Spin-liquid behavior of a simple spin model on the triangular lattice

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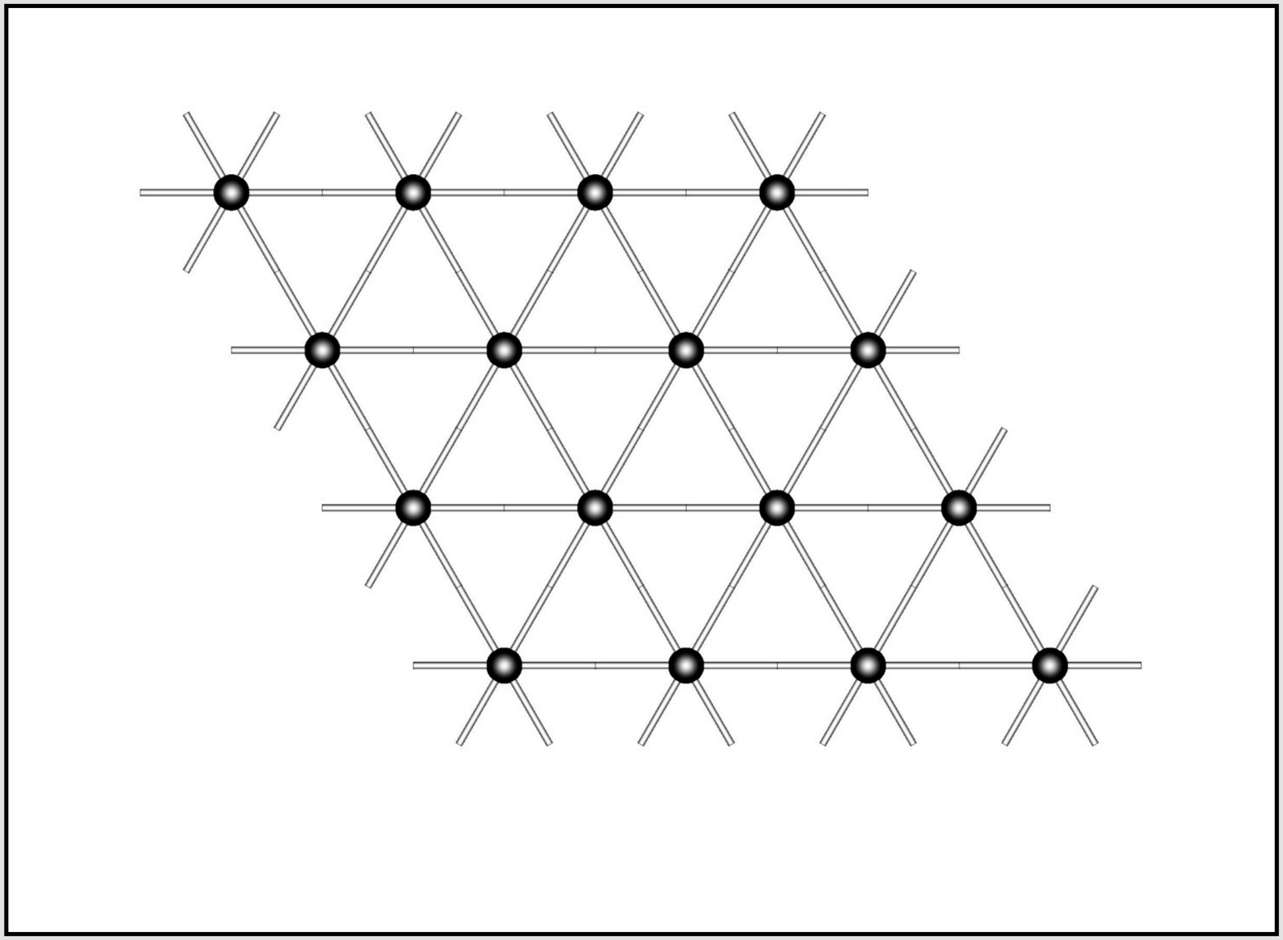
A SIMPLE MODEL OF MANY-BODY QUANTUM MECHANICS

Hilbert Space

Consider any lattice with $N_{\rm site}$ sites.

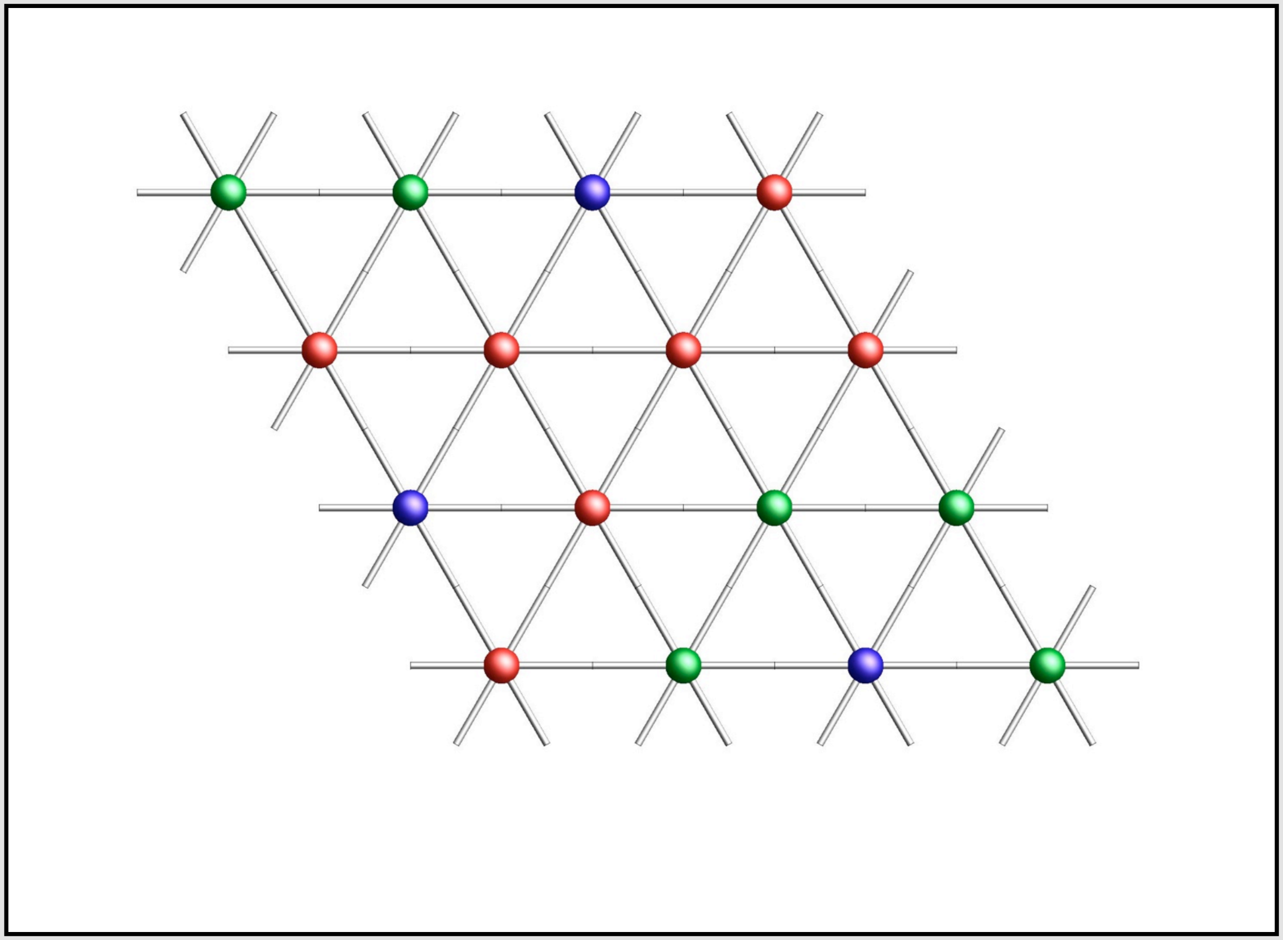
Illustration: 2-d triangular lattice



Hilbert Space

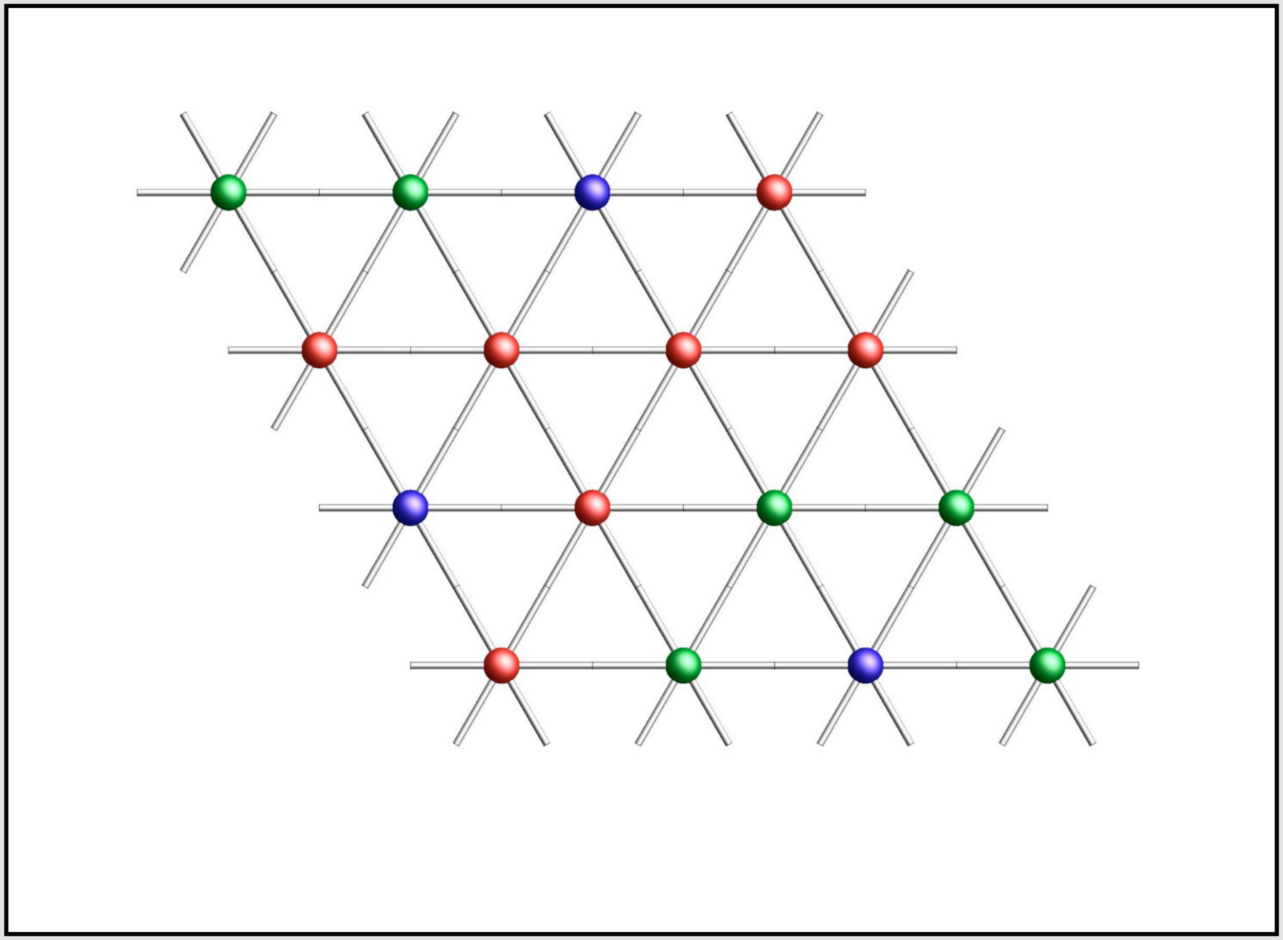
Assign a color to each site (Hilbert space: $N^{N_{\mathrm{site}}}$)

Illustration: $N = 3$



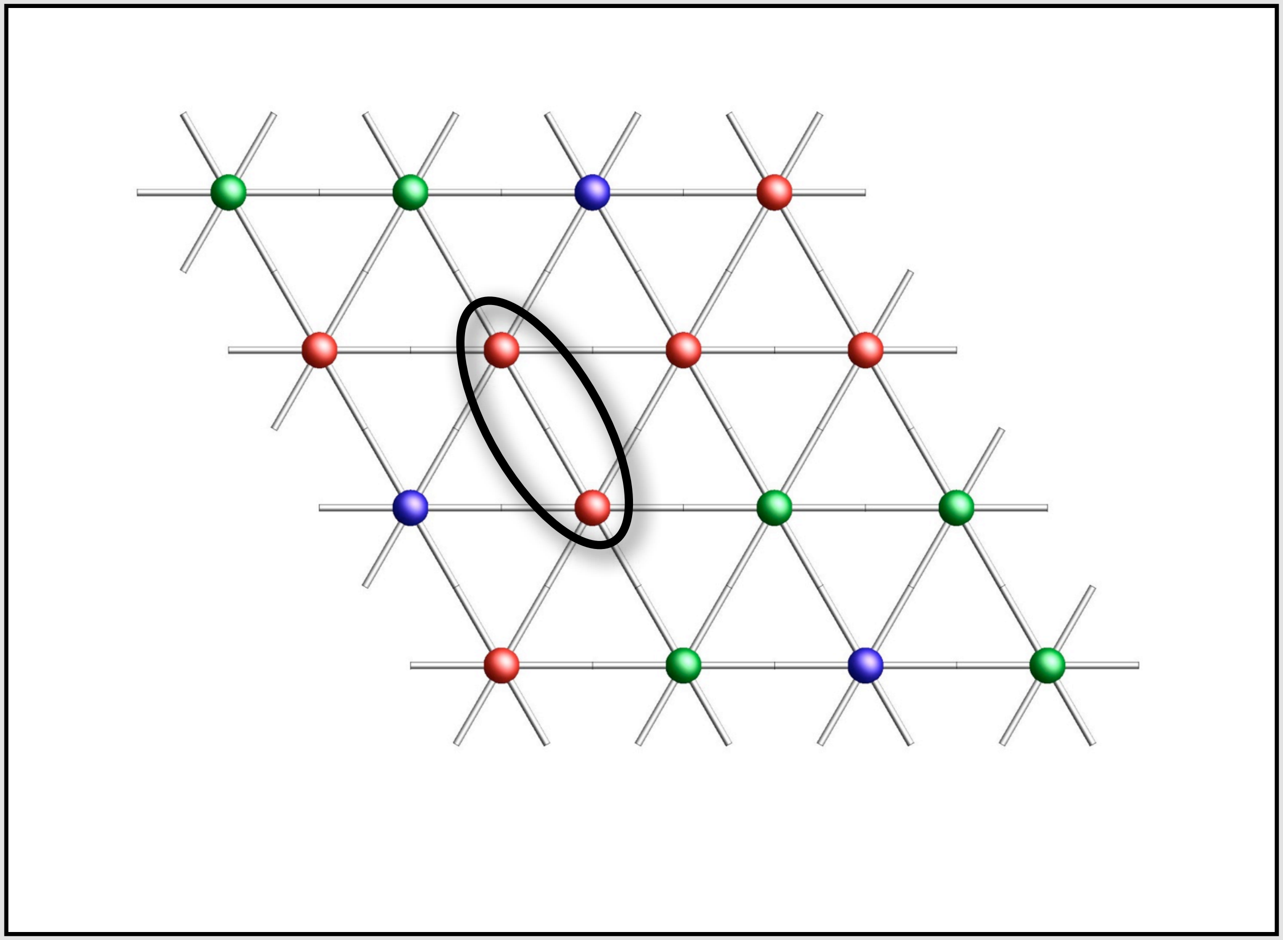
Hilbert Space

To define "Hamiltonian" describe how it acts on Hilbert space.



Quantum Dynamics

$$N = 3$$



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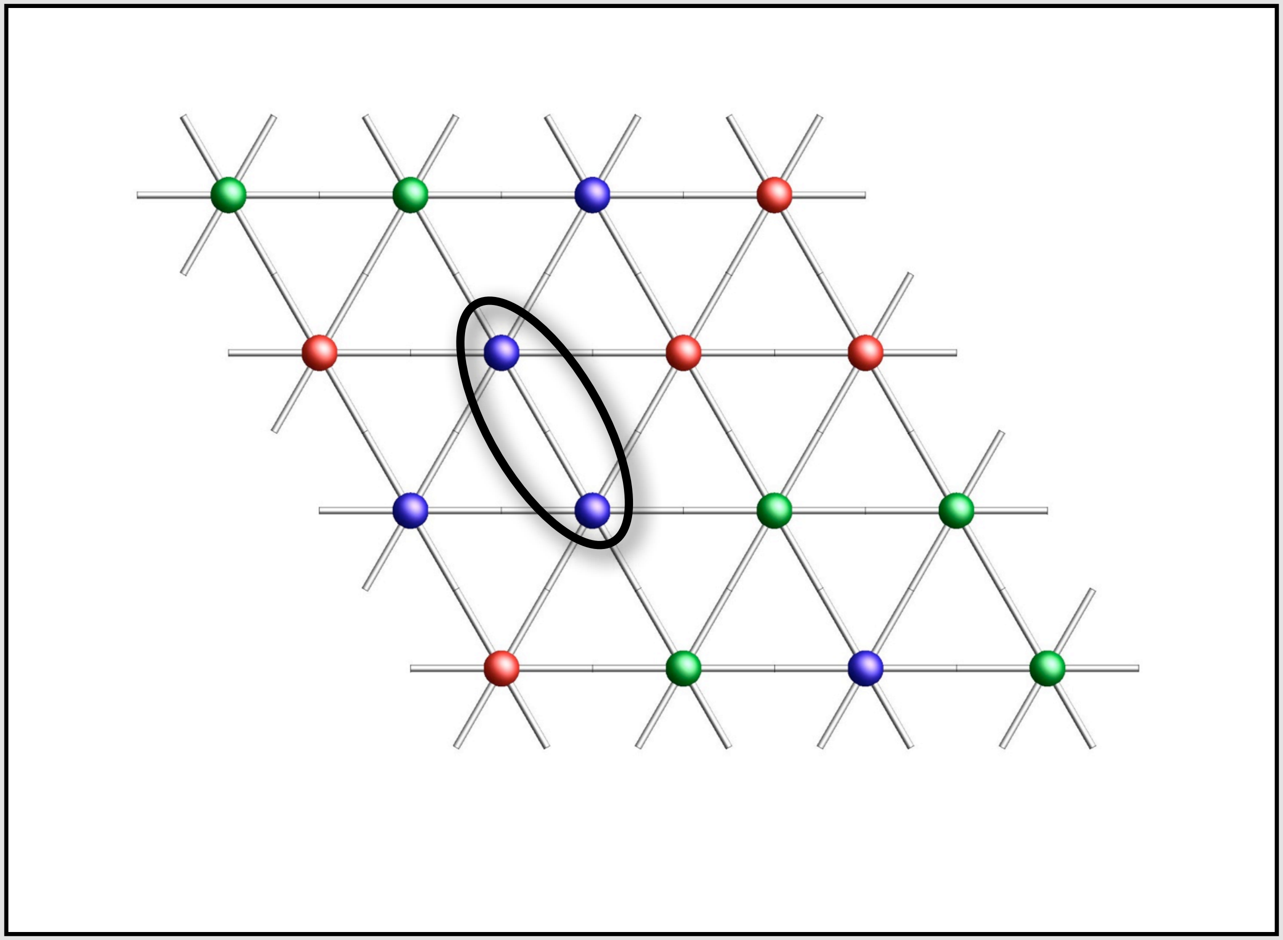

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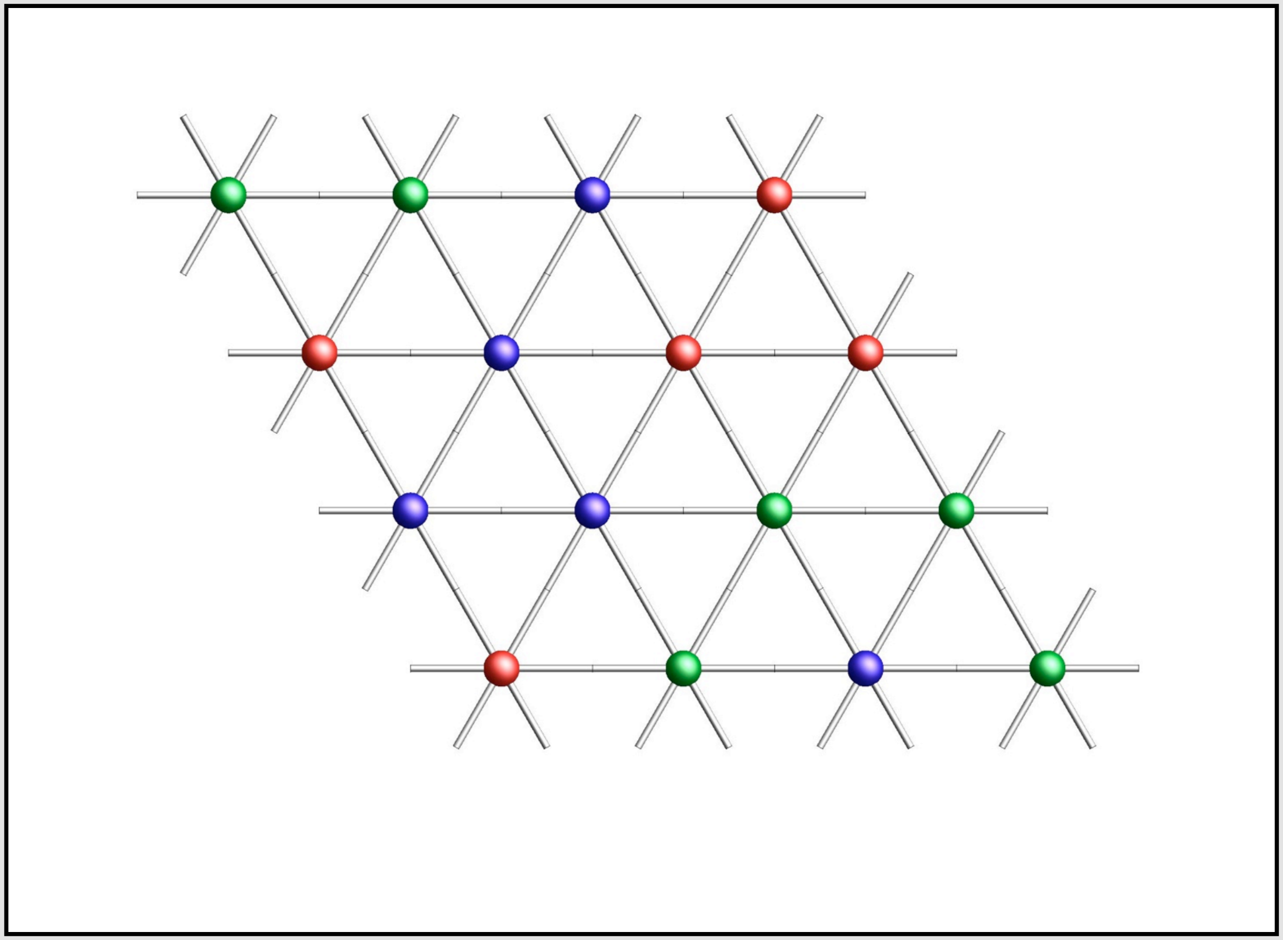

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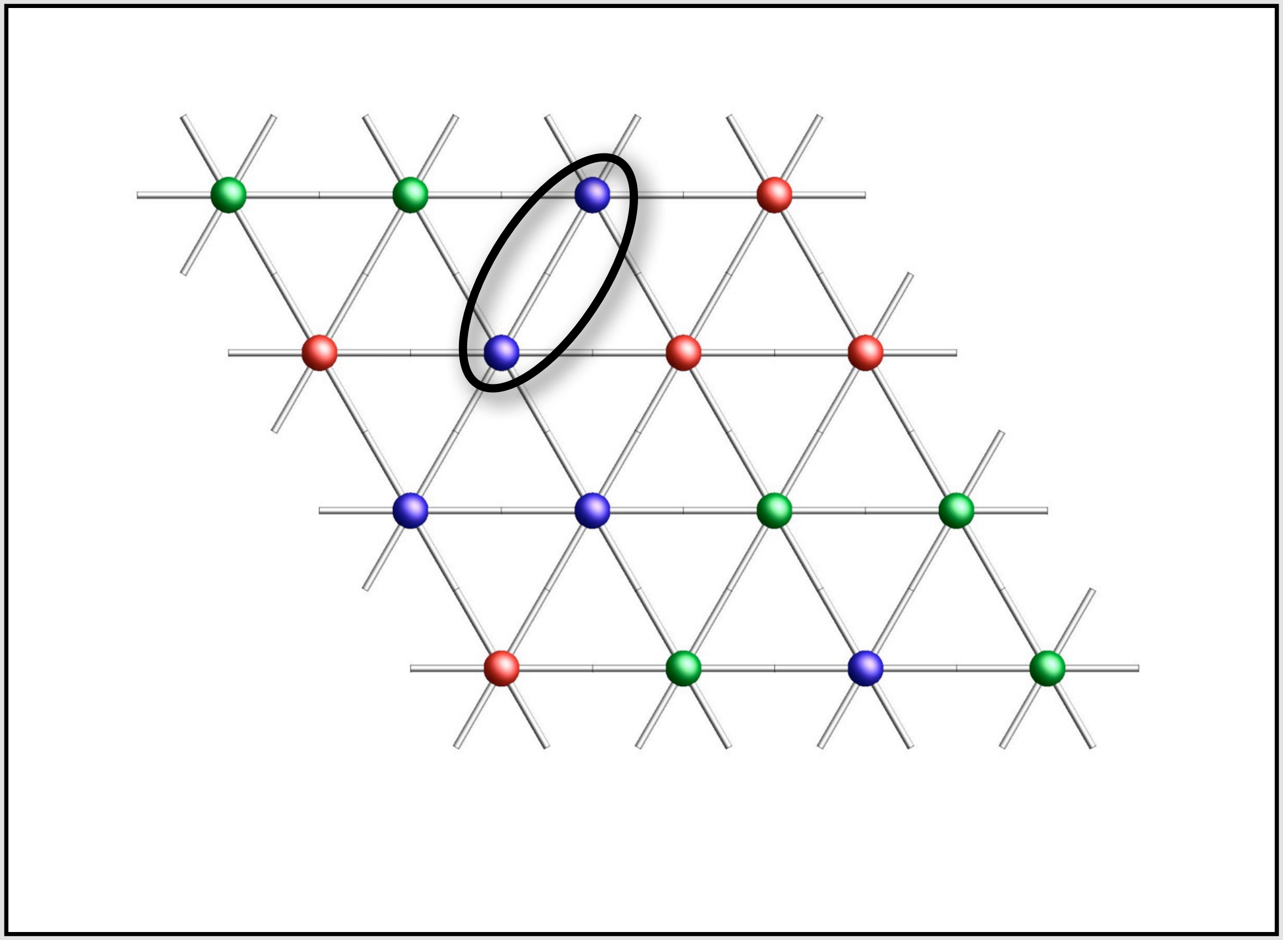

Hamiltonian

Put picture in equations:

$$H_J = -\frac{J}{N} \sum_{\langle ij \rangle, \alpha\beta} |\alpha\alpha\rangle_{ij} \langle\beta\beta|_{ij} \quad (1 \leq \alpha \leq N)$$

Hamiltonian

Put picture in equations:

$$H_J = -\frac{J}{N} \sum_{\langle ij \rangle, \alpha\beta} |\alpha\alpha\rangle_{ij} \langle \beta\beta|_{ij} \quad (1 \leq \alpha \leq N)$$

$$|\mathcal{S}\rangle_{ij} = \frac{1}{\sqrt{N}} \sum_{\alpha} |\alpha\alpha\rangle_{ij}$$

I'll call $|\mathcal{S}\rangle$ a "singlet" state

$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$

Outline of the Talk

$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$

Q: What is the ground state of H_J as a function of N and for different lattices?

$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$

BIPARTITE LATTICES

NON-BIPARTITE LATTICES

$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$

BIPARTITE LATTICES

NON-BIPARTITE LATTICES

Bipartite Lattices

Consider, $N = 2$

$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$

is identical up to a constant to,

$$H_{\mathrm{heis}} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

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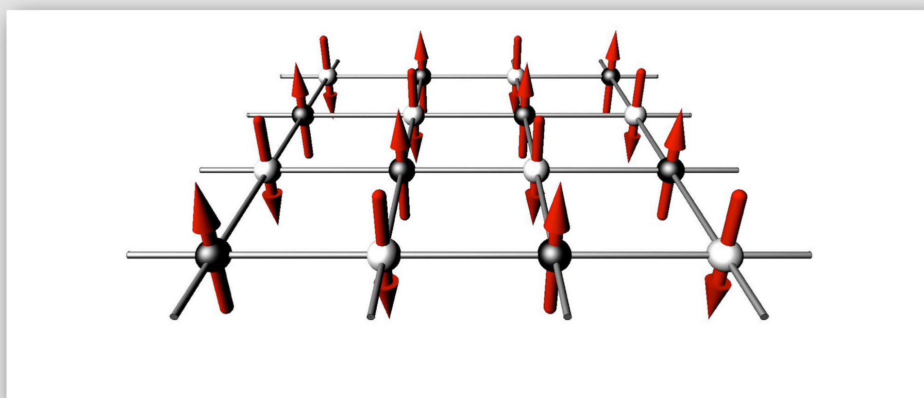


Néel State

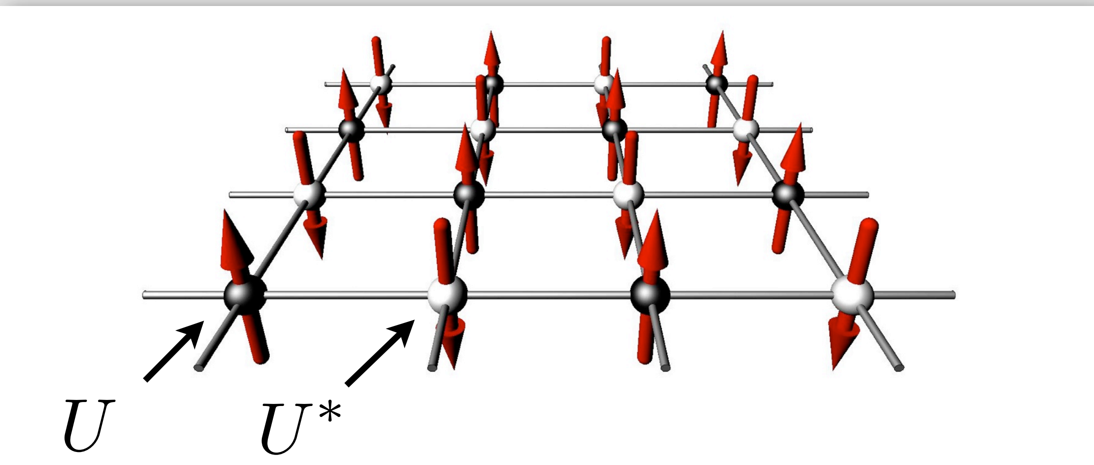
SU(N) model: Singlet Projector

For, $N > 2$

$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$

$$|\mathcal{S}\rangle_{ij} = \sum_{\alpha} |\alpha\alpha\rangle_{ij}$$

SU(N) singlet



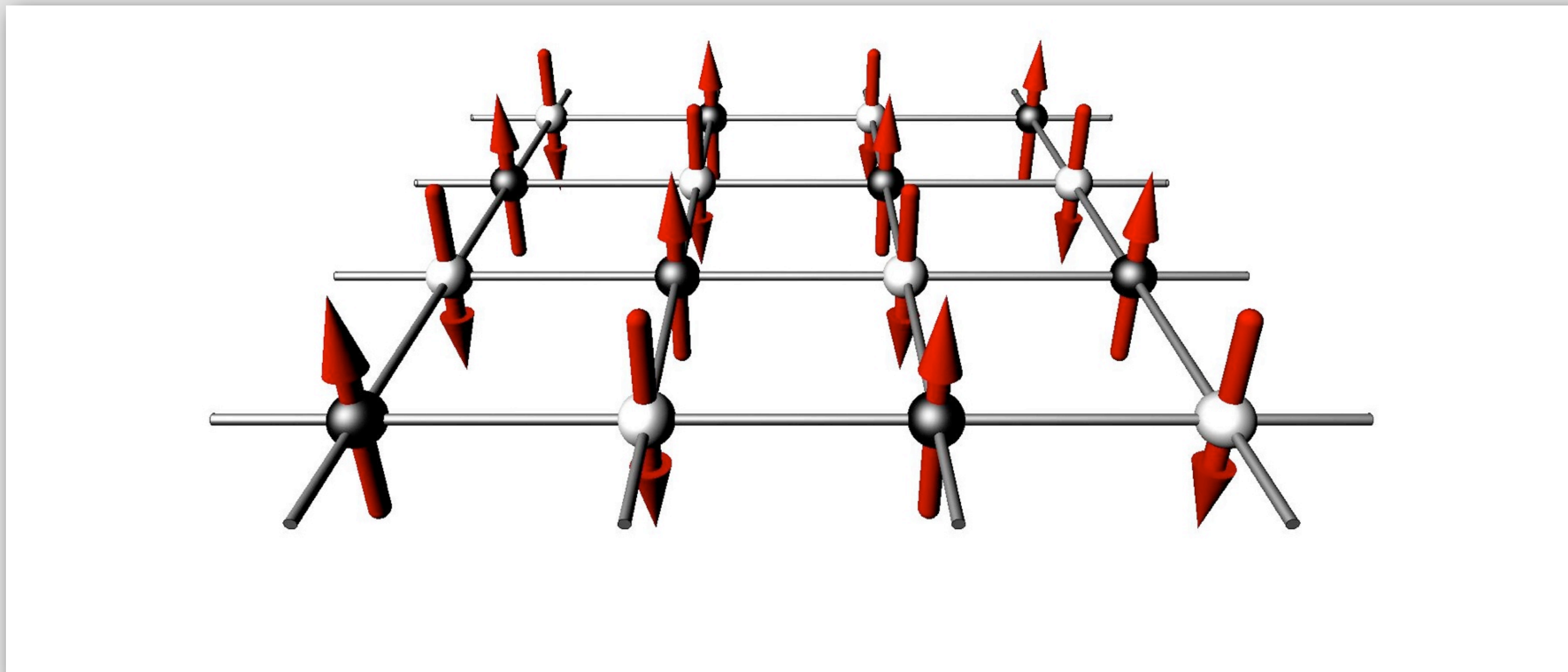
$$|\mathcal{S}\rangle \to |\mathcal{S}\rangle$$

$$H_J = -J \sum_{\langle ij \rangle, a} \hat{T}_i^{*a} \cdot \hat{T}_j^a$$

Affleck (1989)

Small *N*: Magnetic

N=2

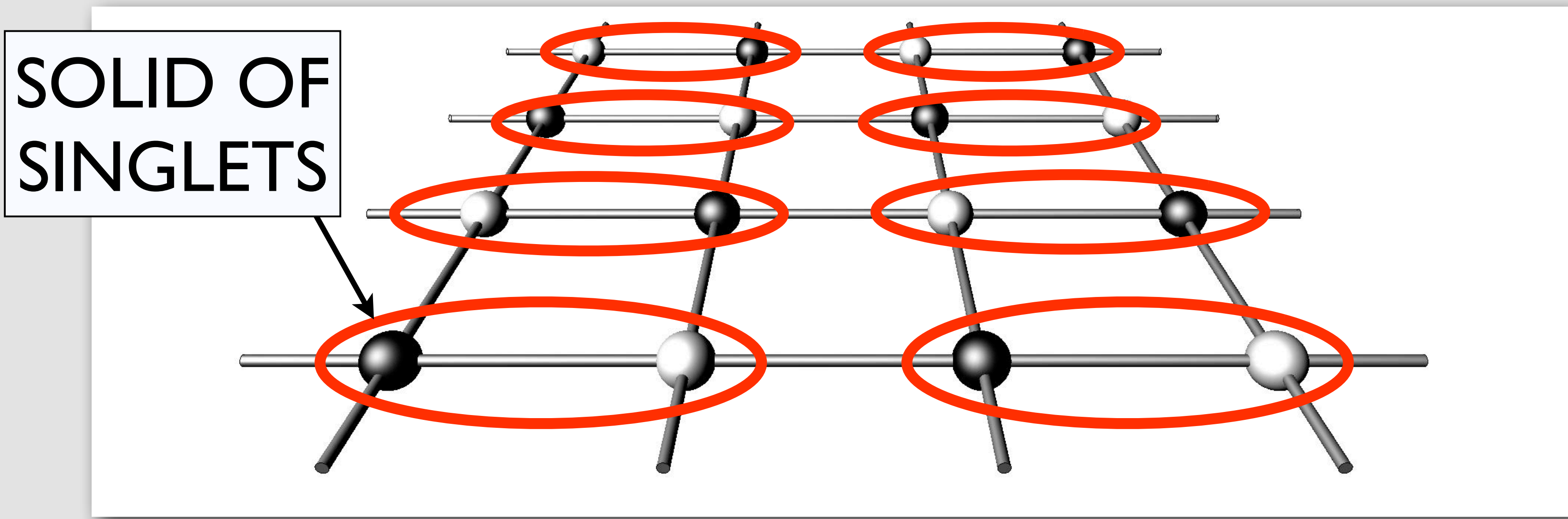


Classical Néel State: Breaks SU(2) symmetry

Large N: Valence Bond Solids

At large-N maps to a quantum dimer model

$$\langle S \rangle = 0 \qquad\qquad O_{\mathrm{VBS}} \sim S_i \cdot S_j$$

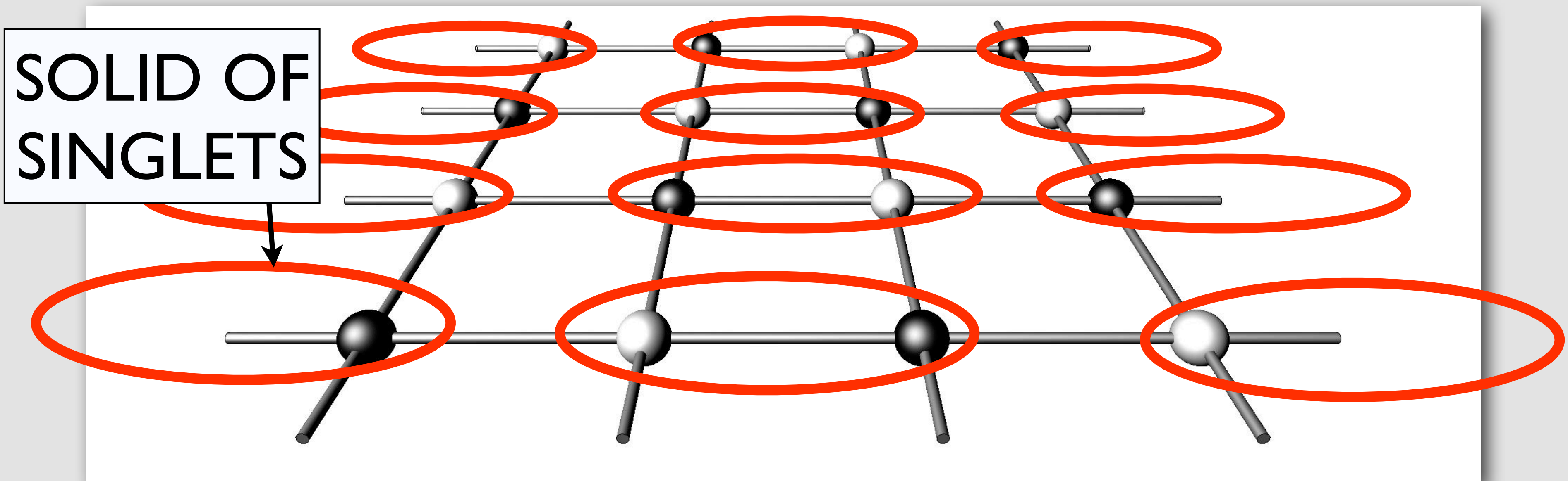


Read, Sachdev Nuc. Phys. B (1986)

VBS: breaks lattice symmetry

Large N: Valence Bond Solids

$\langle S \rangle = 0$ $\qquad O_{\mathrm{VBS}} \sim S_i \cdot S_j$



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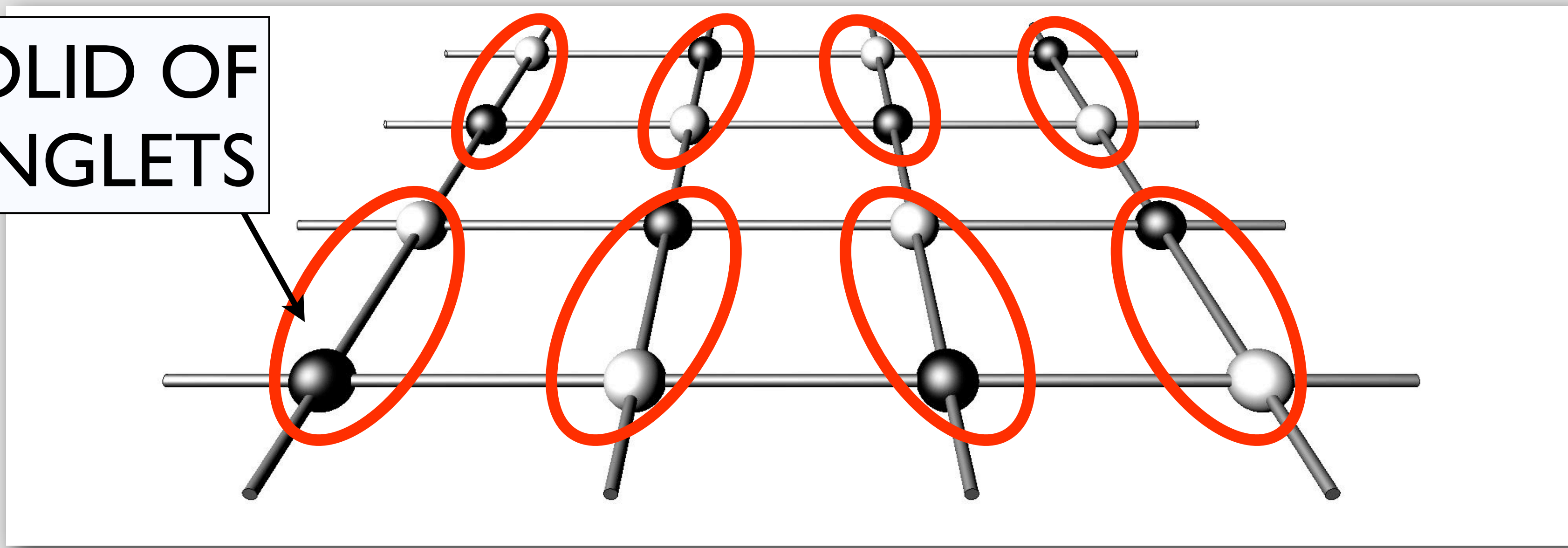
SOLID OF SINGLETS



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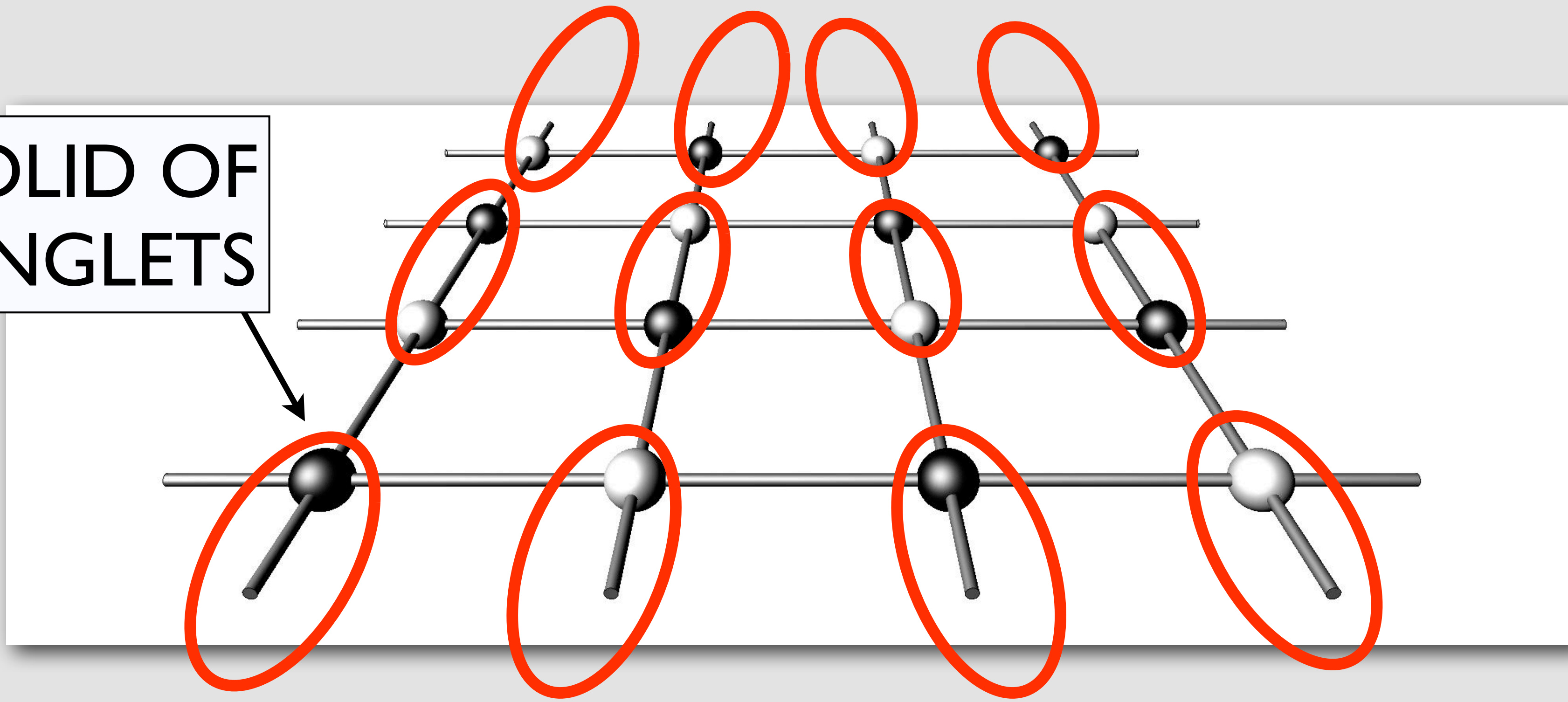

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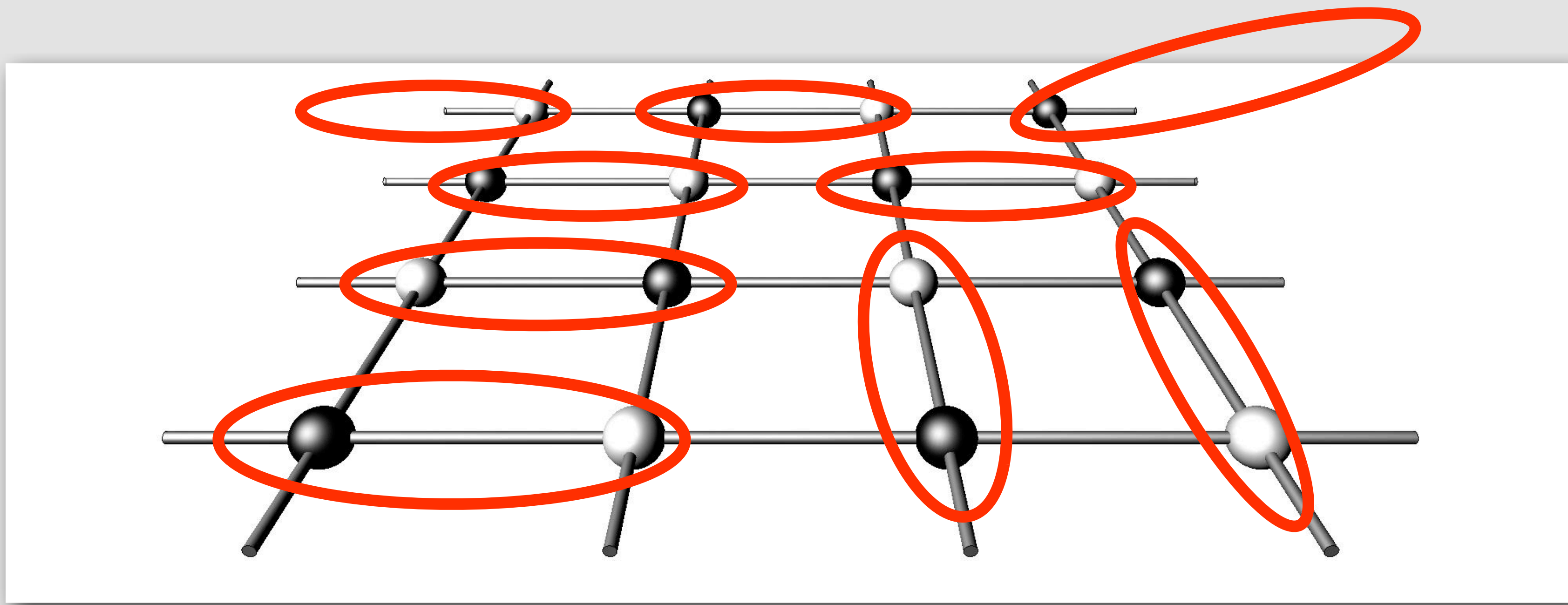
VBS: breaks lattice symmetry

In-between: Anderson RVB?

$$\langle S \rangle = 0$$



Spin Liquid!

Quantum superposition of many singlet coverings

In-between: Anderson RVB?

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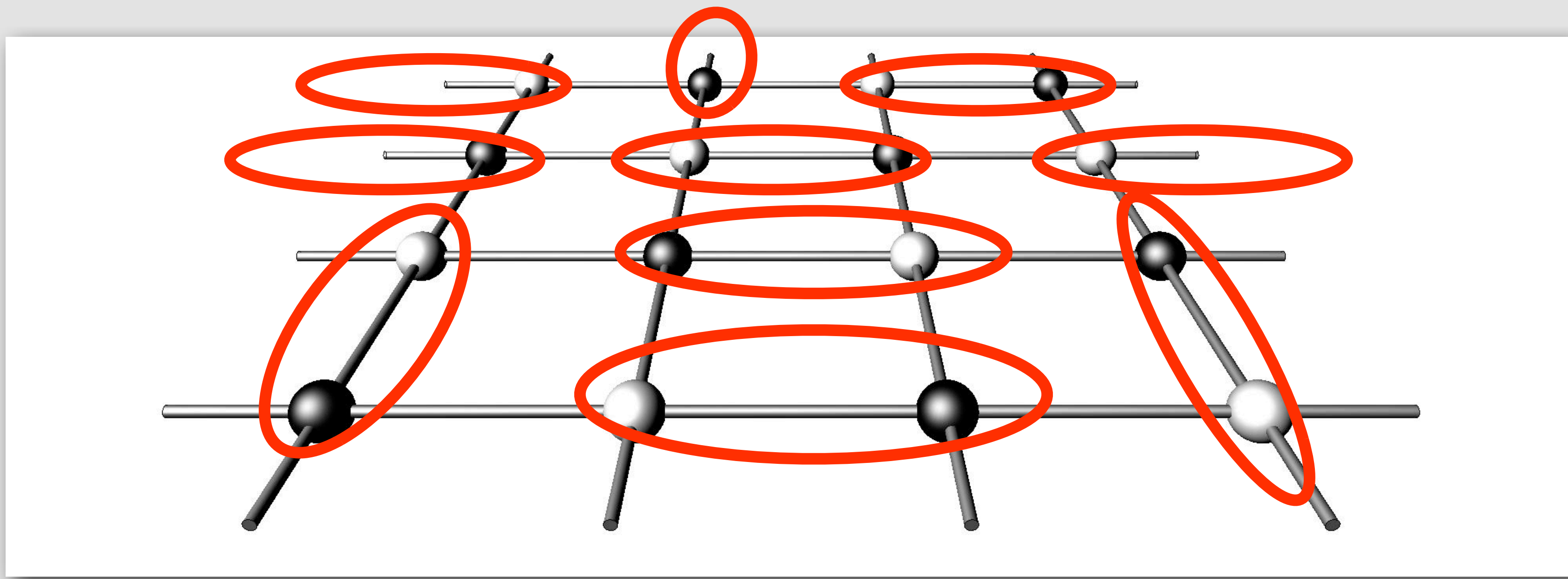

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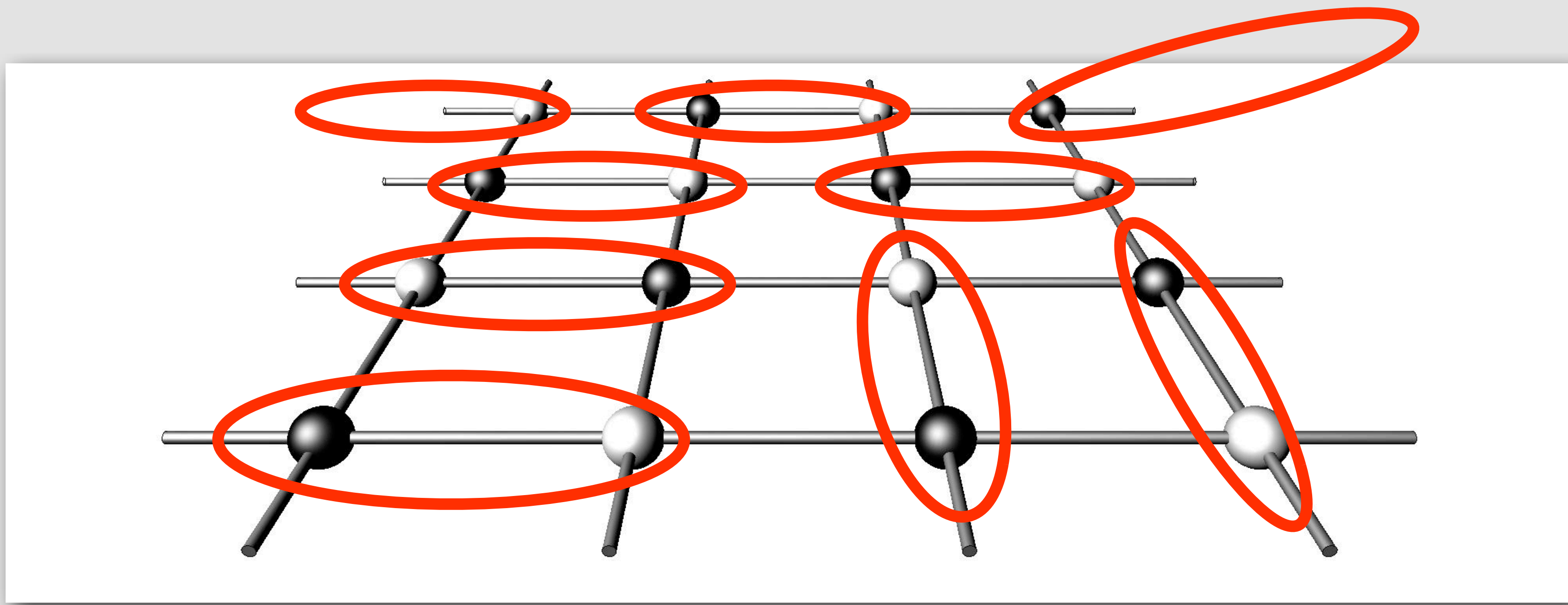

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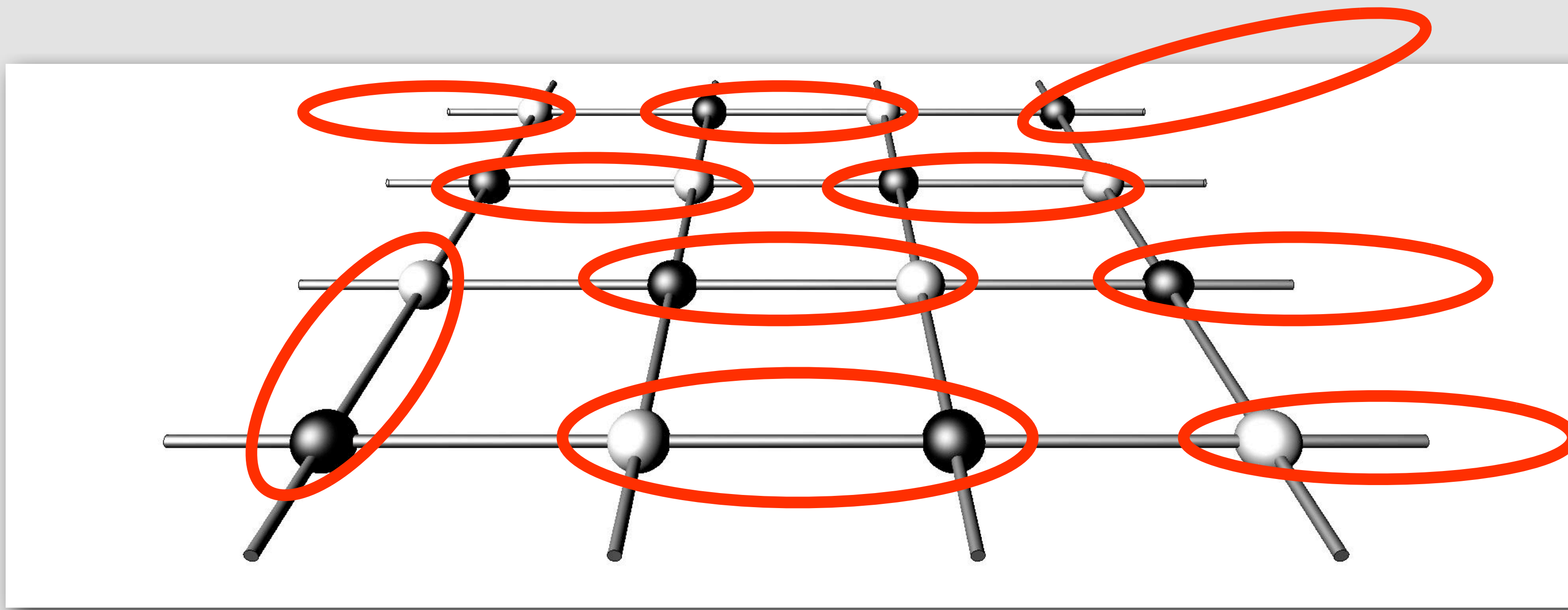
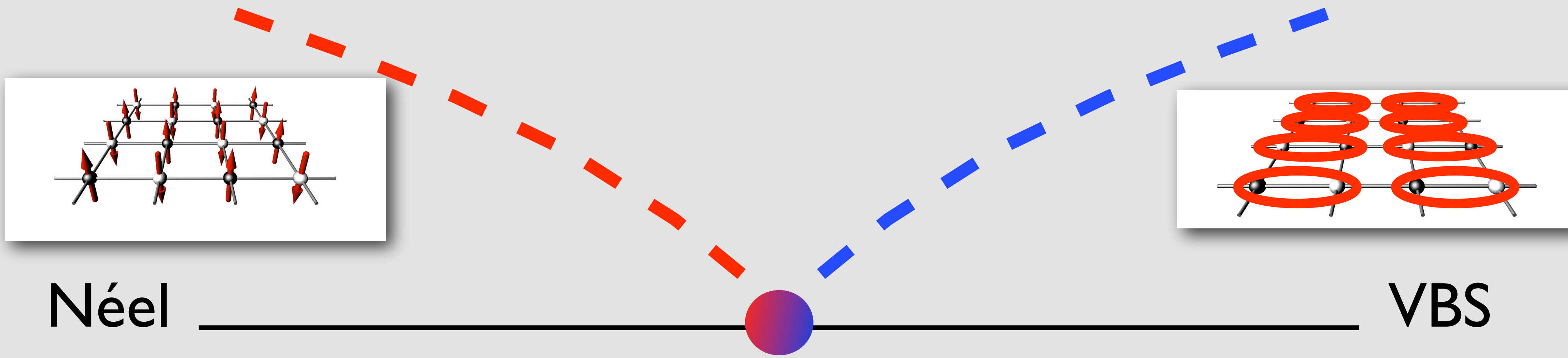


Spin Liquid!

Quantum superposition of many singlet coverings

Deconfined Critical Point

Senthil, Vishwanath, Balents, Sachdev, Fisher. Science (2004)



in an SU(N) anti-ferromagnet:

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2$$

Emergence of scalar-QED right at critical point!

How to study fixed-*N* transition?

A.W. Sandvik, PRL (2007)

$$H_{\mathrm{JQ}} = J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j - Q\sum_{\langle ijkl\rangle}(\mathbf{S}_i\cdot\mathbf{S}_j-\frac{1}{4})(\mathbf{S}_k\cdot\mathbf{S}_l-\frac{1}{4})$$

Lou, Kawashima, Sandvik, PRB (2009);

Kaul, Sandvik PRL (2012)

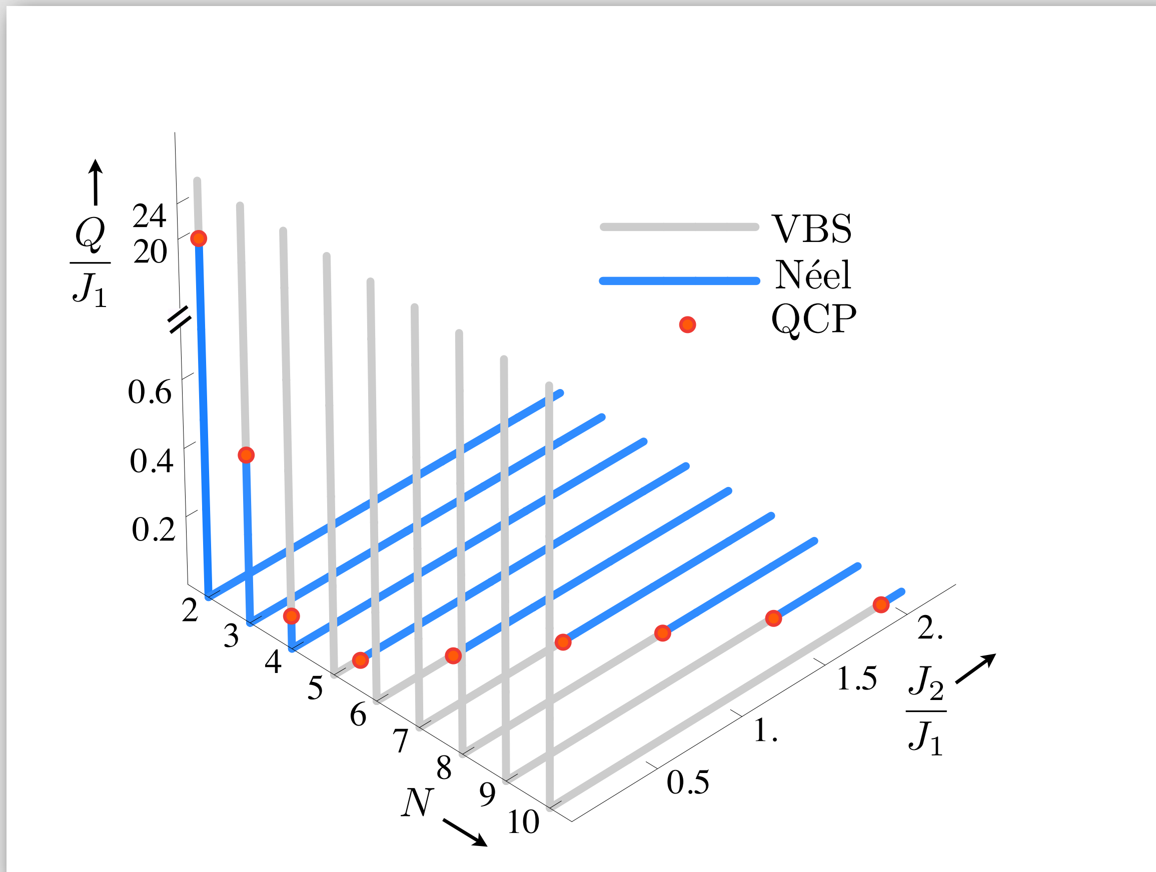
... we can now systematically derive an infinitely-large class of SU(*N*) Hamiltonians that are Marshall positive & hence sign-problem free.

R. K. Kaul, arxiv:1403.5678(2014)

“Designer” Hamiltonians

R. K. Kaul, R.G. Melko, A.W. Sandvik.

Ann. Rev. Cond. Mat. Phys. (2013)



QMC SSE method: Sandvik, AIP (2010)

Universal scaling dimensions

$$C_{N,V}(\mathbf{r},\tau) \sim \frac{1}{(\mathbf{r}^2 + c^2\tau^2)^{(1+\eta_{V,N})/2}}.$$

$1/N$ expansion of CP^{N-1} field theory

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2$$

$$\eta_N = 1 - 32/(\pi^2 N), \qquad 1 + \eta_V = 2\delta_1 N,$$

$$\vec{n} = z_s^* \frac{\vec{\sigma}_{ss'}}{2} z_{s'} \qquad\qquad \delta_1 \approx 0.1246$$

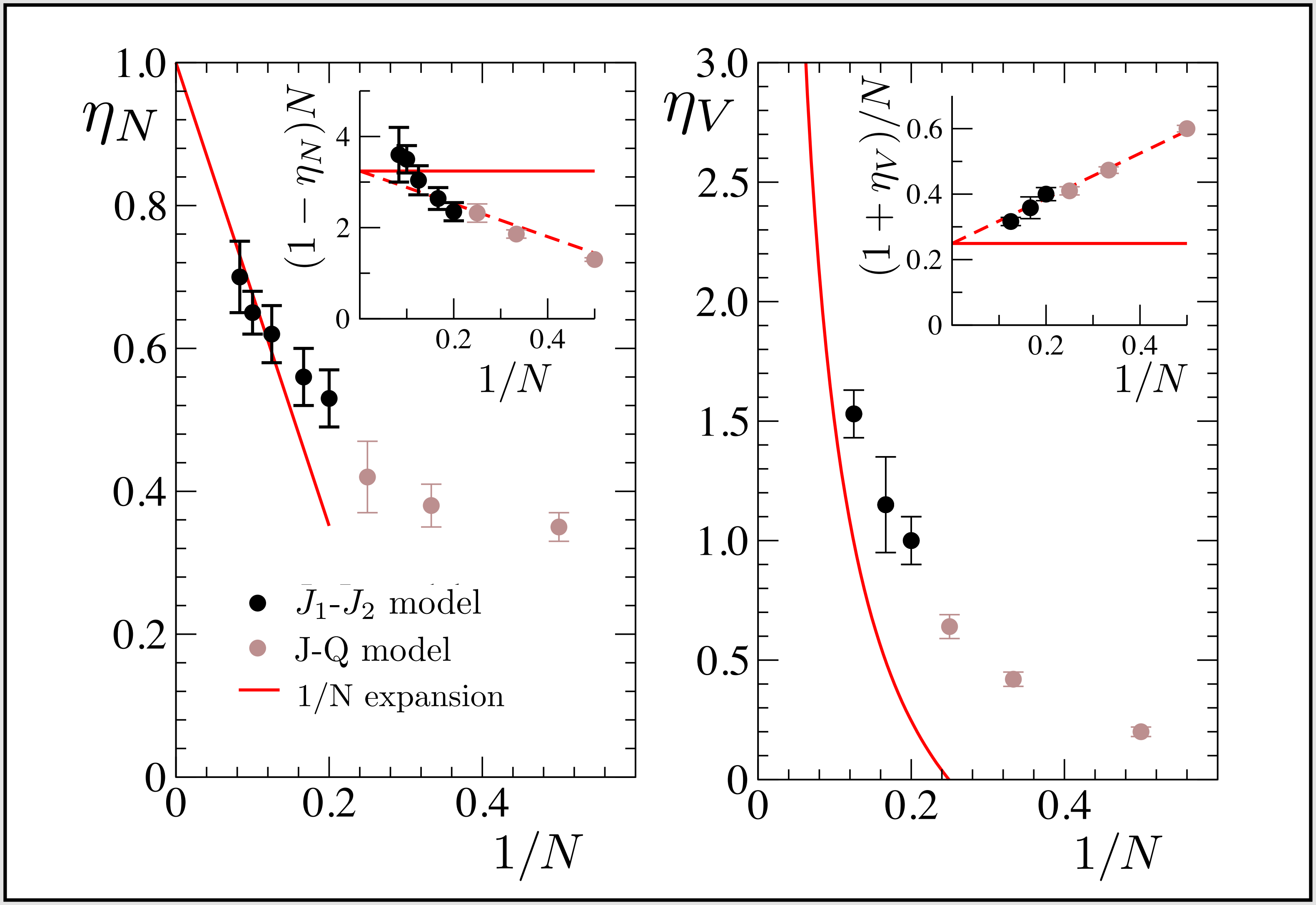
Halperin, Lubensky, Ma PRL (1974);
Kaul, Sachdev (2008);

Murthy, Sachdev (1990);
Borokhov, Kapustin, Wu (2002)
Metlitski, Hermele, Senthil, Fisher (2008).

Néel-VBS in SU(N) magnets

R.K. Kaul & A. W. Sandvik, PRL (2012)

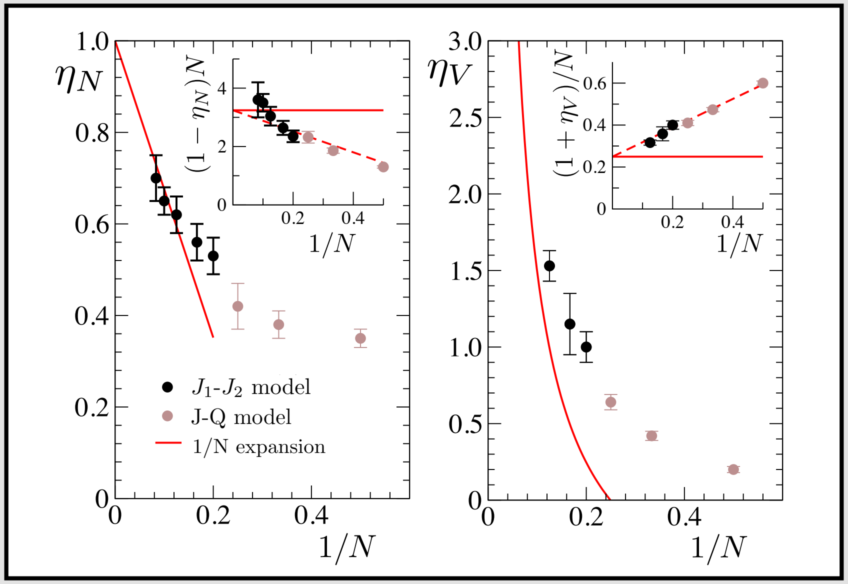
Comparison of exponents with CP^{N-1} *theory.*



Comparison with DQC

R.K. Kaul & A. W. Sandvik, PRL (2012)

$$\eta_N = 1 - \frac{32}{\pi^2 N}$$



$$1 + \eta_V = 2\delta_1 N$$

✓ exponents trends for both $J{-}Q$ & $J_1{-}J_2$ are consistent

✓ η_N extrapolates to 1 in $N \to \infty$ limit.

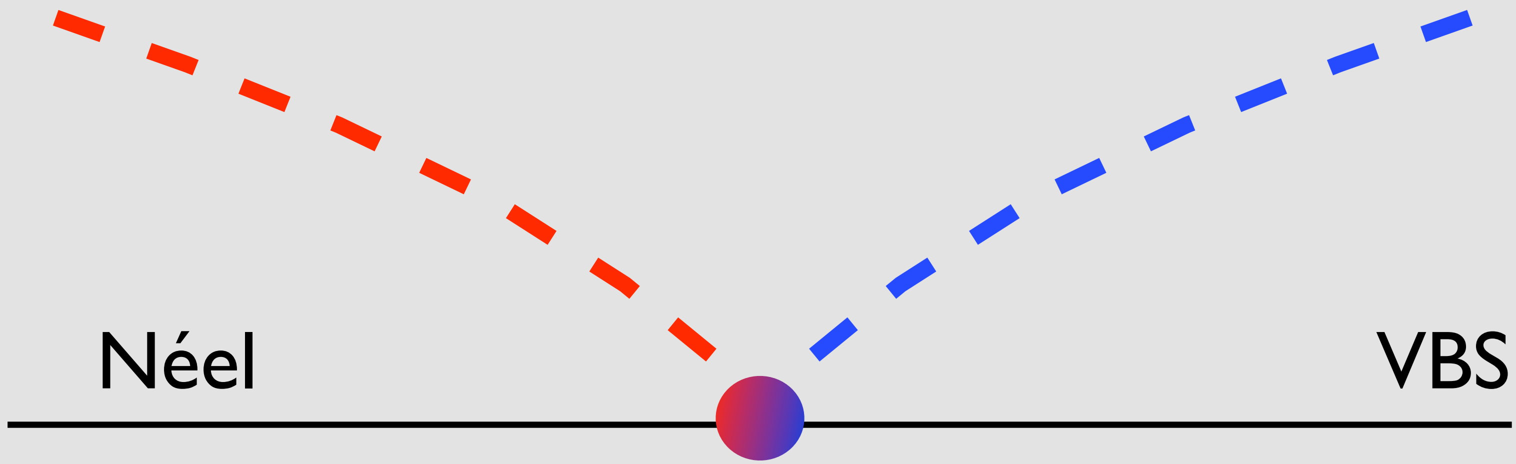
✓ $\frac{1}{N}$ correction for η_N is within few % of large-*N* result.

✓ Leading piece for η_V is within few % of large-*N* result.

? Predictions for next $\frac{1}{N}$ corrections to η_N & η_V

Deconfined Critical Point

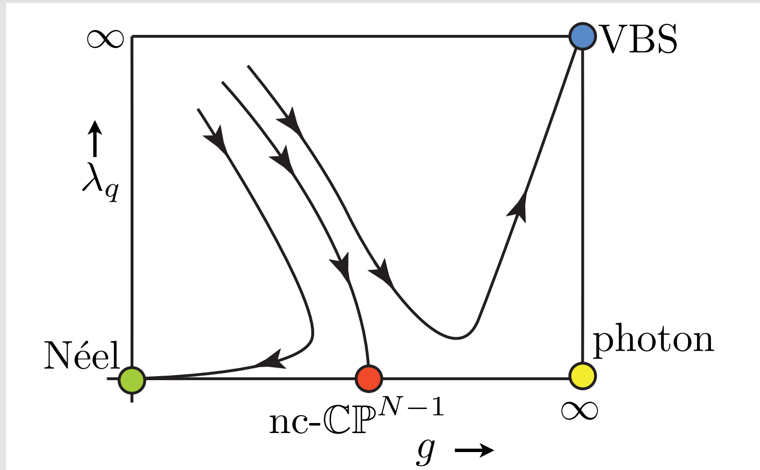
Microscopic sign-problem free models



Strong evidence for a new kind of phase transition:

- Direct continuous transition
- Both order parameters simultaneously critical
- Universality class for $\mathrm{SU}(N) : \mathrm{CP}^{N-1}$ field theory

RG flow q-N



$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + q\text{-fold monopoles}$$

Two central assumptions for deconfined criticality:
(1) existence of fixed point $\mathbb{CP}^{N-1}$.
(2) irrelevance of q-monopoles

N-q phase diagram

M. Block, R.G. Melko & R.K. Kaul, PRL (2013)

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + q\text{-fold monopoles}$$

$N=\infty$							nc-$\mathbb{CP}^{N-1}$
$\dots$							
$N=5$							nc-$\mathbb{CP}^4$
$N=4$							nc-$\mathbb{CP}^3$
$N=3$							nc-$\mathbb{CP}^2$
$N=2$							nc-$\mathbb{CP}^1$
$N=1$							XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\dots$	$q=\infty$	

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$N=\infty$	I	I	I	I	I	I	nc-$\mathbb{CP}^{N-1}$
$\ldots$							
$N=5$						I	nc-$\mathbb{CP}^4$
$N=4$						I	nc-$\mathbb{CP}^3$
$N=3$						I	nc-$\mathbb{CP}^2$
$N=2$						I	nc-$\mathbb{CP}^1$
$N=1$	R	R	R	I		I	XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\ldots$	$q=\infty$	

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$N=\infty$	I	I	I	I	I	I	nc-$\mathbb{CP}^{N-1}$
$\ldots$							
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$N=4$						I	nc-$\mathbb{CP}^{3}$
$N=3$						I	nc-$\mathbb{CP}^{2}$
$N=2$						I	nc-$\mathbb{CP}^{1}$
$N=1$	R	R	R	R		I	XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\ldots$	$q=\infty$	

Halperin/Lubensky/Ma

Kamal/Murthy; Motrunich/ Vishwanath; Kuklov/ Prokofiev/Svistunov/ Troyer

Dasgupta/Halperin

No Monopoles

N-q phase diagram

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + q\text{-fold monopoles}$$

$N=\infty$	I	I	I	I	I	I	nc-$\mathbb{CP}^{N-1}$
$\ldots$							
$N=5$						I	nc-$\mathbb{CP}^4$
$N=4$						I	nc-$\mathbb{CP}^3$
$N=3$						I	nc-$\mathbb{CP}^2$
$N=2$						I	nc-$\mathbb{CP}^1$
$N=1$	R	R	R	I		I	XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\ldots$	$q=\infty$	

N-q phase diagram

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + q\text{-fold monopoles}$$

$N=\infty$	I	I	I	I	I	I	nc-$\mathbb{CP}^{N-1}$
$\dots$							
$N=5$							
$N=4$						I	nc-$\mathbb{CP}^3$
$N=3$						I	nc-$\mathbb{CP}^2$
$N=2$						I	nc-$\mathbb{CP}^1$
$N=1$	R	R	R	I		I	XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\dots$	$q=\infty$	

XY model with n-fold field;
Fukugita/Okawa (1989);
Carmona/Pelissato/Vicari(2000)

Dasgupta/Halperin

N-q phase diagram

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + q\text{-fold monopoles}$$

$N=\infty$	I	I	I	I	I	I	nc-$\mathbb{CP}^{N-1}$
$\ldots$							
$N=5$						I	nc-$\mathbb{CP}^4$
$N=4$						I	nc-$\mathbb{CP}^3$
$N=3$						I	nc-$\mathbb{CP}^2$
$N=2$						I	nc-$\mathbb{CP}^1$
$N=1$	R	R	R	I		I	XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\ldots$	$q=\infty$	

Murthy/Sachdev

N-q phase diagram

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + q\text{-fold monopoles}$$

$N=\infty$	I	I	I	I	I	I	nc-$\mathbb{CP}^{N-1}$
$\dots$							
$N=5$				I		I	nc-$\mathbb{CP}^4$
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$N=2$				I		I	nc-$\mathbb{CP}^1$
$N=1$	R	R	R	I		I	XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\dots$	$q=\infty$	

Square Lattice
Néel-VBS

N-q phase diagram

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + q\text{-fold monopoles}$$

$N=\infty$	I	I	I	I	I	I	nc-$\mathbb{CP}^{N-1}$
$\dots$							
$N=5$				I		I	nc-$\mathbb{CP}^4$
$N=4$				I		I	nc-$\mathbb{CP}^3$
$N=3$				I		I	nc-$\mathbb{CP}^2$
$N=2$				I		I	nc-$\mathbb{CP}^1$
$N=1$	R	R	R	I		I	XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\dots$	$q=\infty$	

q=1,2,3 is different from *4*!

N-q phase diagram

$$\mathcal{L}_{\mathbb{CP}^{N-1}} = \sum_{\alpha=1}^{N} |(\partial_\mu - i a_\mu) z_\alpha|^2 + q\text{-fold monopoles}$$

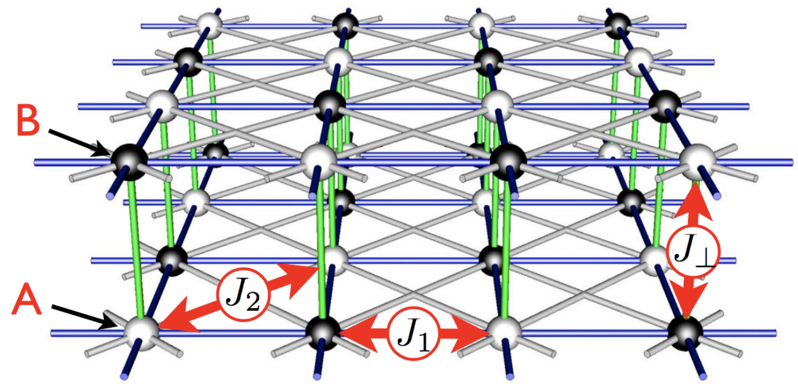
$N=\infty$	I	I	I	I	I	I	nc-$\mathbb{CP}^{N-1}$
$\ldots$							
$N=5$				I		I	nc-$\mathbb{CP}^4$
$N=4$		???		I		I	nc-$\mathbb{CP}^3$
$N=3$				I		I	nc-$\mathbb{CP}^2$
$N=2$				I		I	nc-$\mathbb{CP}^1$
$N=1$	R	R	R	I		I	XY
	$q=1$	$q=2$	$q=3$	$q=4$	$\ldots$	$q=\infty$	

q=1,2,3 is different from *4*!

q=1,2,3

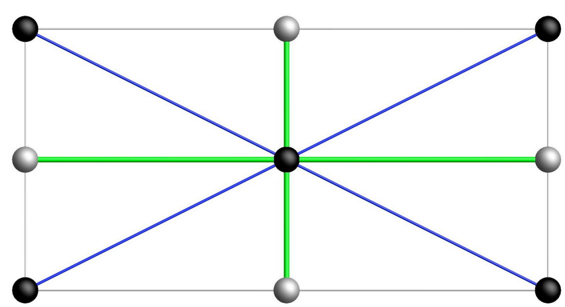
Monopole charge-q is same as min. VBS degeneracy!

Haldane (1989); Read/Sachdev (1990)



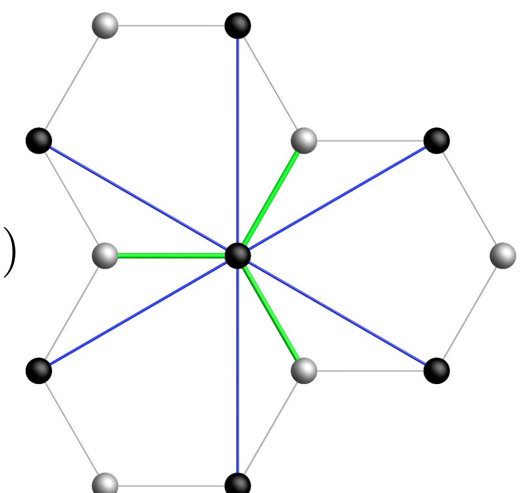
q=1

(b)



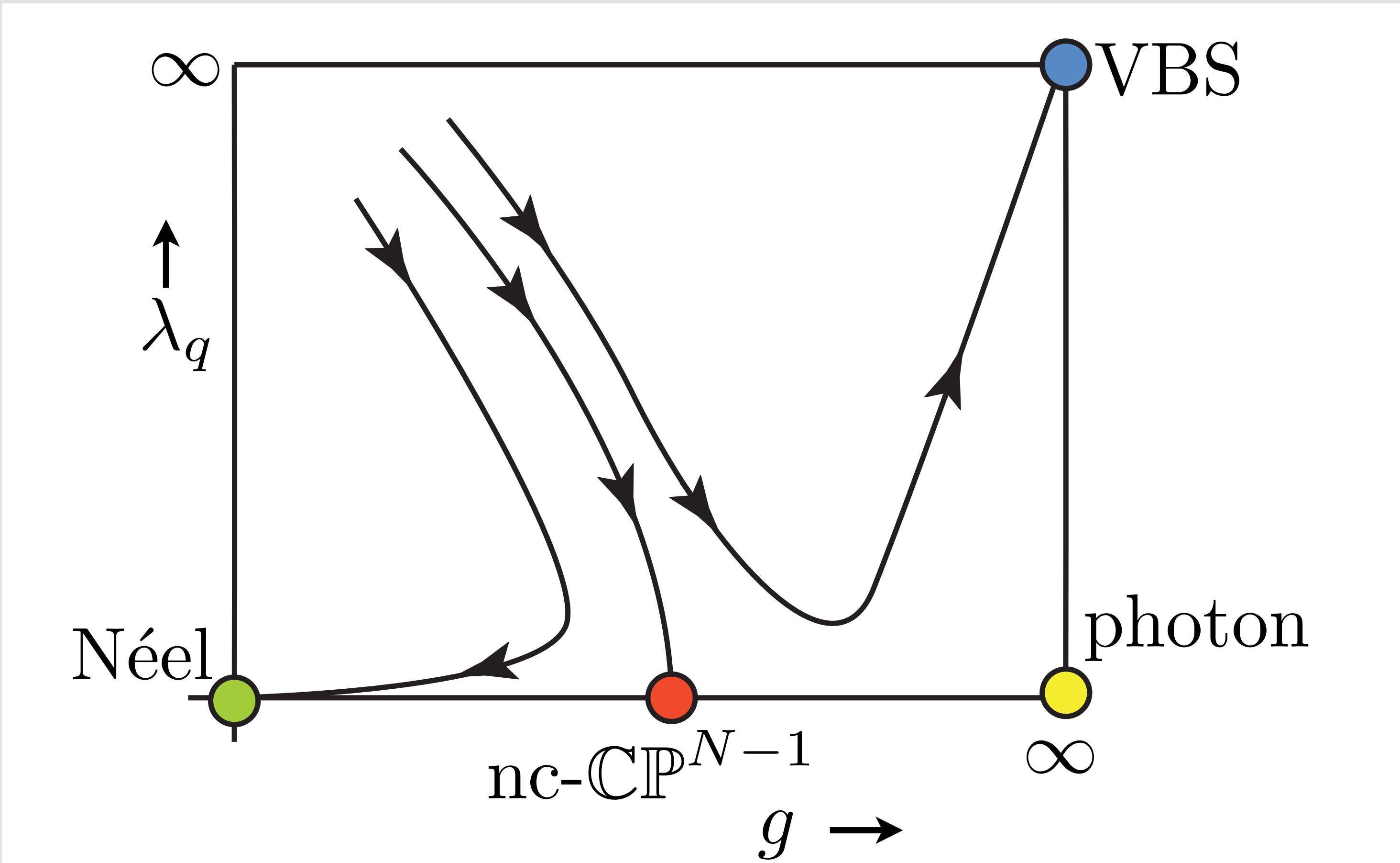
q=2

(c)



q=3

RG flow *q-N*



When is λ_q irrelevant?

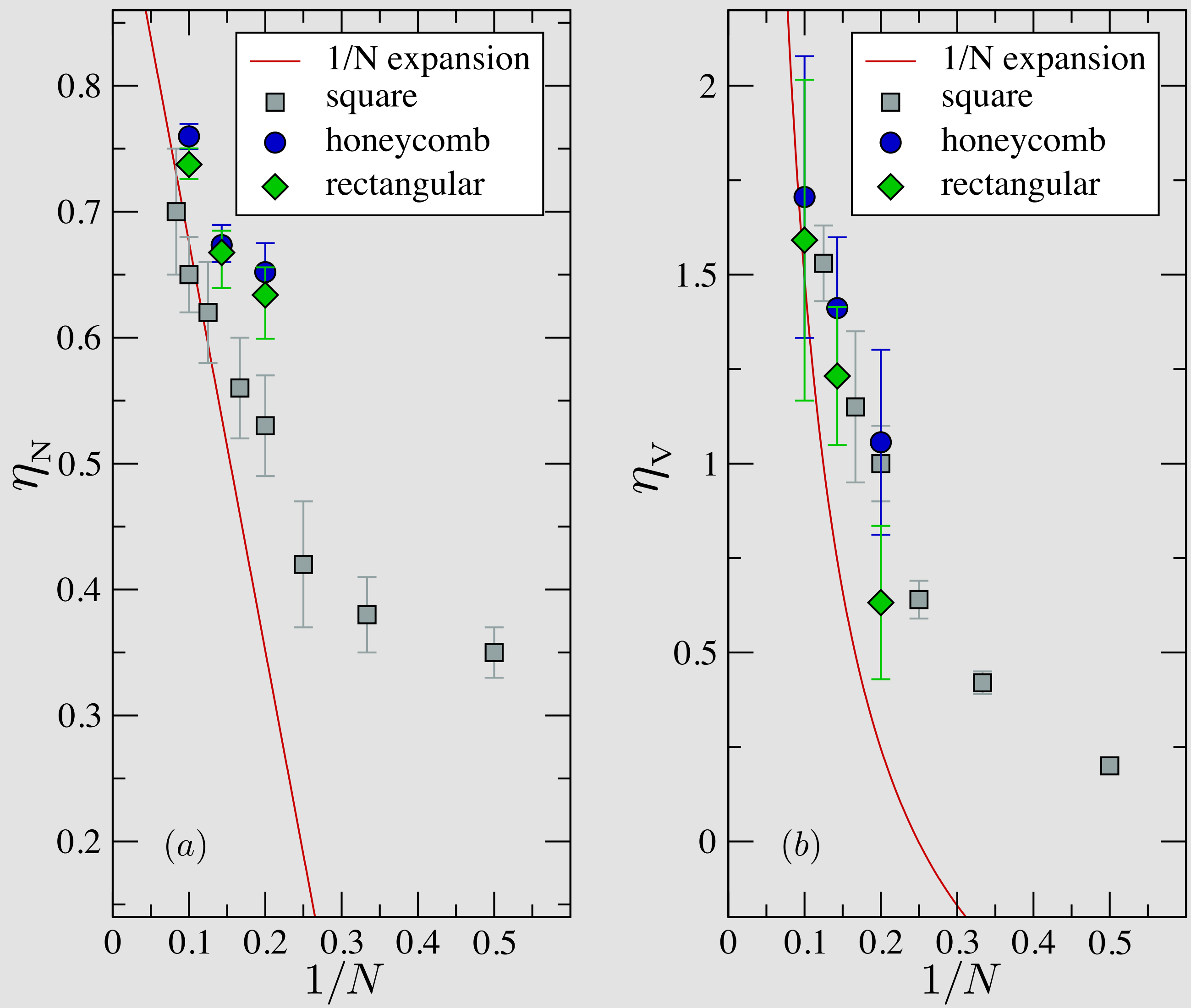
Complete *q-N* phase diagram

M. Block, R.G. Melko & R.K. Kaul, PRL (2013)

$N=\infty, 1/N$	I	I	I	I	$\ldots$	I	nc-$\mathbb{CP}^{N-1}$
$\ldots$							
$N=10$	R	I	I	I		I	nc-$\mathbb{CP}^9$
$N=9$	R	I	I	I		I	nc-$\mathbb{CP}^8$
$N=8$	R	I	I	I		I	nc-$\mathbb{CP}^7$
$N=7$	R	I	I	I		I	nc-$\mathbb{CP}^6$
$N=6$	R	I	I	I		I	nc-$\mathbb{CP}^5$
$N=5$	R	I	I	I		I	nc-$\mathbb{CP}^4$
$N=4$	R	I	I	I		I	nc-$\mathbb{CP}^3$
$N=3$	R	R	I	I		I	nc-$\mathbb{CP}^2$
$N=2$	R	R	I	I		I	nc-$\mathbb{CP}^1$
$N=1$	R	R	R	I		I	XY
$N=0$	R	R	R	R		R	photon
	$q=1$	$q=2$	$q=3$	$q=4$	$\ldots$	$q=\infty$	

Critical Universality?

M. Block, R.G. Melko & R.K. Kaul, PRL (2013)



$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$

BIPARTITE LATTICES

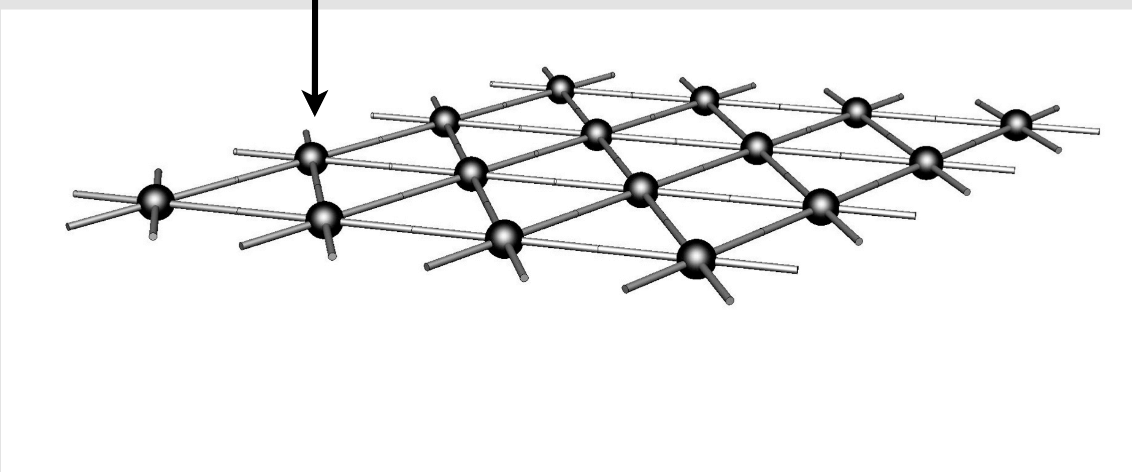
NON-BIPARTITE LATTICES

Non-Bipartite Symmetry

R. K. Kaul (to appear)

$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$

$$1 \le \alpha \le N$$



$$|\mathcal{S}\rangle_{ij} = \sum_{\alpha} |\alpha\alpha\rangle_{ij}$$

O(N) singlet

$$|\mathcal{S}\rangle \to |\mathcal{S}\rangle$$

Special cases on Tri. lattice

$$N = 2 \qquad H_{XXZ} = -\sum_{\langle ij \rangle} \left(\frac{S_i^+ S_j^- + S_i^- S_j^+}{2} - S_i^z S_j^z \right)$$

O(2) broken

Murthy/Arovas/Auerbach, PRB (1997); Melko/Paramekanti/Burkov/Vishwanath/Sheng/Balents ; Heidarian/Damle; Wessel/Troyer. PRL (2005)

$$N = 3 \qquad H_{\mathrm{biq}} = -\sum_{\langle ij \rangle} (S_i \cdot S_j)^2$$

O(3) broken

Tsunetsugu/Arikawa (2005);Bhattacharjee/Shenoy/Senthil (2006). Laeuchli/Mila/Penc PRL (2006); Kaul (2012)

$\vdots$

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Tsunetsugu/Arikawa (2005);Bhattacharjee/Shenoy/Senthil (2006). Laeuchli/Mila/ Penc PRL (2006); Kaul (2012)

$\vdots$

$N = \infty$ $$H_{QDM} = -t \left(|\,=\,\rangle\langle\,\|\;\|\,| + h.c. \right)$$ $\sqrt{12} \times \sqrt{12}$ VBS ordered

Read/Sachdev NPB (1986); Moessner/Sondhi, PRB (2001)

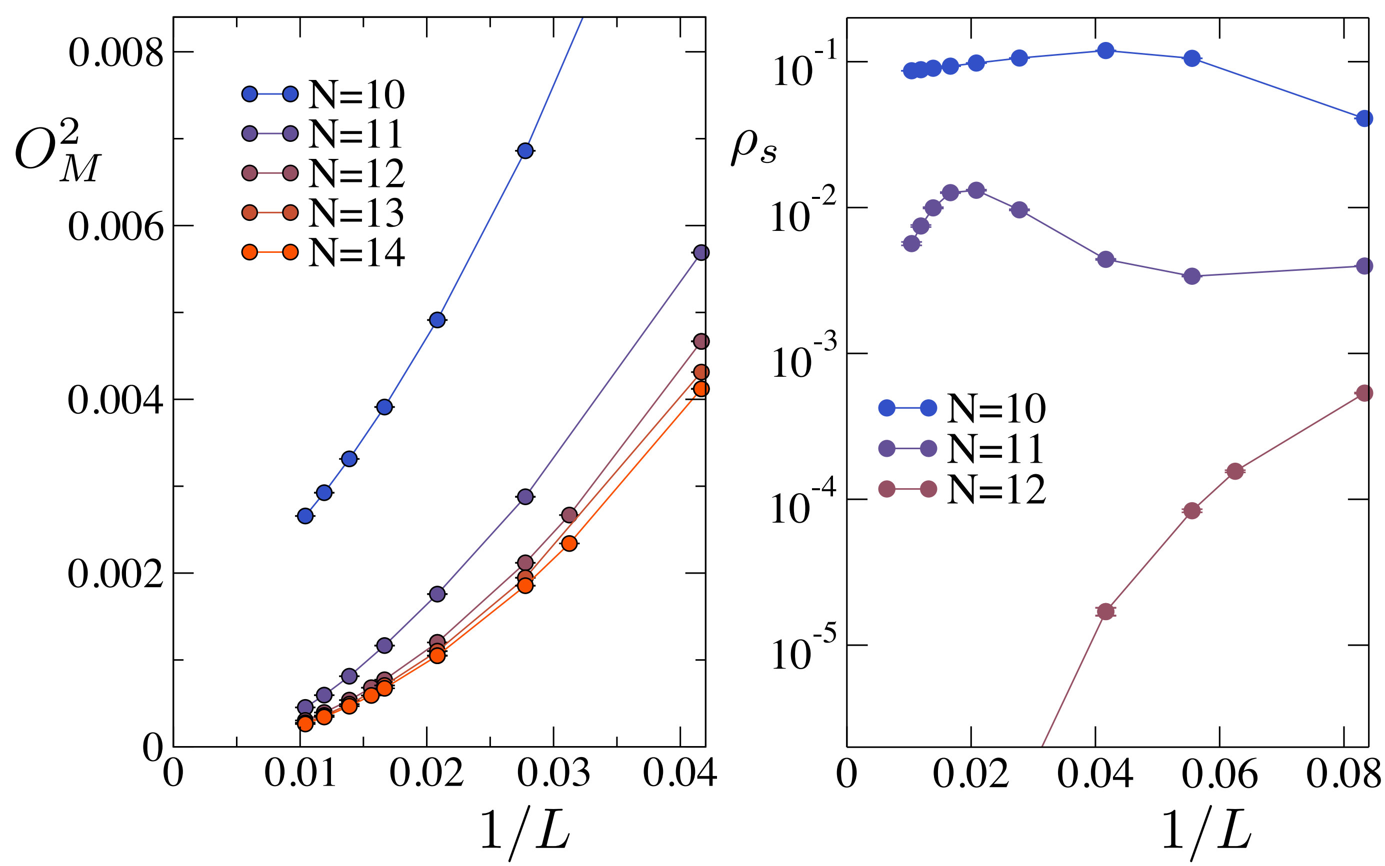
Anderson RVB?

At small-N: Magnetic
At large-N: VBS

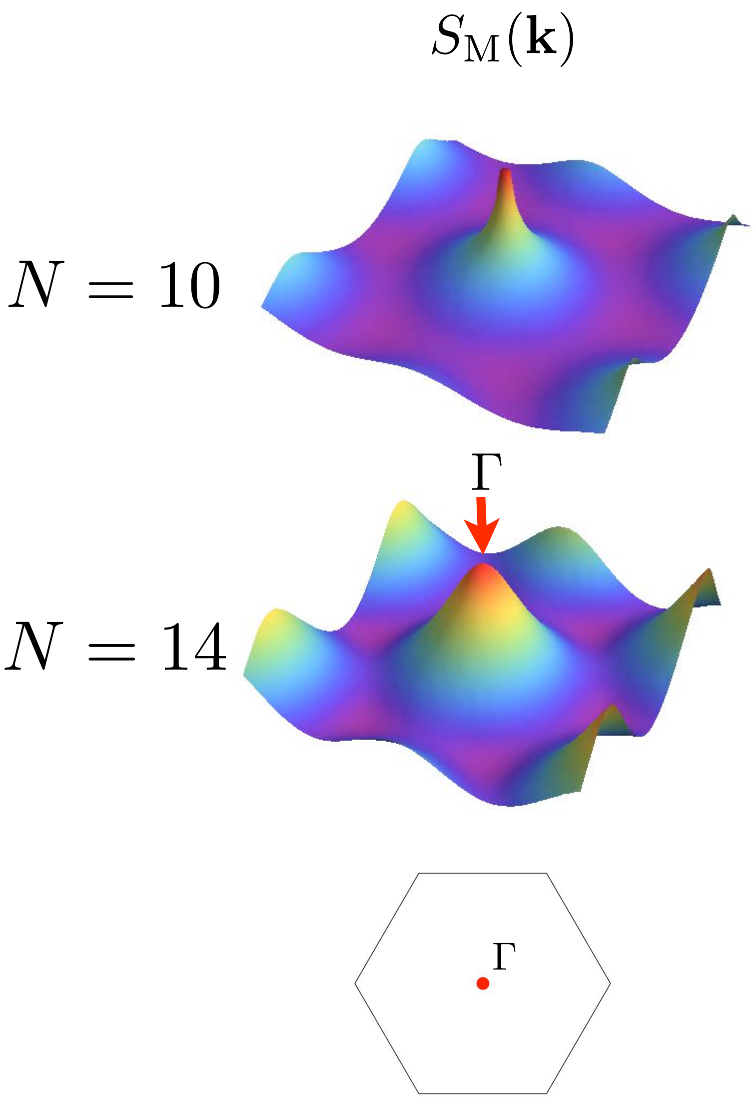
Theory of deconfined criticality does not apply on non-bipartite lattices.

Is there an Anderson RVB in-between?

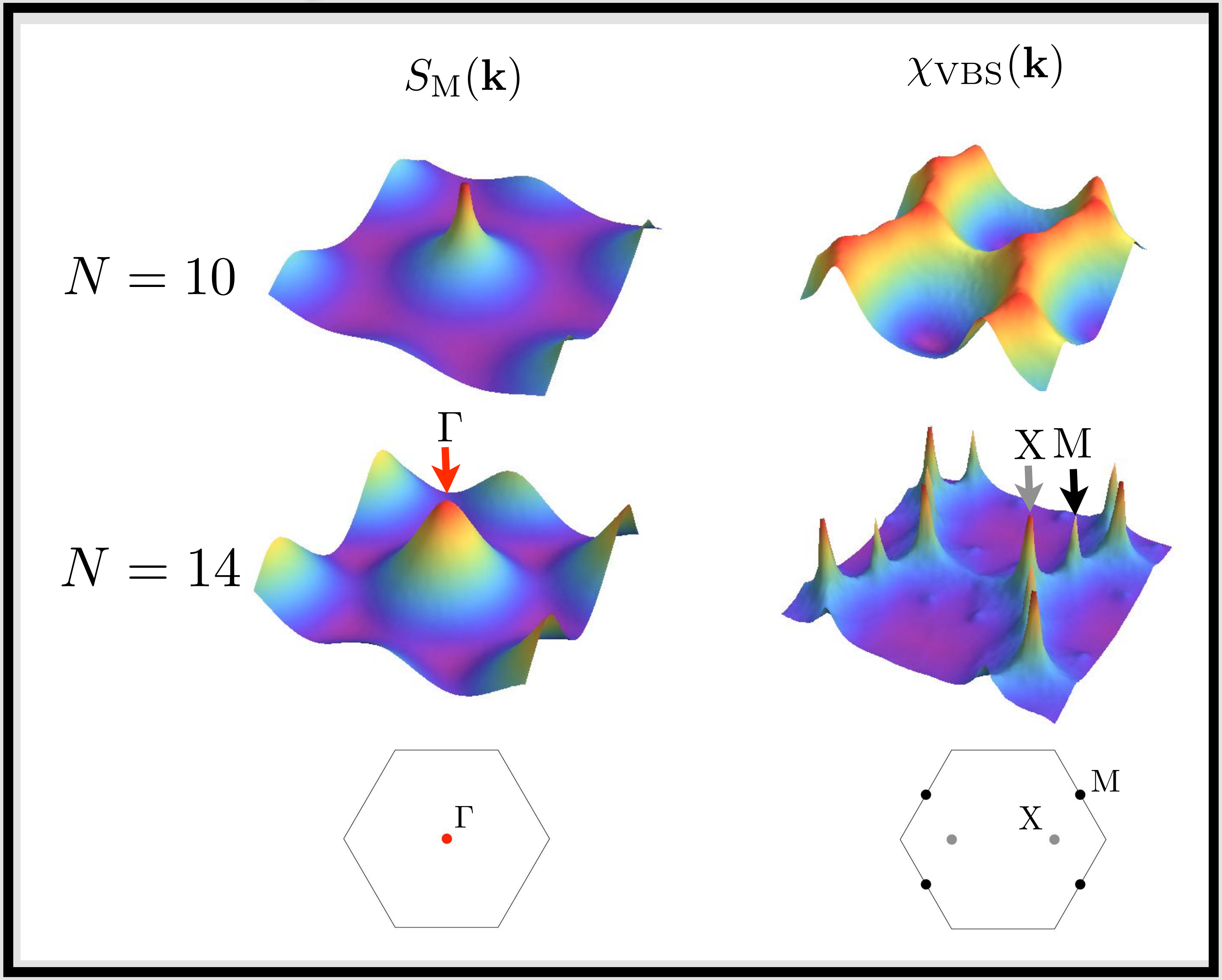
Magnetic Order Parameters



$\mathbf{k}$-space Correlations

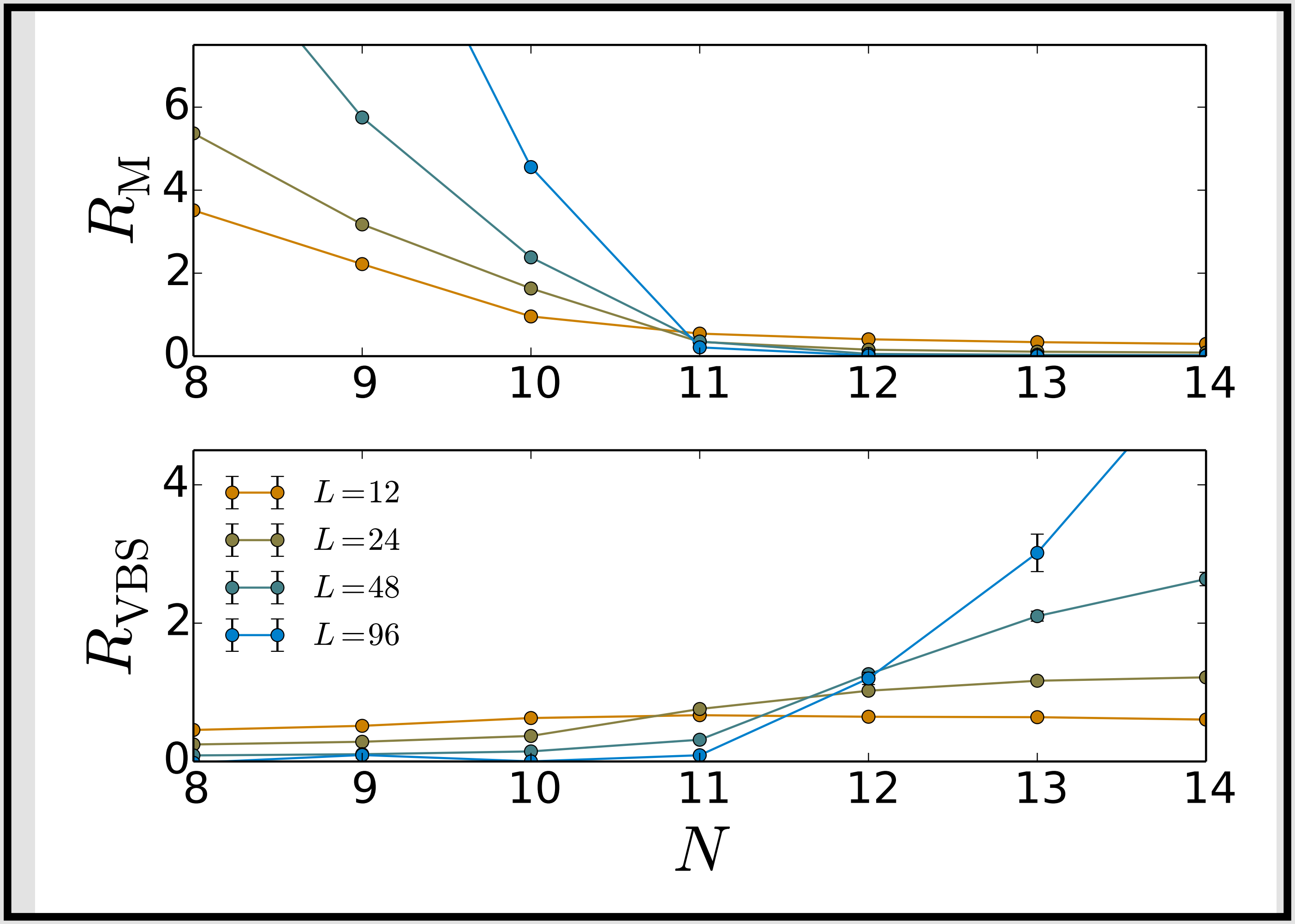


k-space Correlations

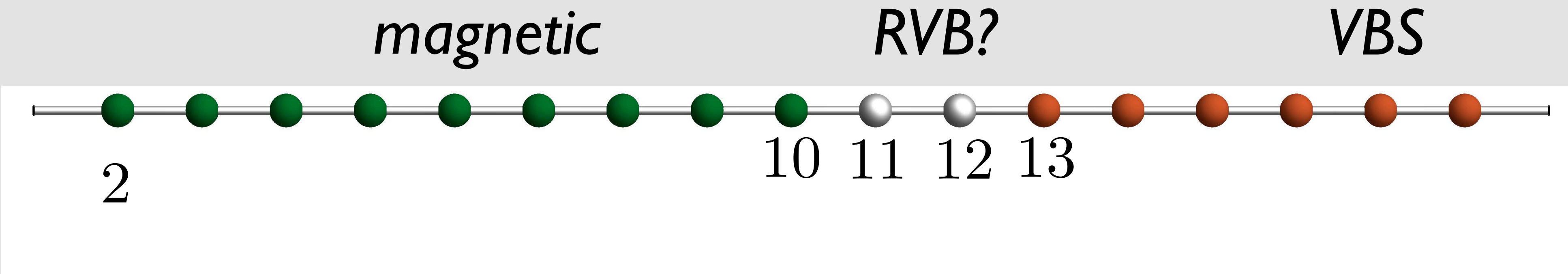


cf. quantum dimer model: Moessner/Sondhi (2001). Ralko/Ferrero/Becca/Ivanov/Mila (2005)

M and VBS ratios

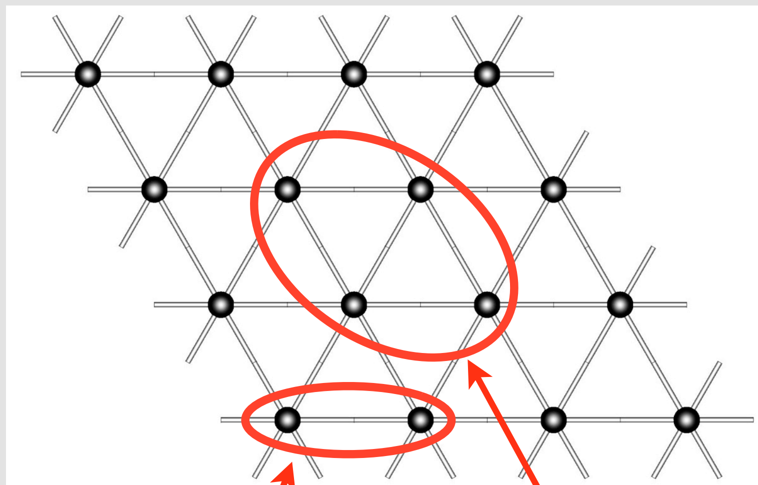


Phase Diagram



J-Q model

A.W. Sandvik, PRL (2007)

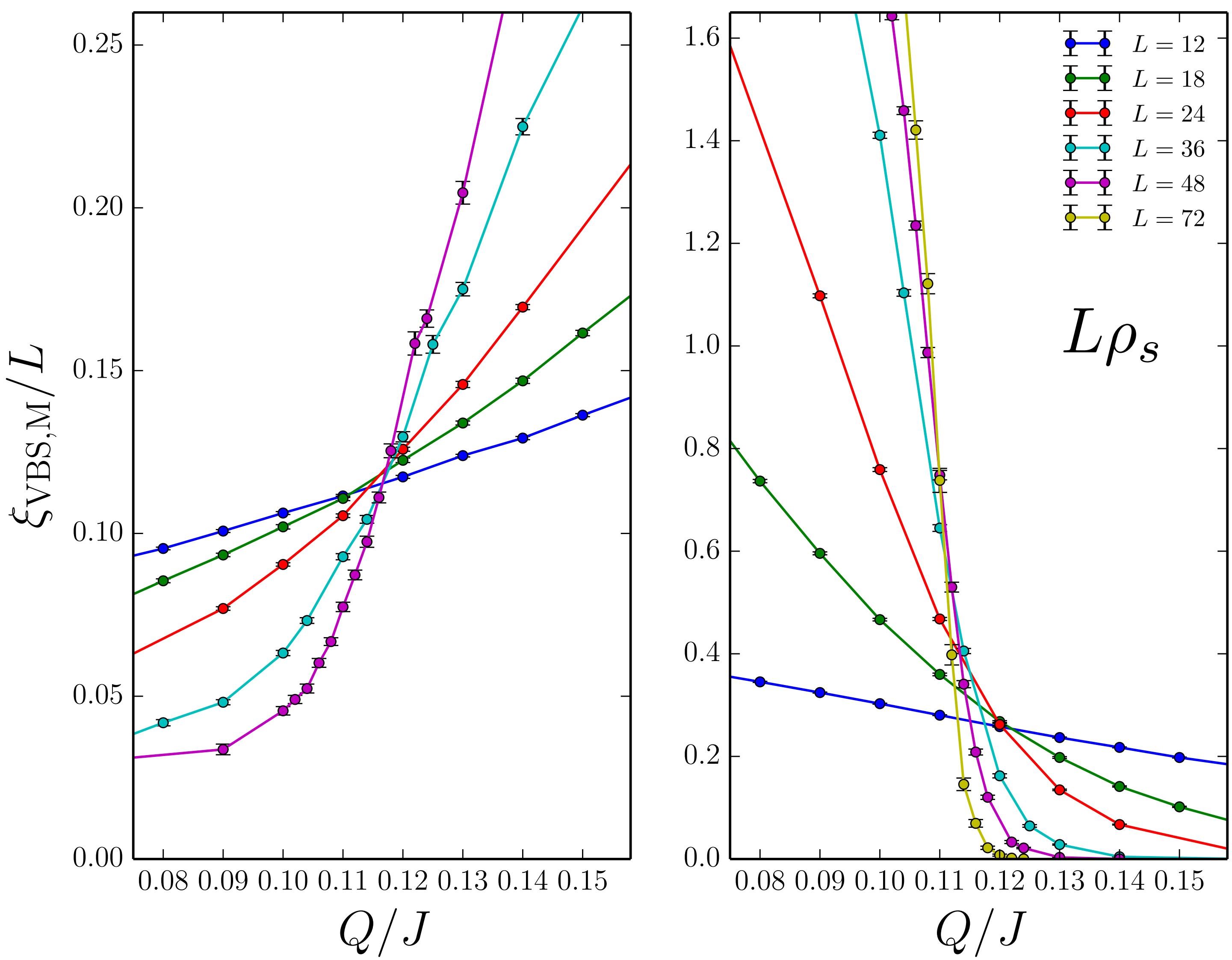


$$H_{J-Q} = -J\sum_{\langle ij\rangle} P_{ij} - Q\sum_{\langle ijkl\rangle} P_{ij}P_{kl}$$

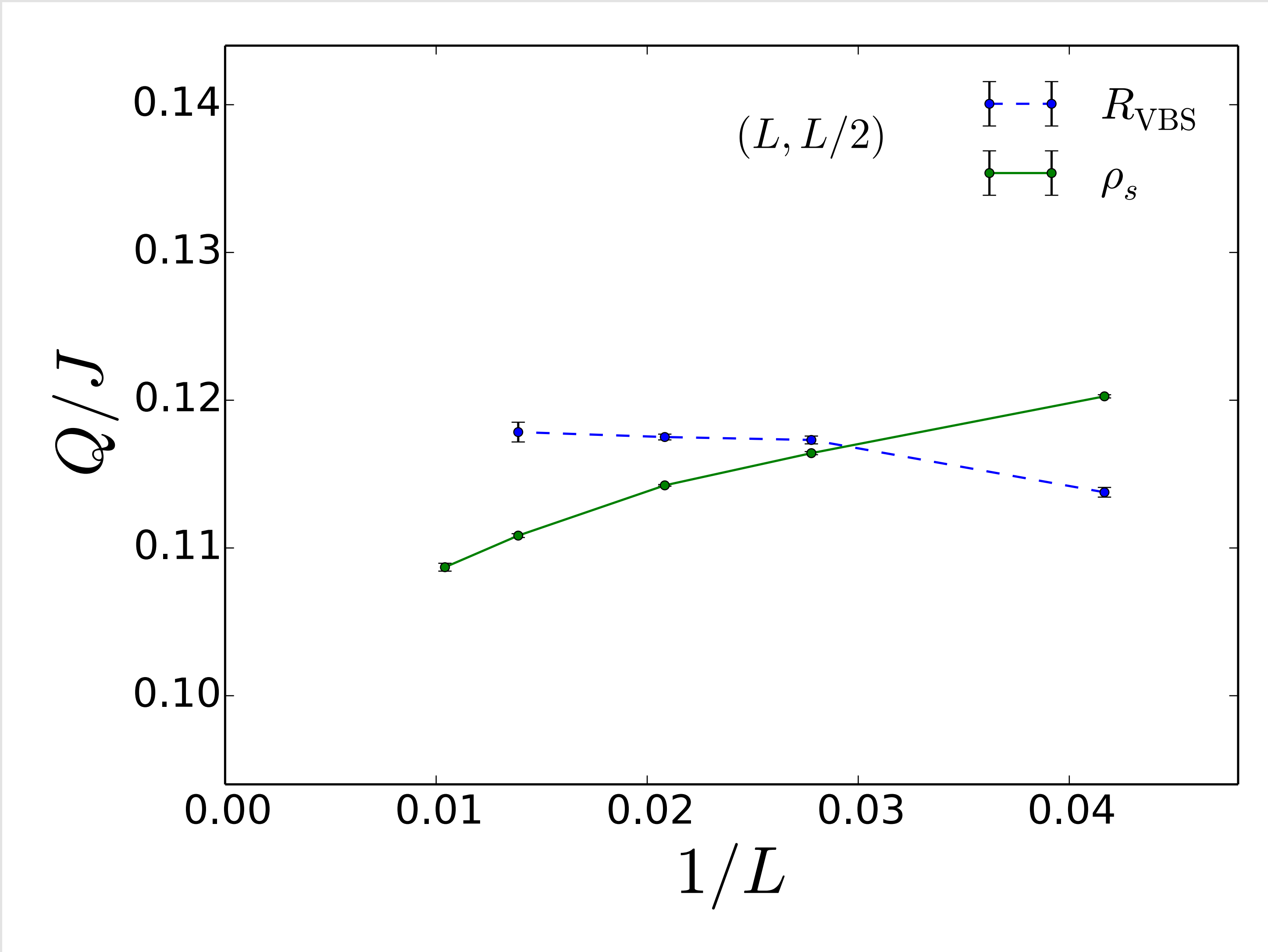
Sign-free way to tune from magnetic to VBS at fixed N!

J-*Q* Triangular Lattice

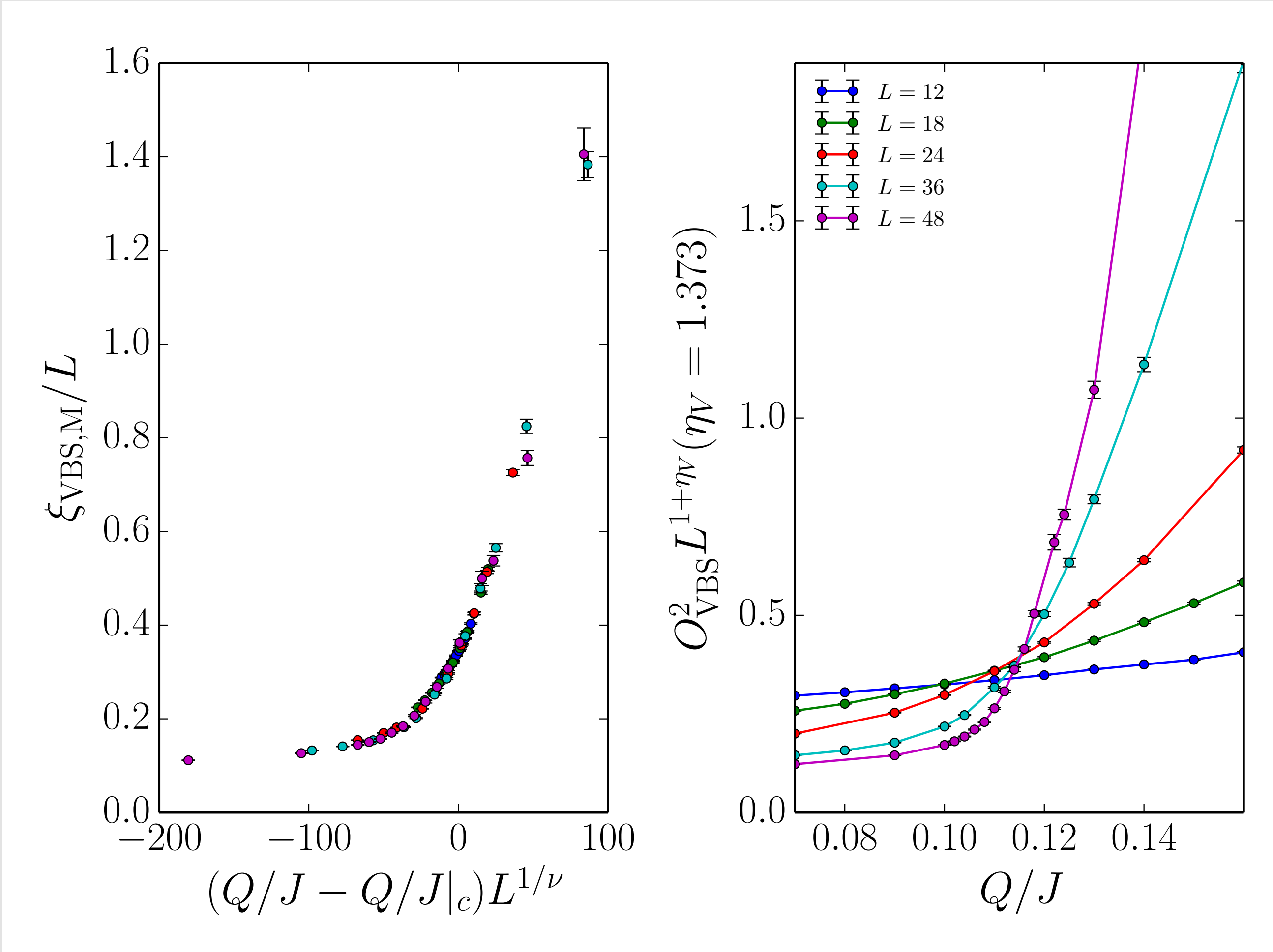
N=10



Two critical points!

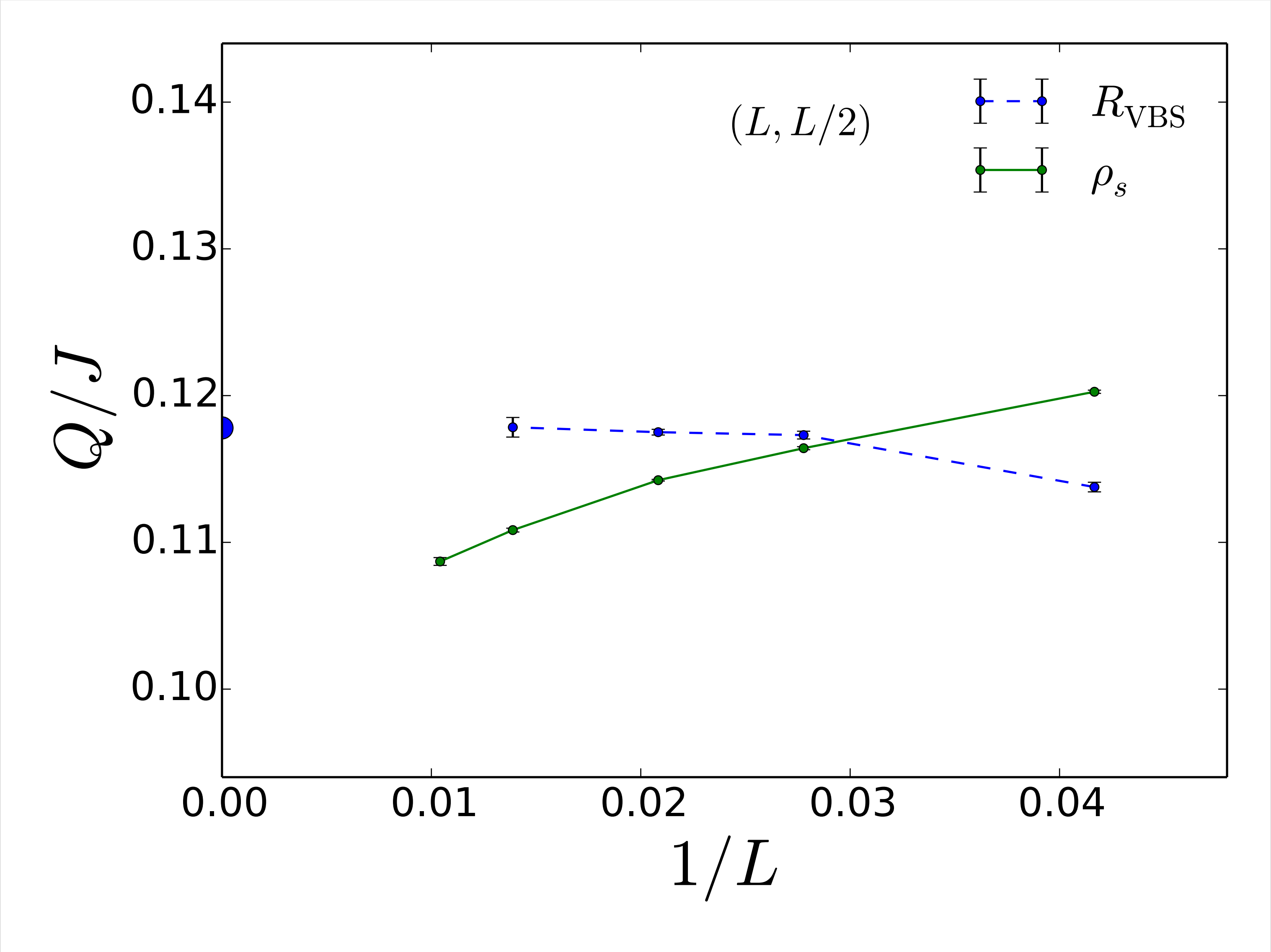


VBS exponents



$\nu = 0.47, Q/J|_c = 0.1178$

Two critical points!



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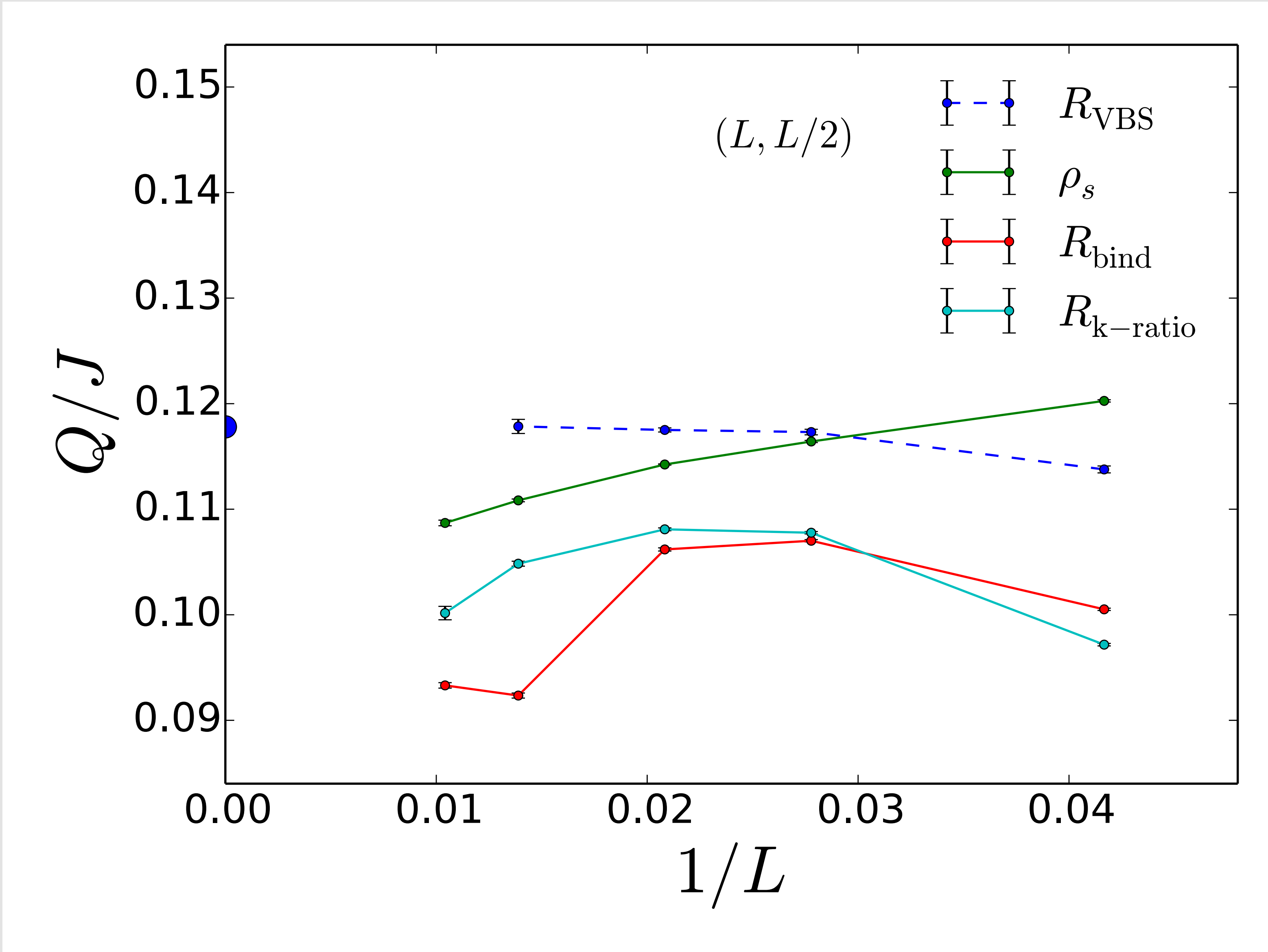

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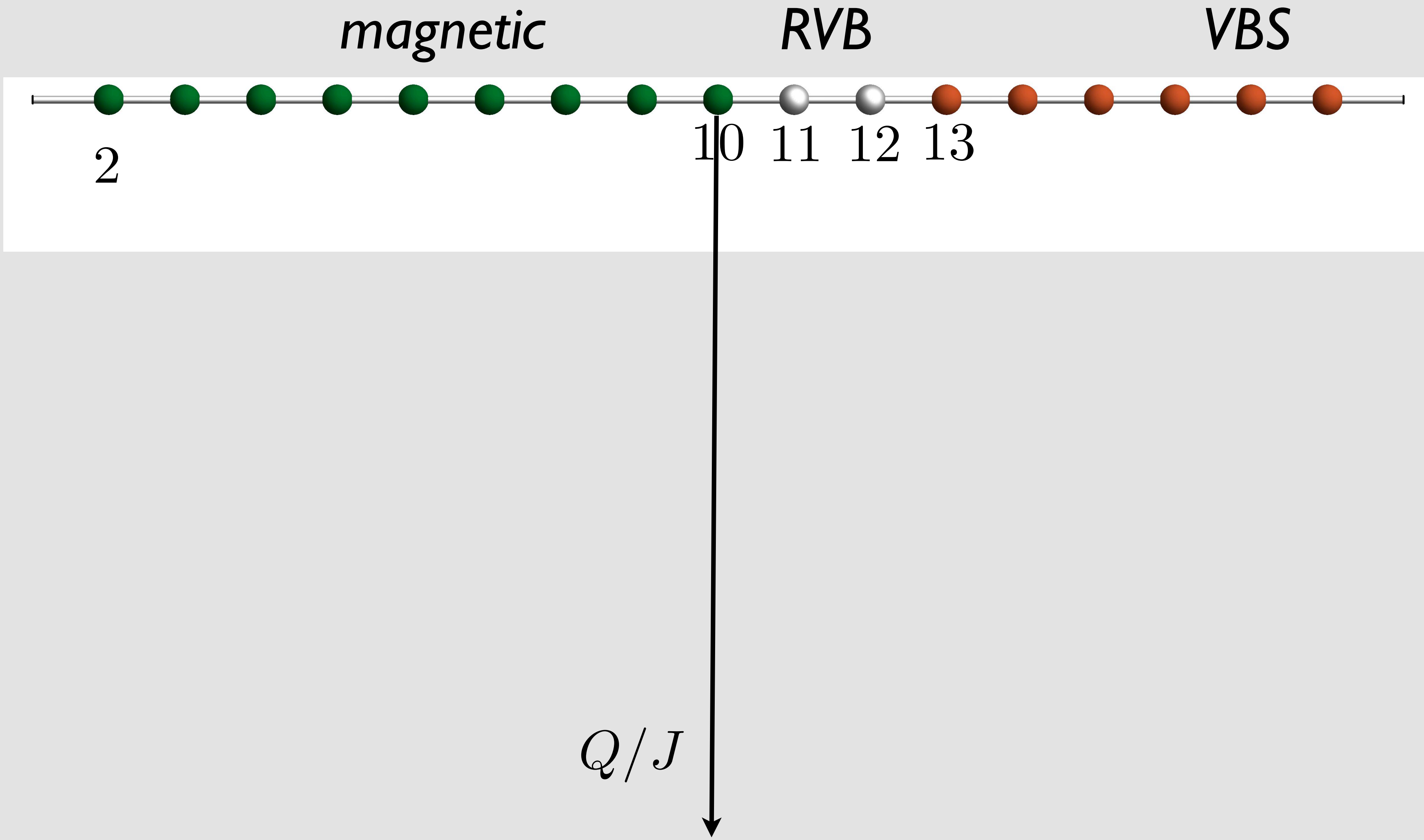
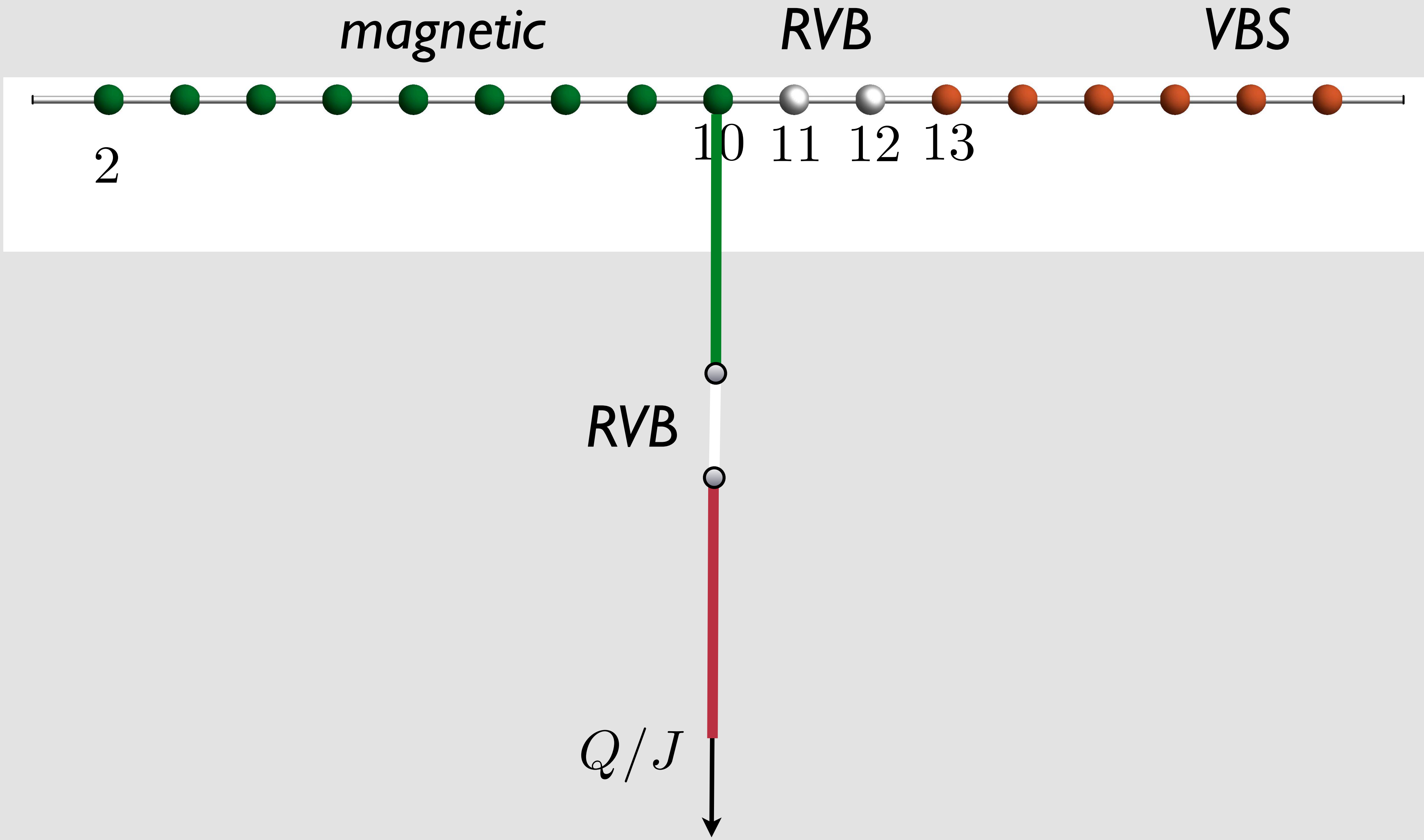


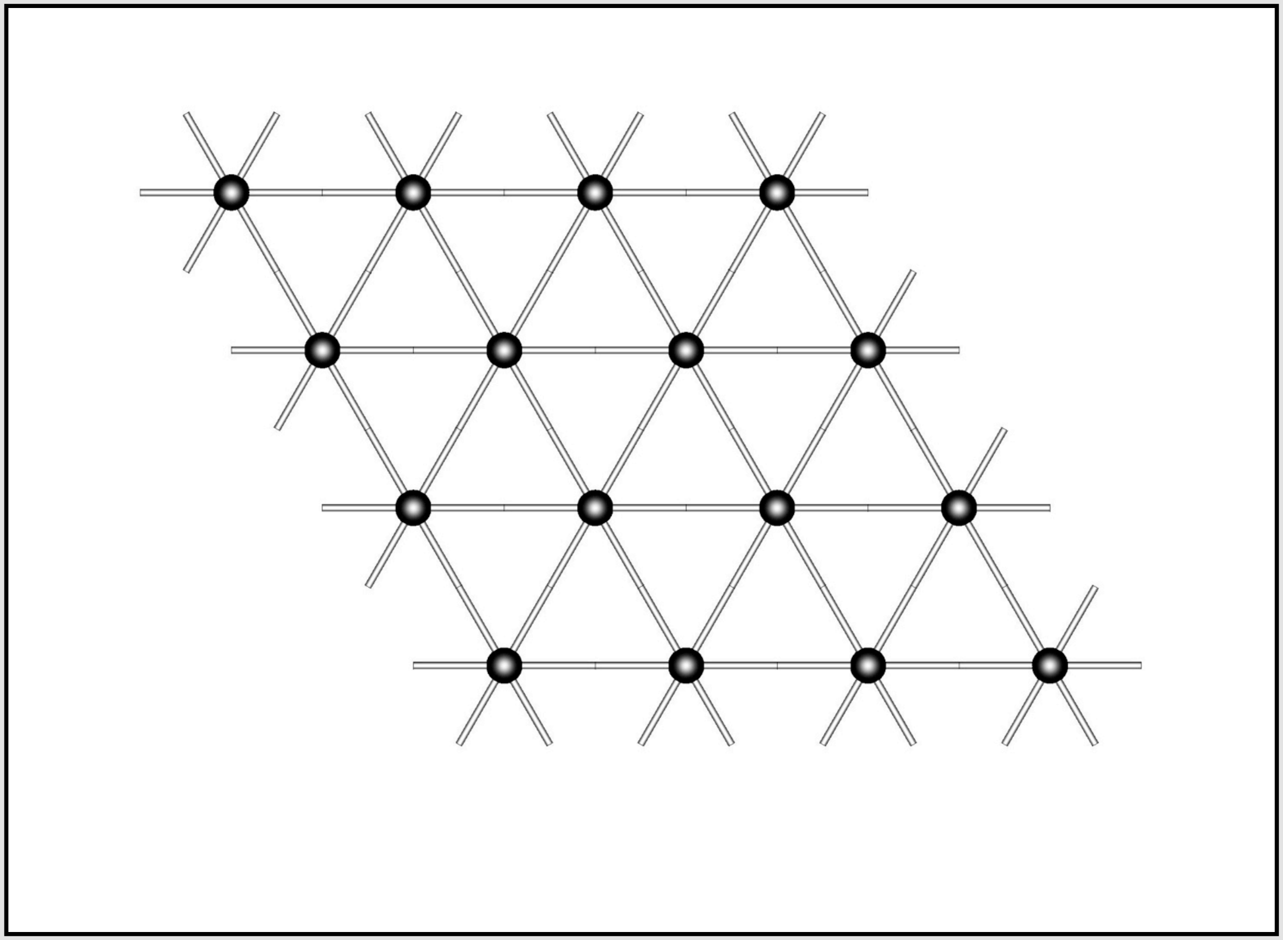
Phase Diagram



Outlook

Model

$$H_J = -J \sum_{\langle ij \rangle} |\mathcal{S}\rangle_{ij} \langle \mathcal{S}|_{ij}$$



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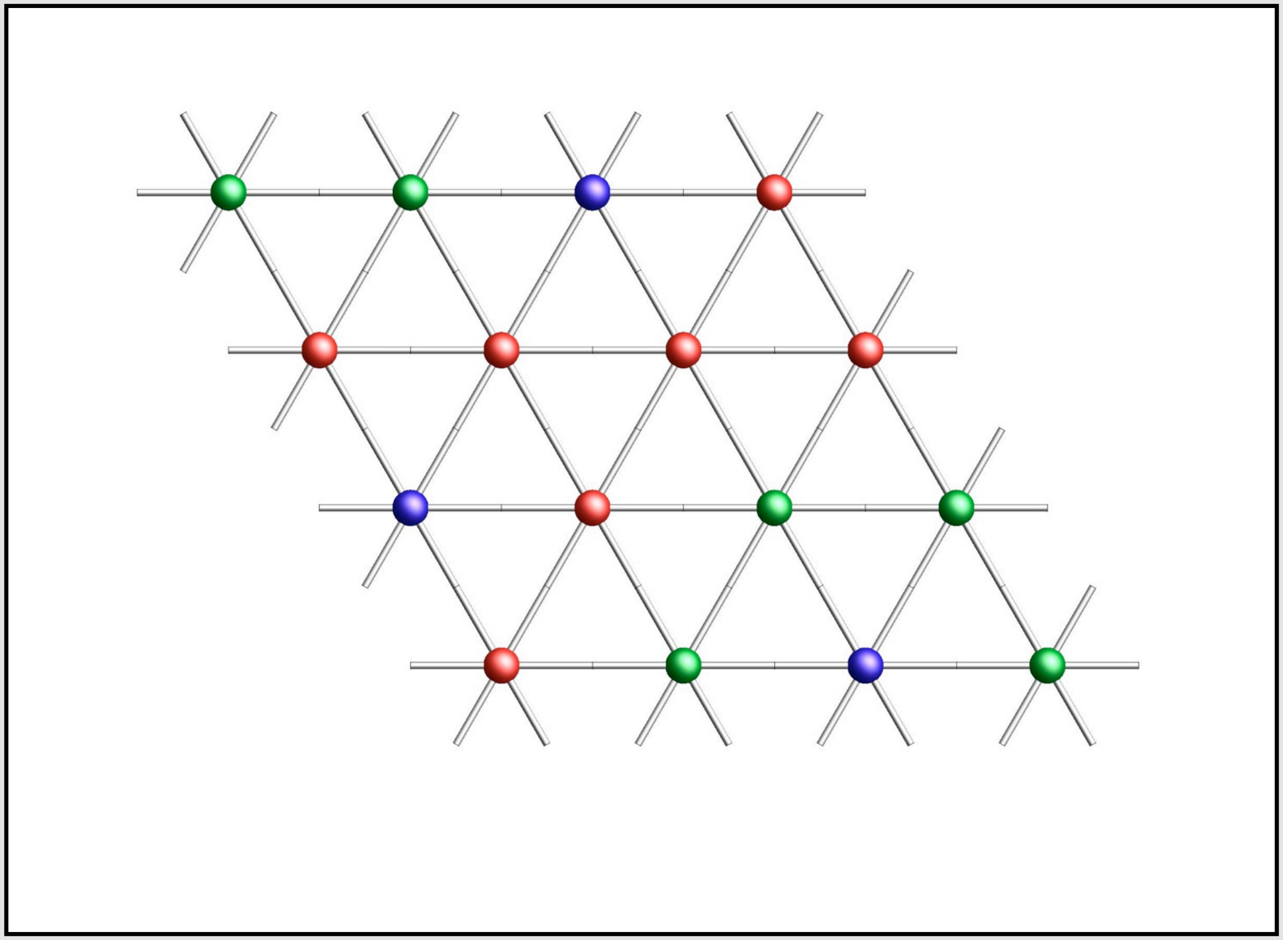

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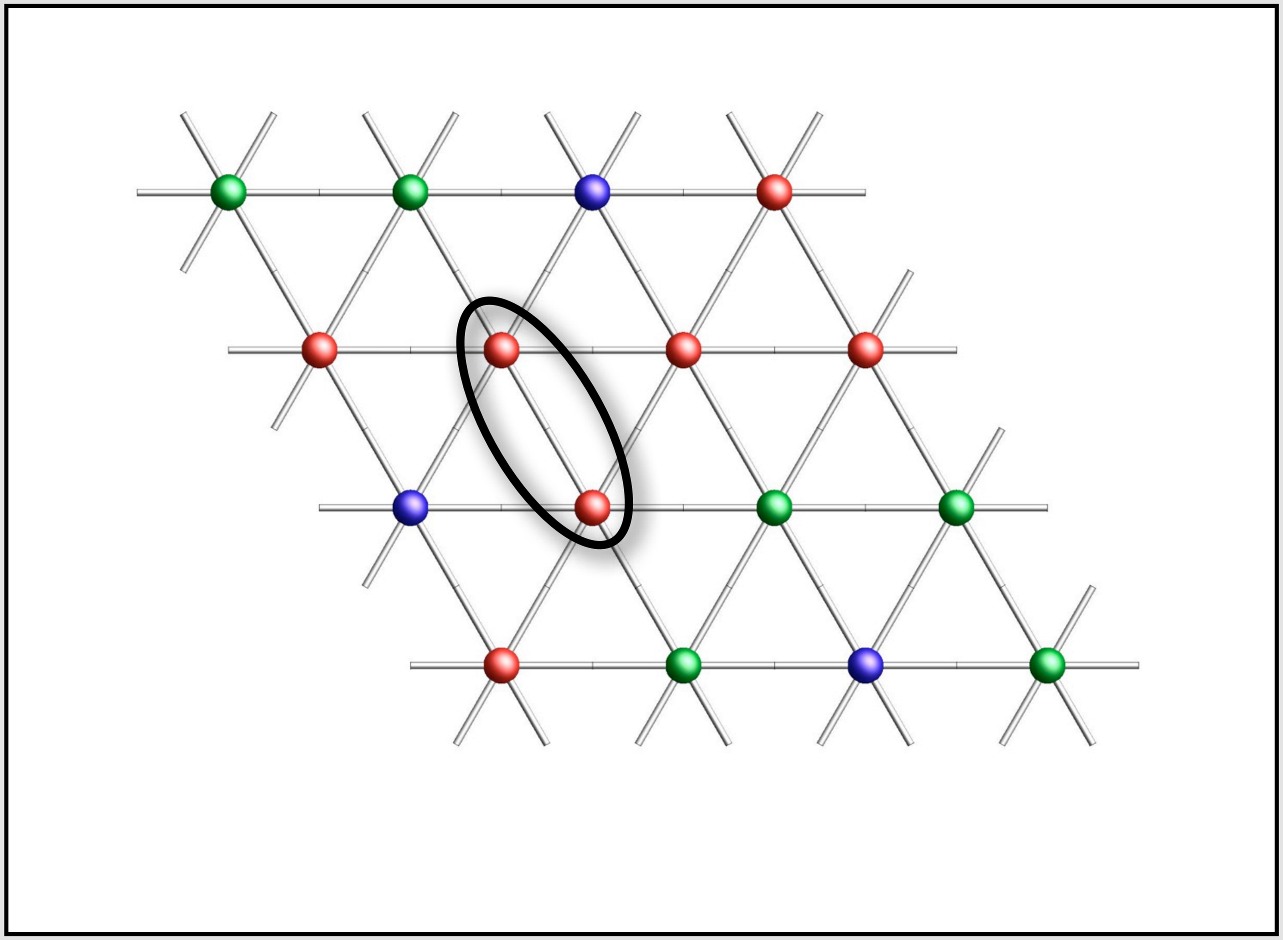

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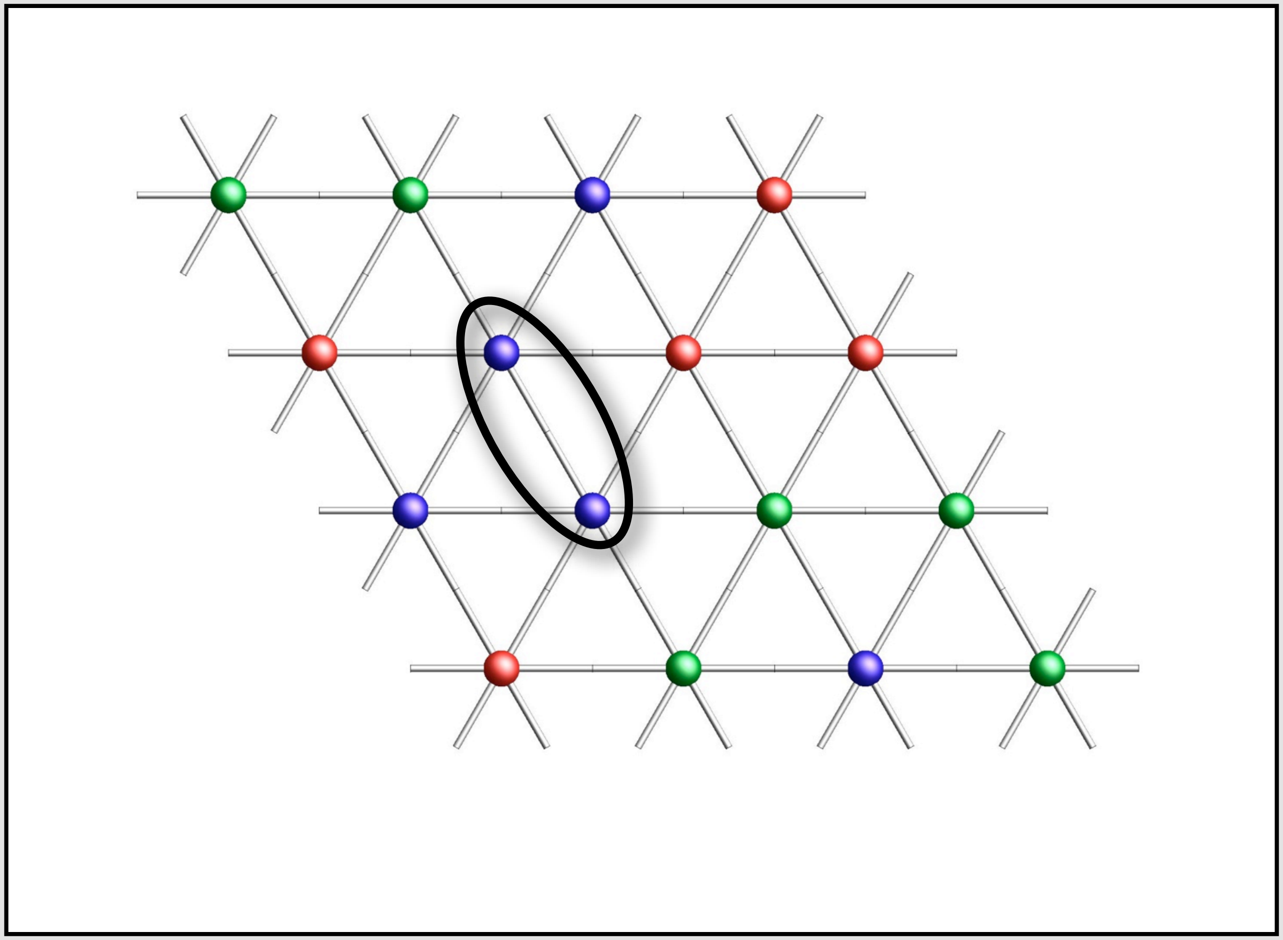

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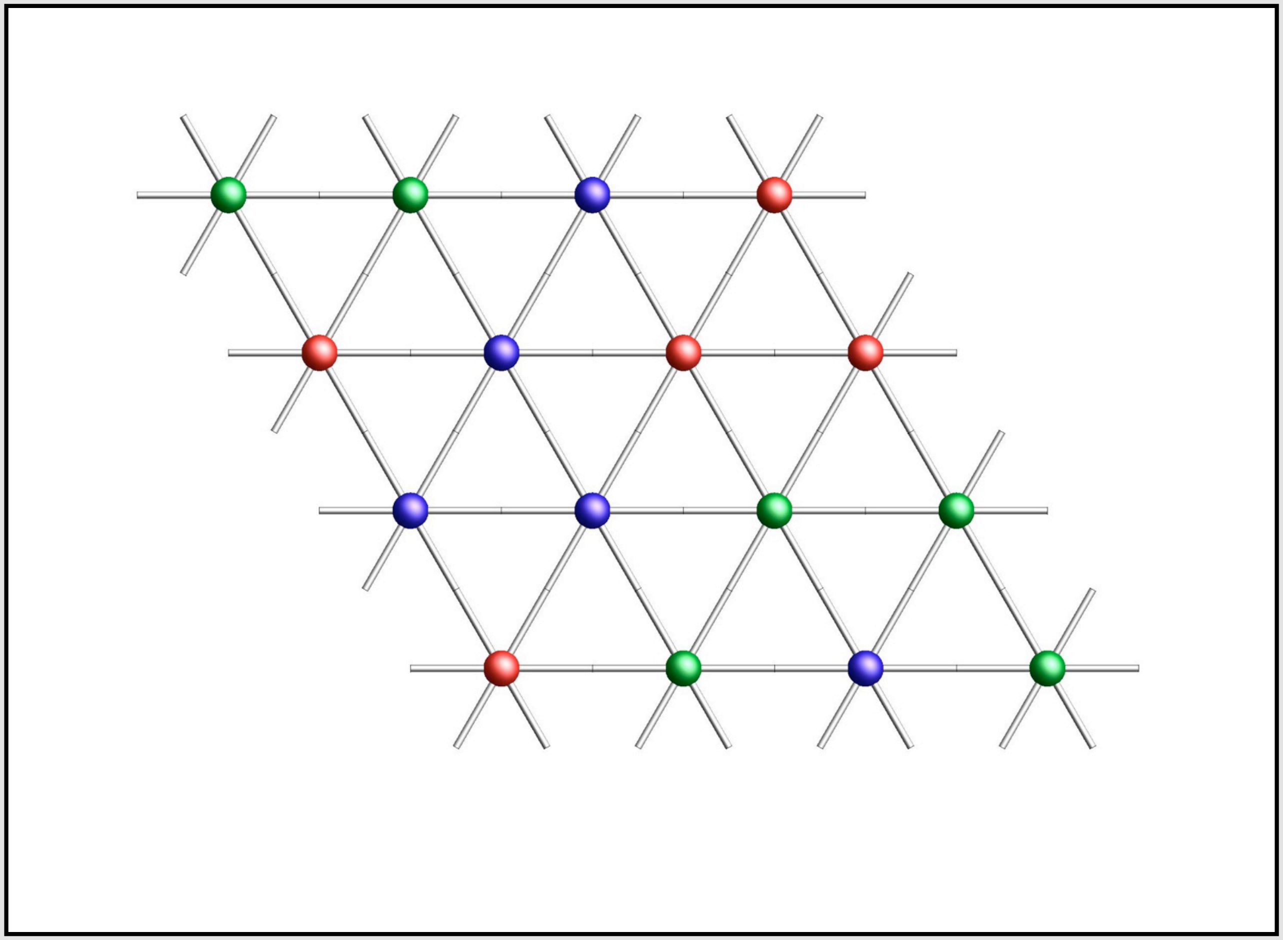

Outlook

Bipartite: Deconfined Criticality

Néel at small-*N*; VBS at large-*N*; direct transition
Evidence consistent with SU(*N*) "deconfined" field theory.
(continuous transition, large-*N* exponents, *q*-*N* phase diagram)
Physical realizations?

Non-Bipartite: Intermediate RVB phase

Magnetic at small-*N*; VBS at large-*N*; intermediate phase
No deconfined criticality
Simplest scenario: A gapped topological spin liquid
Other lattices: Kagome, Pyrochlore?

THE END