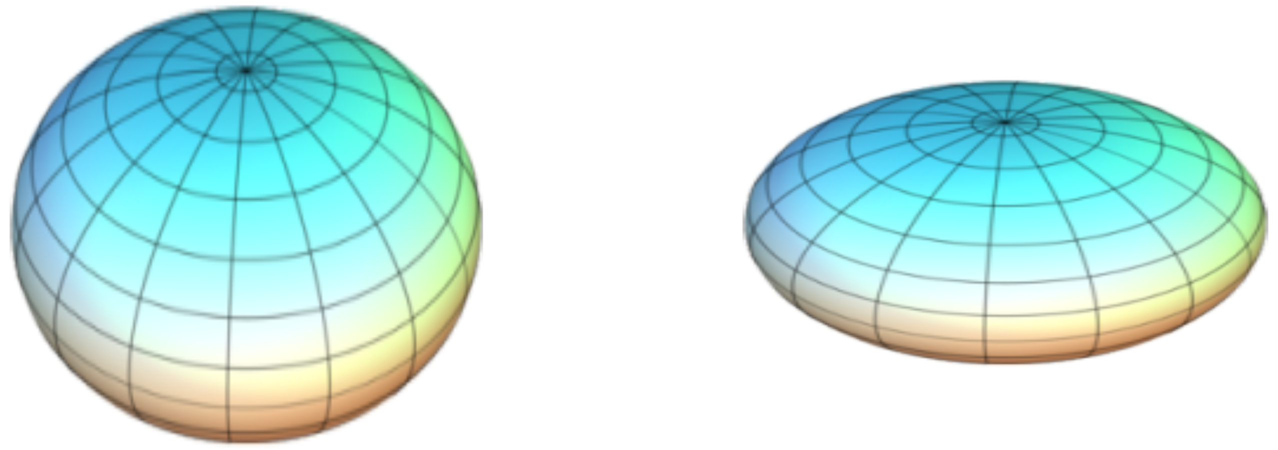
Thermodynamics of the BMN matrix model at strong coupling

Miguel S. Costa

Faculdade de Ciências da Universidade do Porto

Work with L. Greenspan, J. Penedones and J. Santos



Correlations, criticality, and coherence in quantum systems
Évora - October 2014

Motivation

- Gauge/gravity duality as definition of quantum gravity in AdS

  Dual CFT is renormalizable and unitary. Problem: how to decode the hologram?

  Unfortunately field theory is strongly coupled in region of interest for quantum gravity (classical gravity $N \to \infty$, $1/N$ expansion $\equiv$ loop expansion).

- Would like examples where computations in both sides are within reach

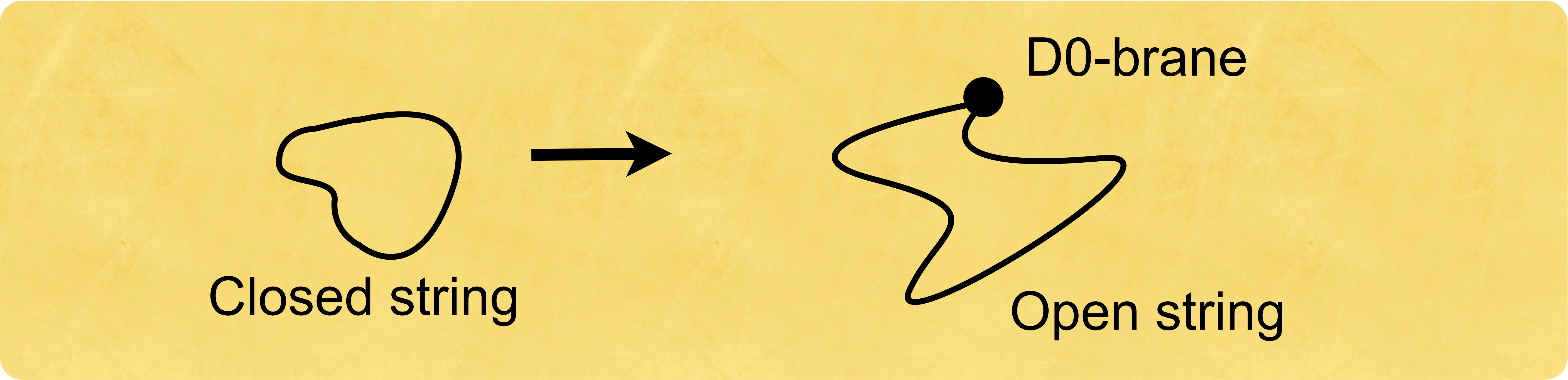
  Test and understand the gauge/gravity duality with observables that are not protected by SUSY and can not be computed using integrability.

  How does gravitation phenomena, like black holes, emerge from gauge theory side?

Idea: Study thermodynamics of black holes dual to Matrix Quantum Mechanics that can be simulated on a computer using Monte-Carlo methods.

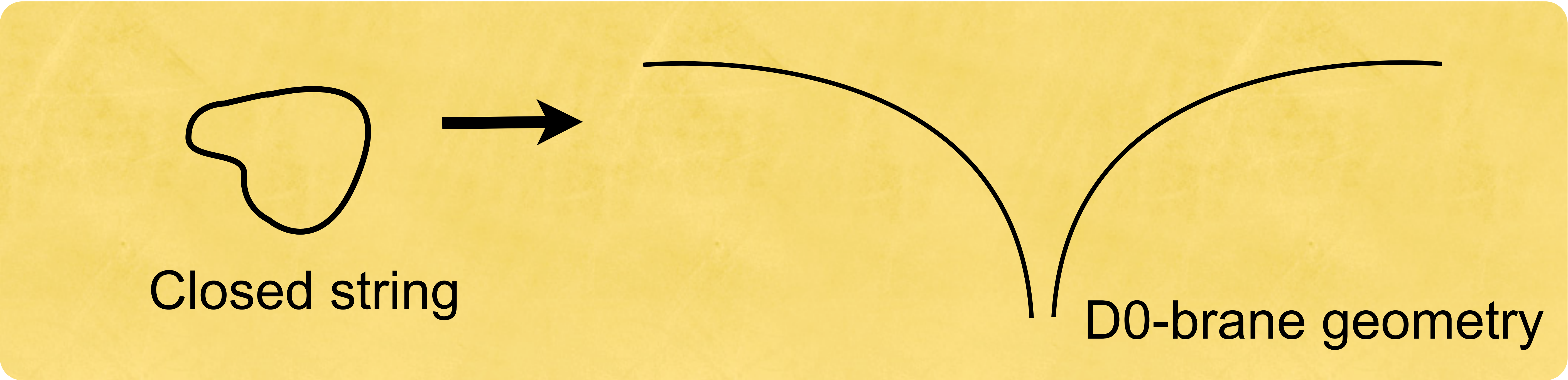
The case of D0-branes

- Closed strings interact with D0-branes in flat space



$$\Updownarrow$$

- Closed strings interact with geometry produced by D0-branes



[Itzhaki, Maldacena, Sonnenschein, Yankielowicz ´98]

D0-branes: field theory description (matrix quantum mechanics) [Itzhaki et al ´98]

$$S_{D0} = \frac{N}{2\lambda} \int dt \, \mathrm{Tr} \left[(D_t X^i)^2 + \Psi^\alpha D_t \Psi^\alpha + \frac{1}{2} \left[X^i, X^j \right]^2 + i \Psi^\alpha \gamma^j_{\alpha\beta} [\Psi^\beta, X^j] \right]$$

$$\begin{aligned} X^i &\equiv SU(N) \text{ bosonic matrices } (i = 1, \ldots, 9) \\ \Psi &\equiv SU(N) \text{ fermionic matrices (16 real components)} \end{aligned}$$

SO(9) global symmetry

- ‘t Hooft coupling is dimensionfull (relevant)

$$\boxed{\lambda = g^2_{YM} N = \frac{g_s N}{(2\pi)^2 l_s^3} \equiv \text{mass}^3}$$

$$\lambda_{eff} = \frac{\lambda}{E^3}$$

$$\begin{aligned} E \to \infty \; (UV) &\equiv \text{weak coupling} \\ E \to 0 \; (IR) &\equiv \text{strong coupling} \end{aligned}$$

- Dual 10D gravitational coupling

$$\boxed{16\pi G_N l_s^{-8} = (2\pi)^{11} \frac{(\lambda l_s^3)^2}{N^2}}$$

• Theory on Euclidean time circle with periodicity $\beta = 1/T$

$$S_{D0} = \frac{N}{2\lambda} \int_0^\beta dt \; \mathrm{Tr}\left[\cdots \right]$$

Dimensionless temperature $\tau = \frac{T}{\lambda^{1/3}}$

Low temperatures is strong coupling

• Can put theory on a computer using Monte Carlo simulations, accessing in particular strongly coupled region.

Dimensionless mean energy $\boxed{\frac{\epsilon}{N^2} = \frac{E}{N^2 \lambda^{1/3}}}$

[Catterall, Wiseman ´07,´08,´09; Anagnostopoulos et al ´07; Hanada et al ´08,´13]

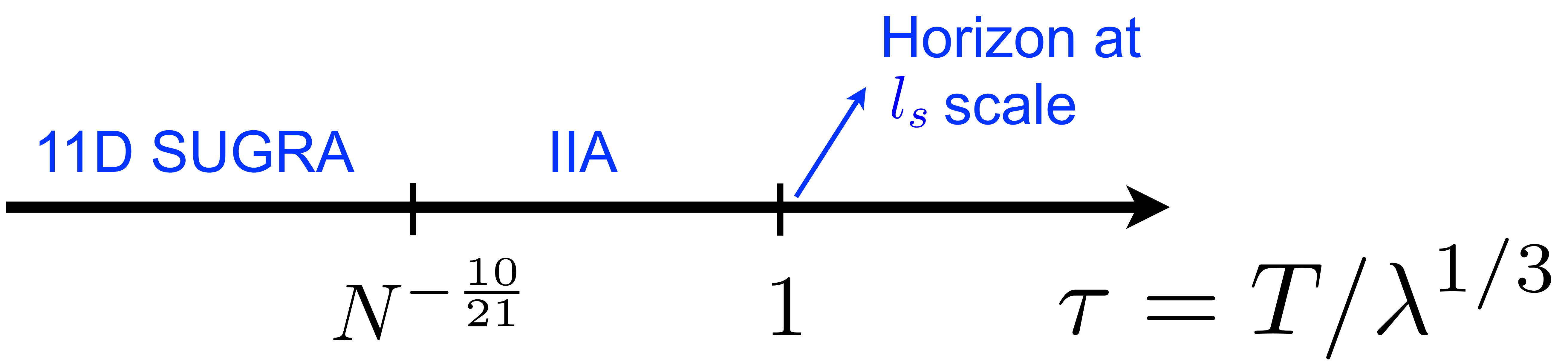
D0-branes: gravitational description

- 11D SUGRA solution (near horizon geometry of non-extremal D0-brane)

$$ds^2 = \frac{dr^2}{f(r)} + r^2 d\Omega_8^2 + \left(\frac{R}{r}\right)^7 dz^2 + f(r)dt\left(2dz - \left(\frac{r_0}{R}\right)^7 dt\right)$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^7 , \qquad \left(\frac{R}{\ell_s}\right)^7 = 60\pi^3 g_s N\,, \qquad \left(\frac{r_0}{\ell_s}\right)^5 = \frac{120\pi^2}{49}\left(2\pi g_s N\right)^{\frac{5}{3}}\tau^2$$

- Classical gravity domain (at horizon)



$$l_s^2 \mathcal{R}(r_0) \ll 1 \;\Rightarrow\; \tau \ll 1$$

$$g_s e^{\phi(r_0)} \ll 1 \;\Rightarrow\; \tau \gg N^{-\frac{10}{21}}$$

- Standard gravitation thermodynamics

$$S = \frac{A_H}{4G_N} = d_1 N^2 \tau^{\frac{9}{5}} \qquad d_1 = 4^{\frac{13}{5}} 15^{\frac{2}{5}} \left(\frac{\pi}{7}\right)^{\frac{14}{5}}$$

$$\boxed{\frac{\epsilon}{N^2} = c_1 \tau^{\frac{14}{5}}} \qquad c_1 = \frac{9}{14} d_1 \quad (\text{because } dE = TdS)$$

- α' corrections give next term in τ expansion, at large N [Hanada et al´08]

$$\frac{1}{16\pi G_N}\int d^{10}x\sqrt{-g}\, e^{-2\phi}\left(\mathcal{R} + \cdots + \alpha'^3 \mathcal{R}^4 + \dots\right) \quad \Rightarrow \quad \frac{S}{N^2} = d_1 \tau^{\frac{9}{5}}\left(1 + d_2 \tau^{\frac{9}{5}}\right)$$

fixes next power in τ expansion

- l_P corrections to 11D SUGRA give 1/N correction (solution purely gravitational)

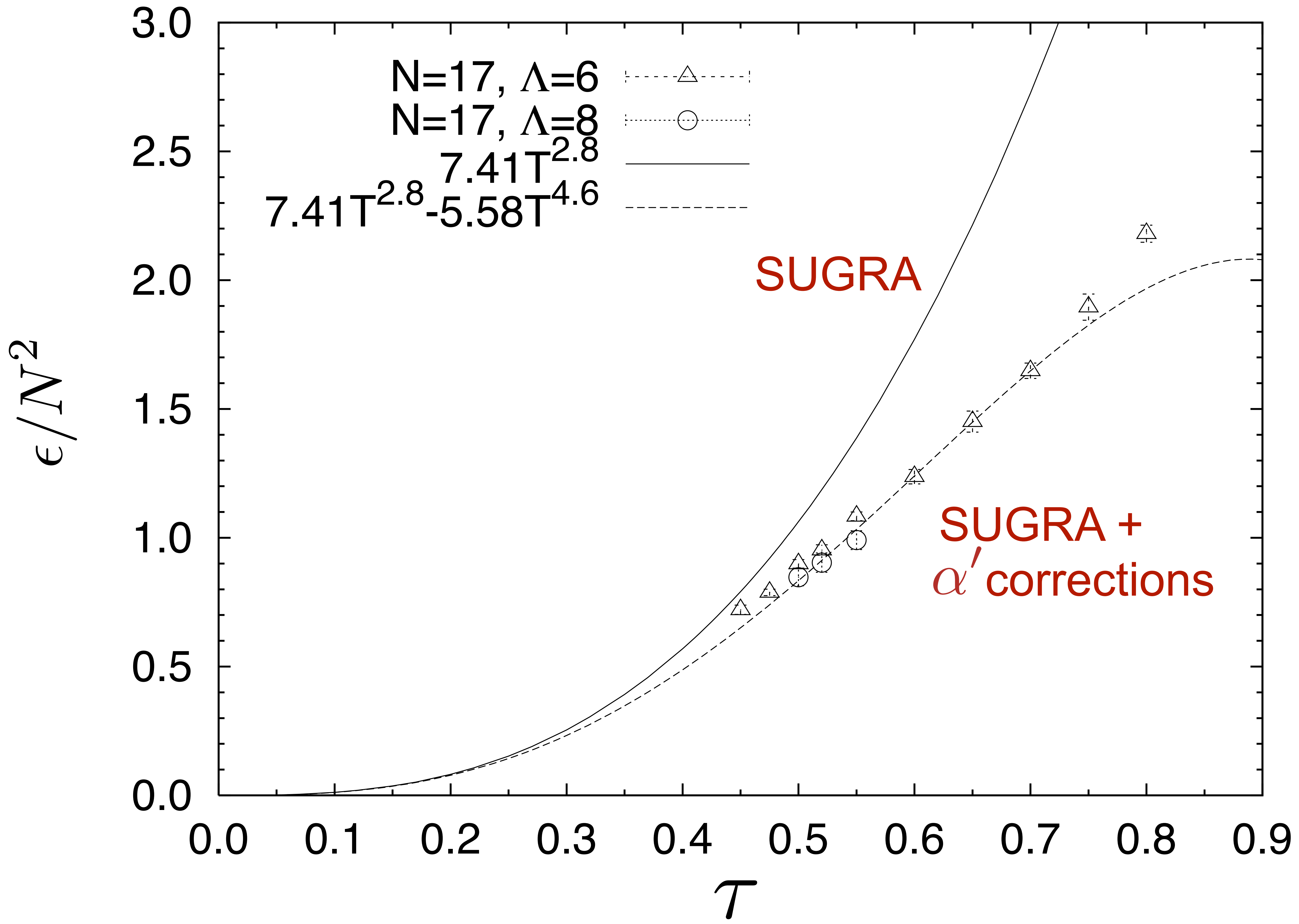
$$\frac{1}{16\pi G_{11}}\int d^{11}x\sqrt{-g}\left(\mathcal{R} + l_P^6 \mathcal{R}^4\right) \quad \Rightarrow \quad \frac{S}{N^2} = d_1 \tau^{\frac{9}{5}} + \frac{1}{N^2} d_3 \tau^{-\frac{3}{5}}$$

fixes both coefficient and power of $1/N^2$ correction

[Hanada et al´13]

- Low temperature expansion predicted from gravity

$$\frac{\epsilon}{N^2} = \left[c_1 \tau^{\frac{14}{5}} + c_2 \tau^{\frac{23}{5}} + \dots\right] + \frac{1}{N^2}\cancel{\left[c_3 \tau^{\frac{2}{5}} + \dots\right]} + \dots$$



Monte-Carlo simulation of MQM

negligible

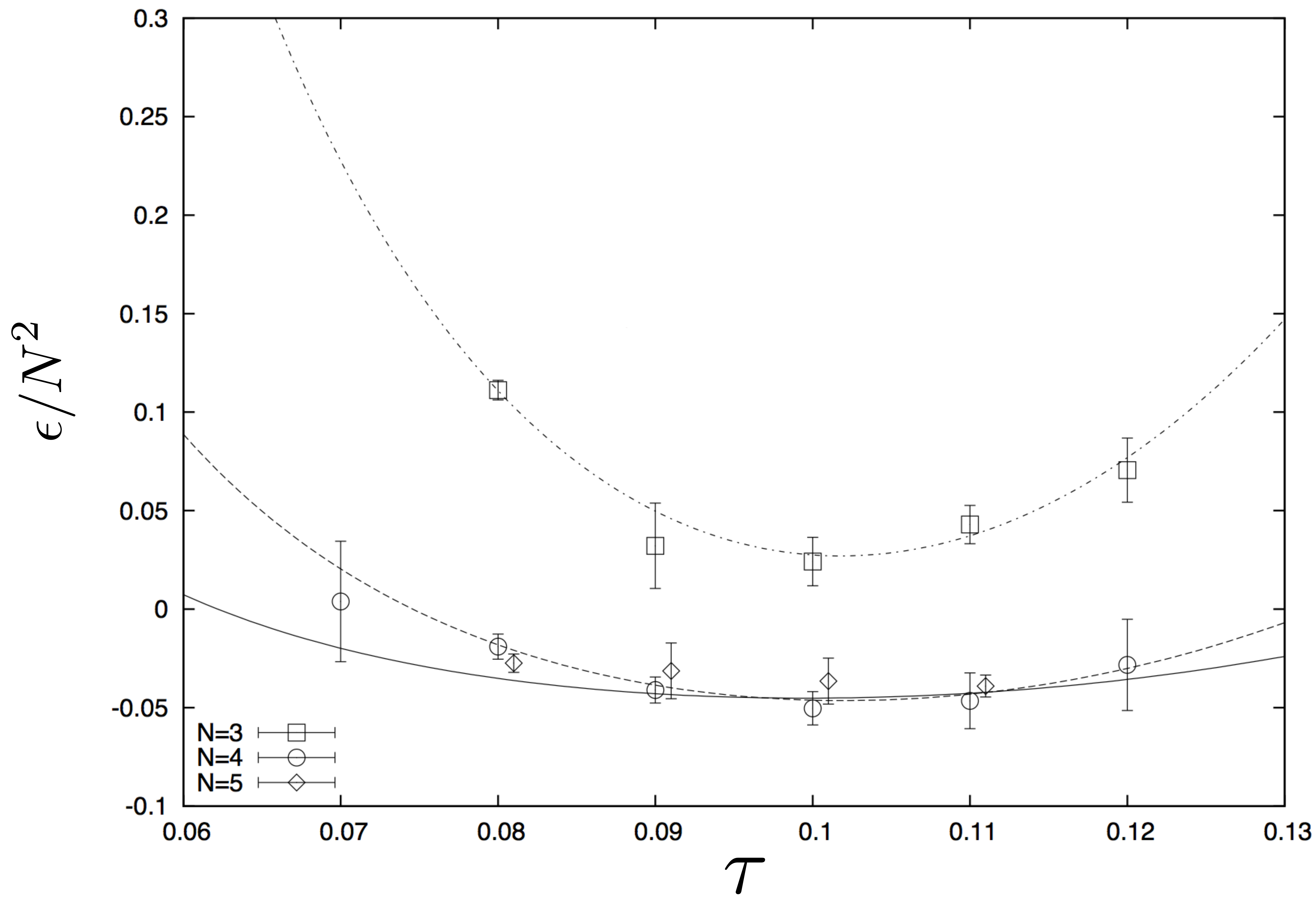
checked

predicted

[Hanada, Hyakutake, Nishimura, Takeuchi ´08]

• Low temperature expansion predicted from quantum gravity

$$\frac{\epsilon}{N^2} = \left[c_1 \tau^{\frac{14}{5}} + \cancel{c_2 \tau^{\frac{23}{5}}} + \dots \right] + \frac{1}{N^2} \left[c_3 \tau^{\frac{2}{5}} + \dots \right] + \dots$$



Monte-Carlo simulation of MQM

$\times$ negligible

$\bigcirc$ checked

[Hyakutake ´13]

[Hanada, Hyakutake, Ishiki, Nishimura ´13]

Today's talk is not about D0-brane matrix model

• Caveat: canonical ensemble ill defined - IR divergences from flat directions in D0-brane moduli space. This is suppressed at large N (metastable state), but it is a source of tension in Monte Carlo simulations [Catterall, Wiseman ´09]

$$\frac{F(T,r)}{N^2} \sim \mathcal{F}_{finite}(T) + \frac{9}{N}\ln r$$

Instability corresponds to Hawking radiation of D0-branes. At large N this is suppressed and black hole is stable (positive specific heat).

• **Today's talk is about BMN matrix model** [Berenstein, Maldacena, Nastase ´02]

Mass deformation resolves IR divergence - canonical ensemble well defined.

Much richer thermodynamics with a 1st order phase transition (at large N there are two dimensionless parameters).

BMN matrix model

$$S = S_{D0} - \frac{N}{2\lambda}\int dt\,\mathrm{Tr}\left[\frac{\mu^2}{3^2}(X^i)^2 + \frac{\mu^2}{6^2}(X^a)^2 + \frac{\mu}{4}\Psi^\alpha\left(\gamma^{123}\right)_{\alpha\beta}\Psi^\beta + i\frac{2\mu}{3}\epsilon_{ijk}X^iX^jX^k\right]$$

Massive deformation of D0-brane MQM. Preserves SUSY but **breaks** $SO(9) \to SO(6) \times SO(3)$

$a = 4, \ldots, 9 \qquad i = 1, 2, 3$

In large N ‘t Hooft limit dimensionless coupling constant $\lambda = \dfrac{g_{\mathrm{YM}}^2 N}{\mu^3}$

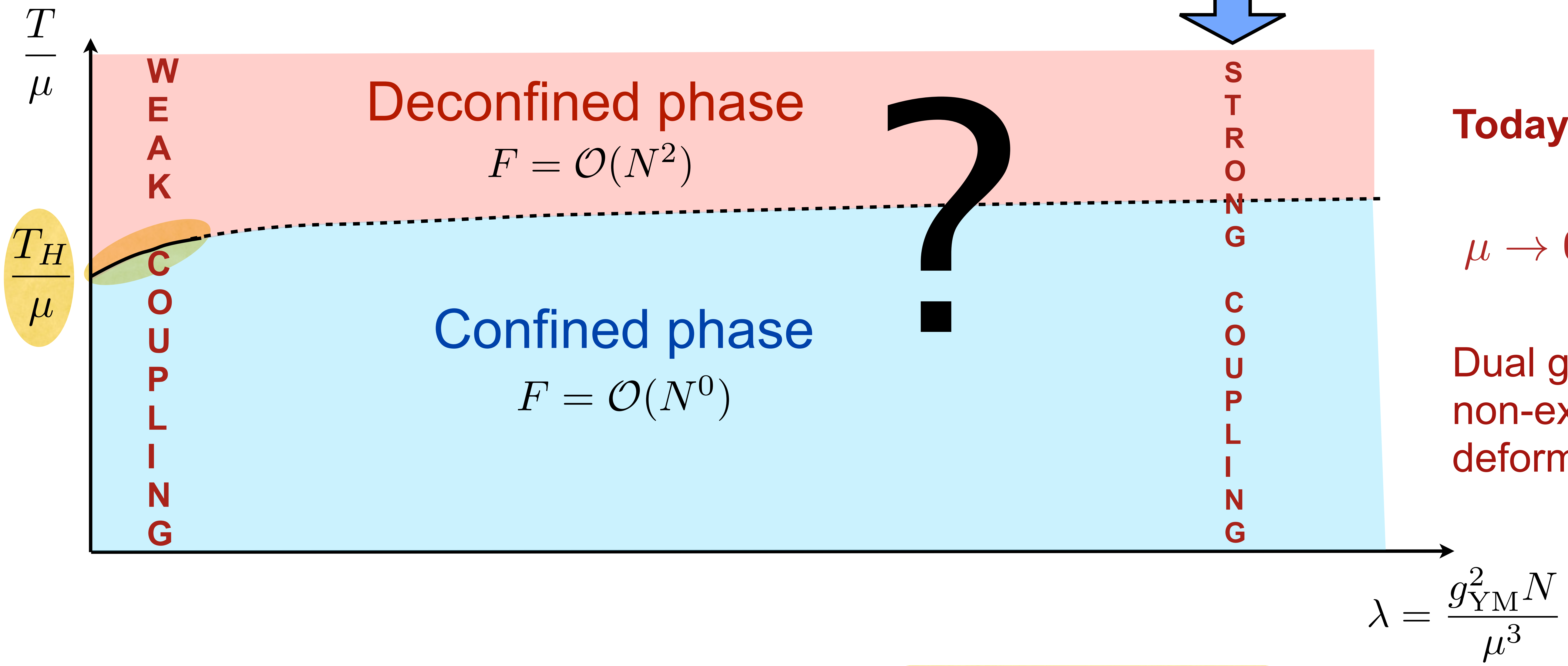
Many vacua $X^a = 0 \qquad X^i = \dfrac{\mu}{3}J^i \qquad [J^i, J^j] = i\epsilon^{ijk}J^k$

Focus on trivial vacuum (single M5-brane) that is SO(9) invariant $X^i = X^a = 0$

Canonical ensemble is well defined and may still be simulated on a computer.

- Thermodynamics ($N \to \infty$)

Start here



Dimensionless coupling $\equiv \lambda = \frac{g^2_{YM} N}{\mu^3}$

Dimensionless temperature $\equiv \frac{T}{\mu}$

Today: strongly coupled limit

$$\mu \to 0\,, \quad \frac{T}{\mu} \text{ fixed and large}$$

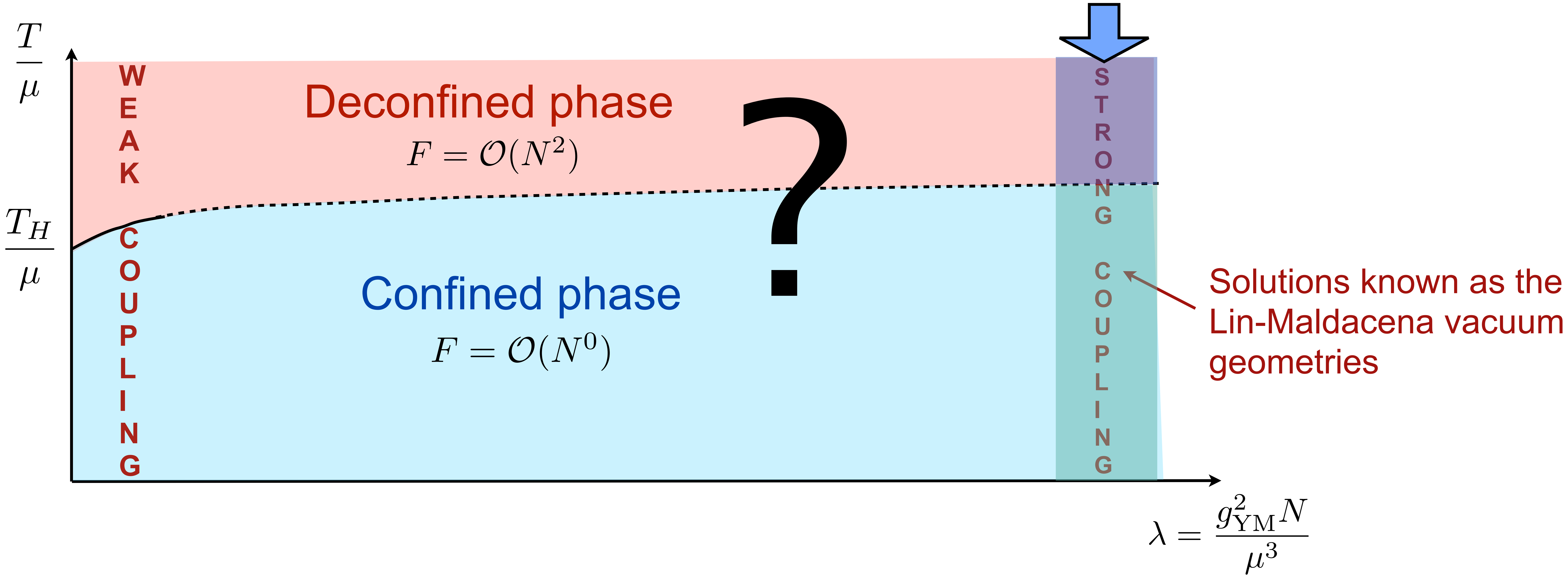
Dual geometry is SO(9) invariant non-extremal D0-brane with deformation turned on

Exponential growth of spectrum with energy $\longrightarrow$ Hagedorn transition

First-order phase transition at

$$\frac{T_c}{\mu} = \frac{1}{12 \log 3}\left[1 + \frac{2^6 5}{3^4}\lambda - c\,\lambda^2 + O(\lambda^3)\right] \approx 0.076 + \mathcal{O}(\lambda)$$

[Hadizadeh, Ramadanovic, Semenoff, Young ’04]



- At strong coupling, for large temperature, dual geometry is SO(9) invariant and is approximately the non-extremal D0-brane solution

$$ds^2 = \frac{dr^2}{f(r)} + r^2 d\Omega_8^2 + \frac{R^7}{r^7} dz^2 + f(r) dt \left(2dz - \frac{r_0^7}{R^7} dt \right)$$

$$dC = \mu \, dt \wedge dx^1 \wedge dx^2 \wedge dx^3$$

Non-normalizable mode responsible for massive deformation

Need back-reaction to decrease temperature and study phase transition at strong coupling. In particular,

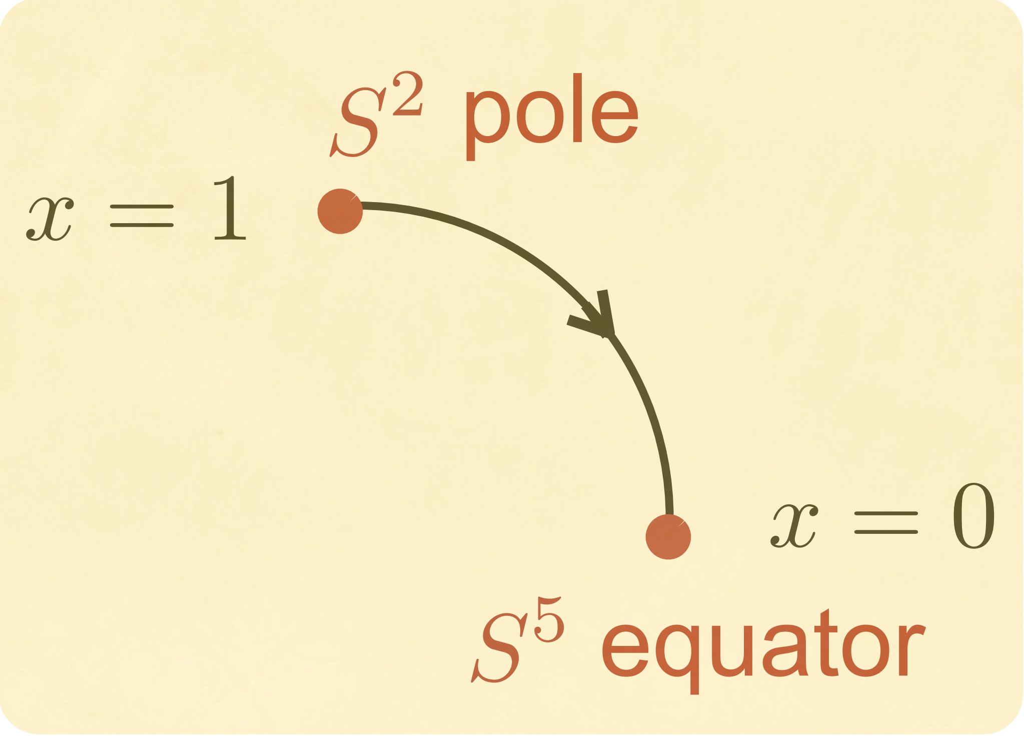
$$SO(9) \rightarrow SO(6) \times SO(3)$$

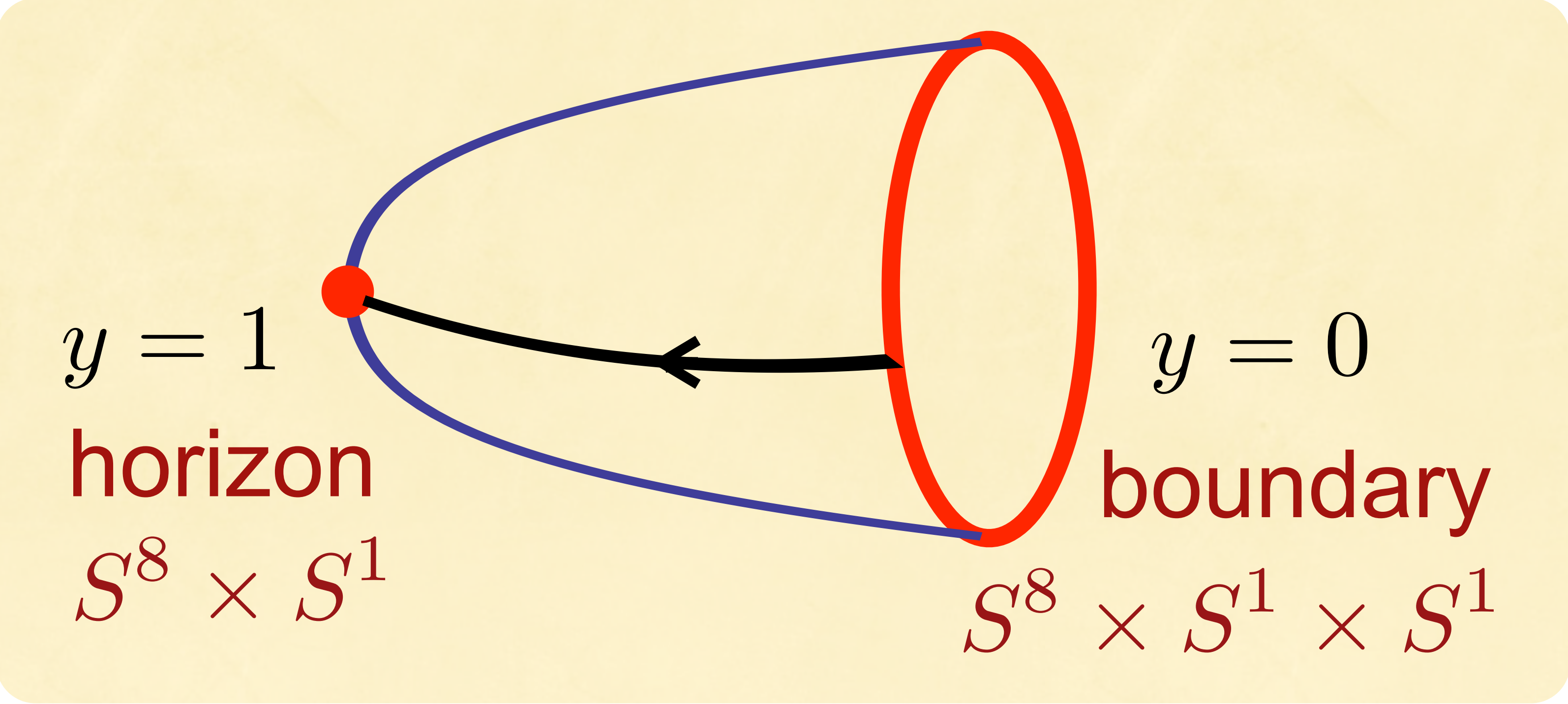
- Ansatz for 11D SUGRA

$$ds^2 = -A\,\frac{(1-y^7)}{y^7}d\eta^2 + T_4\,y^7\left[d\zeta + \Omega\,\frac{(1-y^7)d\eta}{y^7}\right]^2$$

$$+\frac{1}{y^2}\left[B\,\frac{(dy+F dx)^2}{(1-y^7)y^2} + \underbrace{T_1\,\frac{4dx^2}{2-x^2} + T_2\,x^2(2-x^2)d\Omega_2^2 + T_3\,(1-x^2)^2 d\Omega_5^2}_{d\Omega_8^2 \quad \text{if} \quad T_1=T_2=T_3=1}\right]$$

$$C = (M\,d\eta + L\,d\zeta)\wedge d^2\Omega_2$$



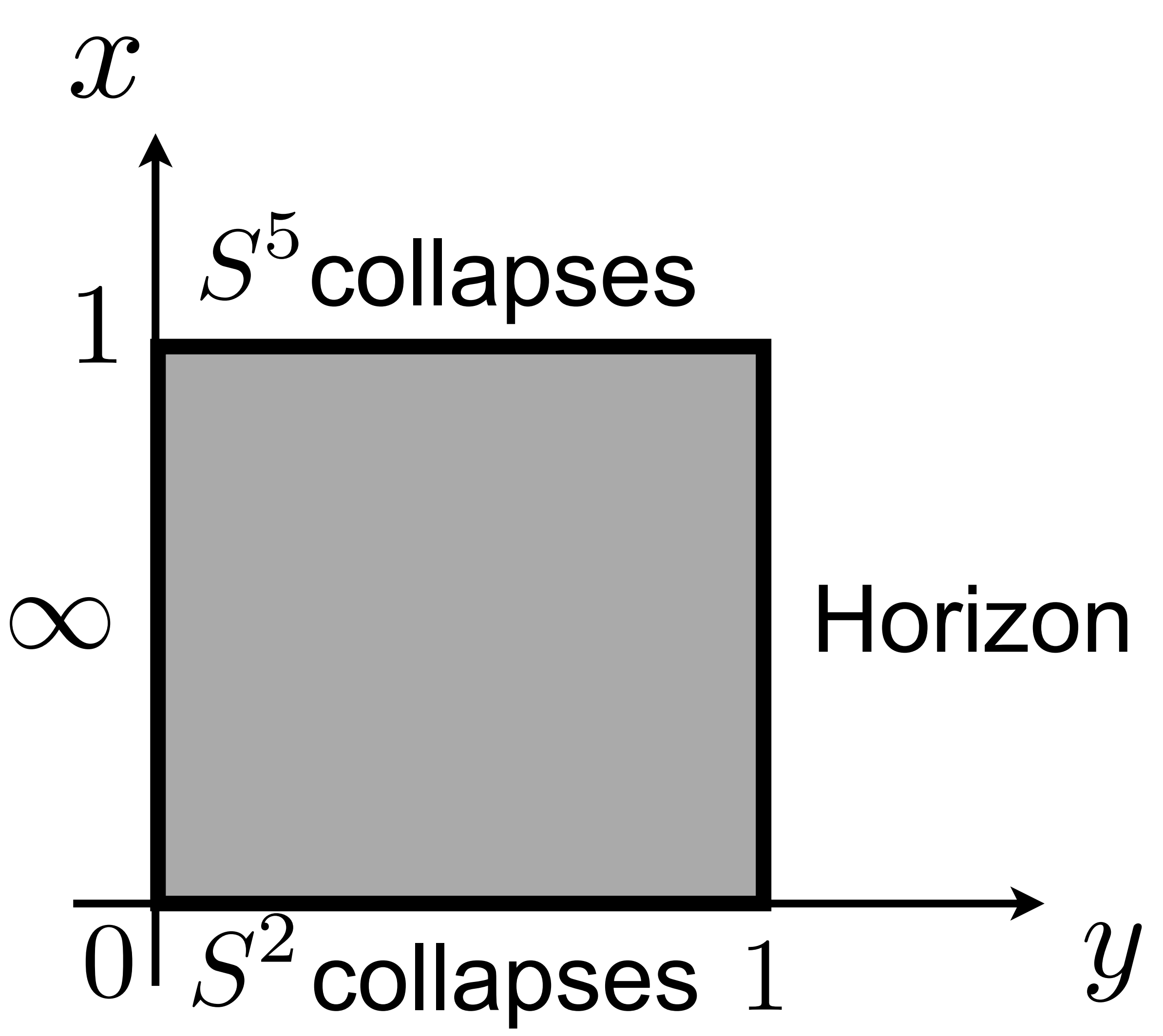


M-theory circle $\zeta \sim \zeta + 2\pi$

x is a angular coordinate on compact 8-dimensional space with S^8 topology

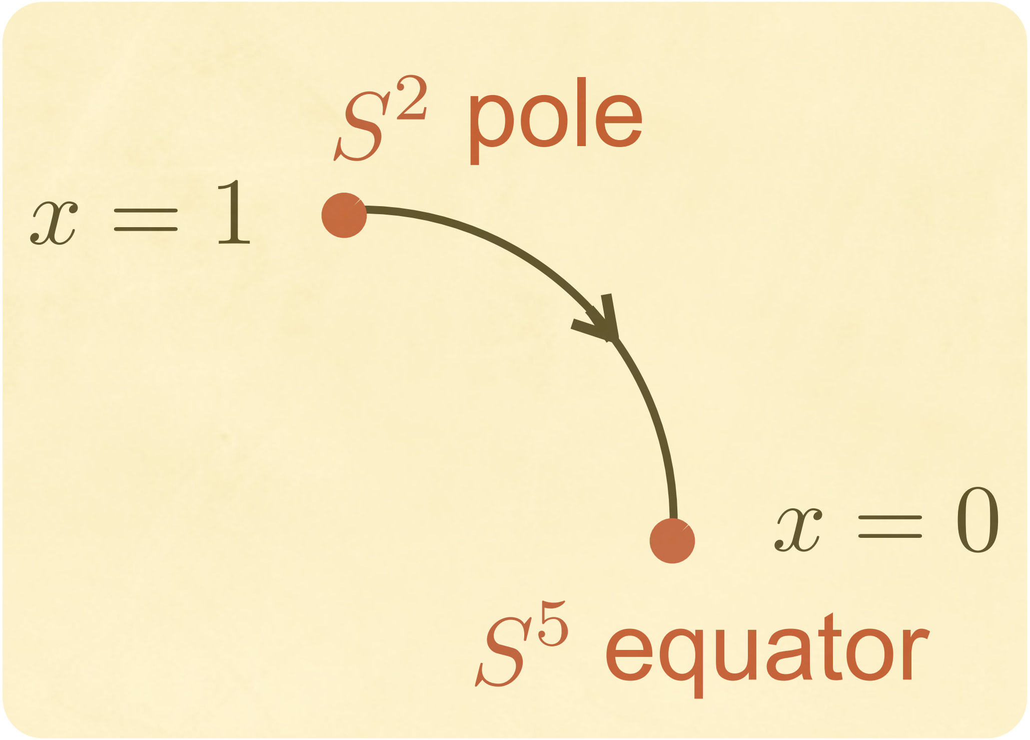
y is a radial coordinate from boundary ($y=0$) to horizon ($y=1$)

$A, B, F, T_1, T_2, T_3, T_4, \Omega, M, L$ are functions of x and y



Tailored to numerical implementation
(domain of unknown is the unit square; everything dimensionless)

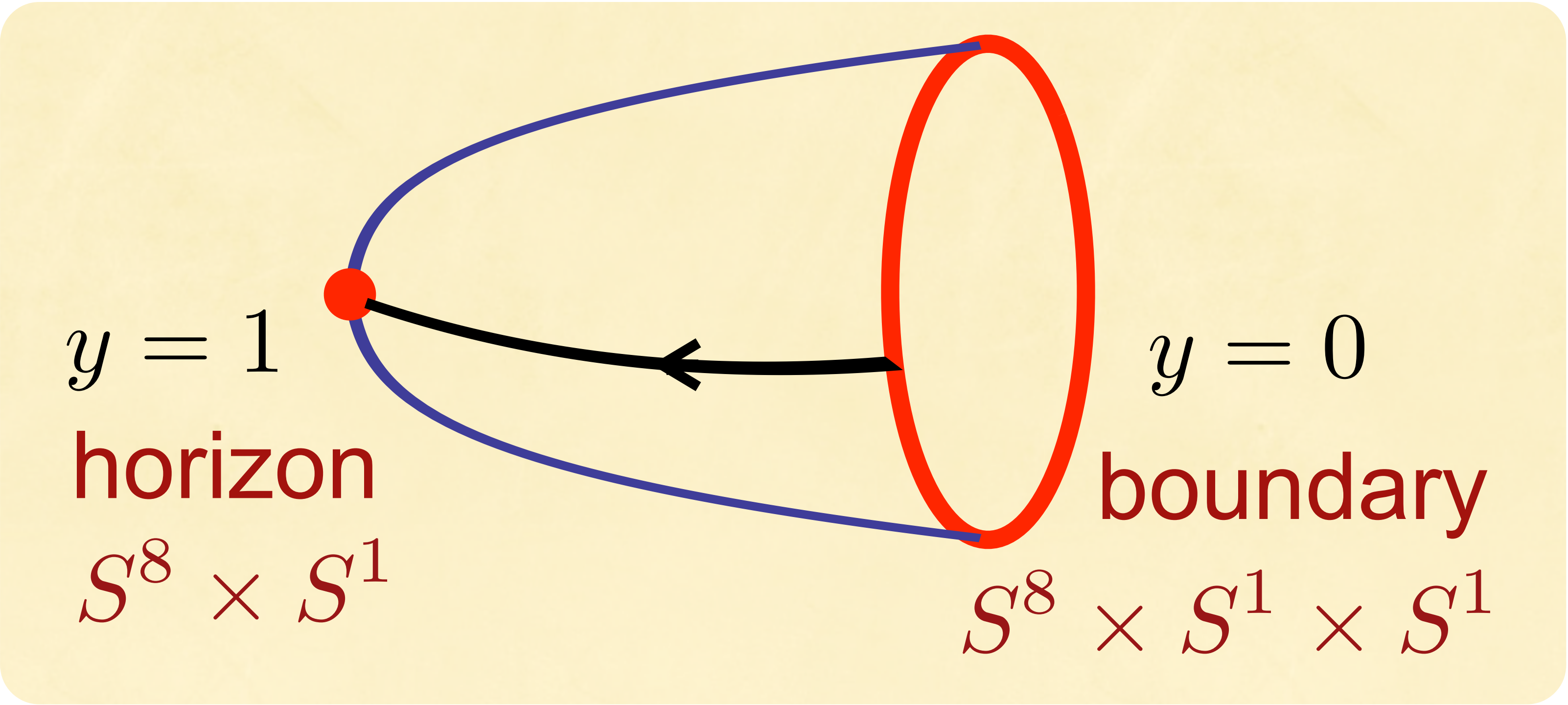
- Ansatz for 11D SUGRA



$$ds^2 = -A\,\frac{(1-y^7)}{y^7}d\eta^2 + T_4\,y^7\left[d\zeta + \Omega\,\frac{(1-y^7)d\eta}{y^7}\right]^2$$

$$+\frac{1}{y^2}\left[B\,\frac{(dy+F dx)^2}{(1-y^7)y^2} + \underbrace{T_1\,\frac{4dx^2}{2-x^2} + T_2\,x^2(2-x^2)d\Omega_2^2 + T_3\,(1-x^2)^2 d\Omega_5^2}_{d\Omega_8^2 \quad \text{if} \quad T_1=T_2=T_3=1}\right]$$

$$C = (M\,d\eta + L\,d\zeta)\wedge d^2\Omega_2$$



Non-extremal D0-brane solution corresponds to

$$A=B=T_1=T_2=T_3=T_4=\Omega=1\,,\qquad F=M=L=0\,,\qquad \beta=\frac{4\pi}{7}\ \text{(Euclidean time circle)}$$

and need to use scaling symmetry of 11D SUGRA action

$$g_{\mu\nu}\to s^2 g_{\mu\nu}\,,\quad C_{\mu\nu\rho}\to s^3 C_{\mu\nu\rho} \quad \Rightarrow \quad I\to s^9 I$$

$$\zeta\sim\zeta+2\pi \ \to\ \zeta\sim\zeta+2\pi s' \quad \Rightarrow \quad I\to s' I$$

with

$$s=r_0$$

$$s'=\left(\frac{R}{r_0}\right)^{\frac{7}{2}}\frac{g_s\ell_s}{r_0}$$

This scaling symmetry will be important later...

- Boundary conditions

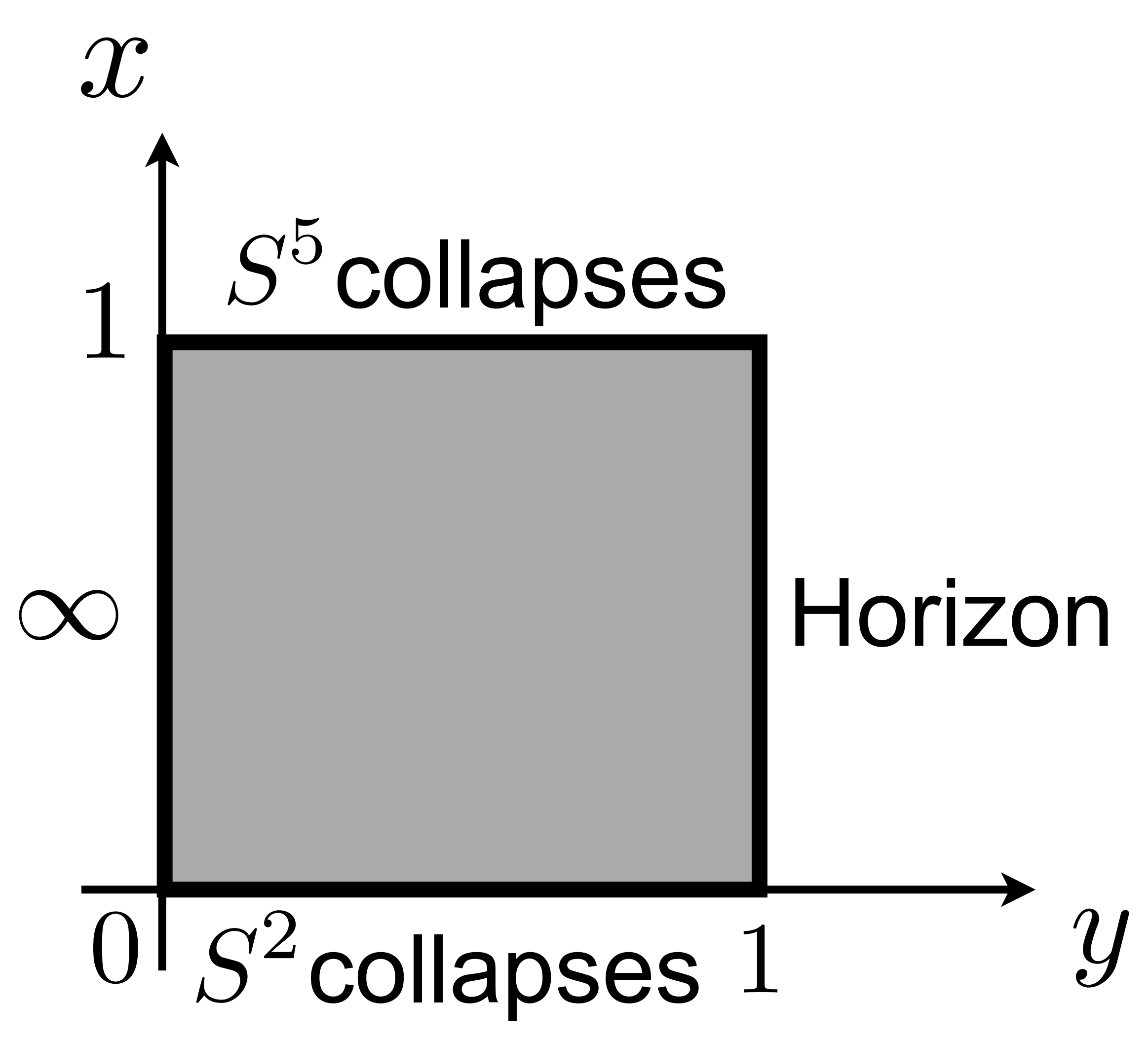
At infinity ($y=0$): $A, B, T_1, T_2, T_3, T_4, \Omega \to 1\,, \quad F \to 0$

$$M \to \hat{\mu}\,\frac{x^3(2-x^2)^{\frac{3}{2}}}{y^3}\,, \qquad L \to \frac{3}{2}\,\hat{\mu}\,y^4 x^3(2-x^2)^{\frac{3}{2}}$$

Recall that

$C = (M\,d\eta + L\,d\zeta)\wedge d^2\Omega_2$

$SO(6)\times SO(3)$ invariant tensor hamonic



Regularity at the axis of symmetry: horizon ($y=1$), S^2 pole ($x=1$) and S^5 equator ($x=0$).

Perform above scalings, then geometry has asymptotics of non-extremal D0-brane with temperature T and mass deformation turned on. There is a single parameter

$$\hat{\mu} = \frac{7}{12\pi}\,\frac{\mu}{T}$$

Important! Just learned that

$$I = \frac{s^9 s'}{16\pi G_N}\,\hat{I}\left(\frac{\mu}{T}\right) = \frac{15}{28}\left(\frac{15}{14^2\pi^8}\right)^{\frac{2}{5}} N^2 \left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{\frac{9}{5}} \hat{I}\left(\frac{\mu}{T}\right)$$

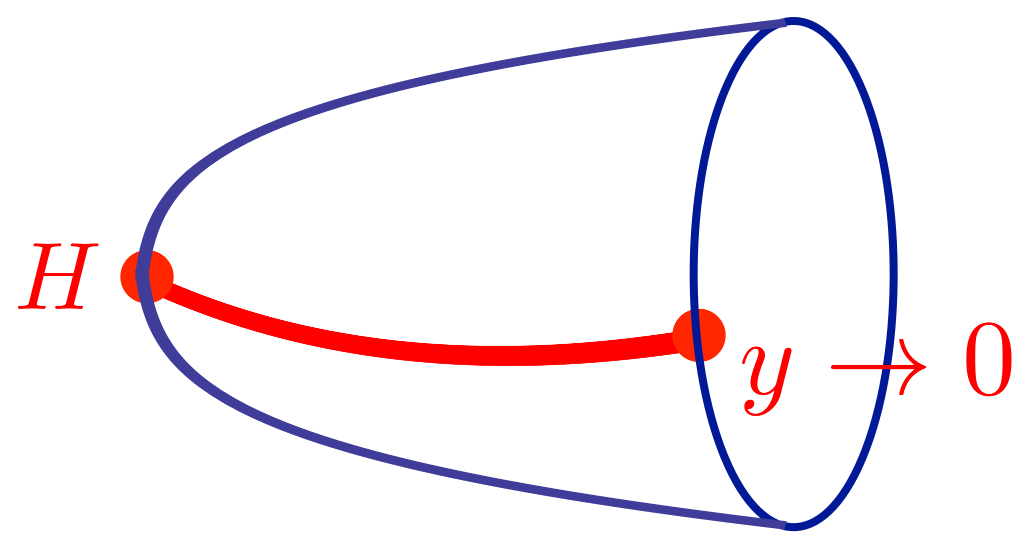
$$S = \frac{s^9 s'}{4G_N}\,\hat{S}\left(\frac{\mu}{T}\right) = \frac{15\pi}{7}\left(\frac{15}{14^2\pi^8}\right)^{\frac{2}{5}} N^2 \left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{\frac{9}{5}} \hat{S}\left(\frac{\mu}{T}\right)$$

Smarr formulae (good to check numerics)

- Let v^μ be a killing vector. From field equations it follows that

$$(K_v)^{\mu\nu} = \nabla^\mu v^\nu + \frac{1}{3} F^{\mu\nu\alpha\beta} v^\gamma C_{\alpha\beta\gamma} + \frac{1}{6} v^{[\mu} F^{\nu]\alpha\beta\gamma} C_{\alpha\beta\gamma}$$

is a conserved antisymmetric tensor, i.e. $\boxed{d\left(\star K_v\right) = 0}$



- Integrate $d\left(\star K_v\right) = 0$ over surface of constant time with $y_1 < y < y_2$

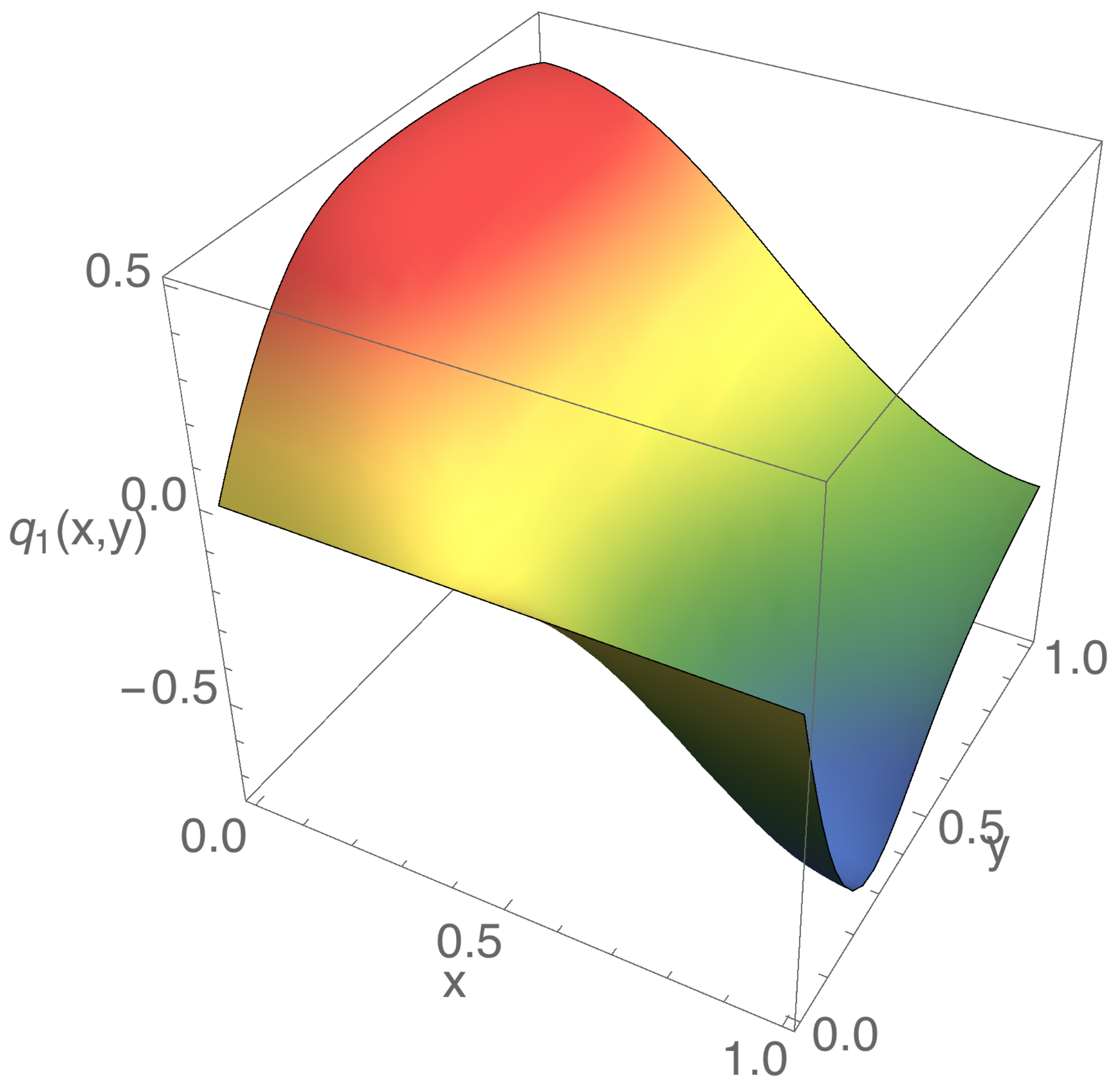
$$0 = \int_{\Sigma_{12}} d\left(\star K_v\right) = \int_{\partial\Sigma_{12}} \star K_v = \int_H \star K_v - \int_{y \to 0} \star K_v$$

- For example take $v = \dfrac{\partial}{\partial \eta}$ (time translations generator)

Smarr formula relates horizon area to boundary data

$$\boxed{\frac{7}{2}\hat{S} = \int_{y \to 0} \star K_v}$$

The solution



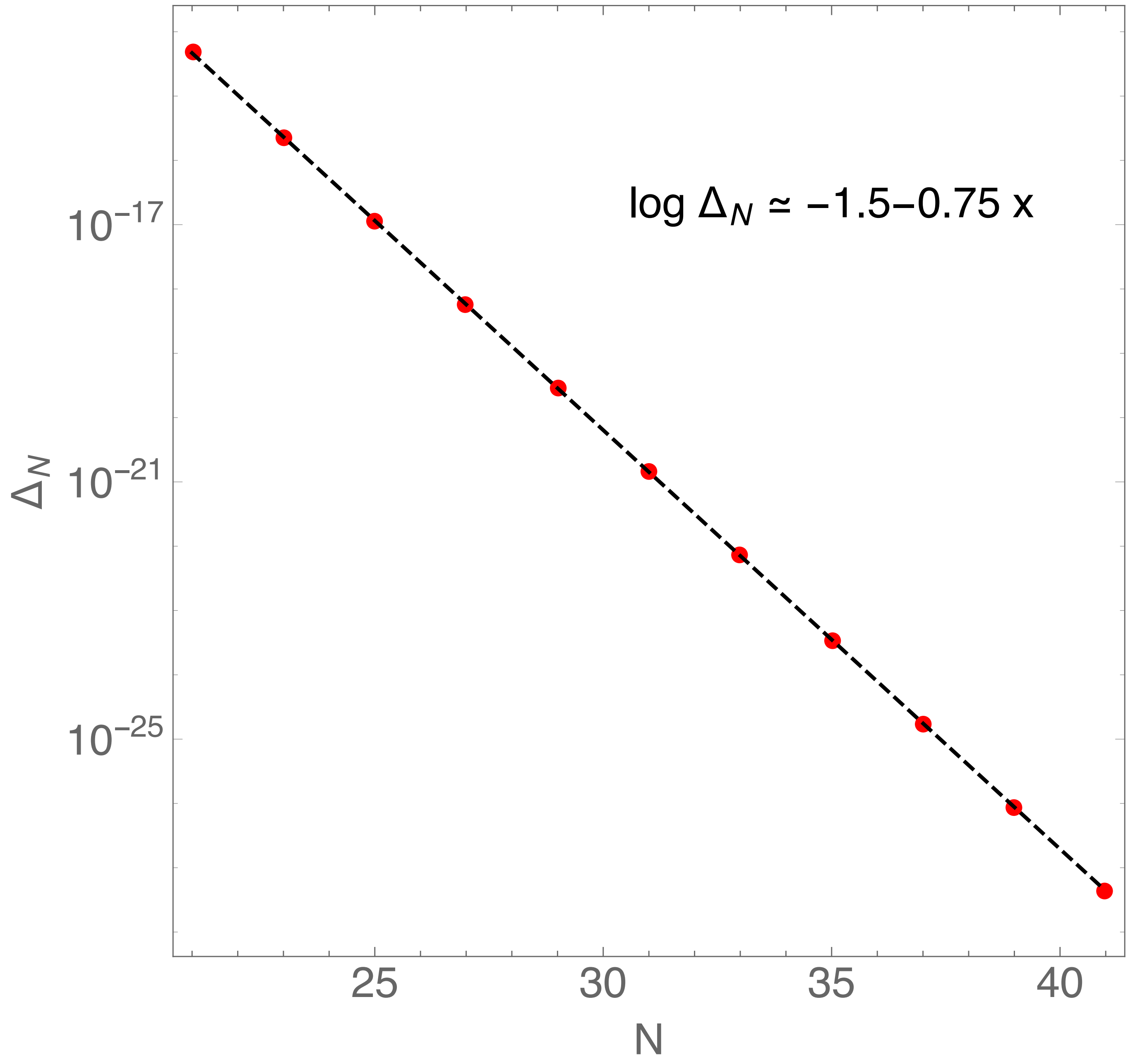
- Einstein-deTurck equations [Headrick, Kitchen, Wiseman '09]

$$R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} = \frac{1}{12}\left(F_{\mu\alpha\beta\gamma}F_{\mu}{}^{\alpha\beta\gamma} - \frac{1}{12}g_{\mu\nu}F^2\right)$$

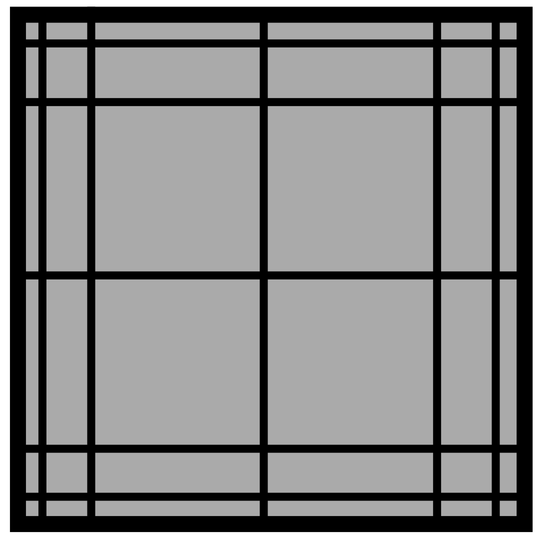
DeTurck term that makes Einstein equations elliptic

$$\xi^\mu = g^{\alpha\beta}\left(\Gamma^\mu_{\alpha\beta} - \tilde{\Gamma}^\mu_{\alpha\beta}\right)$$

$\Delta_N = \left| 1 - \frac{\text{Area}_N}{\text{Area}_{N+1}} \right|$



- Descretize PDEs with $N \times N$ Chebyshev grid

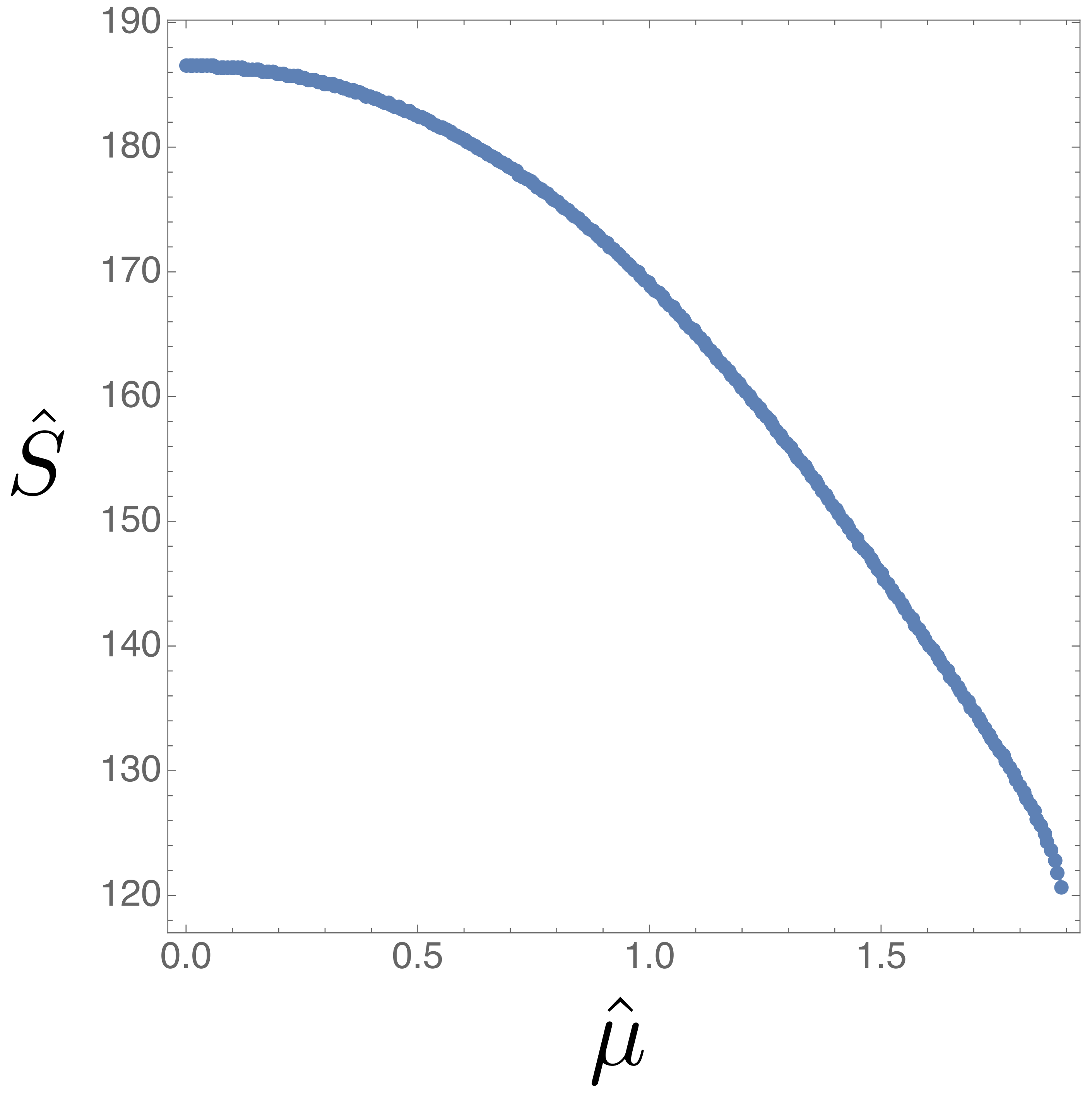


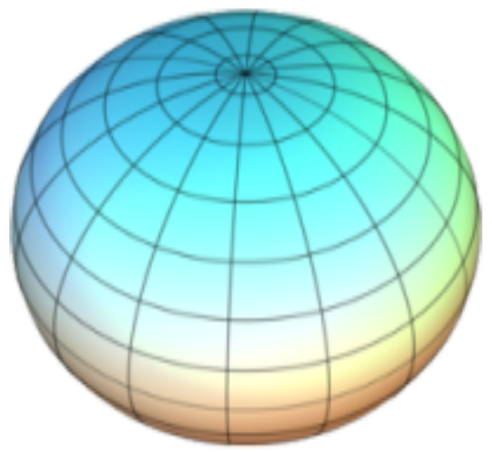
Derivatives are estimated using polynomial approximation that involves all points in the grid

spectral methods - exponential convergence

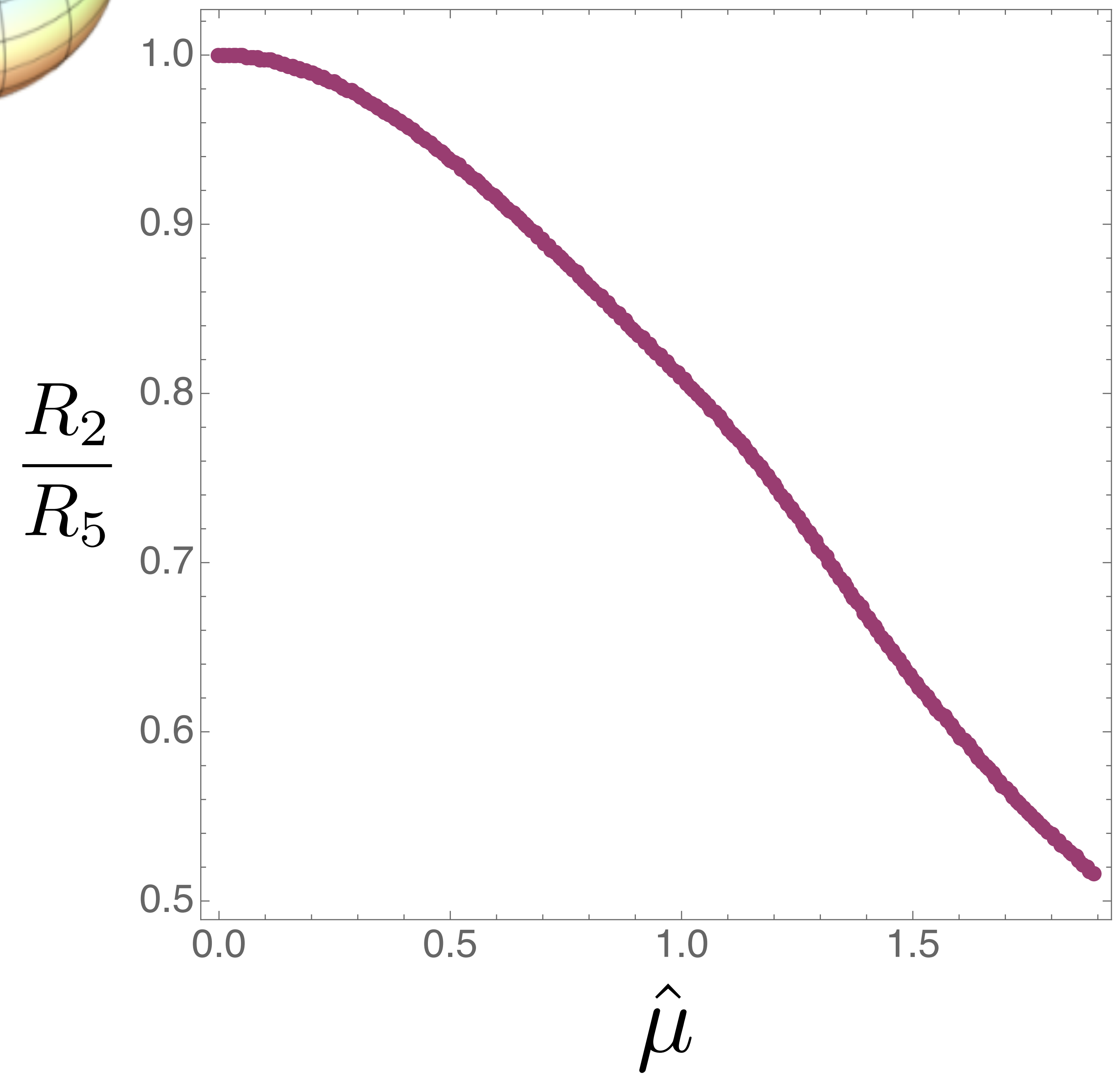
- Horizon area and shape

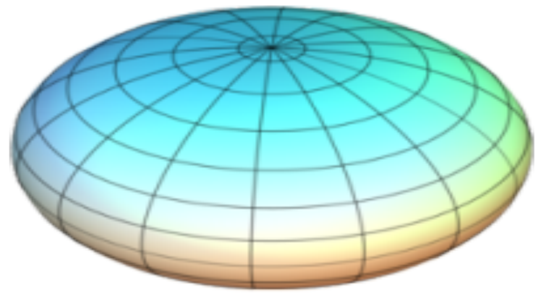
Horizon area





Ratio of maximal radius of S^2 to S^5



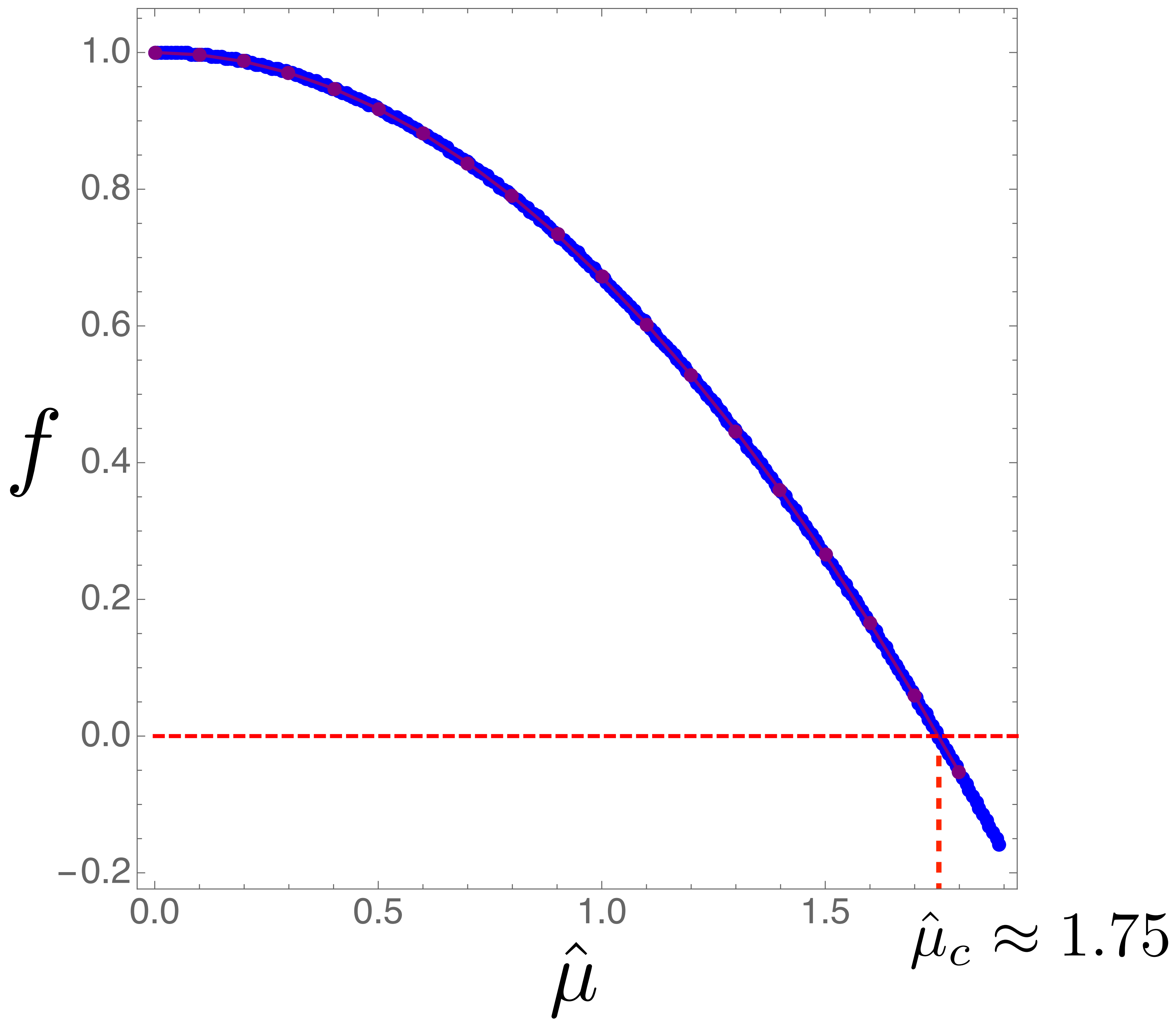


After scaling symmetry to obtain physical metric:

$$S = \frac{15\pi}{7}\left(\frac{15}{14^2\pi^8}\right)^{\frac{2}{5}} N^2 \left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{\frac{9}{5}} \hat{S}\left(\frac{\mu}{T}\right) \qquad R_i = a_i \left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{\frac{2}{5}} \hat{R}_i\left(\frac{\mu}{T}\right)$$

Reproduces scalings predicted from strongly coupled low energy moduli estimate [Wiseman '13]

Black hole thermodynamics - critical temperature



$$F(T,\mu) = F(T,0)\,f(\hat{\mu}) = -c_1 T^{\frac{14}{5}} f(\hat{\mu})$$

both using 1st law or holographic renormalization

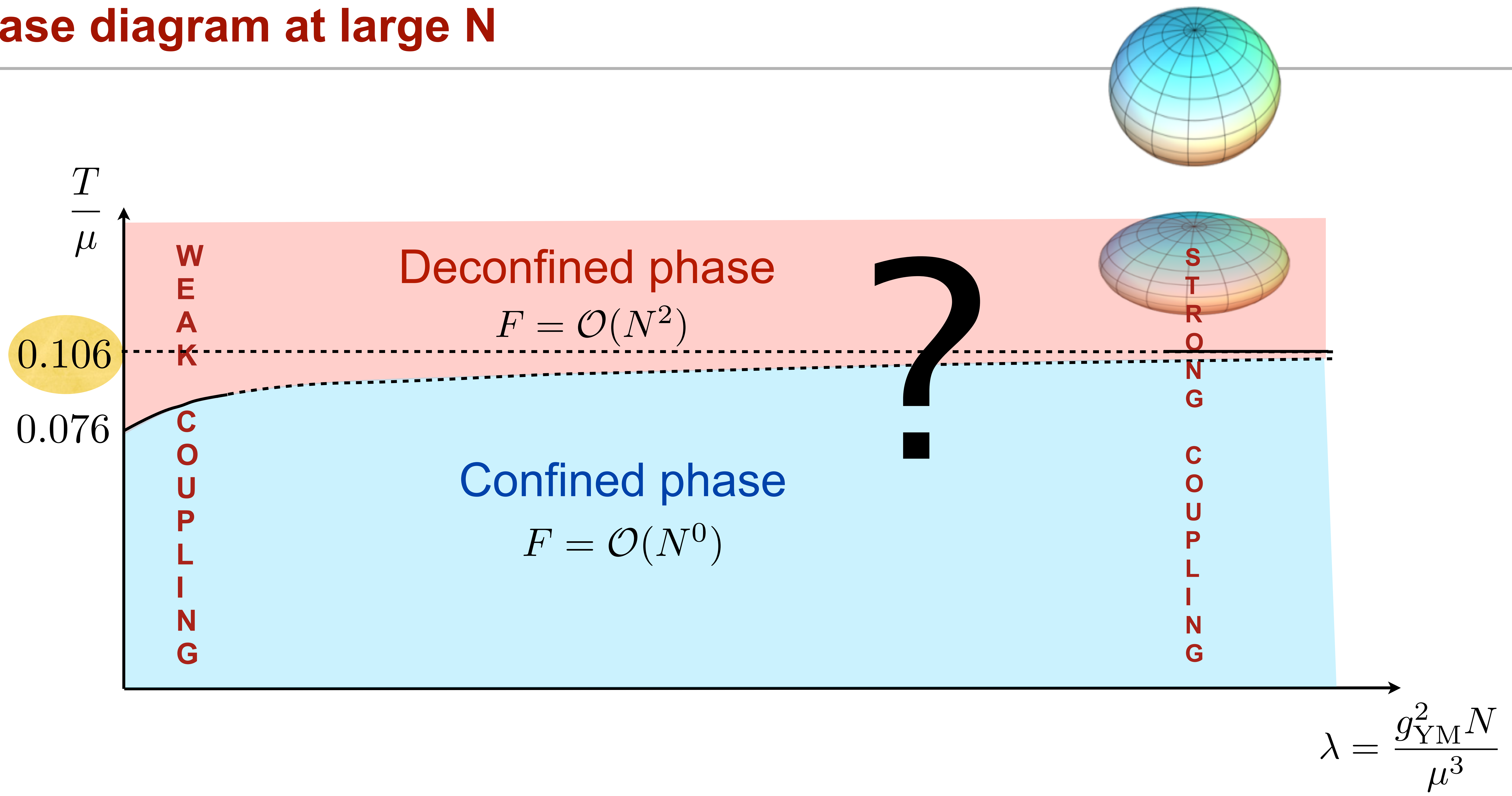
- Phase transition occurs when free energy changes sign, since for $T < T_c$ geometry without horizon is favoured $F \sim \mathcal{O}(N^0)$ [Lin, Maldacena '05]

$$\boxed{\frac{T_c}{\mu} = \frac{7}{12\pi\hat{\mu}_c} \approx 0.106}$$

- BH is thermodynamically stable for $\hat{\mu} < \hat{\mu}_c$

$$c = T\left(\frac{\partial S}{\partial T}\right)_\mu \Rightarrow \frac{c}{S} = \frac{9}{5} - \hat{\mu}\frac{\partial}{\partial\hat{\mu}}\log s(\hat{\mu}) > 0$$

Phase diagram at large N



Very similar to SYM on a 3-sphere $(\mu \equiv 1/R)$

[Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk '03]

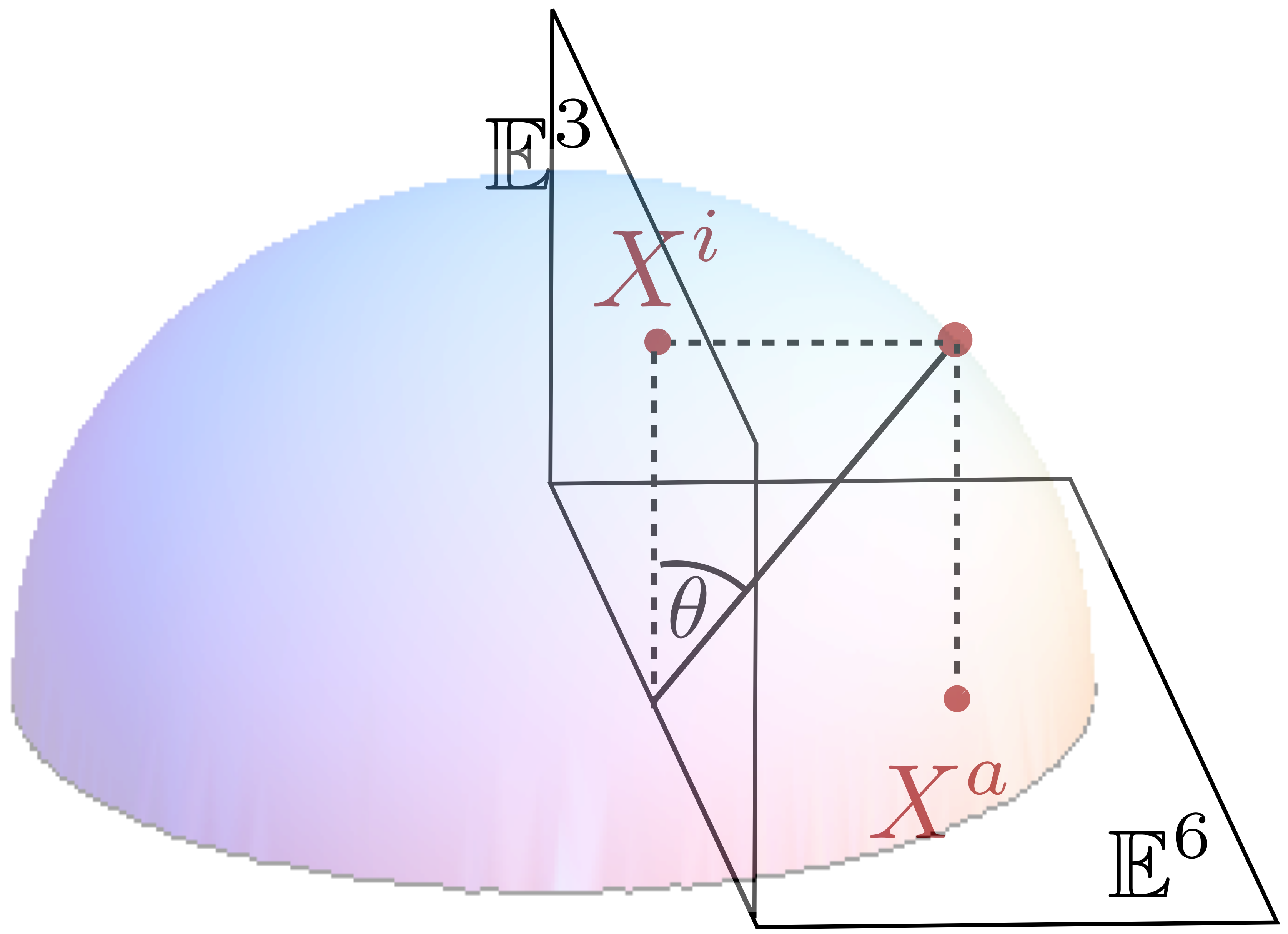
Boundary data

• The 10 functions $Q_i(x,y)$ admit expansion near the boundary $(y=0)$

$$Q_i(x,y) = \sum_j y^j \tilde{Q}_i^j(x)$$

To preserve $SO(6)\times SO(3)$ depends on ratio of radii

$$\sin\theta = \frac{R_5}{R_2} = \left(\frac{X^a X_a}{X^i X_i}\right)^{1/2}$$



• Boundary metric has $SO(9)$ symmetry, so $\tilde{Q}_i^j(x)$ are harmonic functions on S^8. Thus we can classify the $SO(6)\times SO(3)$ invariant perturbations according to $SO(9)$ spin. This helps to establish bulk field / operator correspondence.

- 2- form modes in the asymptotic expansion $C = (M\, d\eta + L\, d\zeta) \wedge d^2\Omega_2$

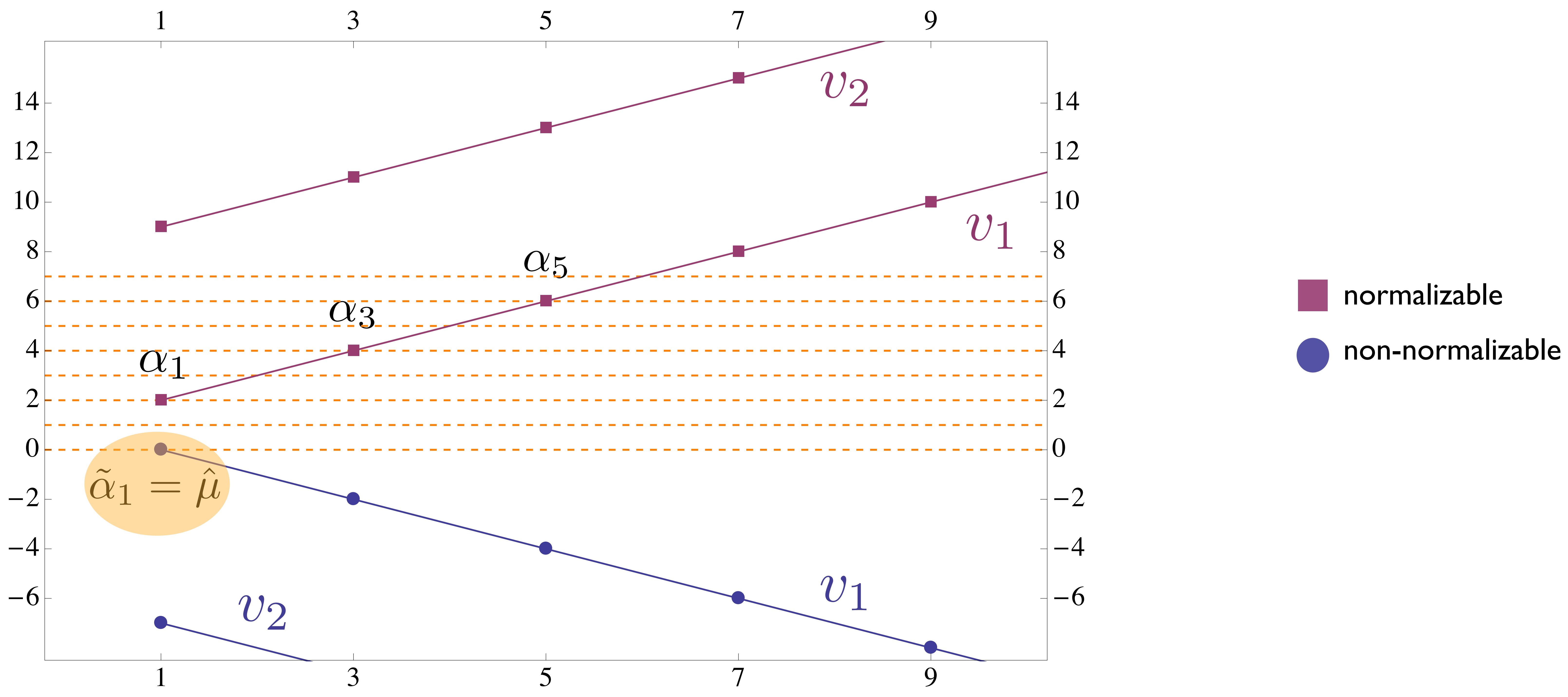
$$v(x,y) = \sum_{l\ odd} \left(\alpha_l\, f_l(y) + \tilde{\alpha}_l\, \tilde{f}_l(y) \right) \mathbb{H}_l(x) \ + \ \text{back reaction}$$

$SO(6) \times SO(3)$ invariant harmonic 2-form

$f_l(y) \sim y^{1+l}$

$\tilde{f}_l(y) \sim y^{1-l}$

$n(l)$

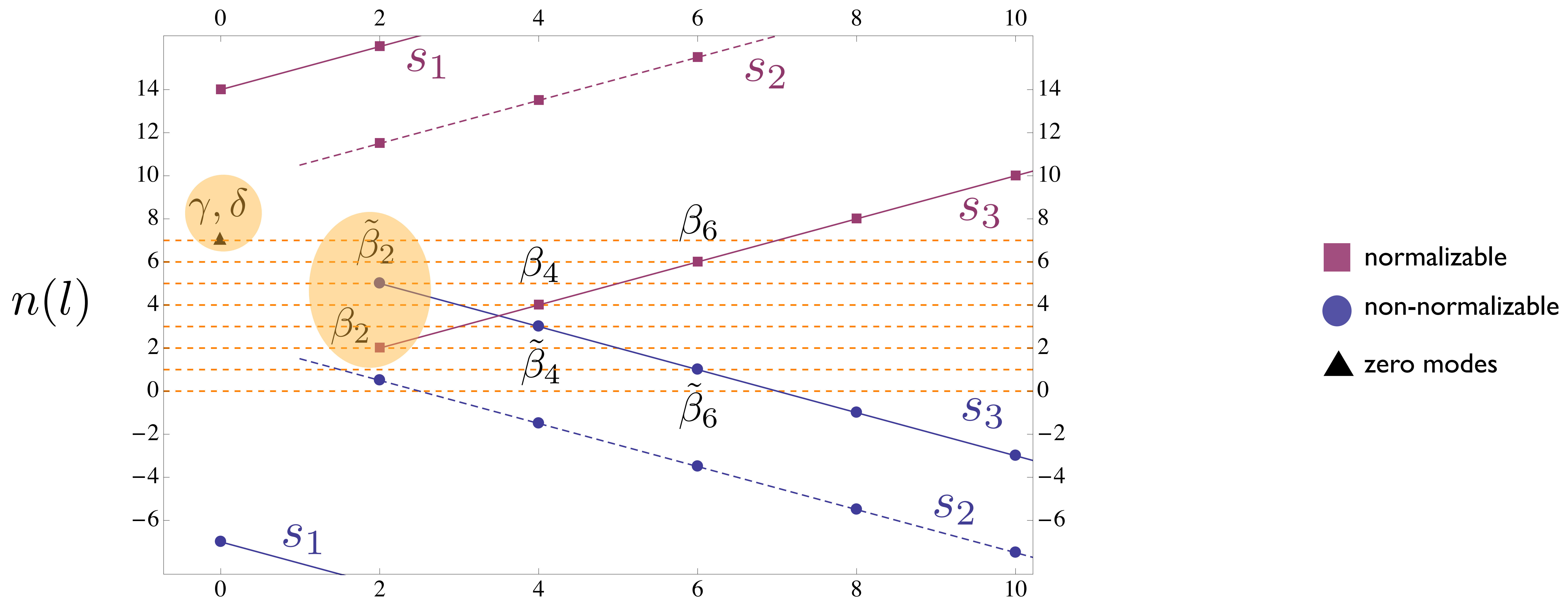


$$\mathcal{O} \sim \epsilon^{ijk} \mathrm{Tr} \left(X_i X_j X_k\, X_{A_1} \ldots X_{A_{l-1}} \right), \qquad l \geq 1 \text{ odd}$$

- scalar modes in the asymptotic expansion

$$s(x,y) = \sum_{l\,even} \left(\beta_l\, g_l(y) + \tilde{\beta}_l\, \tilde{g}_l(y)\right) \mathbb{S}_l(x) \;+\; \text{back reaction}$$

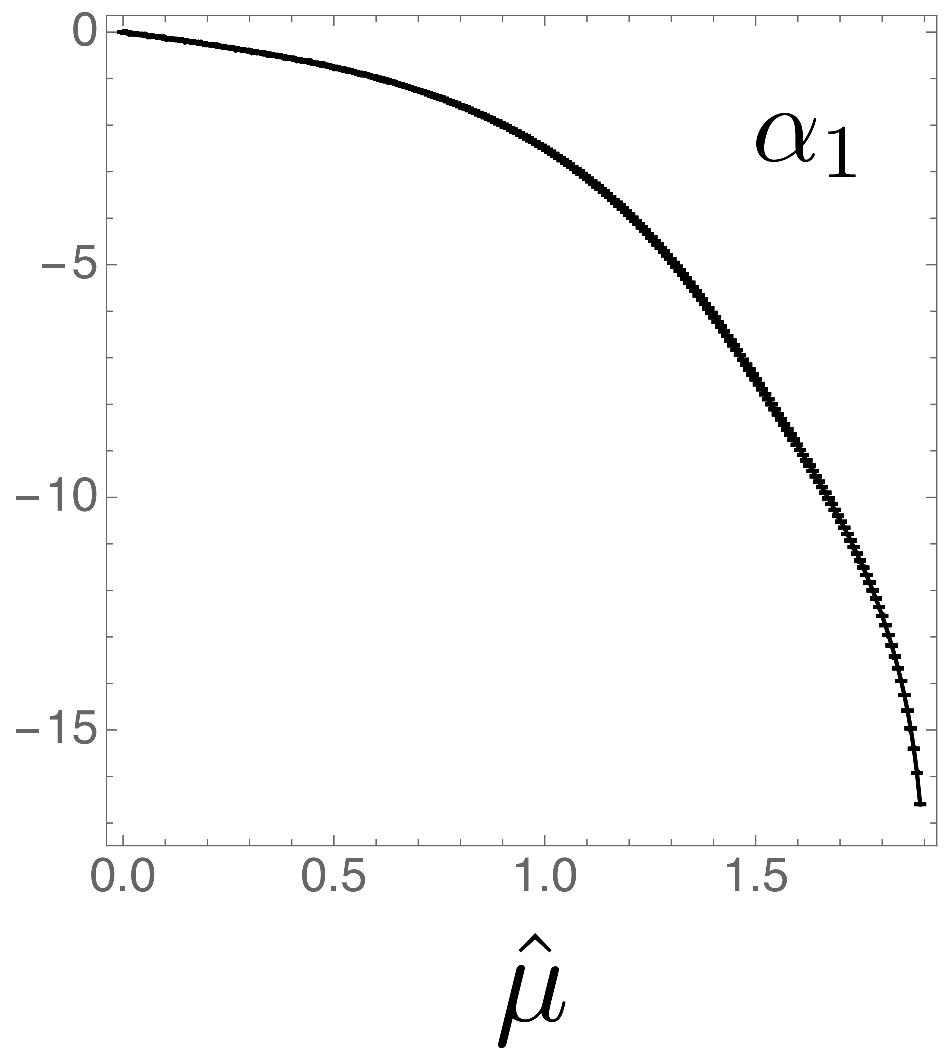
$SO(6) \times SO(3)$ invariant harmonic scalar



$$\mathcal{O} \sim \mathrm{Tr}\left(X_{A_1} \dots X_{A_l}\right), \qquad l \geq 2 \;\; \text{even}$$

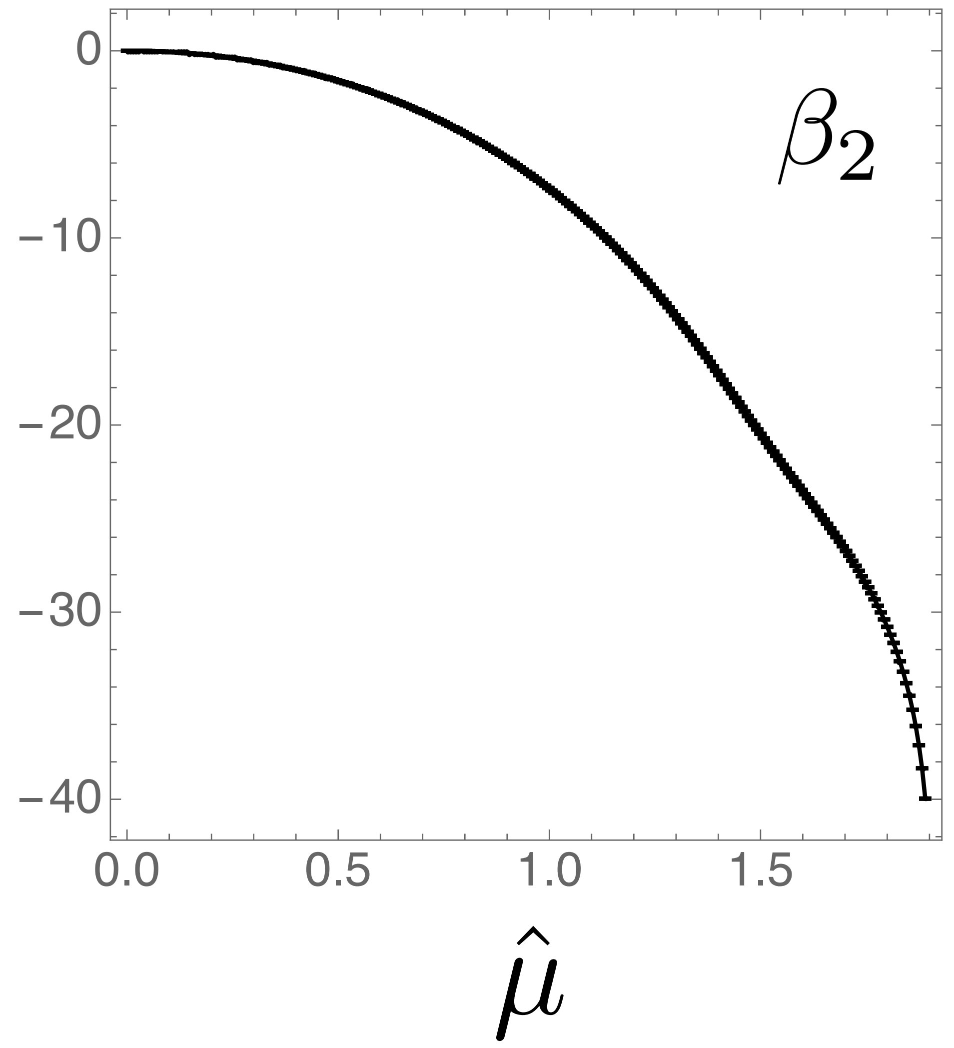
- Vevs read from normalizable modes appear first at order y^2

2-form $(l=1)$



$\epsilon^{ijk}\langle \mathrm{Tr}\,(X_i X_j X_k)\rangle$

Scalar $(l=2)$

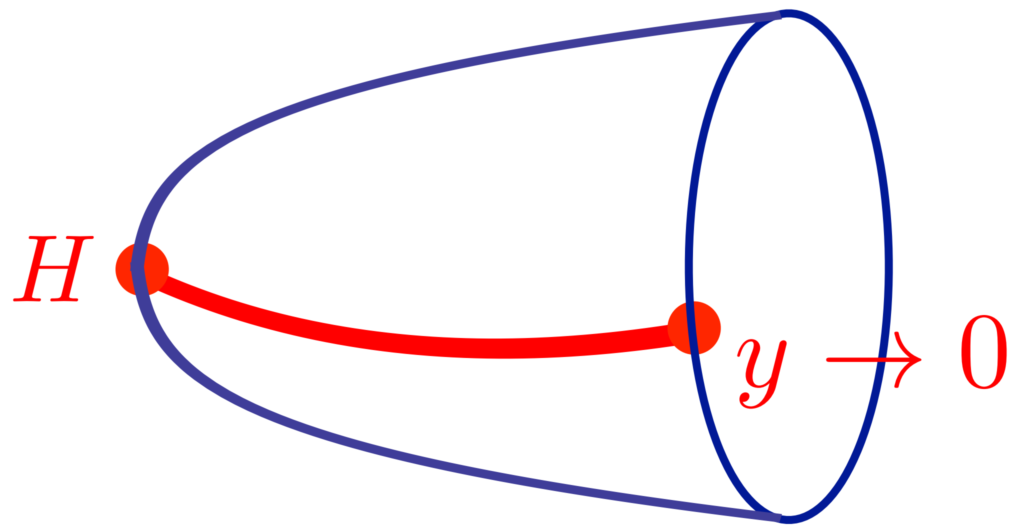


$\langle \mathrm{Tr}\left(2X_i X^i - X_a X^a\right)\rangle$

- Smarr formulae involve coefficients in asymptotic expansion up to order y^7

Numerics pass this highly non-trival
check with 0.05% accuracy

$$d\left(\star K_v\right) = 0$$



Future work

- Confirm phase diagram with Monte-Carlo simulations of PWMM; confirm predictions for expectation values of operators dual to normalizable modes that are turned on

- Study dynamical stability of our BH

- Construct BH duals of other vacua (different horizon topology)
  (caveat: we really only determined upper limit on critical temperature)

- **Deeper question:** What makes the PWMM special?
  What are the minimal ingredients of a quantum mechanical system such that it gives rise to classical gravity in the limit of many degrees of freedom?

THANK YOU