Anomalous transport: from the quark gluon plasma to Weyl semi-metals on a superstring

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CCCQ, Evora, 06-10-2014

Outline

- Quark Gluon Plasma
- Flash review: AdS/CFT
- Anomalous transport model in AdS/CFT
- Map to Weyl semi-metals
- Conclusions

Quark gluon plasma

Please click here for movie on RHICs hot quark soup

Quark gluon plasma

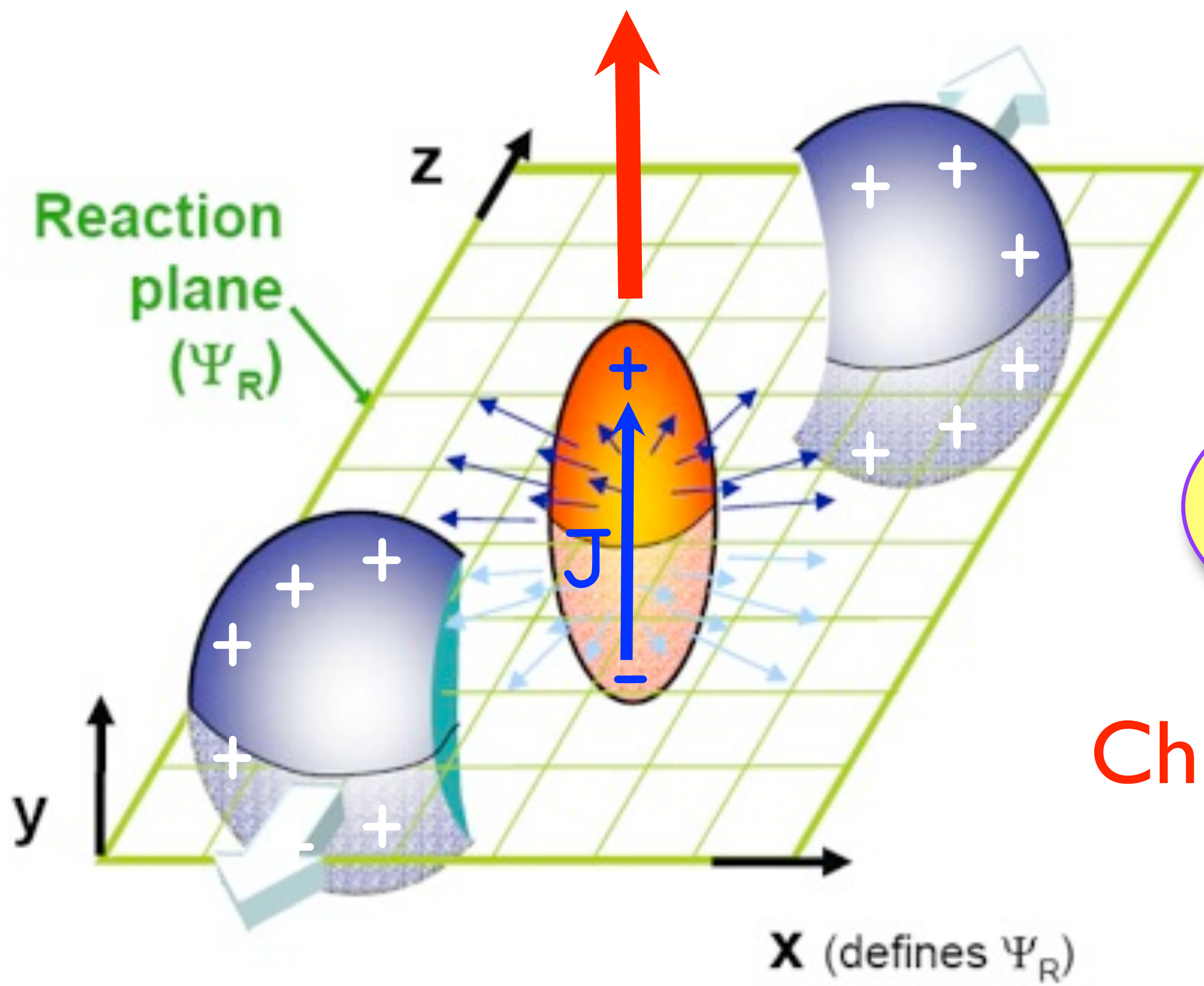
- QCD - confined quarks and gluons
- High T (or ρ): de-confinement and plasma phase
- Smash nucleons agains each other (RHIC, LHC)
- Lowest specific viscosity known!
- Charge separation effect observed
- Possible explanation: Chiral magnetic effect
- $T \sim 3T_c$: deconfined but strongly coupled

Quark gluon plasma

strongest Magnetic field in the Universe 10^{15}T!!!

(QHE: 10 T)

(T ~ 10^{12} K)

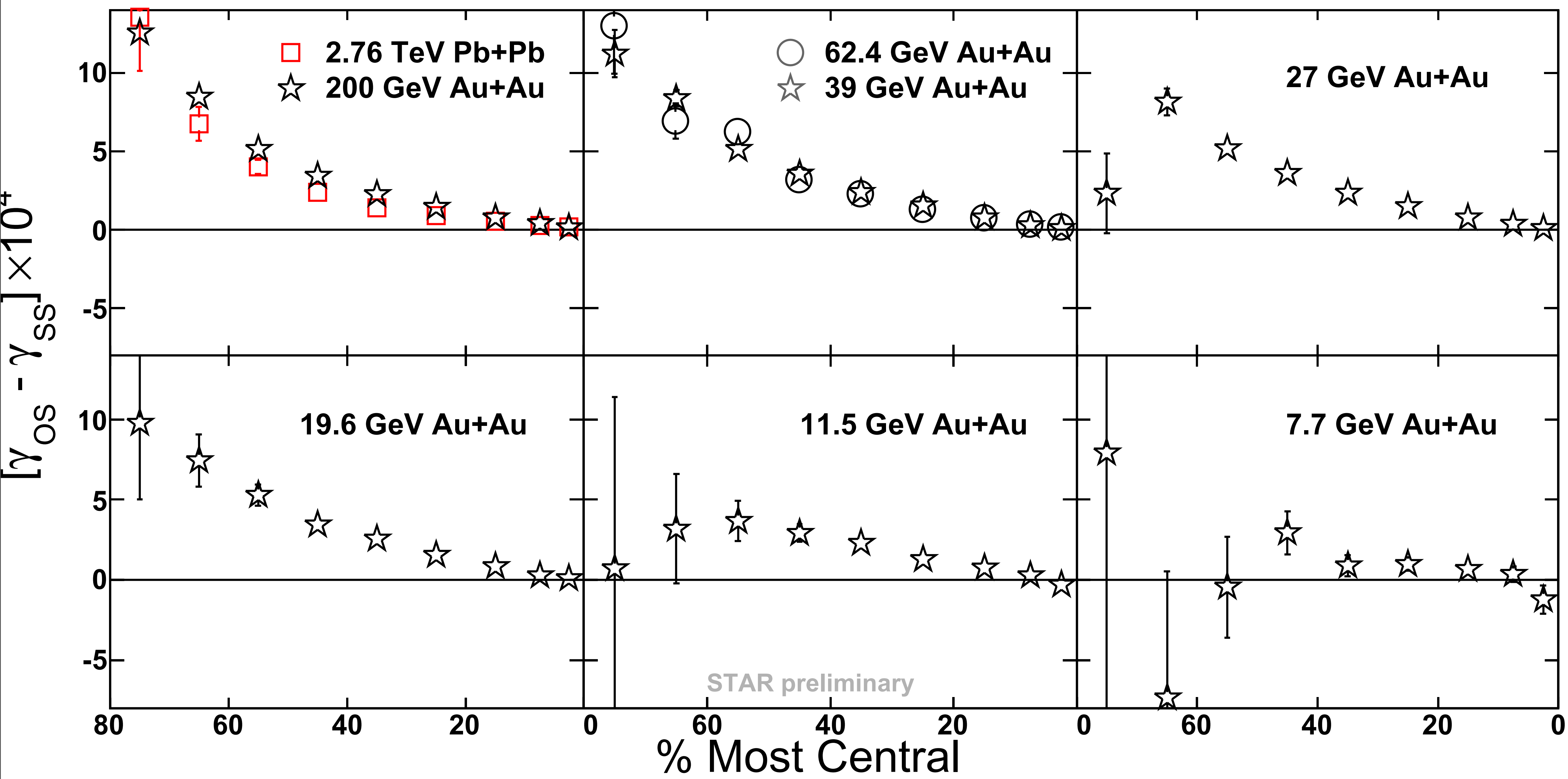


$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$

Chiral Magnetic Effect

[Fukushima, Kharzeev, McLarren]
[Fukushima, Kharzeev, Warringa]

Quark gluon plasma



Quark gluon plasma

- Strongly coupled QCD is difficult
- In need of strongly coupled toy model
- Comes in: AdS/CFT correspondence
- Partially nice results: $\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$
- Understand CME+relatives in AdS/CFT model

AdS/CFT

Moral support:

"... if the gravitational field didn't exist, one could invent it for the purposes of this paper..."

"Theory of Thermal Transport Coefficients"
Luttinger Phys. Rev. 135, A1505, (1964)

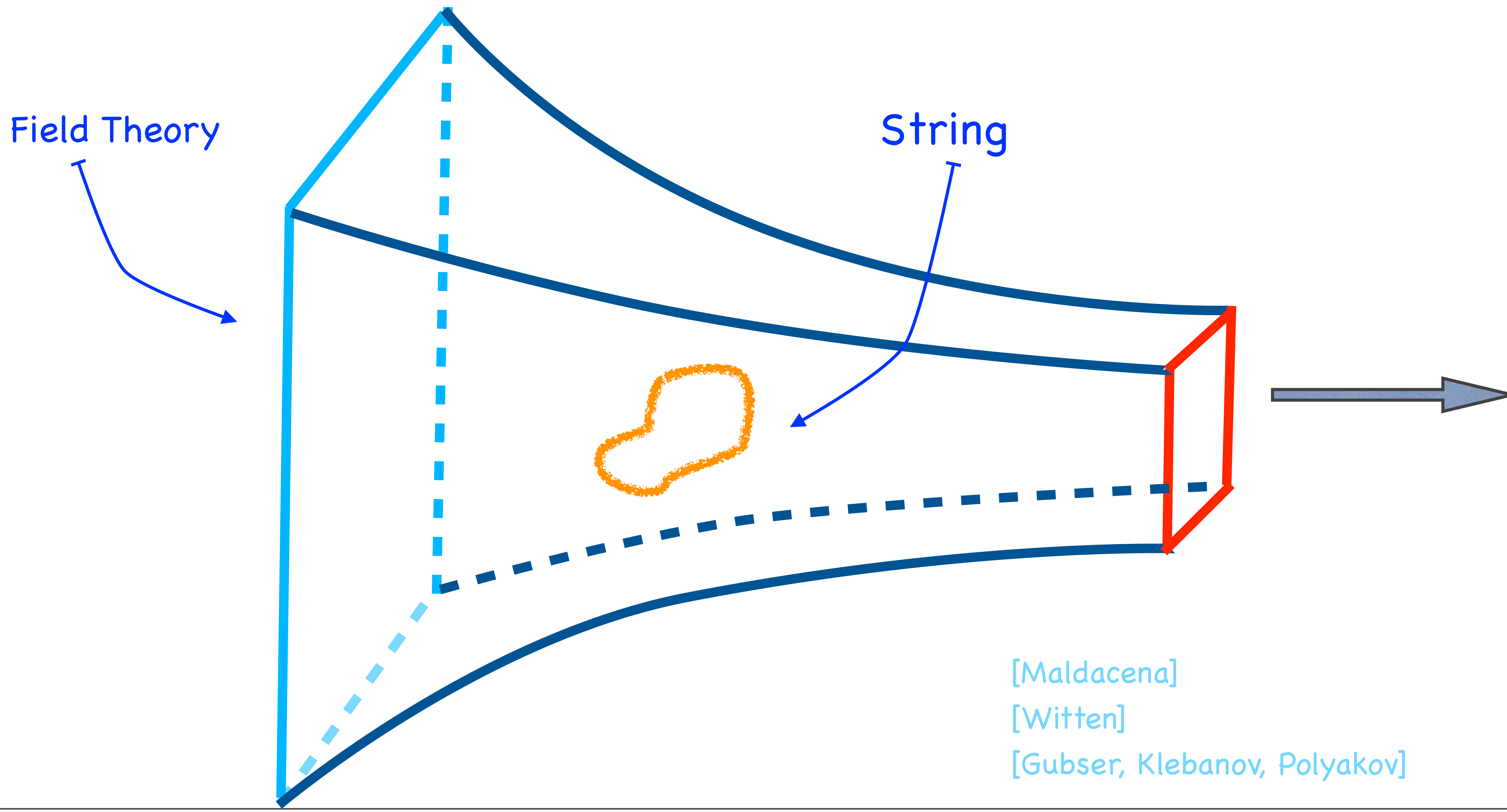
AdS/CFT

"… if the string theory didn't exist, one could invent it for the purposes of computing transport coefficients in strongly coupled theories…"

- Shear viscosity in QGP
- Relativistic 2^{nd} oder hydrodynamics
- Relativistic superfluids
- Parity odd
- ….

AdS/CFT

$$ds^2 = \frac{r^2}{L^2}(dt^2 + d\vec{x}^2) + \frac{L^2 dr^2}{r^2}$$



[Maldacena]
[Witten]
[Gubser, Klebanov, Polyakov]

AdS/CFT

$$\int_{\Phi|_\partial=\Phi_0} D\Phi e^{iS[\Phi]} = e^{iZ[\Phi_0]}$$

$$\frac{\delta^n Z[\Phi_0]}{\delta\Phi_1(x_1)\cdots\delta\Phi_n(x_n)} = \langle O_1(x_1)\ldots O_n(x_n)\rangle$$

Path integral (string theory) on AdS is hard. In practice resort to semi classical limit:

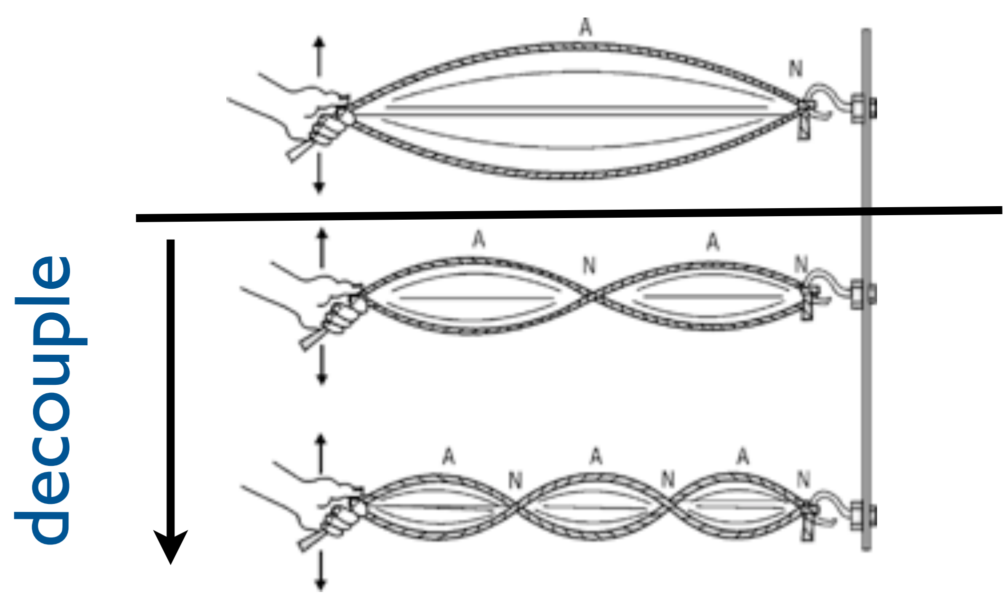
$$S_{grav}[\Phi_0] = Z[\Phi_0]$$

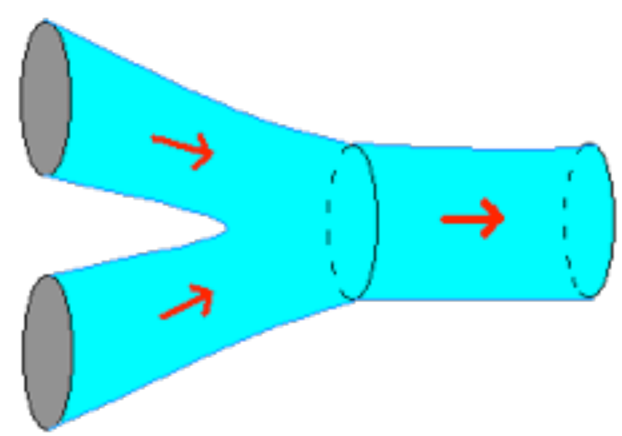
AdS/CFT

- N=4 SYM best understood example: $\{\mathcal{A}_\mu, \Psi^a_\alpha, \phi^I\}$
- All (4-d) fields are NxN matrices (adjoint rep)
- *N*=4 SYM is equivalent to IIB string theory on $AdS_5 \times S^5$

$$g^2_{YM} N = \frac{R^4}{\alpha'^2} \qquad\qquad \frac{1}{N} \propto g_s$$

- *semiclassical gravity limit = large N, large coupling*





AdS/CFT

Dictionary

AdS	Field Theory
five dimensional	four dimensional
strongly coupled	weakly coupled
gravity	no gravity
metric	energy momentum tensor
gauge field	current
...	...

Anomalous transport

- Chiral fermion: $\mathcal{H} = \pm\vec{\sigma}.\vec{p}$

- Classical U(1) symmetry broken by quantum effects

$$\partial_\mu J^\mu = c\,\epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$$

- Model anomaly in 4D via Chern-Simons term in 5D

$$S_{CS} = \int d^5x\,\epsilon^{MNPQR} A_M F_{NP} F_{QR}$$

- Gauge invariant up to boundary term = Anomaly (cfg. QHE)

$$\delta S_{CS} = \int_\partial d^4x\,\epsilon^{\mu\nu\rho\lambda}\,\lambda\,F_{\mu\nu} F_{\rho\lambda}$$

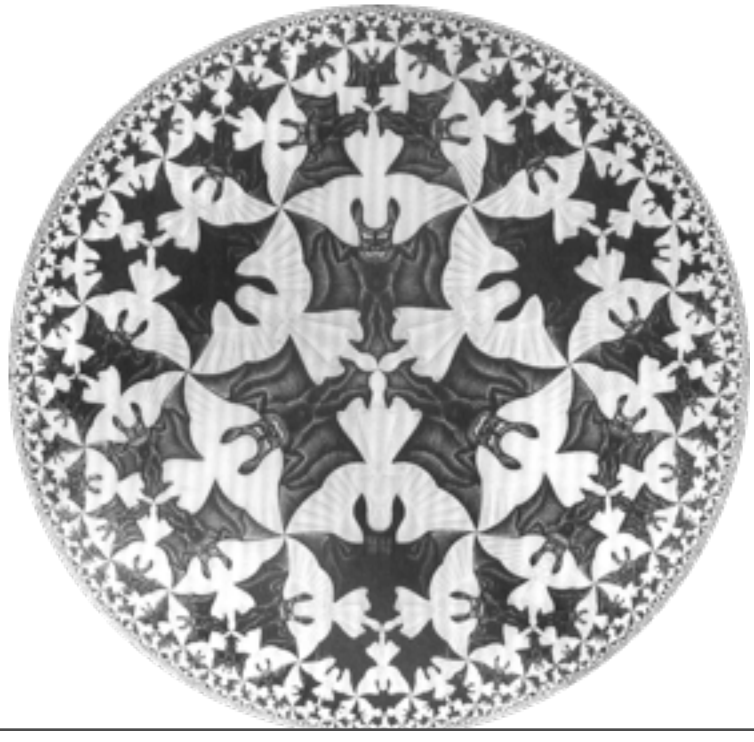
Anomalous transport

Our Model:

$$S_{EM} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN}\right]$$

$$S_{CS} = \frac{1}{16\pi G}\int d^5x\,\epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)$$

In finite T, μ state:
charged black hole in AdS



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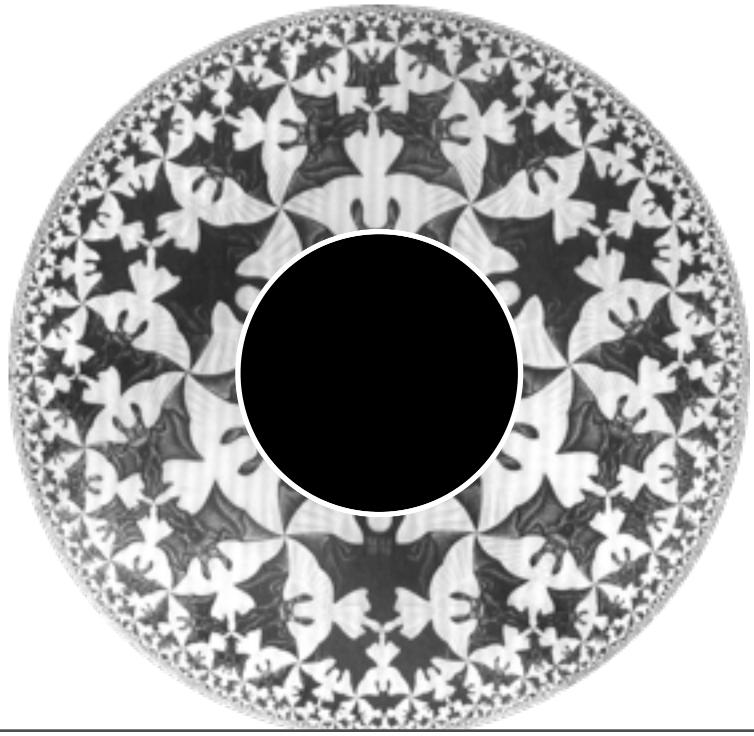

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Anomalous transport

- Compute response to magnetic field and rotation

electric current:

$$\vec{J} = \left(\frac{\mu_5}{2\pi^2} - \frac{A_0^5}{2\pi^2}\right)\vec{B} + \frac{\mu\mu_5}{4\pi^2}\vec{\omega}$$

axial current:

$$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B} + \frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12}\vec{\omega}$$

energy current:

$$\vec{J}_\epsilon = \frac{3\mu_5\mu^2 + \mu_5^3}{6\pi^2}\vec{B} + \frac{T^2\mu_5}{6}\vec{\omega}$$

μ = chemical potential μ_5 = axial chemical potential T = temperature A_0^5 = axial gauge field

Anomalous transport

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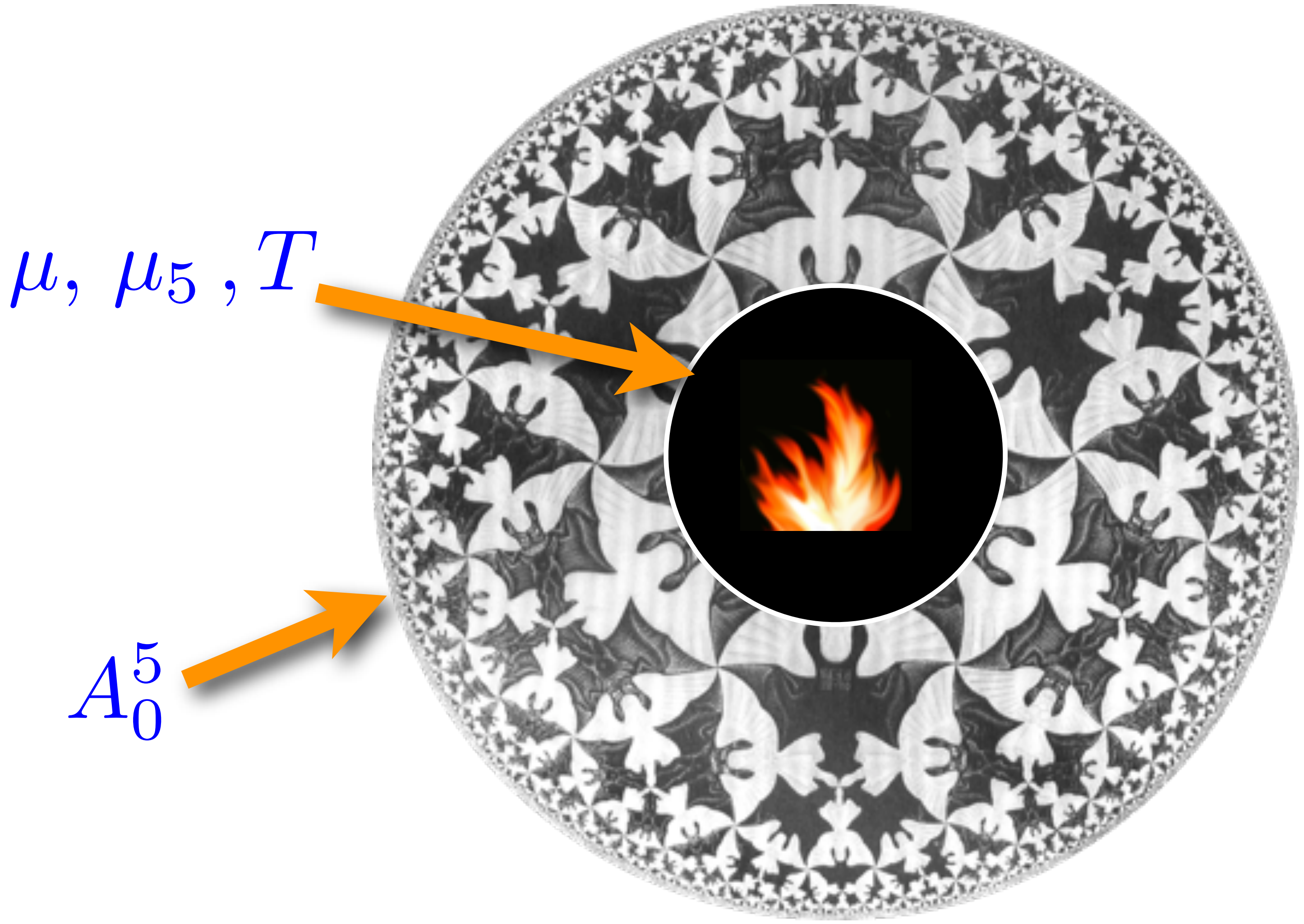
$$(D_\mu J^\mu)^a = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d^{abc}}{32\pi^2}F^b_{\mu\nu}F^c_{\rho\lambda} + \frac{b_a}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right)$$

Anomalous transport

Gauge fields vs state variables

- μ, μ_5, T are state variables, determined by interior of AdS
- A_0^5 is a boundary condition for AdS, a coupling in field theory



Map to WSMs

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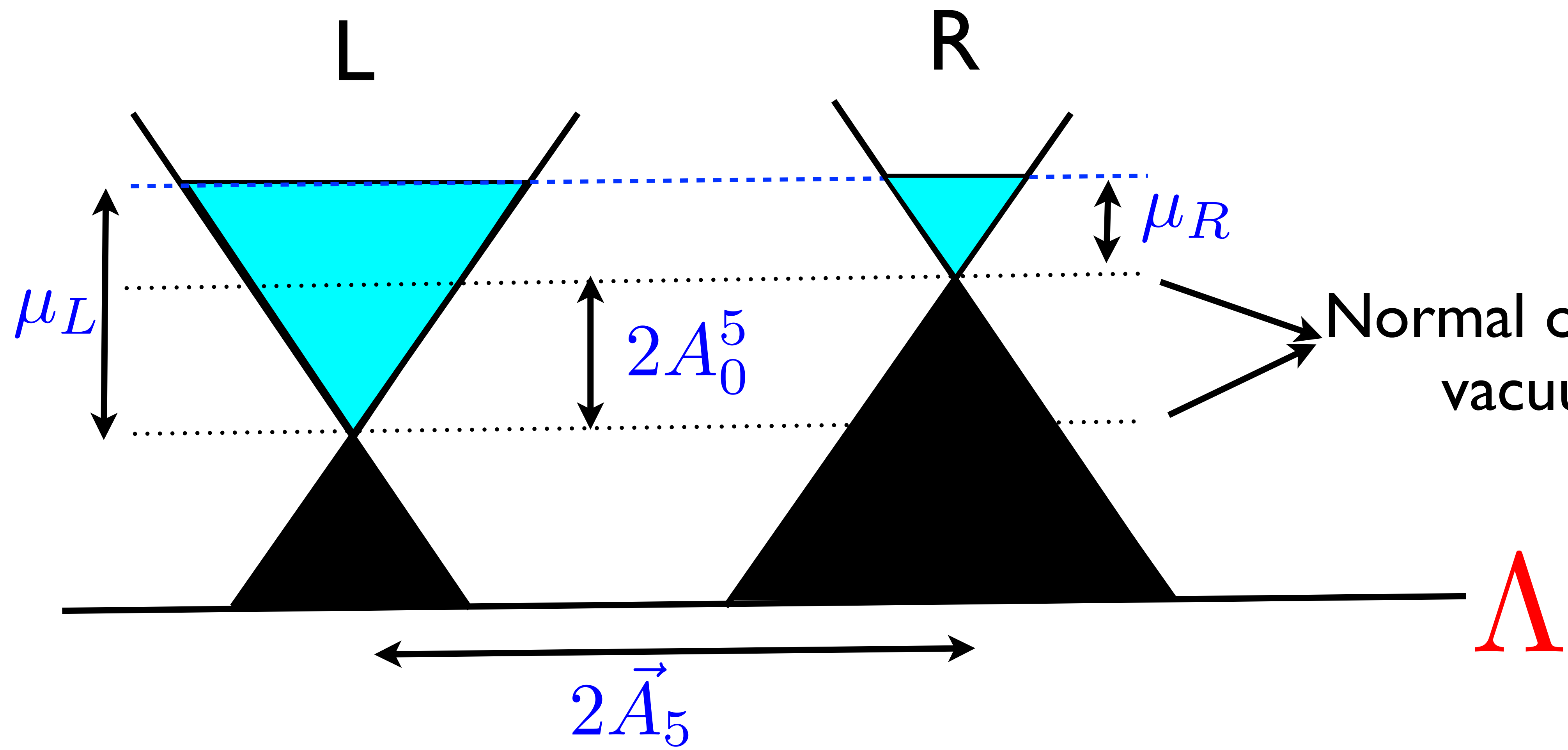

$$\mu_5 = \frac{1}{2}(\mu_L - \mu_R) \qquad \mu = \frac{1}{2}(\mu_R + \mu_L)$$

CME: $\vec{J} = \frac{1}{2\pi^2}\left(\mu_5 - A_0^5\right)\vec{B} = 0$

CVE: $\vec{J} = \frac{\mu\mu_5}{4\pi^2}\omega$

AQHE: $\vec{J} = \frac{1}{2\pi^2}\vec{A}_5 \times \vec{E}$

Map to WSMs

$$\mathcal{L}_{\mathrm{eff}} = \bar{\psi}\gamma^{\mu}\left(i\partial_{\mu} - \gamma_5 b_{\mu}\right)\psi$$

spatial variation = axial magnetic field

- Edge state (Fermi arcs) = LLL of axial magnetic field
- Exotic response patterns (?)

$$\vec{J} = \frac{\mu}{2\pi^2}\vec{B}_5 \qquad \vec{J}_5 = \left(\frac{\mu_5}{2\pi^2} - \frac{A_0^5}{6\pi^2}\right)\vec{B}_5$$

$$\vec{J}_{\epsilon} = \cdots + \frac{T^2}{12}\vec{B}_5 \qquad \vec{J}_5 = \frac{1}{6\pi^2}\vec{A}_5 \times \vec{E}_5$$

[M. Chernodub, A. Cortijo, A. Grushin, K.L., M.A.H. Vozmediono]

Summary

- AdS can give insight in QFT (covariant vs. conserved current)
- Effective toy models for strongly coupled systems
- Particularly suited for transport in relativistic systems
- Anomalies
- (Non)-renormalization theorems
- Possible input for WSM physics? (grav. anomaly measurable in lab?)
- More exotic response patters observable in WSMs?
  (axial magnetic or electric fields)

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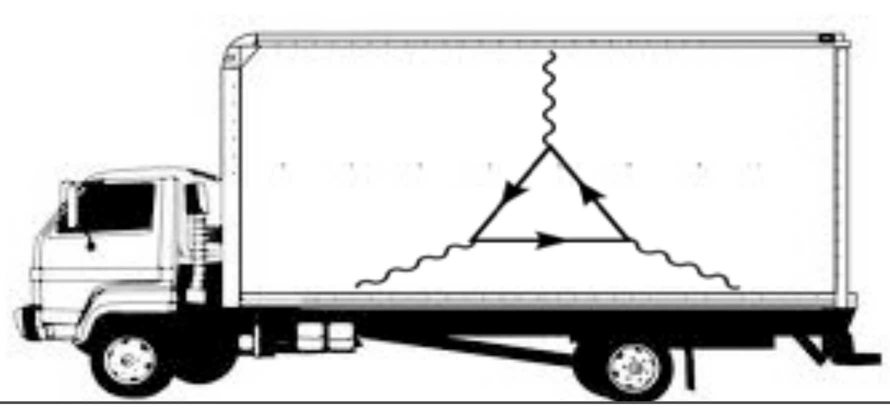

Thank You!