

# Strongly correlated topological states of spinless fermions in 2D lattices

Stefanos Kourtis

"Correlations, criticality, and coherence in quantum systems" – Évora, Portugal, 6/10/14

Work done in collaboration with:

→ J. Venderbos (now @ MIT)

→ M. Daghofer

→ J. van den Brink

→ **new!** C. Loftin (UNC)

→ T. Neupert (Princeton)

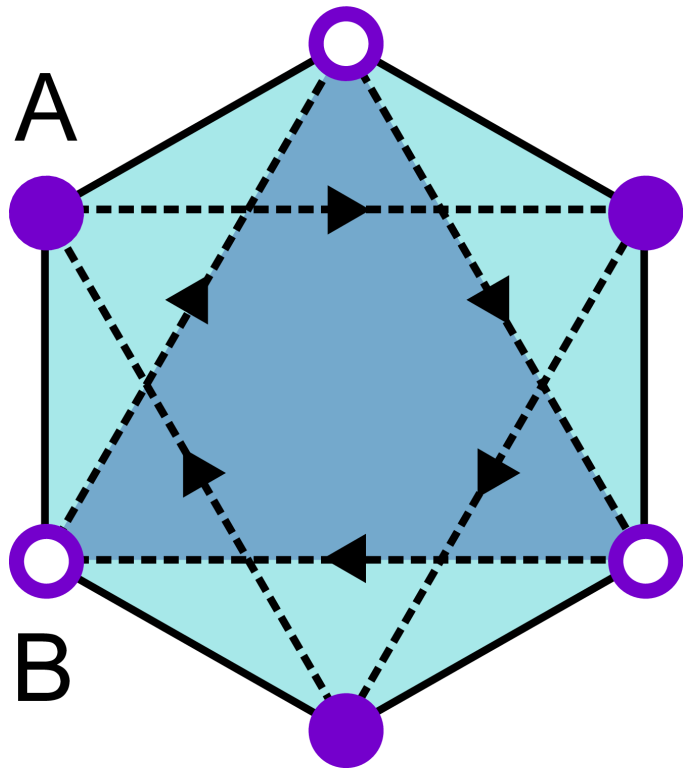
→ C. Chamon (Boston U)

→ C. Mudry (PSI Villigen)



# FQHE in the Haldane model

(Neupert *et al.*, Sheng *et al.*, Regnault & Bernevig, ... , 2011)

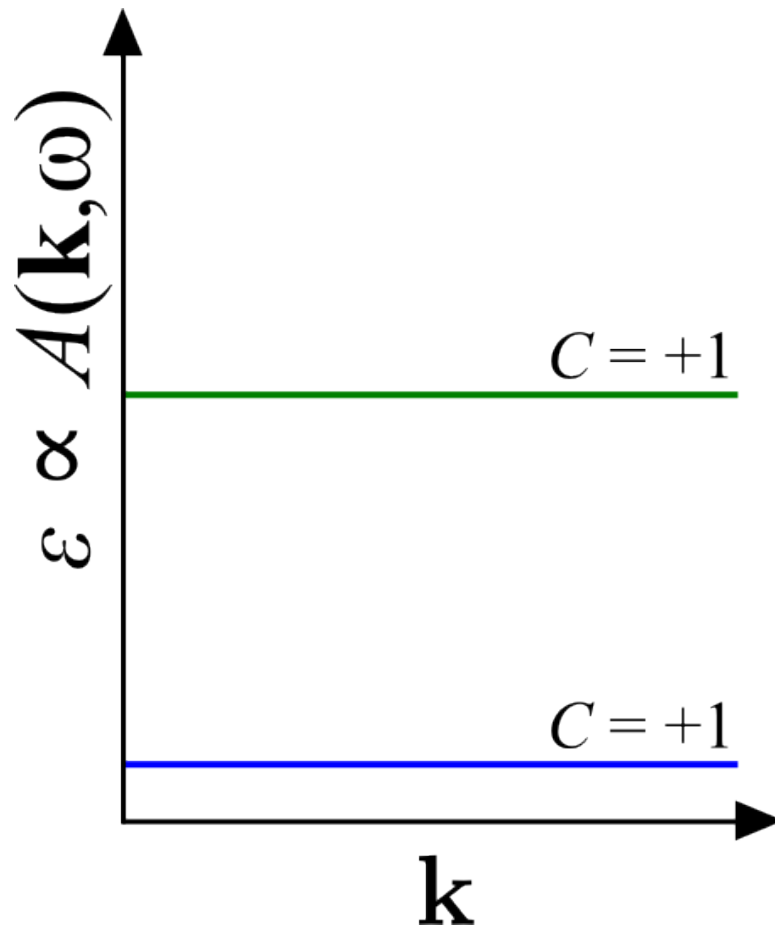


- ▶ spinless fermions
- ▶ filling fraction  $\nu = p/q$
- ▶ **no net flux** through unit cell
- ▶ screened **Coulomb** repulsion
- ▶ Hall conductivity:

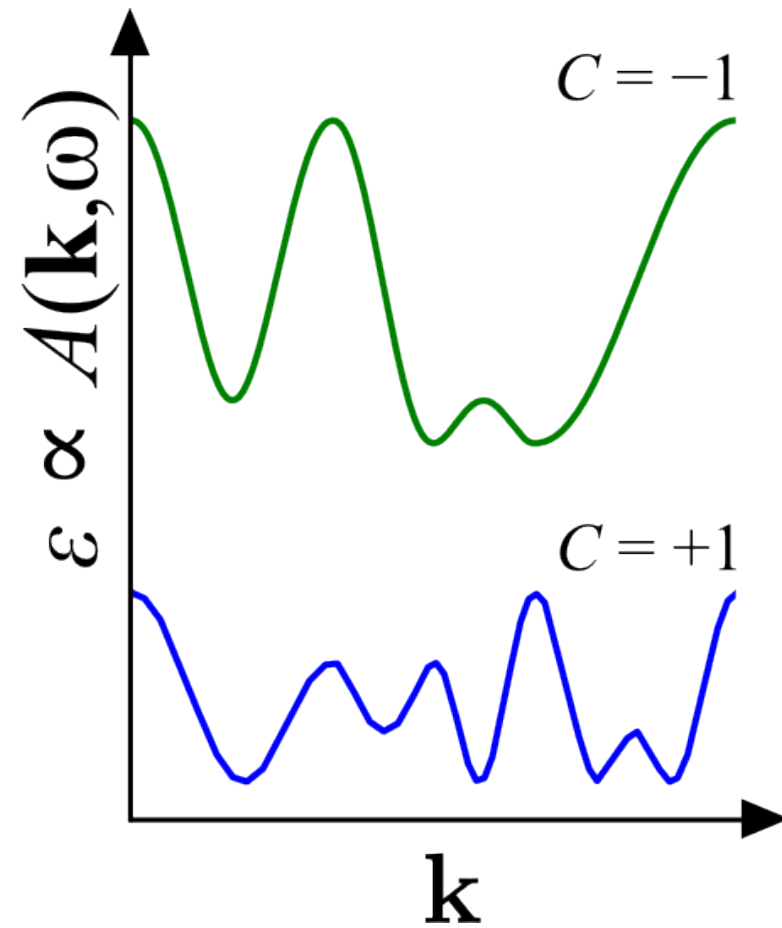
$$\sigma_{xy} = \frac{p}{q} \frac{e^2}{h}$$

# Landau levels vs Chern bands

Landau levels

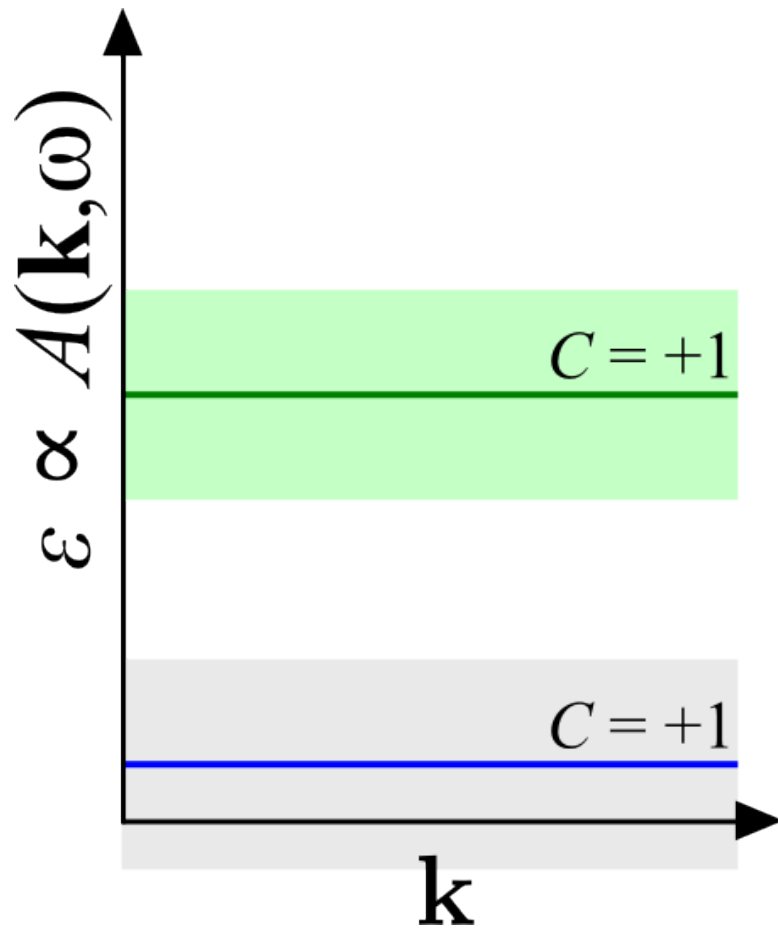


Chern bands

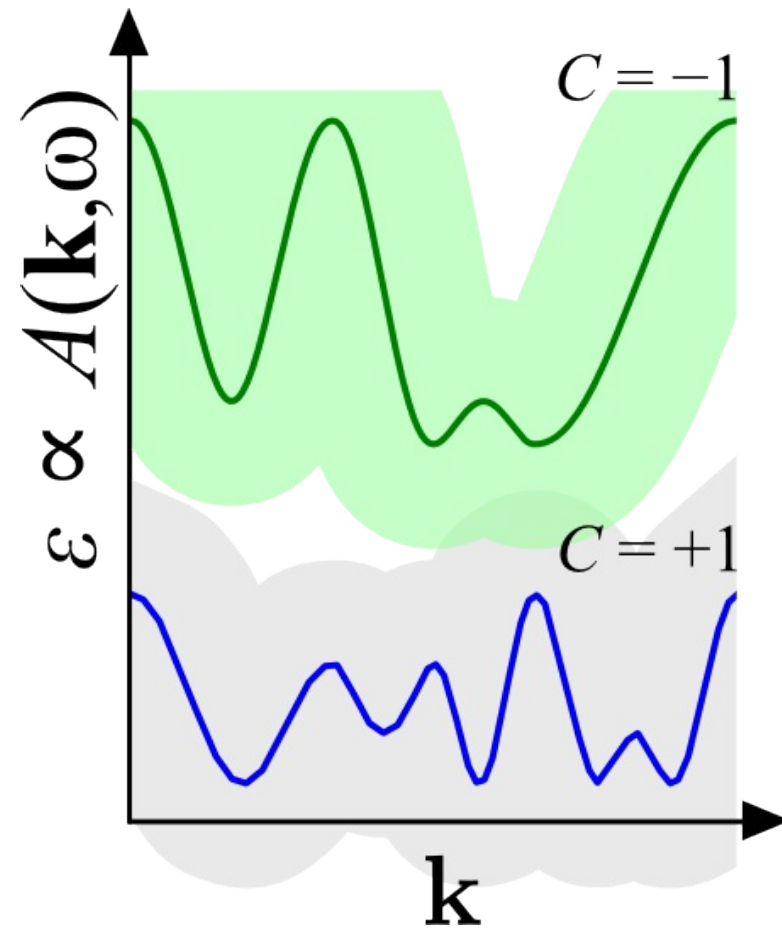


# Landau levels vs Chern bands

Landau levels + V

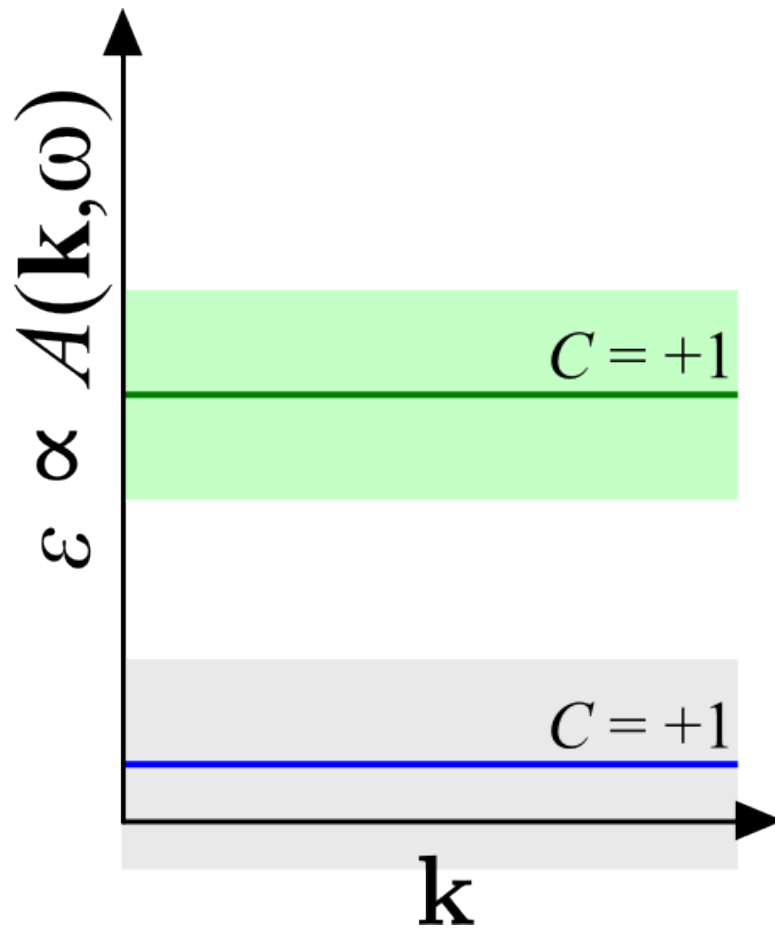


Chern bands + V

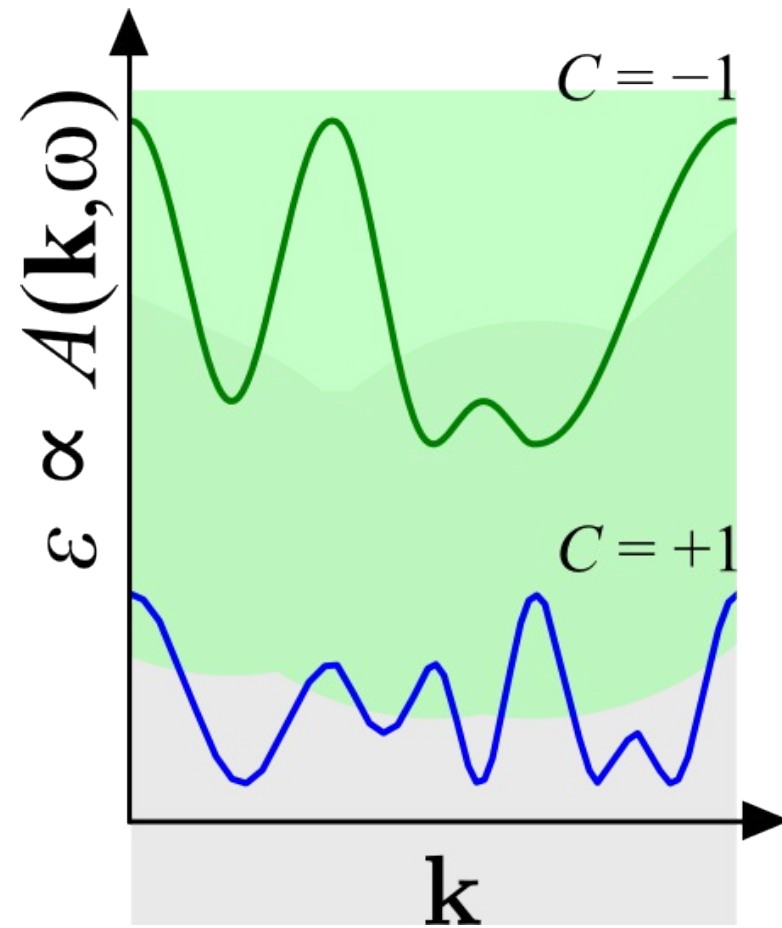


# Landau levels vs Chern bands

Landau levels +  $V$



Chern bands + **large**  $V$



## Correlations → topological order

Real bands are:

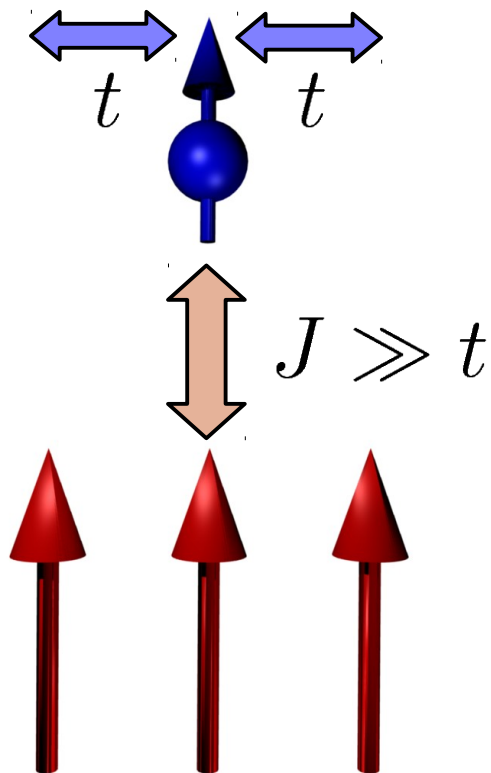
- ▶ multiple
- ▶ dispersive
- ▶ broadened / mixed by interactions

Can the addition of interactions in **real** bands:

- ▶ induce **FQH-like** physics?
- ▶ lead to correlated topological phases **w/o QH counterparts?**  
(topological order w/o underlying topological bands?)

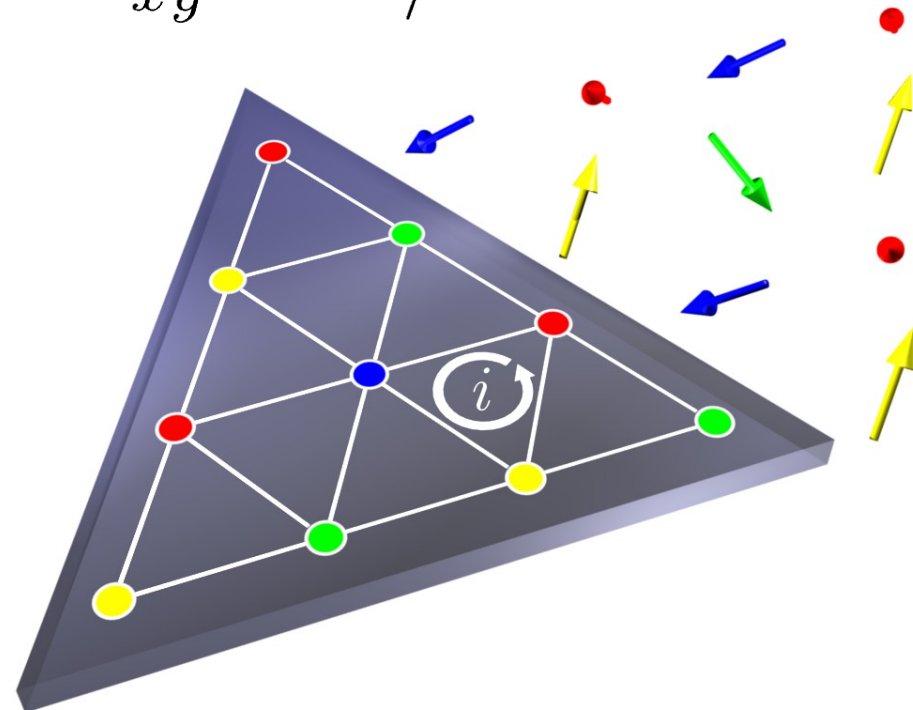
# Chern insulator (CI) on the triangular lattice

## Kondo-lattice model



## Martin & Batista, 2008:

- ▶ at  $\nu = 3/4$   
→ chiral spin pattern
- ▶  $\sigma_{xy} = e^2/h$



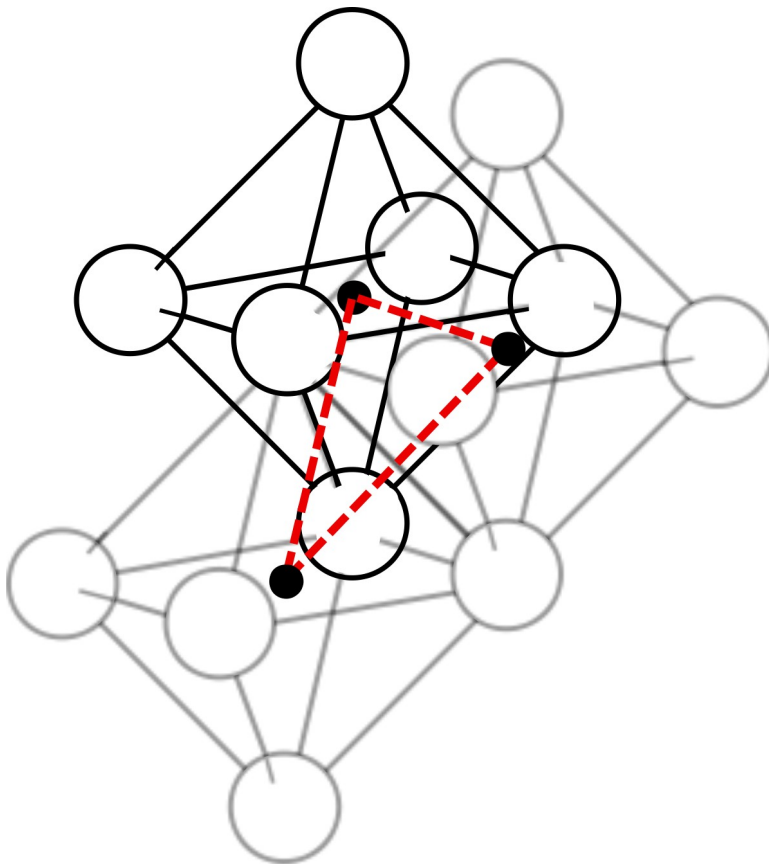
Also:

- ▶ Ohgushi, Murakami & Nagaosa, 2000

# CI on triangular lattice of $t_{2g}$ electrons

3-orbital Hubbard model

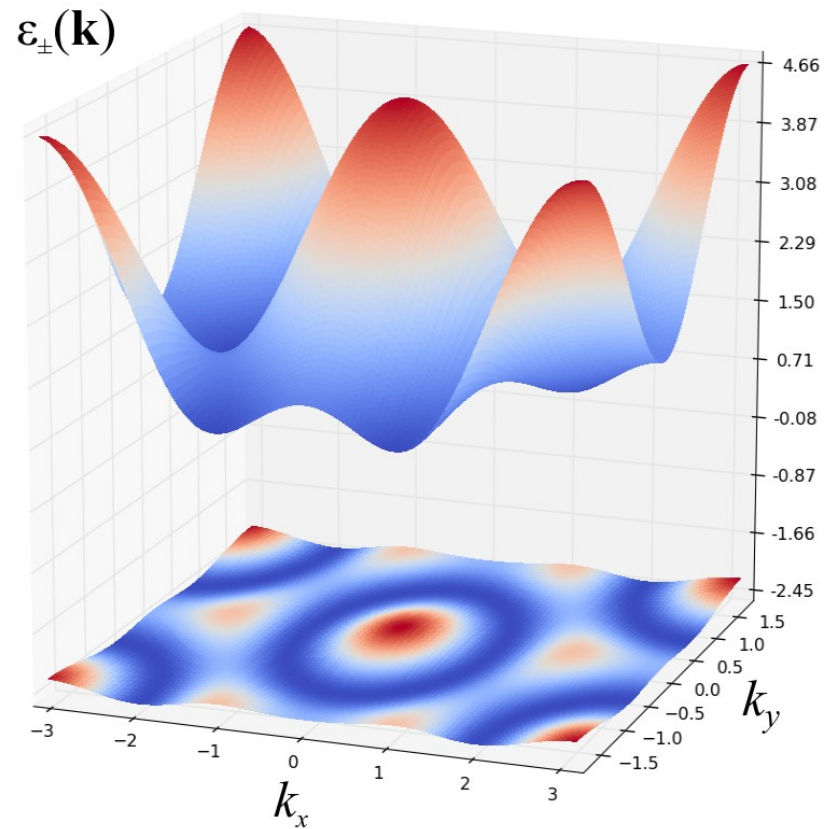
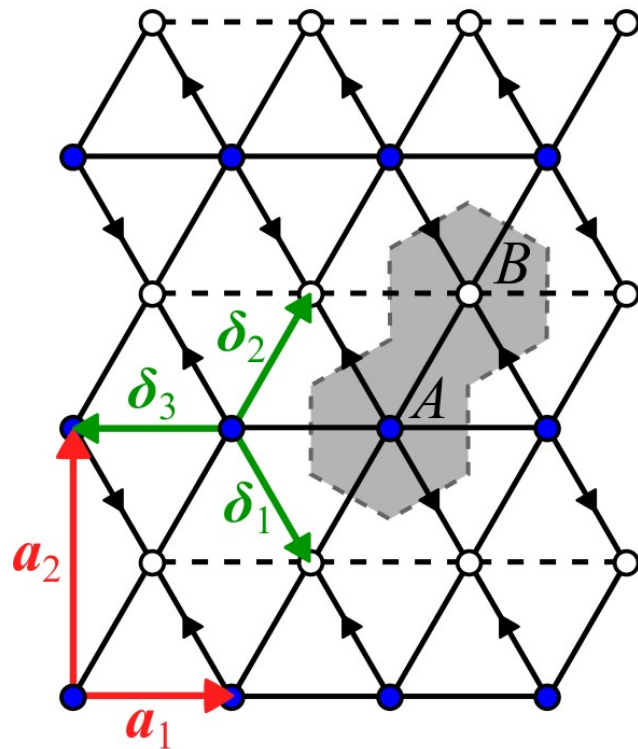
Venderbos *et al.*, 2011; 2012:



- ▶ strong on-site Coulomb effects  
→ mapping to KLM  
→ chiral spin pattern
- ▶ flat isolated single-orbital band
- ▶  $\sigma_{xy} = e^2/h$



# Effective model on the triangular lattice

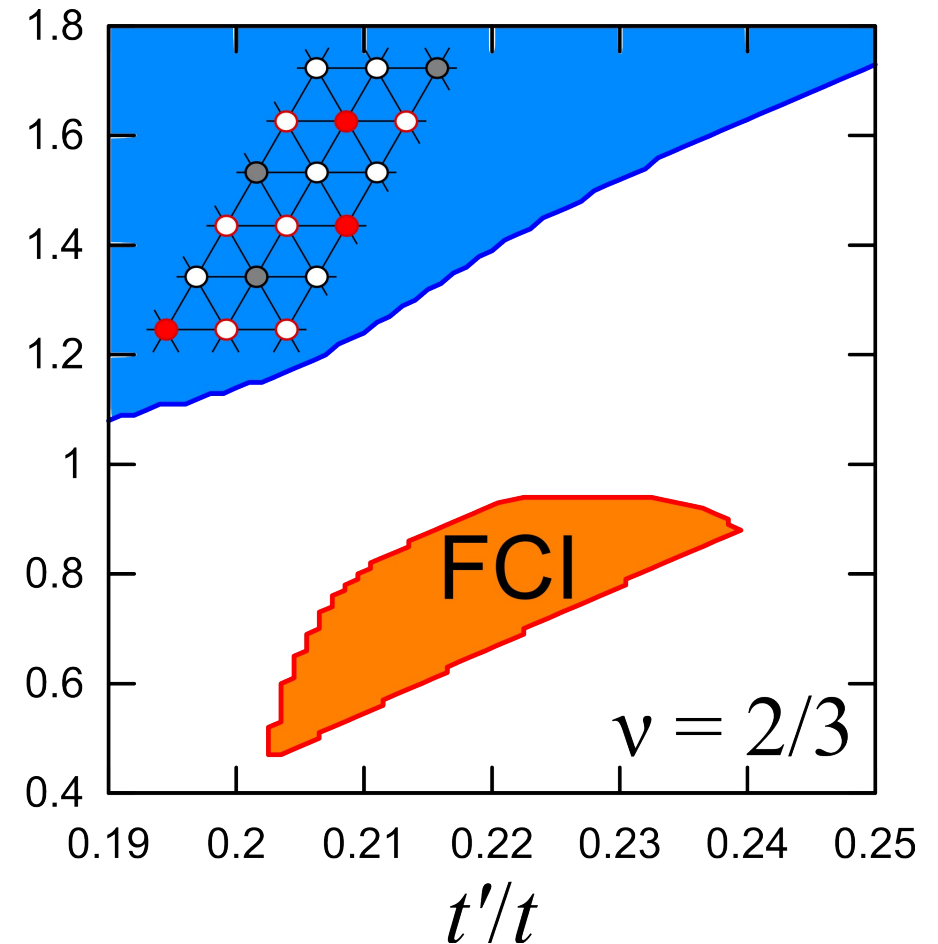
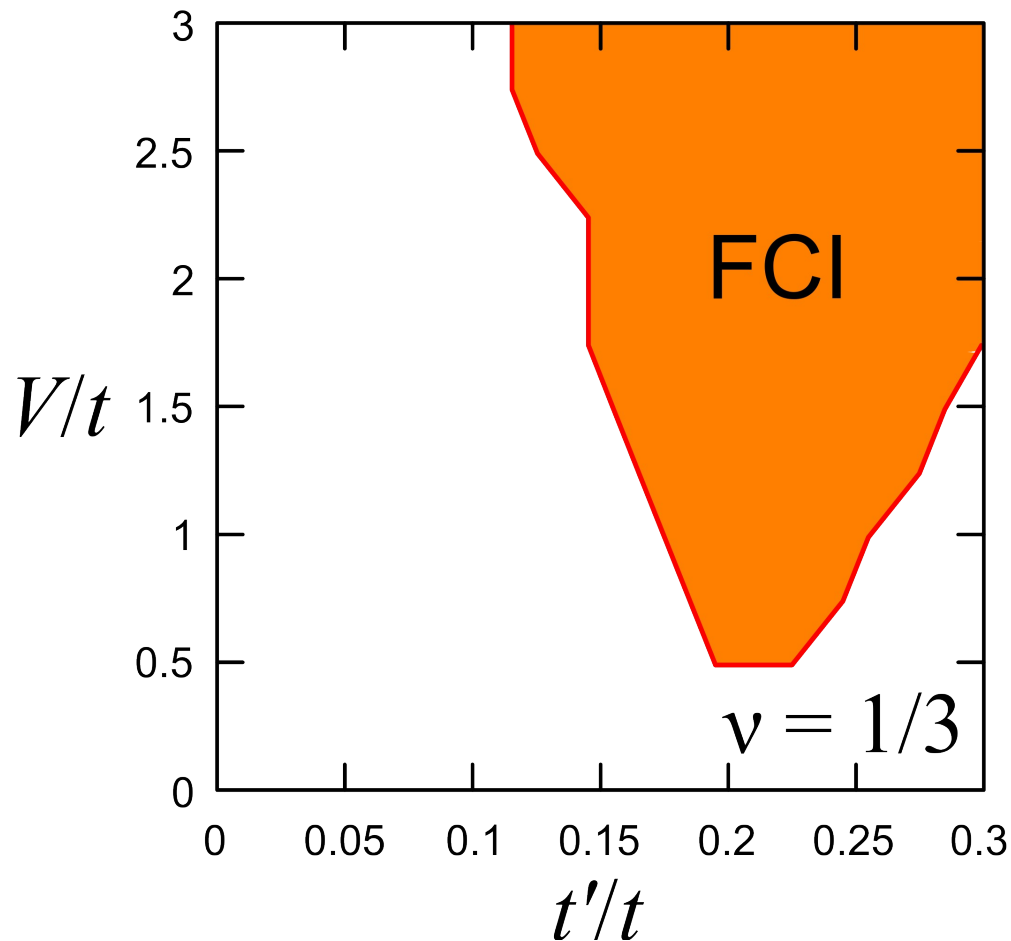


+ partially filled Chern band

+ nearest-neighbor Coulomb interaction  $V$

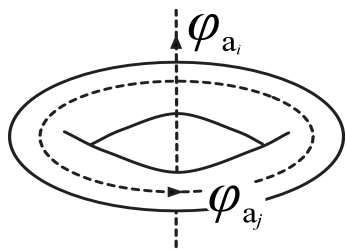
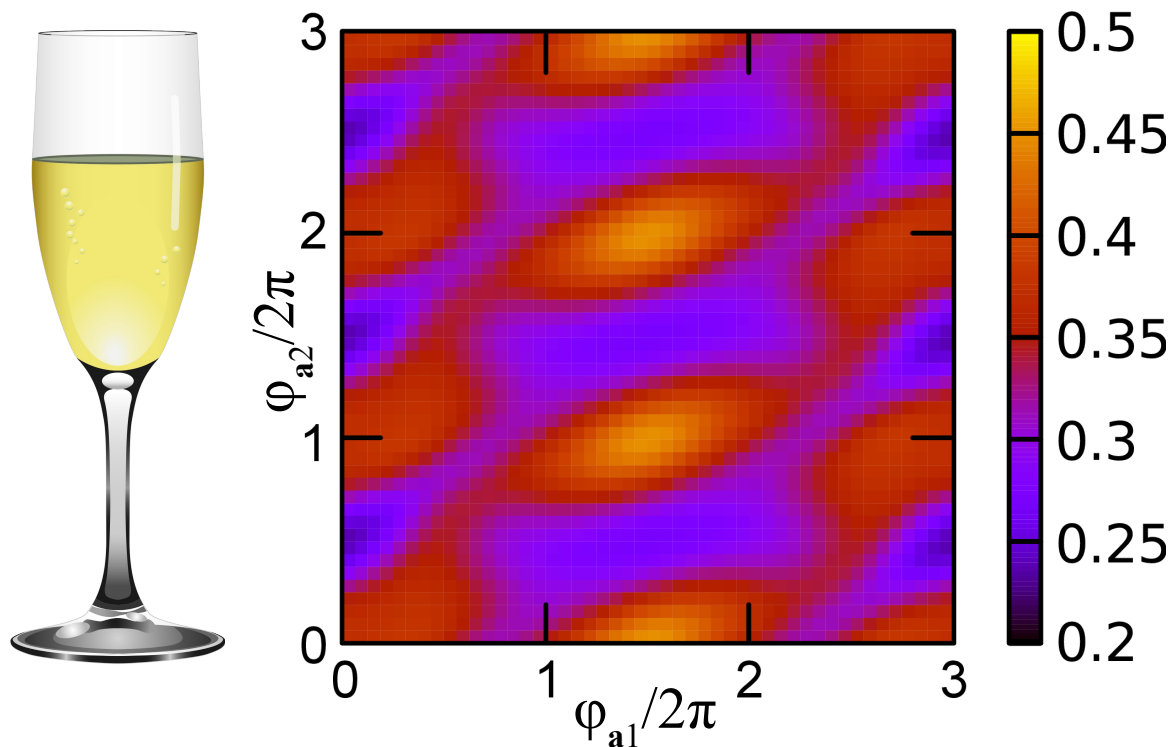
# Phase diagram

Exact diagonalization, 3x6-site cluster, PBC



# Fractional Chern insulator

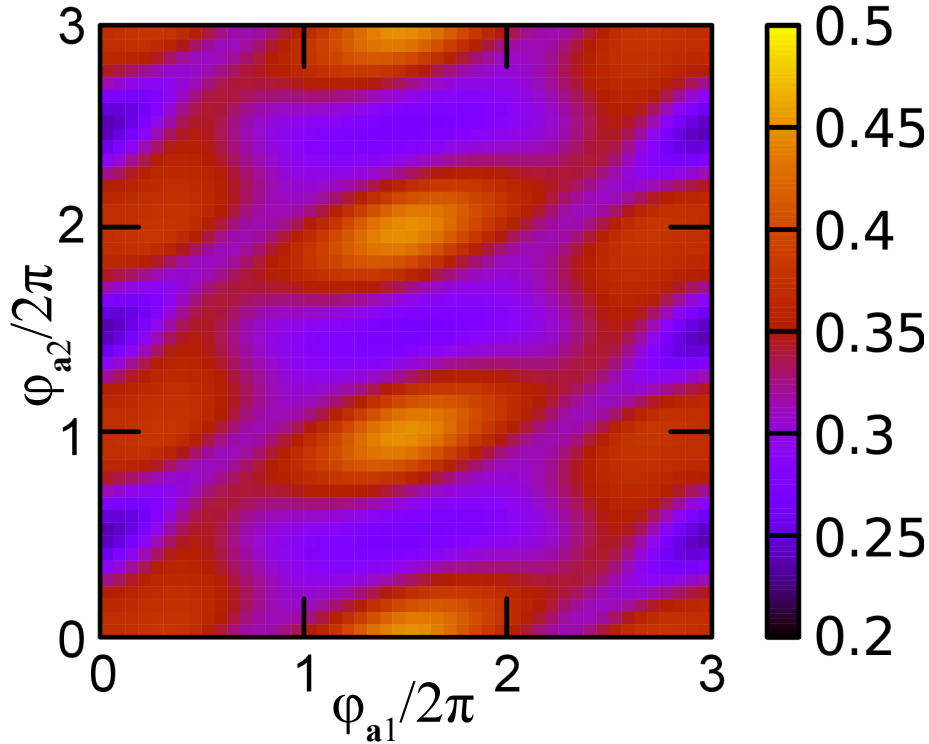
Topological invariant:  $\sigma_H \rightarrow$  integral of **Berry curvature**



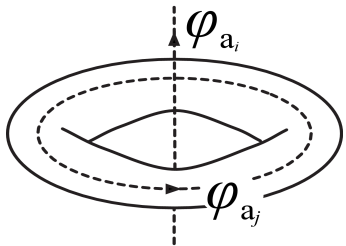
$$\sigma_H = \frac{e^2}{h\pi q} \sum_{n=1}^q \iint_0^{2\pi} d\varphi_{\mathbf{a}_i} d\varphi_{\mathbf{a}_j} \Im \sum_{n' \neq n} \frac{\langle n | \frac{\partial H}{\partial \varphi_{\mathbf{a}_i}} | n' \rangle \langle n' | \frac{\partial H}{\partial \varphi_{\mathbf{a}_j}} | n \rangle}{(\epsilon_n - \epsilon_{n'})^2}$$

# Fractional Chern insulator

Topological invariant:  $\sigma_H \rightarrow$  integral of **Berry curvature**



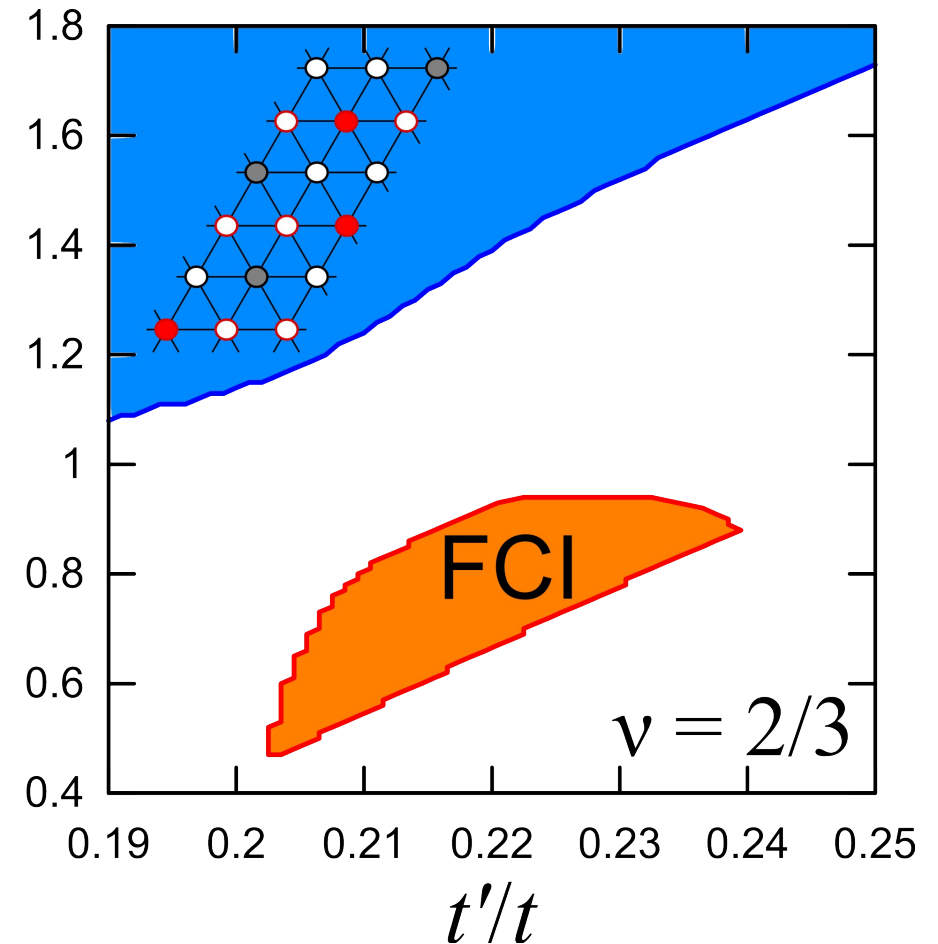
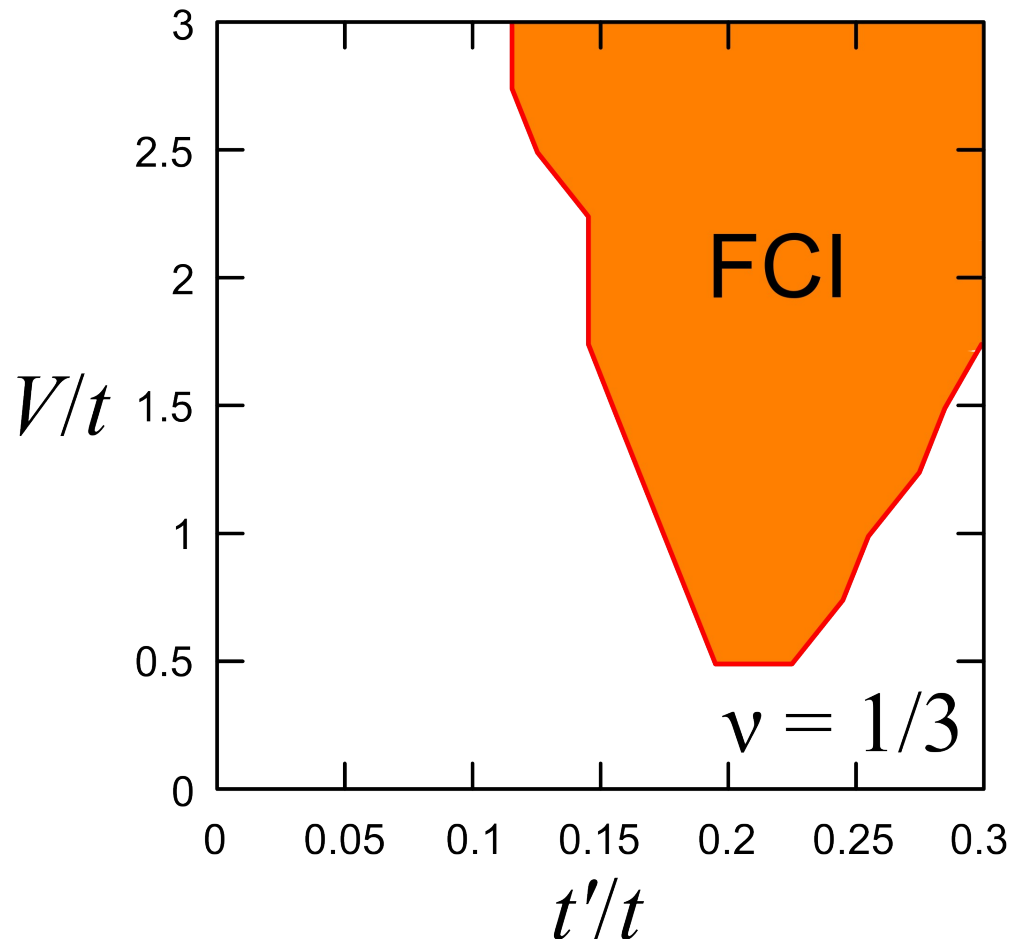
Very precise  
quantization of  $\sigma_H$ ,  
even in finite-size  
numerics



$$\sigma_H = \frac{e^2}{h\pi q} \sum_{n=1}^q \iint_0^{2\pi} d\varphi_{\mathbf{a}_i} d\varphi_{\mathbf{a}_j} \Im \sum_{n' \neq n} \frac{\langle n | \frac{\partial H}{\partial \varphi_{\mathbf{a}_i}} | n' \rangle \langle n' | \frac{\partial H}{\partial \varphi_{\mathbf{a}_j}} | n \rangle}{(\epsilon_n - \epsilon_{n'})^2}$$

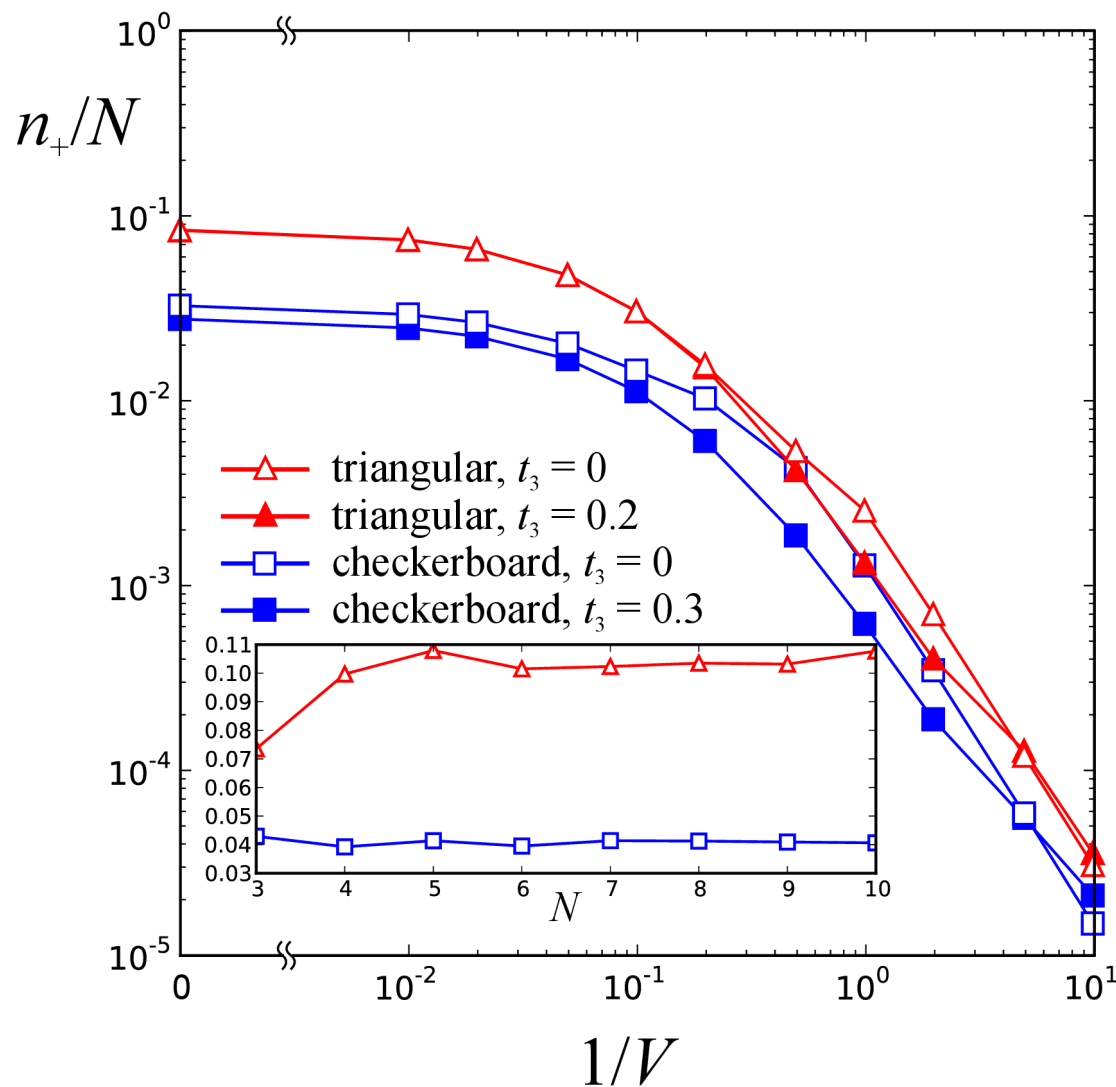
# Phase diagram

Exact diagonalization, 3x6-site cluster, PBC



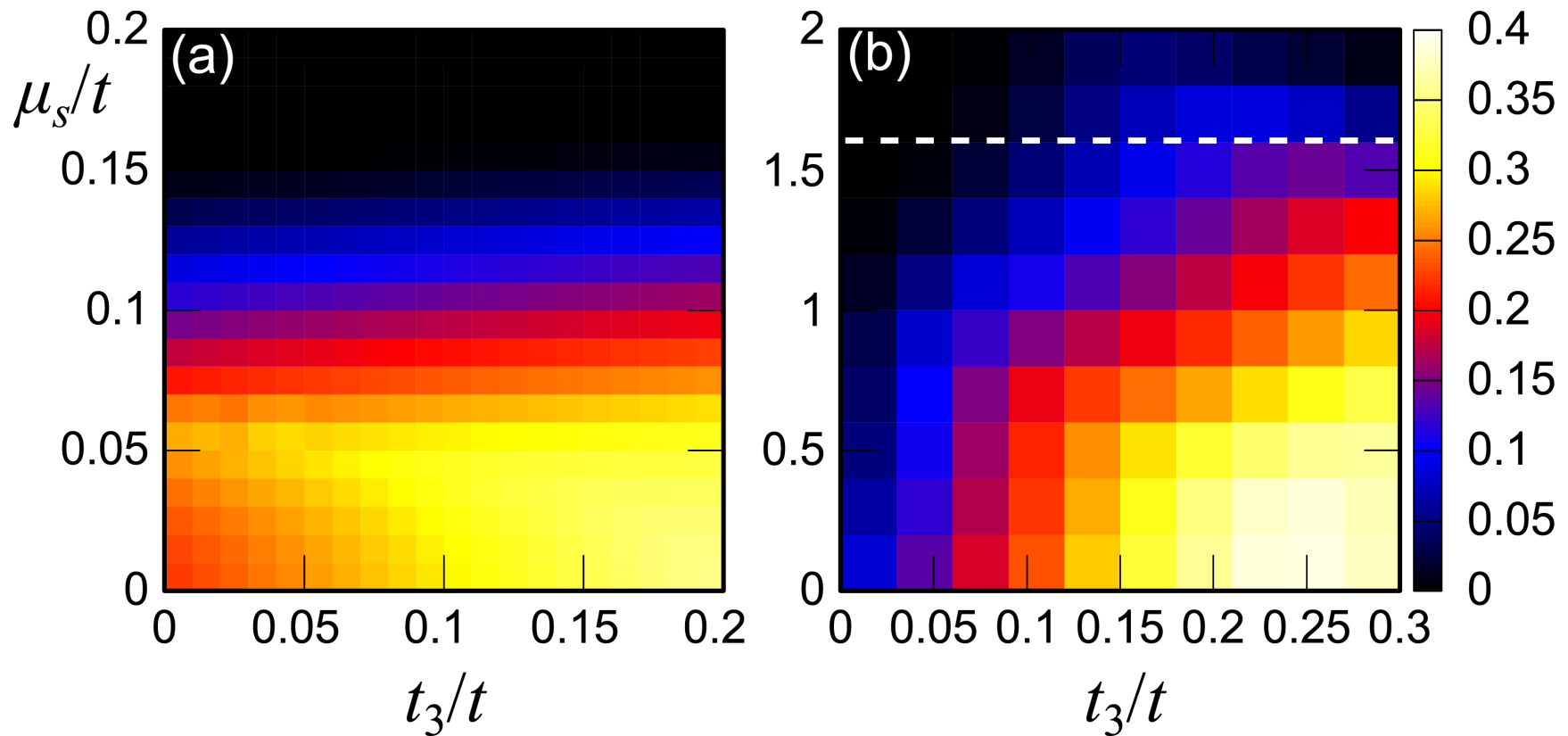
# Upper-band character of FCI states

Exact diagonalization, 6x6-site cluster, PBC



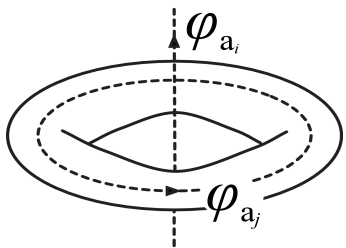
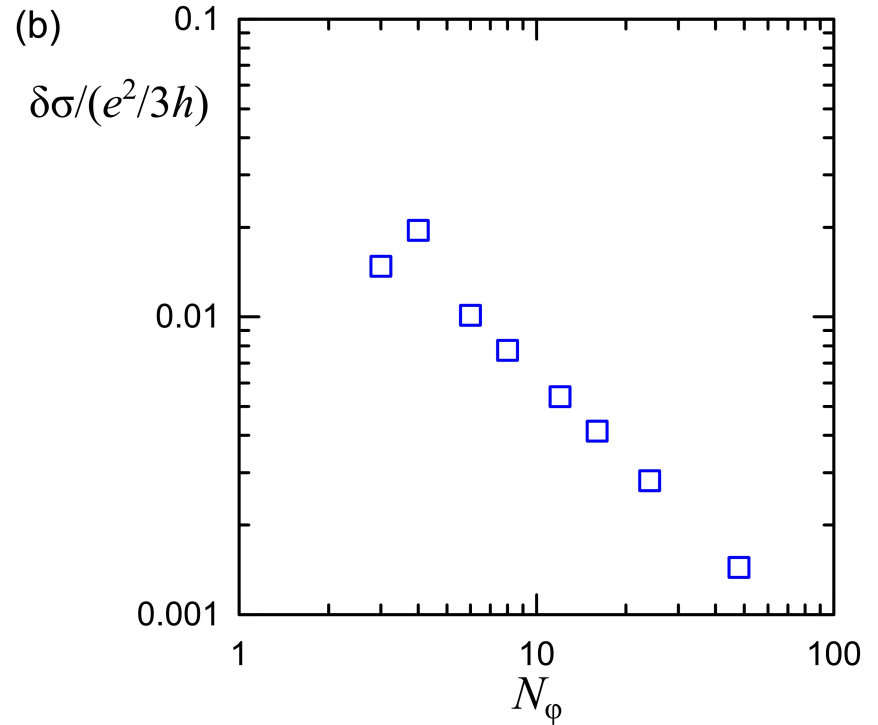
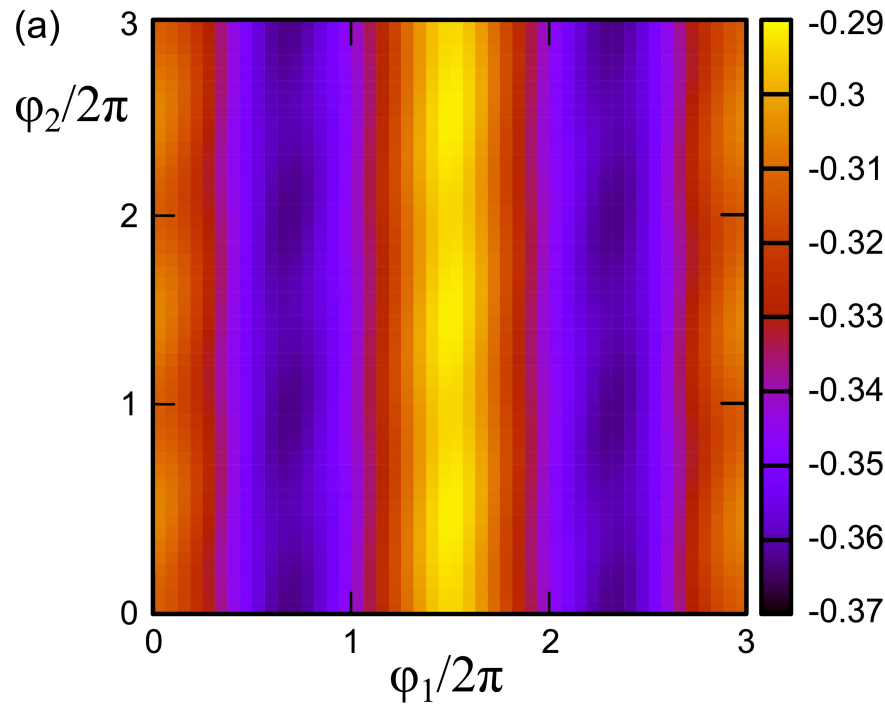
# Phase diagram at $V = \text{infinity}$

Exact diagonalization, 6x6-site cluster, PBC



# Berry curvature at $V = \text{infinity}$

Exact diagonalization, 6x6-site cluster, PBC

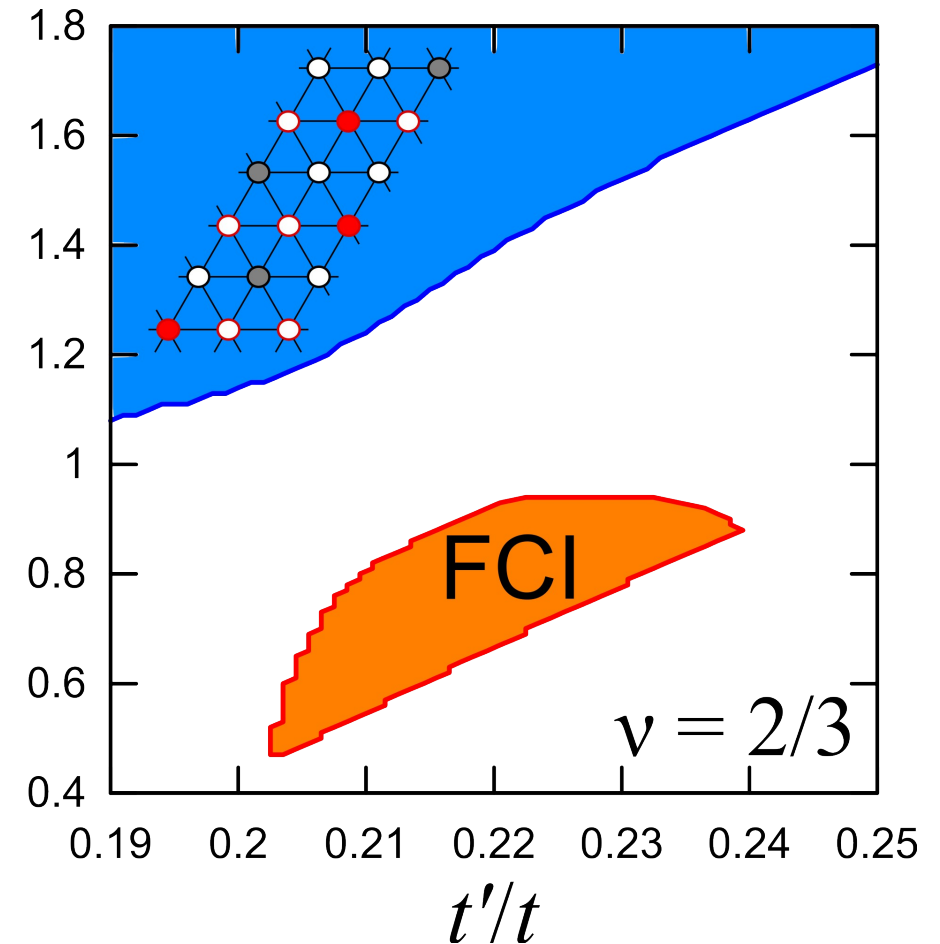
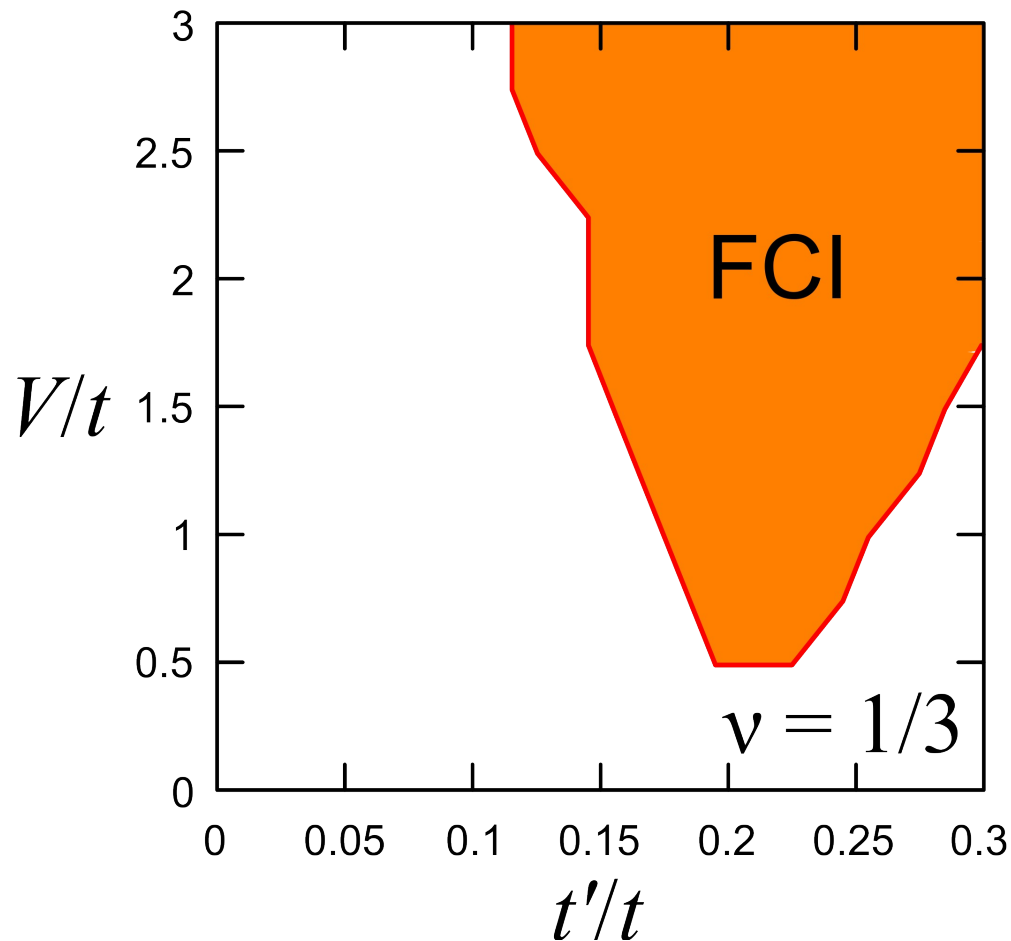


$$\sigma_H = \frac{e^2}{h\pi q} \sum_{n=1}^q \iint_0^{2\pi} d\varphi_{\mathbf{a}_i} d\varphi_{\mathbf{a}_j} \Im \sum_{n' \neq n} \frac{\langle n | \frac{\partial H}{\partial \varphi_{\mathbf{a}_i}} | n' \rangle \langle n' | \frac{\partial H}{\partial \varphi_{\mathbf{a}_j}} | n \rangle}{(\epsilon_n - \epsilon_{n'})^2}$$



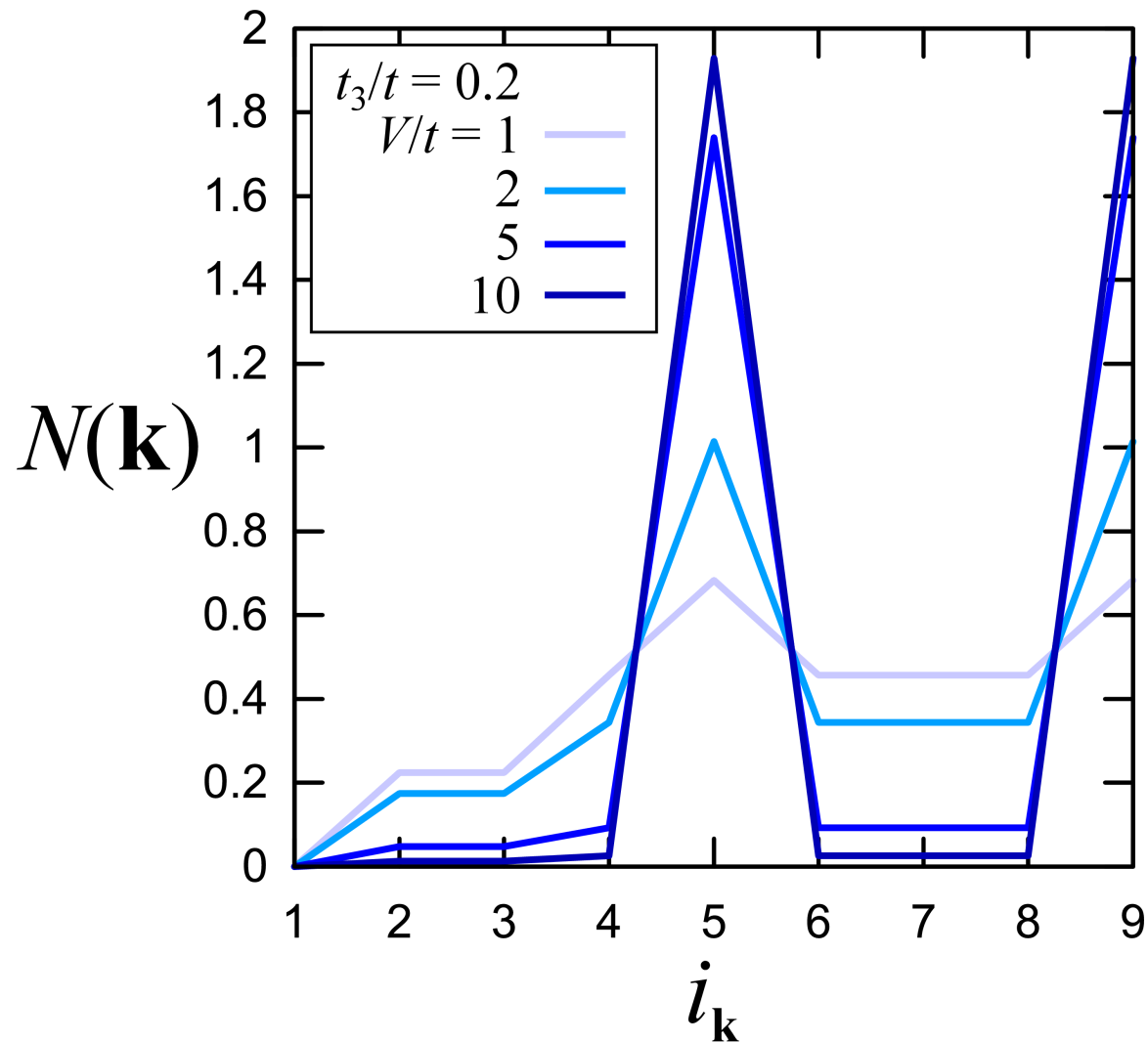
# Phase diagram

Exact diagonalization, 3x6-site cluster, PBC



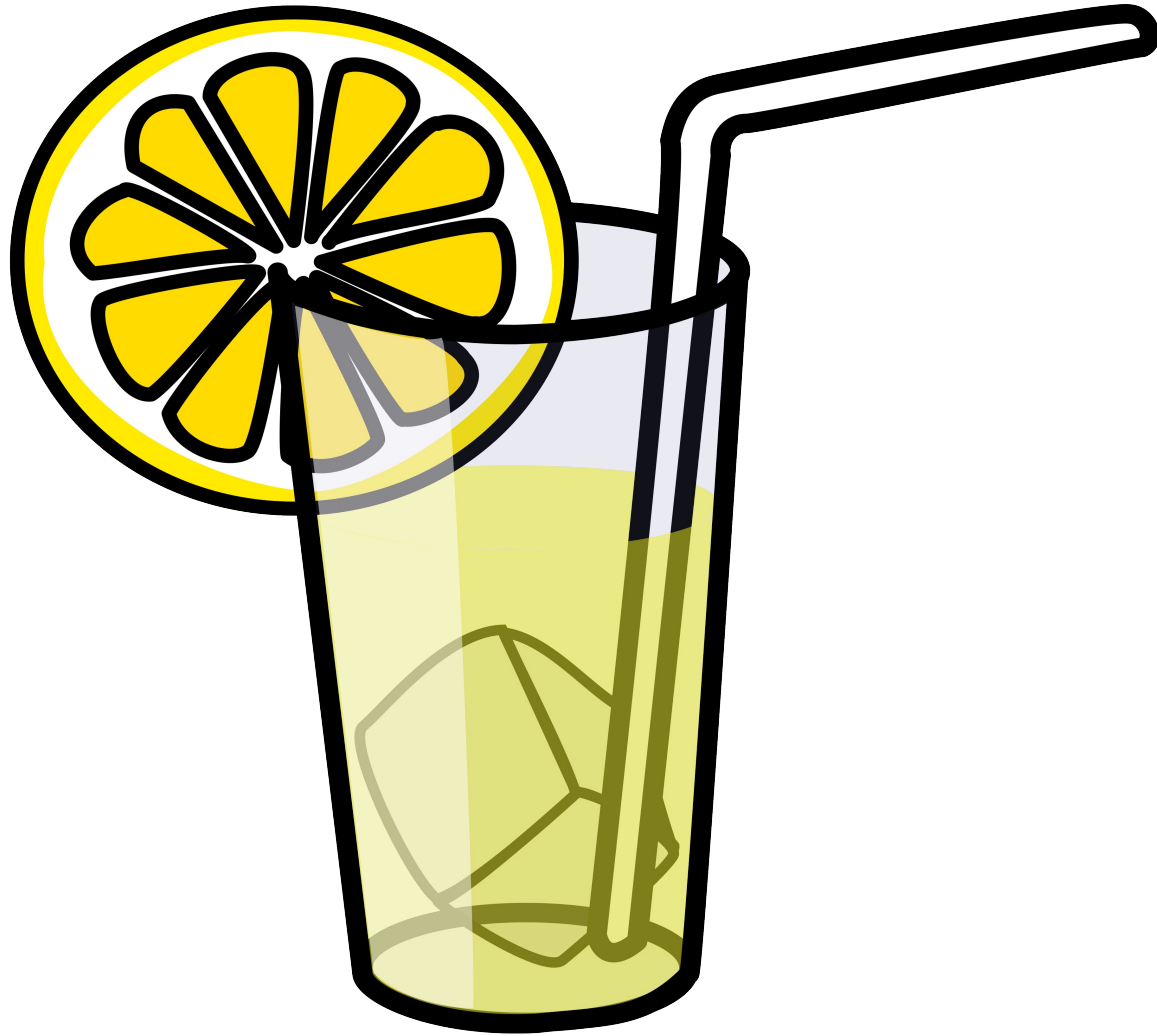
# Charge-density wave

## Static charge-structure factor

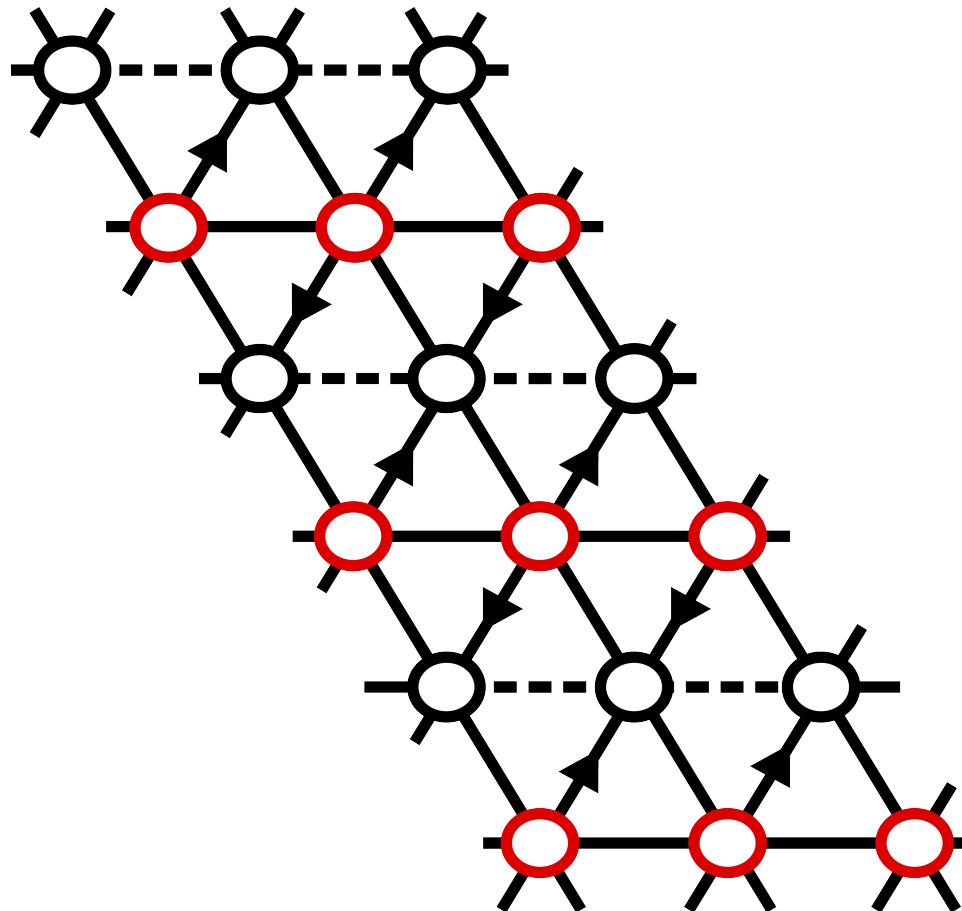




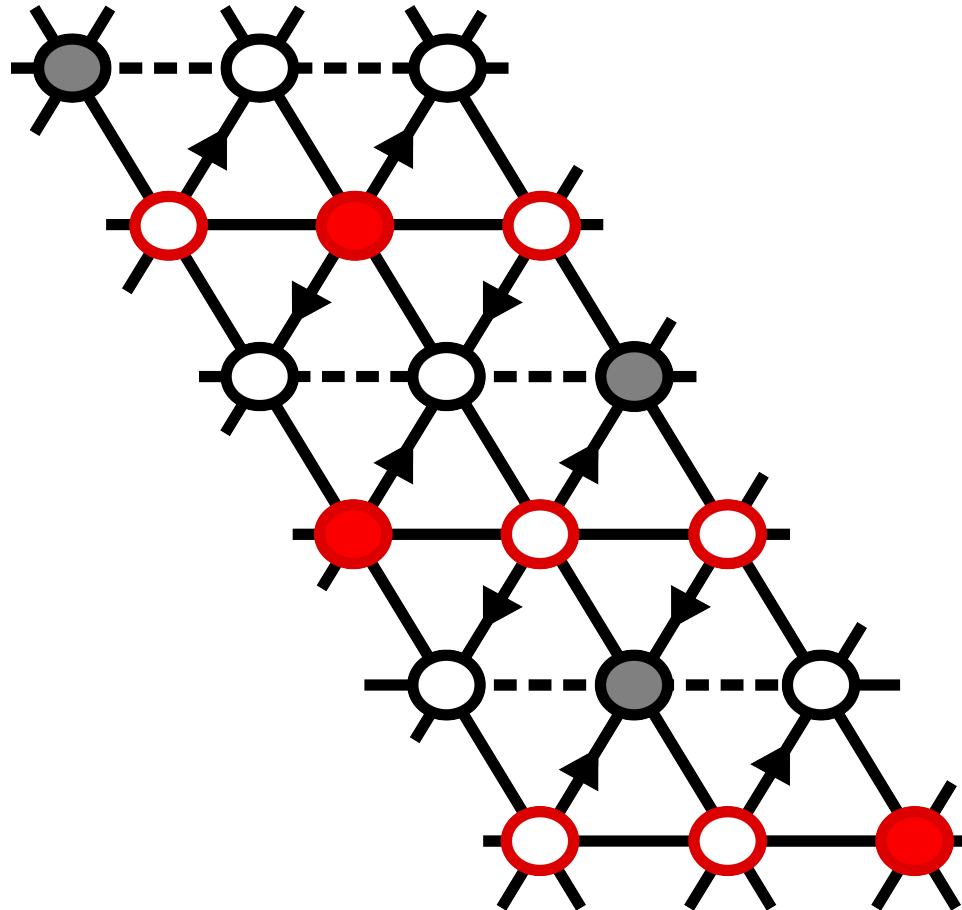




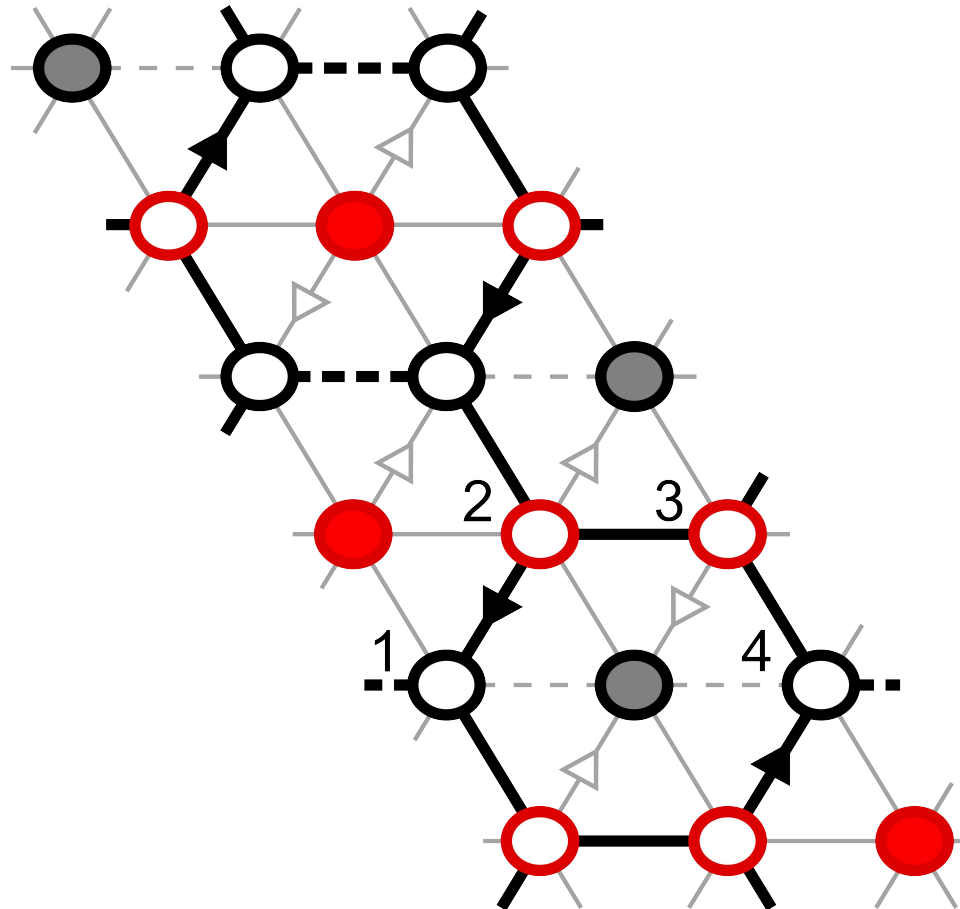
# Topological & Landau order



# Topological & Landau order



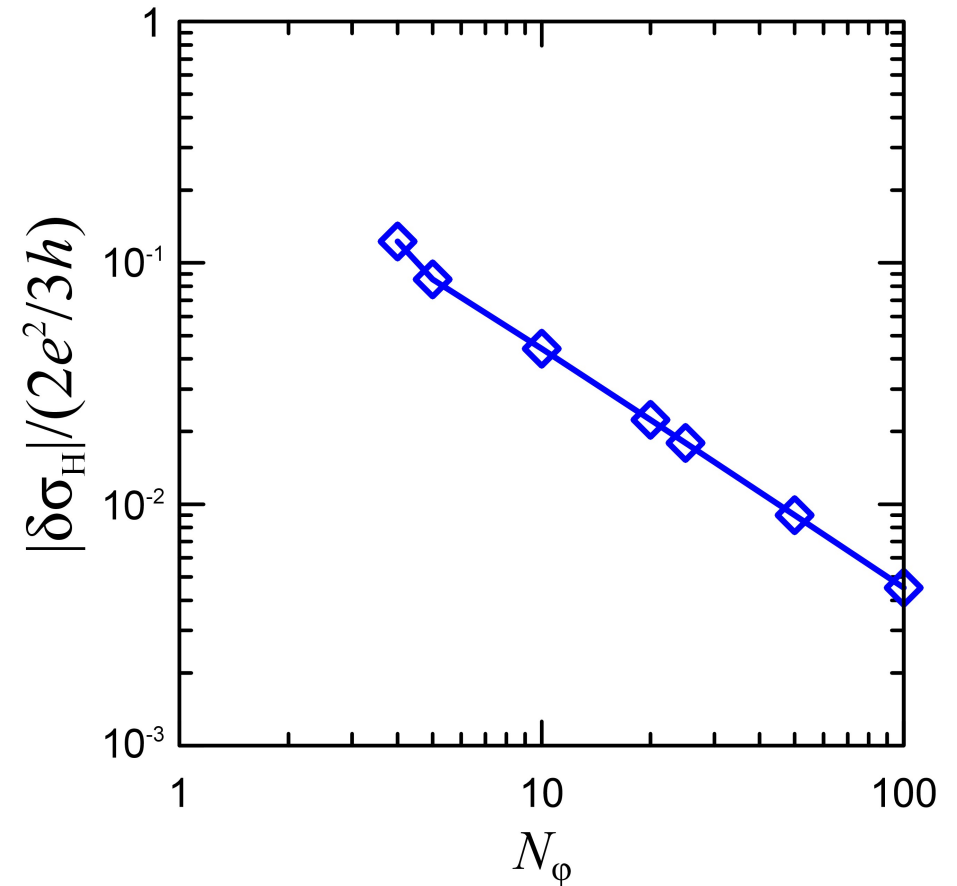
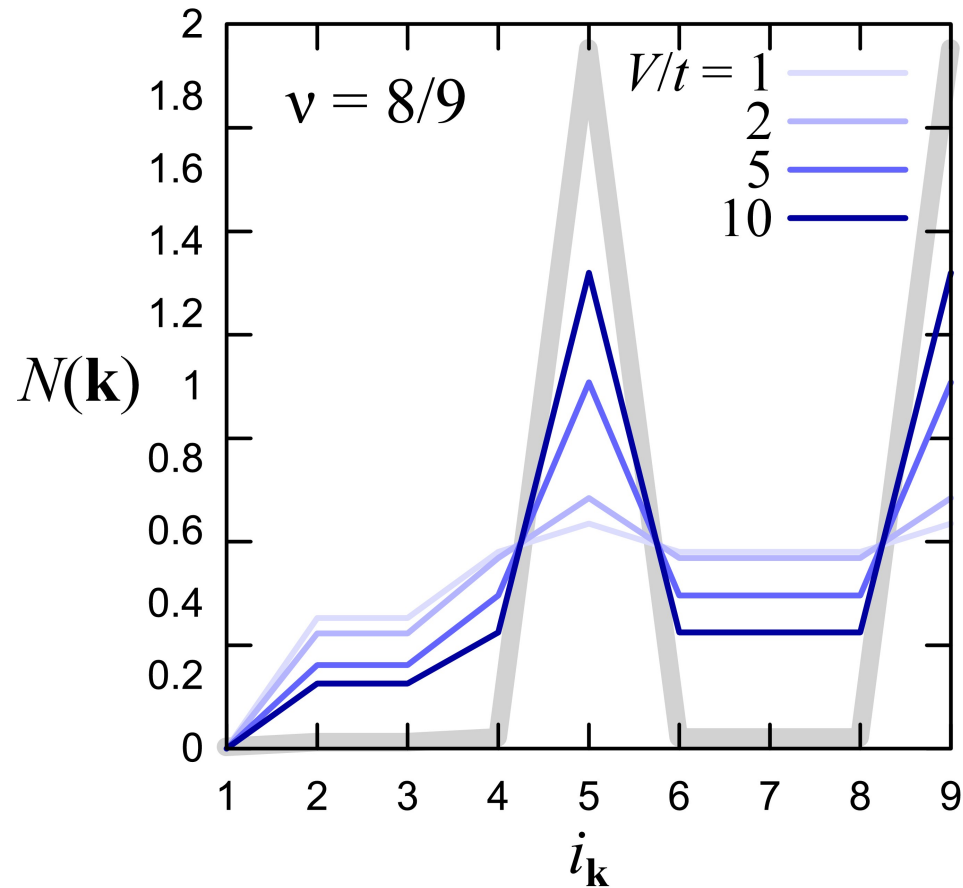
# Topological & Landau order



Similar to pinball liquid → Hotta & Furukawa, PRB 74, 193107 (2006)



# Topological pinball liquid



→ Charge order-induced topological order

# Conclusions

- ▶ FCI states for **arbitrarily strong repulsion...**
- ▶ ... going **beyond** traditional FQH physics (**FCI  $\neq$  FQHE**)
  - J. Venderbos, S. Kourtis, J. van den Brink, and M. Daghofer, Phys. Rev. Lett. **108**, 126405
  - S. Kourtis, J. Venderbos, and M. Daghofer, Phys. Rev. B **86**, 235118
  - S. Kourtis, T. Neupert, C. Chamon & C. Mudry, Phys. Rev. Lett. **112**, 126806
- ▶ **combined** topological & Landau order
  - (potential for topological states from interactions in trivial bands)
  - S. Kourtis & M. Daghofer, arXiv:1305.6948 + work in progress...

# CI on triangular lattice of $t_{2g}$ electrons

TM ion

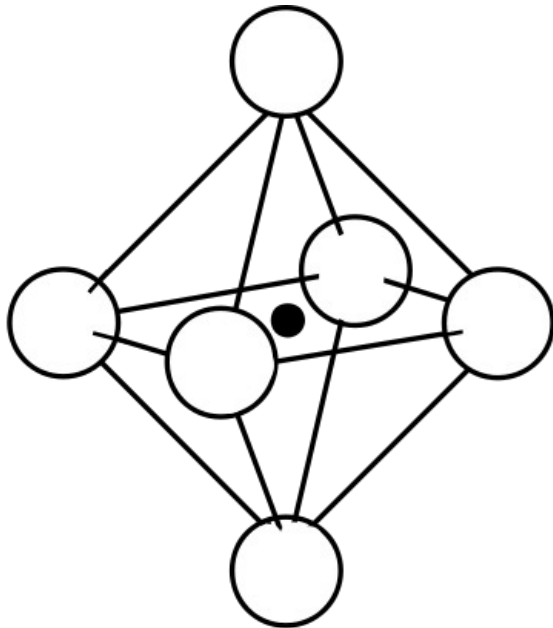


Partially occupied d-shell

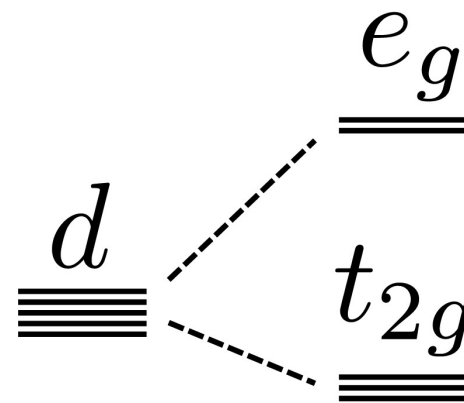


# CI on triangular lattice of $t_{2g}$ electrons

TM oxide

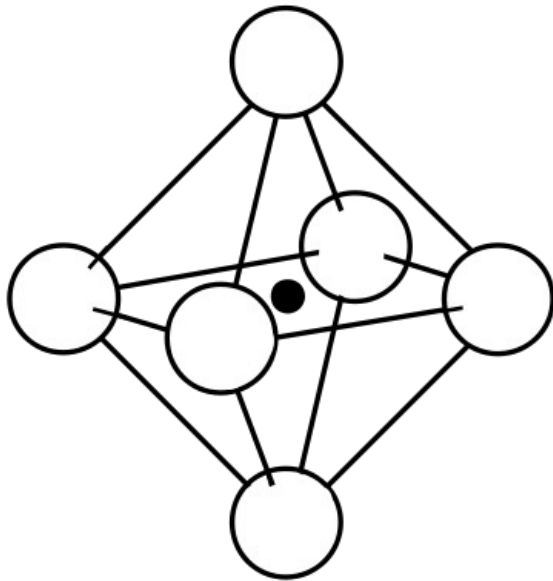


Symmetry lowering

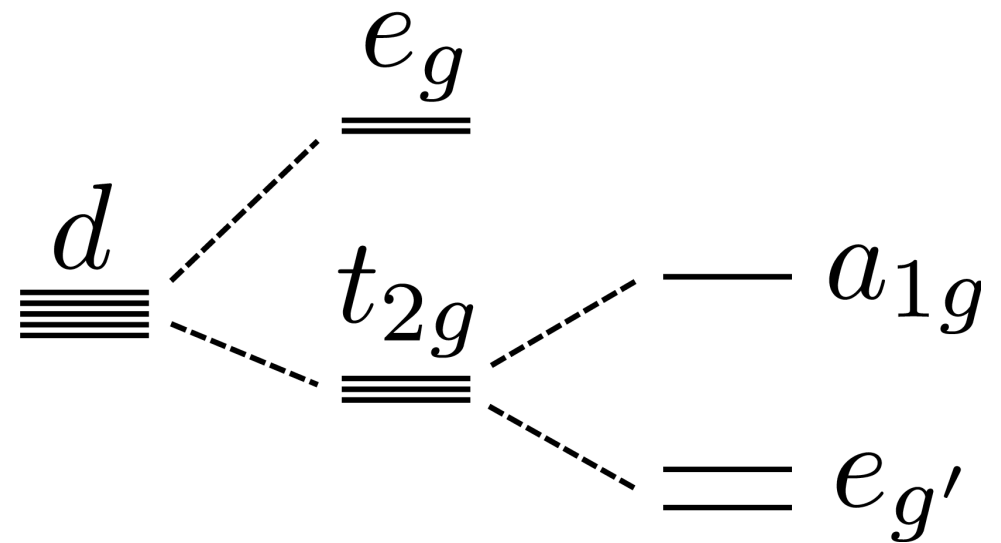


# CI on triangular lattice of $t_{2g}$ electrons

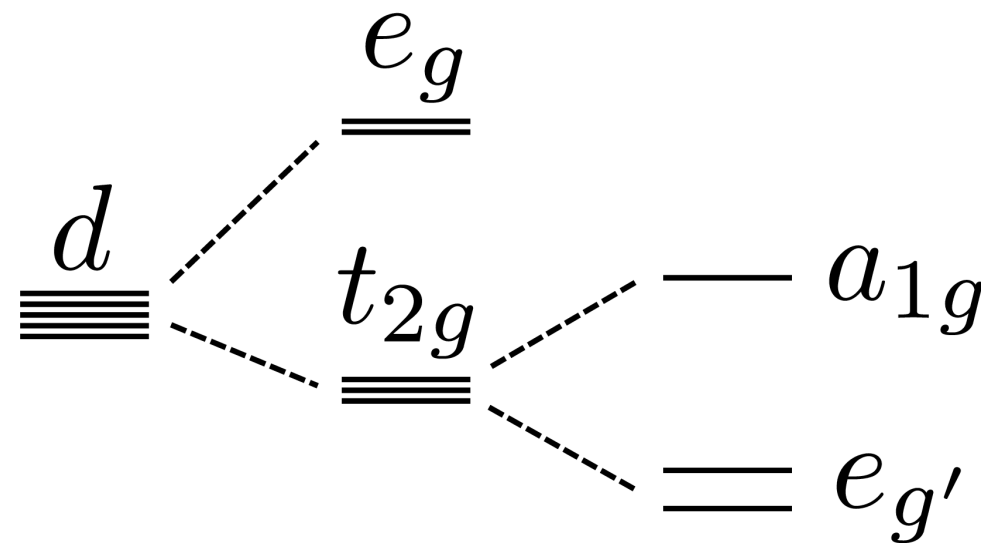
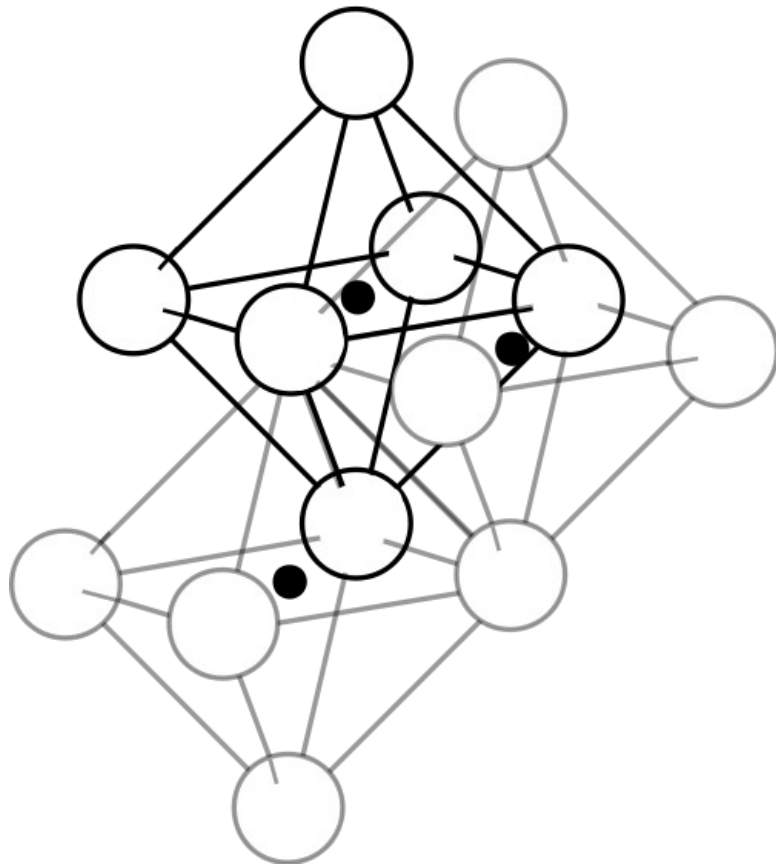
Crystal-field distortion



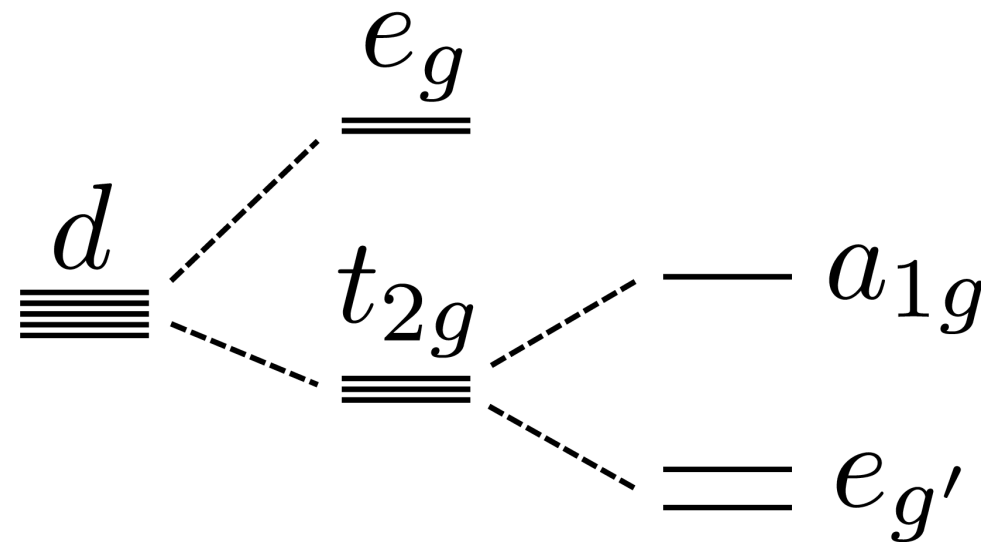
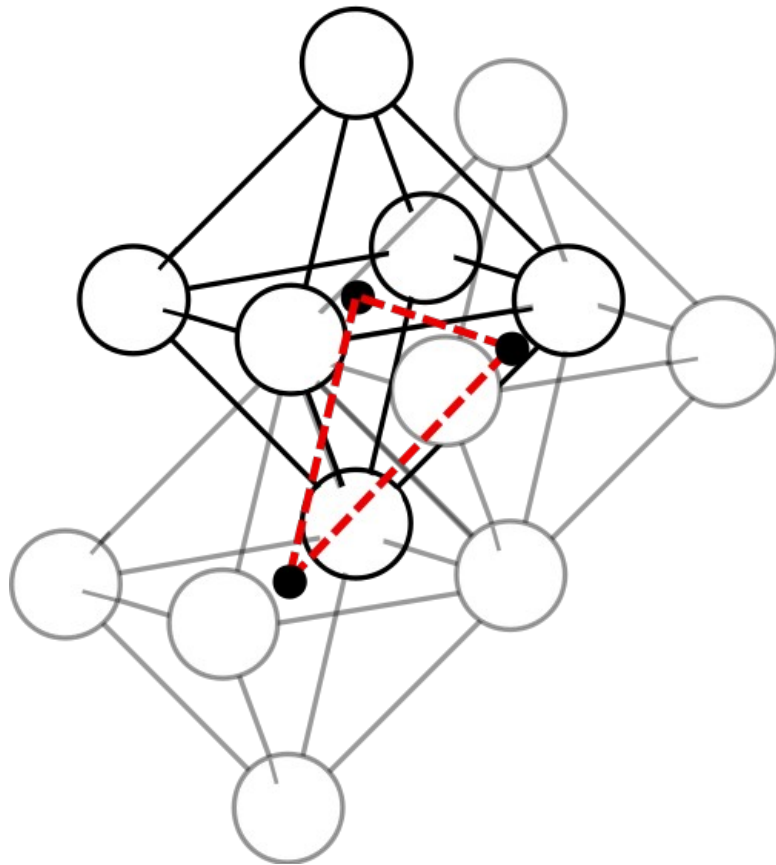
Levels split further



# CI on triangular lattice of $t_{2g}$ electrons

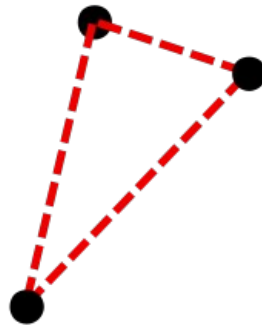


# CI on triangular lattice of $t_{2g}$ electrons

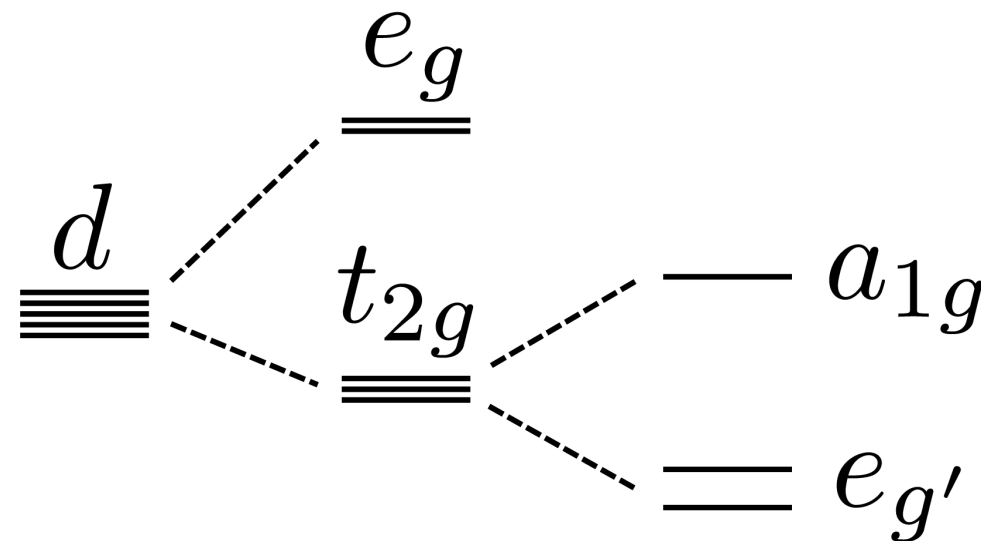


# CI on triangular lattice of $t_{2g}$ electrons

Triangular lattice



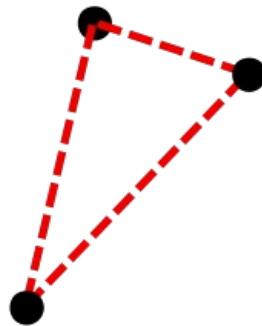
3 orbitals per site



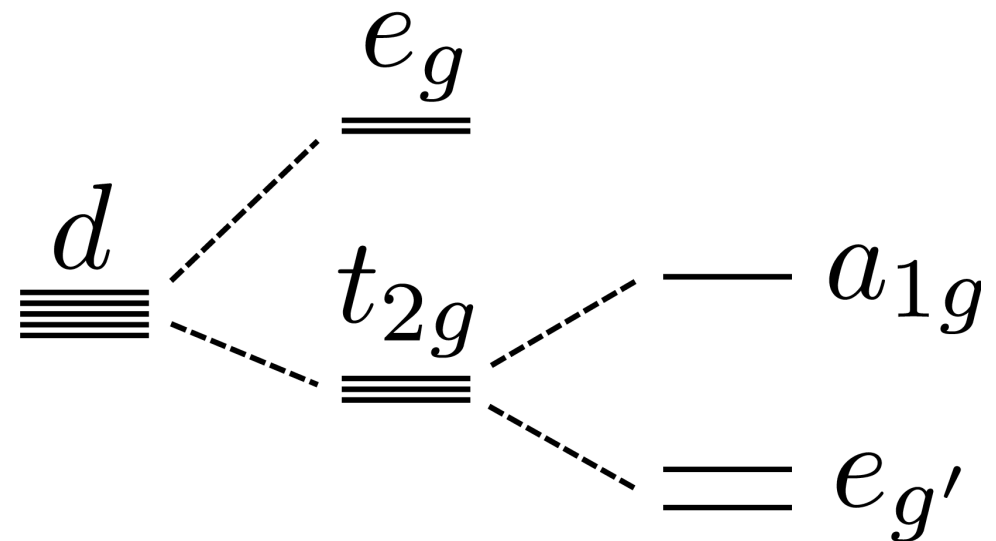


# CI on triangular lattice of $t_{2g}$ electrons

Triangular lattice



3 orbitals per site



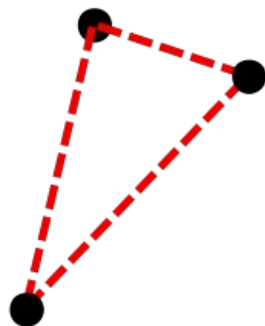
+ on-site Coulomb repulsion

+ Hund's rule coupling

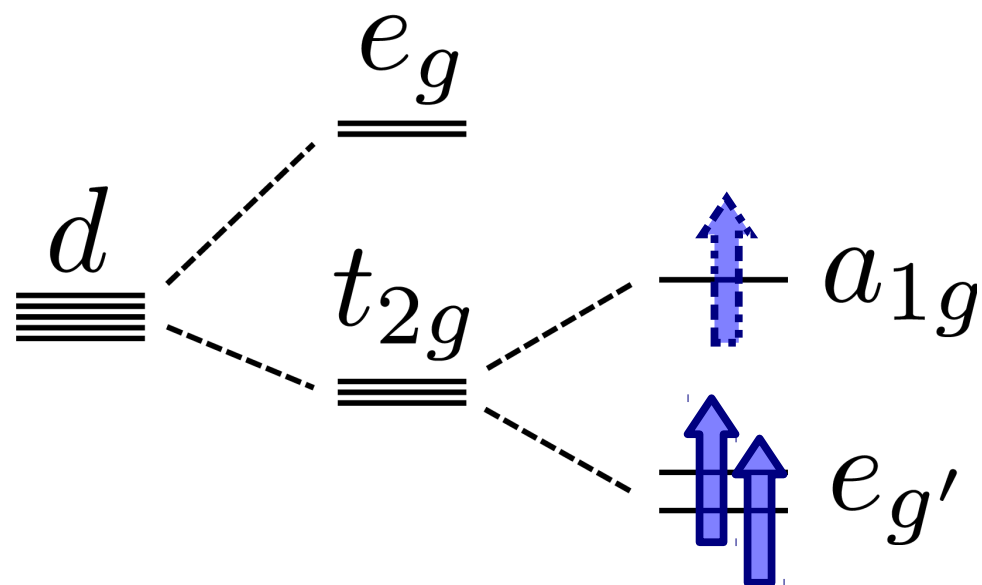
+  $n = 2.5 e^- / \text{site}$

# CI on triangular lattice of $t_{2g}$ electrons

Triangular lattice



3 orbitals per site



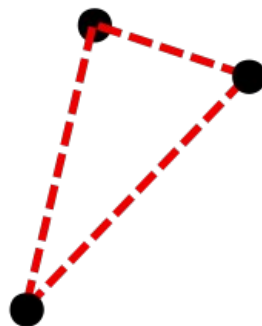
+ on-site Coulomb repulsion

+ Hund's rule coupling

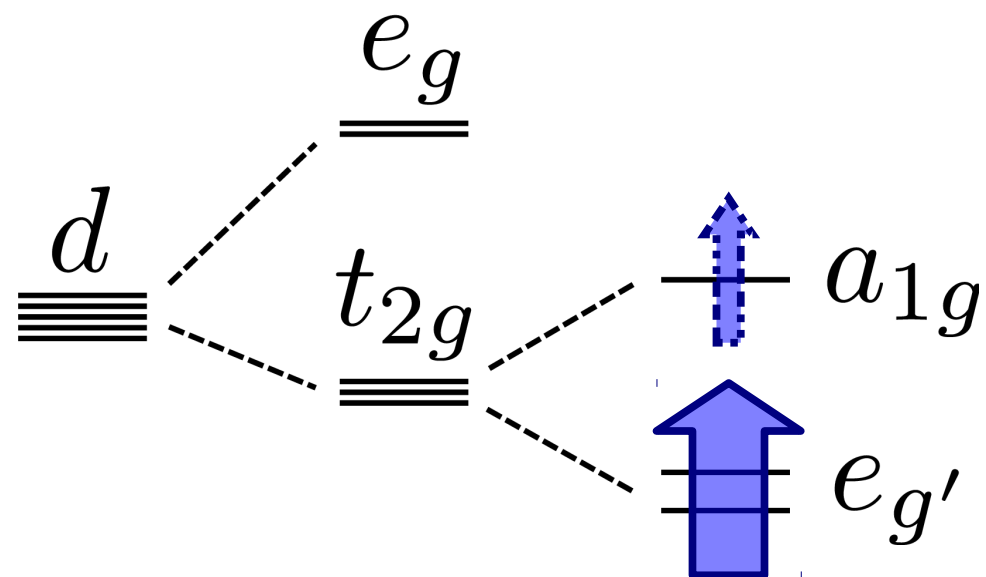
+  $n = 2.5 e^- / \text{site}$

# CI on triangular lattice of $t_{2g}$ electrons

Triangular lattice



3 orbitals per site



+ on-site Coulomb repulsion

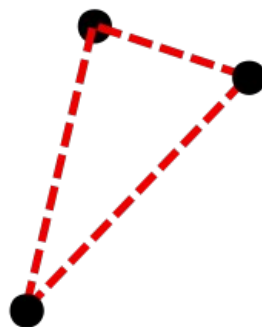
+ Hund's rule coupling

+  $n = 2.5 e^- / \text{site}$

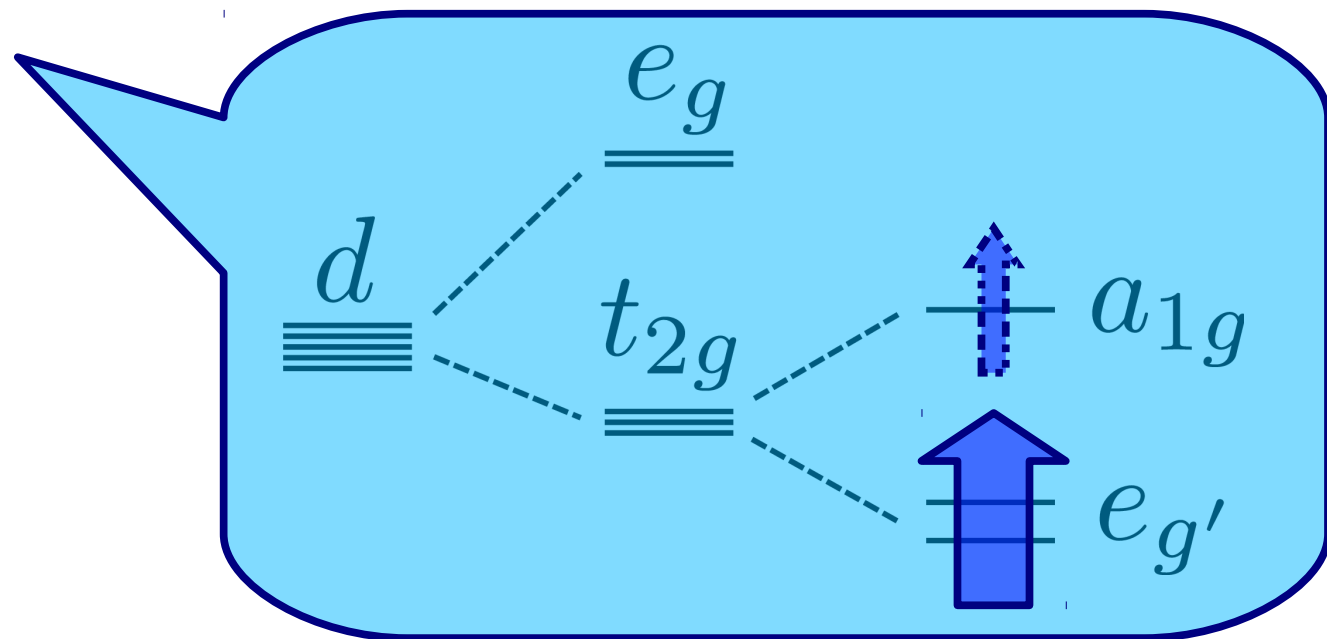
# CI on triangular lattice of $t_{2g}$ electrons

Triangular lattice

Kondo-lattice model



3 orbitals per site



+ on-site Coulomb repulsion

+ Hund's rule coupling

+  $n = 2.5 e^- / \text{site}$

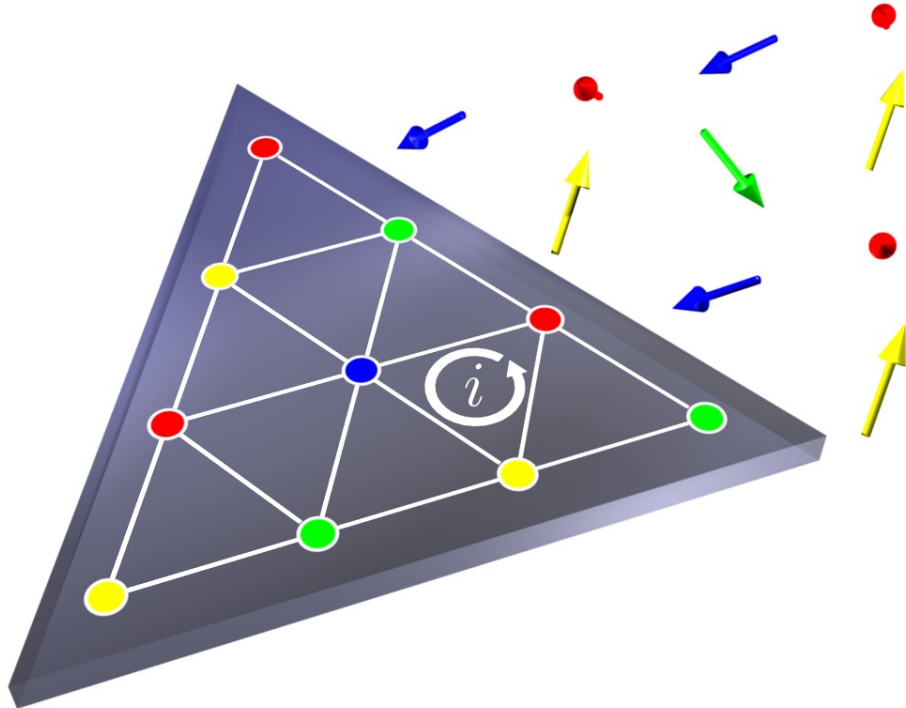
# CI on triangular lattice of $t_{2g}$ electrons

Triangular lattice

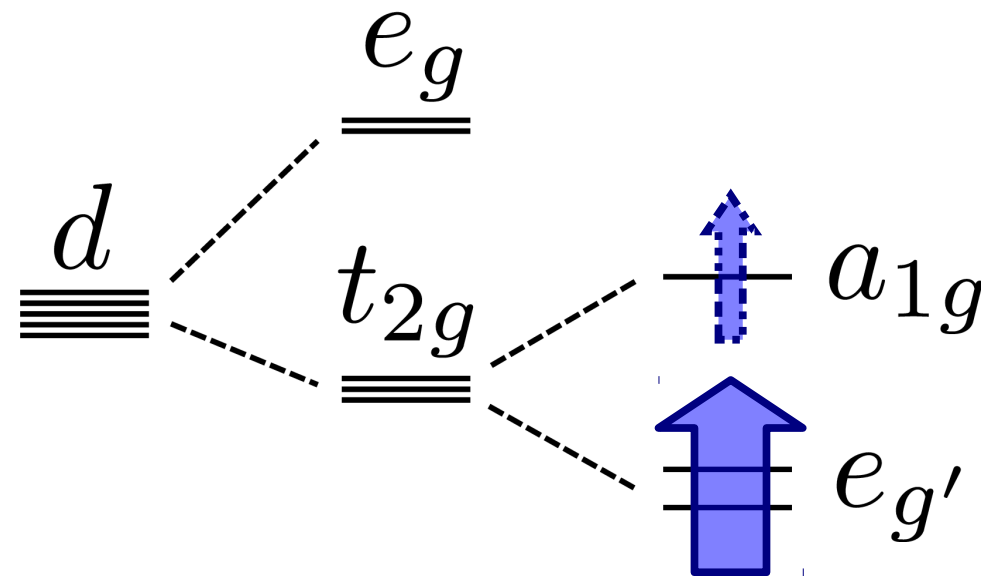
Kondo-lattice model

→ chiral spin pattern

Martin & Batista, 2008



3 orbitals per site



- + on-site Coulomb repulsion
- + Hund's rule coupling
- +  $n = 2.5 e^- / \text{site}$