QUANTUM CORRELATIONS AS MINIMUM LOCAL COHERENCE: APPLICATIONS TO METROLOGY

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IDENTIFYING AND EXPLORING THE QUANTUM-CLASSICAL BORDER

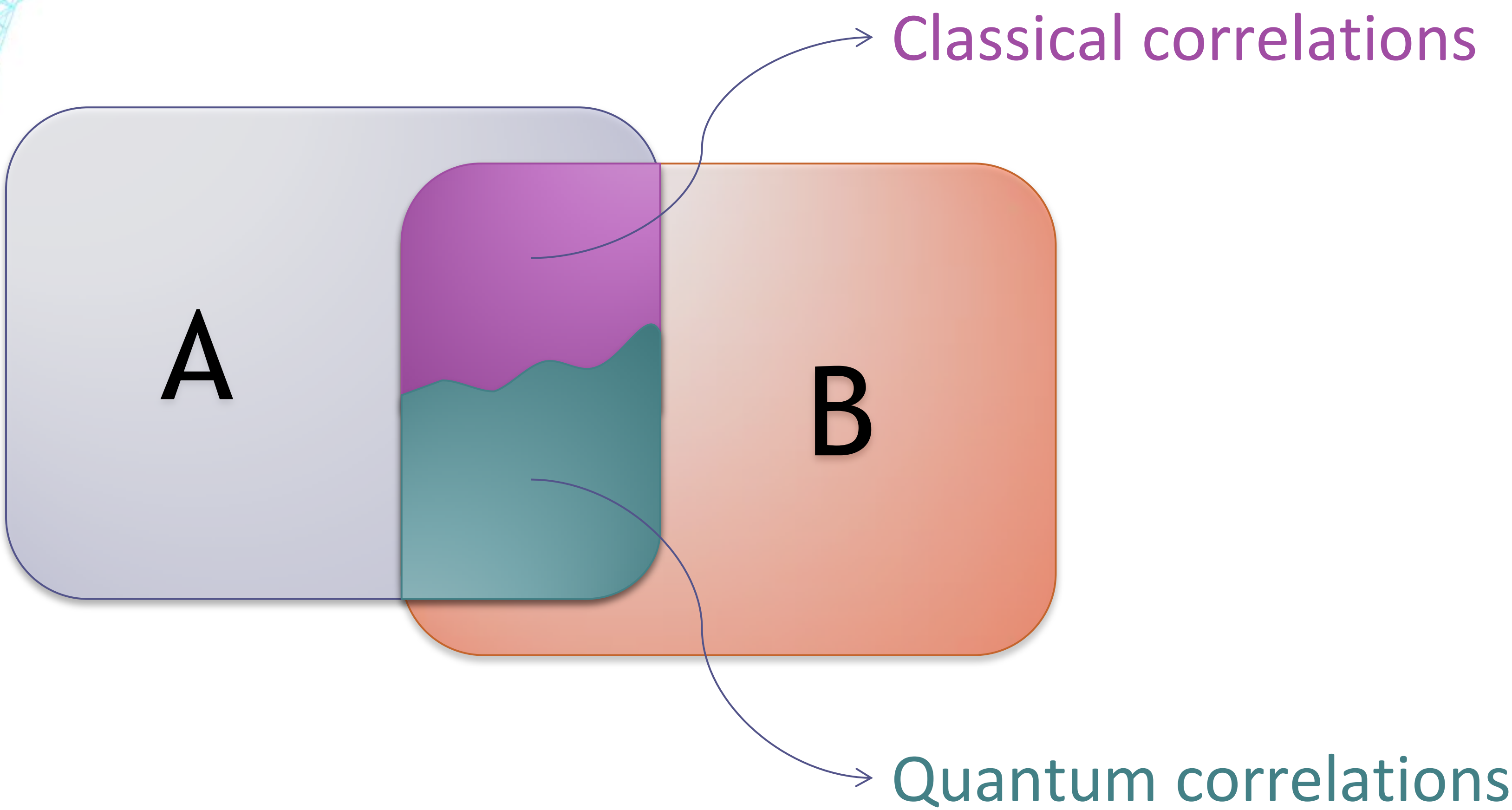
FOCUSING ON CORRELATIONS AMONG COMPOSITE SYSTEMS

OUTLINE

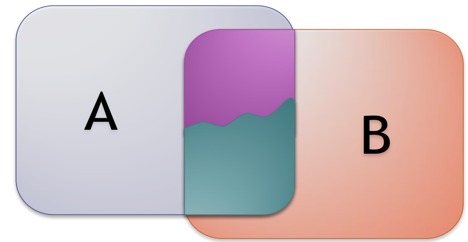
- Quantum correlations
- Quantum metrology
- Interferometric power
- Summary

Pamela Ott - *"Entanglement"* 58x71cm acrylic (2002)

CORRELATIONS

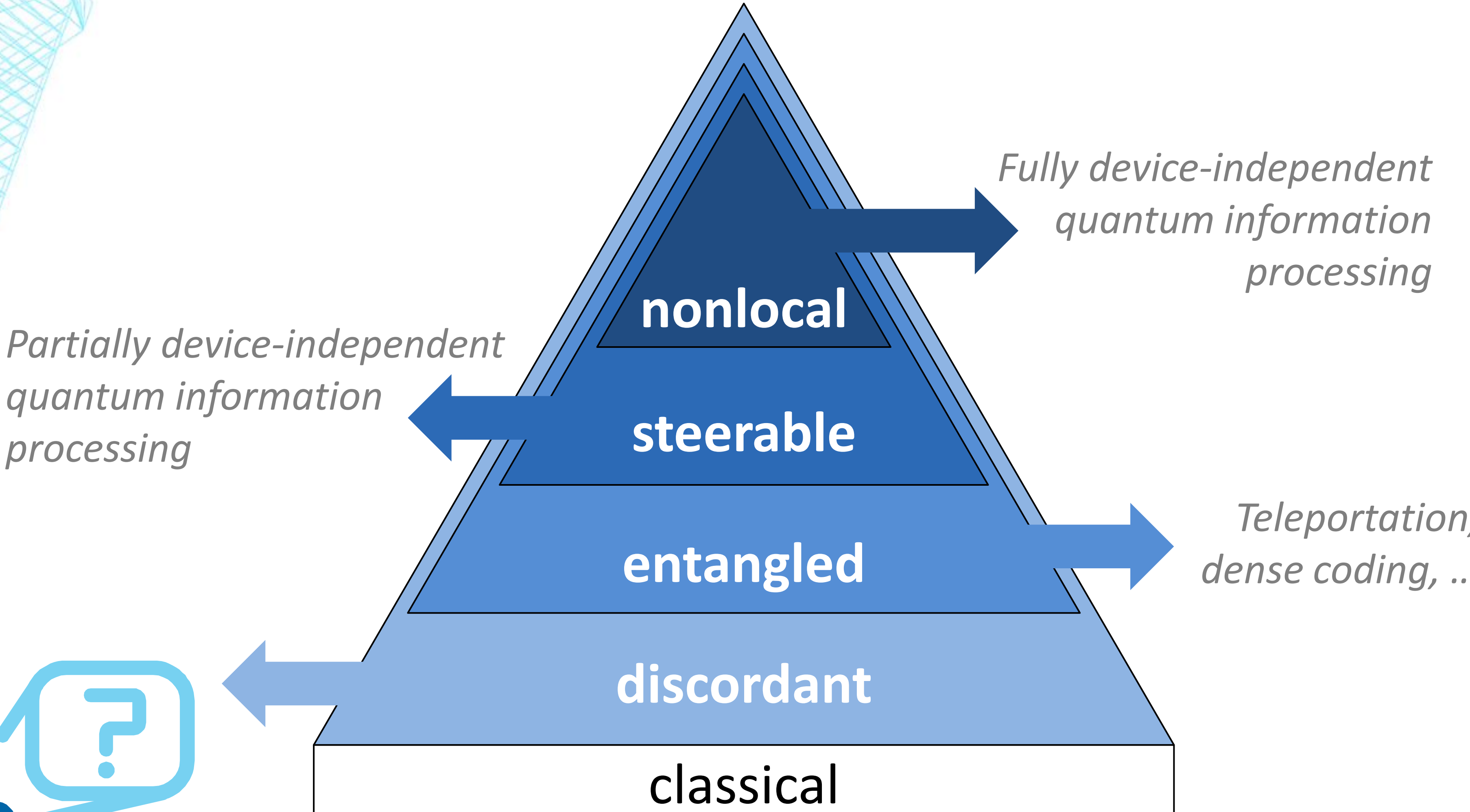


CORRELATIONS

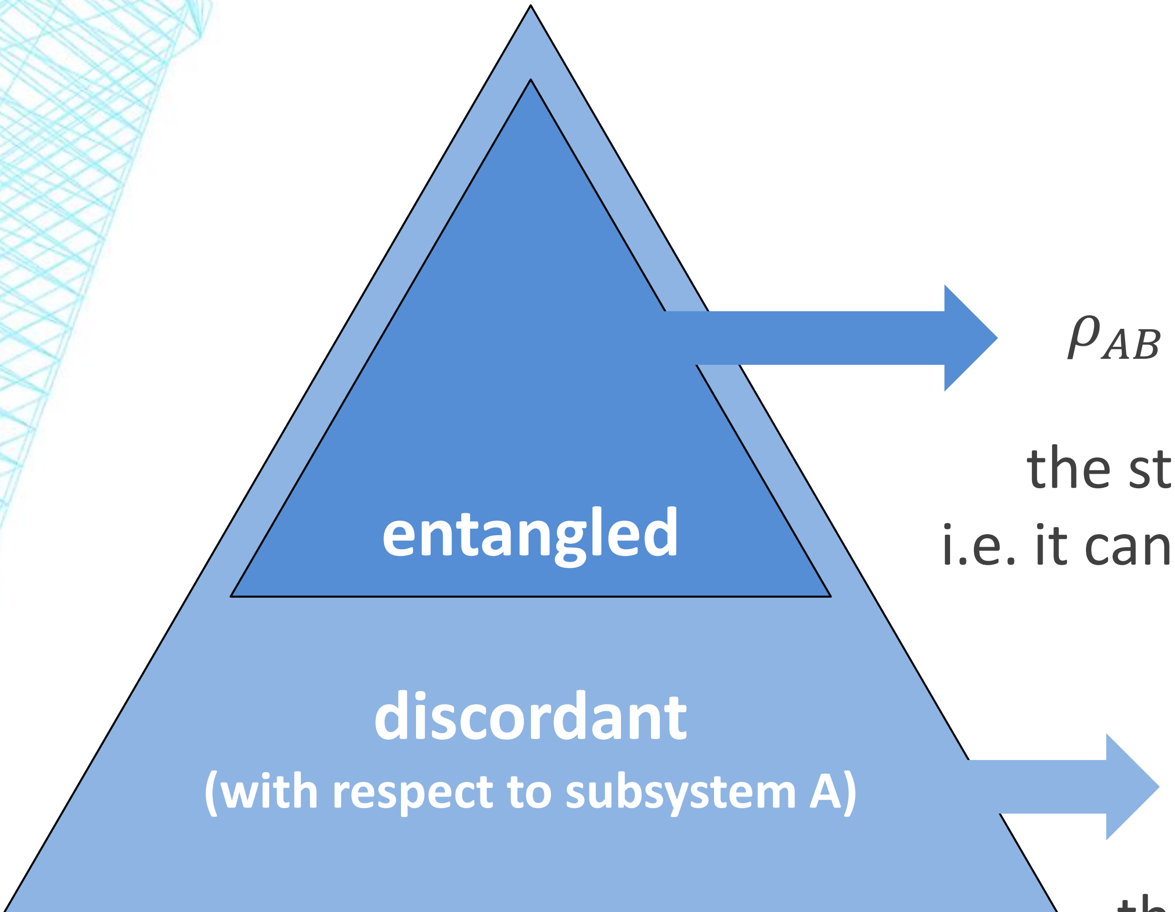


- Pure bipartite states:
  - **entanglement** = nonlocality
    nonclassicality *(one kind of quantum correlations)*
- Mixed bipartite states:
  - *A hierarchy of different quantum correlations…*

MANY SHADES OF QUANTUMNESS



(TWO) SHADES OF QUANTUMNESS



$$\rho_{AB} \neq \sum_k p_k \tau_A^k \otimes \nu_B^k$$

the state is *not* **separable**

i.e. it cannot be created by LOCC

$$\rho_{AB} \neq \sum_k p_k |k\rangle\langle k|_A \otimes \nu_B^k$$

the state is *not* **classical-quantum**

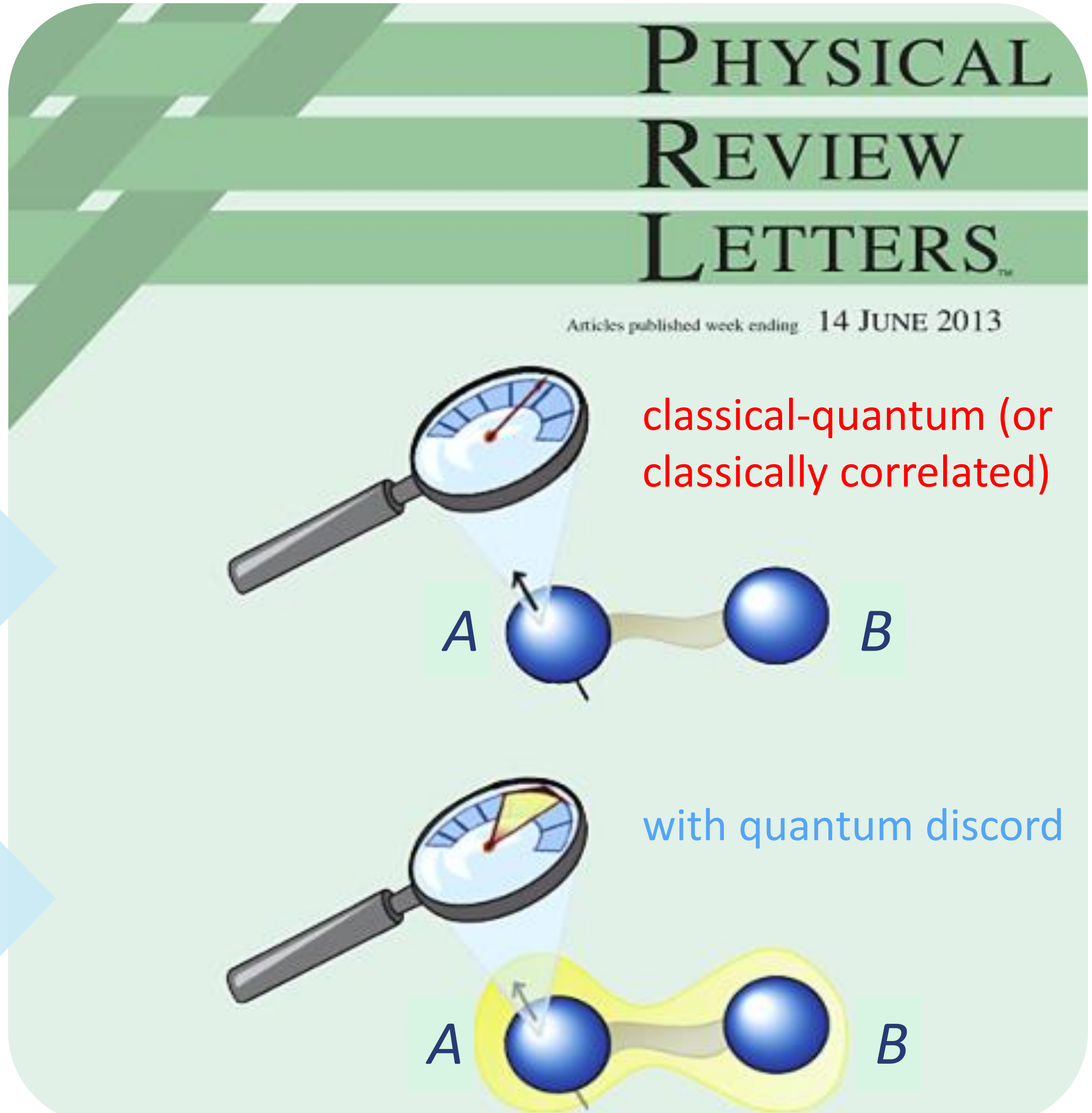
i.e. it is not invariant under any local measurement on party A

DISCORDANT VS CLASSICAL

Ollivier, Zurek, PRL 2001; Henderson, Vedral JPA 2001; Horodecki et al PRA 2005; Groisman et al. arXiv 2007; Piani et al. PRL 2008; Piani et al. PRL 2001; Girolami et al. PRL 2013; 2014; review: Modi et al. RMP 2012… etc.

If there is at least one local measurement I can perform without affecting my state

otherwise



QUANTUM METROLOGY

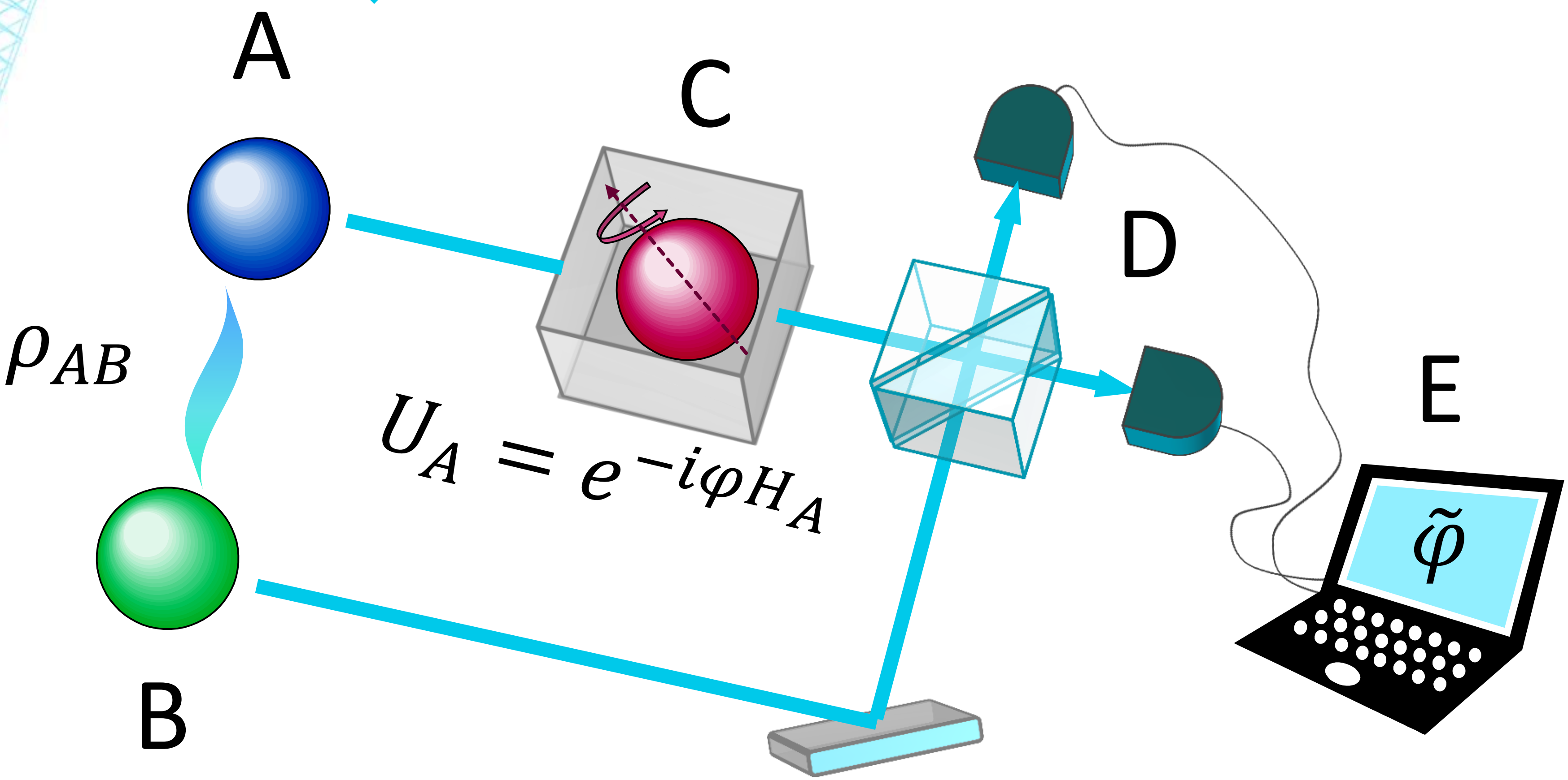
- exploits quantum mechanical features
- to improve the available precision
- in estimating physical parameters

See e.g. S. Huelga *et al*. *Phys. Rev. Lett.* 1997; B. Escher, R. de Matos Filho, L. Davidovich, *Nature Phys.* 2010
For a review, see V. Giovannetti, S. Lloyd, L. Maccone, *Nature Photon.* 2011

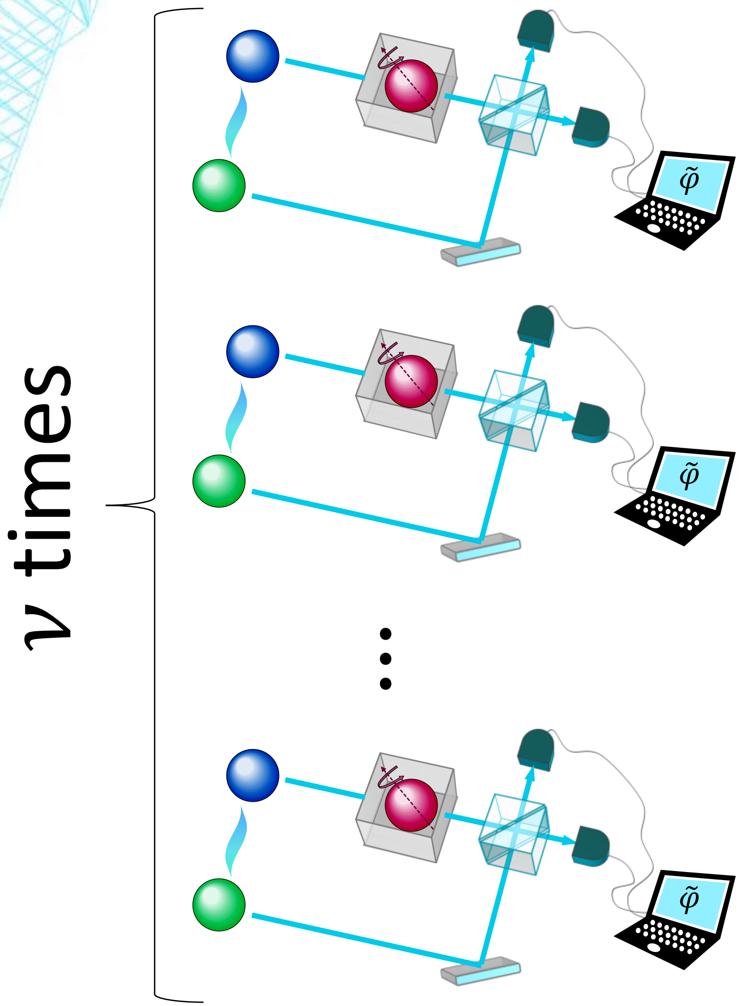
PHASE ESTIMATION

TASK

- Given the generator H_A
- Find the phase φ



PHASE ESTIMATION



Quantum Cramer-Rao Bound

$$\mathrm{Var}(\tilde{\varphi}) \geq 1/[\nu\, F(\rho_{AB}; H_A)]$$

Quantum Fisher Information

$F(\rho_{AB}; H_A)$ measures the precision

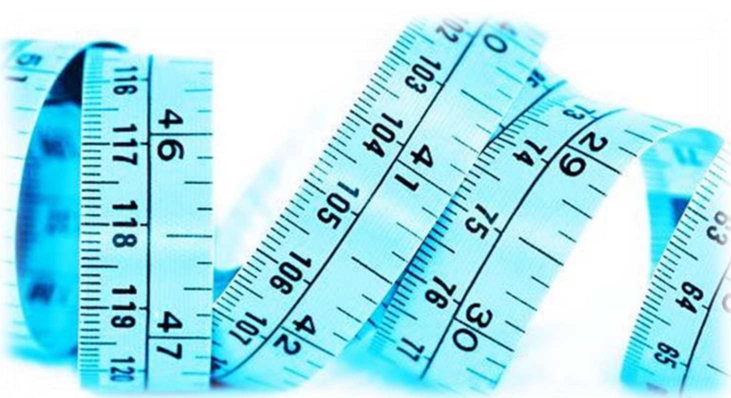
What is the resource?

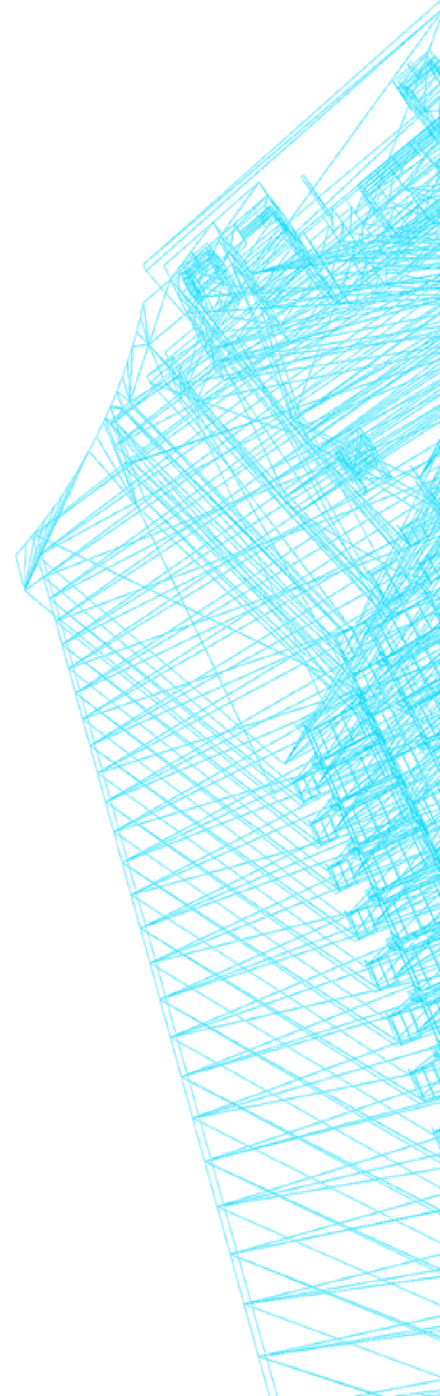
coherence in the eigenbasis of H_A

W. K. Wootters, *Phys. Rev. D* 1981; S. L. Braunstein and C. M. Caves, *Phys. Rev. Lett.* 1994

BLACK BOX QUANTUM METROLOGY

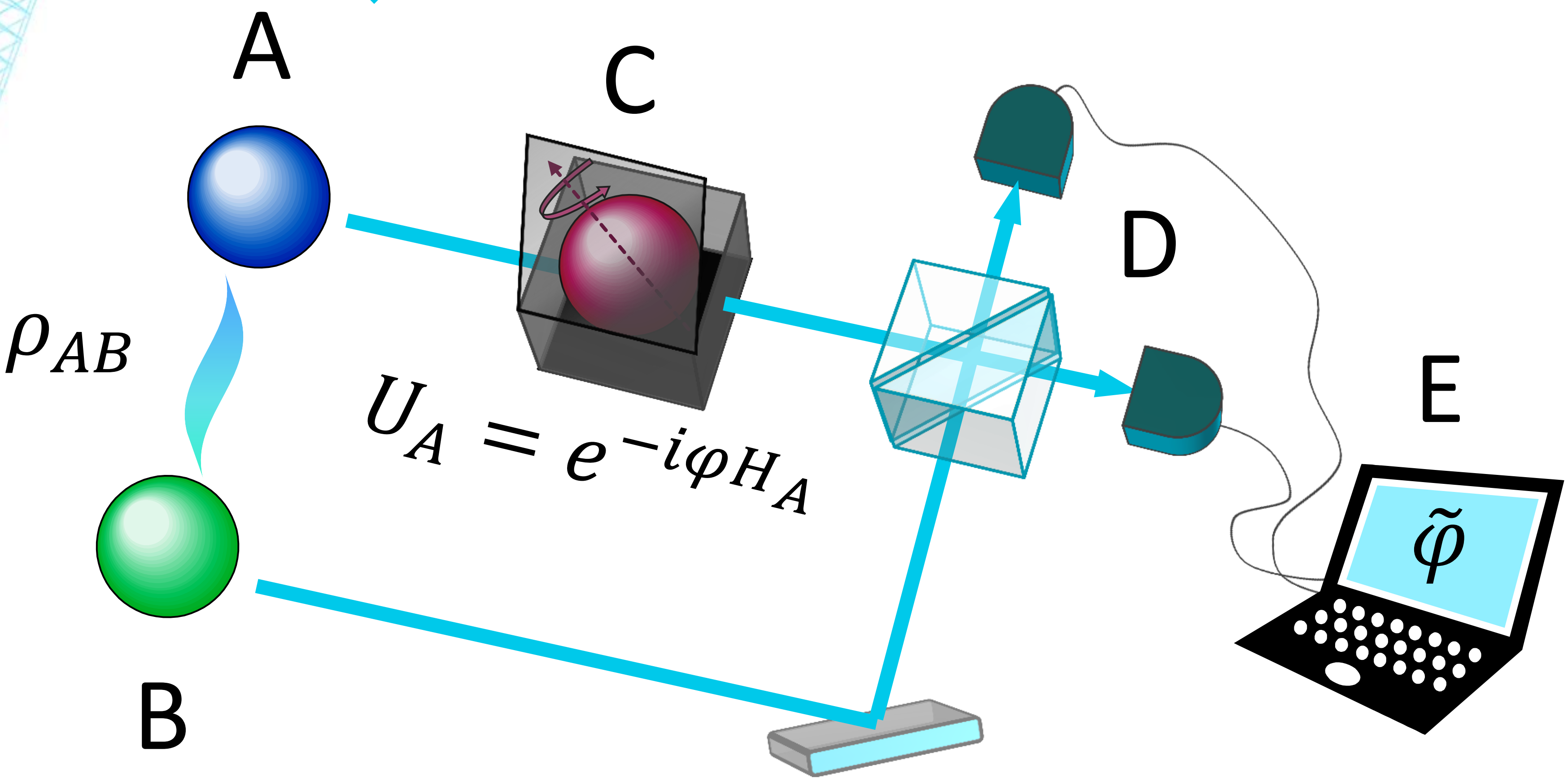
let the phase generator be unknown *a priori*...



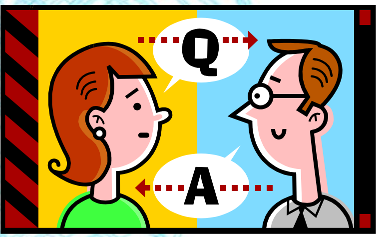


BLACK BOX PHASE ESTIMATION

TASK
- Only the spectrum of H_A is initially known
- Find the phase φ

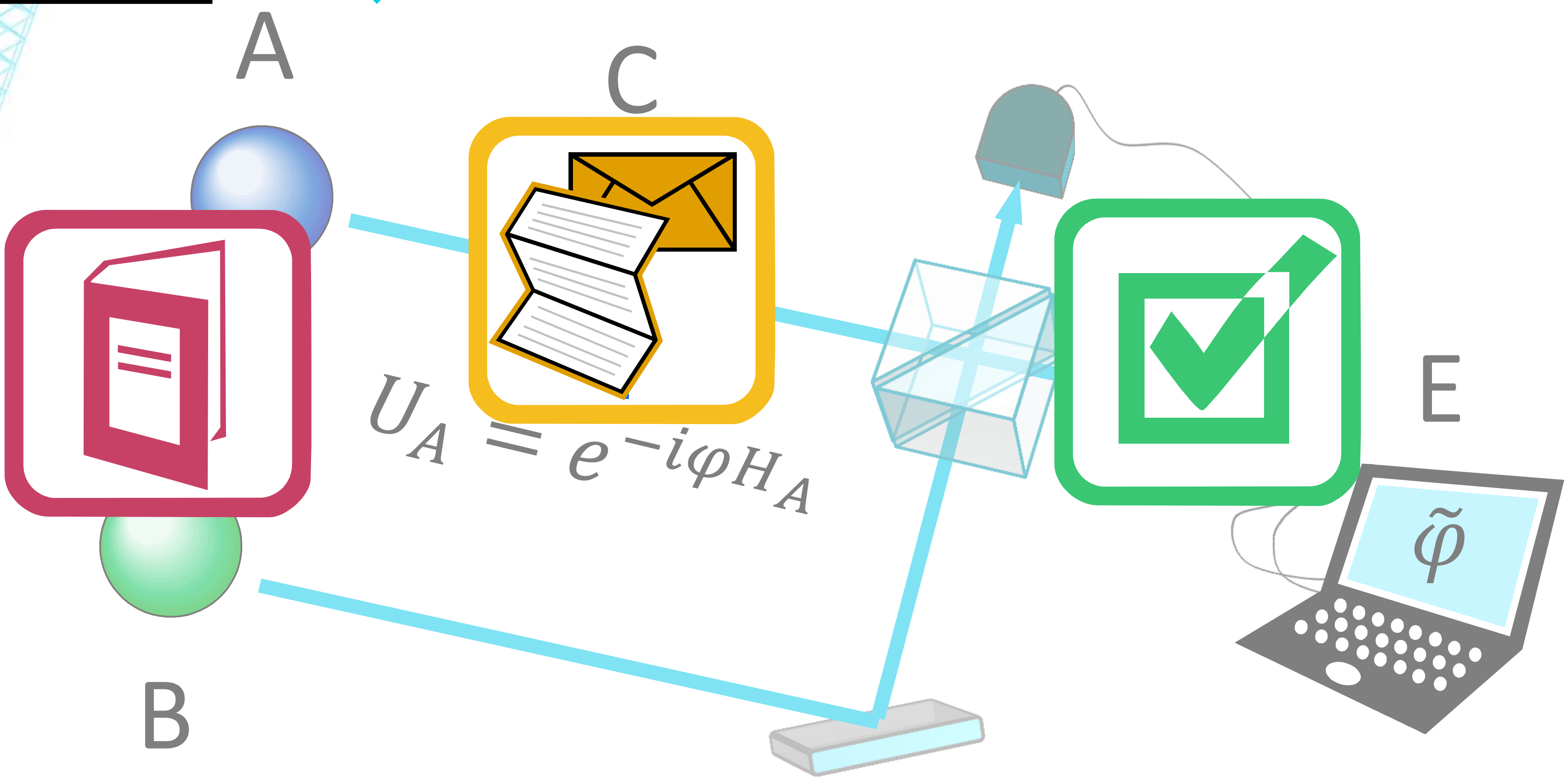


BLACK BOX PHASE ESTIMATION



TASK

- The generator H_A is revealed after ρ_{AB} is prepared
- Find the phase φ

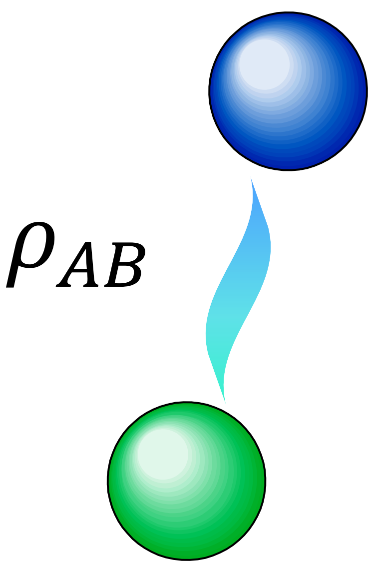


WHICH *PASSE-PARTOUT* INPUT PROBE STATES GUARANTEE A PASS?

WORST-CASE SCENARIO

FIGURE OF MERIT

- How useful the probe state ρ_{AB} is for estimation
- Guaranteed precision for any possible phase generator



$$P(\rho_{AB}) = \frac{1}{4} \inf_{H_A} F(\rho_{AB}\,; H_A)$$

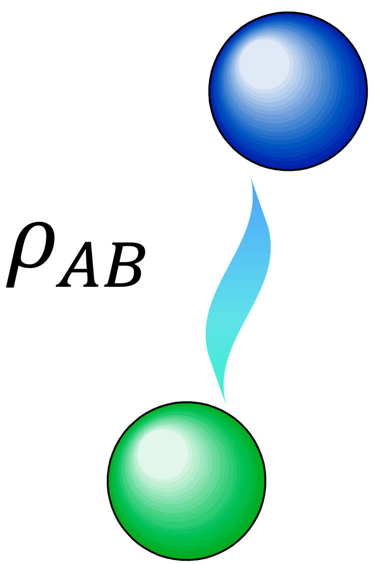
INTERFEROMETRIC POWER

QUANTUM DISCORD IS THE RESOURCE

FIGURE OF MERIT

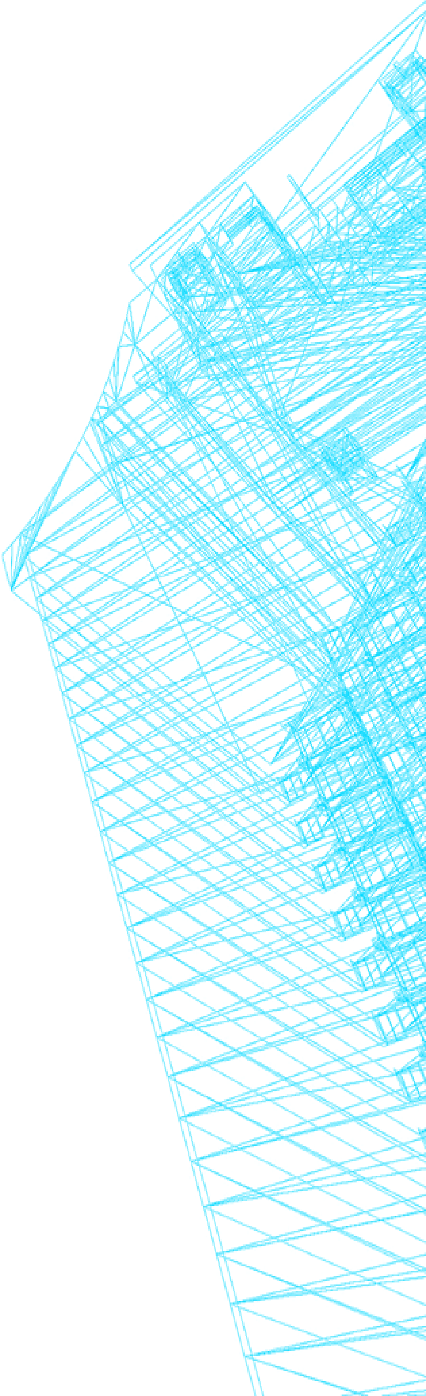
- The task requires coherence in the eigenbases of all H_A's
- **The interferometric power is a measure of discord**

$$P(\rho_{AB}) = \frac{1}{4}\inf_{H_A} F(\rho_{AB}\,; H_A)$$



- P is invariant under local unitaries and nonincreasing under local operations on B
- It vanishes iff ρ is classically correlated, $\rho_{AB} = \sum_i p_i |i\rangle\langle i|_A \otimes \tau_{i\,B}$
- It reduces to an entanglement monotone for pure states
- **It is analytically computable if *A* is a qubit**

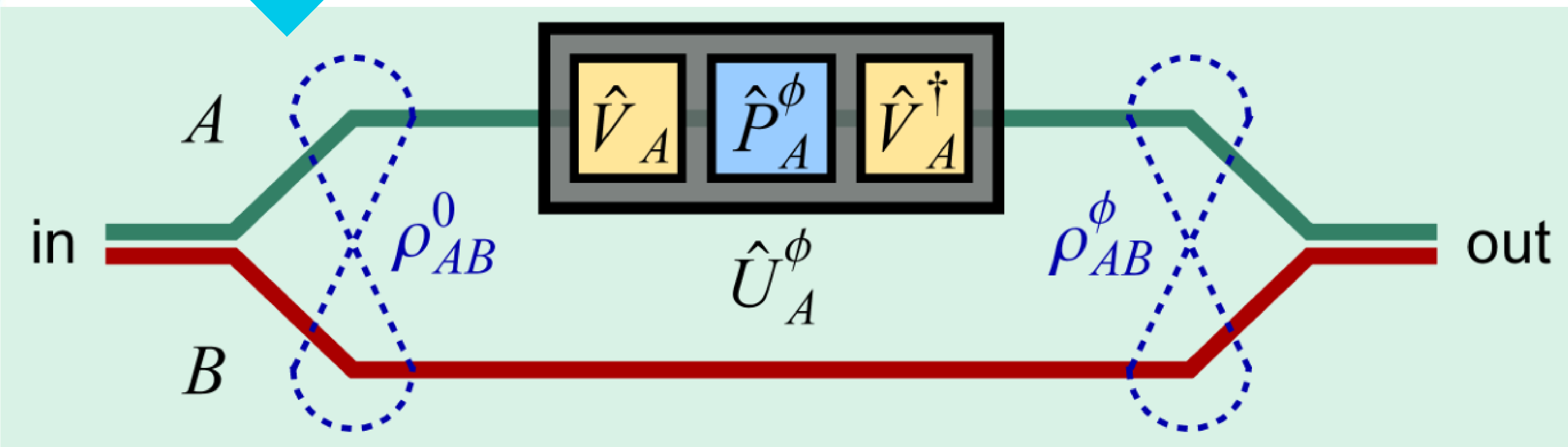
DISCORD-TYPE
QUANTUM CORRELATIONS
= MINIMUM LOCAL COHERENCE
= GUARANTEED PRECISION



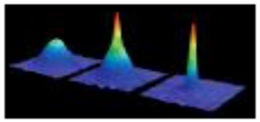
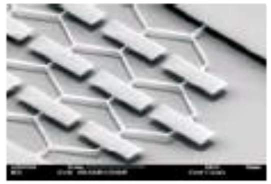
BLACK BOX OPTICAL INTERFEROMETRY

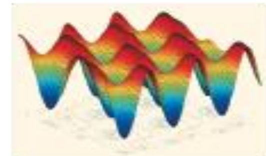
TASK

- Unknown quadratic generator $\hat{U}_A^{\phi}$ (with fixed harmonic spectrum); $\hat{P}_A^{\phi} = \exp(i\,\phi\,\hat{n}_A\,)$
- Find the phase ϕ
- Applications: gravitational wave detectors etc.



GAUSSIAN PROBES

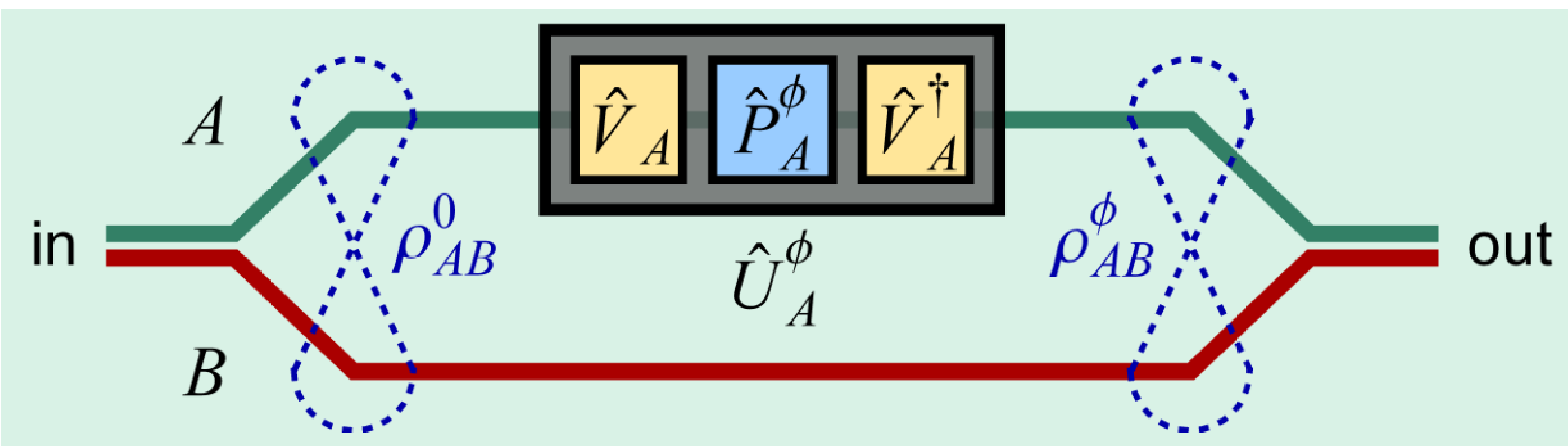





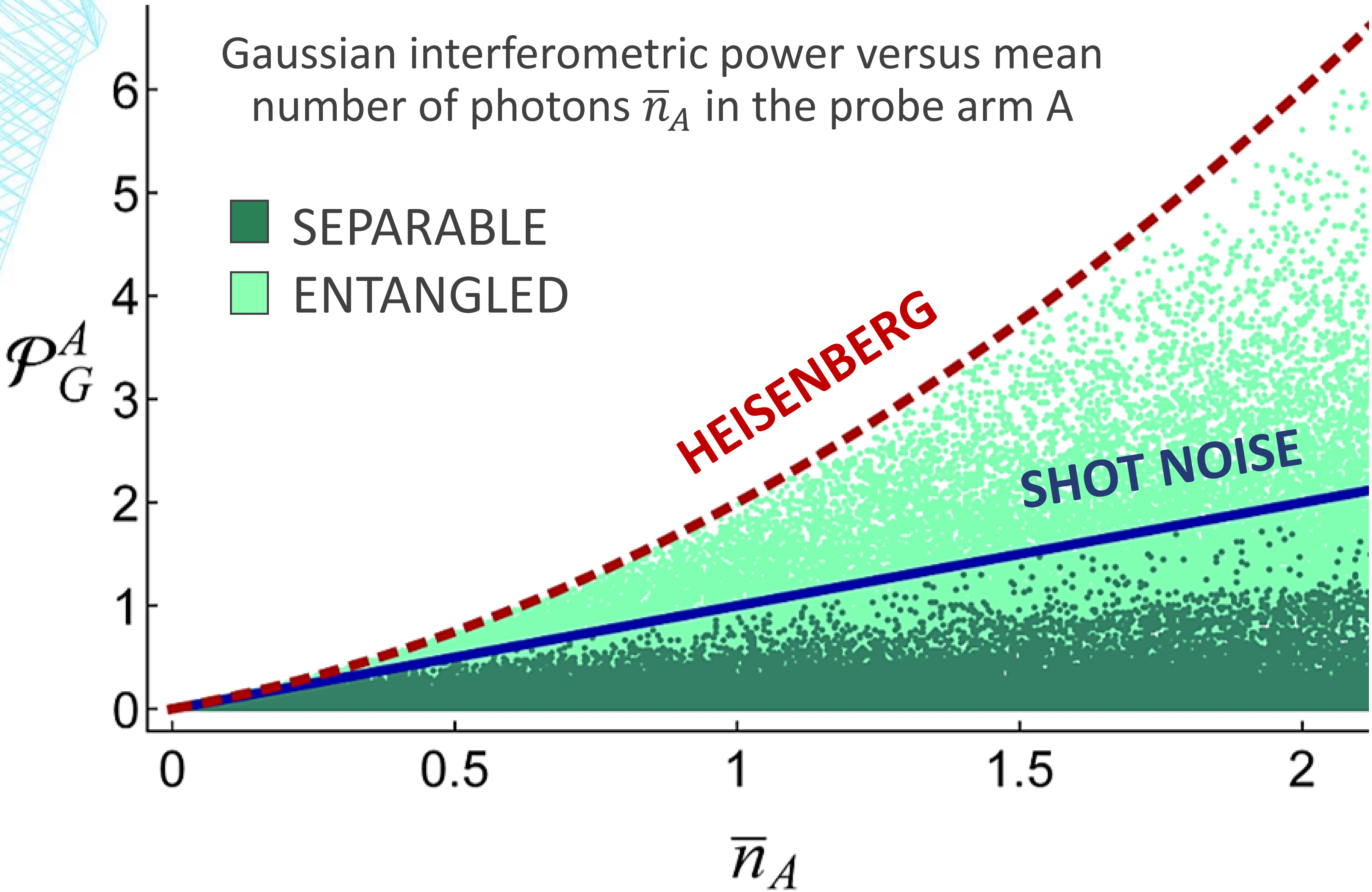

- Very natural: ground and thermal states of all physical systems in the harmonic approximation regime
- Relevant *theoretical* testbeds for the study of structural properties of entanglement and correlations, thanks to the symplectic formalism
- Preferred resources for *experimental* unconditional implementations of continuous variable protocols
- Crucial role and remarkable control in quantum optics
  - coherent states
  - squeezed states
  - thermal states

GAUSSIAN INTERFEROMETRIC POWER

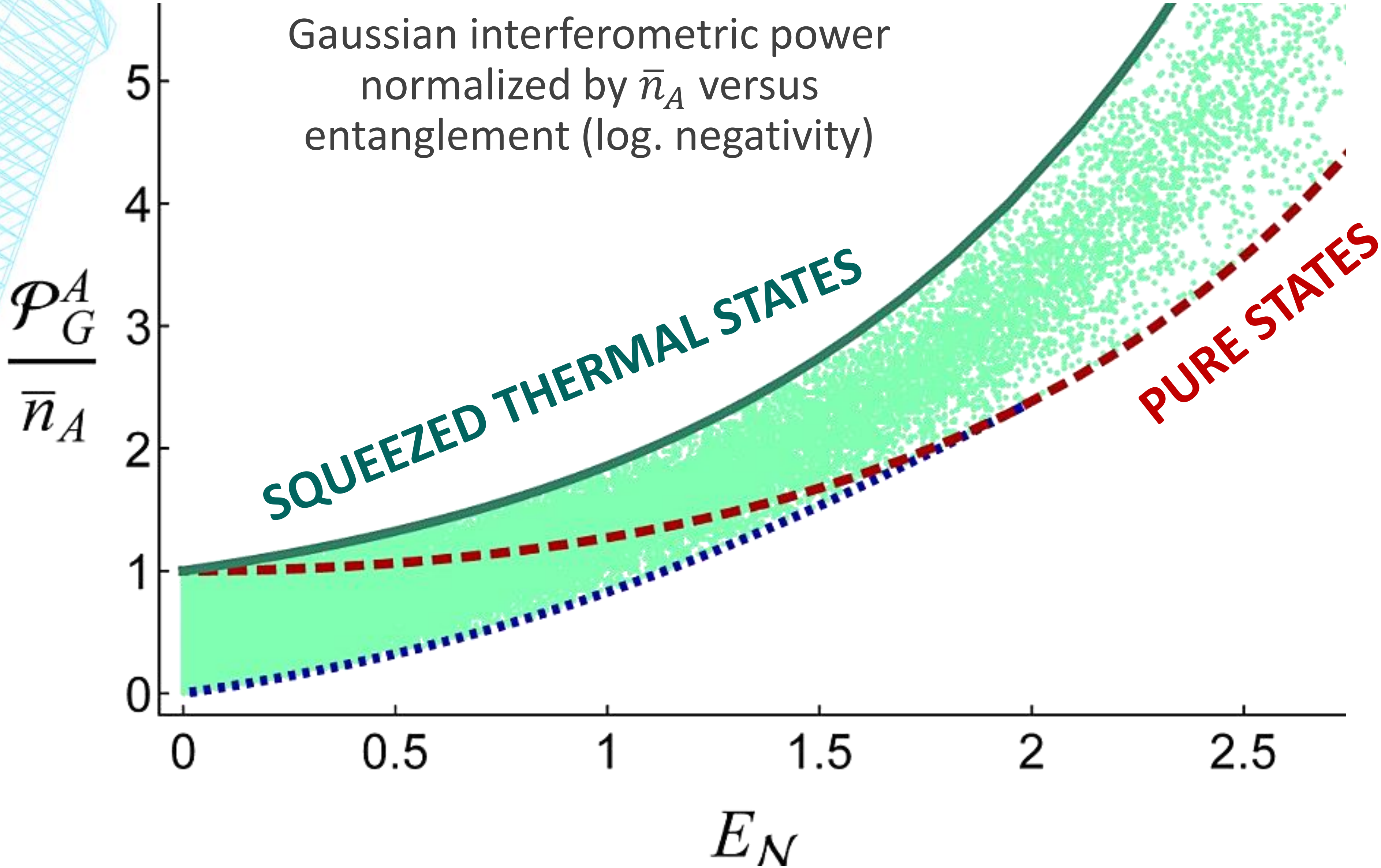
- Defined as for discrete systems, but with optimization restricted to Gaussian generators with harmonic spectrum
- Same properties: it is a measure of discord-type correlations
- **Computable in closed form for two-mode Gaussian states**



METROLOGICAL SCALING

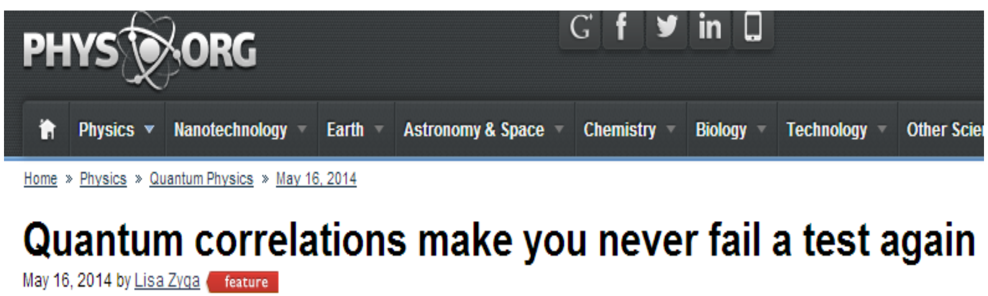


GAUSSIAN IP vs ENTANGLEMENT



SUMMARY

- The most general forms of **quantum correlations** (“discord”) manifest as **coherence** in all local bases
- They can be measured via a faithful, operational and computable quantifier, named **interferometric power**
- Quantum correlations even beyond entanglement guarantee a metrological precision in **phase estimation**, as opposed to classically correlated probes

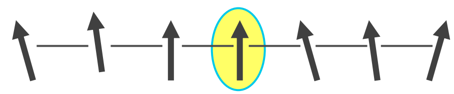


New Scientist Sept 2014

APPENDIX: CLOSED FORMULAE

Qubit-qudit state

ρ_{AB} (2 x d)-dim



$$\mathcal{P}^A(\rho_{AB}) = \varsigma_{\min}[M]$$

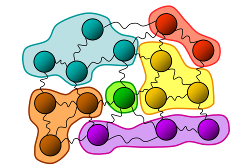
where $\varsigma_{\min}[M]$ is the smallest eigenvalue of the 3×3 matrix M of elements

$$M_{m,n} = \frac{1}{2} \sum_{i,l:q_i+q_l \neq 0} \frac{(q_i - q_l)^2}{q_i + q_l} \langle \psi_i | \sigma_{mA} \otimes \mathbb{I}_B | \psi_l \rangle \langle \psi_l | \sigma_{nA} \otimes \mathbb{I}_B | \psi_i \rangle$$

with $\{q_i, |\psi_i\rangle\}$ being respectively the eigenvalues and eigenvectors of ρ_{AB}, $\rho_{AB} = \sum_i q_i |\psi_i\rangle\langle\psi_i|$.

Phys. Rev. Lett. **112**, 210401 (2014)

Gaussian state with covariance matrix $\boldsymbol{\sigma}_{AB}$ (two-mode)



$$\boldsymbol{\sigma}_{AB} = \begin{pmatrix} \boldsymbol{\alpha} & \boldsymbol{\gamma} \\ \boldsymbol{\gamma}^{\mathrm{T}} & \boldsymbol{\beta} \end{pmatrix}$$

in terms of the four local symplectic invariants of an arbitrary covariance matrix, defined as $A = \det \boldsymbol{\alpha}$, $B = \det \boldsymbol{\beta}$, $C = \det \boldsymbol{\gamma}$, and $D = \det \boldsymbol{\sigma}_{AB}$. The formula reads

$$\mathcal{P}^A_G(\boldsymbol{\sigma}_{AB}) = \frac{X + \sqrt{X^2 + YZ}}{2Y}$$

where

$$\begin{aligned} X &= (A + C)(1 + B + C - D) - D^2 , \\ Y &= (D - 1)(1 + A + B + 2C + D) , \\ Z &= (A + D)(AB - D) + C(2A + C)(1 + B) . \end{aligned}$$

Phys. Rev. A **90**, 022321 (2014)

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THANK YOU

- D. Girolami, T. Tufarelli, <u>G. Adesso</u>; *Characterizing Nonclassical Correlations via Local Quantum Uncertainty*; **Phys. Rev. Lett. 110, 240402 (2013)**
- D. Girolami, A. M. Souza, V. Giovannetti, T. Tufarelli, J. G. Filgueiras, R. S. Sarthour, D. O. Soares-Pinto, I. S. Oliveira, <u>G. Adesso</u>; *Quantum Discord Determines the Interferometric Power of Quantum States*; **Phys. Rev. Lett. 112, 210401 (2014)**
- <u>G. Adesso</u>; *Gaussian interferometric power*; **Phys. Rev. A 90, 022321 (2014)**

http://quantumcorrelations.weebly.com