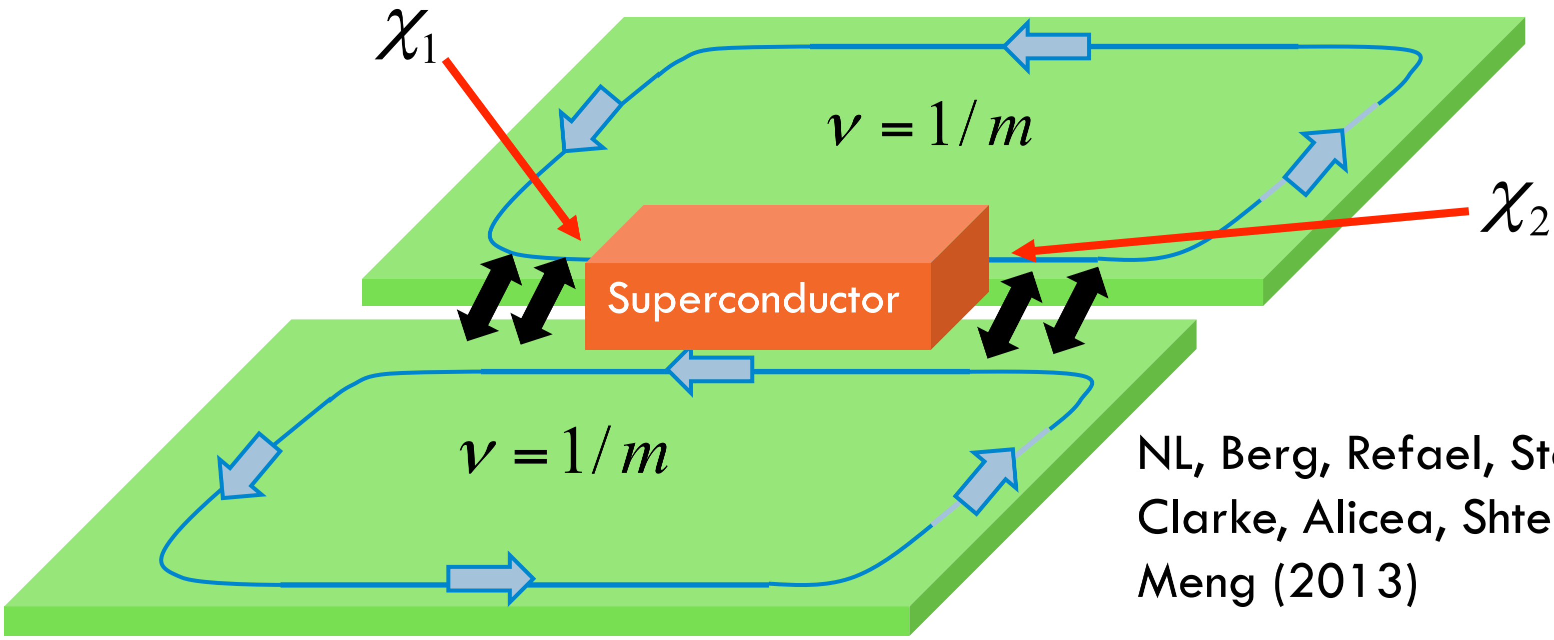
Fractionalized Majorana zero modes

- Fractionalized Majorana zero modes:

$$\left[H,\chi_j\right]=0$$

$$\chi_j\chi_k=e^{i\pi/m\, sign(j-k)}\chi_k\chi_j$$

"Parafermions"



NL, Berg, Refael, Stern (2012)
Clarke, Alicea, Shtengel (2013)
Meng (2013)

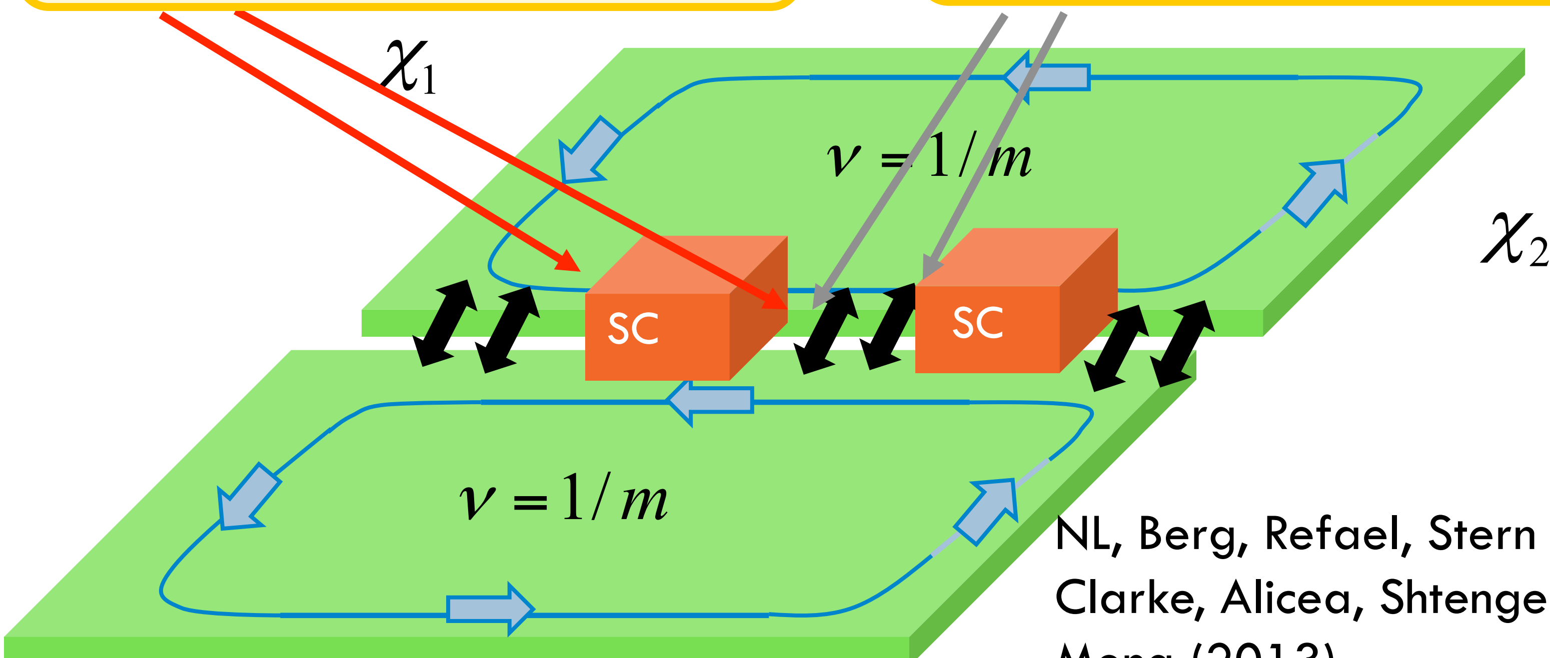
Ground state degeneracy

2m Degenerate ground states per superconducting domain:

$$\text{q.dim}: \ \sqrt{2m}^{N}$$

$$Q = q/m, q = 0,1,..2m$$

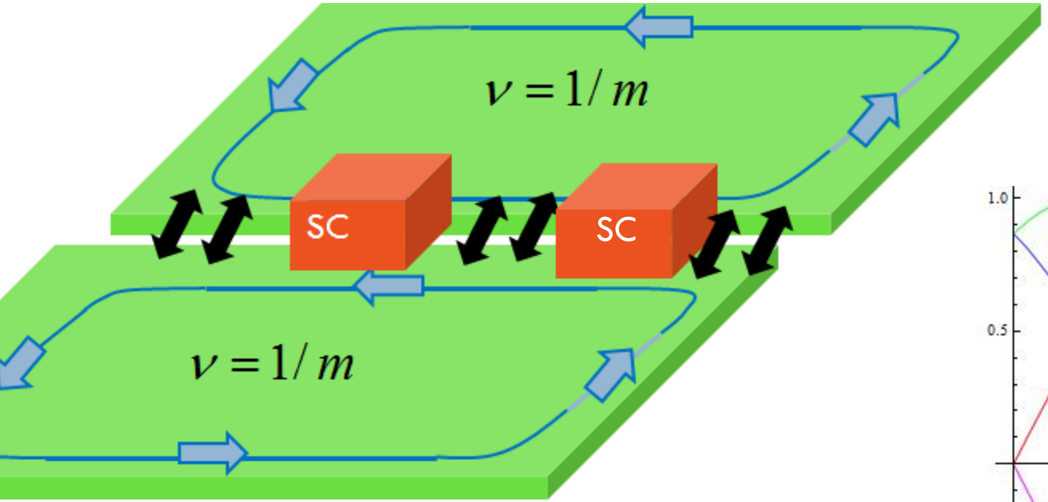
$$S = s/3, s = 0,1,..,2m$$

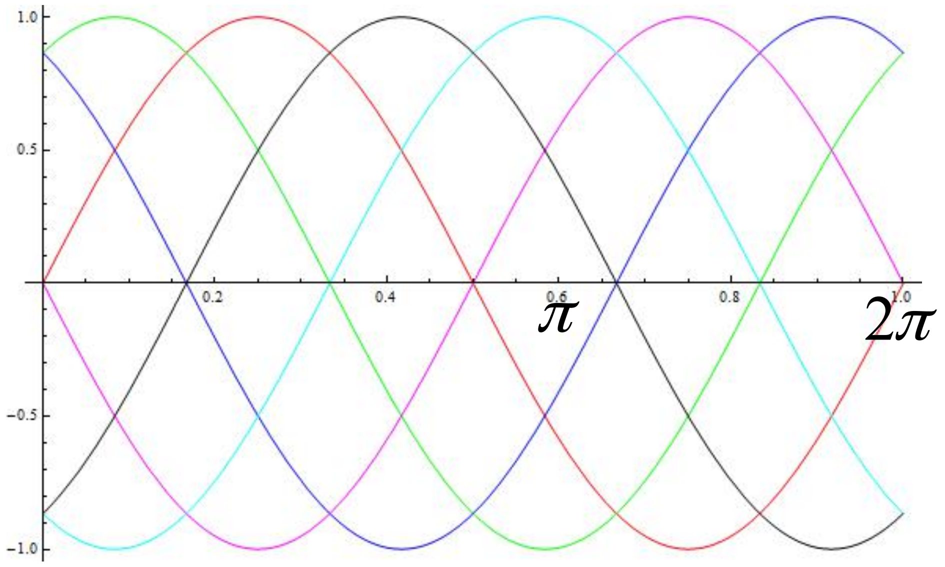


NL, Berg, Refael, Stern (2012)
Clarke, Alicea, Shtengel (2013)
Meng (2013)

Fractional Josephson effect

$$H = t\chi_1\chi^\dagger_{2,\sigma} + h.c. = t\cos(\pi\hat{S} + \delta\phi/2m)$$



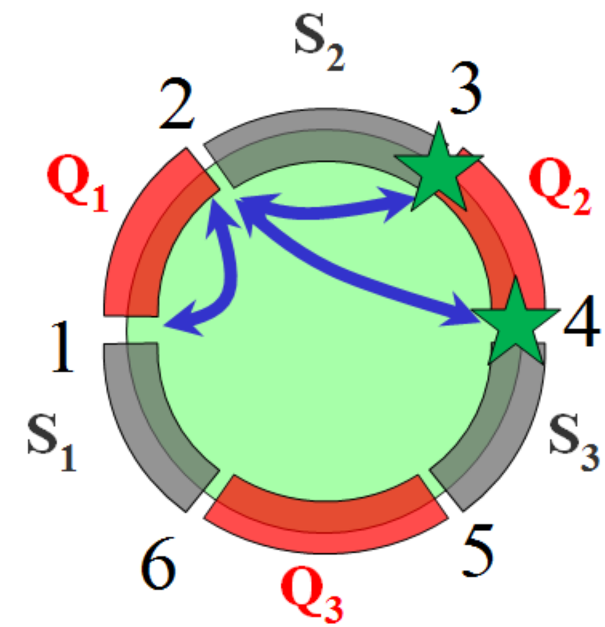


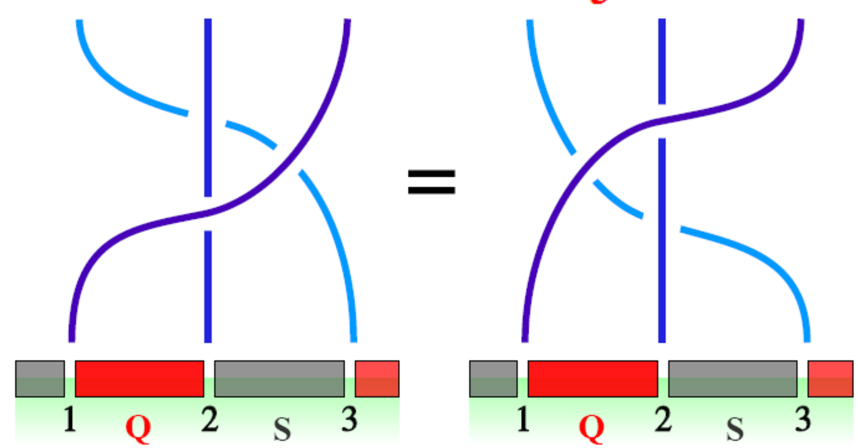
Braiding

$$H(t) = \sum_{ij} \lambda_{ij}(t) \chi_{i,\sigma} \chi^{\dagger}_{j,\sigma} + h.c.$$

$$H(T) = H(0)$$

$$\boxed{U = \exp\left(i \frac{\pi}{2m} q^2 \right)}$$





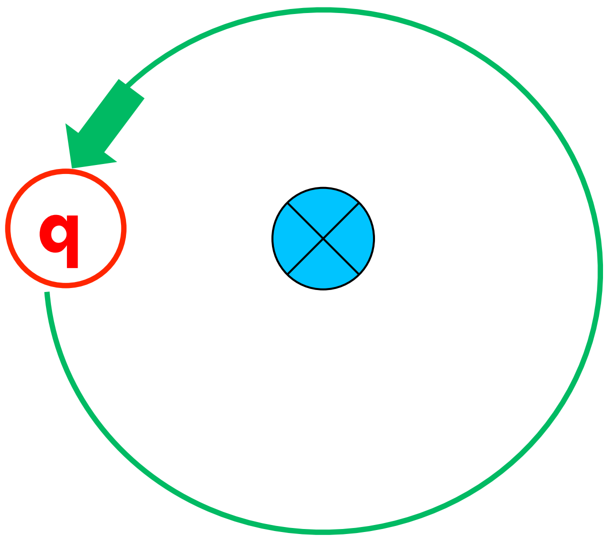
Braiding Properties

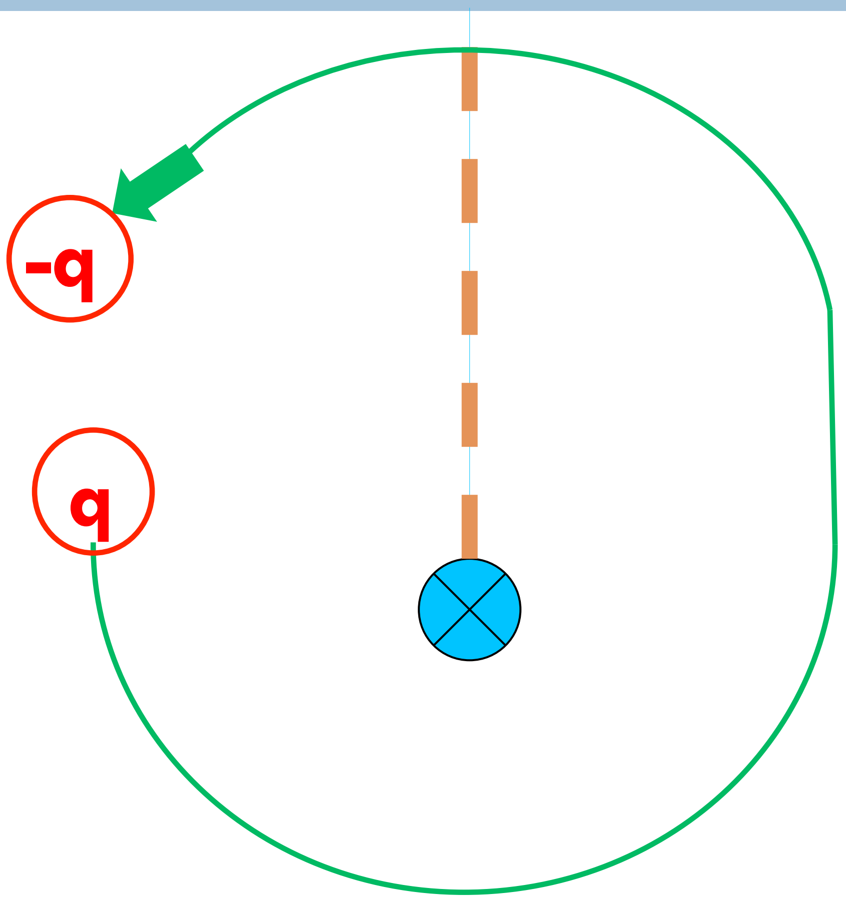
$$\exp\left(i\frac{\pi}{2m}q^2\right) = \exp\left(i\frac{\pi}{2}n_\gamma\right)\boxed{\exp\left(i\frac{2\pi}{m}n_X^2\right)}$$

- Two types of particles:
  - Abelian charges
    $$Q = 0,1,\ldots,m$$
  - Non abelian particle:
    $$X \times X = 0 + 1 + \ldots + m$$

$$\boxed{U = \exp\left(i\frac{2\pi}{m}q^2\right)}$$

Point particles vs. line objects

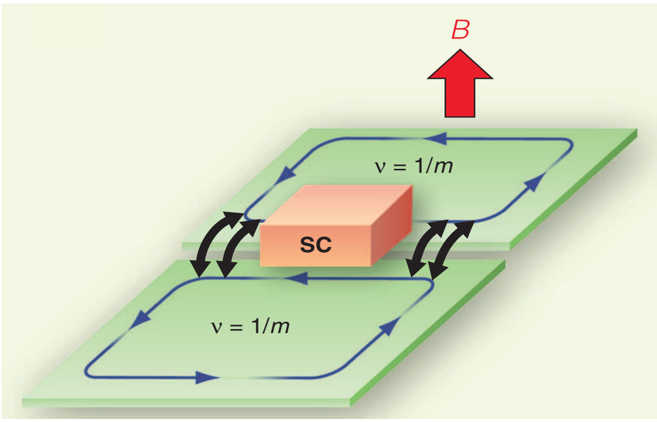


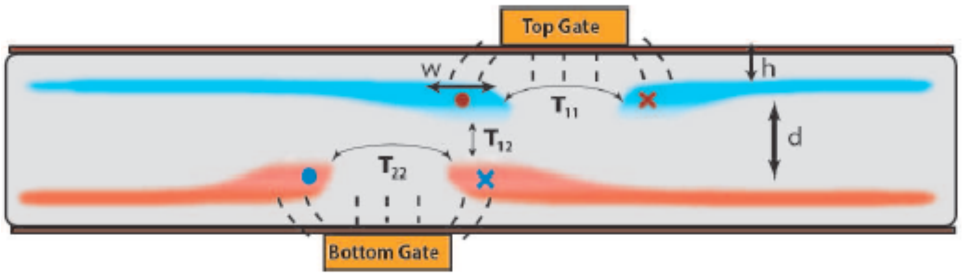


Fidkowski, Lindner, Kitaev (unpublished)

Twist defects in topological phases

- End of "branch cuts" in a top. phase that interchange anyon types





Barkeshli & Qi, arXiv:1302.2673

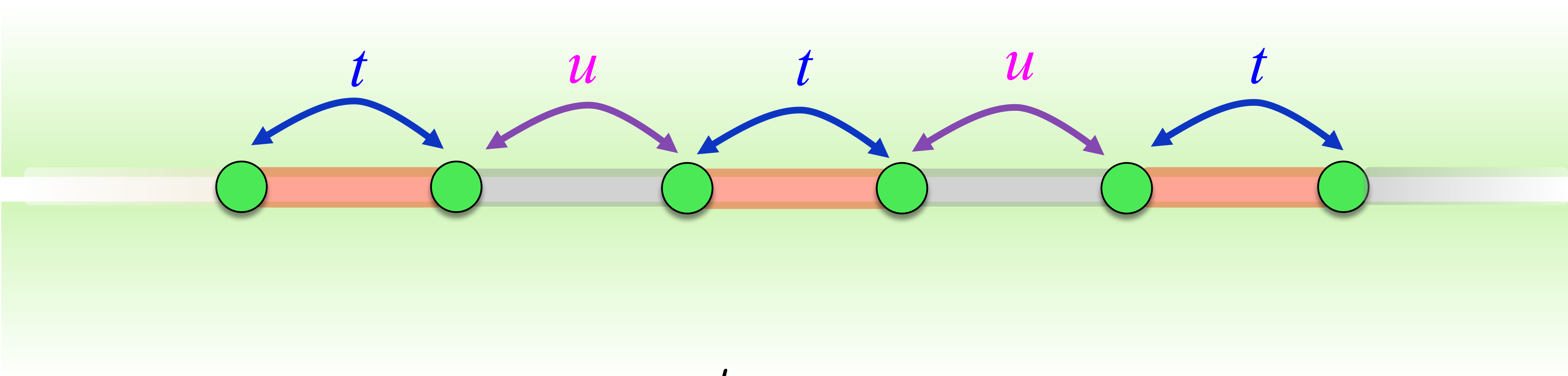
- Controllable non-Abelian systems
- New type of non-Abelian statistics
- Defects in Abelian phases are not QC universal.
  - Defects in non-Abelian phases?

Outline

- Part 1: Fractionalized Majorana zero modes from hybrid superconductor / quantum Hall devices.
  - Majorana Fermions in1D top. superconductors
  - Parafermionic zero modes in Fractionalized 1D superconductors.
  - Twist defects
- Part 2: Fibonacci anyons from a 2D fractionalized superconductor.

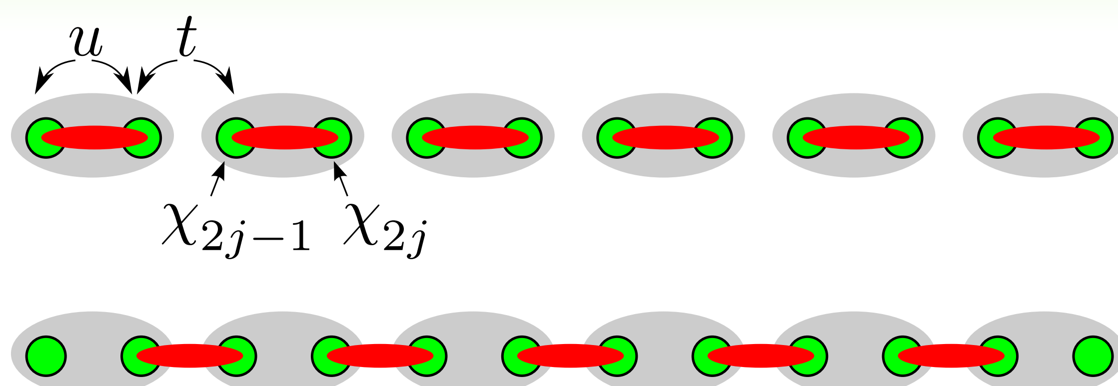
Parafermion chain

- Quasi-particle tunneling: t, u



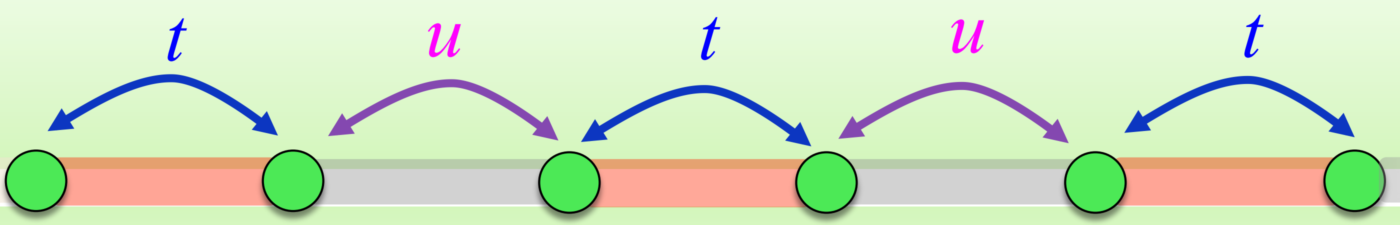
“Trivial” phase

“non-Trivial” phase



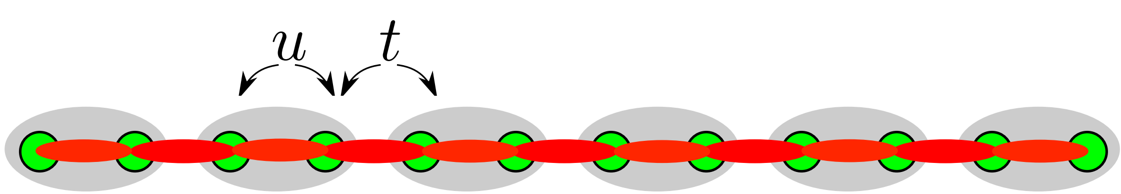
Parafermion chain

Spin unpolarized $\nu = 2/3$ quantum Hall state



- Z_3 Parafermion chain: Quantum critical point: $t = u$

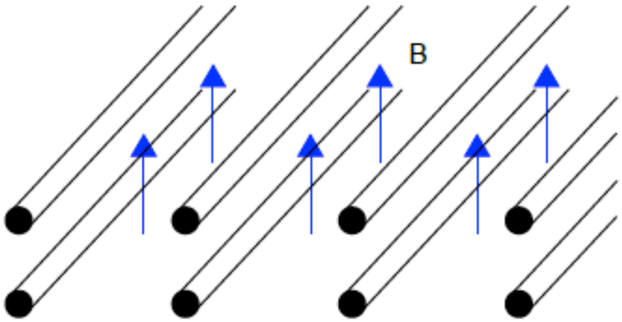
  Z_3 "Parafermion" conformal field theory



From 1D to 2D

From Luttinger liquid to non-Abelian quantum Hall states

Jeffrey C.Y. Teo* and C.L. Kane
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104



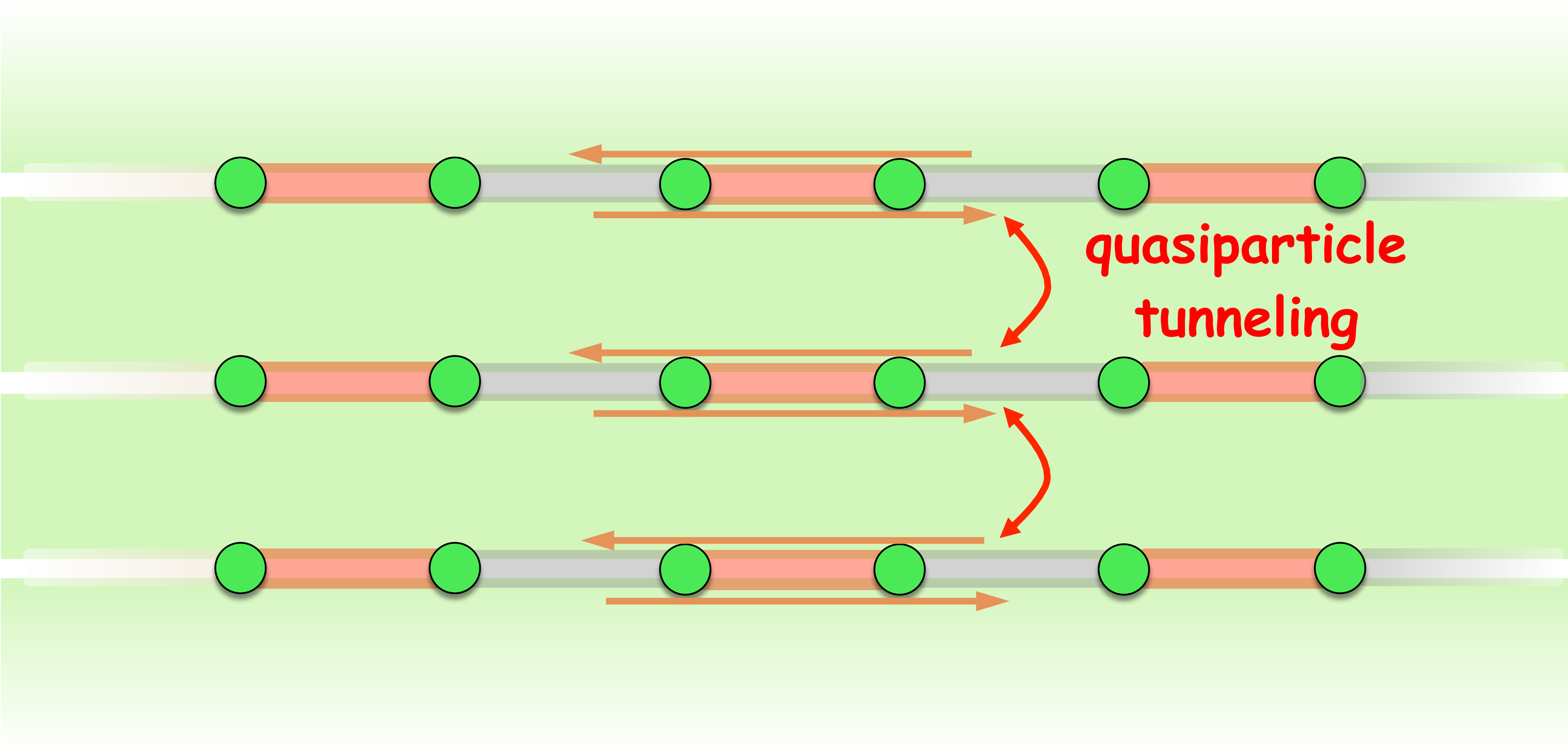
Moore-Read

Z_3 Read-Rezayi

Inspired by Teo & Kane, we consider a system of coupled parafermions chains, to construct a superconducting analog of the Z3 Read-Rezayi state.

Coupled Parafermion chains

- Tune every chain to the critical point:



Fibonacci phase

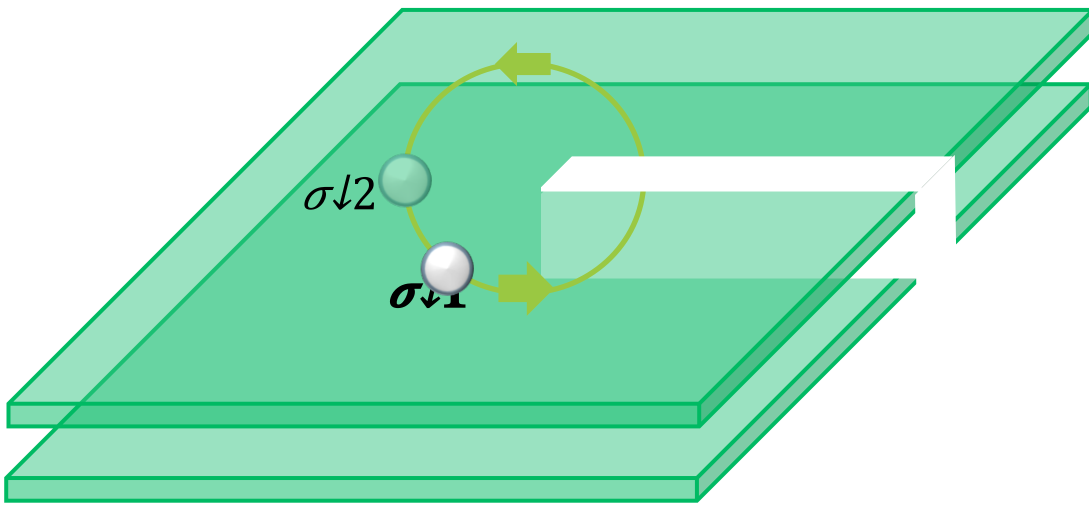
- From bulk-edge correspondence:

  Fibonacci anyons: $\boxed{\varepsilon \times \varepsilon = 1 + \varepsilon}$

- Ground state degeneracy: $1, 1, 2, 3, 5, 8, \ldots \quad \boxed{\sim \left(\frac{1+\sqrt{5}}{2}\right)^N}$

Defect-enriched non-Abelian statistics

Defects can support universal TQC, even when the underlying topological phase is not universal:



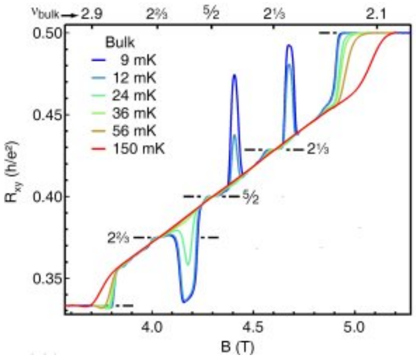
Two layers of Ising anyons

Braid matrix for defects allows for a $e^{i\pi/8}$ phase gate

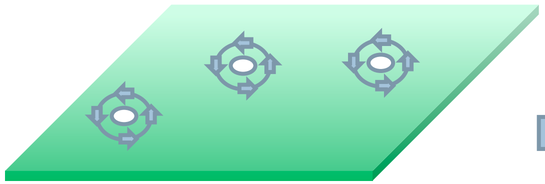
Ising: $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

Zero mode algebra: beyond parafermions

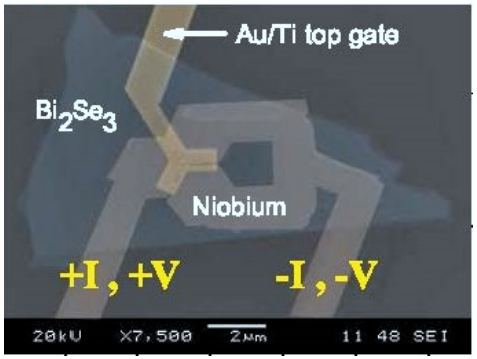
Summary



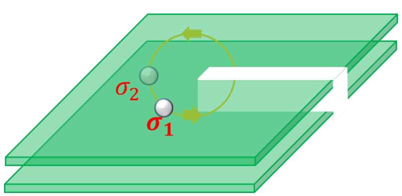
Fractional QH



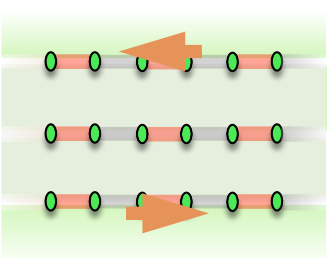
2D p+ip
superconductors



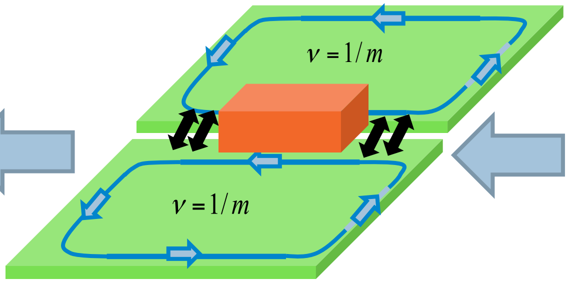
Superconductor - 3D Top. Insulator (Semiconductor) heterostructures



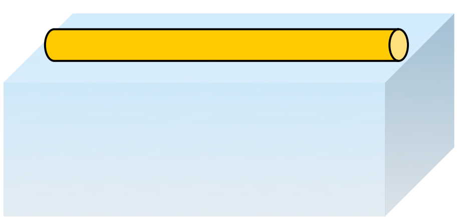
Defects in Top. phases



2D fractionalized superconductors



1D fractionalized superconductors

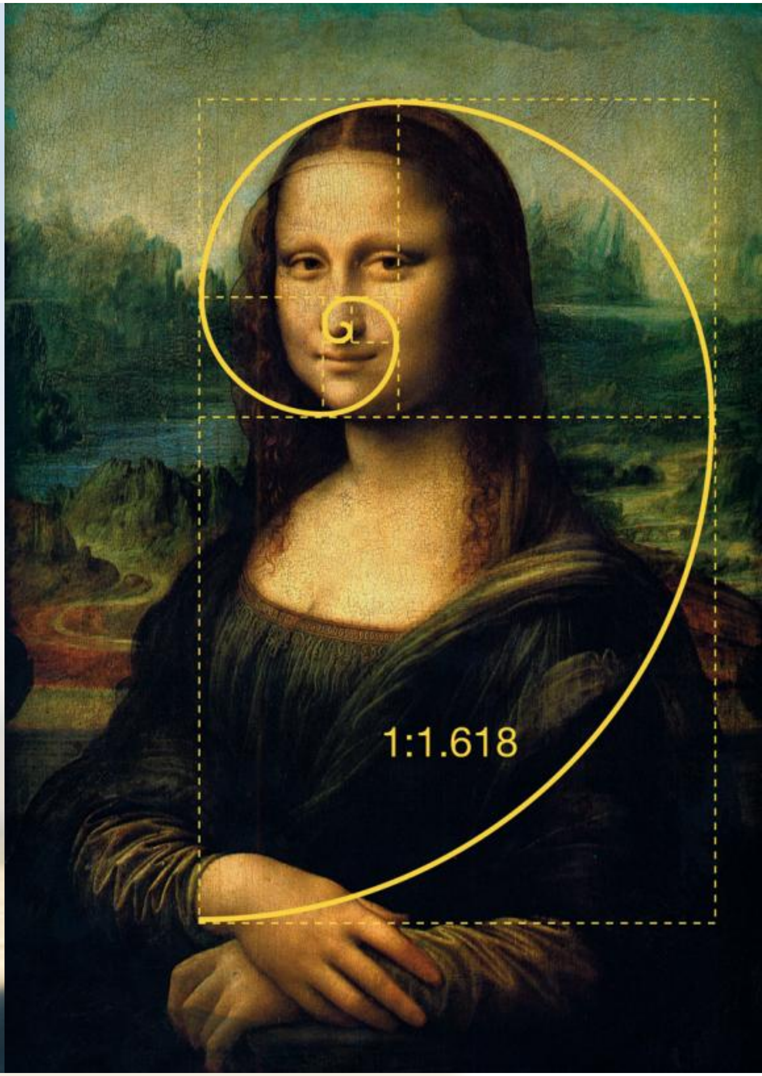


1D Topological superconductors

Summary

- New paradigm for realizing non-abelian systems: defects in two-dimensional topological phases.
- Can be implemented by coupling to superconductors.
- Implementation of universal defects?
- 2D Fractionalized superconductor with “Fibonacci” anyons from lattices of interacting defects.
- How can we trap/manipulate Fib. Anyons?
- Is the fine tuning of the lattice model essential?

Summary



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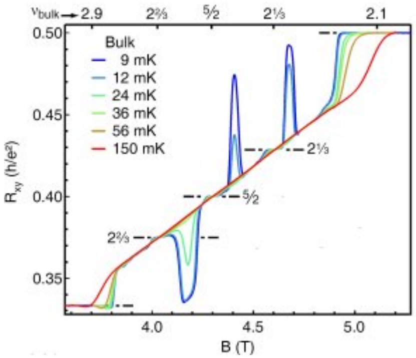
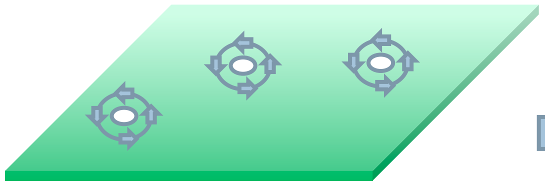
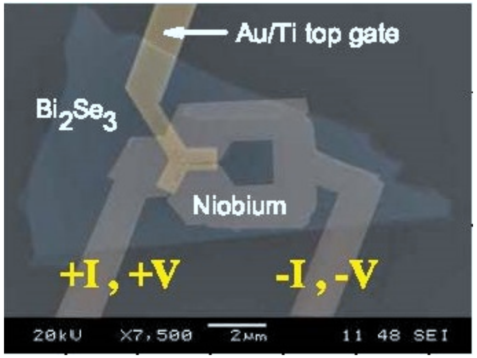


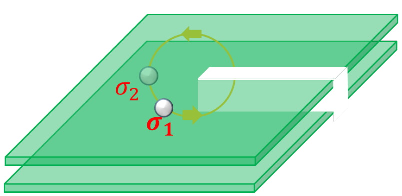
Fractional QH



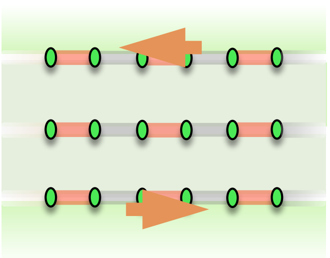
2D p+ip superconductors



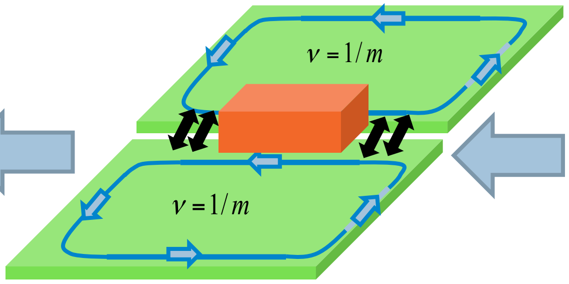
Superconductor - 3D Top. Insulator (Semiconductor) heterostructures



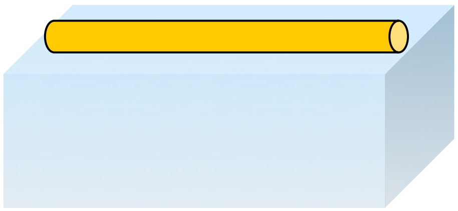
Defects in Top. phases



2D fractionalized superconductors



1D fractionalized superconductors



1D Topological superconductors

Thank you.

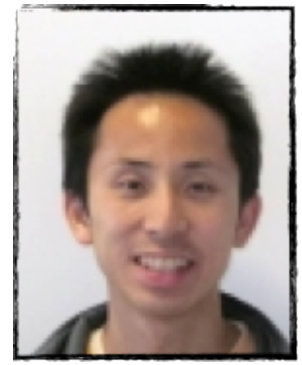
HOW'S YOUR QUANTUM COMPUTER PROTOTYPE COMING ALONG?
GREAT!
THE PROJECT EXISTS IN A SIMULTANEOUS STATE OF BEING BOTH TOTALLY SUCCESSFUL AND NOT EVEN STARTED.
CAN I OBSERVE IT?
THAT'S A TRICKY QUESTION.
Dilbert.com DilbertCartoonist@gmail.com
4-17-12 © 2012 Scott Adams, Inc./Dist. by Universal Uclick

Acknowledgements

Erez Berg

Ady Stern

Jason Alicea



Roger Mong

David Clarke

Yuval Oreg

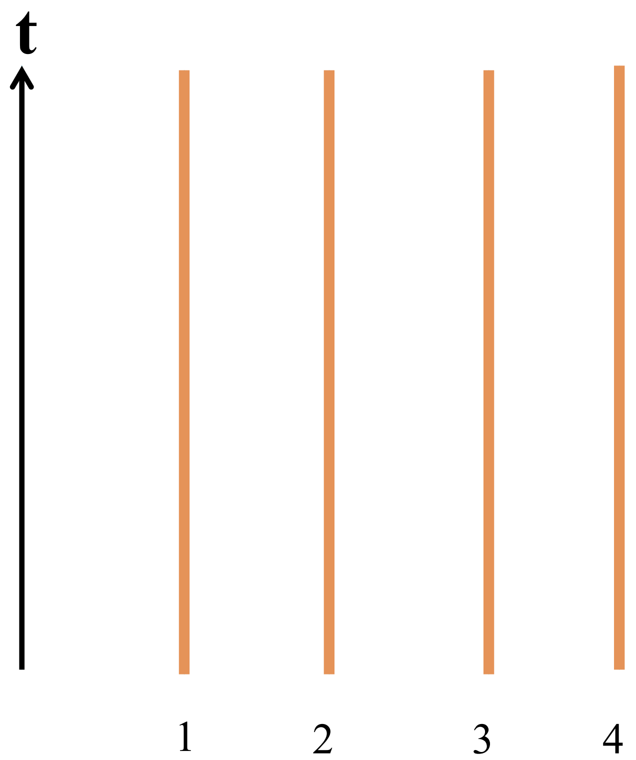
Gil Refael

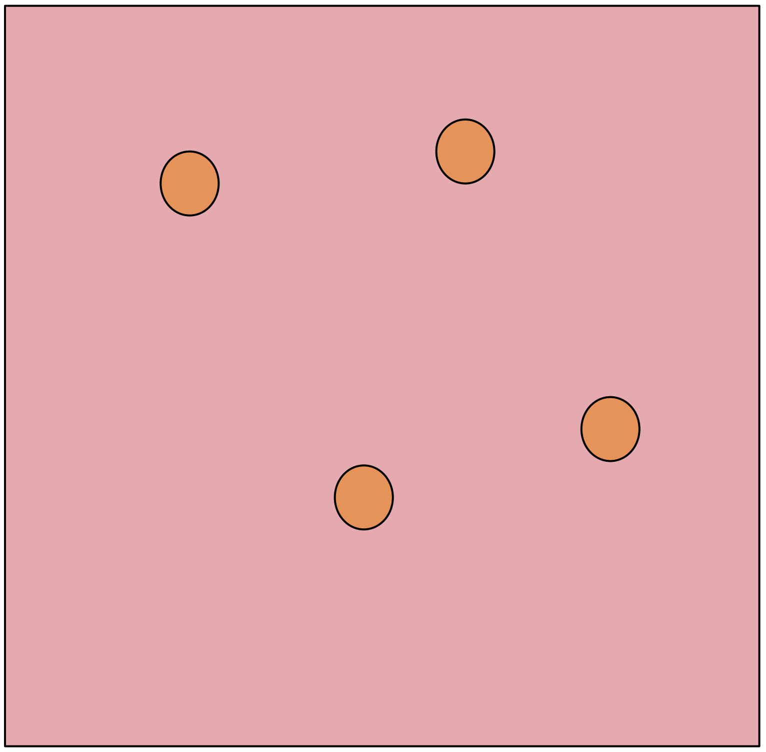
Paul Fendely

Braiding

What are the properties

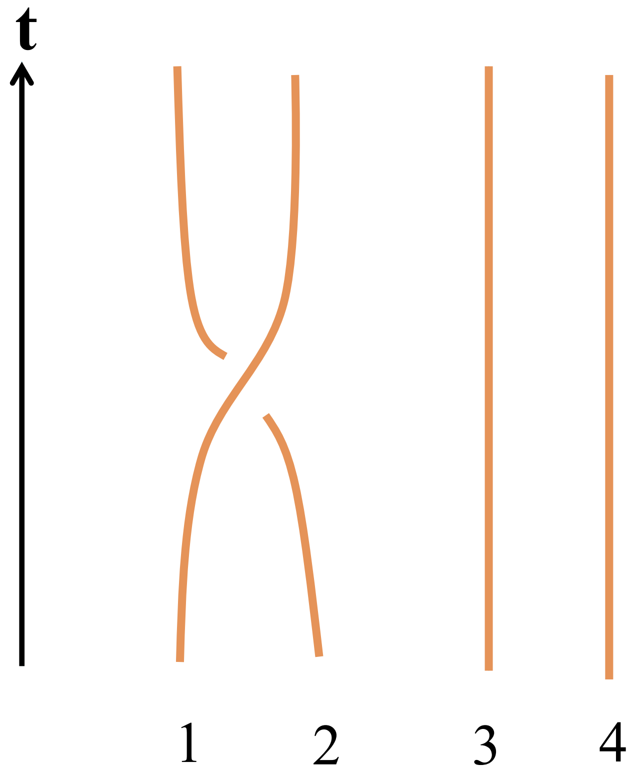
of $\hat{U}_{ij}$?

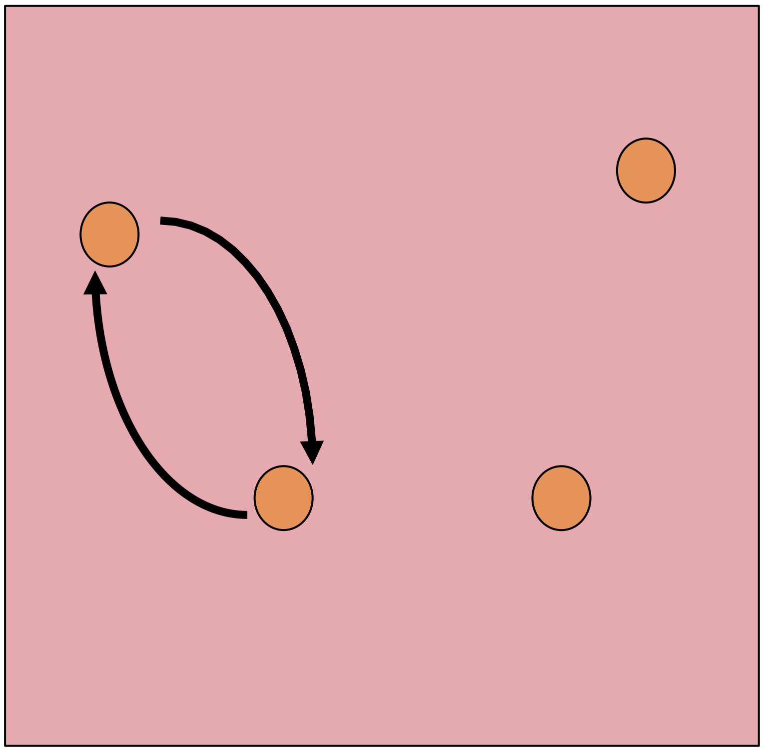




Braiding

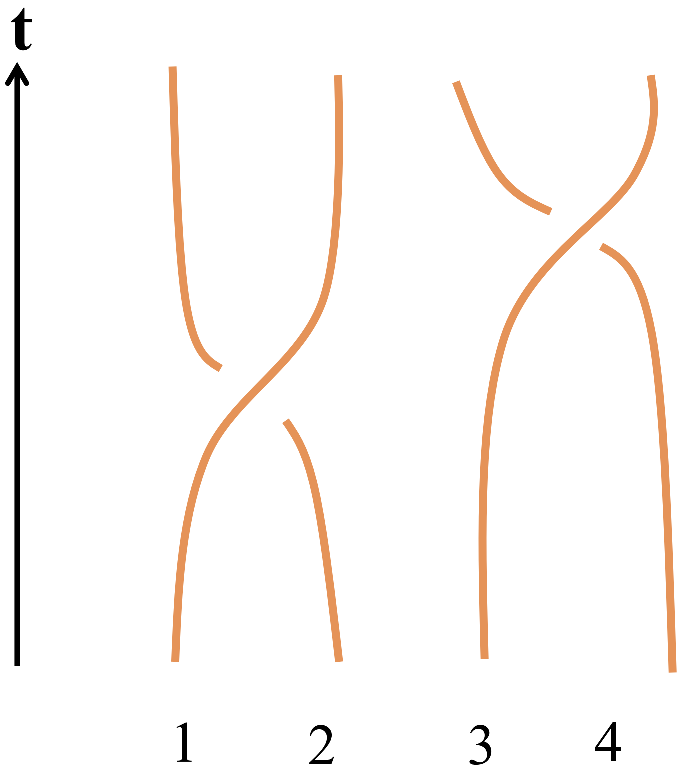
$\hat{U}_{12}$

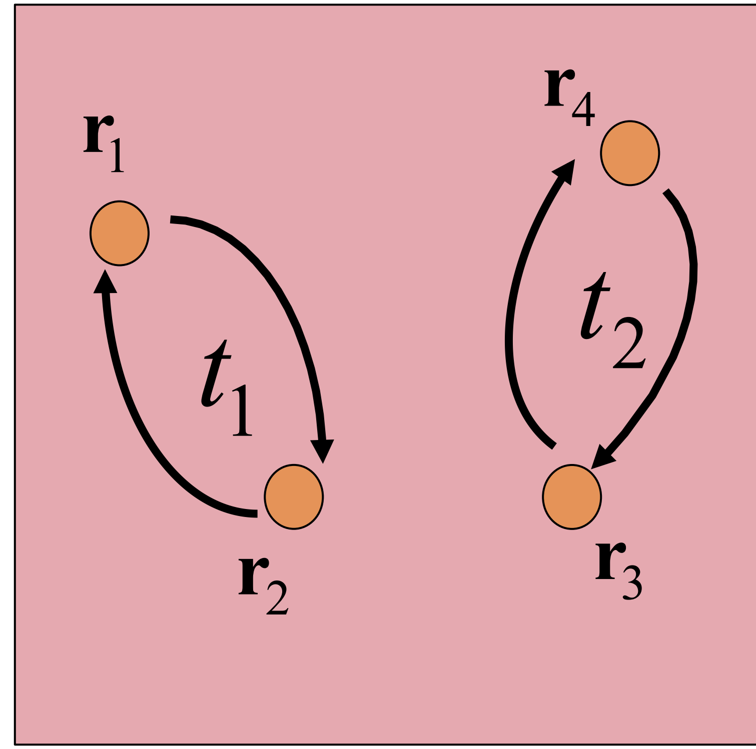




Braiding

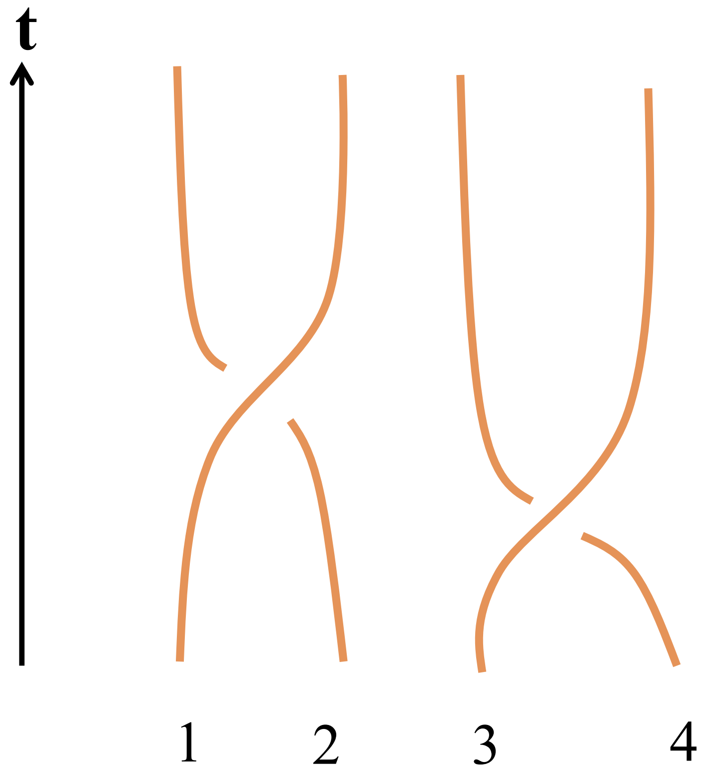
$$\hat{U}_{34}\hat{U}_{12}$$

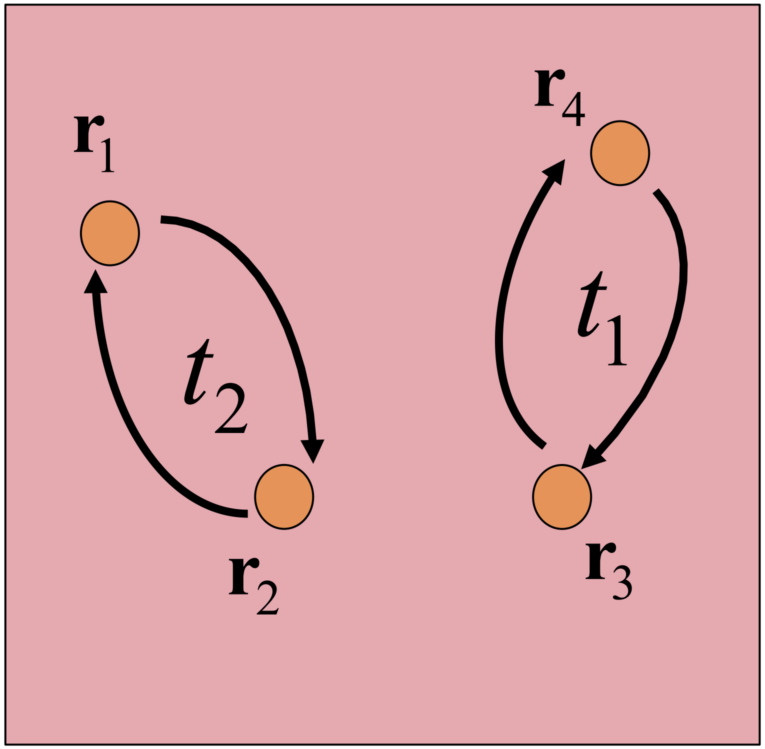




Braiding

$$\hat{U}_{12}\hat{U}_{34}$$

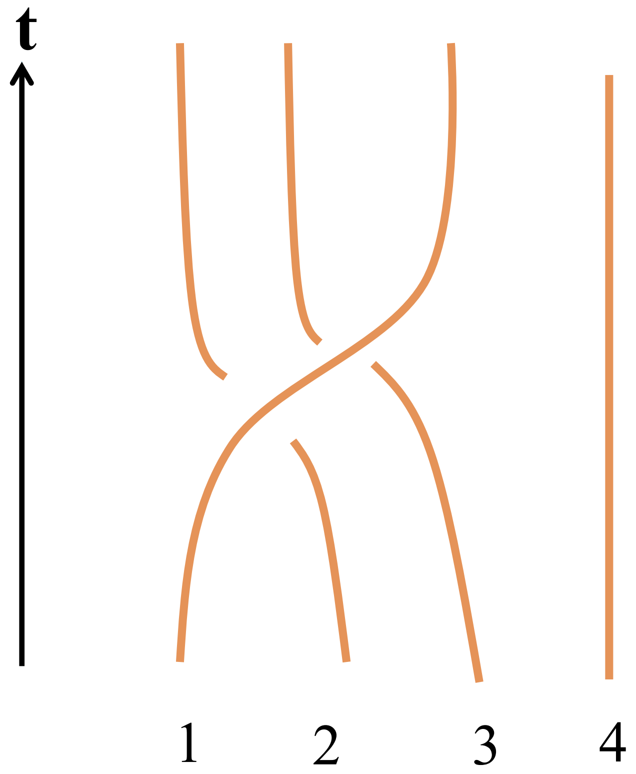


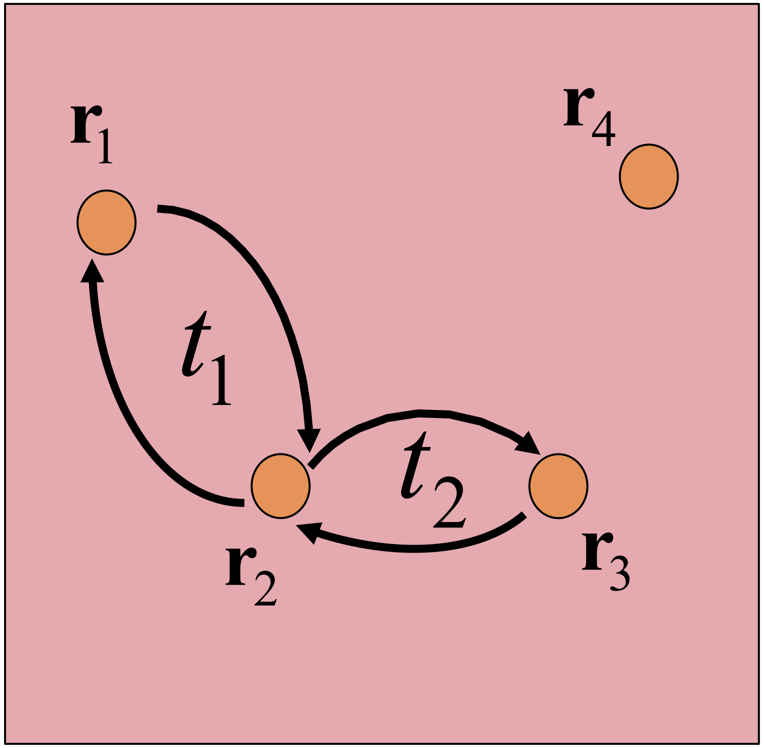


$$\hat{U}_{34}\hat{U}_{12} = \hat{U}_{12}\hat{U}_{34}$$

Braiding

$$\hat{U}_{23}\hat{U}_{12}$$

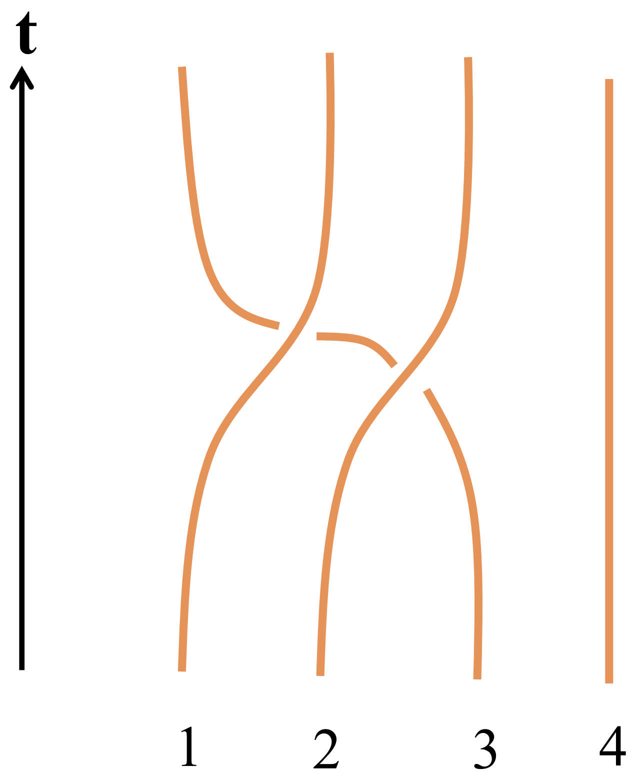


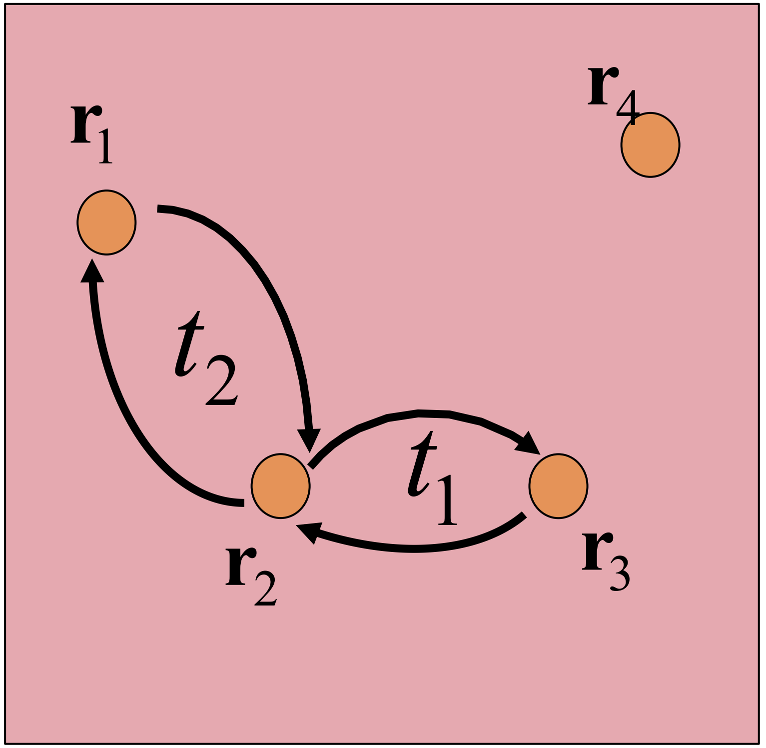


$$t_1 < t_2$$

Braiding

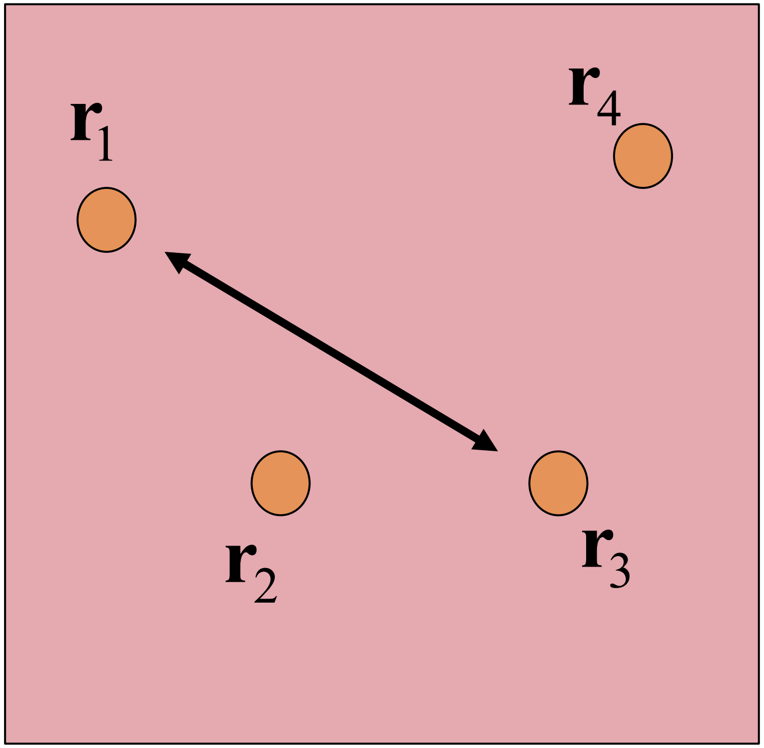
$$\hat{U}_{12}\hat{U}_{23}$$



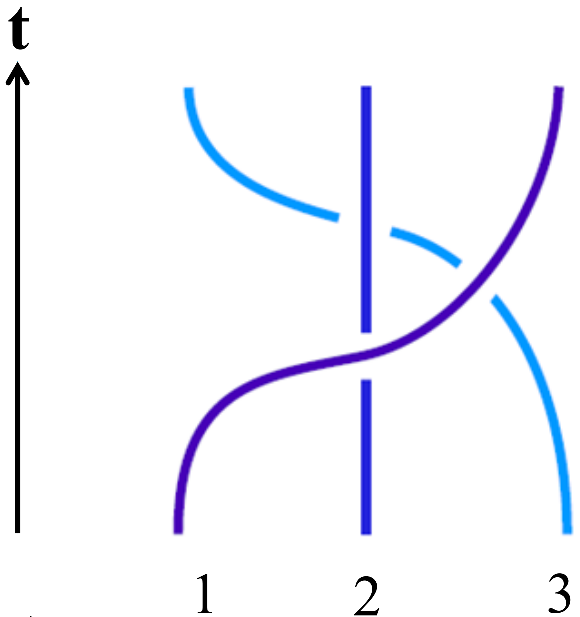


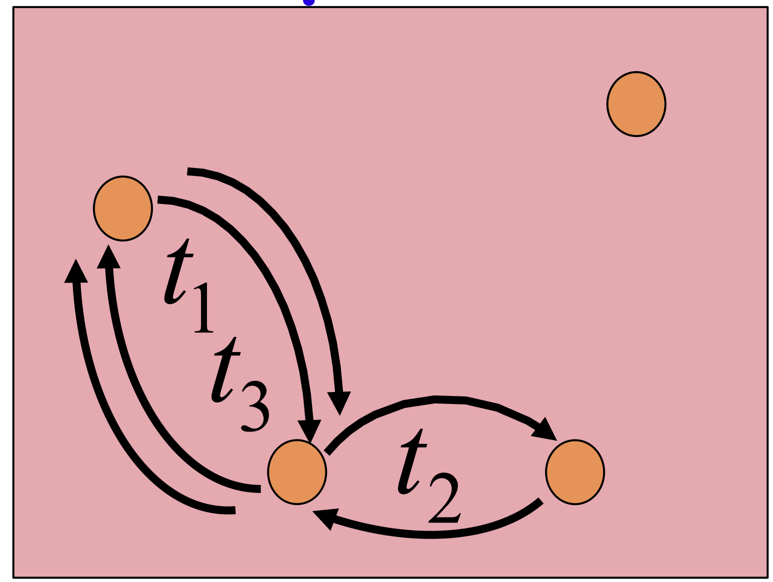
$$\hat{U}_{23}\hat{U}_{12} \neq \hat{U}_{12}\hat{U}_{23}$$

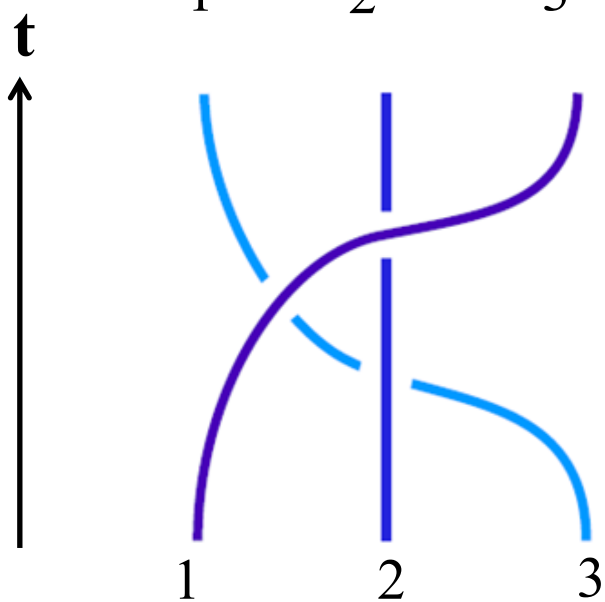
The Braid Group

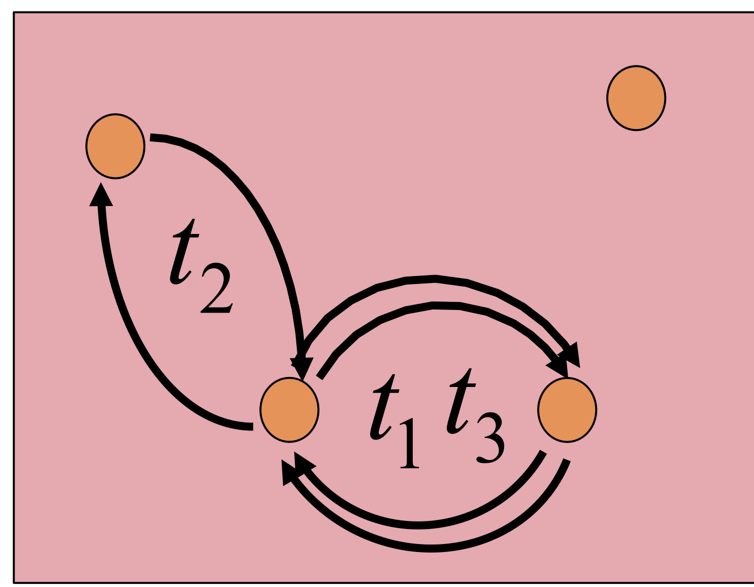


The Braid Group

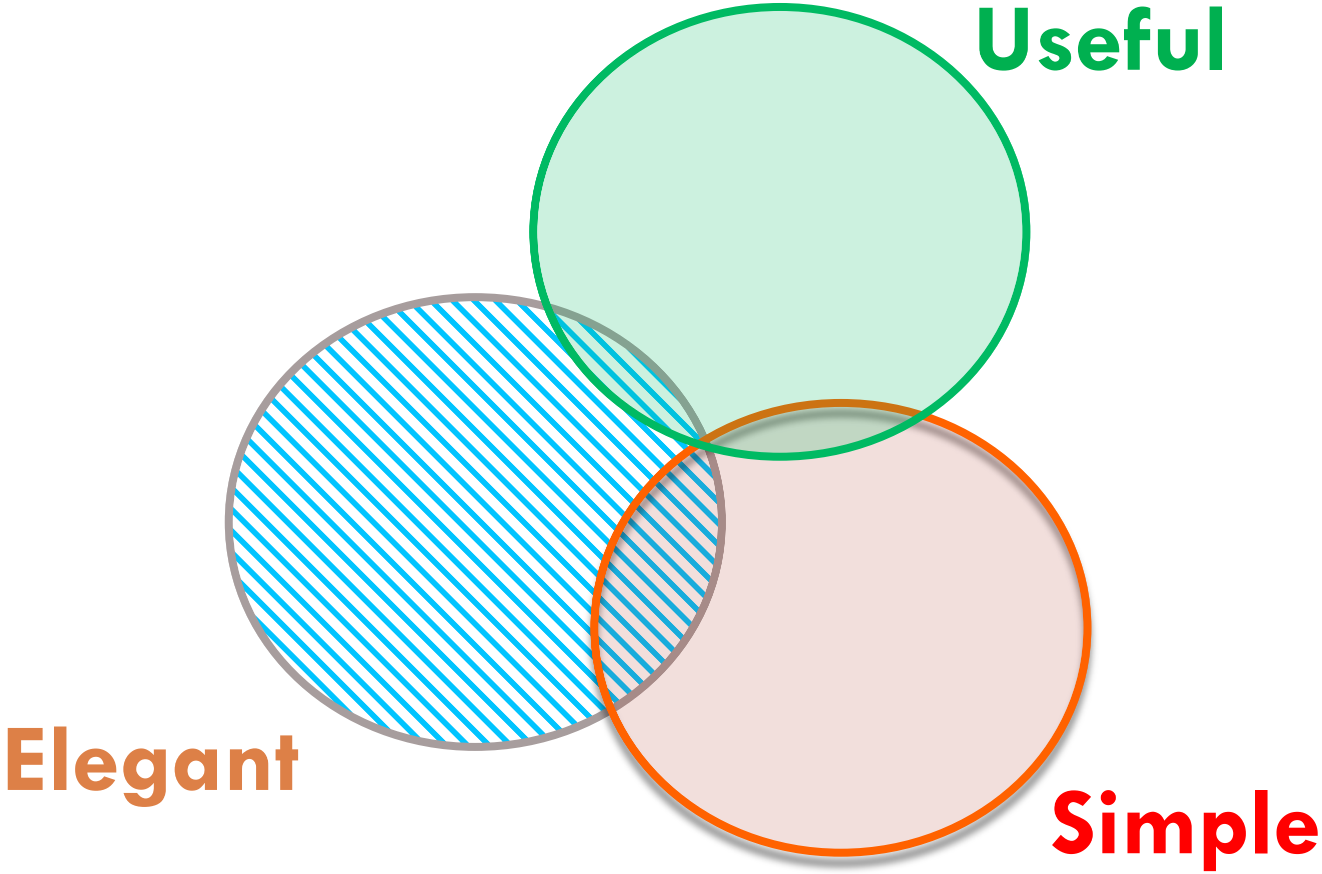








Grad School at the Technion



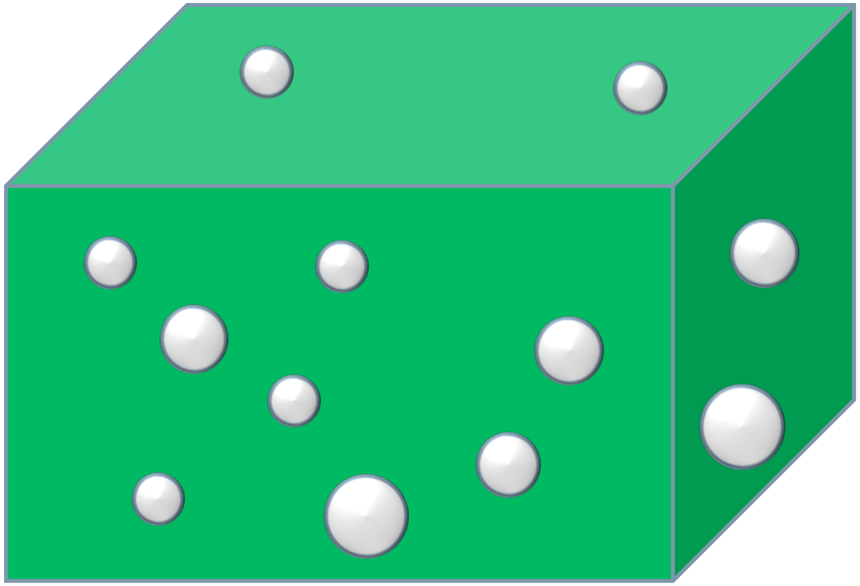
Outline

- Exchange statistics
- Non-Abelian anyons
- Topological quantum computing
- Overview of physical realizations
- Majorana fermions in a 1D superconductor
- Fractionalized superconductors in 1D and 2D

Exchange statistics

Wavefunction transformation under exchange of two identical particles:

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_N) \longrightarrow \psi'(\mathbf{r}_2, \mathbf{r}_1, \ldots \mathbf{r}_N)$$

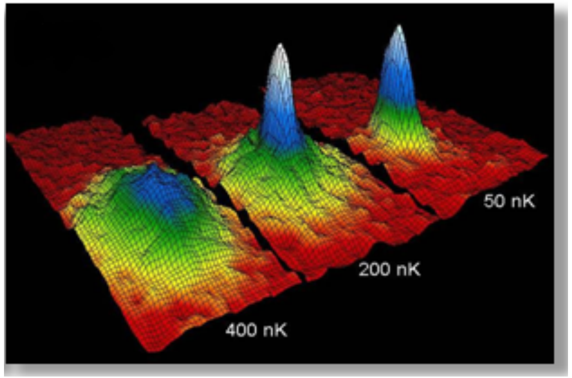


Particle types

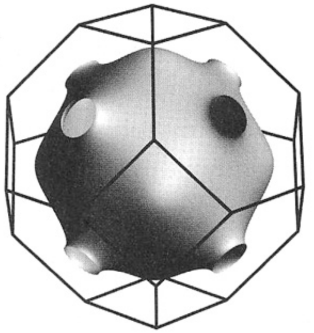
- **Bosons / Fermions**

$$\psi \rightarrow \psi' = \pm\psi$$

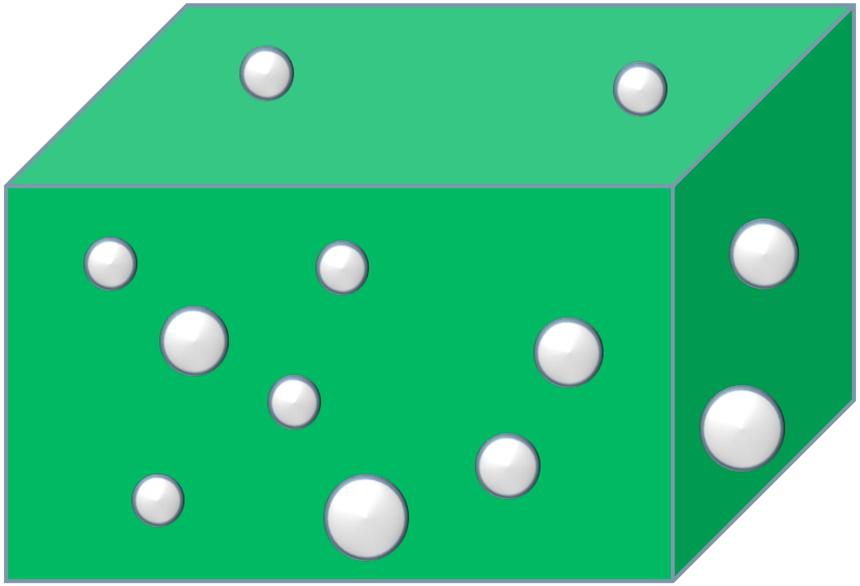
All particles in 3D



Bose-Einstein Condensate



Metal



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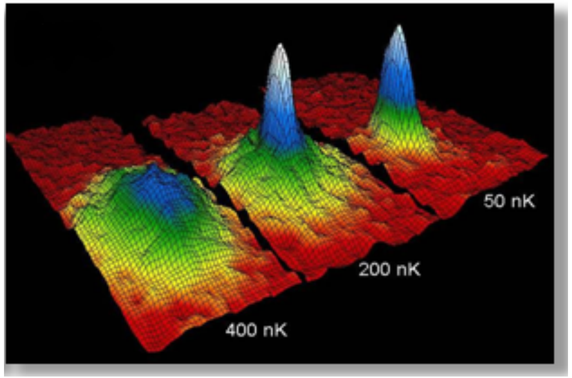
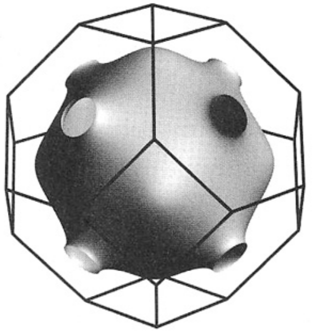
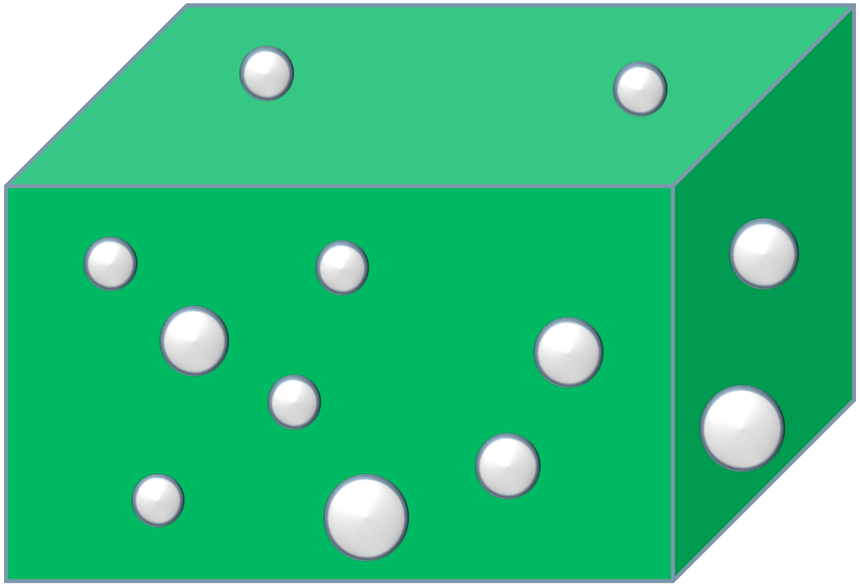
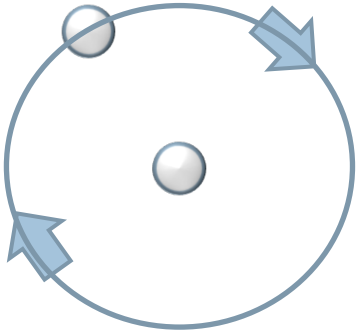


Bose-Einstein Condensate



Metal





Particle types

- **Bosons / Fermions**

  $$\psi \rightarrow \psi' = \pm\psi$$

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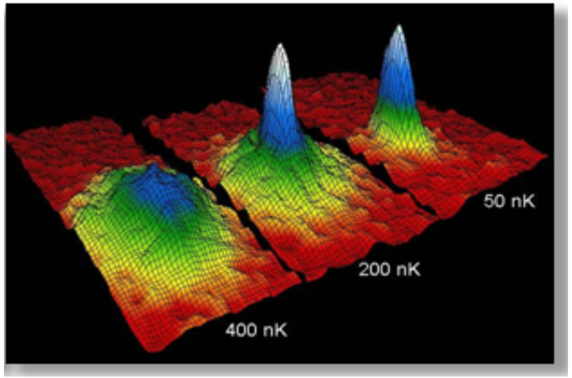

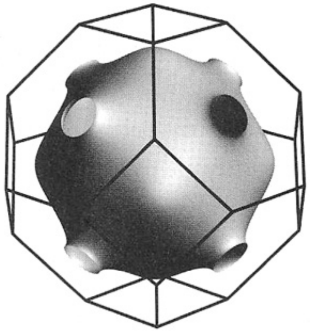

- **"Any-ons"**

  "Emergent" particles in 2D

Abelian anyons

$$\psi \rightarrow \psi' = e^{i\theta}\psi$$

- Exchanges commute
- Most acessible quantum Hall states

Non-Abelian anyons

Outline

- Exchange statistics
- Non-Abelian anyons
- Topological quantum computing
- Overview of physical realizations
- Majorana fermions in a 1D superconductor
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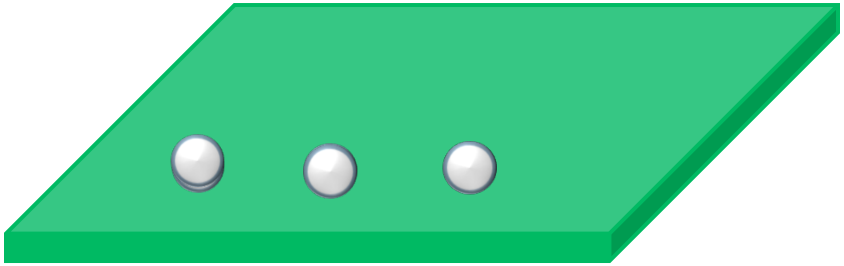
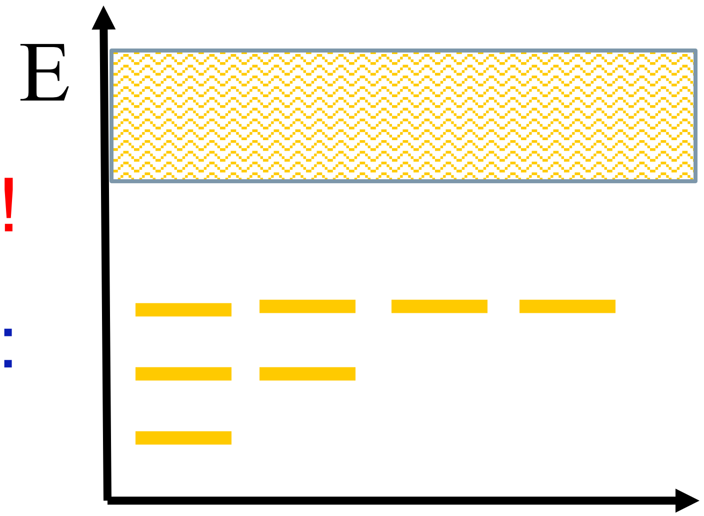
Non-Abelian Anyons

- **Key Properties:**
  - Degeneracy increases with number of anyons



“quantum dimension”

$$\dim H_{GS} \sim \lambda^{N}$$



Robust to perturbations!

  - Non-Abelian statistics:

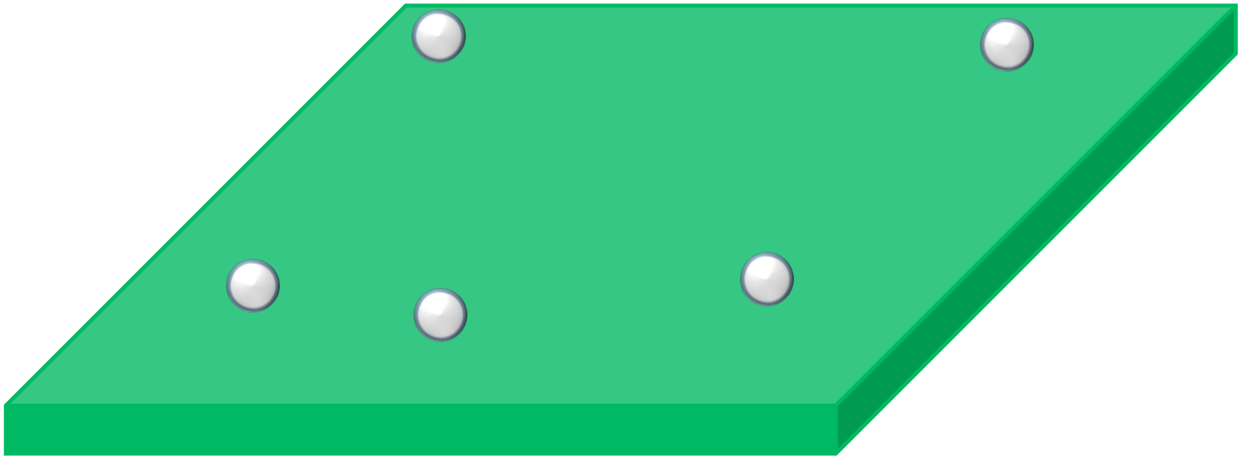
$$\psi_m \rightarrow \psi_n = U_{nm}\psi_m$$

Non-Abelian Anyons

- Non-Abelian statistics:

$$\psi_m \rightarrow \psi_n = U_{nm}\psi_m$$

Transformation depends only on the topology of the path



Non-Abelian Anyons

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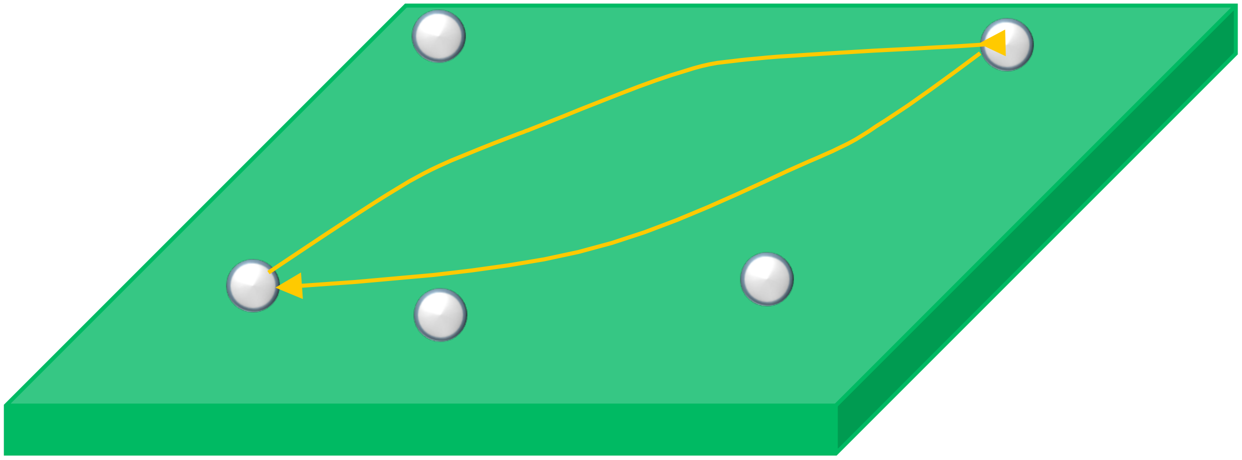

Non-Abelian Anyons

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$$\boxed{\psi_m \rightarrow \psi_n = U_{nm}\psi_m}$$

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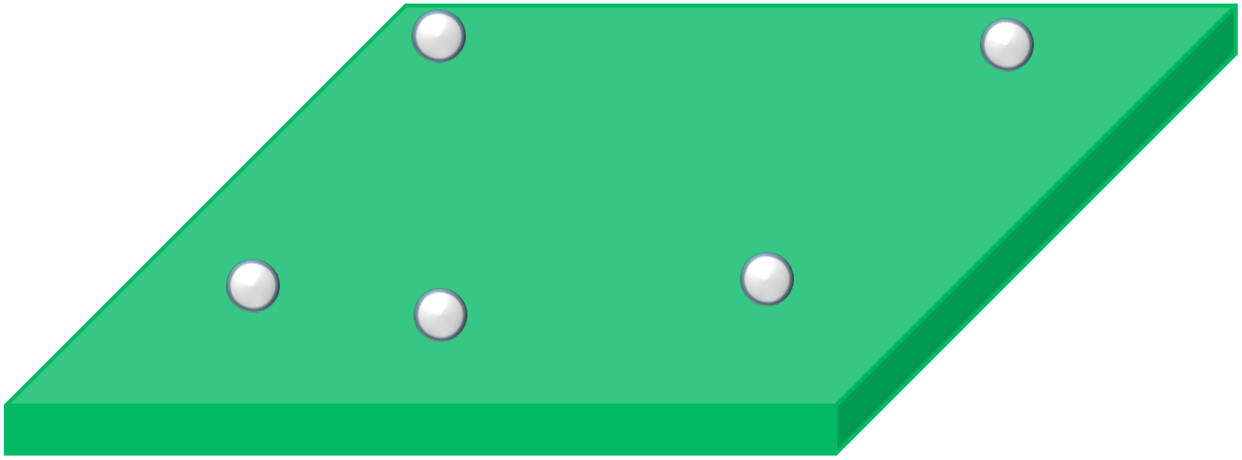

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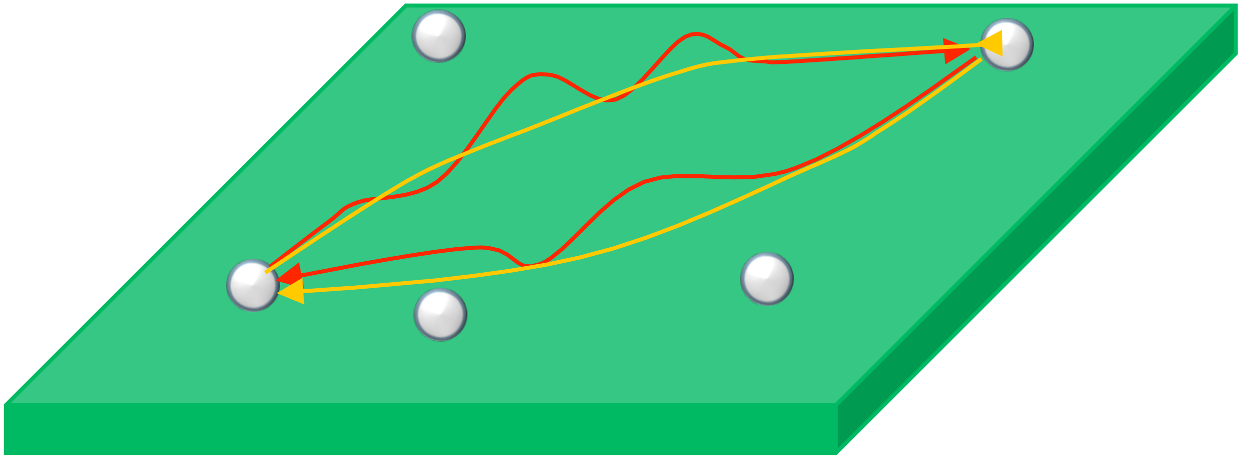

Non-Abelian Anyons

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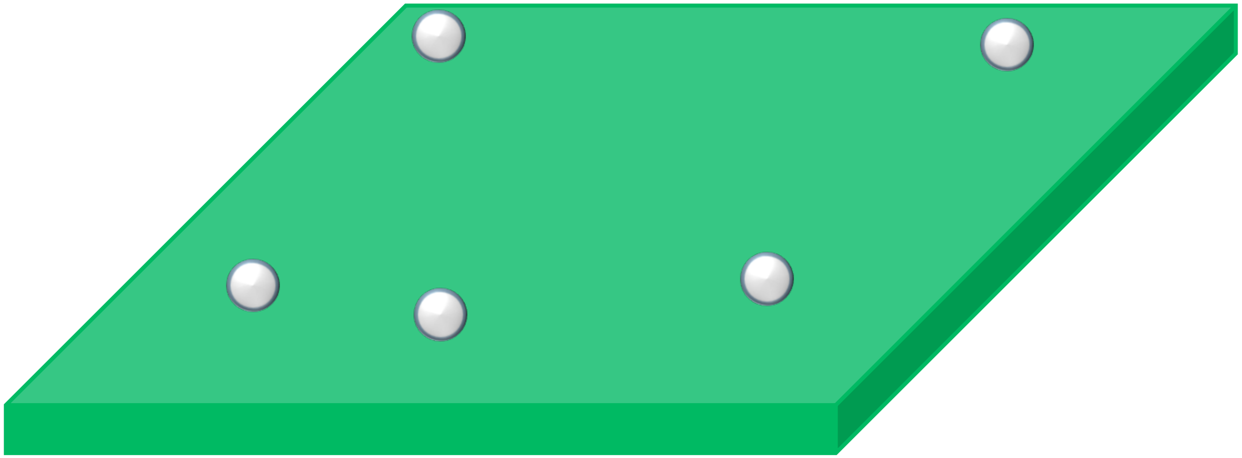
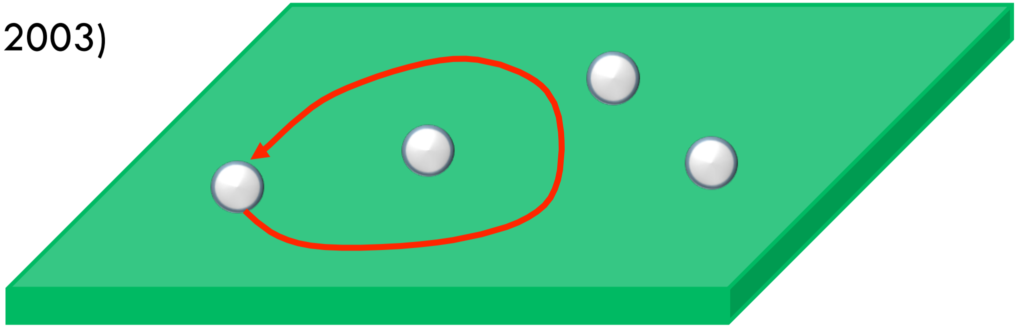
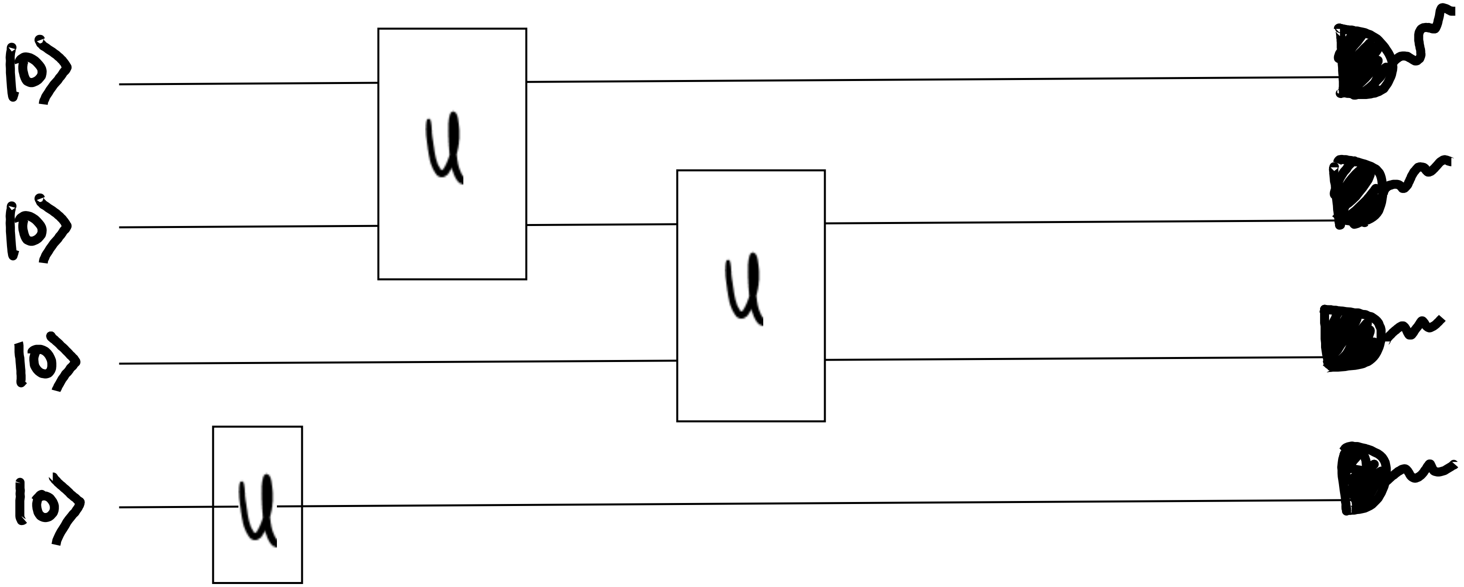


Topological quantum computation

A. Kitaev, Ann. Phys. 303, 2 (2003)



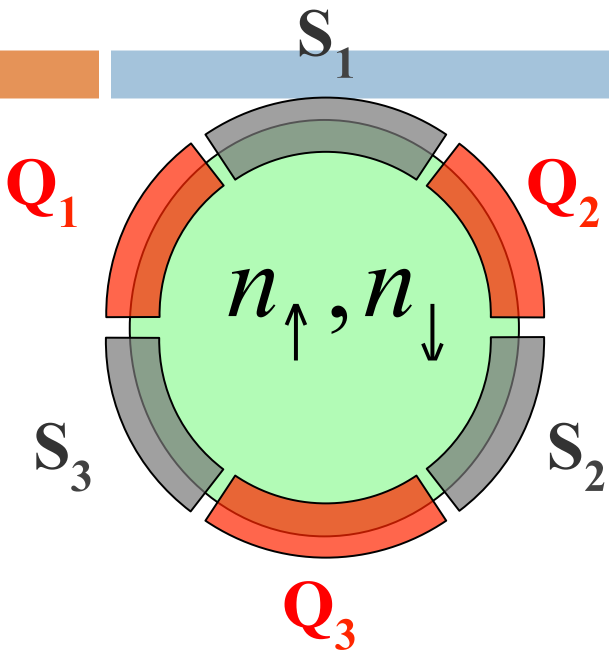


Anyons, anyone?

Still not universal for QC…..

Can we do better?

Ground state degeneracy



Spins, Charges

$$S_j = q/m,\ q = 0,1,..2m-1$$

$$Q_j = q/m,\ q = 0,1,..2m-1$$

$$\boxed{e^{i\pi Q_i} e^{i\pi S_j} = e^{\pm i\pi/m} e^{i\pi S_j} e^{i\pi Q_i}}$$

$\{+\ \ \mathbf{i = j+1}\}$ $\{-\ \ \mathbf{i = j-1}\}$

2N domains, fixed $n_\uparrow, n_\downarrow$ **= Q_{tot}, S_{tot}**

$(2m)^{N-1}$ ground states

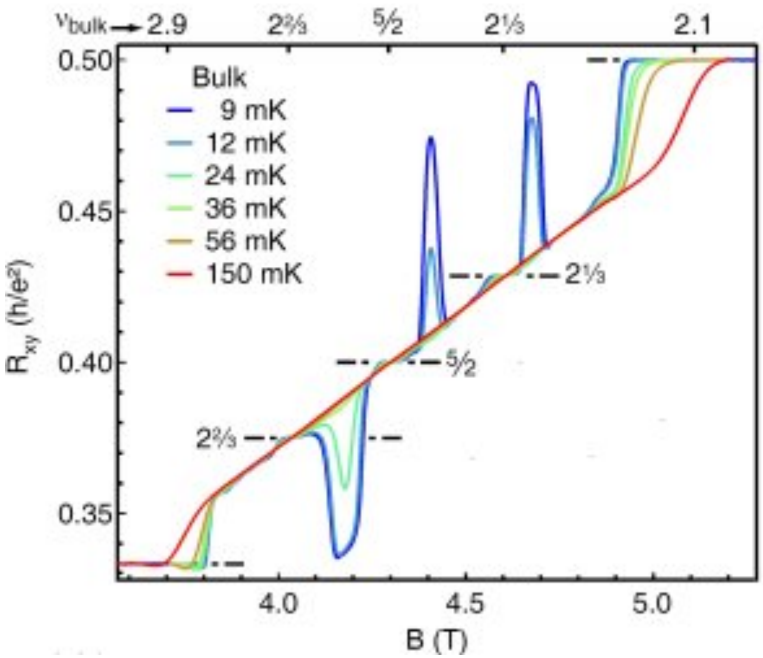
$$= \left(\sqrt{2m}\right)^{2(N-1)}$$

Proposed non-Abelian systems

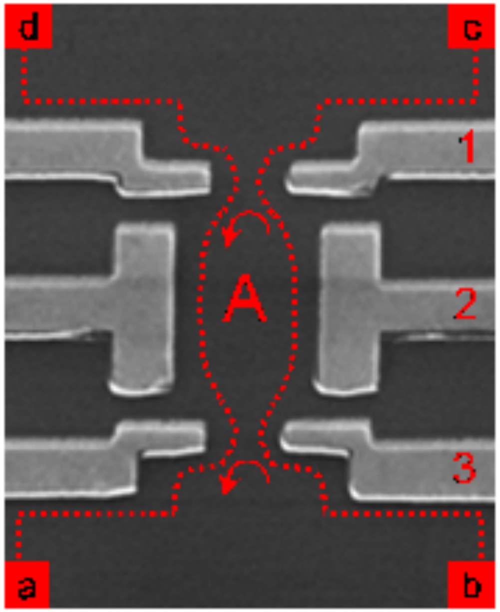
- **Fractional quantum Hall effect at** $\nu = 5/2$

  Willet, Eisentein et al. (1987)

  Moore and Read (1992)



Miller et al., Nature Physics **3**, 561 (2007)



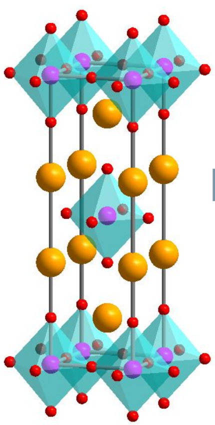
R. L. Willett et al., PRL **111**, 186401 (2013)

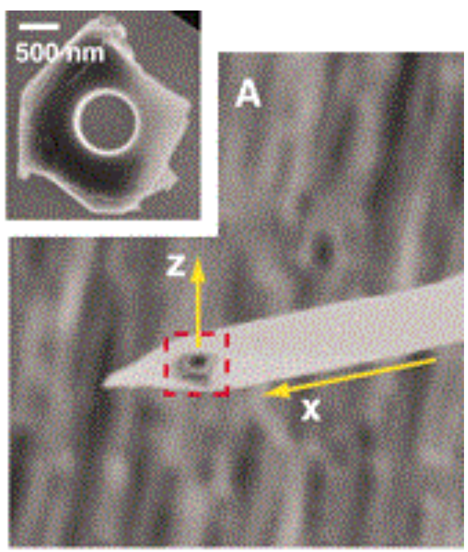
Proposed non-Abelian systems

- **p+ip superconductors**

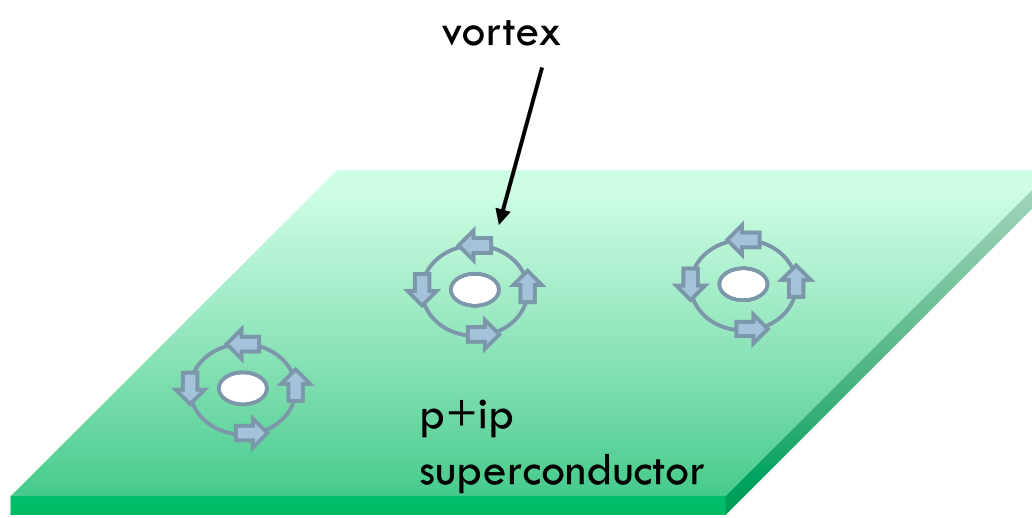
  Read and Green (2001)

$SrRu_2O_4$





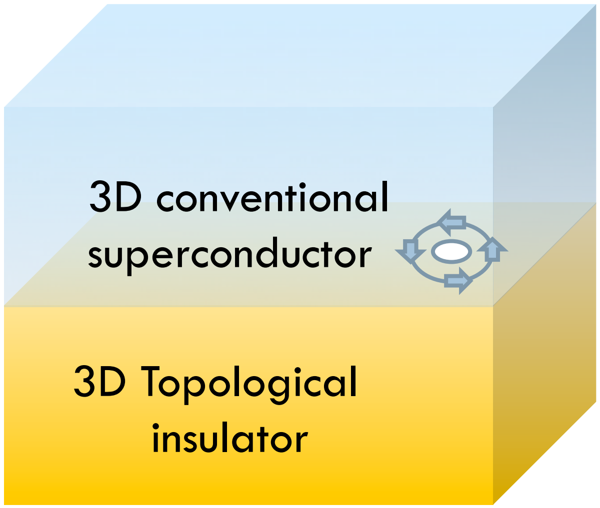
Jang *et al.*, Science **331**, 186 (2001)

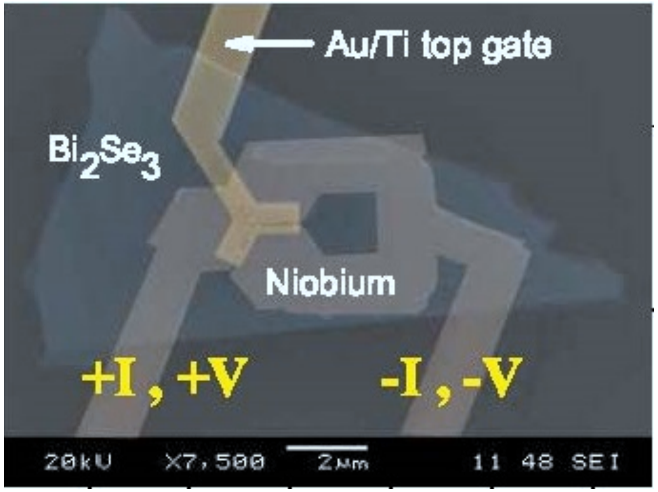


Proposed non-Abelian systems

- **"Engineered" p+ip superconductors**

  Fu & Kane (2008), Sau et al. (2010), Lee (2009), Alicea (2010)





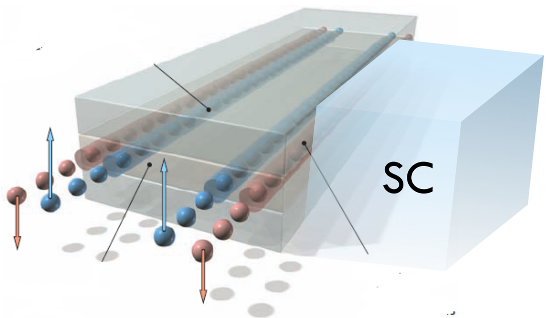
C. Kurter *et al.*, arXiv: 1402.3623

Anyons, anyone?

- **Topological 1D superconductors**

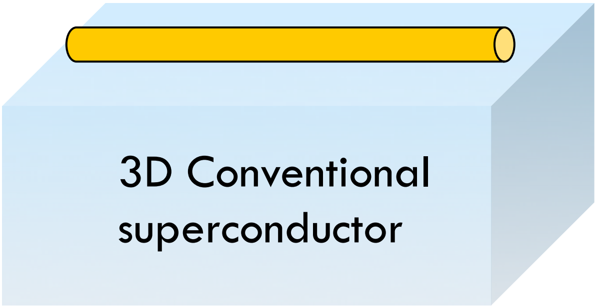
Fu & Kane (2009)

Quantum Spin Hall effect

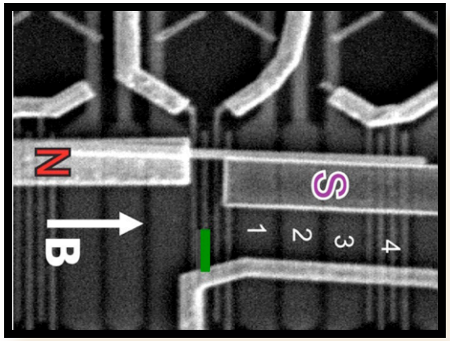


Kitaev (2002)

Oreg et al.,
Lutchyn et al. (2010),



Mourik et al. (2012),
Das et al. (2012).



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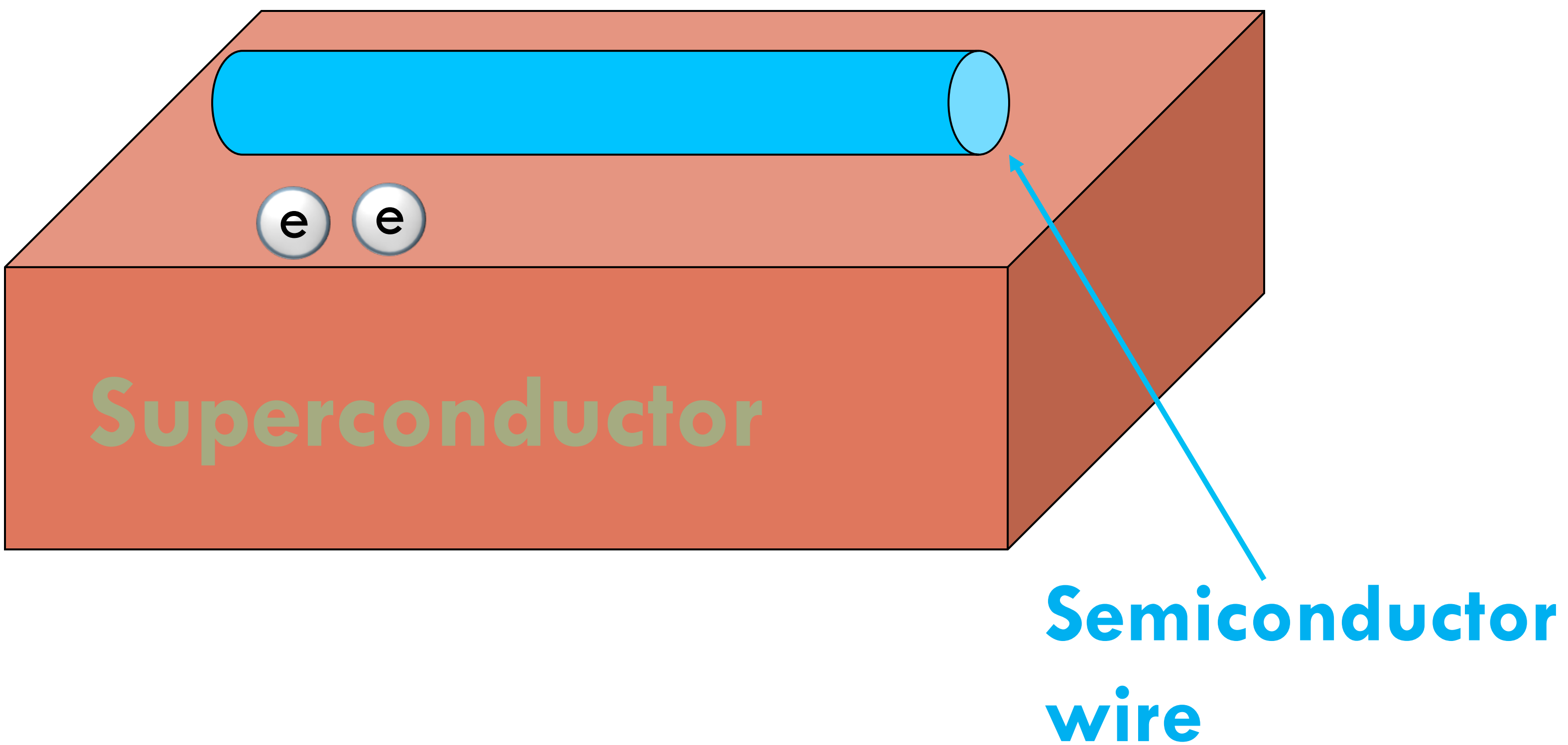
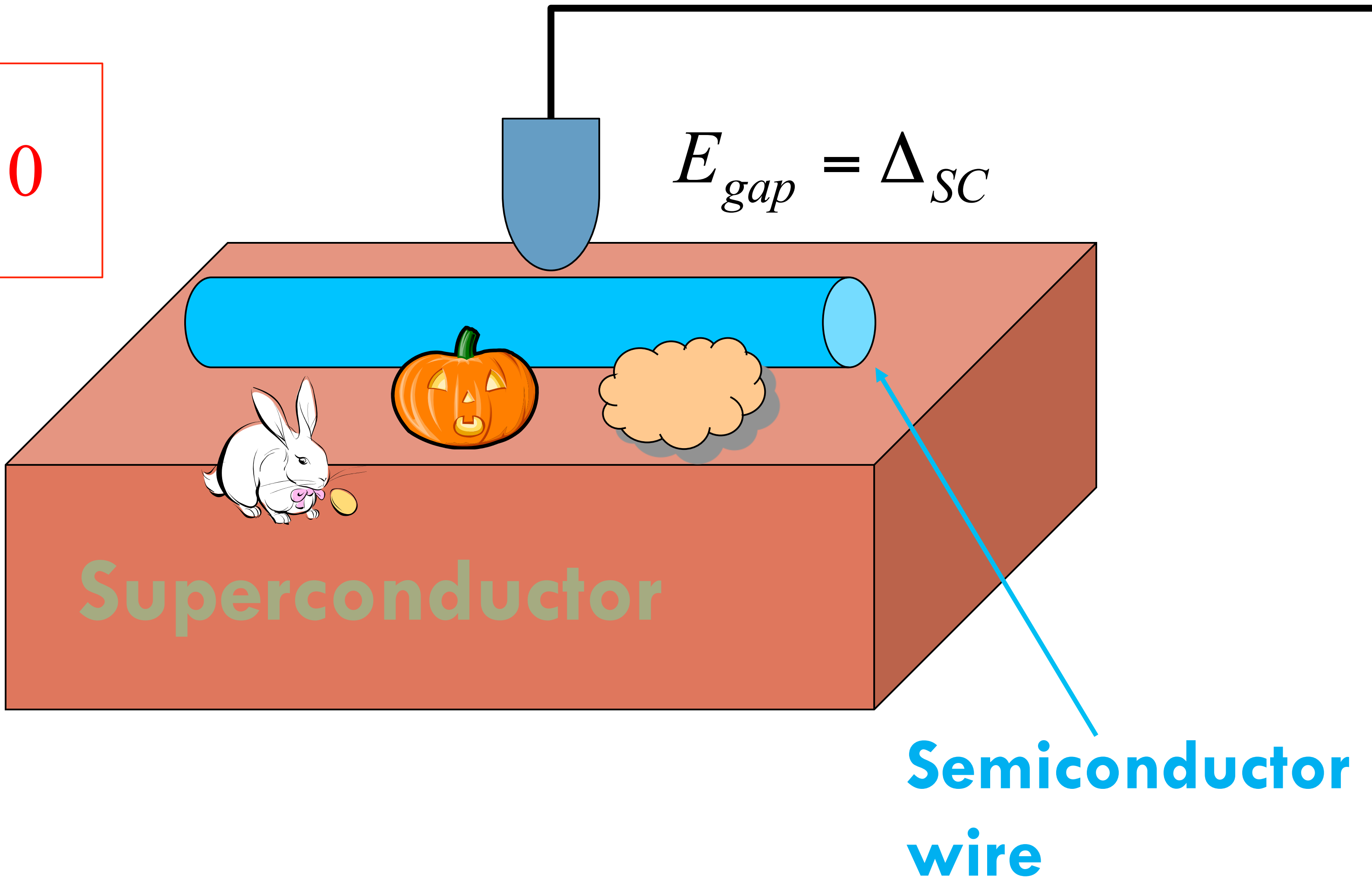
Topological 1D superconductor

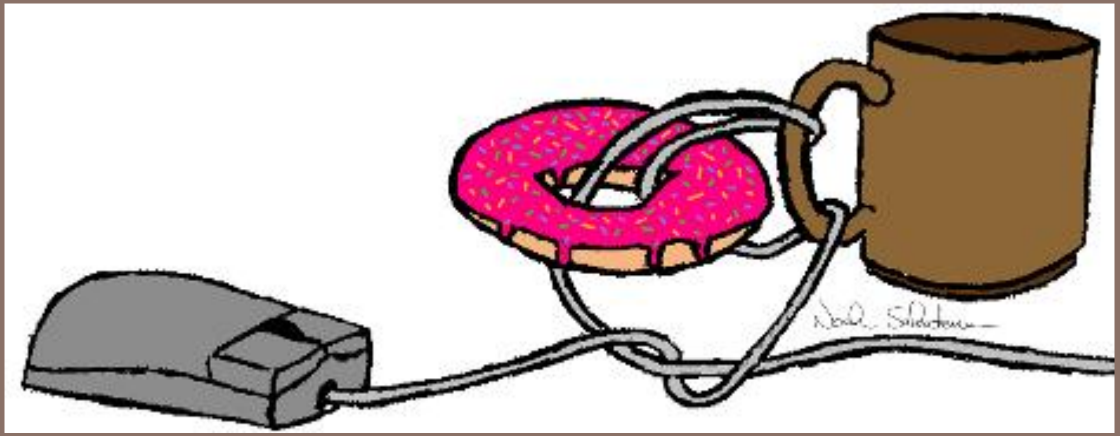
- 1D Semiconductor wire coupled to a bulk superconductor:



Topological 1D superconductor

$$E_{gap} = 0$$





NEW NON-ABELIAN STATES FROM HYBRID QUANTUM HALL-SUPERCONDUCTOR SYSTEMS

EPQHS-5, July 2014

Netanel Lindner

Outline

- Exchange statistics
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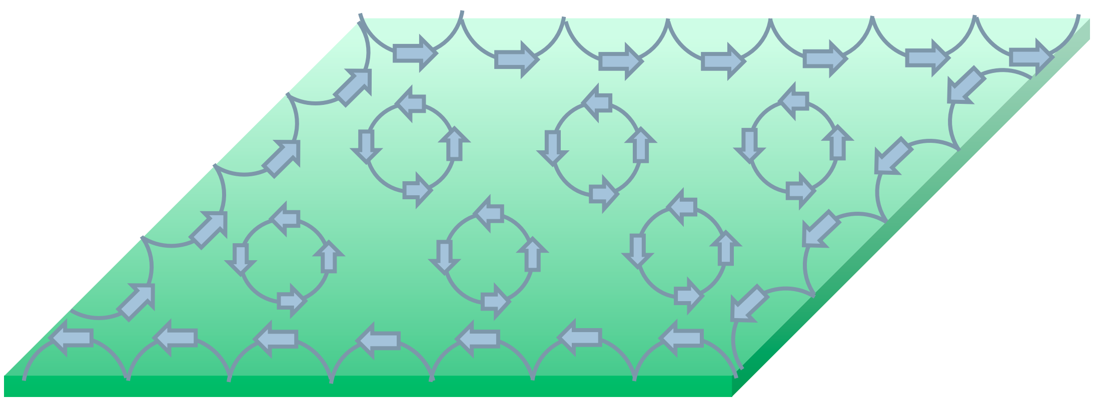
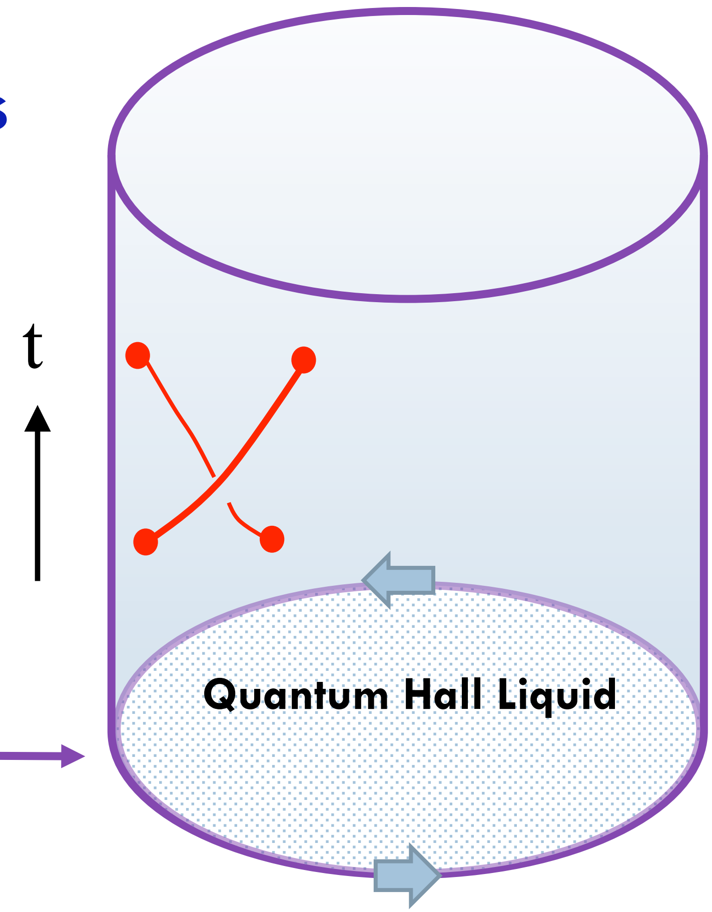
Bulk-edge correspondence

Quantum Hall effect: Edge states

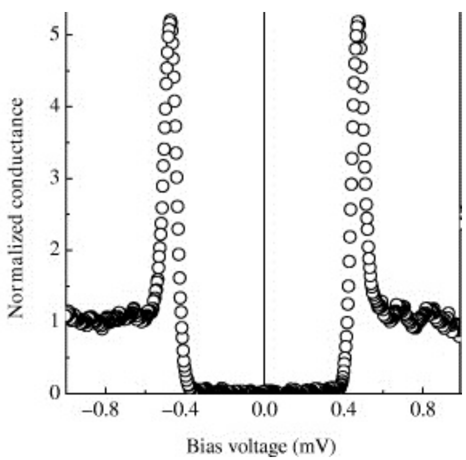




Edge modes
(Described by a conformal field theory)

Pros and Cons

- **Advantages of “Engineered” systems**
  - Energy gap induced by external SC and not by interactions.

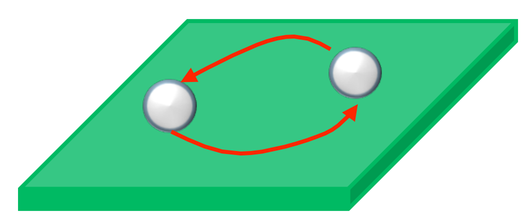


  - Control:
    Anyons can be “easily” localized and manipulated

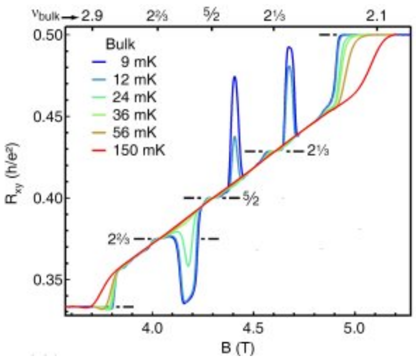
- **Challenges of Ising anyons**
  - Exchange statistics is not rich enough to yield universal quantum computation:

$$\dim H_{GS} : \sqrt{2}^{N}$$

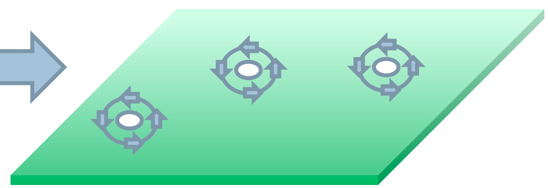
$$U = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$



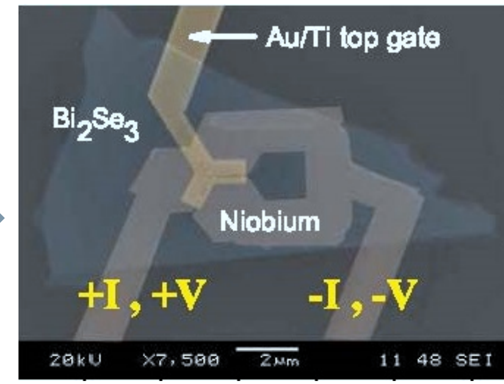
Proposed non-Abelian systems



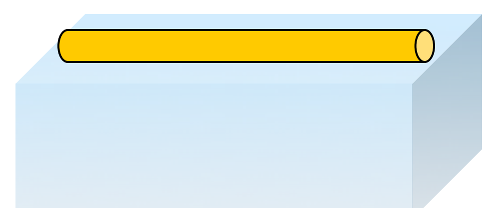
Fractional QH



2D p+ip superconductors



Superconductor - 3D Top. insulator heterostructures



1D Topological superconductors

"Ising" anyons